THE INFLUENCE OF COGNITIVE ABILITIES ON MATHEMATICAL PROBLEM SOLVING PERFORMANCE

by

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DEDICATION

To my wife FEYZA

for her endless support and love

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ABSTRACT

Problem solving has been a core theme in education for several decades. Educators and policy makers agree on the importance of the role of problem solving skills for school and real life success. A primary purpose of this study was to investigate the influence of cognitive abilities on mathematical problem solving performance of students. The author investigated this relationship by separating performance in open-ended and closed situations. The second purpose of this study was to explore how these relationships were different or similar in boys and girls.

Multiple regression analyses were performed to predict students' problem solving performance. Intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability constituted independent variables whereas mathematical problem solving performance scores in closed and open-ended problems were the dependent variables.

The author found that mathematical problem solving performance (MPSP) in closed problems was correlated significantly with cognitive variables including mathematical knowledge, quantitative ability, verbal ability, general intelligence, general creativity, and spatial ability. However, MPSP in closed problems was not correlated at a significant level with working memory and reading ability. Similarly MPSP in open-ended problems was correlated significantly with several cognitive abilities including verbal ability, general creativity, spatial ability, mathematical knowledge and quantitative ability. However, MPSP in open-ended problems was not correlated significantly with working memory, reading ability, and general intelligence.

No significant difference was found between girls and boys in cognitive abilities including general intelligence, general creativity, working memory, mathematical knowledge, reading ability, mathematical problem solving performance, verbal ability, quantitative ability,

and spatial ability. After controlling for the influence of gender, the cognitive abilities explained 51.3% (ITBS) and 53.3% (CTBS) of the variance in MPSP in closed problems as a whole. Mathematical knowledge and general intelligence were found to be the only variables that contributed significant variance to MPSP in closed problems. Similarly, after controlling for the influence of gender, the cognitive abilities explained 51.3% (ITBS) and 46.3% (CTBS) of the variance in mathematical problem solving performance in open-ended problems. General creativity and verbal ability were found to be the only variables that contributed significant variance to MPSP in open problems.

The author concluded that closed and open-ended problems require different cognitive abilities for reaching correct solutions. In addition, when combining all of these findings the author proposed that the relationship between cognitive abilities and problem solving performance may vary depending on the *structure (type)* and *content* of a problem. The author suggested that the content of problems that are used in instruments should be analyzed carefully before using them as a measure of problem solving performance.

CHAPTER I: INTRODUCTION

Problem Solving

No one would deny that humans are social beings who encounter a variety of problems in everyday life. We may not be able to solve every kind of problem, but, as human beings, we have the capacity to devise strategies and procedures to approach problems (Willats, 1990). Butterworth and Hopkins (1988) stated that this capacity appears to be innate. For example, even young babies can communicate their desires to bring caregivers closer to meet their comfort and food needs (Ellis and Siegler, 1994).

In the field of education, the 1930s and 1940s were important eras for educators who believed that school curricula should be redesigned around "real-life" situations (Hiebert et al., 1996). One major concern stated by many educators in the U.S. was that the knowledge acquired in the classroom did not transfer well to the professions, such as medicine, engineering, social work, or education (Boud & Feletti, 1991). Dewey's (1933) ideas about reflective thinking and problem solving provided educators with strong motivation and pathways to redesign school curricula. Dewey believed that reflective thinking was the key to moving beyond the distinction between knowing and doing, thereby providing "a new way of viewing human behavior" (Hiebert et al., 1996). Stemming from Dewey's distinction between knowing and doing, educators produced models for increasing the usefulness of students' knowledge, such as a model of problem-based learning that was designed as "a way of conceiving of the curriculum which is centered on key problems in professional practice" (Boud and Feletti, 1991, p. 14).

Definitions of Problem Solving

Problem solving is a process "to search consciously for some action appropriate to attain a clearly conceived, but not immediately attainable aim" (Polya, 1962, p.117). Kayan (2007) suggested that what may be a problem for one individual may not be a problem for another. For example, dressing for a ceremony might be a big problem for a speaker to solve but not for another person attending the same ceremony.

The term "problem solving" has different meanings depending upon on the domain of the definer. To a psychologist, problem solving means a state of desire for reaching a certain goal from a present condition that either is not directly moving toward the goal, is far from it, or needs more complex logic for finding a missing description of conditions or steps toward the goal (Robertson, 2001). To a mathematician, problem solving means a mathematical situation for which a solution is needed, and for which a direct route to a solution is not known (Polya, 1962). Schoenfeld (1992) adapted Polya's definition to the field of mathematics education, and defined problem solving as a process wherein students encounter a question for which they have no immediately apparent resolution, nor an algorithm that they can directly apply to get an answer. All of these definitions have a commonality when defining problem solving: an act toward making an unknown situation known.

Schoenfeld (1992) stated that any research in problem solving should include the researcher's operational definition of the term *problem solving*. My operational definition for problem solving for this study is *a cognitive act or process directed toward making the unknown situation known in the domain of mathematics*.

Problem Solving Concept in the Field of Mathematics Education

In mathematics education, problem solving has been a core theme for several decades. In the 1940s, problem solving as a theme started to appear in official commission reports on mathematics education. For example, in 1940 the Joint Committee of the Mathematical Association of America and the National Council of Teachers of Mathematics (NCTM) recommended that "the study (of mathematics) should emphasize problem solving and modes of thinking, and should not become a mere sequence of formal and relatively abstract drills" (Joint Commission of the MAA and the NCTM, 1940, p.58). Although the commission's recommendation was determined, very few researchers and educators in the field heard it.

Research on mathematical problem solving has been drawn on, and has evolved from, Polya's (1945) book *How to Solve It*. In his book, Polya provided the outline of a problem solving framework, a hint of the details necessary to implement it, and a description of steps for solving mathematics problems. Polya's problem solving framework included a four-step description of the problem solving process: (a) understanding the problem, (b) devising a plan, (c) carrying out the plan, and (d) looking back. At the first step (understanding the problem) of the process, the problem solver identifies the known and unknown variables. At the second step (devising a plan), the problem solver chooses strategies for responding to what is asked for in the problem, and the solver makes connections between the known and the unknown to develop a plan for the solution (Polya, 1973). At the third step (carrying out the plan), the problem solver implements the plan devised in step 2, and performs any necessary actions or computations. This step is not considered problem solving, but rather the use of mathematics to generate the final result (McAllister 1996). At the last step (looking back), the problem solver checks his plan and solution, and acts upon it to produce the results.

Polya's ideas about problem solving influenced the field of mathematics education for decades. The call of the National Council of Teachers of Mathematics (NCTM) in 1980 for problem solving to become "the focus of school mathematics" was widely echoed in the field of mathematics education (NCTM, 1980, p.1). Later, the members of the council endorsed this recommendation with the statement that "problem solving should underlie all aspects of mathematics teaching to give students experience of the power of mathematics in the world around them" (NCTM, 1989). Most recently, through the Principles and Standards for School Mathematics, the members of the council recommended that "by the end of grade 12 students should be able to (a) build new mathematical knowledge through problem solving, (b) solve problems that arise in mathematics and in other contexts, (c) apply and adapt a variety of appropriate strategies to solve problems, and (d) monitor and reflect on the process of mathematical problem solving" (NCTM, 2000, p.3). One of the reasons why the council members had emphasized problem solving in its reports was that problem solving "encompasses skills and functions that are an important part of everyday life and furthermore it can help people to adapt to changes and unexpected problems in their careers and other aspects of their lives" (NCTM, 2000, p.4). The NCTM pushed successfully for problem solving to become the centerpiece of the mathematics curriculum.

