ENTANGLED QUANTUM SYSTEMS MEANINGFULLY ENCODE INFORMATION: A FORMAL DEMONSTRATION

by

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A Thesis Submitted to the Faculty of the

DEPARTMENT OF PHYSICS

In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE

In the Graduate College

THE UNIVERSITY OF ARIZONA

2005

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ACKNOWLEDGMENTS

This work was supported by a Department of Education Jacob K. Javits graduate Fellowship and University of Arizona BIO5 Collaborative Institute for Bioresearch Graduate Research Award to L.T.T.

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ABSTRACT

"Quantum information" refers to the information-theoretic properties of quantum systems. There is still a general prejudice among physicists against the idea that these systems express information due to the nonlocal behavior of entangled quantum systems. The present paper calculates the information content of simple two-state spin-entangled systems (anti-aligned spin-1/2 electron-positron pair, anti-polarized spin-1 photon pair). This calculation involves a modified application of the entropy formalism of quantum statistical mechanics. In addition, the classical and quantum analogues of mutual information are calculated for the entangled particles and experimental devices used to measure them. It is shown that the amount of information expressed by the spin measurement devices depends upon their parameter settings; the spin information of an entangled system prior to measurement is a constant. These results suggest a distinction between information available to a system's sub-components versus that available to measurement devices outside the system. This "inaccessible" information is likely carried by other physical properties of the system.

1. INTRODUCTION

With the advent of quantum computing it has become increasingly clear that fundamental properties of quantum systems are relevant to information processing. Indeed these properties are so relevant that a new term –*quantum information*- has been designated to describe the information-theoretic properties of quantum systems. These quantum properties include superposition, interference, entanglement, and nonclonability [1]. However, the real physical nature of the 'information' carried by quantum systems has still not been established. That is, outside the field of quantum computation, there is still a general prejudice among physicists against the idea that these systems carry information at all [2]. This prejudice is slowly being reduced due to the empirical demonstration of exotic computational abilities in quantum systems [1,3]. Yet there is a deeper theoretical issue: if quantum systems do not express information, then it is unclear if what they do can be meaningfully called "computation". The meanings of terms like "information" and "computation" as physical processes realized within both quantum *and* classical systems would be thrown into doubt.

The present paper argues that quantum systems do express some physical property that can be operationally quantified in terms of the classical information concept, but the physical property manifests through non-classical behavior. The understanding of the term "physical property" will require, however, an understanding of the relationship between the theoretical concept of information and the physical properties of the quantum states. "Information" has been variously formulated in terms of the uncertainty [4] or certainty [5] in knowledge about a system, which may also be

understood as an index of a system's organization [6]. The more ordered a system, the more uncertainty is possible in its behavior, and thus the more information conveyed by measurements of the system. This paper will follow the Shannon formulation of information because of its close conceptual and mathematical relationship to the thermodynamic Boltzmann entropy S, which has also been used as a fundamental measure of physical organization.[1] One particular information measure, mutual information, is especially relevant because it may be used to index the statistical dependence between two systems without reference to an external observer [7]. This eliminates the objection that information as a concept requires the involvement of a human observer. Two systems may interact, and in effect, "measure" each other [7,8] through their physical interactions. The information so defined is considered a shared resource between the communicating systems. Interestingly, entangled quantum states can also be viewed as shared physical resources [1].

The physical character of entanglement is at the root of the debate over whether or not quantum systems carry information. Entanglement refers to the joint superposition of two or more quantum sub-systems in a manner such that the wavefunction describing the entangled particles cannot be written as the direct product of the wavefunctions representing the individual states of each system. Superposition is the property that a quantum system can exist as a complex linear combination of an elementary basis of

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[1] It is important to remember that other definitions of information may be used, and this may lead to different results when using the calculation procedures discussed in this paper. In as so far as these notions of information can be drawn in under a more general information concept, a direct correspondence between the approaches should be attainable when computing the information of a quantum system. Such a comparison, however, is outside the scope of the present paper.

states that, in the case of quantum computation, represent classical Boolean states. It is generally accepted that non-entangled superpositions of states can encode classical bits of information, while entangled states do not [9]. The reason for this is that entangled systems behave in a manner inconsistent with local physical interactions.

'Locality' is the assumption that all physical interactions are constrained to occur at non-infinite rates. In special relativity, the locality assumption constrains all energy and information exchange directed across space-time to occur at, or below, the velocity of light *c*. In this manner, locality also embodies the principle of temporal causality, i.e. that causes precede effects in time. The principle of locality characterizes the classical concept of "the relative independence of objects far apart in space (A and B): external influence on A has no direct influence on B" (A. Einstein, quoted by Born in [10]], pp. 170-171). It is the notion of 'direct influence' that is the crux of the locality principle. Causal influences carried by an energy signal must take a finite time to propagate from one particle to another before one (or the other's) physical properties and/or behavior can be affected.

Joint measurements of entangled systems, however, demonstrate a statistical interdependence that suggests the determination of one system's properties is somehow dependent upon the determination of the other's. This interdependence does not attenuate with distance, is discriminating in the sense that it only involves the entangled particles, and most notably is apparently instantaneous [9]. This latter property has been demonstrated in experiments where the measurements of the particles are made when they are spacelike separated, i.e. when the particles are located in separate light cone

regions that would require faster-than-light, or even instantaneous, signaling in order for one particle to influence the other. In other words, these systems appear to carry a "direct influence" over each other. It is this apparent direct influence between entangled systems that suggests their interactions are 'nonlocal'. The theoretical and experimental validity of these statistical dependencies have been fairly well-established.[2]

The prejudice against attributing information expressed by these systems arises from the belief that the phenomena do not represent actual "action-at-a-distance", as it is generally assumed that the statistical correlations imply no causal efficacy [9, 11]. The reasons for this are discussed in detail in Section 2.4 below. The field of quantum computation, however, has utilized the properties of entanglement and superposition, in conjunction with classical signals, to process and communicate classical information in a manner that cannot be done purely classically. These effects include significant advances in data compression and computation speed [1, 3]. In addition, there are other effects that seem to emphasize the nonlocal spatiotemporal nature of these quantum systems, for example quantum teleportation [12]. Thus the practical usefulness of these concepts supports the idea that entangled systems actually do convey information and thus may represent a hitherto unknown form of causal physical interaction.

The overall defense of this thesis follows two steps. Step 1 considers simple two-state entangled systems (anti-aligned spin-1/2 electron-positron pair, anti-polarized spin-1 photon pair) demonstrating nonlocal behavior and calculates the information content

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[2] A full review of the experiments involved in testing Bell's Inequality would take the present discussion too far off course. The details of the relevant experimental scenarios underlying the quantum systems under examination will be given when appropriate. A review of the more recent tests may be found in [18].

implicit within their wavefunctions prior to measurement. Non-zero information content indicates that the systems express organization that can be meaningfully labeled "informative". The second step compares this information content with that of the experimental measuring apparatus. Any discrepancy between this "measurement information" and that of the entangled particles indicates that the measurement device is conveying something about the particles, but in an imperfect manner. It will be argued that this "something" is information made available by the interaction of the particles with the measurement device. In this manner, the measurement counts as an "information transaction", which further supports the idea that the entangled particles express information.

This paper is divided into several sections, this introduction being Section 1. Section 2 introduces the two-state systems to be analyzed and shows (via discussion of simple Bell-type inequalities) that they indeed exhibit nonlocal behavior under certain ideal measurement conditions. The following section (Section 3) calculates the information content of the entangled systems themselves via a modified application [8] of the entropy formalism of quantum statistical mechanics. A modified approach is used to circumvent a serious difficulty with the conventional entropy formalism when applied in the context of quantum computation. Section 4 introduces the classical idea of mutual information plus its quantum analogues and calculates their magnitudes for the entangled particles.

