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Investment portfolio analysis: Energy and gold-minerals

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INVESTMENT PORTFOLIO ANALYSIS:
ENERGY AND GOLD-MINERALS

BY

GERARDO MEAVE-FLORES

A thesis Submitted to the Faculty of the

DEPARTMENT OF MINING AND GEOLOGICAL ENGINEERING

In Partial Fulfillment of the Requirements
for the Degree of

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WITH A MAJOR IN MINERAL ECONOMICS

In the Graduate College

THE UNIVERSITY OF ARIZONA

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Professor of Mineral Economics

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ABSTRACT

The purpose of this research is to analyze the impact that a sample of securities blended together would have upon the variance of the expected returns of an energy and a gold-minerals portfolio. A framework based on the Markowitz model, but solved linearly, has been constructed in which the optimal weight of each security in its respective portfolio is determined in order to minimize variance given the expected portfolio returns. The data elaborated for each stock (price, return and dividend) were on an annual basis for a period of 16 years and are the basis from which the projections of both the energy and the gold-minerals portfolio expected returns were derived. The results show that the variance in both portfolios is considerable, because stocks as a group show co-movement, meaning that stocks tend to do well or poorly as a group.

CHAPTER I

INTRODUCTION

After the sharp rise in the price of some minerals such as silver, copper, gold, and molybdenum, as well as that of oil in the early 1970's, too much significance was given to the linkage between raw minerals, energy and the economy as a whole, with the result that corporations and investors increased their investment in minerals and energy in anticipation of realizing large profits. One important reason for this was that energy, in real terms, had been becoming cheaper until the early 1970's and its importance was underestimated. Another reason is that, in the late 1970's, there was a temporary worldwide short supply of molybdenum. Its price jumped rapidly as a result.

The degree of importance as well as the risk involved in expanding investments in these areas, is not the same for everyone, everywhere. The severe economic blow of higher oil and mineral prices in the mid 1970's forced developed and under-developed countries to seek alternative plans for economic development.

In the early 1980's a sharp decline in the price of oil and minerals occurred as a result of an over expansion of world productive capacity. Many countries and corporations remained almost completely dependent on minerals as a source of revenue and, consequently, they were taking heavy losses from earlier investments initiated under much more favorable economic assumptions. Therefore, many mines and petroleum businesses have been closed or are working well below capacity. It will take much time and effort for these countries and corporations to diversify their product lines.

The need to diversify investments in order to minimize risk has been increasing over time. Indeed, U.S. companies, which are among the strongest of the world's oil and mineral companies, are pursuing a new, very aggressive, management strategy. They have taken drastic steps to protect their investments against falling oil and mineral prices. Assets unrelated to main lines of business have been sold and downstream investments added [e.g., acrylonitrile and titanium dioxide manufacturers, and even financial and consumer services]. In fact the new management strategy may allocate to those divisions the primary position. Due to the volatility of world oil and mineral product prices, these same companies have become better commodity traders. They have recently been reducing downside risk by introducing oil-indexed and gold-indexed notes or by trading international currencies. It is the purpose of this paper to examine the effect that diversification would have upon the risk or variance of the expected returns on an energy and gold-minerals securities portfolios.

The two investment portfolios described and solved in the present study are simple in comparison with actual conditions. Most real-life applications are likely to involve choosing from among hundreds of securities to achieve a portfolio with a desired return and minimal associated risk, as for example, the Standard and Poors 500 Index Fund.

Since understanding differences in the behavior of returns and of riskiness between asset classes is important, this study describes the method of calculating the return on the securities and ways of measuring their riskiness used for each of the investment portfolios selected.

Problem and Scope of this Study

The present research is directed to the study of the importance of the allocation of investments among asset classes in a portfolio. Its purpose is to find those securities, selected from stocks listed on the New York Stock Exchange (NYSE), issued by companies that have been active for the past 16 years, which are related to the oil and minerals industries, and that will provide a desired rate of return at given levels of risk. From these we form two portfolios, one for energy and the other for gold-minerals. The model used minimizes the variance of expected return given the restrictions imposed on the model. This model is not dealing with personal risk analysis rather with market conduct.

The data (return, price, variance of return, and dividend) have been recalculated on an annual basis and adjusted to take into account any stock splits that have occurred during the years under study (1970 - 1985).

This study emphasizes the quantitative aspect of investment strategy and the level of risk or uncertainty that such investment takes. For this purpose a model based on Markowitz' method, but solved linearly instead of quadraticaly, has been constructed. There are two reasons for using a linear solution method. First, it simplifies the analysis because it does not require the construction of a series of complex matrices to be solved. Second, despite this simplification, the results appear plausible. This is explained in detail in Appendix A and in Appendix B.

The investments have been divided into energy and gold-minerals portfolios. These are important sectors from the standpoint of their contribution to gross domestic product. Each portfolio involves securities of companies whose activities may include: mining, processing, production, marketing, exploration, and manufacturing, of minerals, energy fuels and downstream products. The energy portfolio focuses on companies engaged in the oil and gas industry; the gold portfolio on companies engaged in gold, silver, copper, aluminum and other mineral production.

Justification of Markowitz' Model

The quality and quantity of data on price, dividends, P/E, and earning/share ratios used in the present study are, in general, adequate for use in Markowitz' model. Most of the data were obtained from Moody's Handbook, a subsidiary of the Dun and Bradstreet Corporation. This does not impose any limitation on the model as it is used in this study. The Markowitz model seems to be most appropriate because good data are available and, for the specific period under study, it is possible to establish some sort of linkage between the data inputs and expected returns. It is possible then, to construct a portfolio investment during this period for which a Markowitz' model can be utilized.

Another argument in favor of the Markowitz model is its theoretical foundations. In general, most investors want to quantify their risk variable. The basic portfolio model developed by Harry Markowitz derives the expected rate of return on a portfolio's assets and a corresponding measure of risk. Markowitz's assumption is that investors are risk averse. Under this assumption a single asset or a portfolio of assets is considered to be efficient if no other asset or portfolio of assets offers a higher expected return with the same (or lower) risk, or a lower risk with the same (or higher) expected return.

One additional advantage of the Markowitz model is its ease of later revision with regard to updated data. Though the results of the Markowitz model provide a picture of actual performance only at a given instant, inferences can be made with regard to the time trend of price, return, variance of return, or of any asset or portfolio of the data input; or new data may be collected. Therefore, the Markowitz model can be used for other periods. This flexibility allows the use the Markowitz model for rebalancing the investment portfolio over time.

A major disadvantage of this model is that it is difficult to use in solving practical problems, mainly because of the quantity of data required by the model. For example, if the portfolio comprises 100 securities, it requires 100 returns, 100 variances and 4950 covariances: because N returns N variances and N(N-1)/2 covariances are needed. This is not a problem for this study. In the present study each portfolio is composed of four securities selected from a small number of sample securities. Therefore, for the present study the Markowitz model is the most sophisticated model available with which to estimate the expected return and variance for each security and its portfolio.

Principal Element in the Investing Program

Conditions today, and probably into the foreseeable future, are not likely to be as favorable for investors as they were two or three decades ago. In hind sight, in those days the prospects for popular investments such as common stocks and real estate were so favorable that it was possible to prosper without much attention to any of the key elements now needed for successful investment.[1] The need for an investment program under today's conditions can be seen by looking at the present, highly volatile stock market.[2]

Investment Policy

The first important step in investment policy is to consider such things as age group, income situation, cash needs, and attitude toward risk taking. As far as income is concerned, an investor with an adequate income

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1. Note: This refers to the average, some investors could lose money while others could make super normal returns.

2. Note: The take over target for companies may be a classic example of this era.

for normal needs derived from a salary, a profession or business, or any other source can be more venturesome than can an investor dependent on investments for all or a major part of his or her income. In this study the amount assumed for each investment portfolio is $10,000.

Investment Strategy

Once an investment policy is established, there remains the question of how best to implement that policy in the face of current investment conditions. Sometimes investment conditions justify a decision to modify, or perhaps suspend for the time being, the basic investment policy (e.g., move into cash). For example, an assessment of current market conditions may show that the market is vulnerable to decline because prices of equities relative to other investments are too high or because interest rates are likely to rise and economic conditions turn unfavorable. In that event, the appropriate strategy for an investor might be to defer at least part of his or her planned equity investments until such securities are available at more "realistic" prices.

Investment Management

Circumstances change both in relation to investments and the circumstances of the investor. Hence, investments must be kept under constant review. This involves not only a continual review of investment policy and strategy, but also review of the prospects for each particular investment.

The Importance of the Portfolio Effect

Each investment decision must be considered in terms of its impact on the overall portfolio. It is the effect of a specific investment on the overall portfolio which really matters. For example, if in the portfolio in which present funds are invested, the overall risk is well below that appropriate from the viewpoint of investment policy and strategy, the reasonable action is to add more risky investments to the portfolio.

Portfolio Management

Portfolio management consists of three major activities: (1) asset allocation, (2) weighing shifts across major asset classes, and (3) security selection within asset classes. Asset allocation can be considered a tactical approach to capitalizing on return opportunities that are perceived to be available in the short term,[3] (daily or monthly). The purpose of the asset allocation process is to put assets together in a portfolio in such a

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3. Note: This depends on the investment policy and strategy of each firm or investor thus, they capitalize such opportunities of return in periods of less than a year.

way as to obtain the highest long-run return (annual, five, or ten year) at the lowest risk. Risk is defined as the uncertainty of future outcomes. It is possible to improve the prospects of maximum returns over the longer-term[4] by objectively determining asset class weighting and by selecting securities within individual asset classes that have above-average return prospects. Since the process of asset allocation should begin by defining the classes of assets available for inclusion in the portfolio we begin by describing and comparing the basic characteristics of the major classes of securities.

Asset Class Characteristics

There is a choice between two major types of investment: fixed income and equity. The first, fixed income, is further subdivided into bonds and preferred stocks. The second, is equity or common stock securities, which do not provide for any specific income in the investment contract. This study deals only with U.S. common stock investments for energy and gold portfolios. Table 1-1 compares the characteristics of the different major asset classes. The columns show the major

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4. Note : This depends on the policy and strategy of investment for each firm or investor thus they, capitalize such opportunities of return over periods longer than a year.

TABLE 1 - 1

Asset Class Characteristics

			BEFORE 1987	
SECURITY CLASSEIFICATION	MATURITY OF SECURITY	FORM OF RETURN	CERTAINTY OF RETURN	TAX STATUS
Cash Equivalents	Short	Discount	High	Fully Taxable
Fixed Income				
Bonds				
U.S. Government	Long	Coupon	Certain	Fully Taxable
Municipal	Long	Coupon	High	Tax-Exempt
Corporate	Long	Coupon	High	Taxable
Preferred Stocks	Perpetual	Dividend	Moderately High	Partial Exclusion
Common Stocks	Perpetual	Dividend & Capital Gains	Least Certain	Some Tax Exclusion

characteristics that are useful in distinguishing among the different securities, including maturity, form of return, certainty of return, and tax status.

Stock Market Indexes

The purpose of this section is to show that tools exist for use in judging a given portfolio's performance, including those of energy and gold-minerals. Stock market indexes measure the behavior of the market. The indexes allow a comparison of the movement of the stock market with such economic indicators as industrial production and changes in money supply. Furthermore, rates of return on the market can be a valuable benchmark for judging the performance of specific portfolios.

Standard and Poor's Composite Index of 500 stocks is a popular market value index that is usually accepted as representative of the market in general. Each stock is weighted in the index according to its market value. An alternative method of index construction is that of equal weighting. This method is based on the assumption that equal dollar amounts are invested in each stock included in the index. An index constructed in this way is more appropriate than that of Standard and Poor's for indicating the movements in price of typical or average stocks. A market-value-weighted index would be appropriate for indicating changes in the aggregate market value of stocks represented by the index.

Figure 1 - 1 shows the performance of two indexes: the Standard and Poor's 500 and the Indicator Digest Average, over the period 1964 - 1980. The Indicator Digest Average is an equally weighted index of NYSE stocks. The correlation between the two indexes indicates that essentially the same information is obtained regardless of the method of weighting used.

Study Outline

A description of the methodology of the present study with emphasis on selecting assets to obtain the desired long-run return at the lowest risk is provided in chapter II. Chapters III and IV contain a description of the data used and results obtained for the energy portfolio (in Chapter III) and the gold-minerals portfolio (in Chapter IV), respectively. Finally, in chapter V a summary of findings and recommendations is provided.

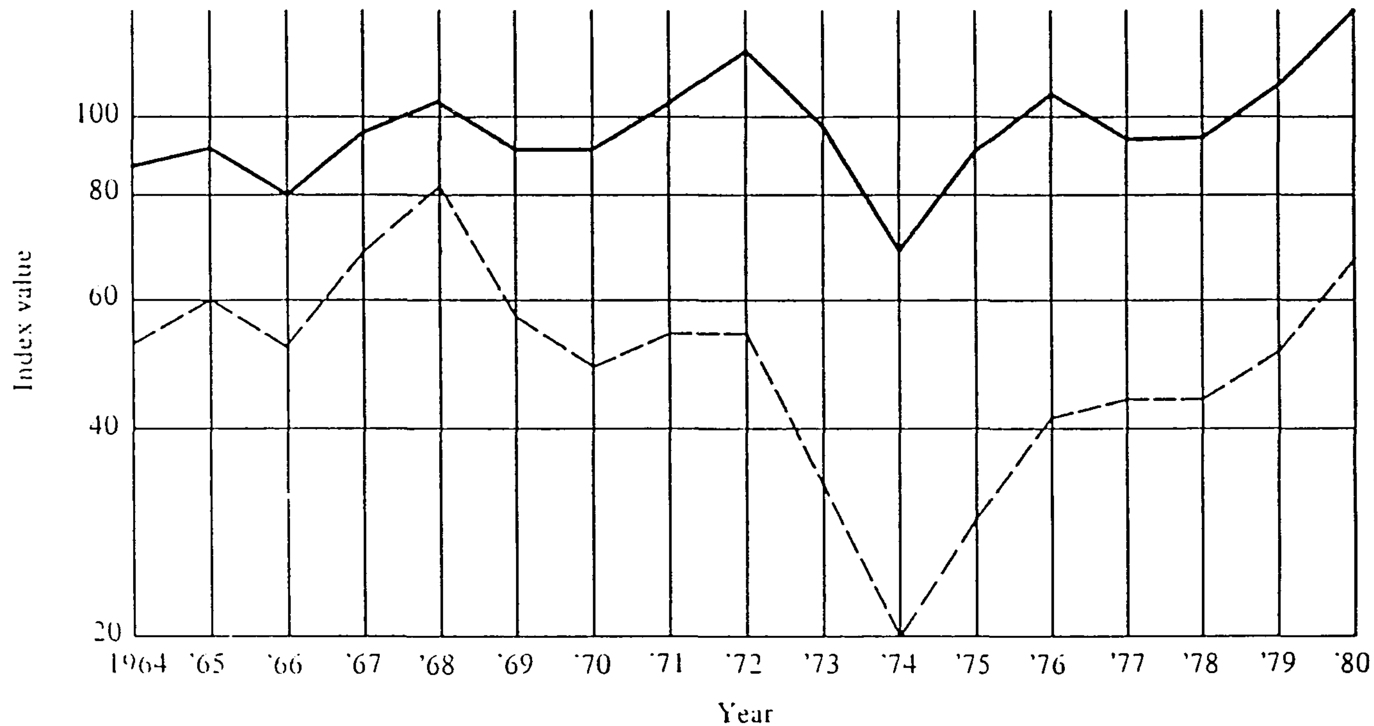

Figure 1-1 Index performance—equally weighted and market-value-weighted, 1964–1980. Solid line= S&P 500; dashed line=Indicator Digest Average. (*Sources: Standard & Poor's Corporation and Indicator Digest Average.*)

CHAPTER II

METHODOLOGY

This study is concerned with blending together major asset classes to obtain the desired long-run rate of return at the lowest risk. This study is also concerned with how to monitor portfolio performance and, with the optimal mix of assets, to form a portfolio. The first model to deal explicitly with risk in a portfolio was devised by Harry Markowitz (1952). In the present study, a mathematical model is set up according to Markowitz' theory, but it is solved using linear equations. The procedure is explained here, in Chapter III, and in Appendix B.

The data for all the securities analyzed were obtained on an annual basis for the years 1970 - 1985 in order to provide a long-run picture of cyclical movements in the areas of energy and gold-minerals stocks. Most financial institutions, including brokerage firms and individual investors, review each portfolio over much shorter periods of time: monthly, weekly, or even on a daily basis. Portfolio management, in general, is an attempt to construct portfolios that have the best chance of meeting long term goals and to improve returns by careful selection of securities. There is no guarantee that these efforts will be successful.

Return

For each security in its respective portfolio, as well as those sample securities from which these securities were selected, annual returns and measured risk were calculated for a period of the last 16 years (1970 - 1985). The goal was to evaluate the behavior of the securities over different economic periods such as business cycles. The mean return over those 16 years for each security was taken as representative of the return that security could have been expected to earn over the period. Indeed, several researchers, most notably Ibbotson and Sinquefel, have calculated the realized return and standard deviation of returns over past periods for such asset classes as corporate bonds, common stocks, treasury bills and long-term government bonds. They wished to determine the real return earned on the various asset classes over a given period. The annual return for each security in its repective portfolio is calculated using the formula provided (2-1). The results are given in Appendix C and Appendix D for all securities in energy and gold minerals, respectively.

$$\text{Return} = \frac{(P_1 - P_o) + D_1}{P_o} \qquad (2\text{-}1)$$

P_1 = Price/share ending value

P_o = Price/share initial value

D_o = Dividend ending value

$$\bar{R} = \text{Mean Return} = \sum_{t=1}^{N} \frac{R_t}{N} \qquad (2-2)$$

R_t = Return for year t

N = Number of years

Risk

Once the mean rate of return is determined (2-2), it is also important to assess the risk involved or associated with earning that return. In other words, the desired rate of return in the investment portfolio cannot be certain despite all of the efforts made to achieve it. The variance and standard deviation of an expected return are alternative statistical measures of the uncertainty or risk for a return. Equation 2-3 shows how to determine the variance for an individual security. The results are given in Appendix C and Appendix D for all securities in energy and gold-minerals, respectively.

$$\text{Variance of Return} = \text{Var (R)} = \sum_{t=1}^{N} [R_t - E(R)]^2 P_i \qquad (2-3)$$

Where:

$Var(R) = S(R)^2$ = Variance of Return

R_t = Return in period of time t

$E(R_t)$ = Mean Return

N = Number of periods under observation, i = 1,2,..., N.

P_t = Probability of occurance of a return at time t

Standard Deviation of Return $S = (Var(R))^{1/2}$ (2-4)

In a portfolio the riskiness of a specific security depends on how it blends with the existing securities and contributes to the overall risk of the portfolio. The covariance of the securities is the statistical measure of the uncertainty or riskiness of a security relative to others in a given portfolio.

$$Cov(Ri\ Rj) = 1/N \sum_{t=1}^{N} [Rti-E(Ri)][(Rtj-E(Rj)] \quad (2\text{-}5)$$

Where:

Cov(RiRj) = Covariance between security i and j.

N = Number of observations.

Rti = Return in period t, for security i.

$\bar{R}_i = E(R_i)$ = Mean return for security i.

Rtj = Return in period t, for security j.

$\bar{R}_j = E(R_j)$ = Mean return for security j.

Technically the divisor in this formula for the sample covariance should be N-1 for there is a loss of one degree of freedom in the calculation, but the unbiased estimate of the unknown population is produced by dividing by sample size N.

The covariance is difficult to interpret for practical application. The correlation coefficient is the measure used to facilitate the interpretation of the covariance. It is the result of dividing the covariance between two securities by the product of the standard deviation of each security.

$$\rho_{i\,j} = \frac{Cov\ (R_i R_j)}{\sigma_i\ \sigma_j} \qquad (2-6)$$

Where:

ρ i,j = Correlation coefficient between securities 1 and 2

σ i = Standard deviation for security 1

σ j = Standard deviation for security 2

Types of Risk

There are four factors that have traditionally been considered important in determining the degree of risk associated with an asset: purchasing-power risk, financial risk, interest rate risk and business risk. These factors impact the future risk of return on assets.

Purchasing-Power Risk

The purchasing-power risk involves choosing a low-risk, but profitable investment. Every investor's goal (at the very least) is to earn more on his or her investments than the rate of inflation, but investors do not always anticipate changes in the rate of inflation.

$$E(R) = E\left[\frac{CF + (P_1 - P_o)}{P_o}\right] \qquad (2\text{-}7)$$

Where:

$E(R)$ = Expected return

CF = Cash flow

P_1 = Price share ending value

P_o = Price share initial value

The present study deals only with U.S. common stocks, the dividends from which are the only cash flow. These are flexible, and the price of the security need not necessarily be adjusted to compensate for unanticipated inflation.

Financial Risk

Financial risk results from the introduction of debt into the capital structure of a corporation. It is generally measured by the percentage of debt compared to equity in the capital structure.[1] Common stocks of companies that finance with debt are subject to this risk. They may fall into financial difficulties and, perhaps, fail because of an unsound financial structure occasioned, for example, by too much reliance on borrowed funds leading to heavy fixed commitments for interest charges and repayment of borrowed funds causing intolerable strains in times of weak business conditions. Both the energy and minerals industries, the focus of the present study, have been affected by sharply declining prices for their products, which is enough reason not to overlook the expected return and the risk involved to achieve it.

Interest-rate Risk

Interest rate fluctuations are a major influence on stock and bond prices.[2] Stock prices are affected in a less direct way than are bond prices. When interest rates rise, investors are able to secure higher returns from

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1. Note: for each of the estimates of trading volume shares in chapters III and IV, the respective debt ratios for that stock is given.

2. Note: this study deals with only common stock securities.

other investments, such as certificates of deposit, treasury bills and money market accounts. Thus, these investments offer greater returns at no additional risk. All market interest rates tend to move together in the long run. These changes in interest rates affect all securities in the same way, because the value of the security is the present value of the security income. The market rate of interest is a component of the discount rate used in calculating present values of securities, and security prices vary inversely with changes in the level of the interest rate. Equation (2-8) indicates that the current price, Po, of the security is the cash flow (dividends) received over the period plus the expected price at the end of the period, P_1, discounted back at the rate k.

$$P_o = \frac{CF + P_1}{(1+k)} \qquad (2\text{-}8)$$

Where:

P_o = Price share initial

P_1 = Price share ending

K = Discount rate

CF = Cash flow

Long term securities show greater variability in price with respect to interest rate changes than do short-term securities; this concept is captured by table 1-1, which shows the relative vulnerability of the different security classes to interest risk.

Business Risk

Business risk is the risk caused by the nature of the firm's business due to internal factors such as inefficiency, poor management, failure to recognize changing conditions, and external factors, such as industry problems, and general economic, political or world trade difficulties. In order to reduce this type of risk, it is necessary to diversify the investment portfolio.

Risk-Return Ratio

The risk-return ratio is the relationship between the expected return from an investment and the amount of risk that one is willing to assume: the greater the risk we are willing to take, the higher the expected return. Figure 2-1 shows a relationship between risk and return. The horizontal axis represents the risk associated with the asset while the vertical axis represents the return expected.

The Markowitz Theory

The first model to deal explicitly with risk in a portfolio sense was devised by Harry Markowitz (1953)[3]. The basic portfolio model developed by Markowitz derived the expected rate of return for a portfolio of assets and an expected risk measure. The fundamental assumption in

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3. Harry Markowitz "Portfolio Selection" Journal of Finance, 7 (1), (March 1952)

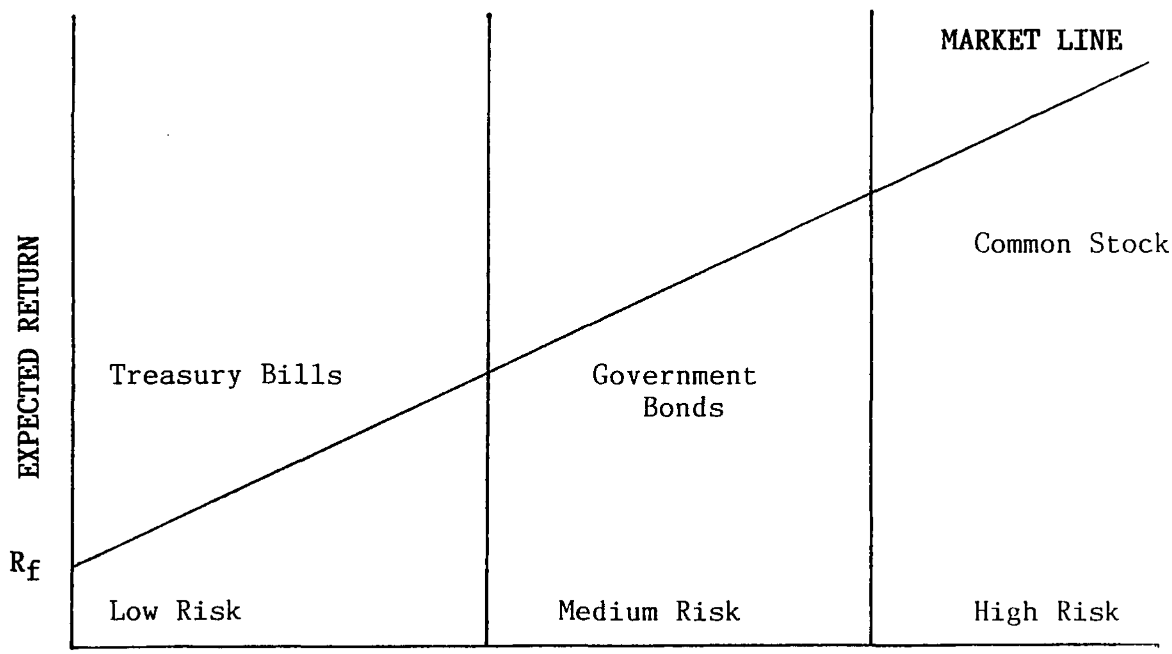

Fig. 2-1 Purchasing-Power Risk, Financial Risk, Interest-Rate Risk, Business Risk.

Markowitz' approach to portfolio analysis assumes that investors are risk averse. This means that an investor will compensate with a higher expected return as risk increases. Markowitz showed that the variance of the rate of return was a proxy measure of risk under a specific set of assumptions and devised the formulas for computing the variance of the portfolios. Markowitz also showed the importance of diversification and how to diversify properly.

The present study is based on several assumptions regarding investor behavior. First, efficient portfolios will be chosen, that is, those that minimize the risk at given investor's desired rate of return. Second, efficient portfolios are identified by properly analyzing information on expected returns for each security. Third, a computer program can be devised to utilize these inputs and to calculate the set of efficient portfolios. Finally, the most significate characteristics of a portfolio are its expected return and some measure of dispersion around the expected return.

Portfolio Return

The selection of the securities to form both energy and gold-minerals portfolios was taken from a selected sample of securities on the New York Stock Exchange (tables 3-1 & 4-1). These securities are stocks of companies whose activities are related to the oil and mineral industries. The expected return of a portfolio is a weighted average of the expected return of each security of which the portfolio is comprised. In chapter one it was mentioned that this study is simpler than most real-life applications. From the sample eight securities were selected, four for each portfolio, that minimize the risk at desired rates of return. The number of securities in these portfolios is arbitrary, it could easily have been more than four, but for simplicity the number is limited to four. In this study, W_i represents each security's proportion of the portfolio and $E(R_i)$ its expected return. The expected return of the portfolio $E(R_{port})$ is calculated as follows:

$$E(R_{port}) = W_1 E(R_1) + W_2 E(R_2) + W_3 E(R_3) + W_4 E(R_4)$$

$$E(R_{port}) = \sum_{i=1}^{N} W_i E(R_i) \qquad (2\text{-}9)$$

Standard Deviation of a Portfolio

The standard deviation of the portfolio is obtained by computing the square root of the weighted average of the variances for the individual assets.

$$\sigma_{port} = \left[\sum_{i=1}^{N} W_i^2 \sigma_i^2 + 2\sum_{i=1}^{N} \sum_{j=1}^{N} W_i W_j Cov_{ij}\right]^{1/2} \quad i \neq j \qquad (2\text{-}10)$$

Where:

σ_{port} = The standard deviation of the portfolio

W_i = The weights of the individual assets in the portfolio, where weights are determined by the proportion of value in the portfolio.

σ_i^2 = The variance of asset i

Cov_{ij} = The covariance between the returns for assets i and j

This formula (2-10) can be stated as the summation of the weighted covariance. In other words, the important factor to consider when adding assets to a portfolio containing a number of other assets is not the individual asset's variance, but its average covariance with all the other assets in the portfolio.

Efficient Frontier for Alternative Portfolios

The efficient frontier is that set of portfolios that has the maximum return for every given level of risk, or the minimum risk for every level of return. Figure 2-2 shows that every portfolio on the frontier has either a higher return for equal risk or lower risk for an equal return. For example, portfolio A has the same return as C but A has lower risk. It was postulated earlier that one determines one's position, based upon one's utility

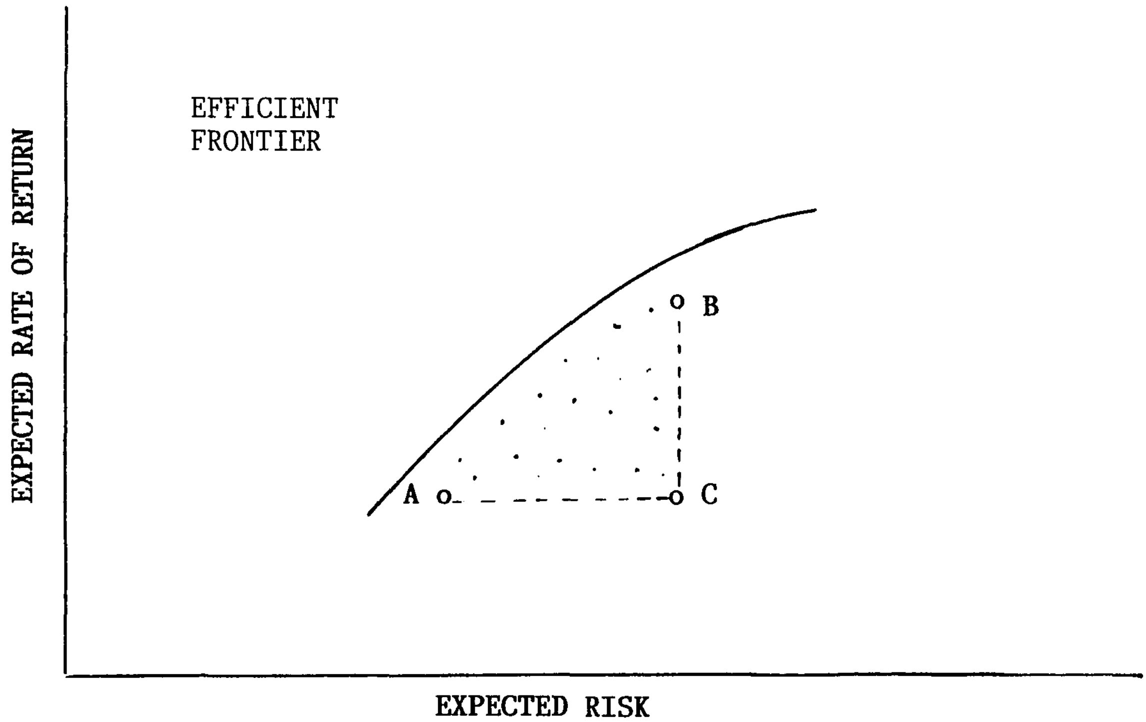

Fig. 2-2 Efficient Frontier for Alternative Portfolios

function and attitude toward risk. One determines where one wants to be along the frontier line based upon risk preferences.

Figure 2-3 shows the efficient frontier and a set of utility curves that may reflect our preference for risk and return. The higher the utility curve, the greater our satisfaction. The ideal investment should be that which lies at the point at which the dashed utility curve touches (is tangent to) the efficient frontier.

Diversification

The purpose of diversification is to reduce the standard deviation of the total portfolio. Several studies have been made on the subject. For example, one by Fisher and Lorie, table 2-1, provides an excellent illustration of the process. They randomly sampled stocks from all those listed on the NYSE to build portfolios ranging from one stock to 500 stocks. Stocks within the portfolios were weighted equally. Also, table 2-1 shows some representative statistics from the study, these simulations showed how the variance of the portfolio was reduced as stocks were added to the portfolio. At the same time, it also showed how quickly the effect of adding stocks exhausted the power of diversification.

Diversification provides substantial risk reduction if the components of a portfolio are

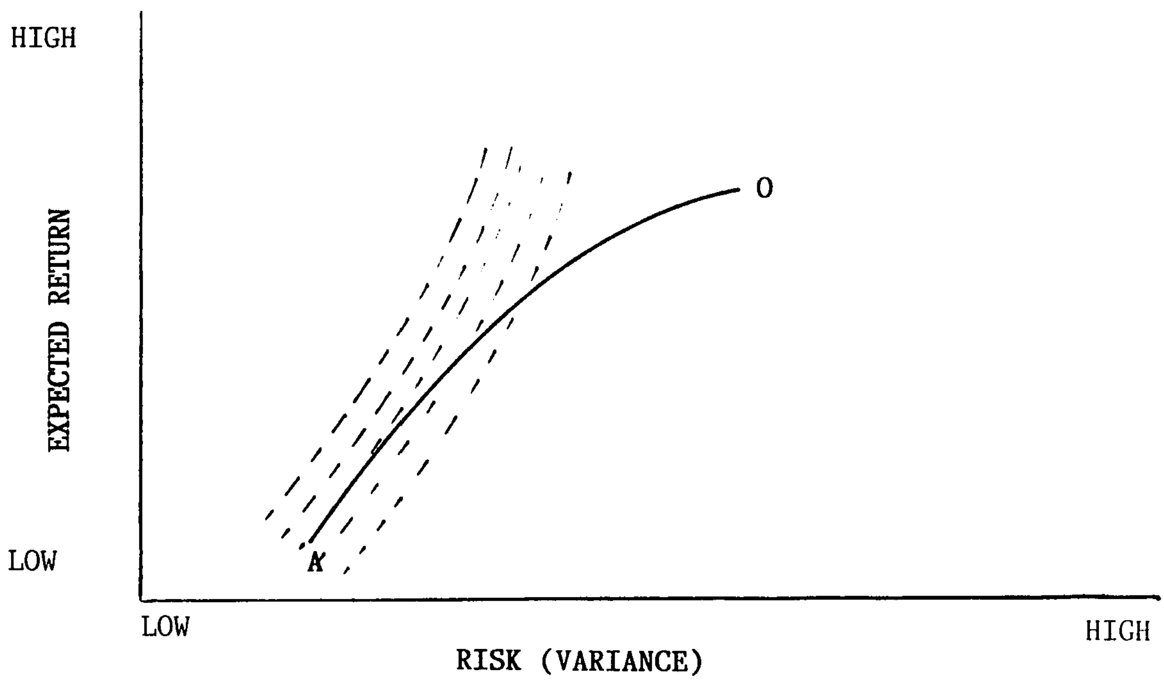

Fig. 2-3 Efficient Frontier and a Set of Utility Curves

TABLE 2 - 1

Portfolio Size - Risk and Return

Portfolio Size	Average Return %	Standard Deviation, %	Diversifiable Risk %	Market Related Risk, %
1	9	40.0	45	55
2	9	32.4	38	62
8	9	25.6	20	80
16	9	94.0	17	88
32	9	23.6	8	92
128	9	22.8	2	98
Indes Fund	9	22.0	0	100

Source: Lawrence Fisher, and James H. Lorie, "Some Studies of Variability of Return on Investments in Common Stocks" Journal of Business, April 1970, pp 99-134 uncorrelated. Figure 2-4 is a graph of the effect of diversification.

Asset Allocation

The purpose of asset allocation is the blending of assets together in a portfolio to maximize return at a level of risk which one is willing to accept. Once the securities are selected the next step is to determine the expected return for these eligible assets. Then, the variance of these individual securities is determined. Fourth, the covariance between the securities in the portfolio is estimated. Fifth, the optimal weights of the securities in the portfolio and sixth, the variance of the portfolio in particular are determined.