In addition to NCTM's emphasiss on problem solving, many other researchers also highlighted the importance of problem solving in mathematics education. For example, according to Cockcroft (1982), problem-solving ability lies "at the heart of mathematics" (p.73) because it is the means by which mathematics can be applied to a variety of unfamiliar situations. Carpenter (1989) expressed the view that teaching problem solving is important to encourage students to refine and build onto their own processes over a period of time as they discard some ideas and become aware of further possibilities. Resnick (1987) asserted that a problem-solving approach contributes to the practical use of mathematics by providing an opportunity for people to develop the facility to be adept when, for instance, technology breaks down.

In summary, both governmental councils and mathematics educators have "elevated a problem solving approach into a position of prominence" (Otten, 2010, p. 14) by locating it at the center of mathematics education. As Otten (2010) pointed out, researchers believe that this emphasis on problem solving would not be only self-contained to the domain of mathematics education but also would transfer into society in positive ways by promoting a knowledgeable citizenry and by creating pathways of advancement for students (Hiebert et al., 1996; Schoenfeld, 2007).

Problem Solving Concepts in the Field of Psychology

The nature of problem solving processes and methods has been studied by psychologists for over one hundred years. In the early 1900s, Thorndike developed a theory of learning and created the ‘law of effect’ based on his studies about animals. According to the law of effect, behaviors are likely to be repeated when they are followed by positive consequences. In his puzzle box experiment, Thorndike put a cat inside a box and observed how a cat used various methods to try to escape. However, the cat could not get out of the box until it hit a lever. After experiencing difficulty, the cats eventually “learned” that the lever opened the box and allowed faster escapes with successive trial, until the rate of escape leveled off. Thorndike (1911) asserted that success in each trial was linked to the success of prior trials. Thorndike greatly influenced the behaviorist view of problem solving. Behavioral psychologists believed that successful problem solving was a process of trial and error.

Contrary to the behavioral psychologists’ approaches to understanding the elements of problem solving processes, gestalt psychologists sought to understand how these elements were organized (Carlson & Heth, 2010). Gestalt psychologists claimed that when thinking about a solution for a problem, problem solvers constructed internal representations of the problem, leading usually to a flash of “insight” that helped them to reach a solution. In his book, *The Mentality of Apes*, Köhler (1917) described his of studies about apes and he found that the apes demonstrated "insight" during problem solving processes. In one of his studies, Köhler observed that chimpanzees figured out to use some tools (e.g., stacked wooden crates as makeshift ladders or long sticks) to reach bananas. Köhler concluded that the chimpanzees had experienced an insight to solve their food problem instead of a trial and error method. The role of insight in the problem solving process was of immense interest to gestalt psychologist (Steele, 2003).

Gestalt theories of perception were criticized for being descriptive rather than explanatory in nature by cognitive psychologists and computational neuroscientists. In his theory, Piaget (1923) proposed a theory of developmental stages, stating that individuals demonstrated distinct common patterns of learning and cognition in each stage of their development. Following Piaget's theory of cognitive development, the cognitive aspects of problem solving have been considered as essential to the notion of intelligence (Resnick & Glaser, 1976; Sternberg, 1982). Later researchers continued to focus on problems that required no background knowledge, but placed greater emphasis on memory and on use of cognitive strategies through the study of multi-step problems (Newell & Simon, 1972). In their book, *Human Problem Solving*, Newell and Simon (1972) described their problem space theory of problem solving. In this theory, individuals are believed to solve problems by searching in a *problem space* that is comprised of *states* including (a) initial state, (b) goal state, and (c) other possible states in between. Newell and Simon claimed that individuals solve problems by moving from one state to another state. According to this theory, cognitive short-cuts, known as heuristics helps them to reach successful solutions.

Types of Problems

Early classifications of problem types appeared in the studies of Minsky (1961) and Reitman (1965). These researchers distinguished between two main types of problems: well defined and ill defined. Minsky (1961) asserted that a well-defined problem had an unambiguous solution that could be presented in a systematic manner. Well defined problems have a definite initial state and the goals and operators are known (Dunbar, 1998). Classic examples of well-defined problems include solving an equation (Dunbar) or calculating the perimeter of a circle. On the other hand, ill-defined problems evoke a highly variable set of responses concerning referents of attributes, permissible operations, and their consequences (Reitman, 1965). Unlike well-defined problems, ill-defined problems are ones in which the solver does not know the operators, the goal, or even the current state (Dunbar). Examples of an ill-defined problem might be finding a cure for cancer (Dunbar) or finding a solution for global warming.

Another problem classification was suggested by Getzels and Csikszentmihalyi (1976), who proposed that the structure, method, and solution of a problem could be used for classification purposes. The key finding from Getzels and Csikszentmihalyi's research was that problem solving could be categorized into three types, based on the interaction between the presenter and solver of a problem (Alhusaini, 2012). The knowledge of both persons about (a) the problem, (b) the method, and (c) the solution made the problem types range from open-ended to closed. Building on Getzels and Csikszentmihalyi's work, Maker and Schiever (1991) proposed the "DISCOVER Problem Continuum" in which six problem types were displayed, along with how much information was known and how much structure was provided for both the problem presenter and the problem solver in each problem type (Table 1.1).

The Problem Continuum was an expanded version of the model developed by Getzels and Csikszentmihalyi (1976, 1967). Maker and colleagues added Problem Types III and IV to provide a more fluid transition between the Types, based on observations during research (Maker, 1978; 1981; 1993; Whitmore & Maker, 1985). In this context, problems were classified as either closed or open based on the number of alternatives available to the problem solver. For example, a problem was defined as closed if it could be solved in only one way and open if it could be solved in an infinite number of ways.

Table 1.1

Problem Continuum

Type		Problem		Method		Solution	
		Presenter	Solver	Presenter	Solver	Presenter	Solver
Closed	I	Specified	Known	Known	Known	Known	Unknown
	II	Specified	Known	Known	Unknown	Known	Unknown
	III	Specified	Known	Range	Unknown	Known	Unknown
Open-Ended	IV	Specified	Known	Range	Unknown	Range	Unknown
	V	Specified	Known	Unknown	Unknown	Unknown	Unknown
	VI	Unknown	Unknown	Unknown	Unknown	Unknown	Unknown

Note: Adapted from Maker, J., & Schiever, W. (2010). *Curriculum development and teaching strategies for gifted learners* (3rd Ed.). Austin, TX: Pro-Ed

Problem Type Examples

Type I. The problem and the method of solution are known to the problem presenter and the problem solver; the presenter knows the (one) correct solution. Solving math problems by a known algorithm or method; following a formula, in language, music, math or science; and performing prescribed body movements, as in dance or sports, are Type I problems.

Type II. In Type II problems, the problem is known by the presenter and the solver, but the method of solution and solution are known only to the presenter. Type II is close to Type I in structure, except that the problem solver does not know the method by which to arrive at a solution. Problems such as mathematical “story problems” requiring the solver to figure out and apply the appropriate method to solve the problem, answering questions about factual material, scientific "experiments" with prescribed materials and variables, playing an instrument while sight-reading the music, and creating a scale drawing are Type II problems.

Type III. The problem is known to the presenter and the solver, but more than one method may be used to arrive at the correct solution, which the presenter knows. Type III problems require a specific solution but many methods may be used to reach this solution. Finding the "key" to mathematical, word, or linguistic patterns; movement sequences created to meet specific requirements; constructions using specified materials; and meeting given criteria are Type III tasks.

Type IV. The problem is known to the presenter and the solver, but the problem may be solved in more than one way and the presenter knows the range of solutions. Problems that can be solved inductively but that have an accepted range of answers, such as geometry problems that may be solved using manipulatives, creating as many equations as possible using three (provided) numbers and the operations of addition and subtraction, and creating music or movement sequences within defined parameters are examples of Type IV problems.