Section 5 calculates the mutual information conveyed about the systems to the experimental measuring apparatus. Using these methods, it is shown that the amount of

information available to the measuring apparatus depends upon the parameters of measurement, while the information content of an entangled system prior to measurement is a constant. These results indicate a distinction between system information available to the system sub-components versus system information available to measuring devices and their observers outside the system. The two information amounts might not be the same; some information available to a system's components may not be available to external observers, at least at the time of measurement. It is suggested that this internal information is most likely carried by the other physical properties that, together with spin, form a complete description of the entangled system.

2. THE NONLOCAL CHARACTER OF ENTANGLED SYSTEMS

2.1 Quantum Information

To understand entanglement, we need to understand two fundamental quantities used in quantum information theory. The first quantity is the two-state system, or "qubit" [13]. For simplicity this paper will restrict analysis to qubits instantiated by the polarization of photons and spin-1/2 particles, as these systems have been used most extensively in the demonstration of the nonlocal properties of entanglement. The spin wavefunction for a spin-1/2 particle may be represented as[3]

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right) \qquad (1)$$

where $|\ \rangle$ expresses an individual spin state and $\uparrow$ or $\downarrow$ represents spin orientation. In this example, the spin states of the particle represent the classical Boolean information states 0 and 1. Similarly the polarization state of a single photon may be represented as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\mathrm{L}\rangle + |\mathrm{R}\rangle\right) \qquad (2)$$

where R and L respectively indicate right-handed and left-handed circular polarization. In this example it is the polarization states of the photons that represent the classical information states 0 and 1.

Two or more qubits may be combined to form a joint system. If this combination is such that the particles are independent of one another, then the joint state may be written as the direct product of the state vectors describing each individual system. If the

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[3] Note that the space terms of the wavefunctions will be suppressed in the present notation, as their presence will not be considered until Section 5.3.

quantum states are not independent, then the joint system state is an entangled state, and the systems together form an "ebit" [3], or entangled superposition of two qubits. For a pair of entangled spin-1/2 particles (an electron-positron pair) the spin portion of the joint wavefunction may be expressed as

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\rangle|\downarrow_2\rangle - |\downarrow_1\rangle|\uparrow_2\rangle\right) \qquad (3)$$

while for an entangled photon pair, the spin wavefunction is

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|R_1\rangle|R_2\rangle + |L_1\rangle|L_2\rangle\right) \qquad (4)$$

Note that these expressions are not factorizable and represent a total spin/polarization of zero for these systems. Entangled states such as these usually arise through interactions between particles, or when particles are simultaneously created under the conservation of some physical property (such as spin or polarization).

2.2 Measuring Entangled Systems

Entangled systems such as these were used by Bohm [14] to more easily elucidate the concerns expressed by the famous EPR argument [15] designed to show the incompleteness of quantum mechanics. The essence of Bohm's argument may be expressed in the following example: consider a situation (Fig. 1A) in which a decay process D emits an electron and positron pair collinearly in the +y and –y directions, with zero net linear and spin-angular momentum. Individual particle spin ± h/2 is polarized along the ± z axis. Then the spin portion of the joint wavefunction may be expressed by equation (3) above.

Along the y axis are two Stern-Gerlach analyzers SG_1 and SG_2 containing inhomogeneous magnetic fields oriented along the z axis. The analyzers will change the trajectories of the particles along the z axis as they pass through the magnetic fields. Now let one Stern-Gerlach device (SG_1; see Fig. 1) be a closer distance to the emitting source of the particles than the other. Since each particle is traveling with the same magnitude of linear momentum, it follows that the closer analyzer will measure its corresponding particle first. Up until this measurement is made, there will be an equal chance that both SG_1 and SG_2 will measure a spin value of $\pm$ h/2. Once this measurement is made, however, the wavefunction collapses and a definite value is measured for SG_1. That is, immediately after measurement the joint particle wavefunction now becomes

$$|\Psi\rangle = |\uparrow_1\rangle|\downarrow_2\rangle \quad \text{or} \quad |\Psi\rangle = |\downarrow_1\rangle|\uparrow_2\rangle \qquad (5)$$

It follows that after measurement of the first particle, the second particle now has a determined state, opposite in spin value, which will be and indeed is measured at SG_2. In this manner, the determination of state (and thus measurement outcome) of particle 2 was "influenced" by the determination of state (and measurement outcome) of particle 1.

The difficulty with this description is that, technically, one must interpret the second particle's determination of state as occurring instantaneously with the measurement (and state determination) of particle 1 (within a constant rest frame; when moving reference frames are considered the situation becomes more complicated; see Section 2.4). Furthermore, the determination of state for particle 2 may presumably occur at arbitrary separation distances large enough for the particles to be outside each other's range of interaction. It can be shown that this distance can be large enough that

no signal could possibly send any information or other influence to affect the measurement of the other [9]. Hence this quantum physical situation appears to violate the locality principle, a conclusion that has been unacceptable to the physical intuition of many physicists.

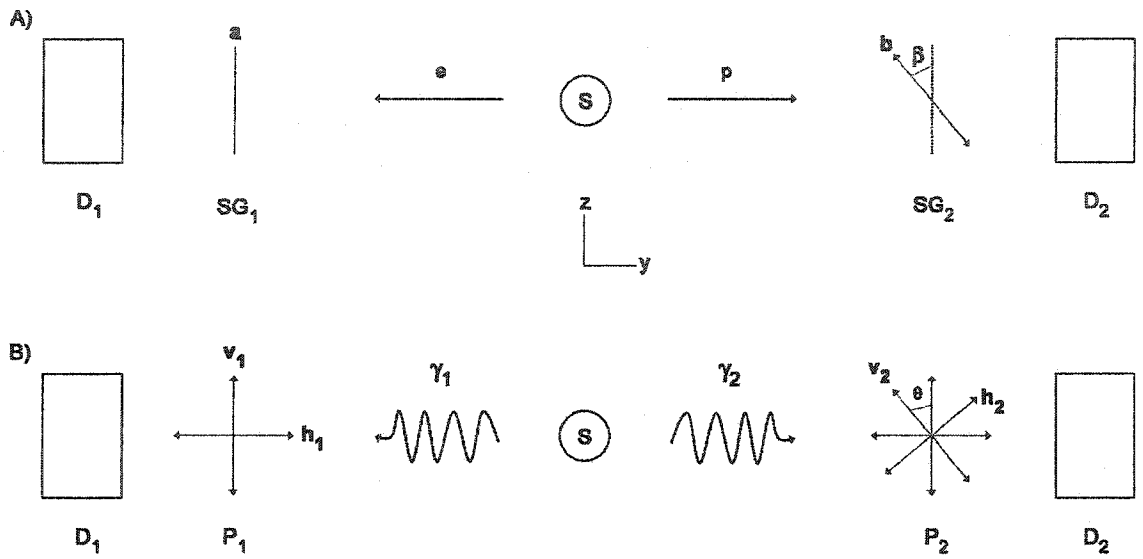

Figure 1. A) EPR thought experiment setup for spin-1/2 electron-positron (S = 0) pair. B) Optical version of the EPR scenario.

2.3 Bell's Inequality

The EPR paper argued that this incompleteness could be resolved with the introduction of local 'hidden' variables that were left unaccounted for by quantum physics. However the physicist J.S. Bell [16] later showed through his celebrated theorem that these variables, whatever they may be, cannot be local. In essence, Bell's theorem calculates an inequality relationship between the correlations of various possible measurements of entangled systems and shows that this relationship is inconsistent. The key assumption in the derivation of the correlations is the placement

of locality constraints on the postulated unknown variables. Thus any inconsistency among the inequality relationships thus calculated implies that the locality assumption is incorrect.