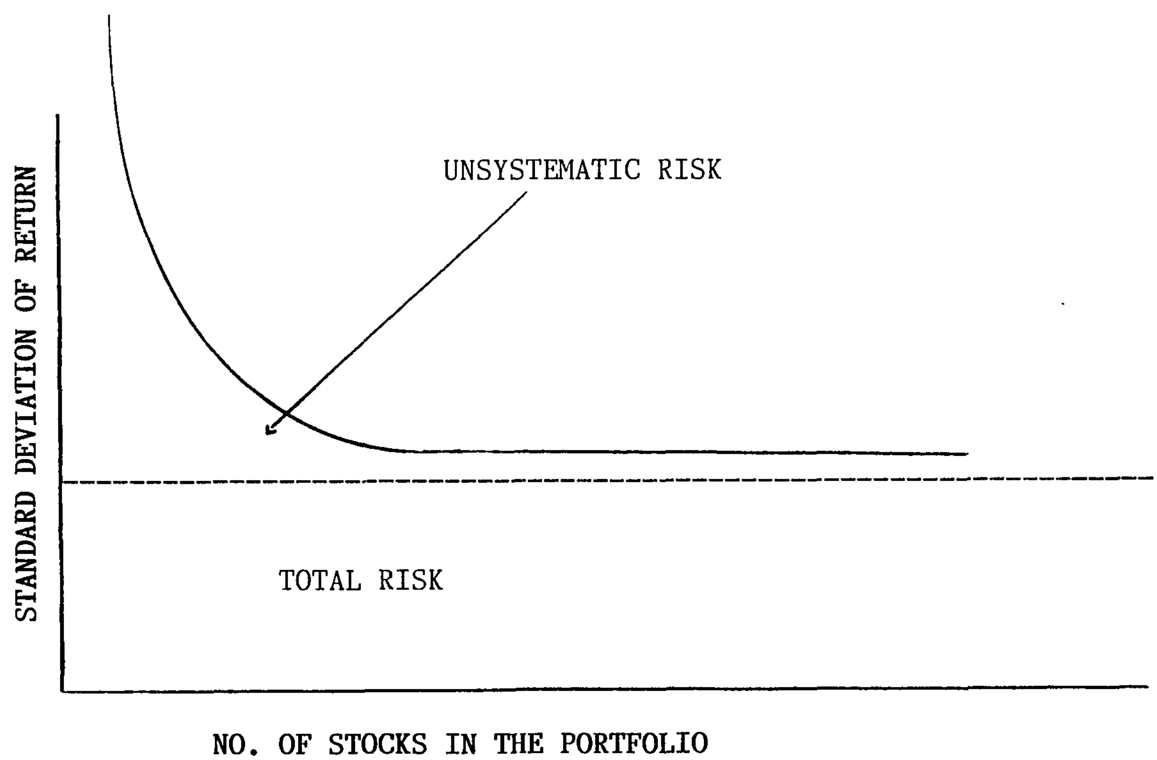

Fig. 2-4 Impact of Number of Stocks in a Portfolio on the Standard Deviation of a Portfolio Return.

CHAPTER III

ENERGY PORTFOLIO

Investment portfolios differ widely in their requirements, cash flow, risk thresholds, and time horizons. The objective of portfolio management is to reconcile these variables in such way as to maximize return and minimize risk. This gives the impression that perhaps these variables are unrelated and that each variable requires a different type of analysis, but in fact the principles of analyzing each variable are essentially identical.

The purpose of this chapter is to construct an energy investment portfolio from a selected sample of securities traded on the NYSE. This chapter also discusses the data input process and the results of applying the model.

The desired return on the energy investment portfolio is 14%. This percentage is the mean return obtained by the selected sample of securities (table 3-1), during the 16 years 1970 to 1985. It is thus reasonable to make the investment at this level of return. The future performance of the investment will likely be close to this average rate of return. It is thus used to match the time

TABLE 3 - 1

Energy Portfolio: Returns, Mean Returns, Variance of Returns and Standard Deviation on Nine Securities Listed in Figure 3 - 1 Through 3 - 9. (Derived from Appendix C, see tables C - 1 to C - 9)

	SECURITY								
RANKING	1	2	3	4	5	6	7	8	9
SECURITY	SOH	XON	PZL	MOB	P	TGT	OXY	TX	KMS
YEAR									
1971	22.74	27.39	14.01	17.09	27.81	26.958	(11.663)	20.838	44.55
1972	1.09	10.57	(15.59)	23.81	19.44	7.52	(12.546)	4.249	29.58
1973	63.58	24.59	9.32	(1.39)	60.86	1.130	(24.46)	4.645	43.52
1974	(0.865)	(12.29)	(8.28)	(21.83)	(5.13)	(11.284)	2.156	(17.018)	(5.35)
1975	7.75	9.14	(0.37)	4.17	(0.28)	21.659	76.926	1.925	13.39
1976	11.02	31.94	37.68	43.75	22.47	77.928	9.61	12.644	(6.44)
1977	14.89	8.10	21.23	27.08	12.75	(13.605)	45.463	16.227	(14.55)
1978	(8.41)	2.15	3.58	2.43	3.27	1.391	(17.725)	(4.693)	(20.11)
1979	89.26	20.85	65.28	53.60	30.81	20.804	15.855	20.874	26.20
1980	100.24	38.80	52.82	55.69	29.94	34.243	36.349	63.875	28.81
1981	(13.26)	9.13	6.58	(13.96)	(1.511)	(5.201)	4.177	(.374)	6.28
1982	(32.50)	(9.74)	(16.34)	(19.76)	(26.780)	(18.493)	(16.319)	(16.760)	15.2
1983	45.27	28.84	5.69	30.46	12.30	29.693	28.569	24.704	8.84
1984	1.71	28.71	9.09	1.52	38.49	12.623	33.22	22.614	0.599
1985	11.11	30.95	44.96	15.37	(9.00)	12.538	7.962	(8.102)	2.97
Mean Return	20.91	16.6	15.31	14.54	14.36	13.19	11.83	9.71	9.53
Variance	1332.866	225.842	562.428	573.854	449.148	549.547	738.59	385.826	410.395
Standard Deviation	36.508	15.028	23.715	23.955	21.193	23.442	21.177	19.64	20.257

and cash flow constraints of the energy portfolio. This chapter also highlights the activities of the companies included in the portfolio. First, a few basic facts about the areas in which these companies are engaged and then a description of stock selection criteria are given, as well as the measures of risk and return for each of them.

An energy portfolio includes securities of companies whose activities are directly related to the field of energy. These companies may engage in the production, marketing, exploration, or refining of energy fuels. Energy companies supply fuels to both producers and consumers. The earnings of these companies are affected by the business cycle, and, most importantly, by trends in the relative price of energy.

The special risk associated with investing in energy related securities is due to exogenous causes, like shifts in international politics, domestic politics and the policies of the Organization of Petroleum Exporting Countries (OPEC). Earnings and dividends of companies in this industry are greatly affected by changes in the prices and quantities of oil and other energy fuels supplied and demanded over a short period of time.

Stock Selection

Given the analytical concepts of portfolio analysis discussed above, the focus here is now on stock selection. This selection deals with both risk and return. The return achieved by the individual stocks is based principally on market effects, unique characteristics of the individual security, and broad market sector or industry trends.

In order for a selection strategy to be successful, there are several critical factors that must be taken into account. For example, one must be able to distinguish between relatively attractive and unattractive stocks. This implies the need for some method of measuring or assessing predictive capability. Second, is the need for a systematic portfolio construction procedure, meaning that predictions for individual stocks are appropriately built into the portfolio. Third, rebalancing the portfolio over time as well as accounting for transactions costs is required.

The last point is beyond the scope of this study. Nevertheless, it is useful and important to comment on it. The selection strategy was predetermined by selecting stocks grouped only in the energy field, and then balancing them based on an evaluation of their risk and return and on a predictive ranking method. The four most relevant stocks (an arbitrary number), were selected from the selected sample of securities, table 3-1, to construct the energy portfolio. A choice based on past performance assumes that average returns for the past are good estimates of the "likely" return in the future, and that the variability of returns in the past is a good measure of the uncertainty of return in the future. In the present study it is assumed that the mean returns on these securities are the "likely" expected returns in the one-year period investment. Variance around the mean rather than at the trend is acceptable because, there is little or no trend.

Table 3-1 shows the mean return and variances of the selected sample of securities in the energy portfolio. No special significance should be attached to this list of securities other than that it will be used for illustrating principals of portfolio analysis. Appendix C indicates how the expected return of the securities in table 3-1 were calculated. Annual returns for the nine securities are listed in table 3-1 and shown in figures 3-1 to 3-9. The nine securities differ in their average rate of return. For example, the average annual rate of return on Exxon Corporation common stock was 16.6% per dollar invested; that for Pennzoil common stock was 15.31% per dollar invested.

The variation of these returns from the expected or mean return is also shown. Figure 3-10 shows the average return and its standard deviation for the

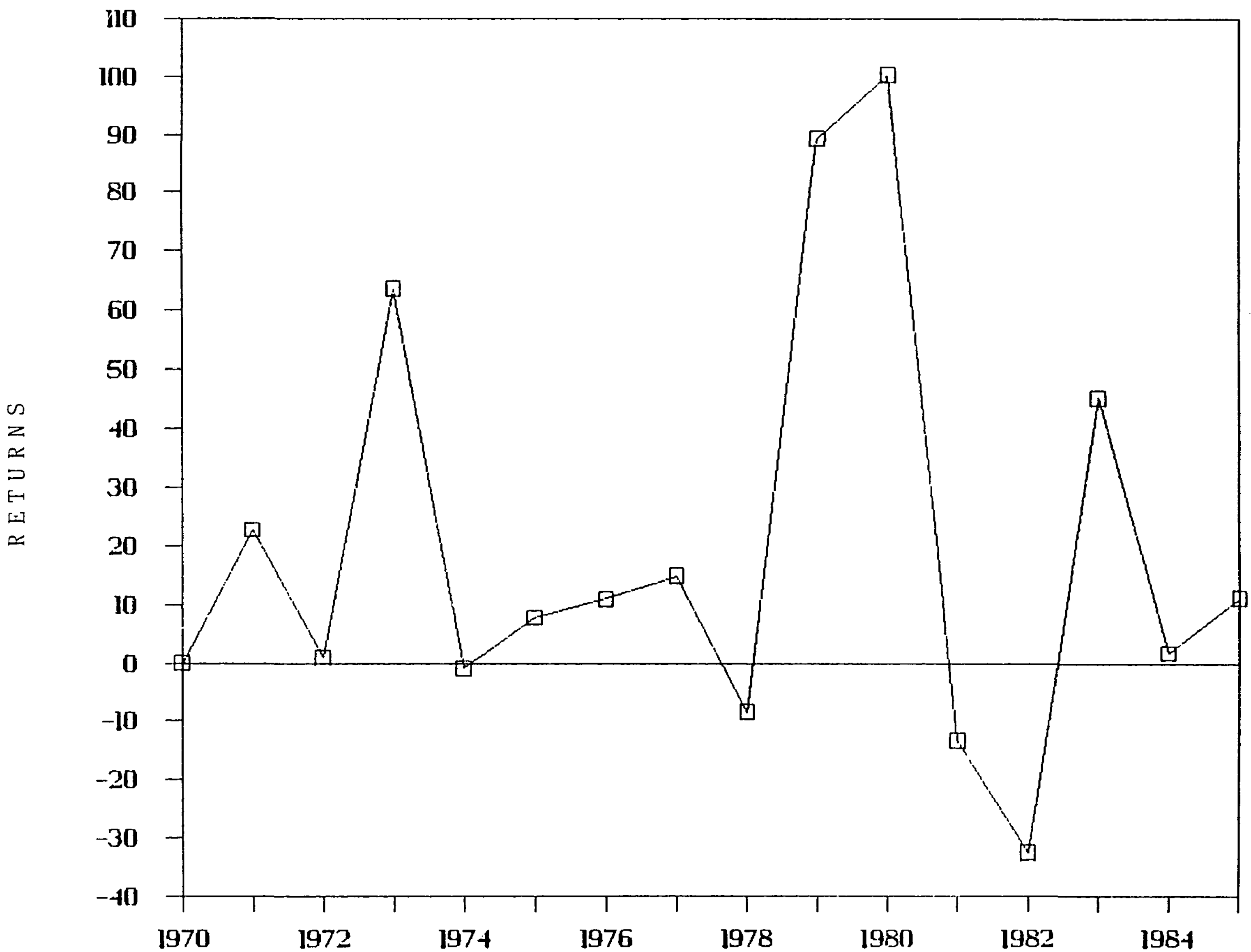

Figure 3-1 Returns on Security 1/Energy Portfolio. --Standard Oil (SOH).

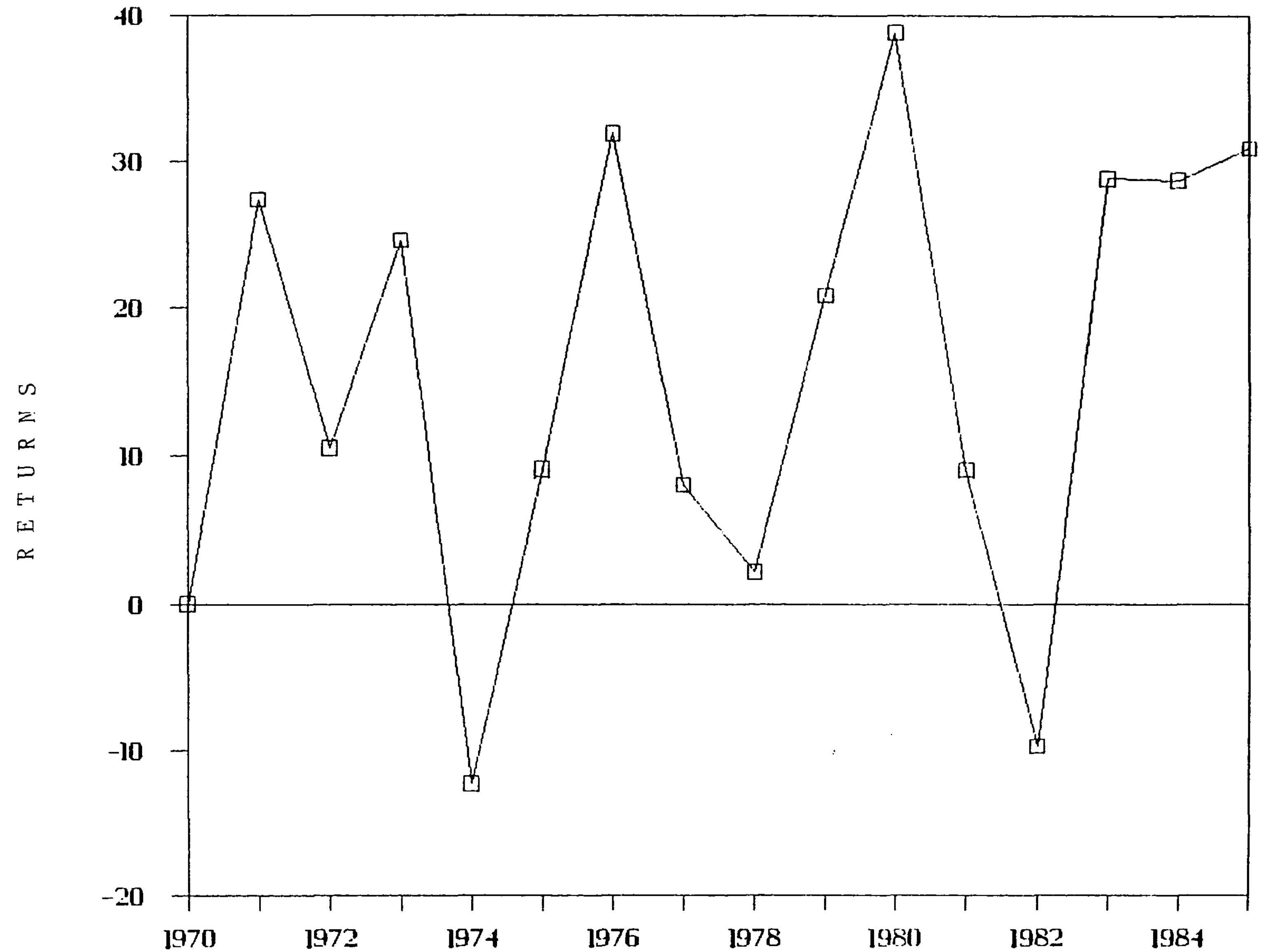

Figure 3-2 Returns on Security 2/Energy Portfolio. --Exxon Corporation (XON).

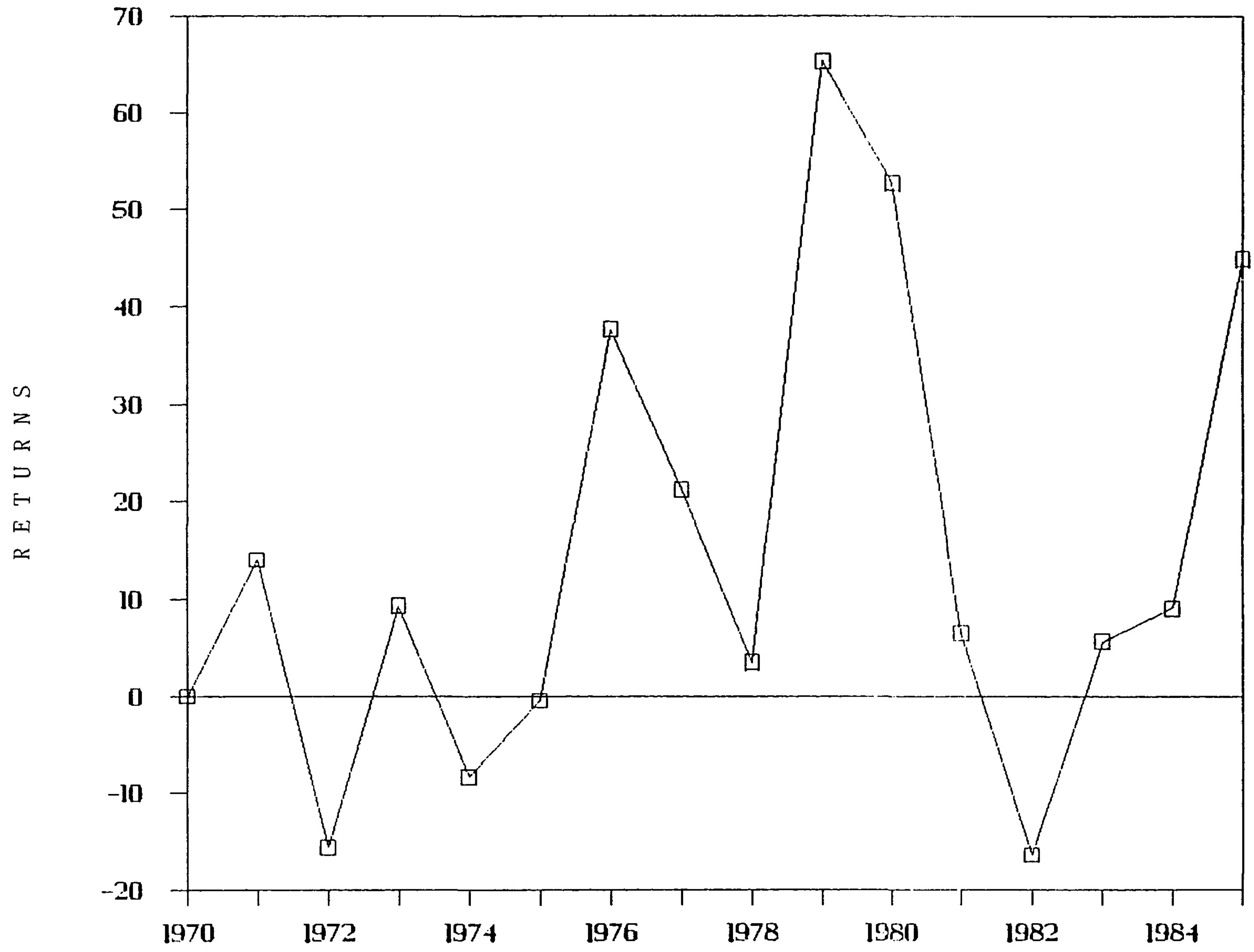

Figure 3-3 Returns on Security 3/Energy Portfolio. --Pennzoil Company (PZL)

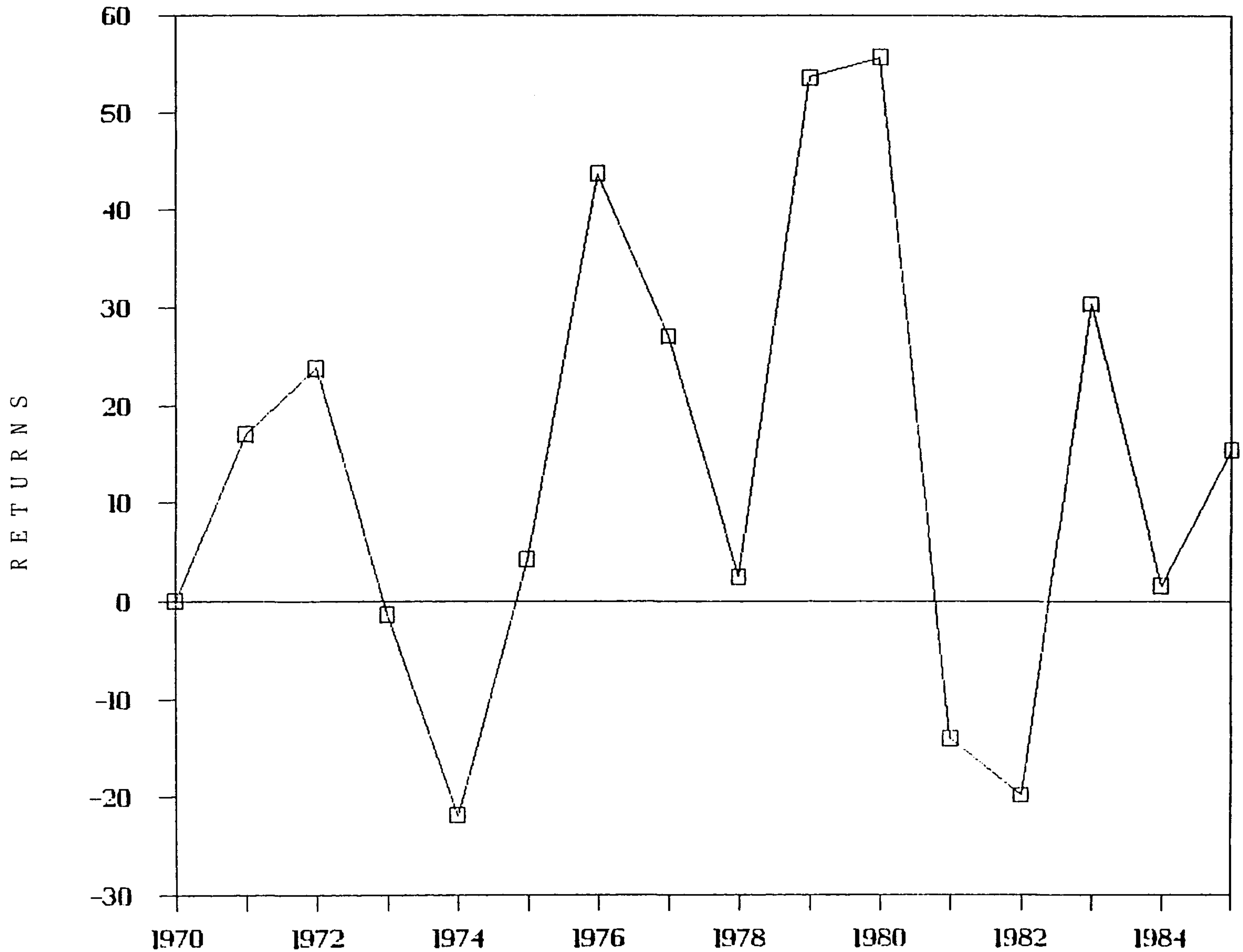

Figure 3-4 Returns on Security 4/Energy Portfolio. --Mobil Corp. (MOB).

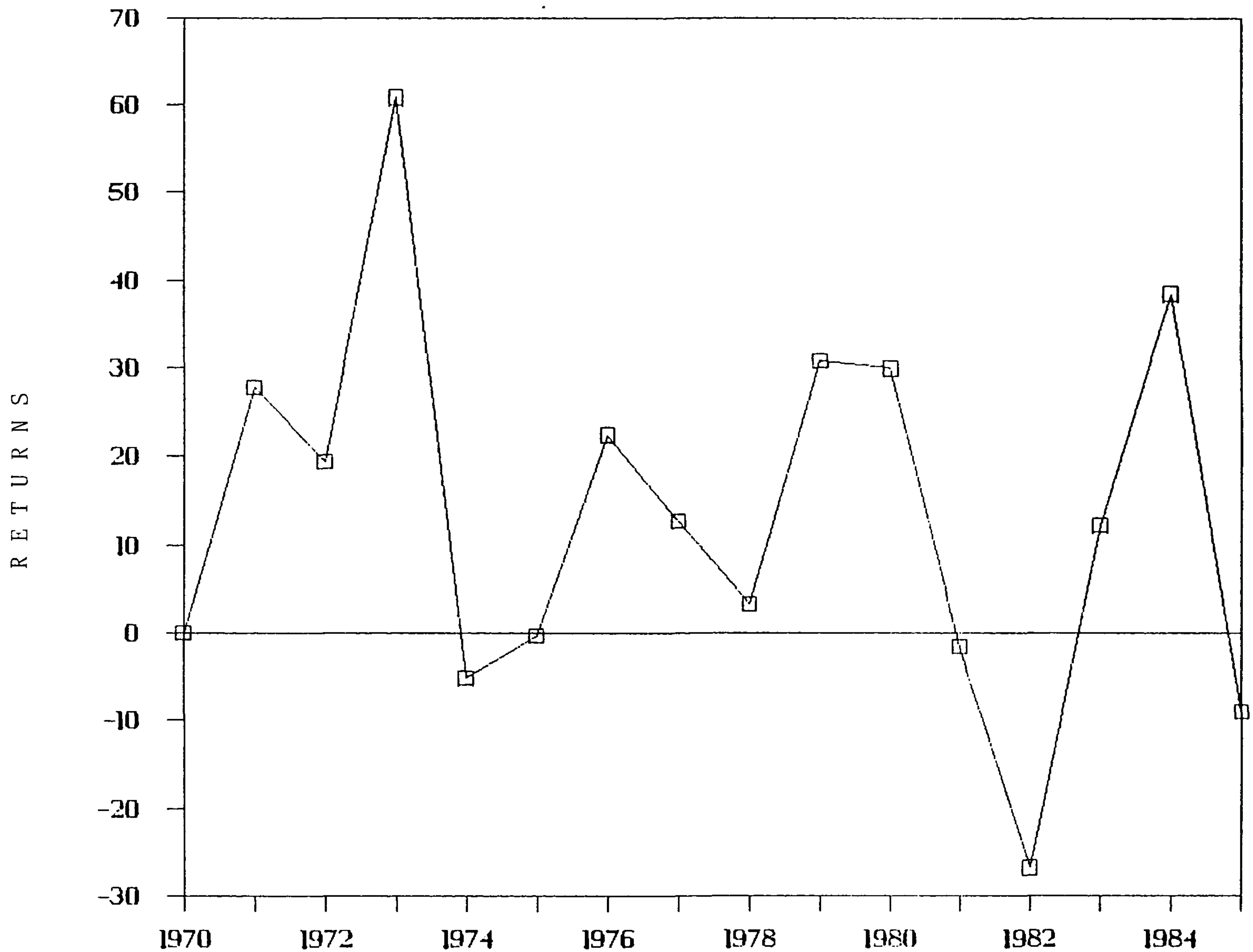

Figure 3-5 Returns on Security 5/ Energy Portfolio. --Phillips Petroleum Company (P).

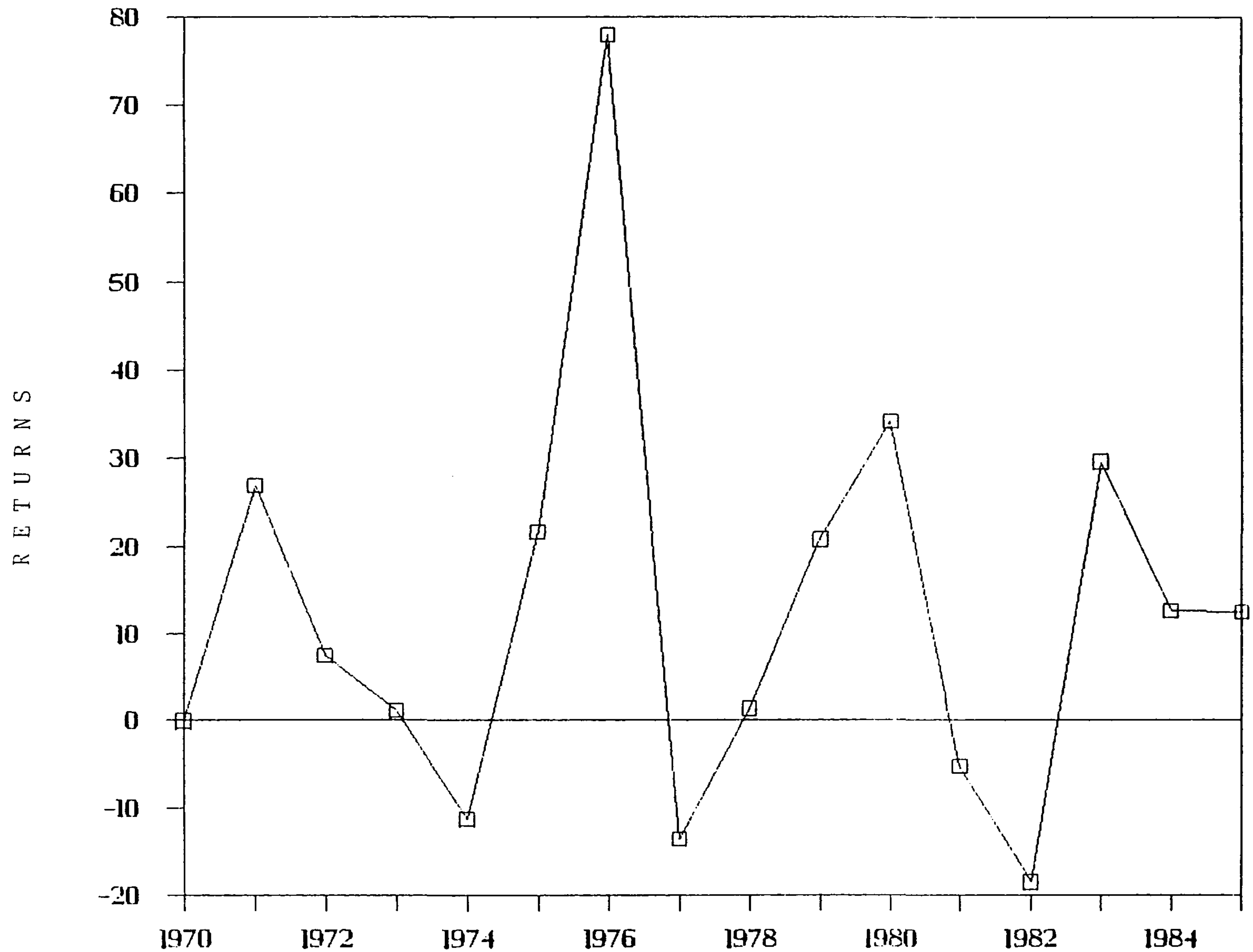

Figure 3-6 Returns on Security 6/Energy Portfolio. --Tenneco Inc. (TGT)

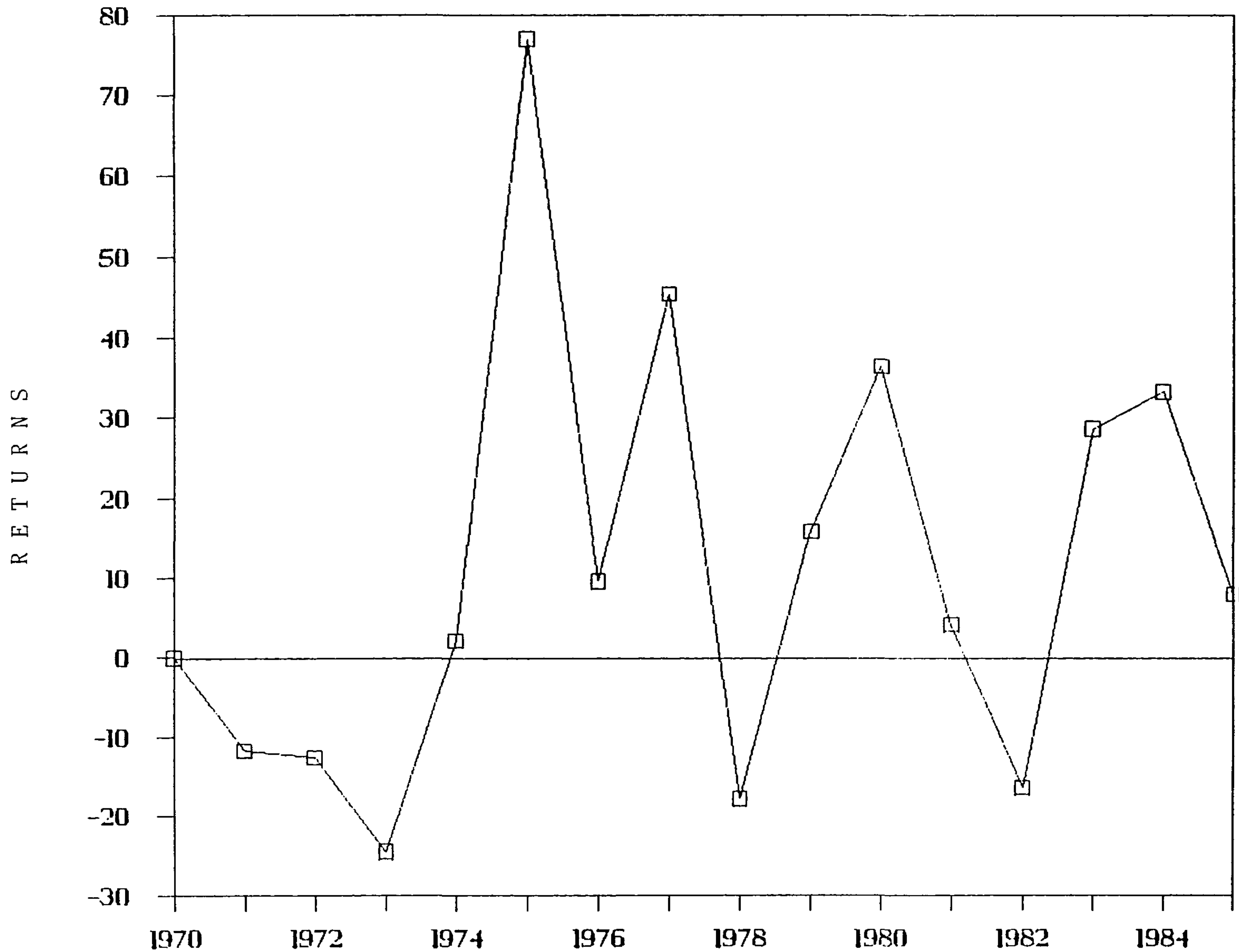

Figure 3-7 Returns on Security 7/Energy Portfolio. --Occidental Petroleum Corporation (OXY).

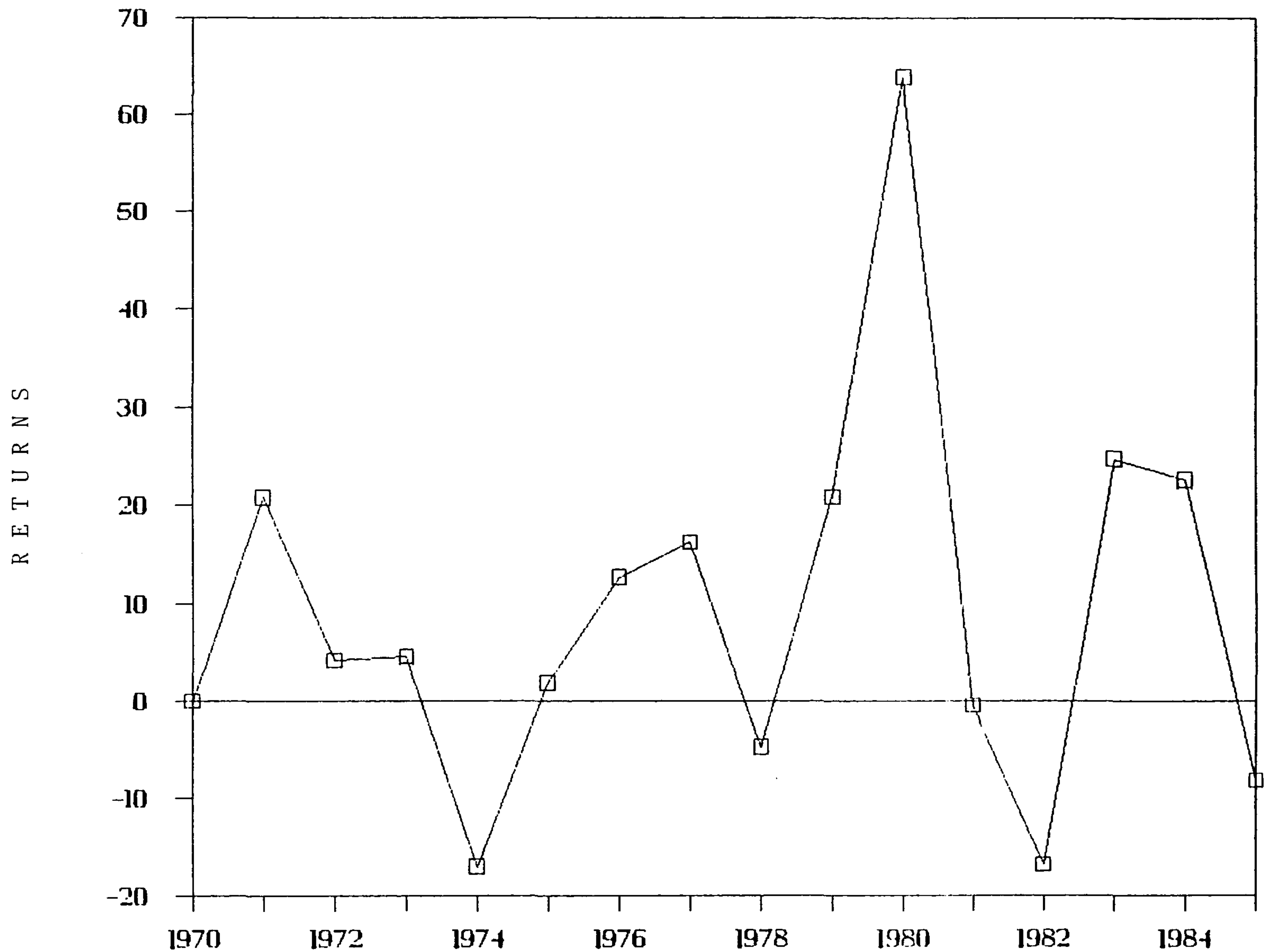

Figure 3-8 Returns on Security 8/Energy Portfolio. --Texaco Inc. (TX).