Type V. The problem is known to the presenter and the solver, but the method and solution are unknown to both. Type V problems are clearly defined, but methods and solutions are open. Questions such as, "In what ways might you share the results of your survey?" define Type V problems, as do constructions using specific materials and meeting pre-set goals (such as building a mousetrap vehicle), creating prose or poetry, making a self-sustainable terrarium or aquarium, writing lyrics to an existing melody, writing a melody for existing lyrics, and finding new ways to apply existing formulas. The problem is known to the presenter and the solver, and the solver is taught the creative problem solving process to use in developing his/her solution, but the solution is unknown to all.

Type VI. The problem is unknown or undefined, and the method and solution are unknown to both presenter and solver. Type VI problems have the least structure; are the most complex; need to be defined and, possibly, redefined; and have numerous possible solutions. These are the problem situations in real life that can be defined in more than one way, and that may need redefining during the problem solving process. Type VI problem situations include those such as environmental pollution; student behavior; ethical behavior and standards; global warming; urban problems; social issues, such as violence or declining literacy; and international border issues.

Significance of the Study

Researchers have agreed that the wide spectrum of cognitive activities presents a dilemma as they attempt to understand the structure of intellectual abilities (Vickers, 2003). How do these cognitive variables influence problem solving performance as a whole? How are boys and girls different or similar in this relation? My strong desire to answer these questions has been the driving force behind this study. Although many researchers investigated the influence of individual factors on problem solving performance, no prior researchers attempted to explore how these cognitive variables influence problem solving performance as a whole because of the ambiguity of the constructs and complexity of the relationships. For this reason, this study contributed valuable information, including the understanding of how problem solving performance is influenced by cognitive variables including intelligence, creativity, knowledge, memory, reading ability, verbal ability, spatial ability, and quantitative ability. Also the findings help to extend the current research on student thought processes. The knowledge about characterization of students' thinking might provide teachers with a plan to implement problem solving-based teaching in classrooms.

This study was different from prior studies because the author distinguished between open-ended and closed problem solving performance. No prior researchers attempted to investigate how the influences of these cognitive variables on mathematical problem solving performance differ in open-ended and closed situations. This approach provides educators and researchers with a comprehensive understanding of the problem solving process.

Recent discourses related to problem solving in mathematics education have been contextualized through sociocultural perspectives. However, problem solving has been defined as a higher-order cognitive process that requires control of many fundamental skills (Goldstein and Levin, 1987). This fact implies that an exploration of problem solving process from a cognitive perspective is needed. The author's approach to investigate the problem solving process through the information processing framework provided researchers in mathematics education with a different 'lens' to understand intellectual abilities. From this perspective, I believe this study had significant findings for the field of mathematics education.

The findings can be applied to the development of teaching methods and materials to be used in future mathematical problem solving classes to facilitate the development of students' abilities and skills for solving open-ended and closed problems. Furthermore, this research was important for curriculum design in that the results supported the creation of curricula that could be more effective and supportive of students.

Statement of Purpose

A primary purpose of this study was to investigate the influences of cognitive abilities including intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability on the mathematical problem solving performance of students.

The author investigated this relationship by separating performance in open-ended and closed situations. The second purpose of this study was to explore how these relationships were different or similar in boys and girls.

Research Questions

The following research questions guided this study:

1. To what extent are cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) and mathematical problem solving performance in open-ended and closed problems related to each other?
2. To what extent are boys and girls similar or different in mathematical problem solving performance in open-ended and closed problems?
3. To what extent do cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) predict mathematical problem solving performance in closed problems?
4. What cognitive variables are the best predictors of the mathematical problem solving performance in closed problems?
5. To what extent do cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) predict mathematical problem solving performance in open-ended problems?
6. What cognitive variables are the best predictors of mathematical problem solving performance in open-ended problems?

Definitions of Terms

The following terms have been used throughout the study, and defined below for clarity in their application to this study.

Problem is a situation in which something is to be found or shown and the way to find or show it is not immediately obvious (Grouws, 1996).

Problem solving is engaging in a task for which the solution method is not known in advance (NCTM, 2000).

Insight is a sudden state of mind that constitutes a quick transition from a state of ‘not knowing’ to a state of ‘knowing’ (Pols, 2002).

Heuristics are rules that determine which moves are to be made in the problem space, as opposed to a random walk (Davidson and Sternberg, 2003).

CHAPTER II: REVIEW OF THE LITERATURE

A primary purpose of this study was to investigate the influences of cognitive abilities such as intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability on the mathematical problem solving performance (MPSP) of students. The author modeled this relationship by separating performance in open-ended and closed problems. Furthermore, I investigated how these relationships were different or similar in boys and girls. This chapter contains a review of previous studies related to problem solving and a brief explanation about the present study's research framework.

Method

The review of literature was conducted in two phases. In the first phase studies that were conducted to explore the relationships between cognitive abilities and MPSP were identified. In the second phase studies that were conducted to explore the gender differences in MPSP were identified.

Phase I

The search for studies that were designed to explore the relationships between cognitive abilities and MPSP was conducted through reviewing articles, dissertations, and theses. The all-purpose databases *Worldcat Local* and *Google Scholar* were used to search for the key words "mathematical problem solving" and "cognitive ability" to find related articles. Other studies were found and collected from bibliographies and reference lists in the studies themselves. The *Proquest Dissertations and Theses* database and *Google Scholar* were used to search for the key words "mathematical problem solving" and "cognitive ability" to find related dissertations and theses. The search yielded 37 articles and 425 dissertations and theses. Each study was screened based on the following four criteria:

1. The study included mathematical problems.

2. The study included information related to the relationship between cognitive abilities and mathematical problem solving performance.
3. The study included numerical outcomes.

Among 37 articles and 425 dissertations and theses, 22 empirical studies were filtered according to the above criteria.

Phase II

The search for studies that were designed to explore gender differences in MPSP was conducted through reviewing articles, dissertations, and theses similar to Phase I. The all-purpose databases *Worldcat Local* and *Google Scholar* were used to search for the key words "mathematical problem solving" and "gender differences" to find related articles. Other studies were found and collected from bibliographies and reference lists in the studies themselves. The *Proquest Dissertations and Theses* database and *Google Scholar* were used to search for the key words "mathematical problem solving", and "gender differences" to find related dissertations and theses. The search yielded 18 articles and 35 dissertations and theses. Each study was screened based on the following four criteria:

1. The study included mathematical problems.
2. The study included information related to the gender differences in mathematical problem solving performance.
3. The study included numerical outcomes.

Among 18 articles and 35 dissertations and theses, 10 empirical studies were filtered according to the above criteria.

Table 2.1 has information about the studies that were conducted to explore the relationships between cognitive abilities and MPSP. Table 2.2 has information about the studies that were conducted to explore the gender differences in MPSP. In this chapter, a brief explanation about the present study's theoretical framework was presented first and then the author included his evaluation and critique of the studies' designs, methods, analysis, findings, and participants while discussing the results of the studies.