To understand Bell's Theorem, consider again the experimental arrangement of Fig. 1A for the electron-positron pair. Let **a** be the unit vector pointing along the axis of SG_1, **b** be the unit vector arranged along the axis of SG_2, and the analyzers oriented with respect to each other at an angle β. Let A and B denote the results of the spin measurements onto the directions of **a** and **b**. The values A and B each have a range of {-1, 1} equivalent to the spin-up and spin-down outcomes. Then

$$E(\mathbf{a},\mathbf{b})_h = \int d\lambda\ p(\lambda)\ A(\mathbf{a},\lambda)\ B(\mathbf{b},\lambda) \qquad (6)$$

represents the correlation between the measurements of A and B under the influence of hidden variable λ. This influence is represented by the average over $p(\lambda)$, which represents the probability distribution of the hidden variable. Note that $|A(\mathbf{a}', \lambda)| \leq 1$ and $|B(\mathbf{b}', \lambda)| \leq 1$ for all $\mathbf{a}'$, $\mathbf{b}'$. In writing expression (6), it has been assumed that the two spin measurements do not influence each other. Mathematically this is expressed by the fact that A (B) is not a function of **b** (**a**), although both are functions of the variable λ.

From these conditions the following conditions can be derived [17], known as Bell's Inequality

$$S = |E(\mathbf{a},\mathbf{b}) - E(\mathbf{a},\mathbf{b}')| + |E(\mathbf{a}',\mathbf{b}') + E(\mathbf{a}',\mathbf{b})| \leq 2$$

where $\mathbf{a}'$ and $\mathbf{b}'$ are arbitrary unit vectors. This expression may be evaluated by computing the correlations in a quantum mechanical manner:

$$E(\mathbf{a},\mathbf{b}) = \langle\Psi|\ (\sigma_1 \bullet \mathbf{a})\,(\sigma_2 \bullet \mathbf{b})\,|\Psi\rangle = -\mathbf{a} \bullet \mathbf{b} \qquad (7)$$

where $|\Psi\rangle$ is given by equation (3). This yields the final relation

$$S = 2\cos\theta - \cos 2\theta \leq 1 \qquad (8)$$

which is violated over the interval (0, pi/2). Thus it appears that the entangled electron-positron pair violates the locality assumption.

Figure 1B displays the experimental setup for measuring a pair of entangled photons emitted from a central atomic decay process that conserves the total angular momentum. This decay process may consist of a $J = 0 \rightarrow J = 1 \rightarrow J = 0$ radiation cascade produced by the action of a coherent light source on a crystalline substance (for example, calcium) [18]. As a result the photons are in the symmetric state

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|R_1\rangle|R_2\rangle + |L_1\rangle|L_2\rangle\right) \qquad (9)$$

where R and L respectively indicate right-handed and left-handed circular polarization. It will be shown in Section 5.2 that (9) has a direct correspondence with (3). Indeed if we designate the range of A and B as {-1,1}, representing the two possible measurement outcomes of the circular polarizations, then a similar argument may be made as in the case of the electron-positron pair to achieve a Bell-inequality for this system [17, 18].

Entangled photons have been the main systems utilized in empirical tests of Bell's inequality. The three generations of tests have been performed which to date have consistently found evidence for locality violations across increasingly large distances [18]. Experimental tests, however, do not deal with ideal circumstances, and they often require the use of additional assumptions, such as the statistical stability of the source and

detector efficiencies. The best experiments so far have circumvented many difficulties [18] with the latter, while also providing stricter tests of locality by randomly switching the orientation angles of the polarizers throughout detection of the photon pairs. These results have been carried through with a new generation of tests involving the behavior of entangled superpositions of photon energy states, instead of entangled spin states [19]. These experiments rely upon the parametric down conversion of an ultraviolet photon into two red photons with different energy states related to each other through the energy conservation of the decay process. The source in these experiments consists of a laser shining through a 'nonlinear' crystal. The photon sources can be manipulated to produce small beams suitable for fiber optic transmission, thus allowing tests of Bell's inequality over very large distances, as has been done by Weihs et al. [20].

2.4 Information or Influence?

This section briefly details why the experimental setups described in the previous section cannot be used to send a signal. The essential argument is drawn from [11] and has come to be known as Eberhard's Theorem.

Recall that in the discussion of Bell's Inequality, the measurement results S_1 and S_2 of the entangled spin-1/2 particles are functions of their respective detector orientations **a**, **b**, plus the hidden variable λ. The probability distribution describing this situation is then given by

$$p(\mathbf{a},S_1,\mathbf{b},S_2)=\sum_{\lambda}\rho(\lambda)p(\mathbf{a},S_1,\lambda)\,p(\mathbf{b},S_2,\lambda) \qquad (10)$$

where it has been assumed that $\sum_{\lambda} \rho(\lambda) = 1$. This expression demonstrates that SG_1 and SG_2 are independent in ($\mathbf{a}$, S_1) and ($\mathbf{b}$, S_2), but not λ. If we then sum across λ, then

$$p(\mathbf{a},S_1,\mathbf{b},S_2) = p(\mathbf{a},S_1)\, p(\mathbf{b},S_2) \qquad (11)$$

where the independence between the detectors is preserved because of the requirement that

$$p(\mathbf{a},S_1) = \sum_{\mathbf{b}} p(\mathbf{a},S_1,\mathbf{b},S_2)$$
$$p(\mathbf{b},S_2) = \sum_{\mathbf{a}} p(\mathbf{a},S_1,\mathbf{b},S_2) \qquad (12)$$

It is the detection probabilities $p(\mathbf{a},S_1)$ and $p(\mathbf{b},S_2)$ that describe the possible measurements when the detectors are in orientations **a** and **b**. If it were possible to influence the measurement outcome at SG_1 by manipulation of SG_2, then the detection probability at SG_1 would become $p(\mathbf{a},S_1,\mathbf{b})$. Careful experimentation with both sides of the measurement apparatus under different conditions **a**, **b** might enable one to determine $p(\mathbf{a},\mathbf{b},S_1)$ and its complement $p(\mathbf{a},\mathbf{b},S_2)$, and then use these distributions to infer the value of **a** (**b**) from measurements at SG_2 (SG_1). Clever manipulation of the detectors could then be used to send a message, which in turn would be detected as deviations in the detection probability of the receiver.

There are two problems with this scenario: first, no experiment has yet demonstrated that the detection probabilities could be modified such that they are each dependent upon the state of the other detector in a manner that overcomes the noise inherent in quantum measurements [11]. As it stands now, the outcomes of detector

measurements are not controllable. However this issue could conceivably be settled by some future technological development or experimental modification. The deeper problem is the following: assume these experiments are taking place in an inertial rest frame and are spacelike separated. A message is sent from SG_1 to SG_2 in the manner outlined above. In this frame, the "transmission" would appear to occur instantaneously. There will be some reference frames in which the manipulation of SG_1 occurs before the "reception" at SG_2. This is no difficulty as forward temporal causality is preserved. However there will be other frames for which the sending of the message at SG_1 occurs after SG_2, so in essence it appears that a message is sent backwards in time. Temporal causality is violated as a consequence. There are some interpretations of quantum mechanics that allow "backwards in time" interactions (to be discussed more in Section 6), but standard interpretation assume that temporal causality holds. This is a reasonable assumption in keeping with conventional physical intuition; indeed most theorems in physics rely upon the temporal causality assumption.