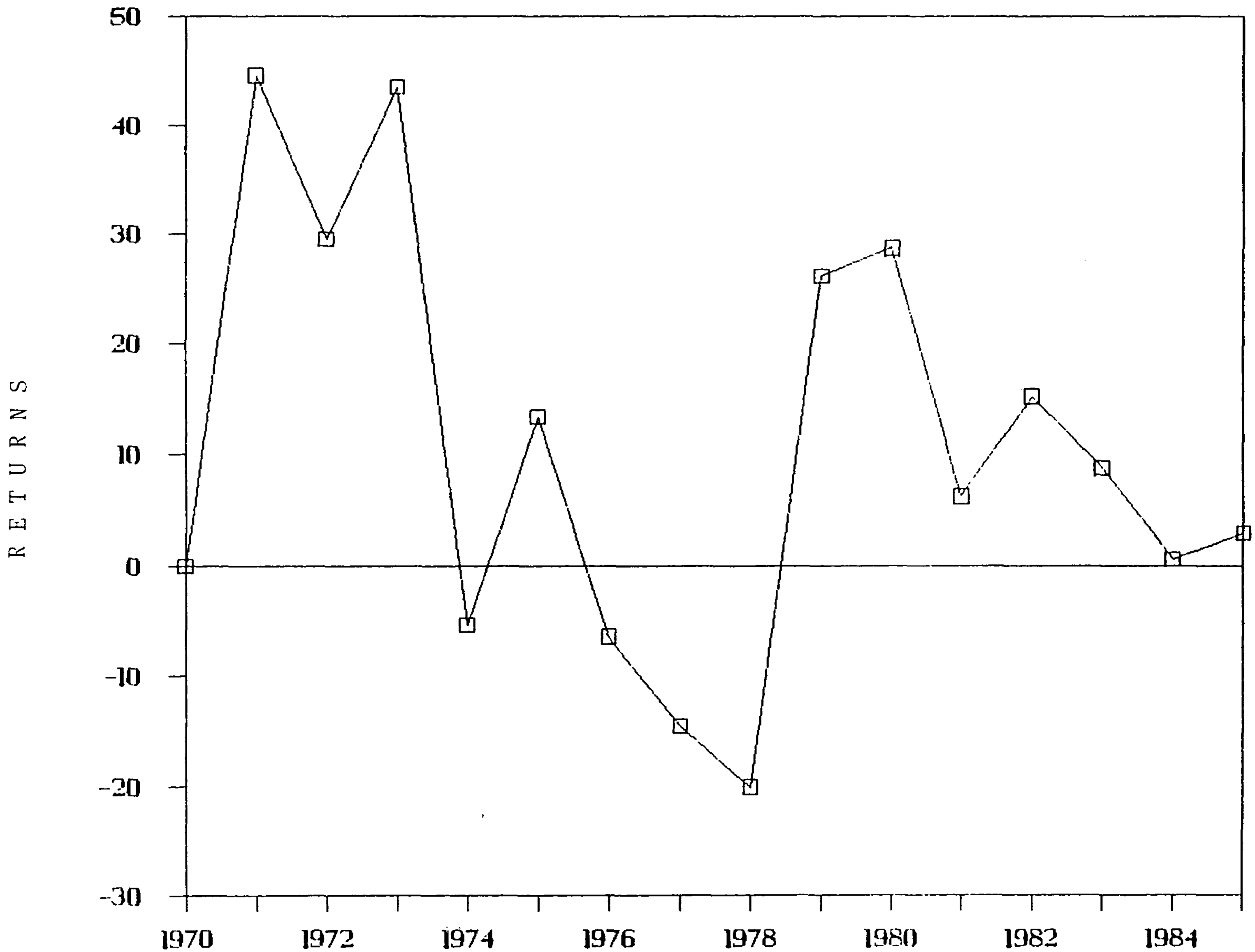

Figure 3-9 Returns on Security 9/Energy Portfolio. --Kerr McGee Corp. (KMS).

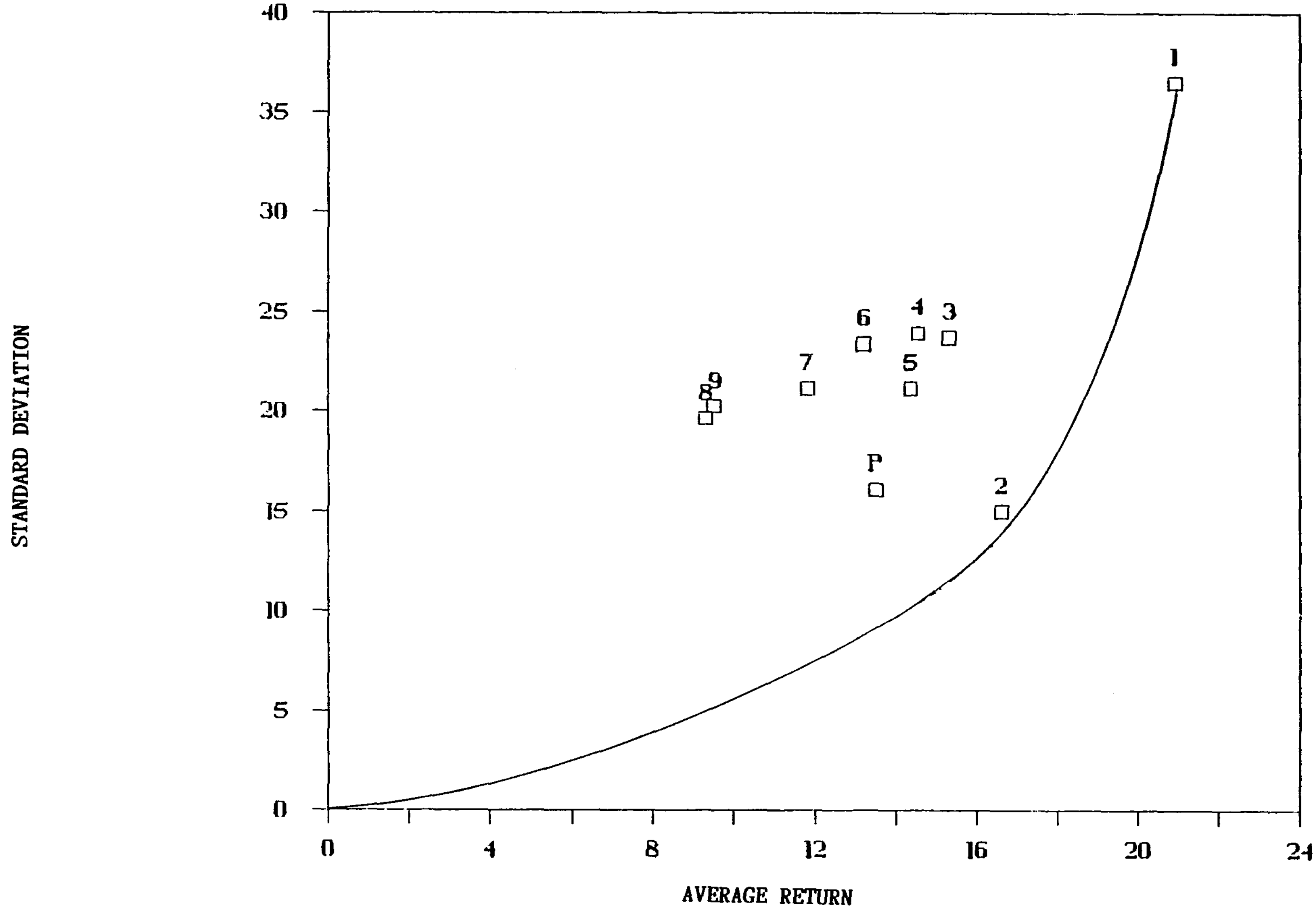

Figure 3-10 Average Return and Standard Deviation for an Energy Portfolio. --Securities 1 through 9.

securities in table 3-1. The horizontal axis represents the standard deviation. Thus the point labelled 1, indicates that security 1 (Standard Oil Company, Ohio common stock) had an average rate of return of 20.91% and a standard deviation of 36.508. Figure 3-10 shows that Exxon Corporation (security 2) had a higher return on average and much lower standard deviation than either security 3 or 5. Security 2 performed better during the period than did 3 or 5 combining a high average with greater stability. Point P indicates the investor-desired rate of return (13.50) and standard deviation (16.11). This portfolio is explained later in this chapter.

The first step in portfolio selection has been accomplished. In fact, once the predictive capacity for these individual stocks is established one can begin to select such stocks for a portfolio. This chapter gives an example of the blending of stocks for a portfolio given the model's constraints. Not all of these securities would appear in the final desired portfolio. They enter the analysis as candidates for a place in the desirable portfolio. Also, results from another blending of alternative securities (2 through 6) is shown later in the chapter.[1]

---

1. Note: In this chapter the results of the six more relevant results of the blending alternatives for optimal mix are commented upon.

Determining the Optimum Mix

Risk, return and the ranking of securities in the random sample were determined in the previous section. The systematic procedure for portfolio construction is shown in Appendix A. The securities considered in alternative portfolio (1) presented in this chapter are: Exxon, Texaco, Phillips Petroleum, and Tenneco. Before determining the optimal proportion of each security in the portfolio, in order to minimize variance for a given expected return (14%), other statistical measures are needed: covariances. Table 3-2 shows these results. The problem that will be dealt with here is extremely simple. However, it will serve to demonstrate the mathematical technique required in a more realistic investment problem. It was decided previously that this portfolio will be composed of four investments. The mathematical methods required to handle this problem are shown in Appendix A.

Thus, inserting the proper constants and constraints (variances, covariances, expected return and E), the equation system from Appendix A, (see page 131) may be expressed in tabular form as table 3-3. With six equations and six unknowns, these equations can be solved, providing us with values for W_1 through W_4: the optimal allocation or weighting of funds to invest in the portfolio with minimal risk. The equation set is solved

TABLE 3 - 2

Energy Portfolio Covariances: Blending Alternative 1

	1	2	3	4						
Year	Exxon	Texaco	Phillips Petroleum	Tenneco						
	$R_E-E(R_E)$	$Rx-E(Rx)$	$Rp-E(Rp)$	$R_t-E(Rt)$	$\sigma_{1,2}$	$\sigma_{1,3}$	$\sigma_{1,4}$	$\sigma_{2,3}$	$\sigma_{2,4}$	$\sigma_{3,4}$
1971	10.79	11.128	13.450	13.788	120.071	145.126	148.773	149.6716	153.433	185.449
1972	(6.03)	(5.461)	5.08	(5.67)	32.972	(30.632)	34.190	(27.777)	30.964	(28.804)
1973	7.99	(5.065)	46.50	(12.05)	(40.469)	371.535	(96.319)	(235.523)	61.03	(560.325)
1974	(28.89)	(26.728)	(19.49)	(24.424)	772.172	563.066	707.054	520.9287	654.141	476.998
1975	(7.46)	(7.785)	(14.64)	8.469	58.076	109.214	(63.179)	113.9771	(65.931)	(123.986)
1976	15.34	2.934	8.11	64.738	45.008	124.407	993.081	23.7947	189.941	525.025
1977	(8.5)	6.517	(1.61)	(26.795)	(55.395)	13.685	227.758	(10.4923)	(174.623)	43.140
1978	(14.45)	(14.03)	(11.09)	(11.799)	202.734	160.251	170.496	155.592	165.539	130.851
1979	4.25	11.164	16.45	7.614	47.447	69.913	32.360	183.648	85.003	125.250
1980	22.2	54.165	15.58	21.053	1202.463	345.876	467.377	843.891	1140.336	328.006
1981	(7.47)	(10.084)	(15.871)	(18.39)	75.328	118.556	137.381	160.043	185.445	291.868
1982	(26.34)	(26.47)	(41.14)	(31.683)	697.219	1083.628	834.530	1088.976	838.649	1303.439
1983	12.24	14.994	(2.06)	16.503	183.478	(25.214)	201.997	(30.879)	247.446	(33.996)
1984	12.11	12.904	24.13	(.567)	156.267	292.214	(6.866)	311.3735	(7.317)	(13.632)
1985	14.35	(17.812)	(23.36)	(.652)	(255.602)	(335.216)	(9.356)	416.08832	11.613	15.231
	Totals				3241.769	3006.409	3779.277	3663.3126	3515.669	2664.464
	Covariance				231.555	214.743	269.948	261.665	251.119	190.319

$$Cov\ (R_i R_j) = 1/n-1 \sum_{\substack{i=1 \\ j=1}}^{N} (R_i - \bar{R}_i)(R_j - \bar{R}_j)$$

Note: Values are derived from Appendix C,
See tables C-2, C-5, C-6 and C-8
( ) are negative values

TABLE 3-3

Energy Portfolio Linear Equations System for Alternative 1

$$225.842\,W_1 + 231.555\,W_2 + 214.743\,W_3 + \tfrac{1}{2}\,1_1\,(16.6\ ) + \tfrac{1}{2}\,1_2\,(1) + 269.948\,W_4 = 0$$

$$231.555\,W_1 + 385.826\,W_2 + 261.665\,W_3 + \tfrac{1}{2}\,1_1\,(\ 9.71) + \tfrac{1}{2}\,1_2\,(1) + 251.119\,W_4 = 0$$

$$214.743\,W_1 + 261.665\,W_2 + 449.1483\,W_3 + \tfrac{1}{2}\,1_1\,(14.36) + \tfrac{1}{2}\,1_2\,(1) + 190.319\,W_4 = 0$$

$$16.6\,W_1 + 9.71\,W_2 + 14.36\,W_3 + 13.19\,W_4 = 14$$

$$1\,W_1 + 1\,W_2 + 1\,W_3 + 1\,W_4 = 1$$

$$269.998\,W_1 + 251.119\,W_2 + 190.319\,W_3 + \tfrac{1}{2}\,1_1\,(13.19) + \tfrac{1}{2}\,1_2\,(1) + 549.547\,W_4 = 0$$

Note: Derived from Appendix A, see pages 130 and 131, Tables 3-1 and Table 3-2 using the method of symmetric single division described by Paul S. Dwyer in <u>Linear Computations.</u> This procedure is explained in detail in Appendix B (table B-2 and B-3). For portfolio alternative (1), at a 14% yield, the full illustration of the values for each step of the method of solution for the system equations (Appendix B, table B-2 and B-3), is not included in this chapter, but is shown for 13.5%. Table 3-4 gives the optimal weights (solution of table 3-3) for each security in energy portfolio alternative (1).

The proportion of the total investment allocated to each of the four securities is given by W_1 through W_4. These are the proportion of total funds that should be allocated to each investment in order to obtain a return of 14% while minimizing variance. But for <u>this particular</u> alternative (1), at 14% desired rate of return, the results show that the optimal portfolio would require a "disinvestment" of security number four (W_4), due to the nonnegative restriction in the W's, a restriction not formally imposed in the model (see Appendix A, page 129). This value of W_4 from table 3-4, W_4 = -0.001659, is thus consistent with the method because of the nonnegativity restriction. This means that no funds would be allocated into investment W_4 at a 14% desired rate of return. Thus, in order to understand these results and devise a way of solution that will meet all restrictions including the

TABLE 3 - 4

Blending Together Alternatives (1 and 2) by Selected Sample Securities from Table 3-1 for Construction of an Energy Portfolio

Desired 14% Portfolio Return

ALTERNATIVE 1					ALTERNATIVE 2				
WEIGHTS W(s)	RANKING POSITION	SYMBOL	EXPECTED RETURN	VARIANCE	WEIGHTS W(s)	RANKING POSITION	SYMBOL	EXPECTED RETURN	VARIANCE
W_1	2	XON	16.6	225.842	W_1	8	TX	9.71	385.826
W_2	8	TX	9.71	385.826	W_2	6	TGT	13.19	549.547
W_3	5	P	14.36	449.148	W_3	5	P	14.36	449.148
W_4	6	TGT	13.19	549.547	W_4	3	PZL	15.31	562.428

SOLUTION

OPTIMAL WEIGHTS (Alternative 1)		OPTIMAL WEIGHTS (Alternative 2)	
W_1 = 0.578742	V_p = -----	W_1 = 0.044585	V_p = 329.085689
W_2 = 0.356632	S_p = -----	W_2 = 0.283446	S_p = 18.140719
W_3 = 0.066279		W_3 = 0.483597	
W_4 = -0.001659	V_{ep} = 278.06631	W_4 = 0.188370	V_{ep} = 320.359518
W_t = 1.000000	S_{ep} = 16.67321	W_t = 1.000000	S_{ep} = 17.898589

WHERE:

V_p = Variance portfolio
S_p = Standard deviation portfolio
V_{ep} = Variance Portfolio (using equal weights/securities 0.25000)
S_{ep} = Standard deviation portfolio (using equal weights/securities)

nonnegativity constraint, variable(s), W(s), may be substituted for another (of the securities listed in table 3-1), and the more efficient securities will be chosen, resulting in an optimal mix of securities to form a portfolio. That is, those securities that minimize the risk at a given rate of return, in this case at 14% yield, and also obtain positive values for all W(s).

Table 3-4, shows model results (W_1 through W_4) for alternative (2). This alternative, would not involve a "disinvestment", and neither would alternatives (3), (4) and (5): see tables 3-5 and 3-6. Note that, the full illustration of the system equations, the values for each step of the solution method (Appendix B, tables B-2 and B-3) and covariance calculations for alternatives (2) through (6), at 14% yield, are not included in this chapter. However, the final values for proportions of individual securities and portfolio variance for each of the alternative portfolios are included (see tables 3-4, 3-5 and 3-6). The alternative without "disinvestment" which offers the least risk at a 14% yield is alternative (2), with a variance of 329.08 (table 3-4).

To illustrate the sensitivity of the W(s), or proportions of securities, and variance portfolio values, Vp(s), if the desired rate of return for the energy investment portfolio were not fixed at 14%, table 3-7 gives the sensitivity of the W(s) and variance portfolio Vp(s)

TABLE 3 - 5

Blending Together Alternatives (3 and 4) by Selecting Sample Securities from Table 3-1 for Construction of an Energy Portfolio

Desired 14% Portfolio Return

ALTERNATIVE 3					ALTERNATIVE 4				
WEIGHTS W(s)	RANKING POSITION	SYMBOL	EXPECTED RETURN	VARIANCE	WEIGHTS W(s)	RANKING POSITION	SYMBOL	EXPECTED RETURN	VARIANCE
W_1	8	TX	9.71	385.826	W_1	8	TX	9.71	385.826
W_2	5	P	14.36	449.148	W_2	6	TGT	13.19	549.547
W_3	4	MOB	14.54	573.854	W_3	3	PZL	15.31	562.428
W_4	3	PZL	15.31	562.428	W_4	1	SOH	20.91	1332.866

SOLUTION

OPTIMAL WEIGHTS (Alternative 3)		OPTIMAL WEIGHTS (Alternative 4)	
W_1 = 0.188638	V_p = 352.411927	W_1 = 0.165720	V_p = 410.368560
W_2 = 0.180028	S_p = 18.772637	W_2 = 0.338286	S_p = 20.257555
W_3 = 0.508406		W_3 = 0.436134	
W_4 = 0.122927	V_{ep} = 362.031943	W_4 = 0.059858	V_{ep} = 481.645993
W_t = 1.00000	S_{ep} = 19.027137	W_t = 1.000000	S_{ep} = 21.946434

WHERE:

V_p = Variance portfolio

S_p = Standard deviation portfolio

V_{ep} = Variance Portfolio (using equal weights/securities 0.25000)

S_{ep} = Standard deviation portfolio (using equal weights/securities)

TABLE 3 - 6

Blending Together Alternatives (5 and 6) by Selecting Sample Securities from Table 3-1 for Construction of an Energy Portfolio.

Desired 14% Portfolio Return

ALTERNATIVE 5					ALTERNATIVE 6				
WEIGHTS W(s)	RANKING POSITION	SYMBOL	EXPECTED RETURN	VARIANCE	WEIGHTS W(s)	RANKING POSITION	SYMBOL	EXPECTED RETURN	VARIANCE
W_1	8	TX	9.71	385.826	W_1	2.	XON	16.6	225.842
W_2	6	TGT	13.19	549.547	W_2	5	P	14.36	449.148
W_3	4	MOB	14.54	573.854	W_3	6	TGT	13.19	549.547
W_4	1	SOH	20.91	1332.866	W_4	3	PZL	15.31	562.428

SOLUTION

OPTIMAL WEIGHTS (Alternative 5)		OPTIMAL WEIGHTS (Alternative 6)	
W_1 = 0.227165	V_p = 452.1204	W_1 = 0.130170	V_p = ---
W_2 = 0.517987	S_p = 21.2631	W_2 = 0.489200	S_p = ---
W_3 = 0.057596		W_3 = 0.477915	
W_4 = 0.197251	V_{ep} = 507.274112	W_4 = (0.097286)	V_{ep} = ---
	S_{ep} = 22.522746		S_{ep} = ---
W_t = 1.000000		W_t = ---	

WHERE:

V_p = Variance portfolio

S_p = Standard deviation portfolio

V_{ep} = Variance Portfolio (using equal weights/securities 0.25000)

S_{ep} = Standard deviation portfolio (using equal weights/securities)

TABLE 3 - 7

Energy Portfolio Alternative (1), Allocation of W's at different Desired Portfolio Rates of Return

		Desired Portfolio Rate-Return			
		12%	13%	13.5%	13.96%
		Optimal Weights			
W_1	=	0.213868	0.396308	0.487528	0.571451
W_2	=	0.585150	0.470891	0.413761	0.361203
W_3	=	0.100027	0.83153	0.074716	0.066954
W_4	=	0.100953	0.049646	0.023993	0.000392
W_t	=	1.000000	1.000000	1.000000	1.000000
V_p	=	295.476131	270.488882	259.781578	250.9826
S_p	=	17.189419	16.446546	16.117741	15.842431
V_{ep}	=	278.066331	278.066331	278.066371	278.066371
S_{ep}	=	16.675321	16.675321	16.675321	16.675321

Where:

$W_{(s)}$ = Securities in the portfolio

W_t = Total funds investable in the portfolio

V_p = Variance of returns in the portfolio

S_p = Standard deviation of returns in the portfolio

V_{ep} = Variance of returns in the portfolio with equal weights (0.25000) in the portfolio

S_{ep} = Standard deviation of returns in the portfolio with equal weights (0.250000).

values, when the desired portfolio rate of return (E) is increased from 12% to 13.96%. This illustration is for the same set of equations as in table 3-3, for alternative (1), but E, the investor desired rate of return (14%) has been replaced by the range of returns mentioned above. In this table only the final values of W(s) are given for optimal selection at minimal risk. Also, table 3-7 shows that at these levels of desired rate of return, all W(s) are positive; no "disinvestment" is required.

Table 3-8 and table 3-9 are complete illustrations of the procedure for the solutions of the equation set from table 3-3 using the method of symmetric single division (Appendix B, table B-2 and B-3, respectively). But the assumed desired portfolio rate of return is 13.5%, in order to obtain positive values for all W(s) and to illustrate one complete sample alternative. Since the amount to be invested is assumed to be $10,000, then the W values from table 3-7 are multiplied by $10,000 as follows:

		DESIRED RATE OF RETURN			
		12%	13%	13.5%	13.96%
		INDIVIDUAL FUNDS ALLOCABLE*			
W_1	=	$2139.00	$3963.00	$4875.00	$5714.00
W_2	=	5852.00	4709.00	4138.00	3612.00
W_3	=	1000.00	832.00	747.00	670.00
W_4	=	1009.00	496.00	240.00	4.00
W_t	=	10000.00	10000.00	10000.00	10000.00

TABLE 3 - 8

Output from the Procedure used to Solve Linear Equations in the Energy Portfolio

W_1	W_2	W_3	$\frac{1}{2}\tau_1$	$\frac{1}{2}\tau_2$	W_4	RHS
225.842	231.555	214.743	16.6	1	269.948	0
231.555	385.826	261.665	9.71	1	251.119	0
214.743	261.665	449.1483	14.36	1	190.319	0
16.6	9.71	14.36	0	0	13.19	13.5
1	1	1	0	0	1	1
269.948	251.119	190.319	13.19	1	549.547	0
1.000000	1.025296	0.950855	0.073502	0.004427	1.195295	0
0	148.413481	41.489765	(7.309921)	(0.025296)	(25.657724)	0
0	41.489765	244.958839	(1.424193)	0.049144	(66.362411)	0
0	(7.309921)	(1.424193)	(1.220145)	(0.073502)	(6.651910)	13.5
0	(0.025296)	0.049144	(0.073502)	(0.004427)	(0.195295)	1
0	(25.657725)	(66.362411)	(6.651910)	(0.195295)	226.879282	0
	1.000000	0.279555	(0.049253)	(0.000170)	(0.172880)	0
	0	233.360158	0.619333	0.056216	(59.189660)	0
	0	0.619333	(1.580186)	(0.074748)	(7.915649)	13.5
	0	0.056216	(0.074748)	(0.004432)	(0.199669)	1
	0	(59.189660)	(7.915649)	(0.199669)	222.443574	0
		1.000000	0.002653	0.000240	(0.253640)	0
		0	(1.581829)	(0.074897)	(7.758561)	13.5
		0	(0.074897)	(0.004445)	(0.185410)	1
		0	(7.758561)	(0.185410)	207.430661	0
			1.000000	0.047348	4.904801	(8.534420)
			0	(0.000899)	.181948	0.360790
			0	.181948	245.484869	(66.214825)
				1.000000	(202.300433)	(401.145626)
				0	282.293217	6.773193
					1	0.023993

Note: This procedure is explained in Appendix B and Table B-2
Derived from Table 3-3, ( ) are negative values

TABLE 3 - 9

Optimal Weights for Blending Together Securities (Alternative 1) in the Energy Portfolio

W_4 = $\frac{6.773193}{282.293217}$ = 0.023993

$\frac{1}{2} l_2$ = -401.145626 - (202.300433 * 0.023993) = -296.291737

$\frac{1}{2} l_1$ = -8.534420 - (0.047348 * -396.291737) - (4.904801 * 0.023993) = 10.111863

W_3 = 0 - (0.002653 * 10.111863) - (0.000240 * -396.291737) - (-.253640 * 0.023993) = 0.074716

W_2 = 0 - (0.279555 * 0.074716) - (-0.049253 * 10.111863) - (-0.000170 * -396.291737) - (-.172880 * 0.023993) =

0.413761

W_1 = 0 - (1.025296 * 0.413761) - (0.950855 * 0.074716) - (0.073502 * 10.111863) - (0.004427 * -396.291737) -

(1.195295 * 0.023993) = 0.487528

W_t = $W_1 + W_2 + W_3 + W_4$

W_t = 1.000000

R_t = 13.5 = 16.6 * 0.487528 + 9.71 * 0.413761 + 14.36 * 0.074716 + 13.19 * 0.023993

Note: The procedure is explained in Appendix B, also see Table B-3
Derived from Table 3-8

Portfolio Variance

The next step is to determine the variance of the portfolio. (Continuing to use alternative 1. See equation 3, Appendix A)

$$V_p = W_1^2 \sigma_1^2 + W_2^2 \sigma_2^2 + W_3^2 \sigma_3^2 + W_4^2 \sigma_4^2 + 2W_1 W_2 \sigma_{12} +$$

$$2W_1 W_3 \sigma_{13} + 2W_1 W_4 \sigma_{14} + 2W_2 W_3 \sigma_{23} + 2W_2 W_4 \sigma_{24} + 2W_3 W_4 \sigma_{34}$$

Next substitute the values: σ_{12} through σ_{34} and W_1 through W_4 from tables 3-2 and 3-9 (alternative #1) as shown in table 3-7, (Vp).

Risk is a consequence of the fact that investment alternatives are governed by exogenous factors beyond one's control. Markets, securities and technologies change. It is not possible to associate a particular rate of return with any given investment alternative, but rather with a range of possible rates of return. Reviewing the basics and all alternative options (1 through 6) in this chapter, alternative (2), offers the lowest risk at an expected rate of return of 14% without disinvestment. (See table 3-4, Vp = 320.35). However alternative 1, at 13.96% yield, and without "disinvestment", has a variance Vp = 250.98 (see table 3-7). It is thus only 0.04% lower in desired portfolio rate of return than alternative 2, but has the advantage of being 21% less risky. Even if the portfolio desired rate of return for alternative (2) is lowered to 13.96%, it will still be 21% riskier than alternative (1) (the calculations are not included in this chapter). The proportions of individual securities in alternative (1) at 13.96% (see page 59) are: W_1 = \$5714.00, W_2 = \$3612.00, W_3 = \$670 and W_4 = \$4.00. This last security proportion, W_4, is so low that it is not possible to buy a single share of this investment. The share price for W_4 (Tenneco Inc.) is \$40.824 (see table C-6, price/share 1985). If the desired portfolio return for alternative 1 is lowered to 13.5%, the proportions of individual securities are listed on page 59 and shown in figure 3-11. It is then possible to buy a couple shares of every individual security (see table C-2, Exxon price/share 1985 is 49.74, W_1, table C-8, Texaco price/share 1985 is 33.726, W_2, table C-5, Phillips Petroleum price/share 1985 is 12.587, W_3 and table C-6, Tenneco price/share 1985 is 40.824). At the same time, it is also reasonable to select alternative (1) at 13.5% instead of alternative (2) at a 14% yield, with only a half percent higher rate of return but 20% more risk. The portfolio variance of alternative (1) at 13.5% is 259.781578 (see table 3-7). While the portfolio variance of alternative (2) at 14% is 329.085689 (see table 3-4). The proper choice of an efficient portfolio depends on the willingness and ability of the investor to assume risk. If

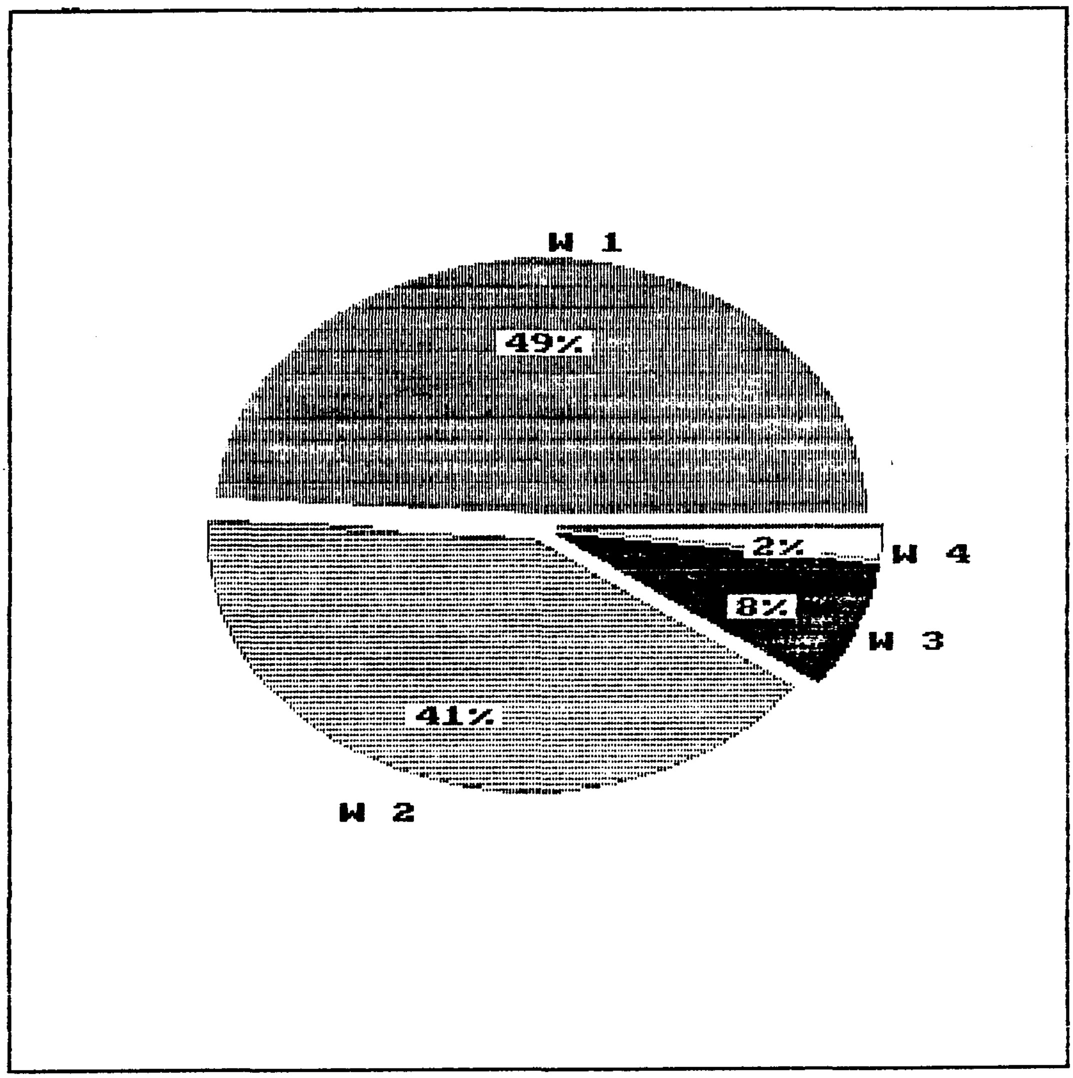

Fig. 3-11 Individual Allocation of Funds/Energy Portfolio

safety is of extreme importance, "likely return" must be sacrificed to decrease uncertainty.

Referring back to figure 3-10, point P represents the investor's required rate of return of 13.5% and a standard deviation of returns of 16.11, as noted earlier (page 59), with $4875.00 of Exxon Corporation, $4138.00 of Texaco, $747.00 of Phillips Petroleum and $240.00 of Tenneco invested ($10,000).

Portfolio P, has both higher average and a lower standard deviation than security 6. Security 2 had a still higher average and lower standard deviation than portfolio P. Since the objective is to rely exclusively on stock selection, there is a need to control the risk associated with the market and group components of return. To control the risk associated with major groups it would be necessary to ensure that the portfolio is well-diversified.