Theoretical Framework

Most contemporary researchers in the field of education have been influenced by theories that have a socio-cultural emphasis. Similarly, in recent studies related to problem solving in mathematics education researchers mostly draw on the work of Wittgenstein and Vygotsky (Ernest, 1994). As Lerman (2000) pointed out, mathematics has been viewed as a social product in these studies. For example, Ernest (1994), as a social constructivist, sees problem solving as the heart of mathematics learning that juxtaposes mathematical discovery and student empowerment (Wilding-Martin, 2009). For Ernest and many other social constructivists, problem solving is an enculturation of students into mathematical forms of life, while raising social awareness and encouraging students to challenge social justice (Wilding-Martin, 2009). However, the author's goal with this study was to explore the cognitive structure of mathematical problem solving processes rather than understating its development in a sociocultural context. In addition, the problem solving concept was referred to by scholars (Resnick & Glaser, 1976; Sternberg, 1982) as a high order thinking process that was composed of major intellectual abilities and cognitive processes. Therefore, approaching the problem through a *cognitive lens* would be more appropriate to answer the research questions of the study.

The theoretical framework of this study was rooted in Newell and Simon's (1972) information-processing (IP) theory of learning. In their theory, Newell and Simon highlighted the similarities between artificial intelligence and human problem solving and they emphasized the role of factors such as working memory capacity and cognitive retrieval of relevant information. They claimed that ability to solve problems successfully depended on a number of factors related to the human information-processing (IP) system. This higher order learning theory has been used to elaborate the cognitive processes of problem solving.

Analysis of the Studies

The studies were analyzed in two sections: (a) cognitive abilities and mathematical problem solving and (b) gender and mathematical problem solving.

Cognitive Abilities and Mathematical Problem Solving

Twenty two studies (Awofala et al., 2011; Bahar and Maker, 2011; Byrnes and Takahira, 1993; Byrnes and Takahira, 1994; Burns et al., 2006; Garderen, 2006; Hegarty and Kozhevnikov, 1999; Heinze, 2005; Huggins, 1988; Hwang et al., 2007; Iguchi, 2008; James and Adewale, 2012; Jordan, 2003; Lee and Chen, 2009; Serafino and Cicchelli, 2003; Sriraman, 2003; Swanson, 2004; Vickers et al., 2004; Vilenius-Tuohimaa et al., 2008; Walker, 2012; Xin and Zhang, 2009) included findings on the relationship between cognitive abilities and mathematical problem solving performance. The summary of the analysis of the studies is presented in Table 2.1.

Findings. Findings within the topic of cognitive abilities and mathematical problem solving performance could be categorized under a cognitive ability that was mainly investigated in the studies: (a) general intelligence, (b) general creativity, (c) mathematical knowledge, (d) working memory, (e) reading ability, and (f) other cognitive abilities.

Table 2.1

Studies Related to the Influence of Cognitive Abilities on Mathematical Problem Solving

Study	Purpose	Mathematical Problem Solving Correlate	Participants' Age/Grade Span	Number of Participants
Huggins (1988)	To investigate the influence of specific thinking skills training on mathematics problem solving performance	Intelligence	Grade 5	132
Hegarty and Kozhevnikov (1999)	To identify how spatial and visual imagery abilities affect problem solving in mathematics	Intelligence, verbal reasoning, non-verbal reasoning, spatial ability	Grade 6	33
Vickers et al., (2004)	To investigate individual differences in performance with several instances of an even more difficult, visually presented problem solving task and to examine their relationship with a measure of intelligence	Intelligence, spatial reasoning, verbal reasoning	17 to 53 years old	69
Burns et al., (2006)	To understand better the previously reported relationship between performance on difficult optimization problems and a measure of intelligence	Intelligence, spatial reasoning, verbal reasoning	Mean age = 25.3, SD = 7.6 years	101

Table 2.1 (Continued)

Studies Related to the Influence of Cognitive Abilities on Mathematical Problem Solving

Study	Purpose	Mathematical Problem Solving Correlate	Participants' Age/Grade Span	Number of Participants
Xin and Zhang (2009)	To explore the associations between students perceived cognitive holding power, fluid intelligence, mathematical achievement, and their realistic problem solving	Intelligence, mathematical achievement	Grades 4 to 6	119
Heinze (2005)	To investigate problem solving strategies and processes of thinking of mathematically gifted elementary students	Intelligence	6 to 10 years old	?
Sriraman (2003)	To investigate the differences in mathematical problem solving performance of gifted and non-gifted students	Intelligence	Grade 9	9
Bahar and Maker (2011)	To investigate the relationship between students’ creative performance and achievement in the mathematical domain through problem solving	Creativity	Grades 2 to 4	78

Table 2.1 (Continued)

Studies Related to the Influence of Cognitive Abilities on Mathematical Problem Solving

Study	Purpose	Mathematical Problem Solving Correlate	Participants' Age/Grade Span	Number of Participants
Hwang et al., (2007)	To explore student multiple representation skills and creativity in solving mathematical problems	Creative ability / thinking	Grade 6	25
Byrnes and Takahira (1993)	To investigate whether prior knowledge and cognitive processes affect problem solving performance while observing gender differences	Mathematical knowledge, computation skill	Grades 9 to 12	40
Byrnes and Takahira (1994)	To investigate whether prior knowledge and cognitive processes affect problem solving performance	Mathematical knowledge, computation skill	Grades 9 to 12	40
Serafino and Cicchelli (2003)	To test the effects of prior knowledge on student mathematical problem solving and transfer to an analogous task	Mathematical knowledge	Grade 5	50
Lee and Chen (2009)	To investigate the effects of type of question prompt and level of prior knowledge on non-routine mathematical problem solving	Knowledge	Grade 9	78

Table 2.1 (Continued)

Studies Related to the Influence of Cognitive Abilities on Mathematical Problem Solving

Study	Purpose	Mathematical Problem Solving Correlate	Participants' Age/Grade Span	Number of Participants
Iguchi (2008)	To examine the relationship between mathematics achievement through problem solving and working memory and whether this relationship changes across levels of math education	Working memory	6 to 16 years old	136
Swanson (2004)	To assess relationship between the components of working memory and mathematical problem solving performance	Working memory, reading ability, numerical skills, and intelligence	Grades 1 to 3	353
Walker (2012)	To examine the relationship between oral reading fluency (ORF) scores and mathematics problem solving scores	Reading ability	Grades 3 to 5	121
Jordan (2003)	To examine mathematical competencies in students with specific mathematics difficulties, and to compare them with students with comorbid mathematics and reading difficulties	Reading ability	7 to 9 years old	180

Table 2.1 (Continued)

Studies Related to the Influence of Cognitive Abilities on Mathematical Problem Solving

Study	Purpose	Mathematical Problem Solving Correlate	Participants' Age/Grade Span	Number of Participants
Vilenius-Tuohimaa et al. (2008)	To investigate the interplay between mathematical word problem skills and reading comprehension	Reading ability	Grade 4	225
Garderen (2006)	To investigate students’ use of visual imagery and its relationship to spatial visualization ability while solving mathematical word problems	Spatial Ability	Grade 6	66
Awofala et al. (2011)	To investigate the effects of verbal ability and cognitive style as moderator variables on mathematical word problem achievement	Verbal abilities	Grade11	450
James and Adewale (2012)	To explore the relationship between the achievement of students in mathematical problem solving and intellectual abilities	Verbal abilities, intelligence, numerical abilities	Grades 9 to 12	206

General intelligence. In the everyday world, the ability to solve practical problems has been regarded generally as an expression of intelligence (Vickers et al., 2004). Similarly, in the field of psychology, the cognitive aspects of problem solving have long been considered as essential to any well-conceived notion of intelligence (Resnick & Glaser, 1976; Sternberg, 1982). In this section, seven studies (Burns et al., 2006; Huggins, 1988; Hegarty and Kozhevnikov, 1999; Heinze, 2005; Sriraman, 2003; Vickers et al., 2004; Xin and Zhang, 2009) related to the relationship between general intelligence and mathematical problem solving were analyzed and a summary of the studies has been shown in Table 2.1.