So the true difficulty in understanding entanglement lies in elucidating the physical mechanism represented by the hidden variable λ that constrains the relative measurement outcomes of the entangled particles. It achieves this in a "spacelike" manner, i.e. the correlations between SG_1 and SG_2 cannot be explained by past conditions in the light cone of either the measuring devices or detectors [11]. What physical properties does λ represent? Answering this question is beyond the scope of the present paper, but one direction to an answer may lie in the development of non-local formulations of quantum mechanics [6].

3. THE INFORMATION CONTENT OF QUBITS AND EBITS

This section calculates the information content of the qubit and ebit systems considered here via the quantum statistical mechanical entropy formalism. There is a profound difficulty, however, inherent with straightforward application of quantum statistical techniques. This is the subject of the next section.

3.1 Difficulties with the Current Quantum Entropy Formalism

The entropy measure primarily used in quantum computation is the von Neumann entropy [21], defined as

$$S(\rho) = -\mathrm{Tr}\,(\rho \log_2 \rho) \qquad (13),$$

where ρ is the density operator for the system. Formally the density operator is defined as

$$\rho = \sum_n w_n |\varphi_n\rangle\langle\varphi_n| \qquad (14)$$

and it represents mixtures of multiple quantum ensembles each characterized by the same state ket.

The problem in applying the von Neumann entropy to calculate the information content of qbits and ebits is that these states are pure states, and the entropy of pure states is zero. For example, consider the following qbit

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow\rangle + |\downarrow\rangle\right) \qquad (15)$$

which is a pure state. The (diagonalized) density matrix of this system is trivial to compute,

$$\rho = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \qquad (16)$$

The von Neumann entropy for this system is then

$$S = -\mathrm{Tr}\begin{pmatrix} \log_2 1 & 0 \\ 0 & 0 \end{pmatrix} = 0 \qquad (17)$$

For an entangled spin-1/2 system (equation 3), the calculation of ρ is slightly more complicated:

$$\rho = |\Psi\rangle\langle\Psi| = \tfrac{1}{2}\left[|\uparrow\downarrow\rangle\langle\uparrow\downarrow| + |\downarrow\uparrow\rangle\langle\downarrow\uparrow| - |\uparrow\downarrow\rangle\langle\downarrow\uparrow| - |\downarrow\uparrow\rangle\langle\uparrow\downarrow|\right]$$
$$= \left[\begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}_1 \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}_2\right] \qquad (18)$$

which when diagonalized and normalized by Tr(ρ) is

$$\rho = \left[\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_1 \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}_2\right] \qquad (19)$$

Thus $\rho \log_2 \rho = 0$ and

$$S(\rho) = -\mathrm{Tr}(\rho \log_2 \rho) = 0 \qquad (20)$$

The von Neumann entropy for the entangled photons is similarly zero.

These results are physically counterintuitive. Even in the case where it is generally agreed that a classical bit of information could be represented by a quantum system (the qbit), the formalism of quantum statistical mechanics returns a null information content. Pure non-superposed states (for example, $|\Psi\rangle = |\uparrow\rangle$ or $|\Psi\rangle = |\downarrow\rangle$) are obviously uninformative because in these cases the states are already determined, and thus known. Pure superposition states are different. Prior to state vector collapse, there

is a fundamental uncertainty as to which state a measurement will determine the system to be in. It is the outcome of this uncertainty that provides the experimenter (or the experiment apparatus) with information about the system. Thus the blind application of the density operator formalism to a pure state, as is done in quantum computing, may be inappropriate.

A likely reason for this difficulty is that the density operator was designed to represent combinations of quantum ensembles in which the information of interest is carried by the relative proportions of kinds of superposed systems within a mixture (for example, a beam of particles consisting of relative proportions of the spin-z and spin-x states). Thus the density operator fails to take into account the information contained within a pure superposed state. Nonetheless, the probabilities calculated from the wavefunction coefficients of a single qubit are derived from the statistics of multiple measurements, and thus describe the variation of a quantum system. Hence a modification of the usual entropy calculation procedure is needed in order to incorporate this variability of qbit and ebit systems.

3.2 Modified Quantum Statistical Mechanical Calculation of Qubit and Ebit Entropy

This section will circumvent the difficulty raised in the previous section by following a procedure introduced by Everett [8]. For the purpose of computing the information content of both qubit and ebit systems, we define a "density matrix" from the component Boolean substates of the systems. That is,

$$\rho_k = a_i^* a_i \delta_{ij} \qquad (21)$$

where the a_i are the amplitudes coefficients of the component states and k indexes each particle individually. For the system expressed by (15), $P = a^*_i a_i = 1/2$. The density matrix calculated in this manner yields

$$\rho = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix} \qquad (22)$$

and the von Neumann entropy is

$$S = -\mathrm{Tr}(\log_2 \rho) = -\mathrm{Tr} \begin{pmatrix} 1/2\log_2 1/2 & 1/2\log_2 1/2 \\ 1/2\log_2 1/2 & 1/2\log_2 1/2 \end{pmatrix}$$

$$= (1/2 + 1/2) = 1 \qquad (23)$$

For the entangled spin-1/2 state (equation 3), $P = a^*_i a_i = 1/2$. The density matrix and the von Neumann entropy are identical to those for the single qubit case. Application of this method to the case of the entangled polarization states (equation 4) also yields a ρ and $S(\rho)$ identical to the spin-1/2 and single qubit cases.

These results are physically intuitive: a two-state system can encode at most 1 bit of classical information, and only when the two states have equal probability. The fact that the ebit only encodes 1 bit among two particles is testament to the fact that these particles are correlated, and thus some degrees of freedom are lost during the course of the entanglement. In essence this result suggests that the entangled particles act as a single particle. If the entangled particles are correlated in this manner, then the mutual information concept introduced in Section 1 should also be able to index this relationship as well. Recall that the mutual information describes the statistical dependence between

two systems without reference to an external observer, and thus counts as a shared resource among interacting systems.

The next section discusses the classical concept of mutual information and its quantum analogues in more detail. These concepts are then applied to the exemplar spin-1/2 and spin-1 systems considered in this paper.

4. THE SHARED INFORMATION RESOURCES OF ENTANGLED STATES

4.1 Mutual Information as a Measure of Statistical Dependence

Mutual information is a classical information measure that may be used to measure the statistical dependence upon component sub-systems, without reference to external observers [7]. Consider two variables X and Y, each with their own marginal densities p(X) and p (Y). The Shannon entropy of X is defined to be

$$S(X) = -\int P(X)\log_2 P(X)\, dX \qquad (24)$$

A similar definition holds for Y. This expression is defined as the information content or magnitude carried by X, and quantifies the uncertainty inherent in making a measurement of X. Classically, the Shannon entropy is formally equivalent to the Boltzmann entropy (when the latter is normalized to dimensionless units).

To the extent that X and Y are related, however, observation of Y reduces the variability in X because now a portion of X can be accounted for by observation of Y. This reduction in variability can be accounted for by use of the conditional density P(X|Y) which, when substituted into (24) gives the conditional entropy [22]

$$S(X \mid Y) = -\int P(X \mid Y)\log_2 P(X \mid Y)\, dX \;\leq\; S(X) \qquad (25)$$

The entropy difference S(X) – S(X|Y) quantifies the information gained (i.e. the reduction in variability attained) by observing Y. The mutual information of X and Y is then defined by averaging this difference over all Y

$$I_{mutual} = \int P(Y)(S(X) - S(X \mid Y))\, dY \qquad (26)$$

and it quantifies the mean information gained by observing Y.