In figure 3-12 the average return for the securities selected are plotted (Exxon, Texaco, Phillips Petroleum, and Tenneco), and the portfolio curve for the mixture (table 3-7) of 0.4875% of Exxon, times the average return/year plus 0.4138% of Texaco, times the average return/year plus 0.747% of Phillips Petroleum, times the average return/year plus 0.24% of Tenneco, times the average return/year. The average portfolio rate of return is 13.5%, the desired rate of return for this investment. This portfolio selection is based on a reasonable

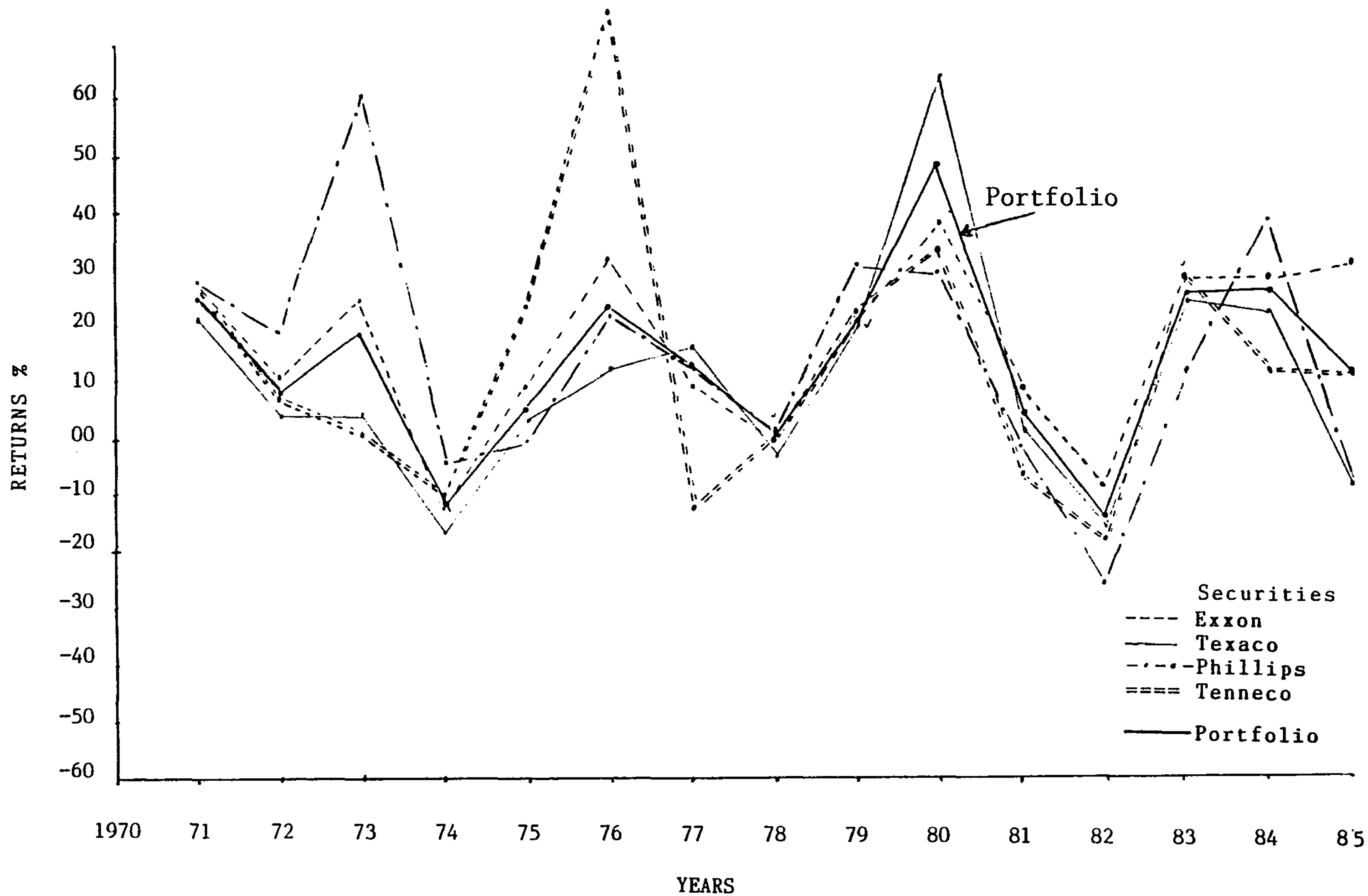

Fig. 3-12 Return of Securities and a Energy Portfolio

expectation about the future, based on past performance, and assumes that the above mix of securities with their respective weights should be maintained, to obtain the same rate of return (13.5%) in the next period, 1986. The reason for plotting the aforementioned portfolio curve is to compare its variability by year, against each one of the securities included in it, for the period analyzed (1970-1985). It is inevitable that the average return on a portfolio lies somewhere between the highest and the lowest average return on the securities contained in the portfolio.

The variance of the portfolio (Vep) with stocks weighted according to alternative 1 (table 3-7) at 13.5% rate of return in this chapter, is 278.06 and the standard deviation is 16.67. These values compare favorably with those of table 2-1.

Table 3-10 shows the correlations among securities in alternative (1). These values measure the extent to which stocks move together and go from +1 to -1 with a midpoint of zero. In the case of alternative (1) the co-movement between the stocks is highly positive. In fact, correlation coefficient values higher than 0.5 are considered high positive correlation. The higher the correlations among security returns, the greater is the standard deviation of the portfolio as a whole.

TABLE 3-10

Energy Portfolio Correlation Coefficients: Alternative 1 (derived from Tables 3-1 and 3-2)

$\rho_{1,2} = 0.7845$	$\rho_{2,3} = 0.6287$
$\rho_{1,3} = 0.6749$	$\rho_{2,4} = 0.5454$
$\rho_{1,4} = 0.7663$	$\rho_{3,4} = 0.3831$

$$\rho_{ij} = \frac{Cov_{ij}}{\sigma_i \sigma_j}$$

Where:

ρ_{ji} = Correlation coefficient for securities ij

Cov_{ij} = Covariance between securities ij

σ_i = Standard deviation for security i

σ_j = Standard deviation for security j

1.	Exxon Corporation	XON
2.	Texaco, Inc.	TX
3.	Phillips Petroleum	P
4.	Tenneco, Inc.	TGT

A brief description of the activities of the companies whose stocks were selected to form an energy investment portfolio follows:

Exxon

Exxon[2] is the world's largest oil company. In 1984, 58% of exploration and production earnings came from foreign sources. About one-half of daily production comes from the Middle East and Africa. Proved developed crude oil reserves were 3,924 million barrels as of 31 December 1984. Sales (and earnings) were derived as follows: petroleum, 93.8% (90.4%); chemicals, 8.7 (7.7%) and other, -2.5% (1.9%).

Net income from fiscal 1985 fell 12% to $4.87 billion due to poor results from chemical operations and excess capacity which depressed margins. Sales were $93.21 billion, down 4%. For the 4th quarter, net income totaled $1.81 billion, up 16% due to stronger earnings in domestic and foreign petroleum and natural gas operations.

Downstream operations continue to be pressured by low demand and overcapacity. Upstream operations are pressured by excess supply. Long term prospects are enhanced by a commitment to increase oil and natural gas

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2. Summary condensed from Moody's Handbook, Spring 1986

reserves. Figure 3-13 shows the trading volume of Exxon shares, 1970 - 1986. The common stock had a split of 2 for 1 in 1976 and 1981. Table C-2 shows the impact on both risk and return of the business cycle 1970 - 1985).

Texaco

Texaco's[3] activities focus on worldwide production, exploration, transportation, refining and marketing of crude oil and its products. The Texaco Chemical Company markets over 500 petrochemical products worldwide. Its net income for 1985 was about $1.23 billion. Its earning reflected a $114 million gain from the sale of a partial interest in Texaco Canada. Total sales were estimated $47.50 billion.

Short-term sales and earnings are adversely affected by declining crude oil and gas prices. Over the long-term, the prospect is for improvement in industry conditions and an emphasis on exploration. Figure 3-14 shows the trading volume of Texaco shares from 1972 to 1986. This figure does not show a stock split during this period. Table C-8 shows the impact on both risk and return of the business cycle (1970 - 1985).

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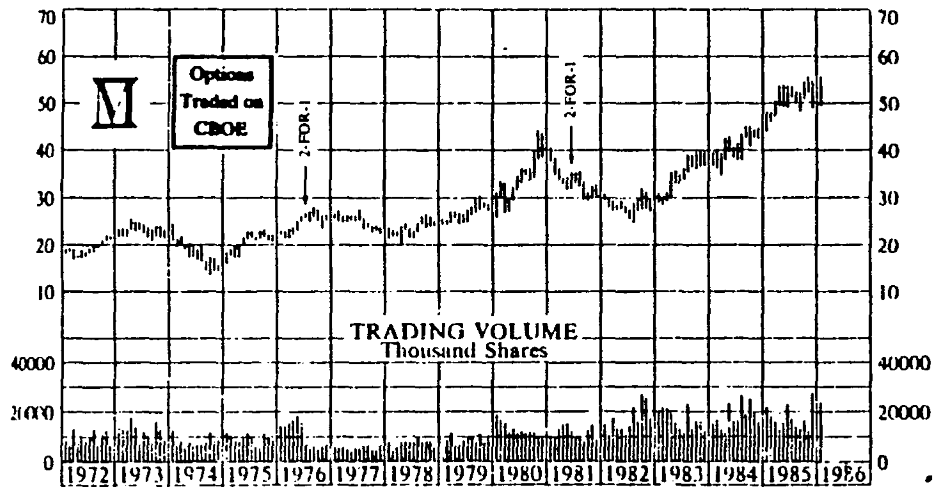
3. Summary condensed from Moody's Handbook, Spring 1986

EXXON CORPORATION

LISTED	SYM.	LTPS•	STPS•	IND. DIV.	REC. PRICE	RANGE (52-WKS.)	YLD.
NYSE	XON	138.0	93.4	$3.60*	51	56 - 45	7.1%

HIGH GRADE. OPERATIONS REFLECT WORLD OIL PRICING AND POLITICAL CONDITIONS. DIVIDENDS HAVE BEEN RISING.



CAPITALIZATION: (12/31/84)

	(000)	(%)
Long-Term Debt	$ 5,105,000	11.5
Def. Inc Tax	8,948,000	20.3
Minority Int.	1,262,000	2.9
Com. & Surp.	28,851,000	65.3
Total	$44,166,000	100.0

Shs. (np)-783,000,000

INTERIM EARNINGS:

Qtr.	3/31	6/30	9/30	12/31
1982	1.43	1.02	1.23	1.71
1983	1.22	1.25	1.41	1.91
1984	1.75	1.63	1.58	1.81
1985a	1.71	0.99	1.33	2.43

INTERIM DIVIDENDS:

Amt.	Dec.	Ex.	Rec.	Pay.
0.85Q	1/30/85	2/4/85	2/8/85	3/11/85
0.85Q	4/24	5/7	5/13	6/10
0.85Q	7/31	8/7	8/13	9/10
0.90Q	10/30	11/5	11/12	12/10
0.90Q	1/29/86	2/[illegible]	2/10/86	3/10/86

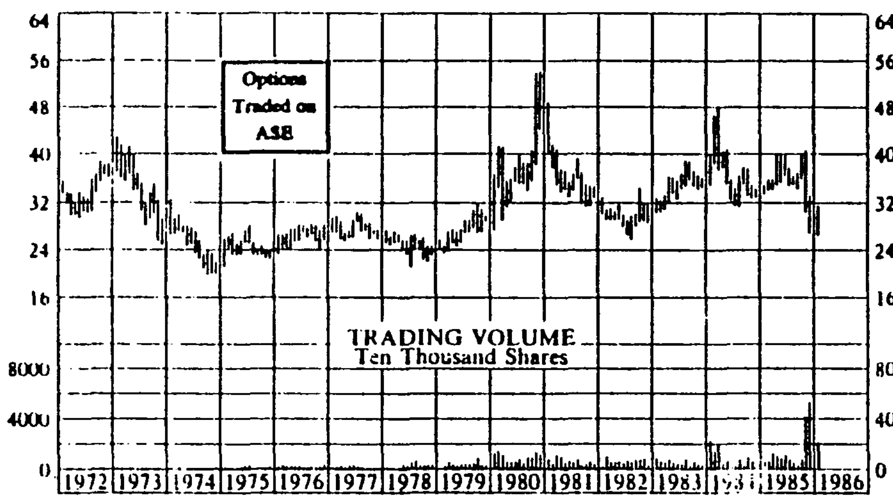
Figure 3-13 Exxon Corporation, Trading Volume Shares.

SOURCE: Moody's Handbook, Spring 1986

TEXACO INC.

LISTED	SYM.	LTPS•	STPS•	IND. DIV.	REC. PRICE	RANGE (52-WKS.)	YLD.
NYSE	TX	100.0	77.0	$3.00*	28	40 - 27	10.7%

UPPER MEDIUM GRADE. AGGRESSIVE EXPLORATION AND PRODUCTION ACTIVITIES AUGURS WELL FOR GROWTH.



CAPITALIZATION:(12/31/84)

	(000)	(%)
Long-Term Debt	$ 10,489,000	39.3
Cap. Lease Oblig.	726,000	2.7
Minority Int.	212,000	0.8
Defer. Inc. Tax	2,160,000	8.1
Com. & Surp.	13,107,000	49.1
Total	$26,694,000	100.0

Shs. ($6.25)-237,667,621

INTERIM EARNINGS:

Qtr.	3/31	6/30	9/30	12/31
1982	1.41	1.17	1.16	1.18
1983	1.24	1.28	1.28	1.00
1984	1.25	1.22	0.91	d2.40
1985e	1.28	1.28	1.26	1.29

INTERIM DIVIDENDS:

Amt.	Dec.	Ex.	Rec.	Pay.
0.75Q	4/26/85	5/1/85	5/7/85	6/10/85
0.75Q	7/26	7/31	8/6	9/10
0.75Q	10/24	10/29	11/4	12/10
0.75Q	1/24/86	1/29/86	2/4/86	3/10/86

Figure 3-14 Texaco Inc., Trading Volume Shares.

Source: Moody's Handbook, Spring]986

Phillips Petroleum[4]

Phillips Petroleum is a large integrated company with operations in 29 countries. The Natural Resources Group discovers and produces crude oil, natural gas and natural gas liquids. The Petroleum Products Group refines, transports and markets petroleum products. Also, the group operates pipelines, a marine shipping business and other product distribution facilities. The Chemical Group manufactures and markets petroleum-based chemical products. As of 31 December 1984, proved developed reserves were: crude oil 714 million barrels; natural gas liquids 179 million barrels, and natural gas, 5734 billion cubic ft.

Income for 1985 plunged from $837 million to $596.00, including foreign currency losses, sales were flat at $15.80 billion.

The refining segment will have decreased margins as the price of crude oil drops. Aggresive sales of assets totaling $2.0 billion in 1986 will help the company reduce its debt from $8.6 billion in 1985 to between $5.5 billion and 6.0 billion. The company's refining and marketing are expected to be strong contributors to earnings. Figure 3-15 shows the trading volume of Phillips Petroleum shares

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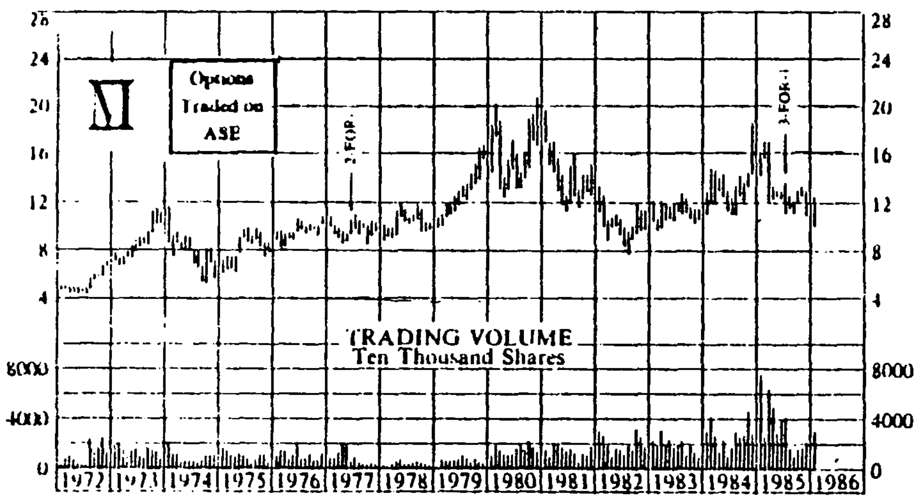
4. Summary condensed from Moody's Handbook Spring 1986

PHILLIPS PETROLEUM COMPANY

LISTED	SYM.	LTPS•	STPS•	IND. DIV.	REC. PRICE	RANGE (52-WKS.)	YLD.
NYSE	P	95.4	83.4	$1.00•	11	17 - 10	9.1%

MEDIUM GRADE. NEW SOURCES OF CRUDE OIL, PARTICULARLY FROM OFFSHORE CALIFORNIA ENHANCE COMPANY'S LONG-TERM OUTLOOK.



CAPITALIZATION: (12/31/85)

	(000)	(%)
Long-Term Debt	$ 6,547,000	64.3
Defer. Inc. Tax	1,673,000	16.4
Minority Int.	36,000	0.3
Preferred	281,000	2.8
Com. & Surp.	1,644,000	16.2
Total	$10,181,000	100.0

Shs ($2.50)-227,141,000

INTERIM EARNINGS:

Qtr.	3/31	6/30	9/30	12/31
1982	0.41	0.31	0.29	0.34
1983	0.28	0.37	0.38	0.54
1984	0.42	0.50	0.47	0.37
1985f	0.25	0.52	0.38	0.39

INTERIM DIVIDENDS:

Amt.	Dec.	Ex.	Rec.	Pay.
3-for-1	...	7/5	5/30	7/3
0.25Q	7/8	7/29	8/2	9/3
0.25Q	10/14	10/28	11/1	12/2
0.25Q	1/13/86	1/27/86	1/31/86	3/1/86

Figure 3-15 Phillips Petroleum Company, Trading Volume Shares.

SOURCE: Moody's Handbook, Spring 1986

from 1972 to 1986. The common stock had a split of 2 for 1 in 1977 and 3 for 1 in 1985. Table C-5 shows the impact on both risk and return of the business cycle (1970 - 1985).

Tenneco Inc.

Tenneco's[5] activities are those of a widely diversified industrial complex. Revenues in 1984 were distributed as follows: oil exploration and production, 15%; natural gas pipelines, 29%; oil and petrochemicals, 18%; processing and marketing, 31%; natural resources, related business and other, 13%. In 1985, income declined 26% due primarily to weakness in the demand for natural gas, plant shut-downs, and a lower level of interest capitalization.

Short term earnings showed some improvement despite a weak market for oil and natural gas which reduced sales. Tenneco will try to increase sales by expanding the Chalorette refineary capacity, producing higher value products and by reducing its operating costs. Figure 3-16 shows the trading volume of shares from 1972 to 1985. This figure does not show any stock split during this period. Table C-6 shows the impact on both risk and return of the business cycle (1970 - 1985).

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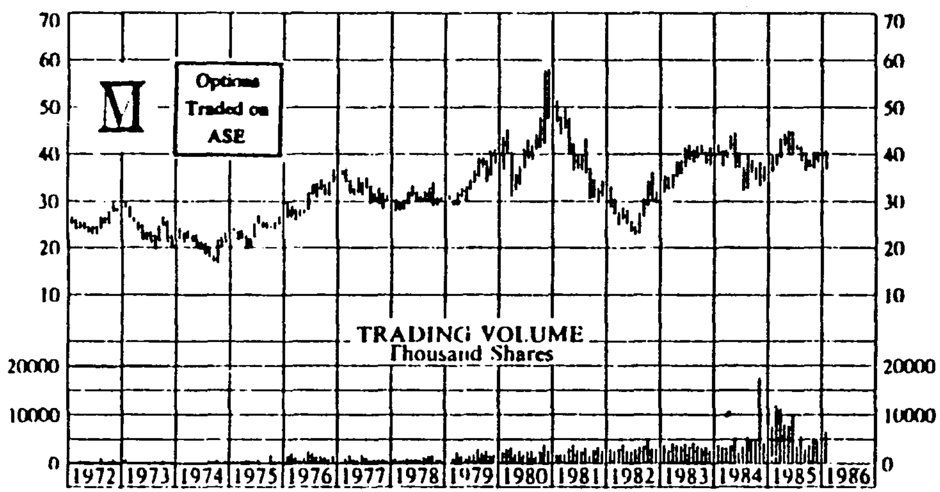
5. Summary condensed from Moody's Handbook Spring 1986

TENNECO, INC.

LISTED	SYM.	LTPS•	STPS•	IND. DIV.	REC. PRICE	RANGE (52-WKS.)	YLD.
NYSE	TGT	104.4	89.0	$3.04*	38	45 - 37	8.0%

UPPER MEDIUM GRADE. DIVERSIFICATION HAS DECREASED SENSITIVITY TO BUSINESS CYCLES.



CAPITALIZATION: (12/31/84)

	(000)	(%)
Long-Term Debt	$ 5,069,000	35.0
Pfd.	572,000	3.9
Defer Inc. Tax	2,704,000	18.7
Com. & Surp.	6,153,000	42.4
Total	$14,498,000	100.0

Shs. (55)-145,597,875

INTERIM EARNINGS:

Qtr.	3/31	6/30	9/30	12/31
1982a	0.88	1.66	1.38	1.96
1983	0.70	1.38	1.04	1.63
1984b	1.00	1.46	0.76	0.80
1985c	0.34	1.03	0.62	0.53

INTERIM DIVIDENDS:

Amt.	Dec.	Ex.	Rec.	Pay.
0.73Q	1/23/85	2/4/85	2/8/85	3/12/85
0.73Q	4/26	5/6	5/10	5/11
0.73Q	7/23	8/5	8/9	9/10
0.76Q	10/19	11/4	11/8	12/10
0.76Q	1/14/86	2/3/86	2/7/86	3/11/86

Figure 3-16 Tenneco Inc., Trading Volume Shares.

SOURCE: Moody's Handbook, Spring 1986

CHAPTER IV

GOLD-MINERALS PORTFOLIO

Portfolio investment management emphasizes that the objective of investment is to have the largest possible pool of assets to finance one's expenditures, either now or later, and that the search for income and for capital gain are only variations of strategic issues of maximizing return and minimizing risk. Thus, all portfolios share this objective, but since the future is uncertain no businessman is able to predict exactly what the value of the assets will be over time. He knows even less about their purchasing power. The degree of risk varies in each case, based on time horizons within which one has to work.

For most investment decisions, the major uncertainty is the probable volatility in the price of the asset and, in particular, its volatility related to the prices of all similar types of assets.

The purpose of this chapter is to form a gold-minerals portfolio from a selected sample of securities traded on the NYSE. This chapter presents, a framework for the data input and results of the model in a literal and a mathematical form.

The desired portfolio rate of return from this investment is (14%). It coincidentaly is the same rate of return as the energy portfolio in chapter III. Also, this rate of return (14%) is the mean return obtained by the selected sample of securities (table 4-1) during 16 years (1970 - 1985). It is reasonable to allocate the investment at this level of risk. The future performance of this investment will likely be close to the mean return.

This chapter presents a brief overview of the activities of the companies selected to form the portfolio. First, a few basic facts about the major areas in which these companies are engaged and then a description of the stock selection criteria are given, as well as emphasizing the rate of risk and return for each of them.

A gold-minerals portfolio includes securities of domestic companies engaged in the exploration, mining, fabrication, processing or marketing of gold, silver, copper, aluminum and other minerals.

Investment related to gold and other minerals are generally considerably highly speculative and are impacted by a host of world-wide economic, financial and political factors. The prices of gold and other precious metals may fluctuate sharply over short periods of time, due to changes in inflation or to expectations regarding inflation; to metal sales by governments, central banks, or international agencies; to changes in governmental economics, policies and to governmental prohibitions or restrictions on private ownership.

TABLE 4 - 1

Gold-Minerals Portfolio: Returns, Mean Returns, Variance of returns and Standard Deviation on Nine Securities Listed in Figures 4-1 Through 4-9

		SECURITY								
YEAR	RANKING SECURITY	1 HM	2 IGL	3 AN	4 FLR	5 NEM	6 AA	7 RLM	8 AR	9 PD
1971		33.193	79.019	16.042	4.03	7.863	(8.868)	(18.151)	(15.025)	(10.914)
1972		(1.01)	22.448	58.055	4.46	1.630	(6.62)	(19.893)	(4.029)	8.913
1973		100.329	49.560	24.071	79.24	(2.593)	37.549	(0.767)	12.00	19.049
1974		114.568	39.047	3.693	(6.45)	(.992)	(1.949)	17.393	0.036	(10.449)
1975		79.164	33.939	2.676	19.34	(6.934)	(0.965)	2.382	(16.481)	(2.556)
1976		(57.939)	(0.582)	17.604	30.59	25.696	31.349	72.829	8.982	22.303
1977		15.96	13.072	10.035	3.81	(10.784)	4.508	16.398	14.285	(21.399)
1978		(7.175)	5.149	2.181	(0.82)	(8.042)	(5.834)	(6.743)	(5.257)	(23.018)
1979		21.918	31.388	43.021	63.23	56.323	22.295	11.864	59.644	23.527
1980		115.97	57.056	102.981	145.57	55.388	27.182	7.330	69.602	44.762
1981		26.628	5.22	(4.424)	(5.43)	42.785	(0.232)	(2.447)	(9.126)	13.664
1982		(31.256)	(31.529)	(27.827)	(49.29)	(30.942)	(4.007)	(20.572)	(32.530)	(33.826)
1983		70.516	27.127	14.659	0.21	31.94	45.498	47.964	47.2	8.133
1984		(6.054)	18.587	23.63	(5.64)	(27.510)	7.067	10.534	(23.67)	(27.876)
1985		(15.041)	3.207	15.97	(8.45)	13.745	(9.02)	3.183	9.425	(8.088)
Mean Return		28.37	23.51	20.16	17.63	9.86	9.2	8.09	7.7	0.14
Variance		2794.683	677.211	859.276	2023.67	702.823	317.192	582.241	837.708	--------
Standard Deviation		52.865	26.023	29.313	44.99	26.51	17.81	24.13	28.943	--------

Note: Derived from Appendix D, see table D-1 to D-9

STOCK SELECTION

On the basis of the background and analytical concepts given above with respect to portfolio analysis, the focus here is on stock selection. The main concern is with both risk and return. The return achieved by the individual stocks is principally derived from the unique characteristics of the individual security, its industry affiliation and by broad market sector forces. At all times stock selection should designate those securities whose risk return ratio is most promising to the investors, given conditions at the specific time of his life. This means, that the risks taken are associated with one's level of income. In the present study the amount assumed for investment ($10,000), is subjectively selected.

It was mentioned in chapter III that in all investments the essential principles of portfolio management are identical. Thus, the critical elements that must be taken into consideration for success in the selection strategy in this chapter are the same as those taken in chapter III: first, distinguish between relatively attractive and unattractive individual stocks. Second, use a systematic portfolio construction procedure. Third, rebalance the portfolio over time. The selection strategy here has been predetermined by selecting stocks grouped only in the gold-minerals field. Then, based on the evaluations of their return and risk and a predictive ranking method, the four most relevant stocks (an arbitrary number) were selected from the selected sample of securities, table 4-1, to construct a gold-minerals portfolio.

Table 4-1 shows the annual returns, mean return, and variances of the selected sample of securities in gold-minerals found in Appendix D. In the present chapter it is also assumed that returns on these securities are based on a one-year investment horizon. Appendix D indicates how the expected return of these securities (table 4-1) is calculated. The extent of the variation of these returns from the expected or mean return is also shown. The limitation for selecting stocks from the selected sample of securities in table 4-1 was arbitrary for ease of illustration.

Annual returns for the nine securities are listed in table 4-1 and shown in figures 4-1 to 4-9. The nine securities differ in their average return. For example, the average annual rate of return on Homestake Mining Co. (HM) was 28.37% per dollar invested; that for International Mineral and Chemical stock was 23.51% per dollar invested.

The variations of these returns are listed in table 4-1 and shown in figure 4-10. The horizontal axis represents the standard deviation. Thus the point labelled 1, indicates that security 1 (Homestake Mining Co. common stock) had an average rate of return of 28.57% and a

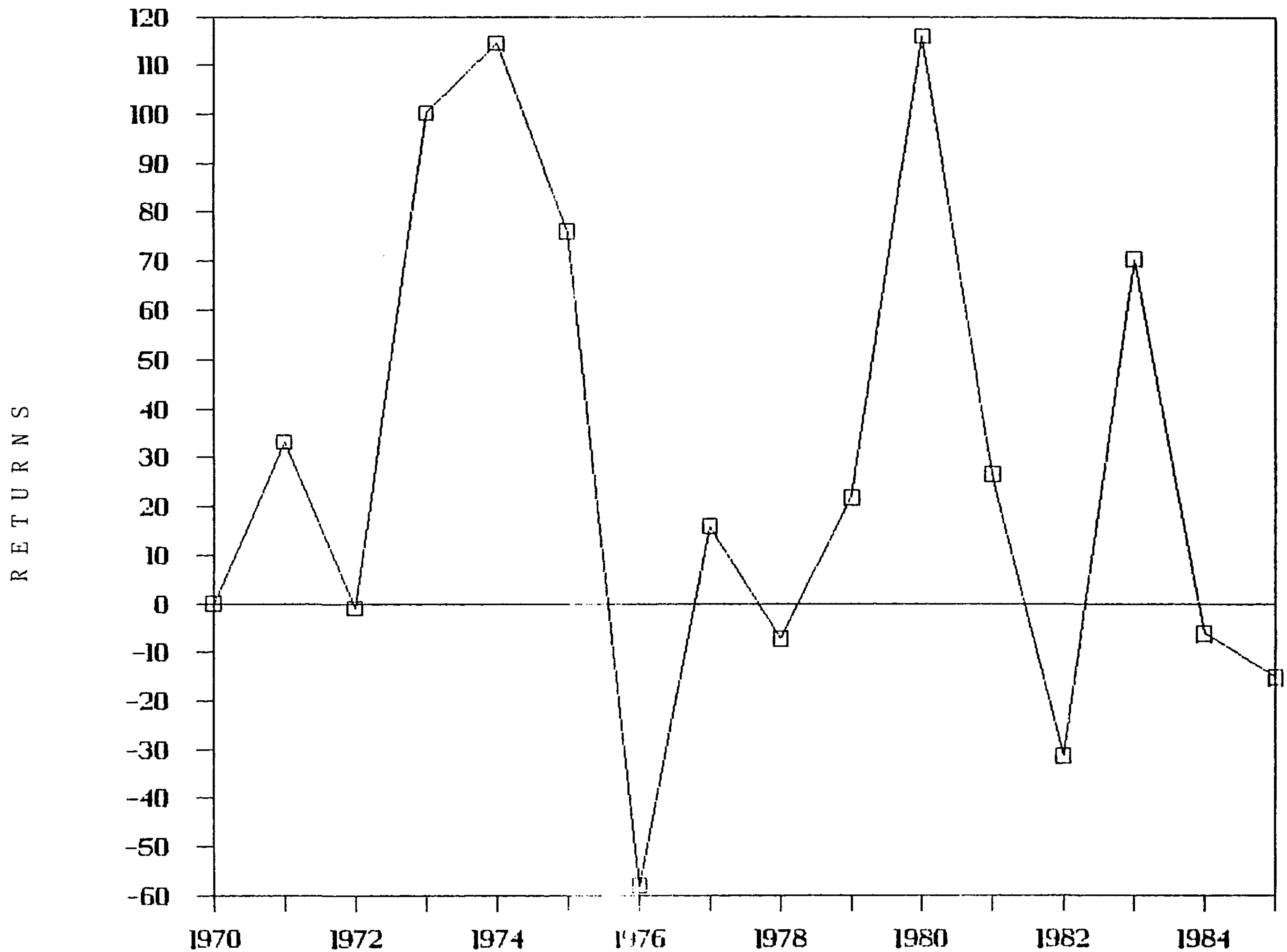


Figure 4-1 Returns on Security 1/Gold-Minerals Portfolio. -- Homestake Mining Company (HM).

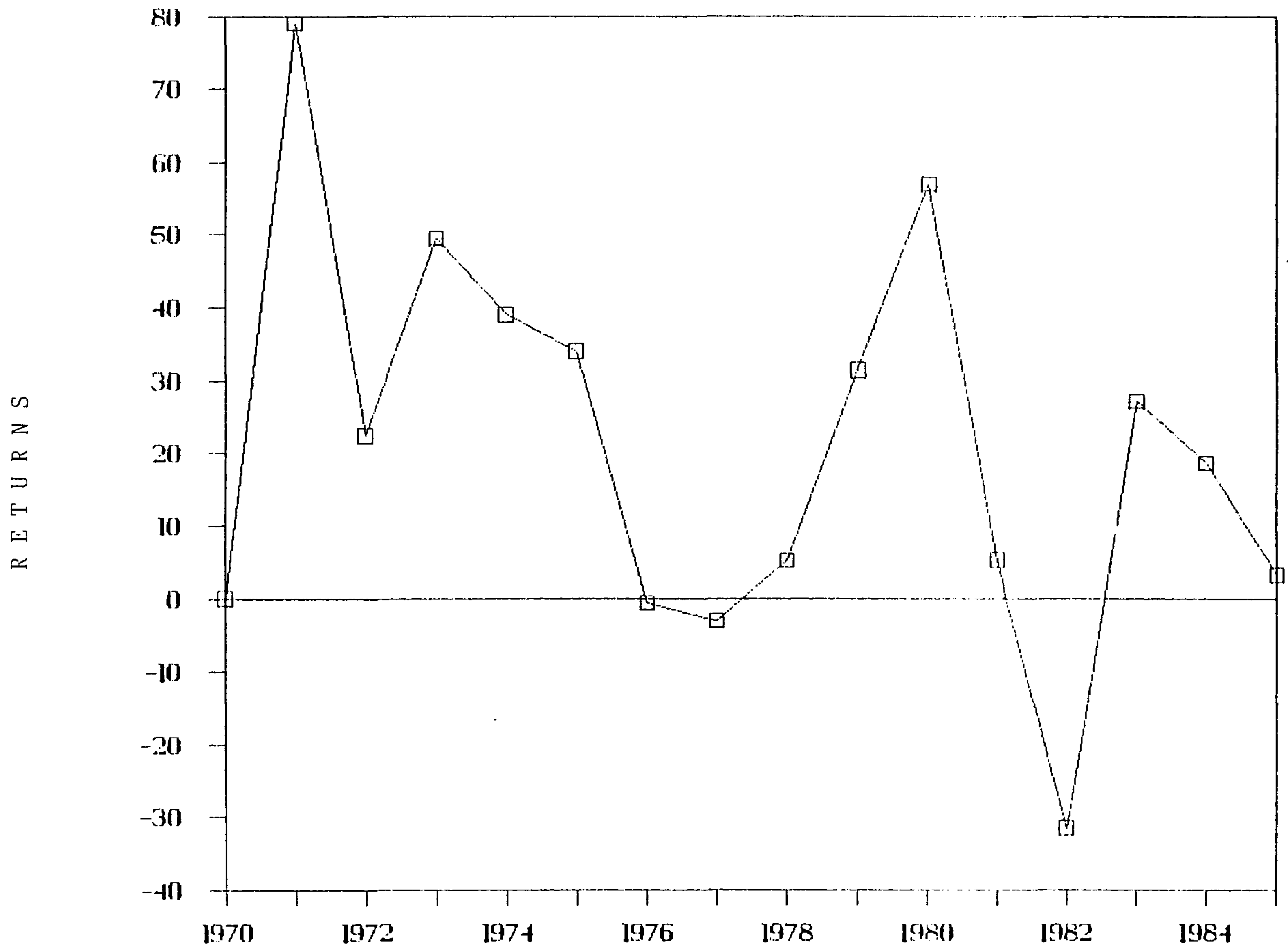


Figure 4-2 Returns on Security 2/Gold-Minerals Portfolio. --International Minerals & Chemical Corporation (IGL).

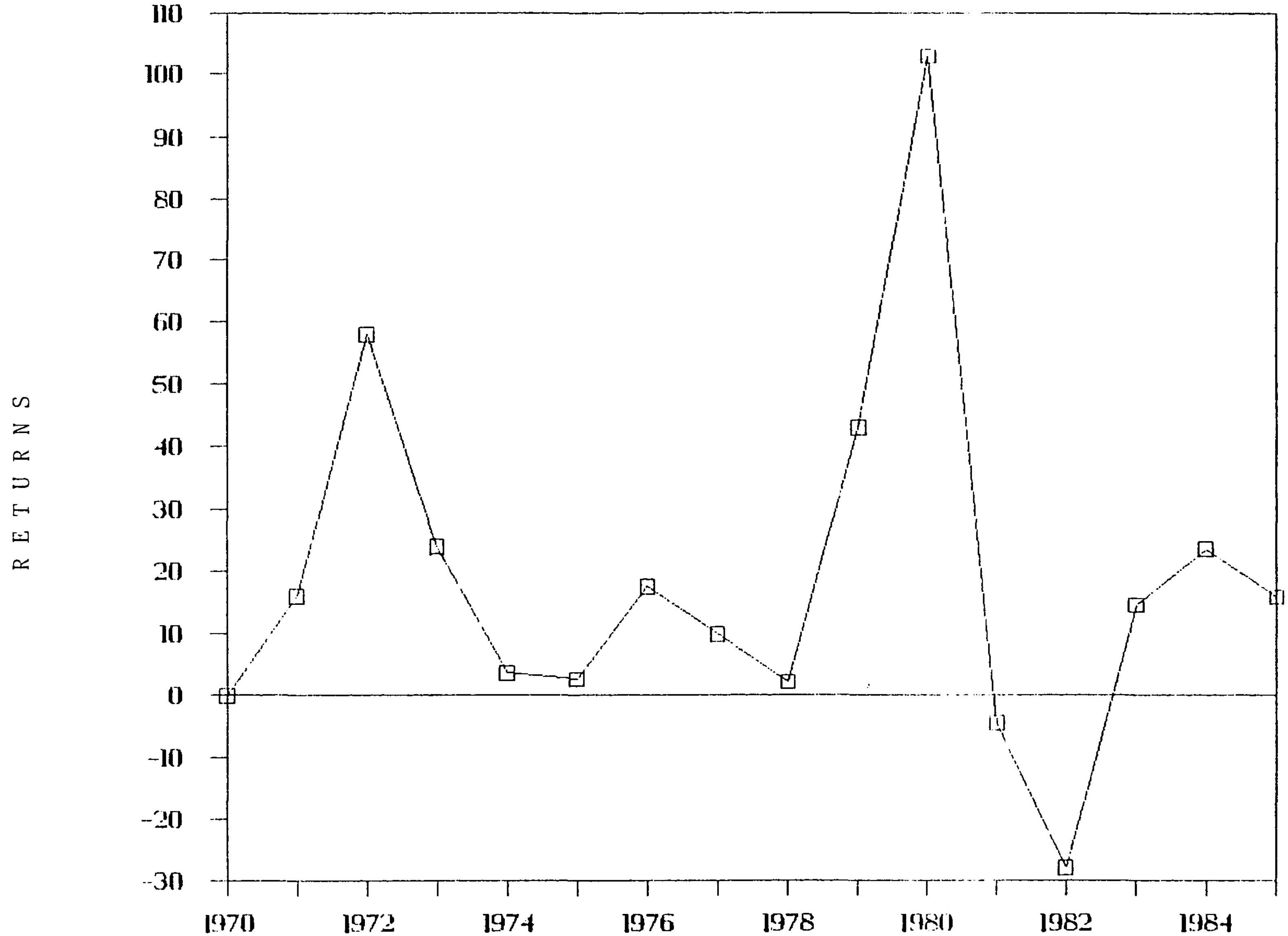


Figure 4-3 Returns on Security 3/Gold-Minerals Portfolio. --Amoco Coproration (AN).