Five of the studies (Burns et al., 2006; Huggins, 1988; Hegarty and Kozhevnikov, 1999; Vickers et al., 2004; Xin and Zhang, 2009) were correlational and two (Heinze, 2005; Sriraman, 2003) were qualitative. Researchers in all five correlational studies used several instruments but only two of them (Huggins, 1988; Hegarty and Kozhevnikov, 2009) documented reliability and validity measures for all the instruments used. Researchers in all of the correlational studies provided information related to sex, age, and grade level of the participants. However, none of the researchers provided a thorough description of participants including demographic background and socio-economic status. Statistical information was reported by all researchers. In addition all researchers interpreted these statistical findings and effect sizes, as suggested by Thompson et al. (2005).

Both researchers (Heinze, 2005; Sriraman, 2003) observed students' problem solving skills to conduct their qualitative studies. They documented information related to participants' age and grade level. However, Sriraman (2003) was the only researcher who provided a thorough description of participants including sample size, demographic background, and socio-economic status. In addition, Sriraman (2003) was the only researcher who used one or more procedures to triangulate the data from his study. He had triangulation of data sources, namely data from students' journal writing, the researcher's journal writing, and the interview protocol. In both of the studies, the authors provided descriptions of the data they collected, providing explanations of participants' narratives and interpretation of results with excerpts from the interviews.

Findings within the topic of general intelligence could be divided in three main areas: (a) influence of intelligence on accuracy of solutions, (b) influence of intelligence on understanding problem complexity, and (c) influence of intelligence on devising problem solving strategies.

Accuracy. The relationship between problem solving accuracy and performance on a measure of intelligence was investigated by researchers (Xin & Zhang, 2009; Vickers et al., 2004; Burns et al., 2006). Regardless of strategies and methods used to solve a problem, when final answers were considered as the only indicator of mathematical problem solving performance, scores on measures of intelligence were found to be correlated with scores on measures of mathematical problem solving at a significant level. In addition, general intelligence was found to be a significant predictor of mathematical problem solving performance (Xin & Zhang, 2009; Vickers et al., 2004; Burns et al., 2006).

Although intelligence was found to be an important factor to reach an accurate answer when solving a problem, no information was provided related to the function of intelligence when solving open-ended problems, which might have numerous possible accurate solutions. When considering that many real life problems have an open-ended structure, the association between intelligence and solving open-ended problem solving performance should be investigated so that the influence of intelligence on problem solving processes can be understood.

Understanding complexity. Mathematically gifted students were found to be more successful at understanding the complexity of problems than were non-gifted students (Sriraman, 2003). In addition, non-gifted students displayed absence of some problem solving and generalization skills found in gifted students. Most commonly lacking were comprehension of the problem situation, assessment of the adequacy of the information given in the problem, identification of the assumptions in the problem situations, and differentiation between interrogative and declarative statements (Sriraman, 2003). When students were asked to solve complex optimization and probability problems, students' problem solving scores and their IQ scores on the Raven's test were found to be correlated at a significant level (Vickers et al., 2004). Also students' IQ scores were found to be correlated significantly with understanding path complexity, which was another aspect of their performance when solving the problem (Burns et al., 2006; Vickers et al., 2004). Furthermore, people's ability to solve complex problems was found to be predicted by their intelligence and it was related to the ability to reason, plan, think abstractly, and comprehend complex ideas (Burns et al., 2006; Sriraman, 2003; Vickers et al., 2004).

Devising strategies. The influence of intelligence on devising strategies to solve mathematical problems was investigated by researchers (Hegarty and Kozhevnikov, 1999; Heinze, 2005; Huggins, 1988). When students were grouped based on their IQ scores, the effect of IQ scores on devising strategies to solve mathematical problems were found to be at a significant level (Huggins, 1988). In addition, when gifted and non-gifted students' mathematical problem solving strategies and thinking processes were compared, gifted students were found to need less time to resolve problems and they showed more systematic and logical strategies in developing solutions to problems than did those were not gifted (Heinze, 2005. In addition, gifted students showed a better ability to verbalize and explain their solutions, and to use their insight to discover the mathematical structure of the problem (Heinze, 2005). Furthermore, the use of schematic spatial representations and strategies was associated with success in mathematical problem solving and also the use of schematic spatial strategies was found to be a significant predictor of mathematical problem solving performance (Hegarty & Kozhevnikov, 1999).

Although the methods and research designs of the studies (Burns et al., 2006; Huggins, 1988; Hegarty and Kozhevnikov, 1999; Heinze, 2005; Sriraman, 2003; Vickers et al., 2004; Xin and Zhang, 2009) that were analyzed in this section varied from study to study, findings were in accordance: Intelligence was a significant predictor of problem solving performance.

General creativity. Many researchers viewed creativity as a special intellectual act to solve problems (Bransford & Stein, 1984; Hayes, 1981; Henle, 1962; Newell, Shawn, & Simon, 1962; Newell & Simon, 1972; Perkins, 1981; Vaughan, 1985). Newell, et al. (1962) defined creativity as “a special class of problem solving activity characterized by novelty, unconventionality, persistence, and difficulty in problem formulation” (p.66), whereas Hayes (1981) defined it as “a special kind of problem solving, that is the act of solving an ill-defined problem” (p. 199). In this section, two studies (Bahar & Maker, 2011; Hwang et al., 2007) related to the relationship between general creativity and mathematical problem solving were analyzed and a summary of the studies has been shown in Table 2.1.

Both of the studies (Bahar & Maker, 2011; Hwang et al., 2007) were correlational. Researchers in both of the studies used several instruments and both of them documented reliability and validity measures for all the instruments used. In both of the studies, researchers provided information related to sex, age, and grade level of the participants. However, only

Bahar & Maker, 2011 provided a thorough description of participants including demographic background and socio-economic status. Statistical information was reported by both researchers. In addition, all researchers interpreted these statistical findings and effect sizes, as suggested by Thompson et al. (2005).

In both of the studies (Bahar & Maker, 2011; Hwang et al., 2007) researchers investigated the relationship between students' creativity and achievement by observing participants' mathematical problem solving performance. Significant relationships were found among all measures of creativity and mathematics achievement. In addition, mathematical creativity was found to be a significant predictor of mathematical achievement scores (Bahar & Maker, 2011; Hwang et al., 2007). Also the ability to elaborate, as an aspect of creativity, was found to be a key factor that influenced students' problem solving performance (Hwang et al., 2007). Furthermore, students with high ability to elaborate took better advantage of peer interactions and teacher guidance to generate more diversified ideas and solutions in problem solving (Hwang et al., 2007). In contrast, students with low ability to elaborate had great difficulty in manipulating their representation skills.

Bahar and Maker's (2011) and Hwang et al.'s (2007) studies were different in their nature. For example, they had different perspectives on the domain of creativity. Bahar and Maker investigated creativity in a specific domain (only in the mathematics domain) whereas Hwang et al. had a domain general perspective. Despite the differences between their nature, when combining Bahar and Maker's (2011) findings with Hwang et al.'s (2007) findings, the author concluded that creativity was an important predictor of mathematical problem solving performance.

Mathematical knowledge. Prior knowledge is defined as an important tool that is used to structure the information in the problem, allows the individual to apply a familiar scaffold to the information, regardless of how helpful or harmful it might be (Pretz, Naples, & Sternberg, 2003). Furthermore, Pretz et al. (2003) asserted that prior knowledge mediates an individual's ability to represent the problem in the most efficient fashion. Specifically, researchers found that individuals who have accumulated considerable knowledge in a domain represent information about problems differently from the ways these problems are represented by individuals without extensive knowledge bases (Chi, Glaser, & Farr, 1988). In this section, four studies (Byrnes & Takahira, 1993; Byrnes & Takahira, 1994; Lee & Chen, 2009; Serafino & Cicchelli, 2003) related to the relationship between mathematical knowledge and mathematical problem solving were analyzed and a summary of the studies has been shown in Table 2.1.