This expression can be put into an explicit form by first noting

that $\int P(Y)\, dY = 1$ and thus

$$I_{mutual} = S(X) - \int P(Y)\, S(X \mid Y)\, dY$$

$$= -\int P(X)\log_2 P(X)\, dX + \int P(Y)\, dY \int P(X \mid Y)\log_2 P(X \mid Y)\, dX \qquad (27).$$

Now

$$P(X) = \int P(X,Y)\, dY = \int P(Y)\, P(X \mid Y)\, dY \qquad (28),$$

thus

$$I_{mutual} = -\int P(Y)dY \int P(X \mid Y)\log_2 P(X)\, dX + \int P(Y)dY \int P(X \mid Y)\log_2 P(X \mid Y)\, dX$$

$$= -\int P(Y)dY \int P(X \mid Y)\,(\log_2 P(X \mid Y) - \log_2 P(X \mid Y))\, dX$$

$$= \int dY\, P(Y) \int dX\, P(X \mid Y)\log_2 \frac{P(X \mid Y)}{P(X)} \qquad (29)$$

Finally if we substitute the definition

$$P(X \mid Y) = \frac{P(X,Y)}{P(Y)} \qquad (30)$$

into (18), then

$$I_{mutual} = \int\int P(X,Y)\log_2 \frac{P(X,Y)}{P(X)P(Y)}\, dXdY \qquad (31).$$

A heuristic method to compute the mutual information is to consider X and Y as being chosen independently from their marginal distributions and compare their entropies to the entropy of the whole system S(X,Y). If X and Y are independent, then

S(X) + S(Y) = S(X,Y). On the other hand if X and Y are correlated, then observing Ygives information about X, and S(X) + S(Y) > S(X,Y). Hence the reduction in uncertainty in X is just

$$I_{mutual} = S(X) + S(Y) - S(X,Y) \qquad (32).$$

This expression is mathematically equivalent to expression (31) as can be shown via manipulation of the originating expression (26):

$$\begin{aligned} I_{mutual} &= \int P(Y)(S(X) - S(X \mid Y))dY \\ &= S(X) - \int P(Y)S(X \mid Y)\, dY \\ &= S(X) - (-\int P(Y) \int P(X \mid Y)\log_2 P(X \mid Y)\, dXdY \\ &= S(X) - (-\int P(Y) \int \frac{P(X,Y)}{P(Y)} \log_2 \frac{P(X,Y)}{P(Y)}\, dXdY \\ &= S(X) - (-\iint P(X,Y)(\log_2 P(X,Y) - \log_2 P(Y))\, dXdY \\ &= S(X) + (-\iint P(X,Y)\log_2 P(Y)\, dXdY) - (-\iint P(X,Y)\log_2 P(X,Y)\, dXdY) \\ &= S(X) + (-\int P(Y)\log_2 P(Y)\, dY) - S(X,Y) \\ &= S(X) + S(Y) - S(X,Y) \end{aligned}$$

It is also possible to calculate the mutual information computed in a more direct quantum mechanical manner, following [8]. Consider a joint quantum system S decomposed into systems S_1 and S_2 such that the density matrices ρ_1 and ρ_2 are diagonal. Then it is always possible to represent the joint state by a single superposition

$$|\psi\rangle = \sum_i a_i |\xi_i\rangle |\eta_i\rangle \qquad (32)$$

Since $|\xi_i\rangle$ and $|\eta_i\rangle$ may represent spin or polarization states, the entangled states (3, 4) are contained within this more general expression. The density matrices are calculated in the manner of (21):

$$\rho_1 = a^*_{1i} a_{1i} \delta_{ij} = P^1_i \delta_{ij} \qquad \rho_2 = a^*_{2i} a_{2i} \delta_{ij} = P^2_i \delta_{ij} \qquad (33)$$

Designate s_i and s_j as the eigenvalues associated with the ith (jth) states $|\xi_i\rangle$ and $|\eta_i\rangle$ of S_1 and S_2, respectively. The joint square amplitude distributions for eigenvalues s_i and s_j are then

$$P(s_i \ \& \ s_j) = P_{ij} = a^*_i a_i \delta_{ij} = P^1_i \delta_{ij} \qquad (34)$$

With these definitions, the *canonical correlation*[4] between S_1 and S_2 is then

$$I^{QM}_{mutual} = \sum_{ij} P(s_i, s_j) \log_2 \left(\frac{P(s_i, s_j)}{P(s_i) P(s_j)} \right) \qquad (35)$$

This is a quantum mechanical version of the mutual information expression as given by (31). The similarity in form between these two expressions indicates that although originally conceived in the context of classical information theory, the mutual information concept also applies in quantum mechanical situations if the probabilities are computed quantum mechanically.

Further evaluation of (35) shows that

$$I^{QM}_{mutual} = \sum_{ij} P_i \delta_{ij} \log_2 \frac{P_i \delta_{ij}}{P_i P_j} = -\sum_i P_i \log_2 P_i$$
$$= -Tr(\rho^{S_1} \log_2 \rho^{S_1}) = -Tr(\rho^{S_2} \log_2 \rho^{S_2}) \qquad (36)$$

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[4] Other more general expressions have also been put forward as quantum mechanical versions of mutual information, for example the *coherent information* [23]

$$I(\rho) \equiv S\left(\frac{\varepsilon(\rho)}{Tr[\varepsilon(\rho)]} \right) - S\left(\frac{A_j \rho A_j^{*T}}{Tr[\varepsilon(\rho)]} \right)$$

where $\varepsilon(\rho) = \sum_i A_i \rho A_i^{*T}$ and the A's are unitary operators with $A_i A_i^{*T} \leq 1$. However the direct comparison of this expression with (36) has yet to be established.

where again the density matrices are defined as in (21) above. Thus the quantum mechanical mutual information is equivalent to the entropy calculated for only one of the particles in the entangled subsystem. This is also a physically intuitive result in that as for the entropy calculations of Section (3.2), the I^{QM}_{mutual} encodes 1 bit among two particles, again suggesting that the entangled particles act as a single shared resource.

Expression (36) above may be used calculate the quantum mechanical information, or since it was shown that expression (26) is equivalent to (31), a simpler form of the quantum mechanical version of (26) is

$$I^{QM}_{mutual} = S(\rho_1) + S(\rho_2) - S(\rho_{12}) \qquad (37)$$

where ρ_{12} is the density matrix calculated from the entangled state via (21). The general forms (26) and (37) of the mutual information will be used in all further calculations.

4.2 The Mutual Information of Ebits

For the spin-1/2 and spin-1 exemplar systems, the density matrices ρ_1, ρ_2, and ρ_{12} are each of the form (see Section 3.2)

$$\rho = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$$

Using expression (37), we then have

$$I^{QM}_{mutual} = 1 + 1 - 1 = 1 \qquad (38)$$

Hence both the spin-1/2 and anti-polarized photon systems each share 1 bit of mutual information between each of the entangled particles.

5. THE MUTUAL INFORMATION CONTENT OF THE MEASUREMENT DEVICE

5.1 Spin-1/2 case

Consider the experimental setup of Fig. 1A and the following wavefunction

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_1\rangle|\downarrow'_2\rangle - |\downarrow_1\rangle|\uparrow'_2\rangle\right) \qquad (39)$$

This wavefunction may be interpreted as representing the possible measurement combinations of the entangled spin-1/2 particles. That is, $|\uparrow_1\rangle|\downarrow'_2\rangle$ represents the experimental outcome of a spin-up particle being measured at SG_1 and a spin-down particle measured at SG2, while the state $|\downarrow_1\rangle|\uparrow'_2\rangle$ represents the opposite situation. This statement is qualified by the fact that SG_2 is now rotated at an angle β with respect to SG_1 (which for simplicity is oriented along the z-axis of the experimental coordinate system; see Fig. 1A). The primes on the second particle kets denote the fact that these kets are rotated with respect to those of SG_1.