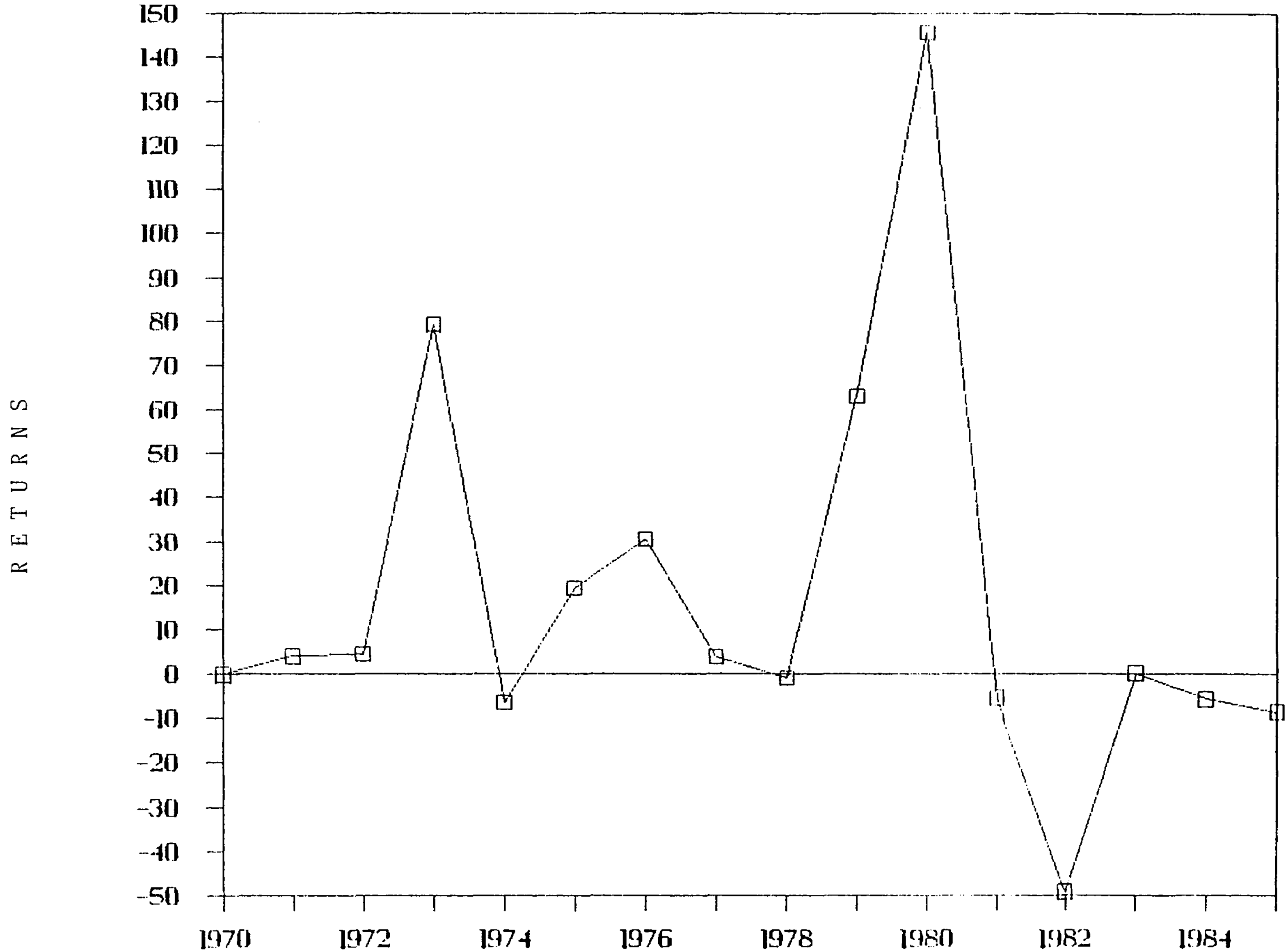


Figure 4-4 Returns on Security 4/Gold-Minerals Portfolio. --Flour Corporation (FLR).

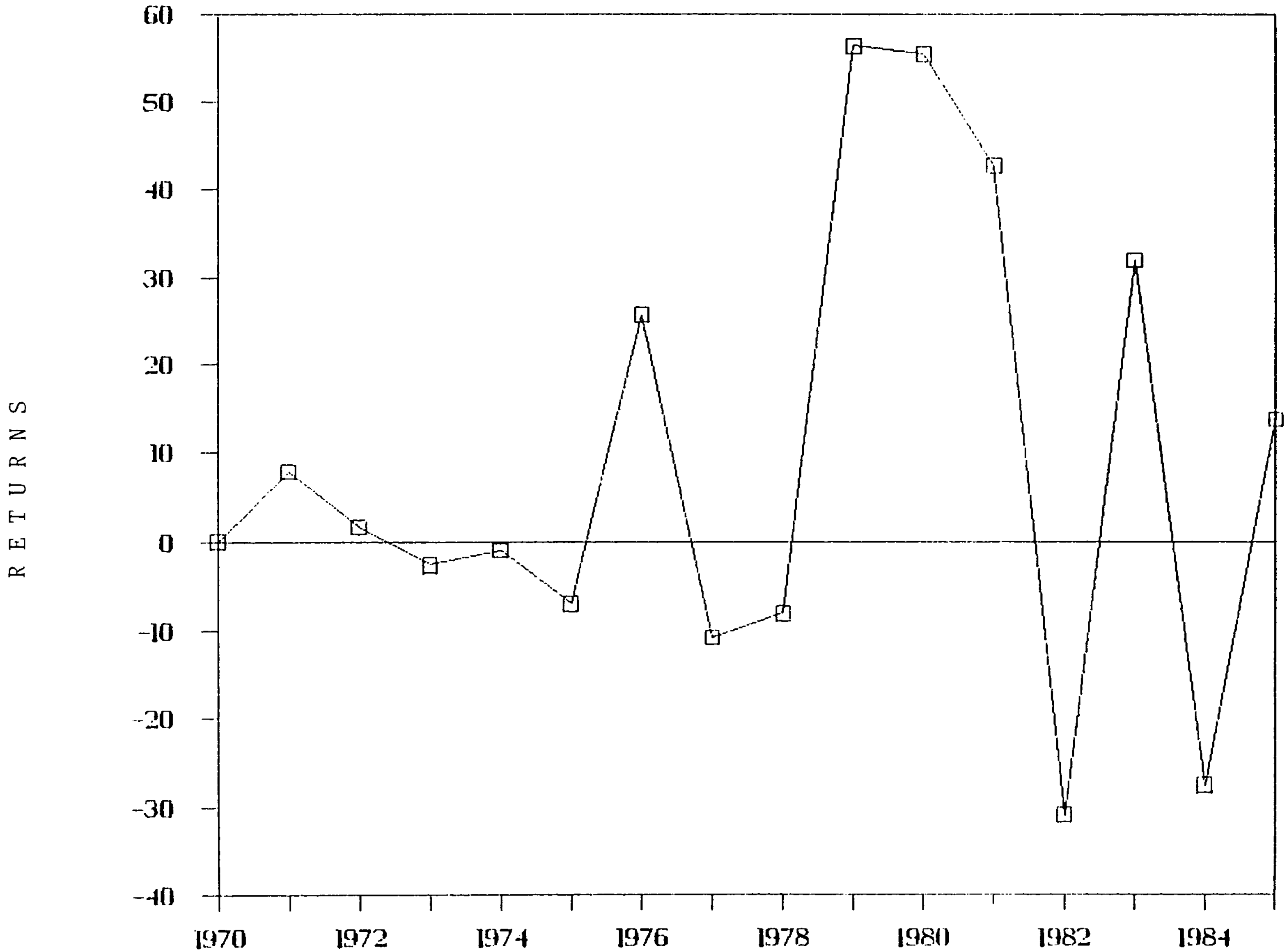


Figure 4-5 Returns on Security 5/Gold-Minerals Portfolio. --Newmont Mining Company (NEM).

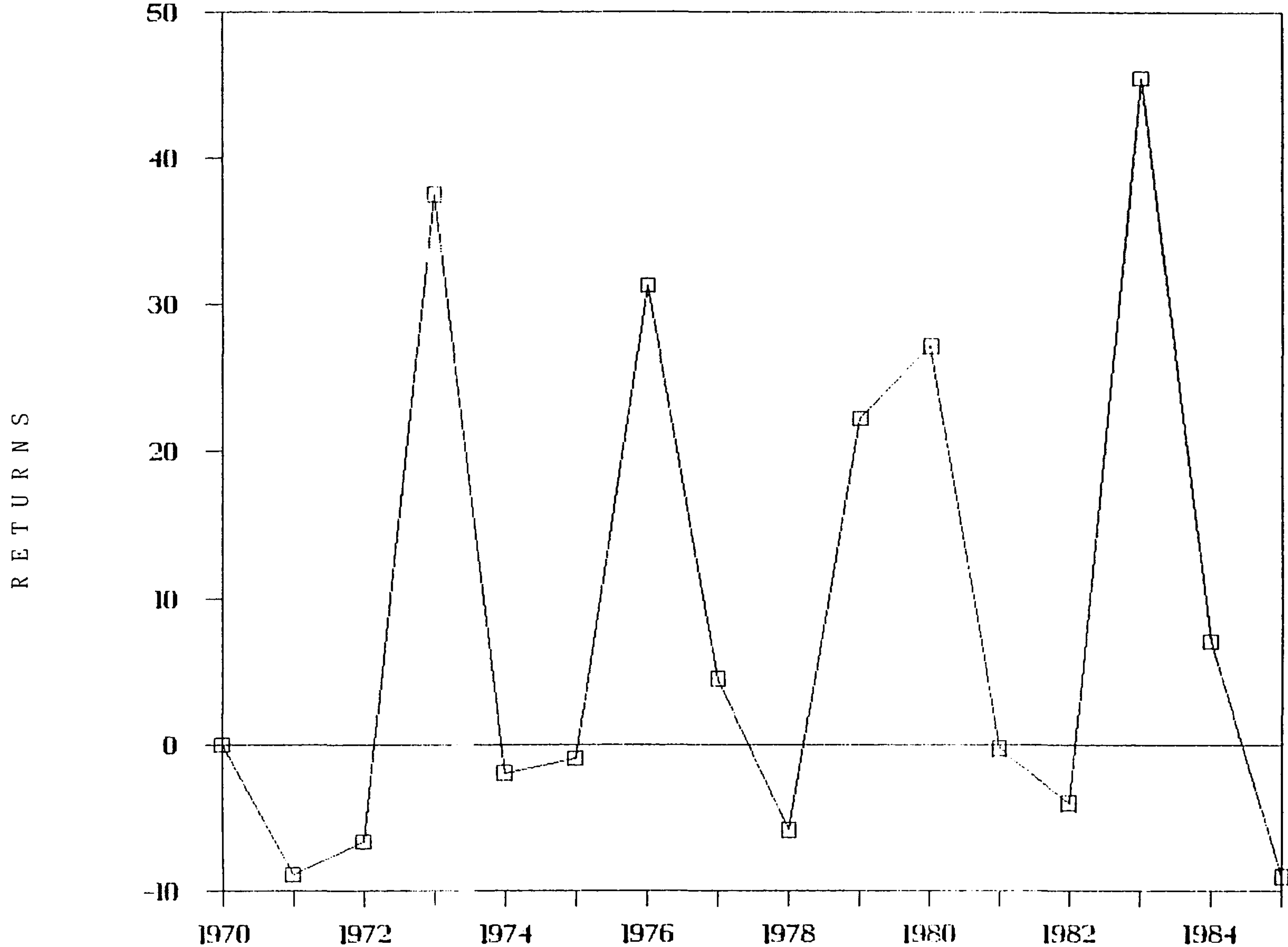


Figure 4-6 Returns on Security 6/Gold-Minerals Portfolio. --Aluminum Company of America (AA).

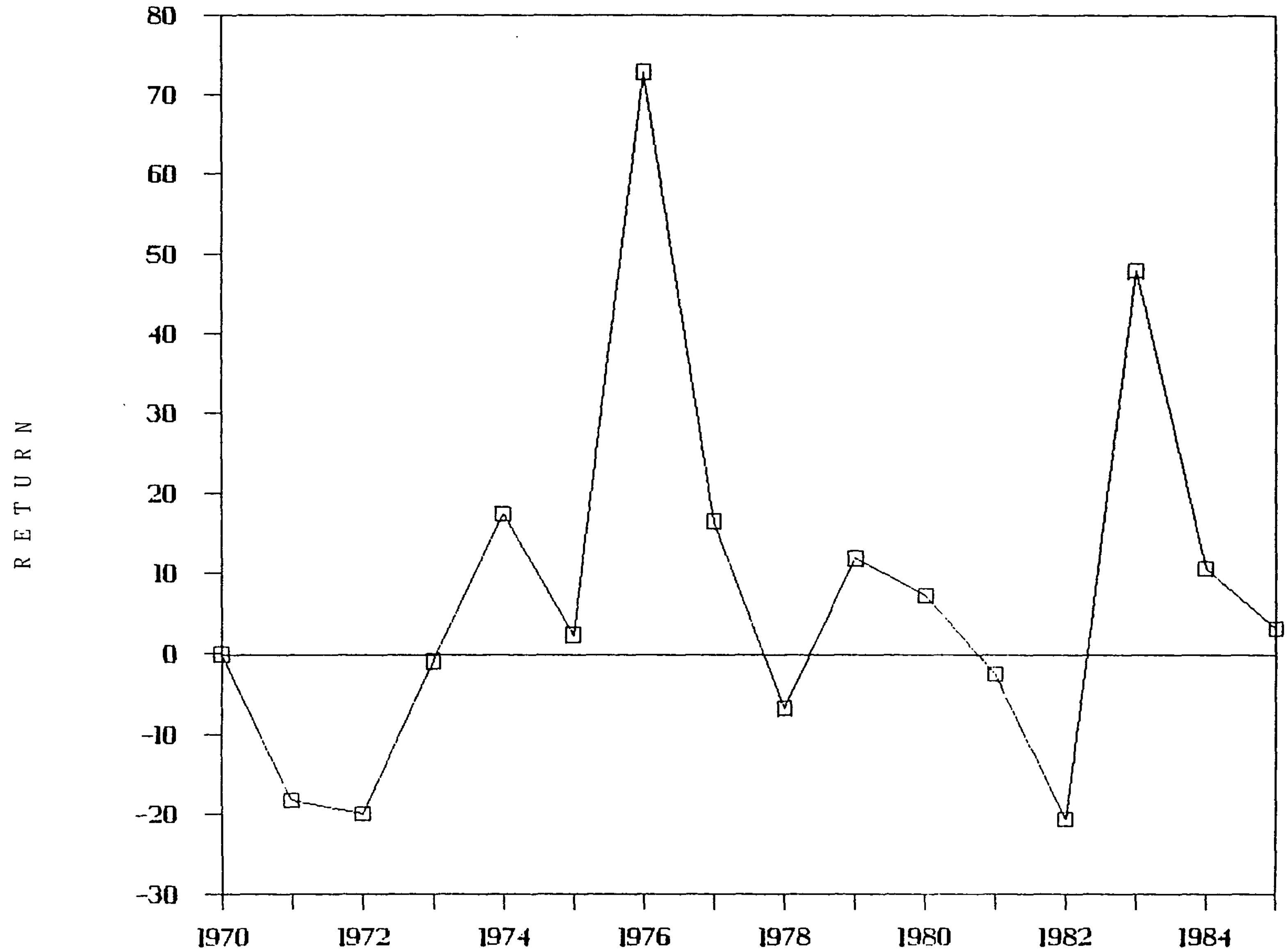


Figure 4-7 Returns on Security 7/Gold-Minerals Portfolio. --Reynolds Metals Company (RLM).

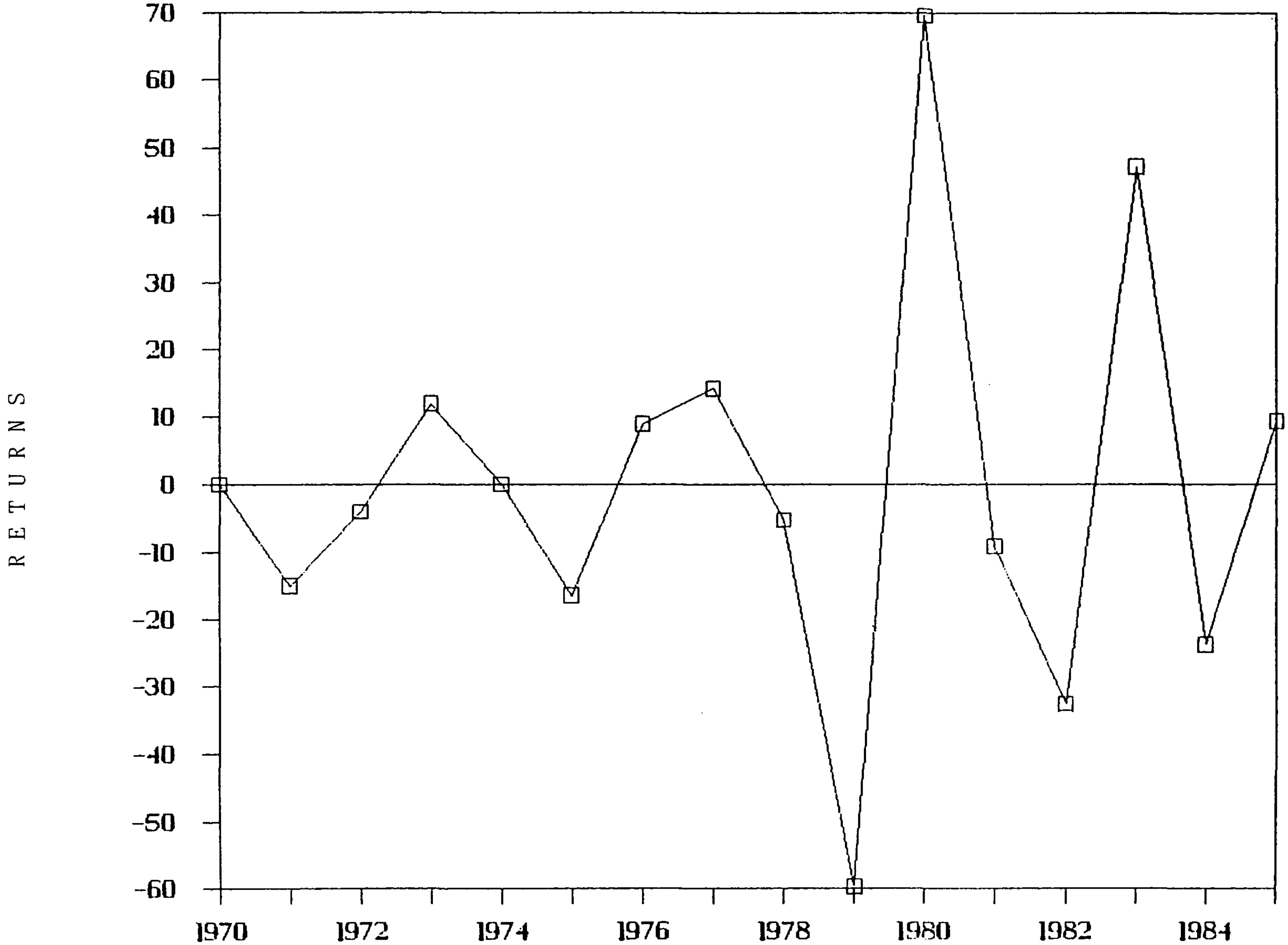


Figure 4-8 Returns on Security 8/Gold-Minerals Portfolio. --Asarco Inc. (AR).

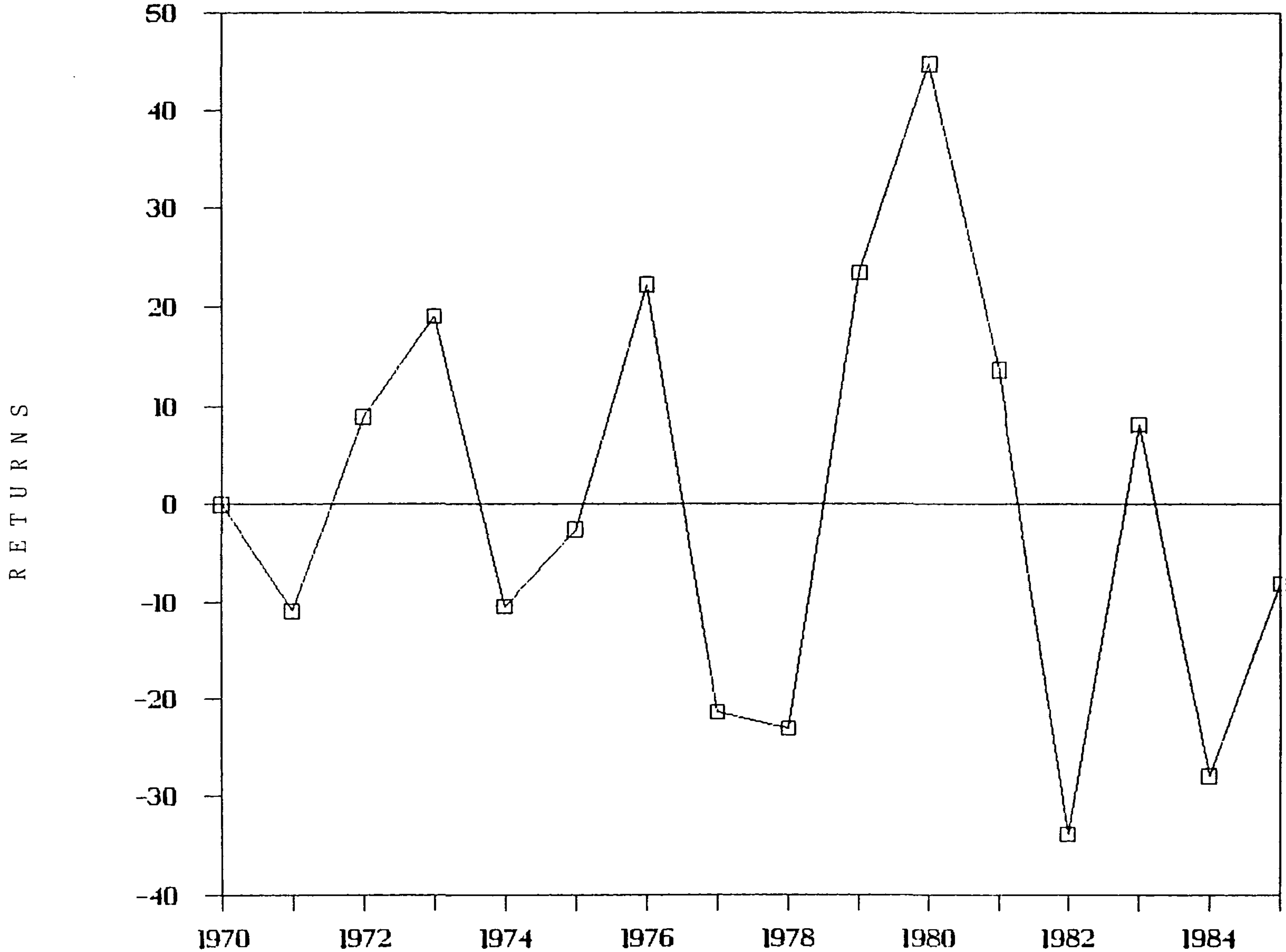


Figure 4-9 Returns on Security 9/Gold-Minerals Portfolio. --Phelps Dodge Corporation (PD).

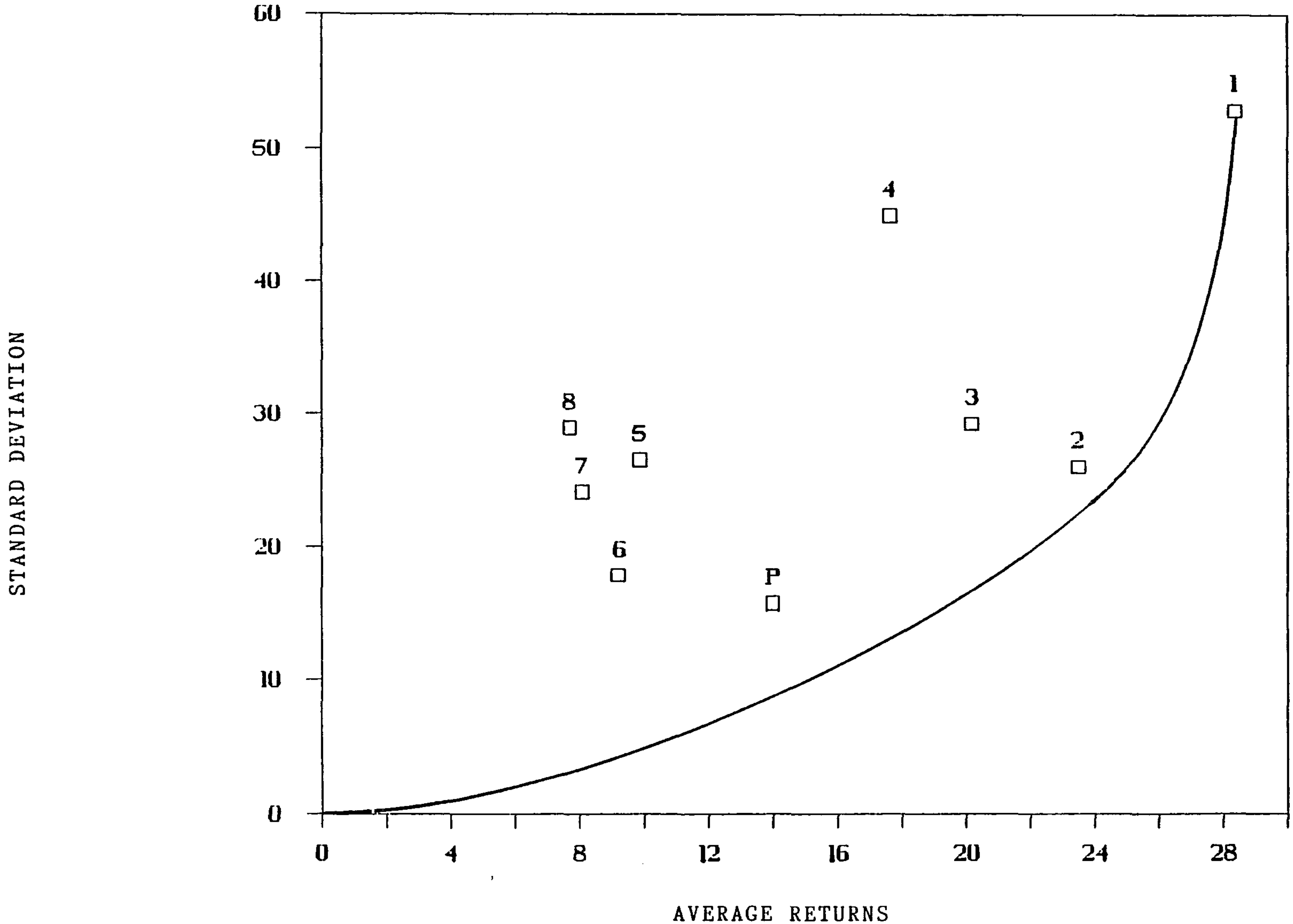


Figure 4-10 Average return and Standard Deviation for an Gold-Minerals Portfolio. --Securities 1 through 8.

standard deviation of 52.865. Figure 4-10 shows that Homestake Mining Co. (security 1) had a higher return on average and much higher standard deviation than either securities 2 or 3. Security 1 performed better during the period than did 2 or 3 combining a high average with greater variability. Point P indicates the investor-desired rate of return (14.%) and standard deviation (15.74); this portfolio is explained later in this chapter.

The first step of portfolio construction has been accomplished. In fact, analysis starts with information concerning individual securities. Next, one can begin to select such stocks for a portfolio. The purpose of the analysis is to find securities which best meet the objectives of the investor. As noted earlier not all of these securities would appear in the final desired portfolio. They enter the analysis as candidates for a place in the desirable portfolio and the four most relevant will be chosen. Also, this chapter is focused on one complete example of the blending of stock alternatives for portfolio construction given the model's constraints.

Determining the Optimum Mix

Risk, return and the ranking for the selected sample of securities (table 4-1) were determined above. The systematic procedure for portfolio construction is shown in Appendix A. The method of solution for this procedure is given in Appendix B (see tables B-2 and B-3).

The securities considered in alternative (1) presented in his chapter are: Aluminum Company of America, Reynolds Metals, International Minerals/Chemical, and Newmont Mining Company.

Before determining the optimal proportion of each security in the portfolio, in order to minimize variance for a given expected return (14%), other statistical measures are needed: covariance. Table 4-2 shows the results for these measures.

The full illustration of the procedure of portfolio construction is focused on alternative (1). The mathematical terms needed in order to handle this problem are shown in Appendix A (as in chapter III, but using different constants). Now, inserting the proper constants for this alternative (1); variances, covariances, expected returns and E, the equation system from Appendix A (see page 130 & 131) can be expressed in a tabular form (table 4-3).

Table 4-4 and table 4-5, are the full illustration of the systematic procedure used to obtain the solutions of the equation system from table 4-3 using the method of symmetric single division (Appendix B, table B-2 and B-3, respectively). Table 4-5 shows the results for the optimal weights for each security in the gold-minerals blending alternative (1). The individual funds to invest in the portfolio with minimal risk is given by W_1 through W_4 .

TABLE 4 - 2

Gold-Minerals Portfolio Covariance: Blending Alternative #1

Year	1 Aluminum Co of Am $R_A - E(R_A)$	2 Reynolds Metals $R_r - E(R_r)$	3 Internt'l Minerals $R_I - E(R_I)$	4 Newmont Mining Co. $R_N - E(R_N)$	$\sigma_{1,2}$	$\sigma_{1,3}$	$\sigma_{1,4}$	$\sigma_{2,3}$	$\sigma_{2,4}$	$\sigma_{3,4}$
1971	(17.988)	(26.241)	55.509	(1.997)	472.023	(998.496)	35.922	(1456.612)	52.403	(110.851)
1972	(15.74)	(27.983)	(1.062)	(8.23)	440.452	16.716	129.540	29.718	230.300	8.740
1973	28.429	(7.323)	26.05	(12.455)	(208.186)	740.575	(354.083)	(190.764)	91.208	(324.577)
1974	(11.069)	9.303	15.537	(10.852)	(102.975)	(171.979)	120.121	144.541	(100.956)	(168.608)
1975	(10.085)	(5.708)	10.429	(16.794)	57.565	(105.176)	169.367	(59.529)	95.86	(175.145)
1976	22.227	64.739	(24.092)	15.836	1438.954	(535.493)	351.987	(1559.692)	1025.207	(381.521)
1977	(4.612)	8.308	(10.438)	(20.344)	(38.316)	48.140	93.827	(86.719)	(169.018)	212.351
1978	(14.954)	(14.833)	(18.361)	(17.902)	221.813	274.570	267.707	272.349	265.540	328.699
1979	13.175	3.774	7.878	46.463	49.722	103.793	612.150	29.732	175.351	366.036
1980	18.062	(0.76)	33.546	45.528	(13.727)	605.908	822.327	(25.495)	(34.601)	1527.282
1981	(9.35)	(10.537)	(18.29)	32.925	98.521	171.012	(307.849)	192.722	(346.931)	(602.108)
1982	(13.127)	(28.662)	(55.039)	(40.802)	376.246	722.497	535.608	1577.528	1169.467	2245.701
1983	36.378	39.874	3.617	22.08	1450.536	131.579	803.226	144.224	880.418	79.863
1984	(2.053)	2.448	(4.923)	(37.36)	(5.026)	10.107	76.700	(12.052)	(91.457)	183.923
1985	(18.14)	(4.917)	(20.303)	3.935	89.194	368.296	(71.381)	99.830	(19.348)	(79.89)
	Totals				4326.798	1382.189	3285.169	(900.219)	3223.443	3109.80
	Covariance				309.057	98.728	234.655	(64.301)	230.246	222.129

$$Cov\,(R_i R_j) = 1/n-1 \sum_{\substack{i=1\\j=1}}^{N} (R_i - R_i)\,(R_j - R_j)$$

Note: Values are derived from Appendix D,
See Tables D-2, D-5, D-6, and D-7
( ) are negative values

TABLE 4 - 3

Gold-Minerals Portfolio Linear Equation System for Alternative 1

$$317.192\,W_1 + 309.057\,W_2 + 98.728\,W_3 + \tfrac{1}{2}\tilde{I}_1(9.2) + \tfrac{1}{2}\tilde{I}_2(1) + 234.655\,W_4 = 0$$

$$309.057\,W_1 + 582.241\,W_2 + 64.301\,W_3 + \tfrac{1}{2}\tilde{I}_1(8.09) + \tfrac{1}{2}\tilde{I}_2(1) + 230.246\,W_4 = 0$$

$$98.728\,W_1 + 64.301\,W_2 + 677.221\,W_3 + \tfrac{1}{2}\tilde{I}_1(23.51) + \tfrac{1}{2}\tilde{I}_2(1) + 222.129\,W_4 = 0$$

$$9.2\,W_1 + 8.09\,W_2 + 23.51\,W_3 + 9.86\,W_4 = 14$$

$$1\,W_1 + 1\,W_2 + 1\,W_3 + 1\,W_4 = 1$$

$$234.655\,W_1 + 230.246\,W_2 + 222.129\,W_3 + \tfrac{1}{2}\tilde{I}_1(9.86) + \tfrac{1}{2}\tilde{I}_2(1) + 702.823\,W_4 = 0$$

Note: Derived from Appendix A, See Page 130 and 131, Table 4-1 and Table 4-2

TABLE 4 - 4

Output from the Procedure used to Solve the Linear Equations (table 4-3) to the Gold-Minerals Portfolio, Alternative (1)

W_1	W_2	W_3	$\frac{1}{2}\tilde{l}_1$	$\frac{1}{2}\tilde{l}_2$	W_4	RHS
317.192	309.057	98.728	9.2	1	234.655	0
309.057	582.241	-64.301	8.09	1	230.246	0
98.728	-64.301	677.211	23.51	1	222.129	0
9.2	8.09	23.51	0	0	9.86	14
1	1	1	0	0	1	1
234.655	230.246	222.129	9.86	1	702.823	0
1.000000	0.974353	0.311256	0.029004	0.003152	0.739788	0
0	281.110362	-160.496930	-0.874048	0.025646	1.609179	0
0	-160.496930	646.481290	20.646442	0.688743	149.091159	0
0	-0.874048	20.646442	-0.266841	-0.029004	3.053945	14
0	0.025646	0.688743	-0.029004	-0.003152	0.260211	1
0	1.609179	149.091159	3.053945	0.260211	529.227925	0
0	1.000000	-0.570939	-0.003109	0.000091	0.005724	0
	0	554.847288	20.147413	0.703386	150.009902	0
	0	20.147413	-0.269559	-0.028924	3.058948	14
	0	0.703386	-0.028924	-0.003155	0.260064	1
	0	150.009902	3.058948	0.260064	529.218713	0
		1.000000	0.036311	0.001267	0.270362	0
		0	-1.001144	-0.054465	-2.388156	14
		0	-0.054465	-0.004046	0.069895	1
		0	-2.388156	0.069895	488.661661	0
			1.000000	0.054403	2.385425	-13.983991
			0	-0.001083	0.199819	0.238349
			0	0.199819	494.358429	-33.395955
				1.000000	-184.411139	-219.969859
				0	531.207396	10.558341
					1	0.019876

Note: This procedure is explained in Appendix B, See table B-2
Derived from Table 4-3

TABLE 4 - 5

Optimal Weights for Alternative 1 at 14% Yield for Gold-Minerals Portfolio

$W_4 = \dfrac{10.558341}{531.207396} = 0.019876$

$\frac{1}{2}\, l_2 = -219.969859 -(-184.411139 * 0.019876) = -216.304481$

$\frac{1}{2}\, l_1 = -13.983991 - (0.054465 * -216.304481) - (2.385425 * .019876) = -2.263660$

$W_3 = 0 - (0.036311 * -2.263660) - (0.001267 * -216.304481) - (0.270362 * .019876) = 0.351035$

$W_2 = 0 - (-0.570939 * 0.351035) - (-0.003109 * -2.263660) - (0.000091 * -216.304481)$

$-(0.005724 * 0.019876) = 0.213002$

$W_1 = 0 - (0.974353 * 0.213002) - (0.311256 * 0.351035) - (0.029004 * -2.263660) - (0.003152 *$

$-216.304481) - (0.739788 * 0.019876) = 0.416086$

$W_t = W_1 + W_2 + W_3 + W_4$

$W_t = 0.999999$

$E = 14 = 9.2*0.416086 + 8.09 * 0.213002 + 23.51 * 0.351035 + 9.86 * 0.019876$

Note: The procedure is explained in Appendix B, Also see table B-3.
Derived form Table 4-4

The particular blending alternative (1) in this chapter met all restrictions, even the nonnegativity requirements (see Appendix A) at a 14% rate of return. The final results ($W_1 \ldots W_4$) of other blending alternatives (2 through 4) are shown in tables 4-6 and 4-7. The ording implies no preference, it is only the sequence in which each of the blending alternatives was analyzed. The more efficient securities were chosen and with them the optimal mix of securities to form a portfolio. That is, those that minimize risk at 14% yield.