All four studies (Byrnes & Takahira, 1993; Byrnes & Takahira, 1994; Lee & Chen, 2009; Serafino & Cicchelli, 2003) were correlational. Researchers in all four studies used several instruments but only two of them (Lee & Chen, 2009; Serafino & Cicchelli, 2003) documented reliability measures for all the instruments used. In addition none of the researchers documented the validity of instruments. All researchers provided information related to sex, age, and grade level of the participants. However, none of the researchers provided a thorough description of participants including demographic background and socio-economic status. Statistical information was reported by all researchers. In addition, all researchers interpreted these statistical findings and effect sizes, as suggested by Thompson et al. (2005).

In these studies prior knowledge was found to be a significant predictor of mathematical problem solving performance (Byrnes & Takahira, 1993; Byrnes & Takahira, 1994; Lee & Chen, 2009; Serafino & Cicchelli, 2003). The influence of mathematical achievement and instructional models on problem solving performance was investigated as well as prior knowledge. Prior knowledge was found to be not only a significant but also the strongest predictor of students' performance in mathematics problem solving performance among these variables (Byrnes and Takahira, 1993).

When the effects of question prompts, prior knowledge, and interaction of question prompts and prior knowledge were investigated in open-ended mathematical tasks, prior knowledge was found to predict problem solving performance at a significant level (Lee & Chen, 2009). In addition, when the effects of prior knowledge and instructional models were evaluated on student mathematical problem solving performance, students with a high prior knowledge had a higher mean in problem solving scores than students with a low prior knowledge regardless of the type of the instruction they were given (Serafino & Cicchelli, 2003). Furthermore, the influence of prior mathematical knowledge on problem solving performance in open-ended situation was analyzed and prior knowledge was found to be a significant predictor of problem solving performance in open-ended situations (Lee & Chen, 2009).

Working memory. Human working memory was declared to be the foundation of mathematical problem solving (Lubienski, 2007). In this section, two studies (Swanson, 2004; Iguchi, 2008) related to the relationship between working memory and mathematical problem solving were analyzed and a summary of the studies has been shown in Table 2.1.

Both studies (Swanson, 2004; Iguchi, 2008) were correlational. Researchers in both of the studies used several instruments and both of them documented reliability and validity measures for all the instruments used. All researchers provided a thorough description of participants including demographic background and socio-economic status. Statistical information was reported by all researchers. In addition all researchers interpreted the statistical findings and effect sizes, as suggested by Thompson et al. (2005).

Working memory was found to predict mathematical problem solving performance at a significant level (Swanson, 2004; Iguchi, 2008). In addition to working memory, fluid intelligence, reading ability, processing speed, and knowledge of algorithms were found to be significant predictors of mathematical problem solving performance. However, working memory was found to predict solution accuracy of word problems independent of measures of fluid intelligence, reading skill, math skill, phonological processing, semantic processing, speed, short-term memory, inhibition, and knowledge of algorithms (Swanson, 2004).

The effect of working memory on three specific areas of math achievement was investigated, including knowledge of basic math facts, calculation skills, and application of math concepts through problem solving (Iguchi, 2008). Greater auditory working memory capacity was found to predict a higher level of math achievement in all areas. Also auditory working memory explained unique variance, above and beyond the contributions of verbal and nonverbal reasoning and processing speed, in overall math achievement, fact fluency, and problem solving, but not calculation skills (Iguchi, 2008). The variance in overall math achievement, fact fluency, and calculation skills explained by variance in working memory remained stable across two age groups representing elementary and secondary levels of education. Researchers concluded that both elementary and secondary level mathematics achievement relied on auditory working memory (Iguchi, 2008).

***Reading ability*.** Reading ability has been considered a basic requirement for success in many academic subjects, including mathematics. Solving mathematical word problems requires strong reading comprehension and educators need to improve students' reading skills to improve mathematics performance (Fuentes, 1998). In this section, three studies (Jordan et al., 2003; Vilenius-Tuohimaa et al., 2008; Walker, 2012) related to the relationship between reading ability and mathematical problem solving were analyzed and a summary of the studies has been shown in Table 2.1.

All three studies (Jordan et al., 2003; Vilenius-Tuohimaa et al., 2008; Walker, 2012) were quantitative. Researchers in all three studies used several instruments and all of them documented reliability and validity measures for all the instruments used. All researchers provided information related to sex, age, and grade level of the participants. However, only one of the researchers (Vilenius-Tuohimaa et al., 2008) provided a thorough description of participants including demographic background and socio-economic status. Statistical information was reported by all researchers. In addition, all researchers interpreted the statistical findings and effect sizes, as suggested by Thompson et al. (2005).

In these three studies, reading ability was found to be associated with mathematical problem solving performance. When participants were grouped based on their reading comprehension ability, the students in the good reading group were found to perform better on mathematical word problems tests than did those students assigned to the poor reader group (Vilenius-Tuohimaa et al., 2008). In addition, a significant correlation was found between reading comprehension scores and mathematics problem solving scores (Walker, 2012). Furthermore, when students with reading disabilities and mathematical disabilities were grouped based on their disabilities, the students with only mathematical disabilities performed better than the students with mathematical and reading disabilities in problem solving but not in calculation (Jordan et al., 2003). Another important finding was reading abilities influenced growth in mathematics achievement whereas mathematics abilities did not influence growth in reading achievement.

One major concern for all three studies (Jordan et al., 2003; Vilenius-Tuohimaa et al., 2008; Walker, 2012) was related to the content of the problems that were used for measuring mathematical problem solving performance. In all three studies, the problems were word problems with long sentences. Therefore, reading comprehension and reading ability would be expected to be a predictor of mathematical problem solving performance. When problems are designed with excessive verbal content, reading achievement would be expected to influence growth in mathematics achievement whereas mathematics abilities do not influence growth in reading achievement. Especially English language learners (ELL) might perform less well on these problems because they read more slowly (Mestre, 1988). Furthermore, ELLs may not recognize vocabulary terms and they may not be familiar with the linguistically complex structure of these problems (Duran, 1989). From this perspective, the influence of reading ability on problem solving performance might vary depending on the problem solver’s familiarity with the language. For this reason, instruments with less verbal complexity have been considered to be more fair as tools to measure student performance.

Other cognitive abilities. Researchers have searched for links between problem solving skills and mental abilities because high-order thinking seemed to be linked to problem solving (Cattell, 1971). Hembree (1992) found several abilities related to problem solving. He reported that creative thinking, critical thinking, memory, perception, reasoning, skill with analogies, skill with inferences, and spatial ability were correlated significantly with mathematical problem-solving measures.

In this section, four studies (Awofala et al., 2011; Garderen, 2006; Hegarty & Kozhevnikov, 1999; James & Adewale, 2012) related to the relationship between other cognitive abilities and mathematical problem solving were analyzed and a summary of the studies has been shown in Table 2.1. All four studies were quantitative. Researchers in all four studies used several instruments and but only two of them (Garderen, 2006; Hegarty & Kozhevnikov, 1999) documented reliability and validity measures for all the instruments used. All researchers provided information related to sex, age, and grade level of the participants. However, only one of the researchers (Hegarty & Kozhevnikov, 1999) provided a thorough description of participants including demographic background and socio-economic status. Statistical information was reported by all researchers. In addition, all researchers interpreted the statistical findings and effect sizes, as suggested by Thompson et al. (2005).