The base kets of the rotated SG_2 device may be expressed in terms of an unrotated basis in the following manner. Let

$$|\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (40)$$

and apply the spin-1/2 rotation operator D to $|\uparrow\rangle$ and $|\downarrow\rangle$ to obtain:

$$\left|\uparrow'_2\right\rangle = D(\alpha = 0, \beta, \gamma = 0)\left|\uparrow_2\right\rangle$$

$$= \begin{pmatrix} \cos\frac{\beta}{2} & -\sin\frac{\beta}{2} \\ \sin\frac{\beta}{2} & \cos\frac{\beta}{2} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \cos\frac{\beta}{2}\left|\uparrow_2\right\rangle + \sin\frac{\beta}{2}\left|\downarrow_2\right\rangle \qquad (41)$$

and

$$\left|\downarrow'_2\right\rangle = -\sin\frac{\beta}{2}\left|\uparrow_2\right\rangle + \cos\frac{\beta}{2}\left|\downarrow_2\right\rangle \qquad (42)$$

If the explicit expressions for the rotated kets are now substituted into equation (39), we have the explicit wavefunction describing the measurement of the entangled spin-1/2 particles:

$$\left|\Psi\right\rangle = \frac{1}{\sqrt{2}}\left(-\sin\frac{\beta}{2}\left|\uparrow_1\right\rangle\left|\uparrow_2\right\rangle + \cos\frac{\beta}{2}\left|\uparrow_1\right\rangle\left|\downarrow_2\right\rangle - \cos\frac{\beta}{2}\left|\downarrow_1\right\rangle\left|\uparrow_2\right\rangle - \sin\frac{\beta}{2}\left|\downarrow_1\right\rangle\left|\downarrow_2\right\rangle\right) \qquad (43)$$

The probabilities describing the four possible measurement combinations are then

$$\begin{aligned} P(\uparrow_1\uparrow_2) &= P(\downarrow_1\downarrow_2) = \frac{1}{2}\sin^2\frac{\beta}{2} \\ P(\uparrow_1\downarrow_2) &= P(\downarrow_1\uparrow_2) = \frac{1}{2}\cos^2\frac{\beta}{2} \end{aligned} \qquad (44)$$

Given the entangled nature of the state, these probabilities are assumed to represent the joint state of the system. The marginal probabilities for a given measurement at each measuring apparatus are computed as

$$P(\uparrow_1) = P(\uparrow_2) = P(\downarrow_1) = P(\downarrow_2) = \frac{1}{2} \qquad (45).$$

These probabilities are based on the assumption that the measurements (and thus the particles) are independent of one another.

To compute the mutual information, we then use the above probabilities in the discrete version of equation (32)

$$I_{mutual} = S(X) + S(Y) - S(X,Y)$$
$$= -\sum_{i=1}^{2} P_i(X)\log_2 P_i(X) - \sum_{i=1}^{2} P_i(Y)\log_2 P_i(Y) + \sum_{k=1}^{4} P_k(X \mid Y)\log_2 P_k(X \mid Y)$$
$$= -2(1/2)\log_2(1/2) - 2(1/2)\log_2(1/2) + 2(1/2\sin^2\frac{\beta}{2}\log_2\sin^2\frac{\beta}{2}) + 2(1/2\cos^2\frac{\beta}{2}\log_2\cos^2\frac{\beta}{2})$$

$$= 1 + \sin^2\frac{\beta}{2}\log_2\sin^2\frac{\beta}{2} + \cos^2\frac{\beta}{2}\log_2\cos^2\frac{\beta}{2} \qquad (46)$$

Expression (46) is a new result that expresses the mutual information conveyed by the measurement apparatus about the entangled spin-1/2 particles. It expresses the average information gained about the outcome at one measurement apparatus by observing the outcome at the other. Although mutual information is a classically defined quantity, the above result is semi-classical in the sense that the probabilities entering into the calculation were quantum mechanical.

The expression (46) is evaluated over an angle $\beta = (0,\pi)$. There is a small mathematical issue, namely the apparent divergence of the logarithmic terms as their trigonometric arguments go to zero. To overcome this difficulty, we make use of L'Hopitals rule. Let $Z = \sin^2\beta/2$ and evaluate

$$\lim_{\sin^2\frac{\beta}{2}\to 0} \sin^2\frac{\beta}{2}\log_2\sin^2\frac{\beta}{2} = \lim_{Z\to 0} Z\log_2 Z = \lim_{Z\to 0}\frac{\log_2 Z}{Z^{-1}} = \lim_{Z\to 0}\frac{f(Z)}{g(Z)}$$

Then by L'Hopital's rule,

$$\lim_{Z\to 0}\frac{f(Z)}{g(Z)} = \lim_{Z\to 0}\frac{f'(Z)}{g'(Z)} = \lim_{Z\to 0}\frac{(\log_2 e)\,Z^{-1}}{-Z^{-2}} = \lim_{Z\to 0}\frac{-Z^2(\log_2 e)}{Z} = \lim_{Z\to 0} -Z\log_2 e = 0$$

Thus at $\beta = 0$, $I_{mutual}(0) = 1 + 0 + \log2(1) = 1 + 0 + 0 = 1$.

At $\beta = \pi$, L'Hopital's rule must be used again to evaluate the cosine logarithmic terms. If we now define $Z = \cos^2\beta/2$, then

$$\lim_{\cos^2\frac{\beta}{2}\to\pi} \cos^2\frac{\beta}{2}\log_2\cos^2\frac{\beta}{2} = \lim_{Z\to\pi} Z\log_2 Z = 0$$

so that $I_{mutual}(\pi) = 1 + \log2(1) + 0 = 1$. Finally, at $\beta = \pi/2$,

$$I_{mutual} = 1 + \sin^2\frac{\pi}{4}\log_2\sin^2\frac{\pi}{4} + \cos^2\frac{\pi}{4}\log_2\cos^2\frac{\pi}{4} = 1 - \frac{1}{2} - \frac{1}{2} = 1 - 1 = 0$$

A continuous plot of the mutual information over $\beta = (0,\pi)$ is given in Fig. 2.

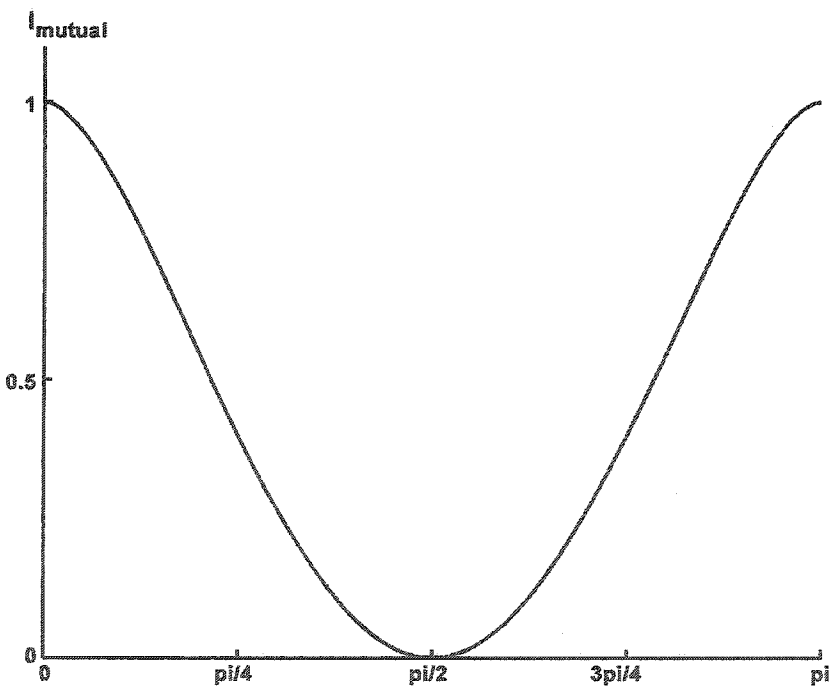

Fig.2

5.2 Spin-1 entangled photons

Consider Fig. 1A and expression (9)