The portfolio variance for alternative 1, is calculated using the equation 3, of Appendix A:

$$Vp = W_1^2 \sigma_1^2 + W_2^2 \sigma_2^2 + W_3^2 \sigma_3^2 + W_4^2 \sigma_4^2 + 2W_1 W_2 \sigma_{12}$$
$$+ 2W_1 W_3 \sigma_{13} + 2 W_1 W_4 \sigma_{14} + 2W_2 W_3 \sigma_{23} + 2W_2 W_4 \sigma_{24} +$$
$$+ 2W_3 W_4 \sigma_{34}$$

Next substitute the values: σ_{12} through σ_{34} and W_1 through W_4 (alternative #1) the portfolio variance is 247.99 at 14% yield (table 4-8).

To illustrate the sensitivity of the W(s) and variance portfolio values, Vp(s), with regard to different desired portfolio rates of return, if the desired rate of return for the gold-minerals portfolio were not fixed at 14% yield, see table 4-8, which shows the final values of W(s), and Vp(s), when the desired portfolio rate of return

TABLE 4 - 6

Blending Together Securities Alternatives (1 and 2) by Selected Sample Securities from Table 4-1 for Contruction of a Gold-Minerals Portfolio

Desired 14% Portfolio Rate of Return

Alternative 1					Alternative 2				
Weights W(s)	Ranking Position	Symbol	Expected Return	Variance	Weights W(s)	Ranking Position	Symbol	Expected Return	Variance
W_1	6	AA	9.20	317.192	W_1	6	AA	9.2	317.192
W_2	7	RLM	8.09	582.741	W_2	2	IGL	23.51	677.211
W_3	2	IGL	23.51	677.211	W_3	5	NEM	9.86	702.823
W_4	5	NEM	9.86	702.823	W_4	8	AR	7.7	837.708

SOLUTION

Alternative 1: OPTIMAL WEIGHTS				Alternative 2: OPTIMAL WEIGHTS			
W_1	=	0.416086	Vp = 247.995721	W_1	=	0.896751	Vp = ------
W_2	=	0.213002		W_2	=	0.253395	
W_3	=	0.351035	Sp = 15.747879	W_3	=	0.439205	Sp = ------
W_4	=	0.019876		W_4	=	-0.589352	
W_t	=	0.999999	Vep = 271.280937	W_t	=	0.999999	Vep = 389.898975
			Sep = 16.470608				Sep = 19.748161

TABLE 4 - 7

Blending Together Securities Alternatives (3 and 4) by Selected Sample Securities from Table 4-1 for Construction of a Gold-Minerals Portfolio.

Desired 14% Portfolio Rate of Return									
Alternative 3					Alternative 4				
Weights W(s)	Ranking Position	Symbol	Expected Return	Variance	Weights W(s)	Ranking Position	Symbol	Expected Return	Variance
W_1	6	AA	9.2	317.192	W_1	6	AA	9.2	317.192
W_2	2	IGL	23.52	677.211	W_2	2	IGL	23.52	677.211
W_3	5	NEM	9.86	702.823	W_3	5	NEM	9.86	702.823
W_4	3	AN	20.16	859.276	W_4	1	HM	28.37	2794.683

Solution

W_1 =	0.637011	Vp =	259.823880	W_1 =	0.647274	Vp =	--------
W_2 =	0.330143	Sp =	16.119053	W_2 =	0.616680	Sp =	--------
W_3 =	0.027605			W_3 =	-0.055932		
W_4 =	0.005239	Vep =	363.527625	W_4 =	-0.208022	Vep =	561.656312
		Sep =	19.066400			Sep =	23.699289
W_t =	0.999999			W_t =	0.999999		

TABLE 4 - 8

Allocation of Optimal Weights for Minimal Risk in the Gold-Minerals Portfolio, Alternative 1, at different Rates of Return

		Desired Portfolio Rate of Return				
		12%	13.5%	14%	14.5%	15%
W_1	=	0.569348	0.454402	0.416086	0.377771	0.339455
W_2	=	0.159086	0.199523	0.21302	0.226480	0.239959
W_3	=	0.204934	0.314510	0.351035	0.387560	0.424085
W_4	=	0.066630	0.031564	0.019876	0.008187	-0.003500
½						
W_t	=	1.000000	1.000000	0.999999	0.999999	1.000000
Vp	=	252.701399	246.592081	247.995721	251.119401	
Sp	=	15.896584	15.703250	15.747879	15.846797	
Vep	=	271.280937	271.280937	271.280937	271.280937	
Sep	=	16.470608	16.470608	16.470608	16.470608	

Where: W(s) = Securities in the portfolio

W_t = Total Fund investable in the portfolio

Vp = Variance Portfolio

Sp = Standard deviation of portfolio

Vep = Variance portfolio with equal weights/ securities (.250000)

Sep = Standard deviation of portfolio with equal weights/securities (.250000) is increased from 12% to 15%. This illustration is for the same set of equations as those of table 4-3 alternative (1). Here, however, the E, the investors desired portfolio return (14%), has been replaced for each one of the new different E (12%, 13.5%, 14.5% and 15%). In this table (4-8) the final W(s) values are given for optimal allocation at minimal risk and their respective portfolio variances Vp(s). For the particular alternative 1, over the range of E (12%, through 15%, table 4-8), not all restrictions have been meet (the nonnegative requirement, see Appendix A). A 15% desired return, requires a "disinvestment" of security four, (W_4). This is also true for alternatives 2 and 4 at 14% yield (table 4-6 and 4-7).

Since the amount assumed to be invested is $10,000, the W(s) values from table 4-8 are multiplied by $10,000.00 as follows:

	DESIRED RATE OF RETURN				
	12%	13.5%	14%	14.5%	15%
W_1 =	$ 5,694	$4,544	$4,161	$ 3,778	3,395
W_2 =	1,591	1,995	2,130	2,265	2,399
W_3 =	2,049	3,145	3,510	3,875	4,241
W_4 =	666	316	199	82	- 35
W_t =	$10,000	$10,000	$10,000	$10,000	$10,000

Reviewing the basics and all alternative options (1 through 4) in this chapter, alternative 1, offers the lowest risk (Vp = 247.99, table 4-8) at an expected rate of return of 14%, without "disinvestment".

The proportions of individual funds at 14% yield as noted earlier are listed on page 102 and shown in figure 4-11. It is possible to buy a couple of shares of every individual security. Appendix D gives the price/share in 1985 for these companies (see tables D-2, D-5, D-6 and D-7).

Referring back to figure 4-10, point P represents the investor required rate of return of 14.00% and a standard deviation of returns of 15.74. Portfolio P has both a higher average and a lower standard deviation than security 6. Security 2 had a still higher average and greater standard deviation than portfolio P.

In figure 4-12 the average return for the securities selected are plotted (Aluminum Co. of America, Reynolds Metals, International Minerals & Chemicals and Newmont Mining Co.) as well as the portfolio curve for the mixture (table 4-5) of 0.416086%, Aluminum Co. times the average return/year, plus 0.213002% of Reynold Metals times the average return/year, plus 0.351035 of International Minerals & Chemicals times the average return/year, plus 0.019876% of Newmont Mining Co. times the average return/year. The average portfolio rate of return is 14%,

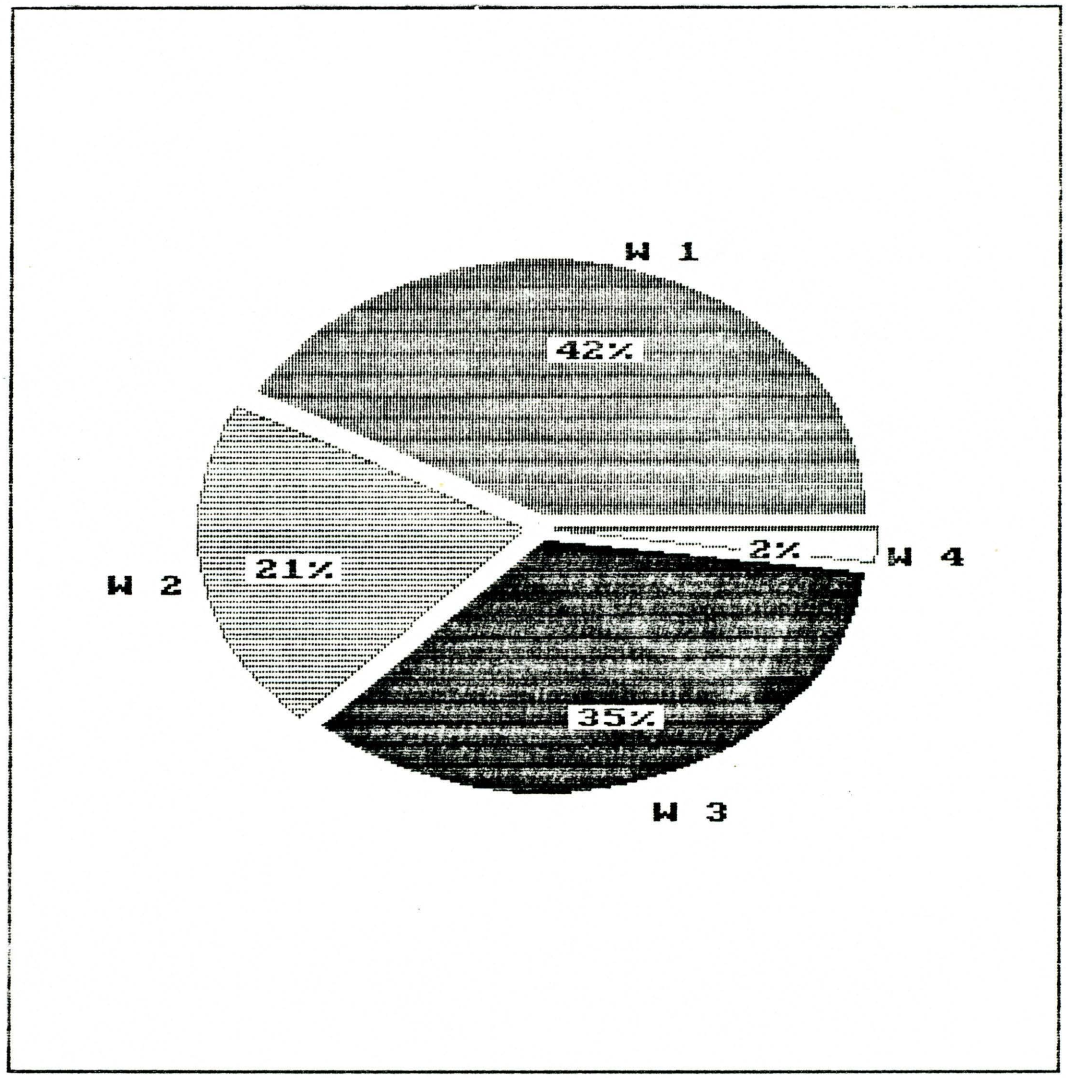


Fig. 4-11 Individual Allocation of Funds/Gold-Minerals Port.

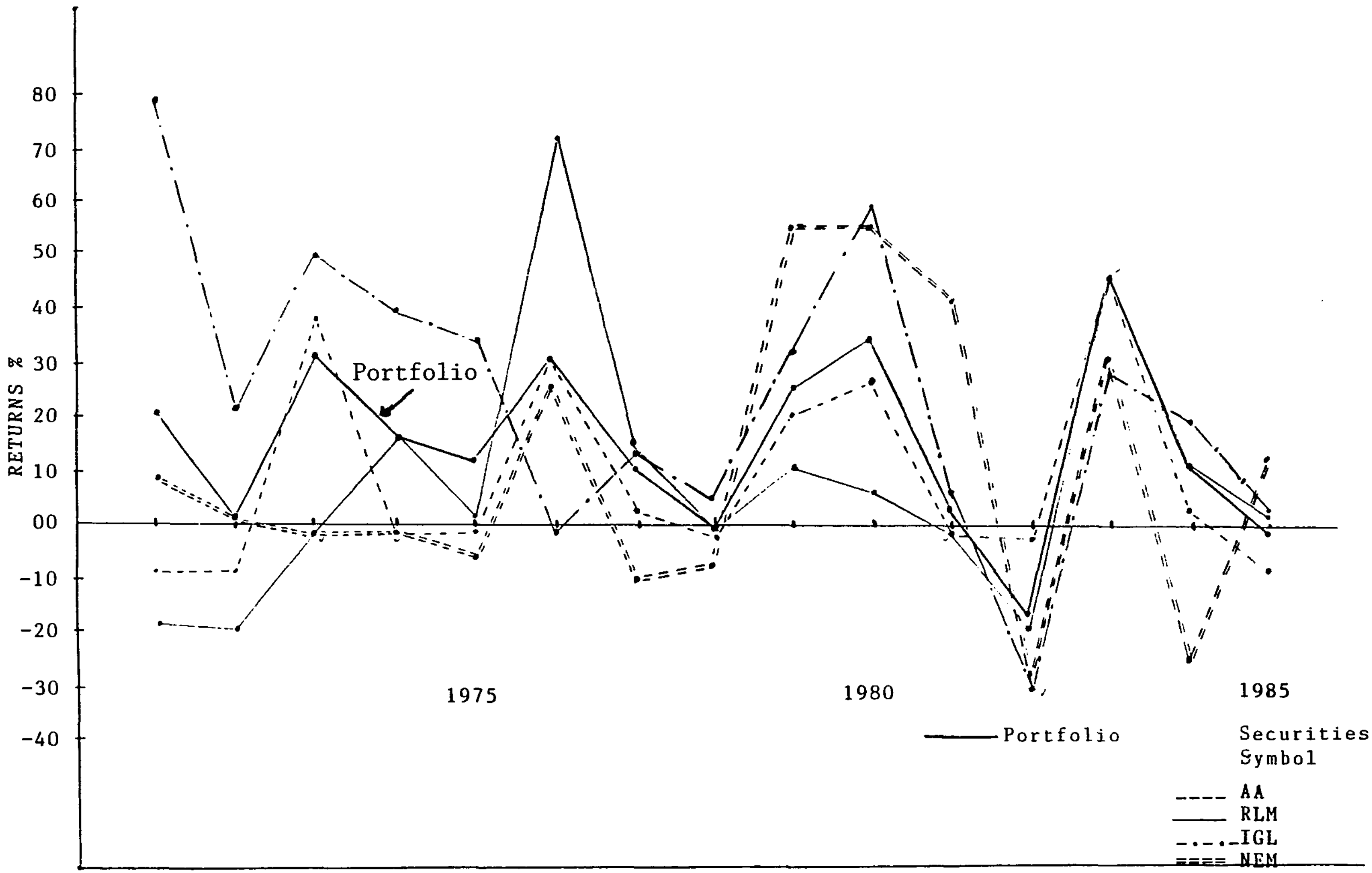


Fig. 4-12 Return of Securities and a Gold-minerals Portfolio

the desired rate of return for this investment. The portfolio selection is based on a reasonable expectation about the future, based on past performance and assumes that the above mix of securities with their respective weights should be maintained, to obtain the same rate of return (14%) in the next period, in this case 1986. The reason for plotting the aforementioned portfolio curve is to compare its variability by year, against each one of the securities included in it, for the period analyzed (1970-1985).

The variance of the portfolio (Vep) with stocks weighted according to alternative 1 (table 4-8) at 14% rate of return in this chapter, is 271.28 and the standard deviation is 16.47. These values compare favorably with those of table 2-1.

Table 4-9 shows the correlations between securities in alternative 1 (Table 4-8). These values measure the extent to which stocks move together ranging from +1 to -1 with a midpoint of zero. In the case of alternative (1) the co-movement between the stocks is positive, but lower than in chapter III. The higher the positive correlation among security returns, the greater is the standard deviation of the portfolio as a whole.

A brief description of the activities of the companies whose stocks were selected to form a gold-minerals investment portfolio follows.

TABLE 4 - 9

Gold-Minerals Portfolio Correlation Coefficients: Alternative 1 (derived from Tables 4-1 and 4-2)

$\rho_{1,2} = 0.7192$	$\rho_{2,3} = -0.1024$
$\rho_{1,3} = 0.2130$	$\rho_{2,4} = 0.3599$
$\rho_{1,4} = 0.4970$	$\rho_{3,4} = 0.3220$

$$\rho_{ij} = \frac{Cov_{ij}}{\sigma_i \sigma_j}$$

Where:

ρ_{ji} = Correlation coefficient for securities ij

Cov_{ij} = Covariance between securities ij

σ_i = Standard deviation for security i

σ_j = Standard Deviation for security j

1. Aluminum Company of America
2. Reynolds Metals Company
3. International Minerals & Chemical Corporation
4. Newmont Mining Company

Aluminum Co. of America

Aluminum Co. of America[1] is the world's leading integrated producer of aluminum products. The activities of the company are mining and processing of bauxite, refining alumina from bauxite, smelting aluminum, and recycling used aluminum products. The company has foreign operations.

In the short-term, a cut in labor benefits and a freeze on wages yield a net income increase to $53.3 million for the quarter ended June 30, 1986, from $40.6 million in 1985. (31.3%)

Alcoa is restructuring the company due to low aluminum prices and over-supplied markets. At the quarter the company has already reduced production to 78% of capacity. The company is also, developing new products, emphasizing high margin fabricated products, and moving into foreign countries. Alcoa is looking for opportunities that are less cyclical and less capital intensive than those of present operations. Figure 4-13 shows the trading volume of shares over the business cycle 1970 - 1985. The common stock had a split of 3 for 1 in 1974 and a 2 for 1 in 1981. Table D-6 shows the results for both risk and return for the period (1972 - 1985).

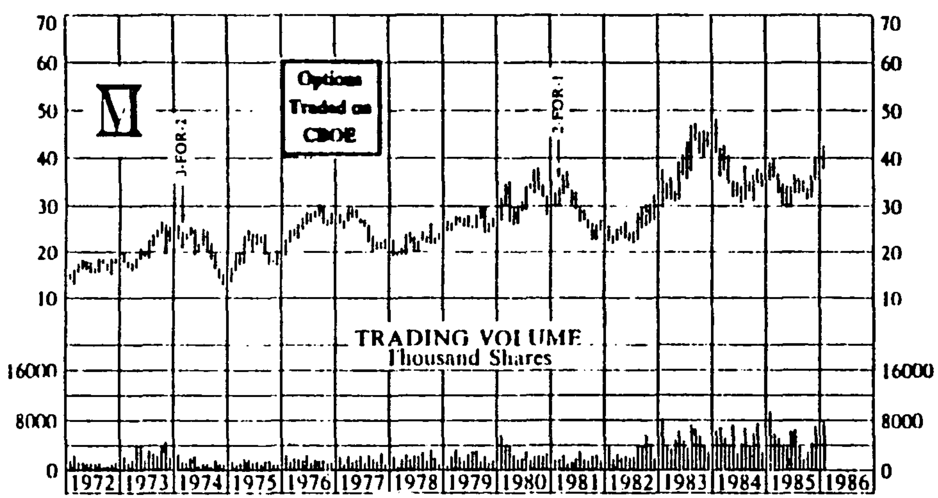
----

1. Summary condensed from Moody's Handbook, fall 1986

ALUMINUM CO. OF AMERICA

LISTED	SYM.	LTPS•	STPS•	IND. DIV.	REC. PRICE	RANGE (52-WKS.)	YLD.
NYSE	AA	104.9	103.0	$1.20*	42	43 - 30	2.9%

MEDIUM GRADE. THIS COMPANY'S OPERATING RESULTS CONTINUE TO BE HURT BY WEAK DEMAND AND PRICES OF ALUMINUM.



CAPITALIZATION: (12/31/84)

	(000)	(%)
Long-Term Debt	b$1,586,500	29.3
Pfd. ($1.00)	66,000	1.2
Defer. Inc. Tax	488,500	9.0
Com. & Surp.	3,277,600	60.5
Total	$5,418,600	100.0

Shs. ($1)-81,140,879

INTERIM EARNINGS:

Qtr.	3/31	6/30	9/30	12/31
1982g	0.31	0.19	d[illegible]	d0.45
1983g	d0.19	0.43	0.60	1.19
1984	1.20	1.38	0.74	d0.14
1985h	0.08	0.49	0.69	d1.49

INTERIM DIVIDENDS:

Amt.	Dec.	Ex.	Rec.	Pay.
0.30Q	1/11/85	1/28/85	2/1/85	2/25/85
0.30Q	4/12	4/29	5/3	5/25
0.30Q	7/12	7/29	8/2	8/25
0.30Q	10/11	10/28	11/1	11/25
0.30Q	1/10/86	2/3/86	2/7/86	2/25/86

Figure 4-13 Aluminum Co. of America, Trading Volume Shares.

SOURCE: Moody's Handbook, Spring 1986

Reynolds Metals Company
[2]

Reynolds Metals Company is a major worldwide, vertically integrated producer of aluminum and a variety of aluminum related products. The company engages in mining and refining bauxite into alumina, production of primary and reclaimed aluminum, and the fabrication of aluminum and aluminum alloys. In 1985, aluminum production and processing totaled 728.700 tons finished products 349,200 tons.

For the quarter ended June 30, 1986, income was $18.2 million compared with $3.6 million a year earlier. Improved results were attributed to a more profitable product mix and higher ingot prices. Reynolds has made moves necessary for recovery. Reynolds closed uneconomical and obsolete plants. The company plans to concentrate on high value-added fabricated and finished products. Aluminum prices are firming but remain weak due to excess world capacity. Recovery will take time but encouraging factors include lower producer and customer inventories, a weak dollar, and increased consumption. Figure 4-14 shows the trading volume of shares over the business cycle 1970-1985. This figure does not show a stock split during this period. Table D-7 shows the results for both risk and return for the period (1972 - 1985).

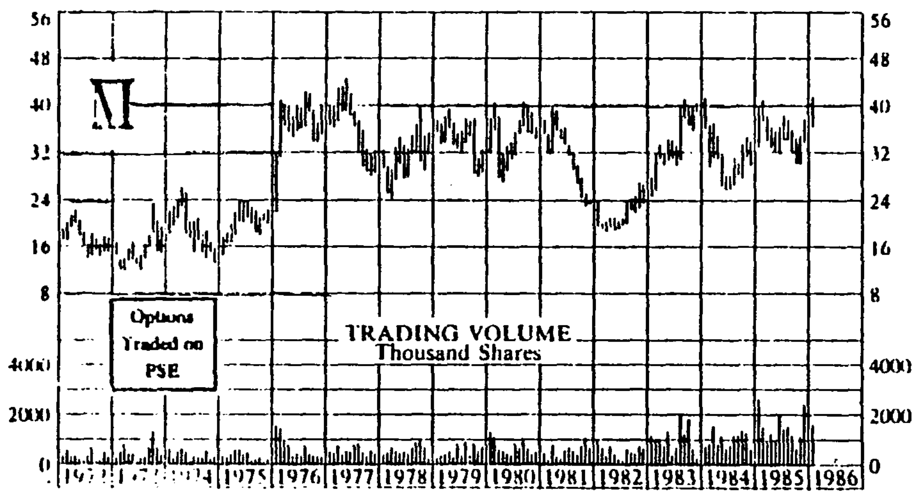
----

2. Summary condensed from Moody's Handbook, Spring 1986

REYNOLDS METALS COMPANY

LISTED	SYM.	LTPS•	STPS•	IND. DIV.	REC. PRICE	RANGE (52-WKS.)	YLD.
NYSE	RLM	107.1	96.9	$1.00•	41	42 - 30	2.4%

LOWER MEDIUM GRADE. DESPITE A GOOD SALES RECORD, COMPETITIVE INDUSTRY CONDITIONS HAVE RESULTED IN AN ERRATIC EARNINGS TREND.



CAPITALIZATION: (12/31/84)

		(000)	(%)
Long-Term Debt	$	1,081,200	42.0
Pfd.		28,400	1.1
Defer. Inc. Tax		154,300	6.0
Com.& Surp.		1,312,700	50.9
Total		$2,576,600	100.0

Shs.(np)-21,558,000

INTERIM EARNINGS:

Qtr.	3/31	6/30	9/30	12/31
1982m	d0.70	1.05	d0.63	d1.08
1983n	---------d4.73---------			
1984s	1.03	1.74	1.82	1.77
1985a	------d13.52-------			d0.13

INTERIM DIVIDENDS:

Amt.	Dec.	Ex.	Rec.	Pay.
0.25Q	2/15/85	3/4/85	3/8/85	4/1/85
0.25Q	5/17	6/3	6/7	7/1
0.25Q	8/16	9/4	9/10	10/1
0.25Q	11/15	11/25	12/2	12/19
0.25Q	2/21/86	3/3/86	3/7/86	4/1/86

Figure 4-14 Reynolds Metals Company, Trading Volume Shares.

SOURCE: Moody's Handbook, Spring 1986

International Minerals & Chemical Corporation[3]

International Minerals & Chemical's operations are organized into four segments: fertilizer products, 61% of sales include phosphate rock, phosphate chemicals, potash and ammonia for inorganic fertilizers, mixed fertilizers and uranium oxide; industry 26%, including the purchase of petroleum coke and ferroalloys for resale, production and sale of ferroalloys, and foundry materials; animal products, 10%; and gas and oil 3%, including exploration, production and development.

For the quarter ended December 31, 1985, net income dropped 72% to $9.8 million. Fertilizer earnings fell 64% due to lower prices. Earnings from animal products were flat. Industry group earnings fell 37%, mainly from tight margins on petroleum coke and price weakness in ferrosilicon and industrial chemicals.

Conditions in the agricultural industry continue to deteriorate. The industry group continues to contend with a lower level of activity in steel, foundry, aluminum, and most other markets. However, these groups should sustain their present earnings performance. Figure 4-15 shows the trading volume of shares over the business cycle 1970 - 1985. The common stock has a 3 for 2 split in 1980.

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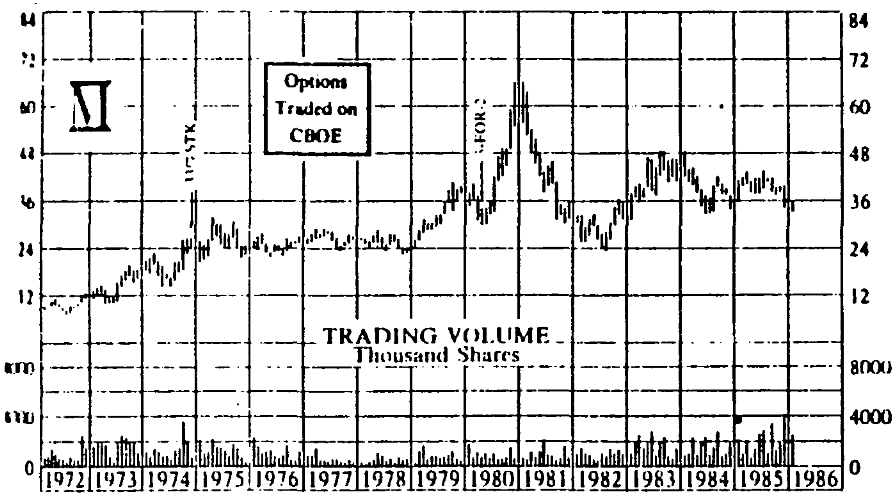
3. Summary condensed from Moody's Handbook, Spring 1986

INTERNATIONAL MINERALS & CHEMICAL CORPORATION

LISTED	SYM.	LTPS•	STPS•	IND. DIV.	REC. PRICE	RANGE (52-WKS.)	YLD.
NYSE	IGL	99.4	83.3	$1.00•	35	44 - 33	2.9%

MEDIUM GRADE. RECENT OPERATING RESULTS REFLECT VARIABLE DEMAND AND PRICES FOR FERTILIZERS.



CAPITALIZATION: (6/30//85)

		(000)	(%)
Long-Term Debt	c$	342,100	22 1
Preferred		10,000	0.6
Defer. Inc. Taxes		117 300	7 6
Common & Surplus		1,082,000	69.7
Total		1,551,400	100.0

Shs.($5)-27,070,915

INTERIM EARNINGS:

Qtr.	9/30	12/31	3/31	6/30
1982-83	0 92	0.81	0 79	0 50
1983-84f	1.10	1 10	1.03	0 94
1984-85	s1.36	1.26	1.14	0.63
1985-86a	0.38	0 36	...	...

INTERIM DIVIDENDS:

Amt.	Dec.	Ex.	Rec.	Pay.
0.65Q	2/20/85	3/11/85	3/15/85	3/29/85
0.65Q	5/15	6/10	6/14	6/28
0.65Q	8/21	9/9	9/13	9/30
0.65Q	10/16	12/11	12/17	1/2/86
0.25Q	2/19/86	3/11/86	3/17/86	3/31

Figure 4-15 International Minerals & Chemical Corp., Trading Volume Shares.

SOURCE: Moody's Handbook, Spring 1986

Table D-2 shows the results for both risk and return for the period (1972 - 1985).

Newmont Mining Co.

Newmont Mining Co.[4] is engaged in the exploration, operation and management of domestic and foreign mining activities. The major interests are gold, energy, and nonferrous metals. Magma Copper Co. handles the North American copper operations, Newmont Mining owns 100%. Newmont controls 87.5% of Foote Mineral Co. and 95% of the Newmont Gold Co.

In the short-term the nonferrous metals segment of this company is likely to incur losses. For the quarter ended June 30, 1986, income increased to $60.8 million compared to $8.0 million a year ago. Gold interest increased gains as prices rose $3.40 per ounce.

In the future gold and gas income are likely to increase while nonferrous metals income continues to be weak. Newmont Mining is making existing operations economically worthwhile. Also, it is expanding its gold production capacity. Figure 4-16 shows the share trading volume picture for 1970 - 1985. This figure does not show a stock split during this period. Table D-5 shows the results for both risk and return for the period 1972-1985.

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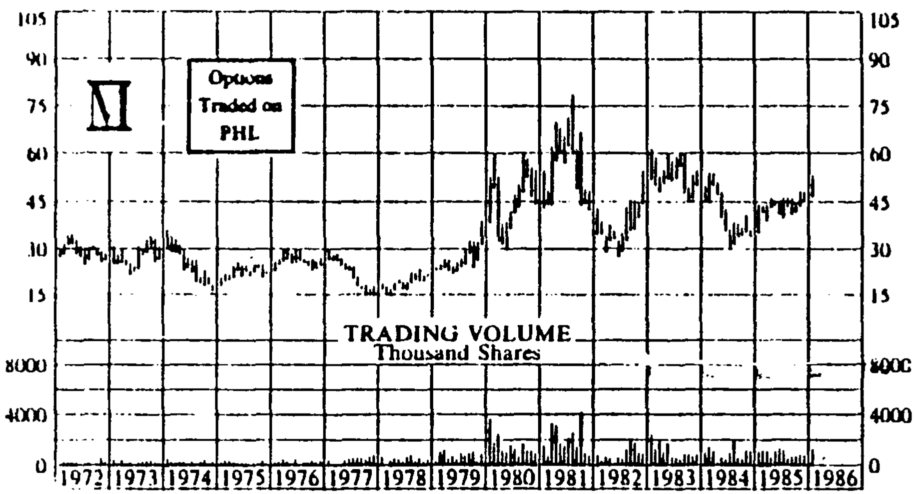
4. Summary condensed from Moody's Handbook, Fall 1986

NEWMONT MINING CORPORATION

LISTED	SYM.	LTPS•	STPS•	IND. DIV.	REC. PRICE	RANGE (52-WKS.)	YLD.
NYSE	NEM	97.7	99.6	$1.00•	52	54 - 39	1.9%

MEDIUM GRADE. THE COMPANY'S MINING INTERESTS MAKE EARNINGS SENSITIVE TO FLUCTUATIONS IN THE METALS MARKETS.



CAPITALIZATION: (12/31/84)

	(000)	(%)
Long-Term Debt	$ 215,982	11.2
Def. Inc Tax	208,136	10.8
Minority Int.	21,471	1.1
Com. & Surp.	1,480,901	76.9
Total	$1,926,490	100.0

Shs.($1.60)-30,313,833

INTERIM EARNINGS:

Qtr.	3/31	6/30	9/30	12/31
1981h	1.18	0.62	0.91	0.32
1982	0.43	0.38	0.32	0.63
1983	0.62	0.52	0.55	0.03
1984	0.39	0.42	0.40	0.18
1985i	------0.86------			d2.03

INTERIM DIVIDENDS:

Amt.	Dec.	Ex.	Rec.	Pay.
0.25Q	4/24/85	5/20/85	5/24/85	6/10/85
0.25Q	7/24	8/20	8/26	9/10
0.25Q	10/23	11/19	11/25	12/10
0.25Q	1/29/86	2/14/86	2/21/86	3/10/86

Figure 4-16 Newmont Mining Company, Trading Volume Shares.

SOURCE: Moody's Handbook, Spring 1986

CHAPTER V

CONCLUSIONS AND RECOMMENDATIONS

We must accept uncertainty, try to understand it, and make it part of our reasoning. It is a basic structural feature of the energy and minerals business environment. This study deals with a technique to minimize risk and allocate financial resources for selected securities and asset classes into portfolios to achieve a desired future rate of return. The main purpose is to use it for decisions related to investment alternative based on expectations for the future. For a given investment strategy a goal must be set up, and the investment strategy pursued should be consistent with it. Given that investment strategy should be directed to reducing the risk involved, the analysis of the results obtained in chapters III and IV should be aimed at examining this goal.

In this chapter, conclusions from our results are used to point out the main features of the allocation of financial resources and the impact of the risk or variance of returns on energy and gold-minerals portfolio investments. Next, some investment recommendations in agreement with the given goal are suggested.

Conclusions

The research suggests the following. In portfolio selection, Markowitz' model and the equations formulated provide efficient computing procedures. Once the problems have been formulated it is possible to solve them by using modern electronic computing machines at relatively low cost.

Programming portfolio construction applies the technique of operations research to an area of finance. It emphasizes the need for research, analysis, and judgment.

Portfolio construction by its use of the equations and computing procedures developed makes it possible to utilize the technique to determine the likely returns and risks of the energy and gold-mineral investment portfolios.

Investing in sector funds such as energy, gold-minerals, etc. is riskier than investing in other equity funds. However, they (sector funds) offer the potential of greater returns if the investor can correctly guess[1] the next hot industry. For the business picture over the years studied (1970 - 1985), it was shown that these sector funds are of high risk and do not offer better long term performance.

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1. "Very few professionals are good at timing, and individuals are less good", says Richard Frucci, portfolio manager of Putnam Information Sciences Trust.

Unecessary investing in sector funds amounts to speculation. One conservative strategy is to invest in income producing utility funds which are not very volatile. Some advisors also suggest that if an investor is looking for a hedge against inflation he might allocate some of his money to a gold fund.

Suppose we believe that past average and standard deviations are reasonable indicators of "most likely" return and "uncertainty" of return in the future. Then, the optimal mix of selected sample securities to form an energy portfolio investment would be alternative 1 (Exxon, Texaco, Pennzoil and Tenneco). The riskiness or portfolio variance of returns for this alternative is 259.78, (Vp), and the standard deviation is 16.11, (Sp), at a 13.5% desired portfolio rate of return (see table 3-7).

The optimal mix of selected sample securities to form a gold-minerals portfolio investment is alternative 1 (Aluminum Co. of America, Reynolds Metals, International Minerals & Chemicals and Newmont Mining Co.). The portfolio variance of returns for this alternative is 247.99, (Vp) and the standard deviation is 15.74, (Sp), at a 14% desired portfolio rate of return (see table 4-6).

Alternative 1, in the gold-minerals portfolio is a more efficient one than alternative 1, in the energy portfolio. The gold minerals alternative 1, as noted before, had both a higher average and a lower standard deviation than energy portfolio alternative 1.

Judgments were employed in choosing one alternative from the set of all efficient portfolios (the discussed alternatives in each chapter). The various efficient combinations of average return and standard deviation were also considered.

If a portfolio is represented by a point on the efficient curve, it is called efficient (fig. 3-10 and 4-10). If a portfolio is "efficient" it is impossible to obtain a greater average return without incurring a greater standard deviation. It is also impossible to obtain a smaller standard deviation without giving up return on the average (see table 4-6, gold-minerals alternative 1). In contrast with table 3-7 (energy portfolio alternative 1) even though is the "best" alternative among those discussed in chapter III, the variance, for example, is lower at a rate of return of 13.96% (250.98) than at 13.5% (259.78). One reason is because security #1 (Exxon Corp.), has both the highest return (16.6%) and the lowest variance (225.84) among the securities listed in table 3-7 (energy portfolio Alternative 1), and the desired portfolio return is close to the average return of this security. Thus, in the first place, a security cannot be efficient if other securities in the portfolio hold a higher return and lower standard deviation.

Another reason is that the impact on the portfolio variance in question is greatly affected by any individual security because it comprises only four securities for this study.