One of the cognitive abilities associated with mathematical problem solving performance was spatial ability. Spatial ability was defined as the ability to generate, retain, and manipulate abstract visual images, and it was described by researchers as one of the important factors influencing problem-solving performance (Lohman, 1979). Gardner (1984) proposed a multifaceted model of intelligence in which spatial abilities were one of seven major components. Gardner described “spatial ability” as the capacity to perceive the visual world accurately, to perform transformations and modifications upon initial perceptions, and to be able to recreate aspects of visual experience even in the absence of relevant physical stimuli. Gardner also stated that spatial ability works in collaboration with logical-mathematical ability (mathematical ability and the ability to reason). Similarly, in her hierarchical "three-stratum theory" of ability, Carroll (1993) demonstrated the positioning of spatial ability in juxtaposition with verbal and mathematical ability.

In two studies (Garderen, 2006; Hegarty & Kozhevnikov, 1999) researchers investigated the effects of spatial ability on solving mathematical problems. Spatial ability was found to be one of the main factors affecting mathematical performance. Individuals with high spatial ability were found to have a wider range of strategies and they were better at determining when to use a particular strategy during problem solving (Hegarty & Kozhevnikov, 1999). Researchers found that the use of schematic images was positively related to success in mathematical problem solving, whereas the use of pictorial images was negatively related to success in mathematical problem solving. In addition the use of schematic imagery was associated with high spatial ability (Hegarty & Kozhevnikov, 1999). In addition, spatial visualization ability was found to be correlated with mathematical problem-solving performance at a significant level (Garderen, 2006). However, I would question the validity of these two studies by referring to the claims of Fennema and Tartre (1985). Fennema and Tartre claimed that a student's spatial ability did not affect his or her likelihood of solving certain math problems correctly, but the students with high spatial ability were still more likely to be "...able to convert word problems to accurate diagrams, and to use those diagrams to get correct solutions" (1985, p. 193). At this point, I would recommend further studies to investigate the role of spatial ability in solving problems that do and do not require use of diagrams to get correct solutions.

Verbal ability was another cognitive ability linked with problem solving performance. Verbal ability has been described as a student's ability in oral and written expression, reading and comprehension skills, and literal understanding and use of words. Verbal abilities were associated with and related to success in mathematical problem solving (Lester, 1980).

In two studies (Awofala et al., 2011, James & Adewale, 2012) verbal ability was found to be an important predictor of problem solving performance. In addition, students with high verbal ability performed significantly better than students with low verbal ability in problem solving performance (Awofala et al., 2011). Also verbal reasoning scores of the students accounted for 58% of the variance in mathematical problem solving scores (James & Adewale, 2012). Although verbal ability had significant effects on students' achievement in mathematical word problems, the accordance of these findings might be because of the similar structure of the problems that were used in these studies. How does the relationship between verbal ability and problem solving performance change when non-verbal problems are used? The author recommends that the relation between verbal ability and mathematical problem solving performance is complex and further studies are needed to explore this relationship.

Gender and Mathematical Problem Solving

In this section 10 studies (Benbow, 1988; Buchanan, 1984; Duffy, Gunther & Walters, 1997; Garrard, 1982; Hembree, 1992; Hyde et al. 1990, Kallam, 1996; Landau, 1984; Lindberg et al., 2010; Paik, 1990) that included findings on the gender differences in mathematical problem solving performance were analyzed and the summary of the analysis of the studies has been presented in Table 2.2.

Six of the studies (Benbow, 1988; Duffy, Gunther & Walters, 1997; Garrard, 1982; Kallam, 1996; Landau, 1984; Paik, 1990) were correlational, three were meta-analysis (Hembree, 1992; Hyde et al. 1990; Lindberg et al., 2010) and one (Buchanan, 1984) was qualitative. Researchers in all six correlational studies used several instruments but only four of them (Benbow, 1988; Garrard, 1982; Landau, 1984; Paik, 1990) documented reliability measures for all the instruments used. Only one of the researchers (Benbow, 1988) documented validity of instruments, mainly content and construct validity. Researchers in all of the correlational studies

Table 2.2

Studies Related to the Gender Differences in Mathematical Problem Solving Performance

Study	Purpose	Participants' Age/Grade Span	Number of Participants
Benbow, 1988	To describe the sex differences in scores on the mathematics section of the Scholastic Aptitude Test (SAT-M) among the intellectually talented and the relation of this sex difference to other attributes and achievement	12 and 13 years old	9927
Buchanan, 1984	To determine if any observable qualitative differences were found in mathematical problem solving performance between girls and boys	Grades 3 to 5	12
Duffy, Gunther & Walters, 1997	To investigate the relationship between mathematical problem solving and gender	12 years old	159
Garrard, 1982	To investigate the mathematical problem solving and spatial visualization ability of female and male students	Grade 8	120
Hembree, 1992	To analyze prior experiments and relational studies in problem solving	Grades 1 to 12 and college students	37022 (meta-analysis of 154 studies)

Table 2.2 (Continued)

Studies Related to the Gender Differences in Mathematical Problem Solving Performance

Study	Purpose	Participants' Age/Grade Span	Number of Participants
Hyde et al. 1990	To find the magnitude of gender differences in mathematics performance	5 to 52 years old	3,175,188 (meta-analysis of 100 studies)
Kallam, 1996	To investigate the differences in mathematical problem solving between males and females	18 to 27 years old	47
Landau, 1984	To investigate how spatial visualization ability, problem presentation format, and the interaction of the two influenced middle school students' performance on mathematical problems and to examine gender-related differences in their performance	Grades 6 to 8	384
Lindberg et al., 2010	To analyze gender differences in recent studies of mathematics performance	All ages	1,286,350 (meta-analysis of 242 studies)
Paik, 1990;	To investigate the influence of individual differences such as gender and cognitive abilities in metacognitive ability and problem solving performance	Grade 10	80

provided information related to sex, age, and grade level of the participants. However, none of the researchers provided a thorough description of participants including demographic background and socio-economic status. Statistical information were reported by all researchers. In addition all researchers interpreted these statistical findings and effect sizes, as suggested by Thompson et al. (2005).

Only one of the researchers (Buchanan, 1984) used qualitative methods to investigate gender differences in mathematical problem solving performance. Buchanan provided a thorough description of participants including sample size, demographic background, and socio-economic status as well as grade level and age. She used triangulation of data sources, namely data from analysis of videotapes, the researcher's writings, and an interview protocol. In addition, the author provided descriptions of the data collected, providing explanations of participants' narratives and interpretation of results with excerpts from the interviews.

Findings. Particularly within the fields of psychology and education, gender differences in mathematics problem solving performance have been studied intensively. Findings within the topic of gender and mathematical problem solving performance could be categorized under schooling age: (a) elementary school period (grades K-6), (b) grade 7 and later years.

In four studies (Buchanan, 1984; Hembree, 1992; Hyde et al. 1990; Lindberg et al., 2010) researchers investigated gender differences in elementary school students' mathematical problem solving performance. No significant differences in problem solving performance between boys and girls were found during the elementary years (K-6). Females were found to perform better in computation skills in grades K-6. In addition, no gender difference was found in understanding of mathematical concepts during elementary and middle school (Hyde et al. 1990).

After grade 7, significant differences between boys and girls were found in mathematical problem solving performance. Boys showed higher performance than girls in mathematical problem solving in high school and college at a significant level ((Benbow, 1988; Duffy, Gunther & Walters, 1997; Garrard, 1982; Hembree, 1992; Hyde et al. 1990, Kallam, 1996; Landau, 1984; Lindberg et al., 2010; Paik, 1990). Boys were found to perform higher than girls in spatial visualization ability scores (Garrard, 1982; Landau, 1984). In addition to spatial ability, boys performed significantly higher than girls in meta-cognition scores (Paik, 1990).