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|R_1\rangle|R_2\rangle + |L_1\rangle|L_2\rangle\right)$$

The circular polarization states of the photons may be expressed in terms of more fundamental horizontal and vertical linear polarizations

$$|L\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle - i|V\rangle\right) \text{ and } |R\rangle = \frac{1}{\sqrt{2}}\left(|H\rangle + i|V\rangle\right)$$

which when substituted into (9) yields

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|H_1\rangle|H_2\rangle + |V_1\rangle|V_2\rangle\right) \qquad (46)$$

This expression indicates that if measurement of photon 1 yields state H_1 then measurement of photon 2 will yield a horizontal measurement H_2. A similar situation obtains if vertical states V_1, V_2 are measured.

Next, consider Fig. 1A and the wavefunction

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|H_1\rangle|H'_2\rangle + |V_1\rangle|V'_2\rangle\right) \qquad (47)$$

This wavefunction may be interpreted as representing the possible polarization measurements made of the entangled photons by the measurement apparatus. That is, $|H_1\rangle|H'_2\rangle$ represents the experimental outcome of a horizontally polarized particle being measured at polarizer 1 and a particle of horizontal polarization at polarizer 1. The state $|V_1\rangle|V'_2\rangle$ represents the case for vertical polarization measurements. This statement is

qualified by the fact that detector 2 is now rotated at an angle θ with respect to detector 1 (the horizontal and vertical axes of the latter are aligned with the y and z axes for simplicity; see Fig. 1B). The primes on the particle 2 kets denote rotation of these kets with respect to those of particle 1. Application of the rotation operator $D = \exp(\pm i\theta)$ to the polarizer 2 kets, followed by substitution into (31) yields [17]

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(\cos\theta|H_1\rangle|H_2\rangle + \sin\theta|H_1\rangle|V_2\rangle + \sin\theta|V_1\rangle|H_2\rangle + \cos\theta|V_1\rangle|V_2\rangle\right) \qquad (48)$$

As for the spin-1/2 particles, expression (48) is the explicit wavefunction describing the measurement possibilities of the entangled system.

The probabilities describing the four possible measurement combinations are then

$$\begin{aligned} P(H_1H_2) &= P(V_1V_2) = \frac{1}{2}\cos^2\theta \\ P(H_1V_2) &= P(V_1H_2) = \frac{1}{2}\sin^2\theta \end{aligned} \qquad (49)$$

while the marginal probabilities for a given measurement at each measuring apparatus are computed as

$$P(H_1) = P(H_2) = P(V_1) = P(V_2) = \frac{1}{2} \qquad (50)$$

These values are the same as for the electron-positron pair, thus demonstrating a 1-to-1 correspondence between the two cases. Thus the mutual information for the entangled photons is the same as found before:

$$\begin{aligned} I_{mutual} &= S(X) + S(Y) - S(X,Y) \\ &= -2\sum_{i=1}^{2} P_i \log_2 P_i + \sum_{k=1}^{4} P_k \log_2 P_k \\ &= 1 + \sin^2\theta\log_2\sin^2\theta + \cos^2\theta\log_2\cos^2\theta \end{aligned} \qquad (51)$$

Again expression (51) is a new result (for photons) that expresses the average information gained about the outcome at one measurement apparatus by observing the outcome at the other.

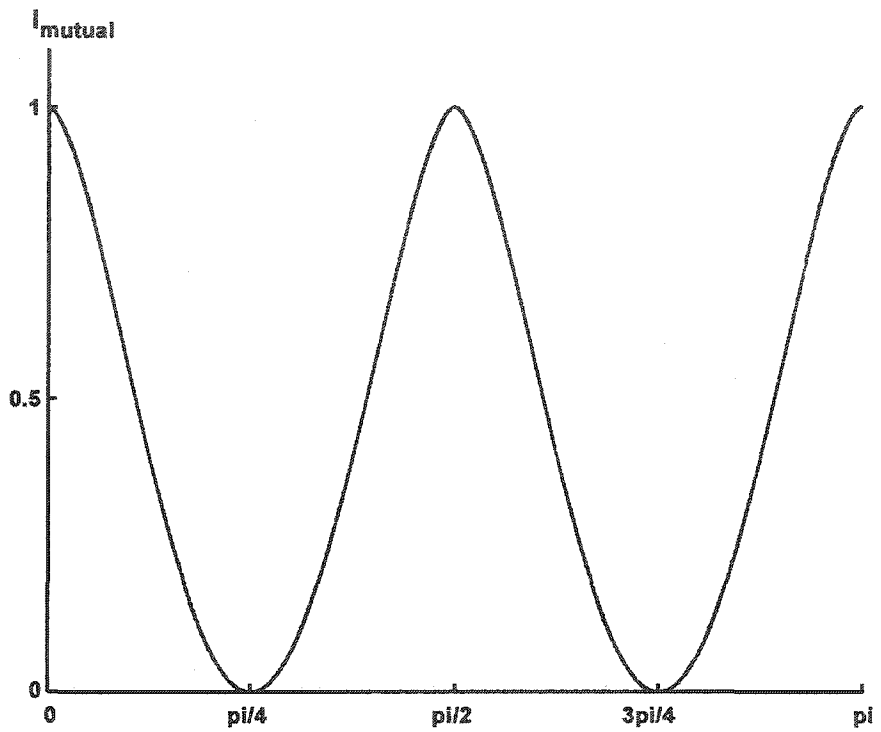

Fig. 3 I_{mutual} for entagled photon case. X axis ranges over 0 to π in order to emphasize the periodicity difference compared to the electron-positron case.

One interesting difference between the entangled photons and the electron-positron pair is that the mutual information of the former has a different periodicity than the latter. It is clear from Fig. BA that the range of θ is $(0,\pi/2)$. Use of L' Hopital's rule reveals that at $\theta = 0$, $I_{mutual} = 1$; $\theta = \pi/4$, $I_{mutual} = 0$; and $\theta = \pi/2$, $I_{mutual} = 1$. A continuous plot of the angular variation in I_{mutual} is given in Fig. 3. This shows that the mutual information in the photon case has a shorter "period of rotation" than the spin-1/2

particles as a consequence of the difference in rotational properties between spin ½ and spin 1 systems.

5.3 System vs. Measurement Information

It is clear from Section 3.2 and 4.2 that the information content of an entangled system prior to measurement is a constant amount. However the mutual information of the measuring apparatus varies with the angle between the detectors. How should this angular dependence be interpreted? Consider the two Stern-Gerlach devices of the spin-1/2 case (the following argument also applies in the spin-1 photon case). When they are perfectly oriented together ($\beta,\theta = 0$), then the two measurements performed by the two devices are in perfect (anti-) correlation; whatever spin measurement made at SG_1, the opposite will be found at SG_2. Orthogonal orientations ($\beta = \pi/2$; $\theta = \pi/4$) yield zero correlations between the measurements, so there is an equal chance for spin up or down measurements (relative to the detector axes) occurring at both detectors. What this illustrates is the angular variation of the capacity of each detector to be used as a predictor of the other's measurement outcomes. It follows that the joint state of both detectors can only provide maximum information about the system at optimum angles ($\beta,\theta = 0$ or $\beta = \pi$, $\theta =$ pi/2). In other words, only at the optimum angles do the detectors reveal the special property of entanglement. At the maximally non-optimum angles ($\beta = \pi/2$; $\theta = \pi/4$) the measurements resemble those that would be emitted from a non-entangled mixture ranging over all possible spin states. An examination of the detector geometry (Fig. 1) reveals why the efficiency of the detectors to express information about

the system is periodic over the rotation angles. As the rotation proceeds beyond the non-optimum angles, the detectors essentially rotate back into perfect alignment.