The fact that security returns are highly correlated, but not perfectly correlated, implies that diversification can reduce risk but not eliminate it.

To find a portfolio with minimum standard deviation at various levels of expected return, an analysis must start with expected returns for each security, variance of return for each security, and covariance of return for each pair of securities. In practical problems of portfolio analysis (hundreds of securities) it is difficult to solve these problems mainly because of the quantity input estimates for the model; for N returns we need N variance and N(N-1)/2 covariances. For example, an analysis of 100 securities requires 100 expected returns, 100 variance and about 4950 covariances. It is not reasonable to ask security analysts for this number of carefully and individually considered covariances. Yet covariances are essential to an analysis of efficient portfolios. They express relationships between securities and guide the analysis to properly diversified portfolios.[2]

To forecast the outcome of results in the future we need all its elements, meaning those elements that have already occurred (or that almost certainly will occur). Suppose, for example, a change in metal prices, legal problems (i.e. Pennzoil's suit against Texaco, as a result the net asset value of the stock has depreciated immediately), company strikes, and shifts in quantities of supply or demand for minerals. Furthermore, identifying predetermined elements is fundamental in planning our portfolio investment. Errors in future studies generally result from poor observation rather than poor reasoning. Today's portfolio management performance practices is a notion of change, adaptation, and innovation.

The ultimate objective of an investment process is to achieve the investor's goals. The performance evaluation of our portfolio investment should provide us with an analysis to emphasize or correct those aspects to contribute the investment goal.

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2. Index models deal with the relevant difficulty by providing a simplified method of representing relationship among securities. There are two types of index models: single index and multi-index. However, for the purpose and simplicity this study offers a full approach to portfolio analysis.

Those simple days in taking a decision investment are over. This is the age of specialization and market portfolios. They express relationships between securities hype. The problem is that choices of investments have multiplied beyond the ability of most investors to cope. It emphasizes the need for systematic methods of developing estimates of returns and risks. Portfolio programming offers an opportunity for improved accuracy in security forecasting and analysis.

The real challenge, of course, is finding those sectors that will lead the stock market and avoid those sectors that are destined to sink at given times.

No cost of capital was involved in this procedure, if we did so, we might get different results for different portfolios. That is another study by itself. An expansion of this study may be utilizing this data to find out those results. Another expansion of this study is to analyze more sector funds, and of course add more stocks in each sector to obtain an optimal real-life investment portfolio.

The use of this technique is not a "beat the market" scheme nor does it guarantee a favorable return on invested funds. The theory simply involves the application of formal mathematical methods to the problem of investment management.

Recommendations

A general strategy to deal with the level of risk on investment in energy and gold-minerals portfolios, would be directed to: First, reduce the risk involved as much as possible without harming our goal by investing in more diversified companies in these areas. Second, most advisors suggest that all but the smallest investors diversify by using several funds. That can include both a selection of funds within a category as well as some from different groups. The end result of this goal would be to monitor the risk more closely. The following recommendations are suggested:

Investment Objectives

The most fundamental of investment principles, is the formulation of specific goal, for any successful investment program.

Whichever category you may be in (energy or gold-minerals), one must come up with an overall strategy and find one or more funds that match one's goal. A systematic review of these goals as often as necessary is strongly recommended to make sure they are still right for one's objective.

Track Record

Comparative performance rankings for periods ranging from the last 10 years to the most recent quarter are easy to obtain. It is recommended that one look at performance results of those securities over both short and long periods of time. One obvious drawback to investing in these portfolios is that it lacks a track record to compare with either the overall market or with the other funds of its type.

Consistent Security Selection

Once our objective is established, we must then determine the amount of risk that we are willing to accept. The more risk we choose to assume, the greater the potential for profit or loss. The specific measurement that indicates the degree of risk involved in these particular investments, is given in this study.

"Realistic" expectations

Essentially, risk is a consequence of the fact that investment alternatives are governed by exogenous factors beyond our control. Furthermore, we cannot associate a particular return with a given investment alternative alone, but only with a range of possible returns.

Adjust to changing markets and conditions

Energy and mineral products and markets that drive the financial services industry change over time as well as economies. These fluctuations demand that the investor recognize and respond to such changes.

Overdiversification

Overdiversification not only can increase the cost of investing but presents management problems in the maintenance of the portfolio.

In addition, selecting too many different securities diminishes the significance of each of the individual holdings. Essentially, the impact of good performers on the overall portfolio is minimized by selecting a large number of different securities.

Underdiversification

It is essential that we do not commit all our funds to a single security. In this study it was possible to accidentally concentrate on a single security, because the portfolio was limited to only four securities. It is recommended further that an investment should not place disproportionate emphasis on any one industry.

Determining the best time to take profits or cut losses

We live in a world of uncertainty. No businessman is able to predict exactly just when to take profits or cut losses. Furthermore, the investor should take time to gather, analyze and interpret information that will affect the future performance of their investments.

Time Value of Money

The investor must recognize the true worth of either Energy or gold-minerals investment portfolios. The reason is that compounding money is a positive force, inflation can be a negative one. It reduces the purchasing power of our investments over time, unless the investments are stocks of a company holding large reserves of oil or of gold and other metals.

At the present time inflation is not a critical factor, although just a few years ago inflation played a dominate role. Then, the invesment goal was to provide a hedge against inflation. With a total rate of return greater than the rate of inflation.

Understanding of Tax Laws

Investors have been forced to reformulate their investment strategies because of tax reform. In fact, the new law reduced or eliminated tax savings opportunities at the same time offered new ways to lower taxes and improve after-tax investment returns.

APPENDIX A

PROCEDURE USED TO DETERMINE THE OPTIMAL WEIGHT FOR EACH SECURITY IN ITS PORTFOLIO

Appendix A is presented to utilize the scheme suggested by Markowitz for portfolio selection.

One major contribution of the Markowitz formulation for portfolio construction is that it has solved the complex mathematical problems involved. Thus, the use of the procedures developed makes it possible to utilize the techniques with a general understanding of the concepts involved.

As computerized trading becomes a way of life for investors each area of business management seeks to utilize the latest development in portfolio construction to improve enterprise operations. Problems that initially appeared impossible have become relatively manageable at relatively low cost by efficient computing procedures.

The portfolio selection in this study seeks to minimize the variance of a given expected return.

The following procedure is used to determine the optimal weight for each security in its repective portfolio, minimizing the variance for a given expected return. The basic equations are from Markowitz's original[1] paper:

Equation 1

$$\sum_{i=1}^{N} W_i = 1$$

$$W_1 + W_2 + W_3 + W_4 = 1$$

Where:

$$W_i > 0 \text{ (for } i = 1 \ldots 4)$$

W_i = Proportion or individual investment of security i in the total investment portfolio

Note: This restriction, that the W's be nonnegative, excludes short sales or borrowing as the case may be. This restriction is consistent with the investment assumptions as short-selling and buying on margin are almost exclusively speculative transactions. Here the purpose is to analyze the holding of a positive stock portfolio.

The Sum of the individual investments or total amount of investable funds in the portfolio equals one.

Equation 2

$$E = \sum_{i=1}^{N} E(R_i)W_i$$

$$E = W_1 E(R_1) + W_2 E(R_2) + W_3 E(R_3) + W_4 E(R_4)$$

Where:

E = The expected return from the four securities, required by the investor's taste for risk.

$W_i E(R_i)$ = The expected yield of each security multiplied by the proportion of funds invested on it.

Equation 3

$$V = W_1^2 \sigma_{11} + W_2^2 \sigma_{22} + W_3^2 \sigma_{33} + W_4^2 \sigma_{44} + 2W_1 W_2 \sigma_{12} + 2W_1 W_3 \sigma_{13}$$

$$+ 2W_1 W_4 \sigma_{14} + 2W_2 W_3 \sigma_{23} + 2W_2 W_4 \sigma_{24} + 2W_3 W_4 \sigma_{34}$$

Where:

V = Variance Portfolio

σ_{11} = Variance of Security 1

.
.
.

σ_{44} = Variance of Security 4

σ_{12} = Covariance of returns between securities 1 & 2

.
.
.

σ_{34} = Covariance of returns between securities 3 & 4

The problem is to minimize portfolio variance given the restrictions imposed in equations 1 and 2. In Markowitz's original paper these equations are solved via quadratic programing. However, for the present study they are solved linearly.

The minimization of a restricted multivariate function may be solved by introducing Lagrangian multipliers and differentiating partially with respect to each variable as follows: (derived from equation 3)

$$\phi = V + \lambda_1 \left(\sum_{i=1}^{4} W_i E(R_i) - E\right) + \lambda_2 \left(\sum_{i=1}^{4} W_i - 1\right)$$

$$\frac{d\phi}{dW_1} = 2W_1\sigma_{11} + 2W_2\sigma_{12} + 2W_3\sigma_{13} + 2W_4\sigma_{14} + \lambda_1 E(R_1) + \lambda_2$$

$$= W_1\sigma_{11} + W_2\sigma_{12} + W_3\sigma_{13} + W_4\sigma_{14} + 1/2\,\lambda_1 E(R_1)$$

$$+ 1/2\,\lambda_2 = 0$$

$$\frac{d\phi}{dW_2} = W_1\sigma_{12} + W_2\sigma_{22} + W_3\sigma_{23} + W_4\sigma_{24} + 1/2\,\tilde{1}_1 E(R_2) + 1/2\,\tilde{1}_2 = 0$$

$$\frac{d\phi}{dW_3} = W_1\sigma_{13} + W_2\sigma_{23} + W_3\sigma_{33} + W_4\sigma_{34} + 1/2\,\tilde{1}_1 E(R_3) + 1/2\,\tilde{1}_2 = 0$$

$$\frac{d\phi}{dW_4} = W_1\sigma_{14} + W_2\sigma_{24} + W_3\sigma_{34} + W_4\sigma_{44} + 1/2\,\tilde{1}_1 E(R_4) + 1/2\,\tilde{1}_2 = 0$$

Now, the equations may be expressed as a system of six linear equations

$$W_1\sigma_{11} + W_2\sigma_{12} + W_3\sigma_{13} + W_4\sigma_{14} + 1/2\,\tilde{1}_1 E(R_1) + 1/2\,\tilde{1}_2 = 0$$

$$W_1\sigma_{12} + W_2\sigma_{22} + W_3\sigma_{23} + W_4\sigma_{24} + 1/2\,\tilde{1}_1 E(R_2) + 1/2\,\tilde{1}_2 = 0$$

$$W_1\sigma_{13} + W_2\sigma_{23} + W_3\sigma_{33} + W_4\sigma_{34} + 1/2\,\tilde{1}_1 E(R_3) + 1/2\,\tilde{1}_2 = 0$$

$$W_1\sigma_{14} + W_2\sigma_{24} + W_3\sigma_{34} + W_4\sigma_{44} + 1/2\,\tilde{1}_1 E(R_4) + 1/2\,\tilde{1}_2 = 0$$

$$W_1 E(R_1) + W_2 E(R_2) + W_3 E(R_3) + W_4 E(R_4) = E$$

$$W_1 + W_2 + W_3 + W_4 = 1$$

The problem has been defined mathematically; E will be replaced by the investor's wanted return and, by solving the six equations in six unknowns, the solution at an optimal allocation of funds among the four investments will be reached.

APPENDIX B

SINGLE DIVISION PROCEDURE FOR THE SOLUTION OF LINEAR EQUATIONS

Appendix B is used to study the method selected to solve the set of linear equations presented in chapters III and IV. Its purpose is to describe the method selected, the justification for using this method, and to present some generalities.

The method used to solve the linear equations is the Single Division method[1]. It is an approximate method. Methods that yield exact solutions (multiplications and subtractions with exact division method of determinants) have one serious drawback: machine capacity which on small computers this is soon exhausted. Thus, we are practically forced to use approximation methods. The machine use here, to calculate the linear equation is a Sharp Scientific Computer Model EL-5500 II.

Division methods are commonly used for the direct reduction of simultaneous equations. In fact, single division methods, applied in this study, provide remarkable speed since each recorded value is the result of an operational unit and can be computed without clearing the products register.

Most of the data provided in Moody's Handbook, from which most of the variable values in these linear equations were derived, had at most five digits and, for practical

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1. Note: generalities of this method are condensed from Dwyer, Paul, Linear Computations, John Wiley and Sons, Inc. New York 1951

interpretation of the final results (W_1 .... W_4), we usually do not need more than seven places ($10,000.00 is the amount assumed to be invested). Thus, we do not need to bother with a number containing many digits. The desired number of places (given machine capacity) used in this study was 11, by using incomplete numbers. This is reasonable because the problem is an approximate one. This introduction of approximate operations is not, as it first appears, in direct violation of Dwyer's argument that approximate operations should be deferred as long as possible. In this single division method, the calculational design calls for the deliberate sacrifice of exactness for the purpose of calculation ease, particularly for those problems in which exact methods become impractical because of machine capacity limitations.

In the single division method, it is conventional to use the first equation as the one to be divided by its leading coefficient. It should be different from zero. The variables are eliminated in 1, 2, 3, 4 order. The method is described in general in table B-2.

The linear equations are solved as follows: First, table B-1 is set up from the linear equations from Appendix A (page 131) that are to be solved using the method:

Step (1) All linear equations are transferred from Appendix A. (Page 131)

TABLE B-1

Linear Equations required to support the Constraints stipulated in this Study.

$$W_1\sigma_{11} + W_2\sigma_{12} + W_3\sigma_{13} + \tfrac{1}{2}\lambda_1 E(R_1) + \tfrac{1}{2}\lambda_2 + W_4\sigma_{14} = 0$$

$$W_1\sigma_{12} + W_2\sigma_{22} + W_3\sigma_{23} + \tfrac{1}{2}\lambda_1 E(R_2) + \tfrac{1}{2}\lambda_2 + W_4\sigma_{24} = 0$$

$$W_1\sigma_{13} + W_2\sigma_{23} + W_3\sigma_{33} + \tfrac{1}{2}\lambda_1 E(R_3) + \tfrac{1}{2}\lambda_2 + W_4\sigma_{34} = 0$$

$$W_1 E(R_1) + W_2 E(R_2) + W_3 E(R_3) + 0 + 0 + W_4 E(R_4) = E$$

$$W_1 + W_2 + W_3 + 0 + 0 + W_4 = 1$$

$$W_1\sigma_{14} + W_2\sigma_{24} + W_3\sigma_{34} + \tfrac{1}{2}\lambda_1 E(R_4) + \tfrac{1}{2}\lambda_2 + W_4\sigma_{44} = 0$$

Derived from Appendix A, Page 130 & 131

TABLE B - 2

Method of Single Division-Symetric adjusted to the present study

STEP	W_1	W_2	W_3	$\frac{1}{2}\tilde{1}_1$	$\frac{1}{2}\tilde{1}_2$	W_4	RHS
(1)	A_{11}	A_{12}	A_{13}	$E(R_1)$	1	A_{14}	0
	A_{12}	A_{22}	A_{23}	$E(R_2)$	1	A_{24}	0
	A_{13}	A_{32}	A_{33}	$E(R_3)$	1	A_{34}	0
	$E(R_1)$	$AE(R_2)$	$E(R_3)$	0	0	$E(R_4)$	E
	A_1	A_1	A_1	0	0	A_1	1
	A_{14}	A_{24}	A_{34}	$E(R_4)$	1	A_{44}	0
(2)	1	b_{12}	b_{13}	$b(R_1)$	b_1	b_{14}	b_o
(3)		$g_{22.1}$	$g_{23.1}$	$gE(R_2)_{.1}$	$g_{1.1}$	$g_{24.1}$	0
		*	$g_{33.1}$	$gE(R_3)_{.1}$	$g_{1.1}$	$g_{34.1}$	0
		*	*	$g_{0.1}$	$g_{0.1}$	$g(R_4)_{.1}$	E
		*	*	*	$g_{o.1}$	$g_{1.1}$	1
		*	*	*	*	$g_{44.1}$	0
(4)		1	$b_{23.1}$	$bE(R_2)_{.1}$	$b_{1.1}$	$b_{24.1}$	0
(5)			$g_{33.12}$	$gE(R_3)_{.12}$	$g_{1.12}$	$g_{34.12}$	0
			*	$g_{0.12}$	$g_{0.12}$	$g(R_4)_{12}$	E
			*	*	$g_{0.12}$	$g_{1.12}$	1
(6)			1	$bE(R_3)_{.12}$	b_{1112}	$b_{34.12}$	0

TABLE B-2

Method of Single Division-Symetric adjusted to the present study--Continued

(7)	$g_{0.123}$	$g_{0.123}$	$g(R_4)_{.123}$	E
	*	$g_{0.123}$	$g_{1.123}$	1
	*	*	$g_{44.123}$	0
(8)	1	$bc_{0.123}$	$b(R_4)_{.123}$	0
(9)		$g_{0.1234}$	$g_{1.1234}$	$gE_{.4}$
		*	$g_{44.1234}$	$g_{1.4}$
(10)		1	$b_{1.1234}$	$bE_{.4}$
(11)			$g_{44.12345}$	$g_{1.45}$
(12)			1	$b_{1.45}$

Step (2) The b(s) value come from dividing the first equation by a_{11} and letting, for example,

$$\frac{a_{1i}}{a_{11}} = b_{1i}, \text{ and } \frac{E(R_1)}{a_{11}} = b(R_1), \text{ etc.}$$

Step (3) let $g_{ig.1} = a_{ij} - a_{i1} b_{1j}$

Step (4) The b(s) values are obtained by dividing the first equation of step 4 by $g_{22.1}$ letting, for example,

$$\frac{g_{23.1}}{g_{22.1}} = b_{23.1}, \text{ etc.}$$

Step (5) Let $g_{ij.12} = g_{ij.1} - g_{i2.1} b_{2j.1}$

Step (6) The b(s) values are obtained by dividing the first equation of step 5 by $g_{33.12}$ and letting, for example,

$$\frac{gE(R_{3.12})}{g_{33.12}} = bE(R_{3.12}), \text{ etc.}$$

Step (7) let $g_{ij.123} = g_{ij.123} - g_{i3.12} b_{3j.12}$ etc.

Step (8), (10)&(12) Follow the same characteristic procedure outlined in steps (3), (5) and (7)

Finally, table B-3 is the solution for the allocation of funds ($W_1 \ldots W_4$). This is carried out by substituting in the equations using the b's as coefficients. A symmetric form of the solution is solved in detail in tables 3-9 and 4-5 for energy and gold-minerals portfolios, respectively. One way to check the exactness of the results achieved using the single division method for solving the set of linear equations in the present study is that both the final sum of all the variables' values, W_1 through W_4, that are the proportions of each security in the portfolio, should equal one. The model constraint and the expected return of the portfolio should be equal to the sum of the expected return of each security multiplied by the proportion of funds invested in it, the second model constraint.

The single division method has been developed independently by many authors. Recently, Aitken, Banchiewicz, Dwyer, and Crout, all have emphasized the use of the method, or variations of it, in connection with non-symmetric problems.

In the present study the purpose is to address exactness up to 6 decimal places ($10,000) with respect to the model constraints and the specifications above. This is shown in tables 3-4, 3-5, 3-6, 3-7, 3-9, 4-4, 4-5, 4-6, 4-7 and 4-8. (See values of W_1 through W_4 and W_t).

TABLE B-3

Back Solution for the Variables Values

W_1	$+ b_{12} W_2$	$+ b_{13}W_3$	$+ b(R_1)(\frac{1}{2}\tilde{1}_1)$	$+ b_1(\frac{1}{2}\tilde{1}_2)$	$+ b_{14}W_4$	$= 0$
W_2	$+ b_{23.1}W_3$	$+ bE(R_2)_{.1}(\frac{1}{2}\tilde{1}_1)$	$+ b_{1.1}(\frac{1}{2}\tilde{1}_2)$	$+ b_{24.1}$	$+ W_4$	$= 0$
W_3	$+ bE(R_3)_{12}(\frac{1}{2}\tilde{1}_1)$	$+ b_{1.12}(\frac{1}{2}\tilde{1}_2)$	$+ b_{34.12}(W_4)$			$= 0$
$(\frac{1}{2} 1_1)$	$+ bc_{o.123}(\frac{1}{2}\tilde{1}_2)$	$+ b(R_4)_{.123} W_4$				$= 0$
$(\frac{1}{2} 1_2)$	$+ b_{1.1234} W_4$					$= bE_{.4}$
W_4						$= b_{1.45}$

APPENDIX C

ENERGY PORTFOLIO: COMPUTATION OF RETURN, MEAN RETURN AND VARIANCE OF RETURN FOR INDIVIDUAL SELECTED SAMPLE SECURITIES

(The calculation values have been derived from Moody's Handbook)

TABLE C-1

Standard Oil

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_s					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_s)$	$R_s - E(R_s)$	$[R_s - E(R_s)]^2$	P_s	$[R_s - E(R_s)]^2 P_s$
1970	10 1/2 - 6 5/8	0.6	0.34	14.3	8.58	-	-	-	-	-	-
1971	11 1/2 - 8 7/8	0.54	0.34	18.7	10.192	22.74	20.91	1.832	3.355	0.067	0.225
1972	12 2/8 - 7 1/2	0.41	0.34	24.3	9.963	1.09	20.91	(19.81)	392.77	0.067	26.32
1973	21 3/8 - 10 1/2	0.51	0.34	31.6	15.958	63.58	20.91	42.672	1820.86	0.067	121.99
1974	21 1/2 - 18 7/8	0.86	0.34	18.0	15.480	(0.865)	20.91	(21.77)	474.07	0.067	31.76
1975	21 3/8 - 11	0.86	0.34	19.0	16.340	7.75	20.91	(13.15)	173.14	0.067	11.600
1976	20 1/4 - 15 3/8	0.89	0.34	20.0	17.8	11.02	20.91	(9.888)	97.78	0.067	6.551
1977	22 7/8 - 17 1/4	0.91	0.34	22.1	20.111	14.89	20.91	(6.01)	36.22	0.067	2.426
1978	21 3/4 - 14 3/8	2.0	0.42	9.0	18.	(8.41)	20.91	(29.32)	859.45	0.067	57.58
1979	46 1/2 - 20	4.92	0.61	6.8	33.456	89.26	20.91	68.35	4671.94	0.067	313.02
1980	91 1/2 - 39 5/8	7.37	1.40	8.9	65.593	100.24	20.91	79.33	6293.50	0.067	421.66
1981	72 3/4 - 36 1/8	7.92	2.25	6.9	54.648	(13.26)	20.91	(34.16)	1167.48	0.067	78.22
1982	42 1/4 - 26 1/2	7.63	2.55	4.5	34.335	(32.50)	20.91	(53.40)	2852.46	0.067	191.114
1983	58 7/8 - 35	6.14	2.60	7.7	47.278	45.27	20.91	24.36	593.48	0.067	39.76
1984	50 1/2 - 40	6.14	2.65	7.4	45.436	1.71	20.91	(19.19)	368.58	0.067	24.69
1985	55 3/4 - 39 5/8	1.31	2.80	36.4	47.684	11.11	20.91	9.79	96.00	0.062	5.95
						20.91				1.00	1332.866

$$R_s = \frac{(P_1 - P_2) + D_1}{P_o}$$

$$\bar{R} = E(R_s) = \sum_{i=1}^{N} \frac{R_s}{N} = 20.91$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} \left[R_i - E(R_i)\right]^2 P_i$$

$$\text{Standard Deviation} = \sqrt{\sum_{i=1}^{N} \left[R_i - E(R_i)\right]^2 P_i}$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R_s = Annual Return

R_s = Annual Return

N = # of years

$\bar{R}$ = Mean Return

$E(R_s)$ = Expected Return

Variance (σ^2) = 1332.866

Standard Deviation (σ) = 36.508

TABLE C-2

Exxon Corporation

	DETERMINE RETURN						DETERMINE VARIANCE				
YEARS	PRICE RANGE	E EARN/SHARE	D DIV/SHARE	P/E RATIO	P PRICE	R_E RETURN	$E(R_E)$	$R_E-E(R_E)$	$[R_E-E(R_E)]^2$	P	$[R_E-E(R_E)]^2P$
1970	18 3/8-12 1/2	1.465	0.94	10.5	15.3825	-	-	-	-	-	-
1971	20 5/8-16 3/4	1.695	0.95	11.0	18.645	27.39	16.6	10.79	116.424	0.067	7.800
1972	22 3/8-17	1.71	0.95	11.5	19.665	10.57	16.6	(6.03)	36.36	0.067	2.436
1973	25 5/8-21	2.725	1.065	8.6	23.435	24.59	16.6	7.99	63.84	0.067	4.277
1974	25 -13 3/4	3.51	1.25	5.5	19.305	(12.29)	16.6	(28.89)	834.632	0.067	55.920
1975	23 1/2-16 1/4	2.80	1.25	7.1	19.88	9.14	16.6	(7.46)	55.652	0.067	3.729
1976	28 1/2-21 3/8	2.95	1.37	8.4	24.78	31.94	16.6	15.34	235.316	0.067	15.766
1977	27 7/8-22 3/8	2.69	1.50	9.4	24.286	8.10	16.6	(8.5)	72.25	0.067	4.841
1978	26 7/8-21 1/2	3.10	1.65	7.8	24.18	2.15	16.6	(14.45)	208.80	0.067	13.989
1979	30 5/8-24 3/8	4.87	1.95	5.6	27.272	20.85	16.6	4.25	18.063	0.067	1.210
1980	44 3/8-26	6.51	2.70	5.4	35.154	38.80	16.6	22.2	492.84	0.067	33.020
1981	41 -29 1/2	6.43	3.00	5.5	35.365	9.13	16.6	(7.47)	55.80	0.067	3.739
1982	32 1/4-24 7/8	4.82	3.00	6.0	28.920	(9.74)	16.6	(26.34)	693.80	0.067	46.484
1983	39 3/4-28 1/2	5.79	3.10	5.9	34.161	28.84	16.6	12.24	149.817	0.067	10.038
1984	45 1/2-36 1/8	6.77	3.35	6.0	40.62	28.71	16.6	12.11	146.652	0.067	9.8257
1985	55 7/8-44 1/8	6.46	3.45	7.7	49.74	30.95	16.6	14.35	205.92	0.062	12.767
										1.00	225.841

$$R = \frac{(P_1 - P_2) + D_1}{P_o}$$

$$\bar{R}_E = E(R_E) = \frac{\sum_{i=1}^{N} R}{N} = 16.6$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R = Annual Return

R_E = Annual Return

N = # of years

$\bar{R}_E$ = Mean Return

$E(R_E)$ = Expected Return

$$\text{Standard Deviation} = \sigma^2 = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$$

Variance = σ^2 = 225.841

Standard Deviation = σ = 15.028

TABLE C-3

Pennzoil Company

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_z					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_z)$	$R_z - \bar{E}(R_z)$	$[R_z - \bar{E}(R_z)]^2$	P_z	$[R_z - E(R_z)]^2 P$
1970	24 - 10	1.98	0.53	8.7	17.226	-	-	-	-	-	-
1971	26 1/8 - 12 3/8	0.97	0.53	19.7	19.109	14.01	15.31	(1.3)	1.69	0.067	0.113
1972	18 5/8 - 12 3/8	1.20	0.53	13.0	15.6	(15.59)	15.31	(30.9)	954.81	0.067	63.972
1973	21 5/8 - 11 7/8	1.62	0.53	10.2	16.524	9.32	15.31	(5.99)	35.880	0.067	2.403
1974	20 3/8 - 8	2.45	0.70	5.9	14.455	(8.28)	15.31	(23.59)	556.488	0.067	37.285
1975	15 7/8 - 11 3/8	2.03	0.80	6.7	13.601	(0.37)	15.31	(15.68)	245.862	0.067	16.473
1976	22 5/8 - 12 5/8	2.88	0.87	6.2	17.856	37.68	15.31	22.37	500.417	0.067	33.528
1977	23 7/8 - 17 3/8	2.23	1.13	9.2	20.516	21.23	15.31	5.92	35.046	0.067	2.348
1978	22 1/4 - 17 5/8	2.49	1.33	8.0	19.92	3.58	15.31	(11.73)	137.592	0.067	9.219
1979	41 5/8 - 20 7/8	4.69	1.50	6.7	31.423	65.28	15.31	49.97	2497.001	0.067	167.299
1980	62 1/2 - 30	5.90	2.0	7.8	46.02	52.82	15.31	37.51	1407.00	0.067	94.269
1981	58 1/4 - 35 1/4	4.23	2.1	11.1	46.95	6.58	15.31	(8.73)	76.21	0.067	5.106
1982	50 1/4 - 23 5/8	3.00	2.2	10.3	37.08	(16.34)	15.31	(31.65)	1001.723	0.067	67.115
1983	42 1/2 - 31 1/4	3.03	2.2	12.2	36.96	5.69	15.31	(9.62)	92.544	0.067	6.200
1984	45 3/4 - 30 3/4	3.89	2.2	9.8	38.12	9.09	15.31	(6.22)	38.688	0.067	2.592
1985	72 - 34	3.96	2.2	13.4	53.06	44.96	15.31	29.65	879.123	0.062	54.506
										1.000	562.428

$$R = \frac{(P_1 - P_2) + D_1}{P_o}$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R = Annual Return

$$\bar{R}_z = E(R_z) = \sum_{i=1}^{N} \frac{R}{N} = 15.31$$

R_z = Annual Return

N = # of years

$\bar{R}_z$ = Mean Return

$E(R_z)$ = Expected Return

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$$

$$\text{Standard Deviation} = \sigma = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$$

Variance = σ^2 = 562.428

Standard Deviation = σ = 23.715

TABLE C-4

Mobil Corporation

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_M					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_M)$	$R_M-E(R_M)$	$[R_M-E(R_M)]^2$	P_M	$[R_M-E(R_M)]^2 P_M$
1970	14 1/2 - 9	1.20	0.60	9.9	11.88	-	-	-	-	-	-
1971	15 - 11 1/2	1.34	0.64	9.9	13.27	17.09	14.54	2.55	6.502	0.067	0.436
1972	19 - 12 1/2	1.42	0.67	11.1	15.76	23.81	14.54	9.27	85.93	0.067	5.756
1973	19 - 10 1/2	2.09	0.70	7.1	14.84	(1.39)	14.54	(15.93)	253.765	0.067	17.00
1974	14 1/8 - 7 3/4	2.57	0.80	4.2	10.80	(21.83)	14.54	(36.37)	1322.777	0.067	88.626
1975	12 1/4 - 8 1/2	2.00	0.85	5.2	10.40	4.17	14.54	(10.37)	107.537	0.067	7.205
1976	16 1/8 - 11 7/8	2.27	0.88	6.2	14.07	43.75	14.54	29.21	853.224	0.067	57.166
1977	17 3/4 - 14 1/2	2.38	0.98	7.1	16.90	27.08	14.54	12.54	157.252	0.067	10.536
1978	18 1/8 - 14 1/2	2.66	1.08	6.1	16.23	2.43	14.54	(12.11)	146.652	0.067	9.826
1979	30 1/4 - 17	4.73	1.28	5.0	23.65	53.60	14.54	39.06	1525.684	0.067	102.221
1980	44 3/4 - 24 7/8	6.62	1.73	5.3	35.09	55.69	14.54	41.15	1693.323	0.067	113.453
1981	41 1/4 - 24 1/8	5.72	2.00	5.7	32.60	(13.96)	14.54	(28.5)	812.25	0.067	54.421
1982	28 5/8 - 19 1/2	3.31	2.00	7.3	24.16	(19.76)	14.54	(34.3)	1176.49	0.067	78.825
1983	34 3/8 - 24 1/8	3.69	2.00	8.0	29.52	30.46	14.54	15.92	253.446	0.067	16.981
1984	32 1/8 - 23 1/8	3.12	2.20	8.9	27.77	1.52	14.54	(13.02	169.52	0.067	11.358
1985	34 3/8 - 25 1/2	2.55	2.20	11.7	29.84	15.37	14.54	0.83	0.689	0.062	0.043
										1.00	573.853

$$R_M = \frac{(P_1 - P_2) + D_1}{P_o}$$

$$\bar{R} = E(R) = \frac{\sum_{i=1}^{N} R_M}{N} = 14.54$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$$

$$\text{Standard Deviation} = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R_M = Annual Return

R_M = Annual Return

N = # of years

$\bar{R}$ = Mean Return

$E(R_M)$ = Expected Return

Variance σ^2 = 573.853

Standard Deviation σ = 23.955

TABLE C-5

Phillips Petroleum Company

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_p					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_p)$	$R_p-E(R_p)$	$[R_p-E(R_p)]^2$	P_p	$[R_p-E(R_p)]^2P$
1970	5 1/8 - 3 3/8	0.26	0.22	16.1	4.186	-	-	-	-	-	-
1971	5 7/8 - 4 1/2	0.30	0.22	17.1	5.13	27.81	14.36	13.45	180.903	0.067	12.1205
1972	7 1/2 - 4 3/8	0.33	0.22	17.9	5.907	19.44	14.36	5.08	25.806	0.067	1.7290
1973	11 3/4 - 6 3/4	0.51	0.22	18.2	9.282	60.86	14.36	46.50	2162.25	0.067	144.8708
1974	11 7/8 - 5 1/4	0.94	0.25	9.1	8.554	(5.13)	14.36	(19.49)	379.86	0.067	25.4506
1975	10 - 6 1/8	0.75	0.43	10.8	8.1	(0.28)	14.36	(14.64)	214.3472	0.067	14.3613
1976	11 - 8 1/4	0.90	0.29	10.7	9.63	22.47	14.36	8.11	65.772	0.067	4.4067
1977	12 5/8 - 8 1/4	1.12	0.33	9.4	10.528	12.75	14.36	(1.61)	2.592	0.067	0.1737
1978	12 1/8 - 8 7/8	1.54	0.40	6.8	10.472	3.27	14.36	(11.09)	122.988	0.067	8.2402
1979	16 7/8 - 9 7/8	1.92	0.45	6.9	13.248	30.81	14.36	16.45	270.603	0.067	18.1304
1980	20 7/8 - 12 1/2	2.34	0.60	7.1	16.614	29.94	14.36	15.58	242.736	0.067	16.263
1981	19 3/4 - 11 3/8	1.93	0.73	8.1	15.633	(1.511)	14.36	(15.871)	251.889	0.067	16.8765
1982	13 5/8 - 7 3/4	1.41	0.73	7.6	10.716	(26.78)	14.36	(41.14)	1692.499	0.067	113.3974
1983	12 7/8 - 9 3/4	1.57	0.73	7.2	11.304	12.30	14.36	(2.06)	4.244	0.067	0.2843
1984	18 3/4 - 11 1/8	1.75	0.78	8.5	14.875	38.49	14.36	24.13	582.257	0.067	39.0112
1985	17 1/8 - 11	2.07	0.95	6.8	12.586	(9.00)	14.36	(23.36)	545.689	0.062	33.8327
										1.00	449.148

$$R_p = \frac{(P_1 - P_2) + D_1}{P_o}$$

$$R_p = E(R_p) = \sum_{i=1}^{N} \frac{R_p}{N} = 14.36$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R = Annual Return

R_p = Annual Return

N = # of years

$\bar{R}_p$ = Mean Return

$E(R_p)$ = Expected Return

$$\text{Standard Deviation} = \sigma = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$$

Variance = σ^2 = 449.148

Standard Deviation = σ = 21.19

TABLE C-6

Tenneco Incorporated

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_t					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_t)$	$R_t - E(R_t)$	$[R_t - E(R_t)]^2$	P_t	$[R_t - E(R_t)]^2 P_t$
1970	24 7/8 - 17 1/2	1.88	1.32	11.3	21.244	-	-	-	-	-	-
1971	29 5/8 - 21 5/8	2.27	1.32	11.3	25.651	26.958	13.19	13.788	189.558	0.067	12.700
1972	29 5/8 - 23	2.50	1.33	10.5	26.25	7.520	13.19	(5.67)	32.149	0.067	2.154
1973	30 3/8 - 19 3/4	2.86	1.38	8.8	25.168	1.130	13.19	(12.055)	175.323	0.067	9.737
1974	24 3/4 - 16 3/4	4.08	1.52	5.1	20.808	(11.284)	13.19	(24.474)	598.977	0.067	40.131
1975	27 3/8 - 19 3/4	4.15	1.66	5.7	23.655	21.659	13.19	8.469	71.724	0.067	4.806
1976	37 3/8 - 26	4.33	1.82	9.3	40.269	77.928	13.19	64.738	4191.009	0.067	280.798
1977	37 1/4 - 28 1/2	4.38	1.94	7.5	32.85	(13.605)	13.19	26.795	717.972	0.067	48.104
1978	34 1/2 - 28	4.53	2.05	6.9	31.257	1.391	13.19	(11.799)	139.216	0.067	9.327
1979	41 1/2 - 29	5.30	2.25	6.7	35.51	20.804	13.19	7.614	57.973	0.067	3.884
1980	58 3/8 - 31 1/4	5.95	2.45	7.6	45.22	34.243	13.19	21.053	443.229	0.067	29.696
1981	51 7/8 - 29 7/8	6.01	2.00	6.8	40.868	(5.201	13.19	(18.391)	338.229	0.067	22.661
1982	36 1/2 - 27 7/8	5.90	2.63	5.2	30.68	(18.493)	13.19	(31.683)	1003.812	0.067	67.255
1983	42 3/8 - 31 7/8	4.75	2.74	7.8	37.05	29.693	13.19	16.503	272.349	0.067	18.247
1984	41 3/4 - 32 7/8	4.01	2.83	9.7	38.897	12.623	13.19	(.567)	0.321	0.067	0.021
1985	45 1/4 - 36 1/2	2.52	2.95	16.2	40.824	12.538	13.19	(.652)	0.425	0.062	0.026
										1.00	549.547

$$R_t = \frac{(P_1 - P_2) + D_1}{P_o}$$

$$R = E(R_t) = \sum_{i=1}^{N} \frac{R_t}{N} = 13.19$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} \left[R_i - E(R_i) \right]^2 P_i$$

$$\text{Standard Deviation} = \sqrt{\sum_{i=1}^{N} \left[R_i - E(R_i) \right]^2 P_i}$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R_t = Annual Return