Thus if we contrast the constant information value of the entangled system prior to measurement with the information conveyed by the detectors after measurement, we see that there is a discrepancy at certain angles. This supports the interpretation that the entangled systems do in fact carry information whose physical existence is independent of the detection event. If the information content of the particles relied upon the detector setup then there should be a similar angular variation, which there is not. Note that in constructing the rotated wavefunctions (43, 48) a passive rotation was assumed; the rotation was applied to the measurement device not the system (active rotation). This is reasonable because the angular variation is a physical property of the experimental apparatus.[5]

So it seems as if there are cases when information shared across an entangled quantum system is not fully available to the experimental apparatus. Thus a distinction should be made between system information available to the system itself (and its' sub-

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[5] Interestingly, if an active rotation is applied then it can be shown that the mutual information expressed by the system is still a constant. In this case (for the spin-1/2 system), we would first construct a density operator from (43)

$$\rho = 1/2\cos^2\frac{\beta}{2}|\uparrow\downarrow\rangle\langle\uparrow\downarrow| + 1/2\cos^2\frac{\beta}{2}|\downarrow\uparrow\rangle\langle\downarrow\uparrow| + 1/2\sin^2\frac{\beta}{2}|\uparrow\uparrow\rangle\langle\uparrow\uparrow| + 1/2\sin^2\frac{\beta}{2}|\downarrow\downarrow\rangle\langle\downarrow\downarrow|$$

$$= \begin{pmatrix} \cos^2\frac{\beta}{2}+\sin^2\frac{\beta}{2} & \\ & \cos^2\frac{\beta}{2}+\sin^2\frac{\beta}{2} \end{pmatrix}_1 \begin{pmatrix} \cos^2\frac{\beta}{2}+\sin^2\frac{\beta}{2} & \\ & \cos^2\frac{\beta}{2}+\sin^2\frac{\beta}{2} \end{pmatrix}_2 = \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}_1 \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}_2$$

which when normalized by Tr(p) yields

$$\rho = \begin{pmatrix} 1/2 & \\ & 1/2 \end{pmatrix}_1 \begin{pmatrix} 1/2 & \\ & 1/2 \end{pmatrix}_2$$

Next apply (36) to yield $I^{QM}_{mutaul} = 1$, a constant. Note that in applying (36) an active rotation of the system is implied.

components) versus system information available to measuring devices and observers outside the system. The two information amounts might not be the same; some information available to a system's components may not be available to external observers, at least at the time of measurement. The information shared between the particles is a semi-private affair; observers can only "eavesdrop" on the interparticle interaction to a limited extent. This may account for the inability of these systems to be used to send signals via spin-measurements. However the possibility remains that this information may be carried by other properties of the system, for example energy or momentum. Spin and these additional properties together form a set of variables that completely describe the state of the system. Thus measurement of these properties in conjunction with spin may provide the link to the "inaccessible" information carried by quantum systems.

5.4 Measurement as an Energetic Interaction

It is instructive to consider the physics of the measurement process in order to better understand how the detectors provide information about the particle spin states. In the abstract quantum mechanical description, the measurement of one of the entangled particles "collapses" the system wavefunction into one of the possible outcomes. This "collapse" arises through the energetic interaction between the detecting devices and the particles.

For example, in the case of the electron-positron pair the Stern-Gerlach analyzers contain inhomogeneous magnetic fields **B** pointed in the **z**-direction. The spins **S** of the

electrons and positrons is associated with a magnetic moment μ which is proportional to **S**, and oppositely directed. The interaction between the magnetic field **B** and the magnetic moment of the particles produces a net force

$$\mathbf{F} = -\nabla E = -\nabla(-\mu \bullet \mathbf{B})$$

on the particles. This force serves to deflect the particles away from their incident path while they are within the **B** field. In the SG apparatus, the predominant component of **B** is in the **z**-direction, so only the **z** components of the magnetic moment play a role in the interaction. Thus

$$\mathbf{F} = \mu_z \frac{\partial B}{\partial z}\mathbf{z}$$

and since the magnetic moments are proportional to the **S**-states of the particles, it follows that the interaction is ultimately dependent upon the **z** components of the **S**-states, i.e. $\mu_z \sim S$. This force performs work on the particles changing the total energy of the particles by an amount $E = -\mu \bullet \mathbf{B}$, called the *interaction energy* of the system. This energy not only changes the trajectories of the particles in the **z** direction, but also increases their kinetic energy.[6]

In the case of the entangled photons, the polarizing devices often consist of layered glass or laminated dielectrics that cause some light to be transmitted and some reflected, each with orthogonal linear polarizations [24]. This is due to the fact that

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[6] This is because **B** appears as a time varying magnetic field in the rest frame of the particles (or equivalently, in the lab frame the particles experience different values of **B** from point to point over time). Forces arising from magnetic fields of this sort can perform work on the particle, and hence change their energies, in contrast to forces arising from constant magnetic fields which cannot [24].

the photons are originally in a circular polarization state (4), the superposition of two linear polarization states. Reflection or transmission of a photon involves the selection of one or the other of these components. In this manner, the momenta of the incident particle/waves are changed (in direction, although not necessarily in magnitude).

What is important about these examples is that the reduction of an entangled system (or any superposed system, for that many) relies upon an energetic interaction between the quantum system and measurement apparatus. The information carried by the system cannot be known outside of the system without the expenditure of energy, a fact consistent with the second and third laws of thermodynamics. Thus it is ultimately the possible energy states realized by the interaction between the system, the detection device, and the surrounding environment that physically realize the information of the system. After a measurement the classical information conveyed by the particle is clearly destroyed in that a subsequent measurement will reveal the same value of the observable as before (or a value consistent with the time evolution of the particle). Thus within the limits set by the complementarity principle, there is no more uncertainty in the state. Only until the state eventually evolves through time under unitary evolution by interacting with the rest of the universe will a state of superposition be restored. This suggests that all physical interaction could be generally described as a transfer of information among systems, an idea that has been expressed elsewhere [5,8].

6. CONCLUSION

This paper has formally demonstrated that entangled quantum systems do express spin-associated information (Sections 3 – 5) that appears to be expressed by the system in a manner that does not violate relativistic causality. Furthermore, some of this information seems to be inaccessible via spin measurements, and is only accessible to subcomponents of a quantum system itself. This is consistent with other analyses arguing that much of the quantum information of a system is largely inaccessible to measurement, yet can still be utilized to perform the manipulation of classical information states [27]. However this unknown information is likely carried by other physical properties of the system aside from the property of spin, such as momentum and energy. Thus the statement "inaccessible to measurement" may need to be refined to incorporate the fact that determination of the total information carried by a system may be inherently limited by the ability to extract that information via measurements of a particular type of physical property (i.e. spin). A complete set of measurements may be required to extract the total amount of information in a system. Future theoretical work should investigate the relationship between spin-associated information and the quantum statistical mechanical concept of entropy, as the latter, based on the conventional definition of the density matrix, fails to adequately account for information instantiated through spin properties (as shown in Section 3.1).

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