R = Annual Return

N = # of years

$\bar{R}$ = Mean Return

$E(R_t)$ = Expected Return

Variance $(\sigma)^2$ = 549.547

Standard Deviation (σ) = 23.44

TABLE C-7

Occidental Petroleum Corporation

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_o					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_o)$	$R_o - E(R_o)$	$[R_o - E(R_o)]^2$	P_o	$[R_o - E(R_o)]^2 P$
1970	26 3/8 - 13	2.88	0.99	6.8	19.584	-	-	-	-	-	-
1971	22 3/4 - 9 7/8	1.22	1.00	-	16.3	(11.663)	11.83	(23.497)	552.1090	0.067	36.9913
1972	18 1/4 - 10	0.01	0.13	-	14.125	(12.546)	11.83	(24.38)	594.384	0.067	39.8237
1973	13 1/2 - 7 3/4	1.10	-	9.7	10.67	(24.46)	11.83	(36.294)	1317.254	0.067	88.2560
1974	14 5/8 - 7 3/8	4.74	-	2.3	10.90	2.156	11.83	(9.678)	93.663	0.067	6.2754
1975	22 5/8 - 12 3/8	2.65	1.0	6.9	18.285	76.926	11.83	65.09	4237.046	0.067	283.882
1976	24 1/2 - 13 1/2	2.76	1.0	6.9	19.044	9.61	11.83	(2.224)	4.9461	0.067	0.3314
1977	31 1/8 - 21	2.92	1.13	9.1	26.972	45.463	11.83	33.63	1130.977	0.067	75.775
1978	26 3/8 - 14 7/8	0.39	1.25	-	20.612	(17.725)	11.83	(29.55)	873.734	0.067	58.54
1979	29 5/8 - 15 3/4	7.30	1.25	3.1	22.63	15.855	11.83	4.021	16.168	0.067	1.083
1980	39 1/8 - 19	8.82	1.75	3.3	29.106	36.349	11.83	24.515	600.985	0.067	40.266
1981	34 7/8 - 21 5/8	7.77	2.35	3.6	27.972	4.177	11.83	(7.656)	58.6174	0.067	3.9274
1982	24 3/4 - 17	0.69	2.50	30.3	20.907	(16.319)	11.83	(28.15)	792.541	0.067	53.103
1983	27 1/2 - 21 1/4	1.15	2.50	21.2	24.38	28.569	11.83	16.73	280.06	0.067	18.764
1984	35 3/4 - 24 3/8	3.08	2.30	9.8	30.18	33.22	11.83	21.386	457.36	0.067	30.643
1985	36 3/4 - 23 1/8	4.49	2.50	6.7	30.083	7.962	11.83	(3.872)	14.99	0.062	0.9295
										1.00	738.590

$$R_o = \frac{(P_1 - P_2) + D_1}{P_o}$$

$$R_o = E(R_o) = \frac{\sum_{i=1}^{N} R_o}{N} = 11.83$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R_o = Annual Return

R_o = Annual Return

N = # of years

$\bar{R}_o$ = Mean Return

$E(R_o)$ = Expected Return

$$\text{Standard Deviation} = \sigma = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$$

Variance = σ^2 = 738.590

Standard Deviation = σ = 27.177

TABLE C-8

Texaco Incorporated

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_x					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_x)$	$R_x-E(R_x)$	$[R_x-E(R_x)]^2$	P_x	$[R_x-E(R_x)]^2P$
1970	35 1/2 - 24	3.02	1.60	9.9	29.898	-	-	-	-	-	-
1971	39 5/8 - 29 5/8	3.32	1.60	10.4	34.528	20.838	9.71	11.128	123.832	0.067	8.297
1972	39 1/8 - 29 1/4	3.27	1.66	10.5	34.335	4.249	9.71	(5.461)	29.823	0.067	1.998
1973	43 1/8 - 25	4.75	1.73	7.2	34.200	4.645	9.71	(5.065)	25.654	0.067	1.719
1974	32 7/8 - 20	5.84	2.10	4.5	26.280	(17.018)	9.71	(26.728)	714.386	0.067	47.864
1975	28 3/8 - 21 1/8	3.06	2.00	8.1	24.786	1.925	9.71	(7.785)	60.606	0.067	4.06
1976	28 3/4 - 23 5/8	3.20	2.00	8.1	25.970	12.644	9.71	2.934	8.608	0.067	0.577
1977	30 5/8 - 25 3/4	3.43	2.00	8.2	28.126	16.227	9.71	6.517	42.471	0.067	2.846
1978	27 3/4 - 22 1/2	3.14	2.00	7.9	24.806	(4.693)	9.71	(14.403)	207.446	0.067	13.899
1979	32 1/8 - 23 3/4	6.48	2.12	4.3	27.864	20.874	9.71	11.164	124.635	0.067	8.351
1980	54 3/8 - 27 1/2	8.31	2.45	5.2	43.212	63.875	9.71	54.165	2933.847	0.067	196.568
1981	49 1/8 - 31 1/2	8.75	2.8	4.6	40.250	(.374)	9.71	(10.084)	101.687	0.067	6.813
1982	34 7/8 - 26	4.92	3.00	6.2	30.504	(16.760)	9.71	(26.47)	700.661	0.067	46.944
1983	39 1/8 - 30 1/2	4.80	3.00	7.3	35.04	24.704	9.71	14.994	224.82	0.067	15.063
1984	48 3/4 - 31 1/2	1.03	3.00	38.8	39.964	22.614	9.71	12.904	166.513	0.067	11.156
1985	40 1/4 - 27	5.11	3.00	6.6	33.726	(8.102)	9.71	(17.812)	317.267	0.062	19.671
										1.00	385.826

$$R_x = \frac{(P_1 - P_2) + D_1}{P_o}$$

$$\bar{R} = (R_x) = \sum_{i=1}^{N} \frac{R_x}{N} = 9.71$$

$$\text{Variance } 6^2 = \sum_{i=1}^{N} \left[R_i - E(R_i)\right]^2 P_i$$

$$\text{Standard Deviation} = \sqrt{\sum_{i=1}^{N} \left[R_i - E(R_i)\right]^2 P_i}$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R_x = Annual Return

R_x = Annual Return

N = # of years

$\bar{R}$ = Mean Return

$E(R_x)$ = Expected Return

Variance $(6)^2$ = 385.826

Standard Deviation (6) = 19.64

TABLE C-9

Kerr McGee Corporation

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_k					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_k)$	$R_k - E(R_k)$	$[R_k - E(R_k)]^2$	P_k	$[R_k - E(R_k)]^2 P$
1970	19 1/8 - 9 7/8	0.79	0.25	18.1	14.3	-	-	-	-	-	-
1971	24 5/8 - 15 7/8	0.89	0.29	22.9	20.38	44.55	9.53	35.02	1226.400	0.067	82.169
1972	33 1/4 - 19	1.07	0.30	24.4	26.108	29.58	9.53	20.05	402.00	0.067	26.934
1973	48 - 26 1/2	1.26	0.30	29.5	37.17	43.52	9.53	33.99	1155.32	0.067	77.406
1974	46 1/4 - 23 1/2	2.32	0.38	15.0	34.8	(5.35)	9.53	(14.88)	221.414	0.067	14.834
1975	47 1/2 - 30	2.58	0.50	15.1	38.96	13.39	9.53	3.86	14.899	0.067	0.998
1976	41 3/8 - 30 1/4	2.60	0.57	13.8	35.88	(6.44)	9.53	(15.97	255.04	0.067	17.087
1977	37 1/4 - 22 1/2	2.31	0.63	13.0	30.03	(14.55)	9.53	(24.08)	579.846	0.067	38.849
1978	26 5/8 - 19 7/8	2.29	0.63	10.2	23.36	(20.11)	9.53	(29.64)	878.529	0.067	58.86
1979	34 1/4 - 23	3.09	0.74	9.3	28.74	26.20	9.53	16.67	277.89	0.067	18.619
1980	46 1/2 - 26	3.51	0.87	10.3	36.15	28.81	9.53	19.28	371.712	0.067	24.905
1981	43 7/8 - 30 3/4	4.07	0.98	9.2	37.44	6.28	9.53	(3.25)	10.562	0.067	0.708
1982	38 1/2 - 22 1/2	3.98	1.10	7.7	30.65	(15.2)	9.53	(24.73)	611.573	0.067	40.975
1983	37 1/2 - 27 1/8	2.24	1.10	14.4	32.26	8.84	9.53	(0.69)	0.476	0.067	0.032
1984	36 3/8 - 26 3/8	1.22	1.10	25.7	31.35	0.59	9.53	(8.94)	79.92	0.067	5.350
1985	36 - 26 1/4	2.62	1.10	11.9	31.18	2.97	9.53	(6.56)	43.03	0.062	2.669
										1.00	410.395

$$R_k = \frac{(P_1 - P_2) + D_1}{P_o}$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R = Annual Return

$$R_k = E(R_k) = \sum_{i=1}^{N} \frac{R_k}{N} = 9.53$$

R_k = Annual Return

N = # of years

$\bar{R}_k$ = Mean Return

$E(R_k)$ = Expected Return

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$$

$$\text{Standard Deviation} = \sigma = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$$

Variance = σ^2 = 410.395

Standard Deviation = σ = 20.258

APPENDIX D

GOLD-MINERAL PORTFOLIO: COMPUTATION OF RETURN, MEAN RETURN AND VARIANCE OF RETURN FOR INDIVIDUAL SELECTED SAMPLE SECURITIES

(The calculation values have been derived from Moody's Handbook)

TABLE D-1

Homestake Mining Company

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_H					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_H)$	$R_H-E(R_H)$	$[R_H-E(R_H)]^2$	P_H	$[R_H-E(R_H)]^2 P_H$
1970	4 5/8 - 2 3/4	0.10	0.07	36.1	3.100						
1971	5 1/4 - 2 5/8	0.09	0.07	45.1	4.059	33.193	28.37	4.823	23.261	0.067	1.559
1972	5 - 3	0.21	0.07	18.8	3.948	(1.01)	28.37	(29.38)	863.184	0.067	57.833
1973	11 7/8 - 3 7/8	0.67	0.07	11.7	7.839	100.329	28.37	71.959	5178.098	0.067	346.933
1974	23 1/4 - 9 3/4	1.00	0.42	16.4	16.4	114.568	28.37	86.198	7430.095	0.067	497.816
1975	18 1/2 - 10 1/2	0.71	0.42	40.1	28.471	76.164	28.37	77.794	2284.266	0.067	153.046
1976	14 3/4 - 8 1/4	0.65	0.34	17.9	11.635	(57.939)	28.37	(86.309)	7449.243	0.067	499.099
1977	14 5/8 - 11 1/2	0.76	0.42	17.2	13.072	15.96	28.37	(12.41)	154.008	0.067	10.319
1978	13 3/8 - 9 3/4	0.92	0.45	12.7	11.684	(7.175)	28.37	(35.545)	1263.447	0.067	84.651
1979	17 1/4 - 9 7/8	1.81	0.67	7.5	13.575	21.918	28.37	(6.452)	41.628	0.067	2.789
1980	44 - 13 1/4	2.98	0.71	9.6	28.608	115.97	28.37	87.6	7673.76	0.067	514.142
1981	35 1/2 - 18	0.84	0.65	31.7	26.628	(4.649)	28.37	(33.019)	1090.254	0.067	73.049
1982	27 7/8 - 8 3/8	0.51	0.20	35.5	18.105	(31.256)	28.37	(59.626)	3555.260	0.067	238.202
1983	37 3/4 - 23 1/2	1.08	0.20	28.4	30.672	70.516	28.37	42.146	1776.285	0.067	119.011
1984	36 3/4 - 20 1/2	0.59	0.20	48.5	28.615	(6.054)	28.37	(34.424)	1185.012	0.067	79.396
1985	28 3/4 - 20 1/8	0.47	0.20	51.3	24.14	(15.041)	28.37	(43.411)	1884.515	0.062	116.840
										1.00	2794.683

$$R_H = \frac{(P_1 - P_2) + D_1}{P_o}$$

$$\bar{R}_H = E(R_H) = \sum_{i=1}^{N} \frac{R_H}{N} = 28.37$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R = Annual Return

R_H = Annual Return

N = # of years

$\bar{R}_H$ = Mean Return

$E(R_H)$ = Expected Return

$$\text{Standard Deviation} = \sigma = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$$

Variance = σ^2 = 2794.683

Standard Deviation = σ = 52.865

TABLE D-2

International Mineral & Chemical

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_I					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_I)$	$R_I-E(R_I)$	$[R_I-E(R_I)]^2$	P_I	$[R_I-E(R_I)]^2P_I$
1970	07 5/8 - 04	0.09	-	52.8	4.752	-	-	-	-	-	-
1971	10 5/8 - 6 5/8	0.49	0.03	17.3	8.477	79.019	23.51	55.509	3081.249	0.067	206.444
1972	7 - 7 1/2	0.82	0.13	12.5	10.25	22.448	23.51	(1.062)	1.128	0.067	0.076
1973	20 1/8 - 10 1/8	1.05	0.21	14.4	15.12	49.560	23.51	26.05	678.603	0.067	45.466
1974	27 - 14 3/8	2.39	0.47	8.6	20.554	39.047	23.51	15.537	241.398	0.067	16.174
1975	32 1/2 - 20 5/8	6.61	1.09	4.0	26.44	33.939	23.51	10.429	108.764	0.067	7.287
1976	28 1/4 - 21 7/8	5.17	1.47	4.8	24.816	(0.582)	23.51	(24.092)	580.424	0.067	38.888
1977	29 3/4 - 23 1/2	4.06	1.67	6.5	26.39	13.072	23.51	(10.438)	108.952	0.067	7.300
1978	29 - 22 5/8	4.41	1.73	5.9	26.019	5.149	23.51	(18.361)	337.176	0.067	22.587
1979	41 - 23 1/2	4.48	1.93	7.2	32.256	31.388	23.51	7.878	62.063	0.067	4.158
1980	66 1/2 - 30	5.38	2.24	9.0	48.42	57.056	23.51	33.546	1125.334	0.067	75.397
1981	66 1/2 - 30 3/8	5.63	2.53	8.6	48.418	5.22	23.51	(18.29)	334.529	0.067	22.413
1982	36 7/8 - 23 1/2	4.56	2.60	6.7	30.552	(31.529)	23.51	(55.039)	3029.292	0.067	202.963
1983	41 - 31 3/8	3.02	2.60	12.0	36.24	27.127	23.51	3.617	13.083	0.067	0.877
1984	49 - 32 7/8	4.17	2.60	9.8	40.376	18.587	23.51	(4.923)	24.236	0.067	1.624
1985	44 - 34 1/4	4.39	2.60	8.9	39.071	3.207	23.51	(20.303)	412.212	0.062	25.557
										1.00	677.211

$$R_I = \frac{(P_1 - P_2) + D_1}{P_o}$$

$$\bar{R}_I = E(R_I) = \sum_{i=1}^{N} \frac{R_I}{N}$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R_I = Annual Return

R_I = Annual Return

N = # of years

$\bar{R}_I$ = Mean Return

$E(R_I)$ = Expected Return

Variance $\sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$

Standard Deviation $= \sigma = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$

Variance $= \sigma^2 = 677.211$

Standard Deviation $= \sigma = 26.023$

TABLE D-3

Amoco Corporation

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_{Ac}					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_{A_c})$	$R_{A_c}-E(R_{A_c})$	$[R_{A_c}-E(R_{A_c})]^2$	P_{A_c}	$[R_{A_c}-E(R_{A_c})]^2 P_{A_c}$
1970	13 1/2 - 8 1/8	1.14	0.58	9.7	11.058						
1971	17 1/2 - 13	1.24	0.58	12.3	12.252	16.042	20.16	(4.118)	16.958	0.067	1.136
1972	22 1/8 - 15 1/4	1.35	0.60	13.9	18.765	58.055	20.16	37.895	1436.031	0.067	96.214
1973	26 - 19	1.84	0.65	12.3	22.632	24.071	20.16	3.911	15.296	0.067	1.025
1974	27 1/2 - 17	3.43	0.83	6.6	22.638	3.693	20.16	(16.467)	271.162	0.067	18.168
1975	26 5/8 - 18	2.68	1.00	8.3	22.244	2.676	20.16	(17.484)	305.690	0.067	20.481
1976	29 1/2 - 20 1/2	3.05	1.15	8.2	25.010	17.604	20.16	(2.556)	6.533	0.067	0.438
1977	29 7/8 - 22 3/8	3.45	1.30	7.6	26.220	10.035	20.16	(10.125)	102.516	0.067	6.869
1978	29 - 21 7/8	3.68	1.40	6.9	25.392	2.181	20.16	(17.929)	323.244	0.067	21.657
1979	42 7/8 - 26 3/4	5.12	1.50	6.8	34.816	43.021	20.16	22.861	522.625	0.067	35.016
1980	99 1/2 - 38	6.54	2.00	10.5	68.670	102.981	20.16	82.821	6859.318	0.067	459.574
1981	80 - 47 1/2	6.56	2.60	9.7	63.632	(4.424)	20.16	(24.584)	604.373	0.067	40.493
1982	51 7/8 - 33 1/2	6.25	2.80	6.9	43.125	(27.827)	20.16	(47.987)	2302.752	0.067	154.284
1983	55 - 38 1/4	6.39	2.80	7.3	46.647	14.659	20.16	(5.501)	30.261	0.067	2.027
1984	60 5/8 - 48 1/8	7.70	3.00	7.1	54.670	23.630	20.16	3.47	12.040	0.067	0.807
1985	70 1/4 - 50 1/4	7.42	3.30	8.1	60.102	15.972	20.16	(4.188)	17.539	0.067	1,087
										1.00	859.276

$$R_{Ac} = \frac{(P_1 - P_2 + D_1)}{P_o}$$

$$\bar{R} = E(R_{Ac}) = \sum_{i=1}^{N} \frac{R_{Ac}}{N} = 20.16$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^{n} \left[R_i - E(R_i)\right]^2 P_i$$

$$\text{Standard Deviation} = \sqrt{\sum_{i=1}^{n} R_i - E(R_i)^2 P_i}$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D = Dividend Ending Value

R_E = Annual Return

R_{A_c} = Annual Return

N = # of years

R = Mean Return

E (R_{Ac}) = Expected Return

Variance σ^2 = 859.276

Standard Deviation = 29.313

TABLE D-4

Fluor Corporation

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_f					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_f)$	$R_f - E(R_f)$	$[R_f - E(R_f)]^2$	P_f	$[R_f - E(R_f)]^2 P_f$
1970	6 1/2 - 3 3/8	0.33	-	15.1	4.983	-	-	-	-	-	-
1971	6 - 4	0.27	-	19.2	5.184	4.03	17.63	(13.6)	184.96	0.067	12.392
1972	7 3/8 - 3 1/2	0.15	-	36.1	5.415	4.46	17.63	(13.17)	173.448	0.067	11.621
1973	13 3/8 - 6	0.23	-	42.2	9.706	79.24	17.63	61.61	3795.792	0.067	254.318
1974	13 - 5 1/8	0.70	0.05	12.9	9.03	(6.45)	17.63	(24.08)	579.846	0.067	38.850
1975	16 3/8 - 5	0.96	0.12	11.1	10.656	19.34	17.63	1.71	2.924	0.067	0.1959
1976	15 1/8 - 10	1.30	0.24	9.7	12.61	20.59	17.63	2.96	8.762	0.067	0.587
1977	14 3/8 - 11 1/8	1.50	0.34	8.5	12.75	3.81	17.63	(13.82)	190.99	0.067	12.796
1978	14 1/2 - 9 7/8	1.55	0.40	7.9	12.245	(0.82)	17.63	(18.45)	340.402	0.067	22.807
1979	27 3/4 - 11 1/8	2.01	0.49	9.7	19.497	63.23	17.63	45.6	2079.36	0.067	139.317
1980	71 - 24 1/4	2.73	0.65	17.3	47.229	145.57	17.63	127.94	16368.64	0.067	1096.699
1981	61 7/8 - 26	2.83	0.80	15.5	43.865	(5.43)	17.63	(23.06)	531.76	0.067	35.628
1982	30 1/2 - 11 7/8	1.94	0.80	10.9	21.146	(49.25)	17.63	(66.68)	4472.934	0.067	299.687
1983	25 3/8 - 16 1/8	1.02	0.80	20.3	20.706	0.21	17.63	(17.42)	303.456	0.067	20.33
1984	23 3/8 - 14 1/2	0.01	0.60	-	18.938	(5.64)	17.63	(23.27)	541.493	0.067	36.28
1985	20 1/8 - 13 3/4	7.25	0.40	-	16.938	(8.45)	17.63	(26.08)	680.166	0.062	42.17
										1.00	2023.67

$$R_f = \frac{(P_1 - P_2) + D_1}{P_o}$$

$$\bar{R}_f = E(R_f) = \sum_{i=1}^{N} \frac{R_f}{N} = 17.63$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R = Annual Return

R_f = Annual Return

N = # of years

$\bar{R}_f$ = Mean Return

$E(R_f)$ = Expected Return

$$\text{Standard Deviation} = \sigma = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$$

Variance = σ^2 = 2023.67

Standard Deviation = σ = 44.99

TABLE D-5

Newmont Mining Company

	DETERMINED RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_N					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_N)$	$R_N - E(R_N)$	$[R_N - E(R_N)]^2$	P_N	$[R_N - E(R_N)]^2 P_N$
1970	37 1/2 - 21 1/2	3.0	1.04	9.8	29.400	–	–	–	–	–	–
1971	39 1/4 - 22 1/4	2.13	1.04	14.4	30.672	7.863	9.86	(1.997)	3.988	0.067	0.267
1972	35 1/4 - 25	1.86	1.04	16.2	30.132	1.630	9.86	(8.23)	67.733	0.067	4.538
1973	34 3/4 - 21 3/4	4.15	1.13	6.8	28.22	(2.595)	9.86	(12.455)	155.127	0.067	10.394
1974	36 5/8 - 16 1/4	4.55	1.55	5.8	26.39	(0.992)	9.86	(10.852)	117.766	0.067	7.8903
1975	26 1/2 - 18 1/4	2.05	1.60	11.2	22.96	(6.934)	9.86	(16.794	282.038	0.067	18.897
1976	31 - 23	1.88	1.60	14.50	27.26	25.696	9.86	15.836	250.779	0.067	16.802
1977	30 - 16	0.62	1.40	37.10	23.002	(10.784)	9.86	(20.344)	413.878	0.067	27.730
1978	24 1/4 - 14 1/2	1.28	0.80	15.90	20.352	(8.042)	9.86	(17.902)	320.482	0.067	21.472
1979	39 3/4 - 21 1/2	7.05	1.50	4.3	30.315	56.323	9.86	46.463	2158.810	0.067	144.640
1980	60 7/8 - 30 1/2	7.64	2.03	5.9	45.076	55.388	9.86	45.528	2072.799	0.067	138.878
1981	79 1/4 - 42 5/8	3.06	2.55	20.2	61.812	42.785	9.86	32.925	1084.056	0.067	72.632
1982	55 1/2 - 27 5/8	1.76	1.15	23.6	41.536	(30.942)	9.86	(40.802)	1664.803	0.067	111.542
1983	62 - 45 1/2	1.73	1.00	31.1	53.803	31.94	9.86	22.08	487.526	0.067	32.664
1984	54 7/8 - 31	1.23	1.00	30.9	38.007	(27.510)	9.86	(37.36)	1395.770	0.067	93.517
1985	49 - 35 1/2	1.19	1.00	–	42.25	13.795	9.86	3.935	15.484	0.062	0.96
										1.00	702.823

$$R_N = \frac{P_1 - P_o) + D_1}{P_o}$$

$\bar{R} = E(R_N) = \sum_{i=1}^{N} \frac{R_N}{N}$

$\bar{R} = 9.86$

Variance $\sigma^2 = \sum_{i=1}^{N} \left[R_i - E(R_i)\right]^2 P_i$

Standard Deviation $= \sqrt{\sum_{i=1}^{N} R_i - D(R_i)^2 P_i}$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R_n = Annual Return

R_N = Annual Return

N = # of years

$\bar{R}$ = Mean return

$E(R_N)$ = Expected Return

Variance $(\sigma)^2$ = 702.823

Standard Deviation = 26.51

TABLE D-6

Aluminum Company of America

	DETERMINE RETURN										
		E	D	P/E	P	R_A					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_A)$	$R_A-\bar{E}(R_A)$	$[R_A-\bar{E}(R_A)]^2$	P_A	$[R_A-E(R_A)]^2 P_A$
1970	24 3/4 - 15 3/4	1.14	0.6	11.6	20.184	–	–	–	–	–	–
1971	23 3/8 - 12	0.82	0.6	21.7	17.794	(8.868)	9.20	(18.068)	326.453	0.067	21.872
1972	19 1/8 - 13	1.54	0.6	10.4	16.016	(6.62)	9.20	(15.82)	250.272	0.067	16.768
1973	26 5/8 - 16	1.55	0.64	13.8	21.390	37.549	9.20	28.349	803.666	0.067	53.846
1974	26 3/8 - 14 3/8	2.57	0.67	7.9	20.303	(1.949)	9.20	(11.149)	124.300	0.067	8.328
1975	25 1/8 - 13 3/8	.93	0.67	20.9	19.437	(0.965	9.20	(10.165)	103.327	0.067	6.923
1976	30 5/8 - 19 1/4	2.07	0.69	12.0	24.840	31.349	9.20	22.147	490.440	0.067	32.863
1977	29 5/8 - 20 3/8	2.79	0.85	9.0	25.11	4.508	9.20	(4.692)	22.015	0.067	1.475
1978	26 1/2 - 19 1/4	4.45	0.95	5.1	22.695	(5.834)	9.20	(15.034)	226.021	0.067	15.144
1979	30 1/4 - 23 1/8	7.15	1.30	3.7	26.455	22.295	9.20	13.095	171.479	0.067	11.489
1980	38 1/4 - 26 1/8	6.54	1.60	4.9	32.046	27.182	9.20	17.982	323.352	0.067	21.665
1981	37 1/2 - 22 5/8	3.97	1.80	7.6	30.172	(0.232)	9.20	(9.432)	88.963	0.067	5.960
1982	32 3/4 - 21 7/8	0.15	1.65		27.313	(4.007)	9.20	(13.207)	174.425	0.067	11.686
1983	47 3/4 - 29 1/4	2.05	1.20	18.8	38.54	45.498	9.20	36.298	1317.545	0.067	88.276
1984	48 5/8 - 30 3/4	3.13	1.20	12.8	40.064	7.067	9.20	(2.133)	4.550	0.067	0.305
1985	40 3/4 - 29 3/4	0.23	1.20		35.25	(9.02)	9.20	(18.22)	331.968	0.062	20.582
										1.00	317.182

$$R_A = \frac{(P_1 - P_2 + D_1}{P_o}$$

$$\bar{R} = E(R_A) = \sum_{i=1}^{N} \frac{R_A}{N}$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^{n} \left[R_i - E(R_i)\right]^2 P_i$$

$$\text{Standard Deviation} = \sqrt{\sum_{i=1}^{n} \left[R_i - E(R_i)\right]^2 P_i}$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D = Dividend Ending Value

R_A = Annual Return

R_A = Annual Return

N = # of years

R = Mean Return

E (R_A) = Expected Return

Variance σ^2 = 317.182

Standard Deviation = 17.810

TABLE D-7

Reynolds Metals Company

	DETERMINE RETURN						DETERMINE VARIANCE				
		E	D	P/E	P	R_r					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_r)$	$R_r-E(R_r)$	$[R_r-E(R_r)]^2$	P_r	$[R_r-E(R_r)]^2 P_r$
1970	37 - 22 1/2	2.50	1.10	11.9	29.75	-	-	--	-	-	-
1971	33 1/4 - 13 3/4	0.13	0.85	-	23.5	(18.151)	8.09	(26.241)	688.590	0.067	46.136
1972	22 3/4 - 14	0.18	0.45	-	18.375	(19.893)	8.09	(27.983)	783.048	0.067	52.464
1973	23 3/4 - 12	2.41	0.40	7.4	17.834	(0.767)	8.09	(7.323)	53.626	0.067	3.593
1974	26 1/2 - 13 1/8	6.23	1.00	3.2	19.936	17.393	8.09	9.303	86.546	0.067	5.799
1975	24 1/4 - 14 5/8	3.29	1.00	5.9	19.411	2.382	8.09	(5.708)	32.581	0.067	2.183
1976	42 5/8 - 22 1/8	4.16	1.10	7.8	32.448	72.829	8.09	64.739	4191.138	0.067	280.806
1977	44 7/8 - 28 1/4	4.61	1.35	7.9	36.419	16.398	8.09	8.308	69.023	0.067	4.625
1978	39 3/4 - 24 1/2	6.11	1.58	5.3	32.383	(6.743)	8.09	(14.833)	220.018	0.067	14.741
1979	39 5/8 - 28 1/4	9.25	2.00	3.7	34.225	11.864	8.09	3.774	14.243	0.067	0.954
1980	40 7/8 - 27 1/4	9.32	2.25	3.7	34.484	7.33	8.09	(0.76)	0.576	0.067	0.039
1981	39 3/4 - 23 1/8	4.40	2.40	7.1	31.24	(2.447)	8.09	(10.537	111.028	0.067	7.439
1982	27 3/8 - 18 3/4	1.34	1.75	-	23.063	(20.572)	8.09	(28.662)	821.510	0.067	55.041
1983	41 3/8 - 24 7/8	4.73	1.00	-	33.125	47.964	8.09	39.874	1589.936	0.067	106.526
1984	41 5/8 - 26	6.36	1.00	5.6	35.616	10.538	8.09	2.448	5.993	0.067	0.402
1985	41 1/4 - 30 1/4	13.65	1.00	-	35.75	3.183	8.09	(4.907)	24.079	0.062	1.493
										1.00	582.241

$$R_r = \frac{(P_1 - P_2) + D_1}{P_o}$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R_r = Annual Return

$$\bar{R}_r = E(R_r) = \sum_{i=1}^{N} \frac{R_r}{N}$$

R_r = Annual Return

N = # of years

$\bar{R}_r$ = Mean Return

$E(R_r)$ = Expected Return

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$$

$$\text{Standard Deviation} = \sigma = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$$

Variance = σ^2 = 582.241

Standard Deviation = σ = 24.130

TABLE D-8

Asarco, Inc.

	DETERMINE RETURN										
		E	D	P/E	P	R_{as}					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_{as})$	$R_{as}-E(R_{as})$	$[R_{as}-E(R_{as})]^2$	P_{as}	$[R_{as}-E(R_{as})]^2 P_{as}$
1970	36 1/8 - 21 1/8	3.32	1.90	8.6	28.552	---	---	------	-------	------	------
1971	28 3/4 - 16 3/8	1.72	1.73	13.1	22.532	(15.025)	7.7	(22.725)	516.426	0.067	34.6
1972	23 1/2 - 17 1/2	1.84	1.20	11.1	20.424	(4.029)	7.7	(11.729)	137.569	0.067	9.217
1973	26 1/8 - 17 1/4	4.25	1.20	5.1	21.675	12.00	7.7	4.3	18.49	0.067	1.239
1974	27 3/8 - 13	4.71	1.43	4.3	20.253	0.036	7.7	(7.664)	58.737	0.067	3.935
1975	19 3/4 - 12	0.95	1.05	16.7	15.865	(16.481)	7.7	(24.818)	584.721	0.067	39.176
1976	20 - 13 1/8	1.58	0.70	10.5	16.590	8.982	7.7	1.282	1.644	0.067	0.110
1977	23 5/8 - 13	1.10	0.70	16.6	18.260	14.285	7.7	6.585	43.362	0.067	2.905
1978	20 3/8 - 13 3/8	1.69	0.40	10.0	16.900	(5.257)	7.7	(12.957)	167.883	0.067	11.248
1979	37 7/8 - 13 5/8	8.56	1.30	3.0	25.68	59.644	7.7	51.944	2698.179	0.067	180.778
1980	58 1/2 - 25 1/2	8.02	1.85	5.2	41.704	69.602	7.7	61.902	3831.858	0.067	256.734
1981	48 1/4 - 24 3/4	1.54	1.40	23.7	36.498	(9.126)	7.7	(16.826)	283.114	0.067	18.969
1982	31 - 17 1/4	2.40	0.50	---	24.125	(32.53)	7.7	(40.23)	1618.453	0.067	108.436
1983	44 1/2 - 25 5/8	1.54	0.40	22.8	35.112	47.2	7.7	39.5	1560.25	0.067	104.537
1984	34 1/2 - 18 1/2	12.56	0.30	----	26.5	(23.67)	7.7	(31.3)	979.69	0.067	65.639
1985	27 3/4 - 15 3/4	2.87	----	----	35.625	9.425	7.7	1.725	2.976	0.062	0.1845
										1.00	837.708

$$R = \frac{(P_1 - P_2) + D_1}{P_o}$$

$$R_{as} = E(R_{as}) = \sum_{i=1}^{N} \frac{R_{as}}{N}$$

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R = Annual Return

R_{as} = Annual Return

N = # of years

$\bar{R}_{as}$ = Mean Return

$E(R_{as})$ = Expected Return

$$\text{Standard Deviation} = \sigma = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$$

Variance = σ^2 = 837.708

Standard Deviation = σ^2 = 28.943

TABLE D-9

Phelps Dodge Corporation

	DETERMINE RETURN										
		E	D	P/E	P	R_D					
YEARS	PRICE RANGE	EARN/SHARE	DIV/SHARE	P/E RATIO	PRICE	RETURN	$E(R_D)$	$R_D - E(R_D)$	$[R_D - E(R_D)]^2$	P_D	$[R_D - E(R_D)]^2 P_D$
1970	56 1/2 - 34	5.36	2.10	8.4	45.024	----	---	**------	--------	-----	-------
1971	48 - 28	3.62	2.10	10.5	38.01	(10.914)	0.14	------	--------	-----	-------
1972	44 1/2 - 34 3/8	4.01	2.10	9.8	39.298	8.913	0.14	------	--------	-----	-------
1973	50 1/4 - 38 3/4	5.31	2.18	8.4	44.604	19.049	0.14	------	--------	-----	-------
1974	49 7/8 - 25 1/2	5.47	2.20	6.9	37.743	(10.449)	0.14	------	--------	-----	-------
1975	40 1/4 - 29	2.26	2.20	15.3	34.578	(2.556)	0.14	------	--------	-----	-------
1976	45 3/4 - 34 3/8	2.11	2.20	19.0	40.09	22.303	0.14	------	--------	-----	-------
1977	41 1/4 - 18 1/8	0.73	1.80	40.7	29.711	(21.399)	0.14	------	--------	-----	-------
1978	27 1/8 - 17 5/8	1.16	0.60	19.2	22.272	(23.018)	0.14	------	--------	-----	-------
1979	32 - 20 7/8	5.06	1.20	5.2	26.312	23.527	0.14	------	--------	-----	-------
1980	48 1/4 - 25	4.20	1.55	8.7	36.54	44.762	0.14	------	--------	-----	-------
1981	48 1/2 - 31 1/8	2.61	1.60	15.3	39.933	13.664	0.14	------	--------	-----	-------
1982	34 - 18 1/4	3.59	0.30	----	26.125	(33.826)	0.14	------	--------	-----	-------
1983	34 - 22 1/2	2.76	----	----	28.250	8.133	0.14	------	--------	-----	-------
1984	27 7/8 - 12 7/8	8.81	----	----	20.375	(27.876)	0.14	------	--------	-----	-------
1985	24 - 13 1/2	0.61	----	30.7	18.727	(8.088)	0.14	------	--------	-----	-------

$$R_D = \frac{(P_1 - P_2) + D_1}{P_o}$$

P_1 = Price/Share Ending Value

P_o = Price/Share Initial Value

D_1 = Dividend Ending Value

R = Annual Return

$$\bar{R}_D = E(R_D) = \sum_{i=1}^{N} \frac{R_D}{N}$$

R_D = Annual Return

N = # of years

$\bar{R}_D$ = Mean Return

$E(R_D)$ = Expected Return

** THE MEAN EXPECTED RETURN IS SO LOW THAT FURTHER CALCULATIONS ARE NOT NEEDED TO CATCH AUTHOR'S INTEREST.

$$\text{Variance } \sigma^2 = \sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i$$

$$\text{Standard Deviation} = \sigma = \sqrt{\sum_{i=1}^{N} [R_i - E(R_i)]^2 P_i}$$

Variance = σ^2 =

Standard Deviation = σ =

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