At this point, three comprehensive meta-analyses (Hembree, 1992; Hyde et al. 1990; Lindberg et al., 2010) provided valuable information to explain the inconsistent findings among these studies. In these three meta-analysis studies researchers found that the gender differences in problem solving were not significant in the elementary and middle school years but in the high school and college years a moderate effect size favored males. Although these meta-analyses seemed to indicate clear understanding of the relationship between gender and problem solving performance, Zhu (2007) recommended further studies to investigate how different factors interact to produce gender differences in mathematical problem solving performance. Furthermore, he questioned the ability of test scores to indicate the real differences in cognitive abilities.

In his review of research, Zhu analyzed the influences of factors involved in mathematical problem solving including cognitive abilities, speed of processing information, and many complex variables related to problem solving, such as (a) physiological differences in brains, (b) influences of sex hormones, (c) learning styles, (d) learners’ attitudes, (e) stereotype threat in mathematics tests, (f) differences in socialization, and (g) socioeconomic variables. He concluded that the findings of the studies were complex and inconsistent. Based on the difficulty of explaining the inconsistencies between the studies, Zhu recommended analyzing the relationship between gender and mathematical problem solving from a comprehensive perspective. The author suggested further studies to explore how biological and psychological variables interact with both experience and the environment to contribute to gender differences in mathematical problem solving patterns.

Conclusion

Problem solving is referred to an "extremely complex form of human endeavor that involves much more than the simple recall of facts or the application of well-learned procedures" (Lester 1994, p. 668). Researchers concluded that to be a good problem solver in mathematics, students should be able to select and use task-appropriate cognitive strategies for understanding, representing, and solving problems (Mayer, 1992; Schoenfeld, 1985).

Problem solving has come to be viewed as a process that requires use of many cognitive abilities and processes including intelligence (Polya, 1973; Resnick & Glaser, 1976; Sternberg, 1982), creativity and originality (Polya, 1953), reading ability (Hite, 2008), spatial ability (Booth and Thomas, 1999), verbal ability (Dodson, 1972), working memory (Swanson 2004), and knowledge (Lester, 1980). In the studies that were analyzed in this chapter researchers found that cognitive abilities influenced mathematical problem solving at significant levels. However, this analysis showed a need for exploring the relationship between these cognitive variables and problem solving performance as a whole instead of individual relationships because the interactions between cognitive abilities could impact the relationships. To fill this gap in the studies, the author investigated the influence of several cognitive abilities including general intelligence, general creativity, working memory, mathematical knowledge, reading ability, spatial ability, quantitative ability, and verbal ability on mathematical problem solving performance as a whole.

Another gap in the studies was that prior researchers did not consider that the relationship between cognitive abilities and problem solving performance could vary depending on the type of the problem. To fill this gap, the author modeled this relationship by separating performance in open-ended and closed problems. By doing so, he aimed to investigate how problems with different structures might require different cognitive abilities for reaching successful solutions.

CHAPTER III: METHOD

A primary purpose of this study was to investigate the influences of cognitive abilities such as intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability on the mathematical problem solving performance (MPSP) of students. The author modeled this relationship by separating performance in open-ended and closed problems. Furthermore, the author investigated how these relationships were different or similar in boys and girls. This chapter included the research methodology and design, participants and sampling, settings, and instruments sections. The data collecting procedures and data analyses sections were presented in this chapter.

Research Method and Design

The purpose of this study was to investigate the influences of cognitive abilities on the mathematical problem solving performance of students. For the exploration of potential relationships among these variables, numerical data were analyzed. Therefore, this study was classified as quantitative research. According to Burns and Grove (2005), in quantitative research, numerical data are used and statistical analyses are employed to obtain information about the world, giving the opportunity to describe and examine possible relationships among variables.

The research design of this study was a non-experimental and descriptive correlational study. Correlational studies have been used to examine the relationships among two or more variables, and they provide an opportunity to determine the pattern and the strength of the existing relationships and also allow for hypotheses generation. A correlational relationship indicates association between variables in a synchronized manner that does not imply causal relationship. Non-experimental studies are very common in the field of education, because many human characteristics cannot be manipulated experimentally due to natural and ethical reasons. Studies that combine descriptive and correlational characteristics are used to examine variables and to describe relationships among them (Burns & Grove, 2005; Polit & Beck, 2004; Trochim, 2001).

Participants and Sampling

The data in the study were collected during the STEP-UP and DISCOVER Project. The participants included 67 students in grade 3. The participants were from four schools in a southwestern state in the U.S. as presented in Table 3.1. All schools were located in the Diné Nation, and all were in rural, low-income areas. At least 94% of the students at each of these schools came from low income families.

Table 3.1

The Number of Students at Each School

		Gender	
School	Number	Male	Female
School A	20	7	13
School B	13	6	7
School C	17	12	5
School D	17	11	6
Total	67	36	31

School A was a K-8 school near a small town with approximately half its students being boarders (some of whom went home each weekend, and some of whom went to homes of relatives and friends only during extended vacations) and half its students being day students. Of the 630 children, 98-99% were Diné (Navajo), and the only non- Diné were the children of the staff. Approximately 70% of the children spoke Diné as their home and dominant language, and some were bilingual in English and Diné. All early elementary teachers spoke Diné and used it when necessary to provide directions to children.

School B was a K-6 public school located in a small community. Of the 390 students, approximately 99% were Diné and others were children of teachers or other staff members. Many of the students spoke Diné as their first or dominant language, so early elementary teachers either spoke Diné or worked with an instructional assistant who did.

School C was a small community controlled school funded by Bureau of Indian Affairs. The school had 430 students in grades K-12 located in a Diné Indian Reservation. Many of the students spent one to three hours riding the school bus each day. Approximately 98% of the students were Diné, with the only non- Diné students being those whose parents were teachers or administrators at the school. The faculty and staff were 90% Diné, and most came from local community. The school was unique on the Diné nation, being the only school in which total bilingualism and literacy in Diné was taught K-12.

School D was a K-6 school located in a small community near the border. Of the 574 students, 99% were Diné and 1% Caucasian. Although most of the children spoke some English, many of them would not have been considered proficient in either language when they entered school. Most teachers were Caucasian, but teachers in primary grades were assisted by a bilingual teaching assistant. The bilingual instructional model would be considered transitional or English immersion, with all or most teachers having an endorsement in teaching English as a second language (ESL).

Settings

The data were collected from the archives of the DISCOVER projects at the University of Arizona. Specifically, the author selected students who were enrolled in classrooms that were part of the Systematic Training of Educational Programs for Underserved Pupils (STEP-UP) project, a research and development grant that was federally funded. In general, the project involved four states and 12 local school districts. The main purpose of the STEP-UP project was to develop and test procedures for identifying gifted students who came from diverse backgrounds and then provide them with appropriate curricula. In the DISCOVER project archives, longitudinal data about students who enrolled in the STEP-UP project's classrooms were available for researchers, including results that were collected using a variety of tests. In the Southwestern region of the United States, the coordinator of the STEP-UP project agreed to organize the classrooms to be self-contained in four different schools (A, B, C, and D) to monitor students' development over time, provide them with appropriate curricula, and collect longitudinal data. More details about the STEP-UP project in the Southwestern region of the United States are in the STEP-UP project report (Maker, 1993).

Operational Definitions of Variables

General intelligence. General intelligence was defined as a person's capacity to apprehend meaningless figures presented for his/her observation, see the relations between them, conceive the nature of the figure completing each system of relations presented, and so doing, develop a systematic method of reasoning (Raven, 1988).

General creativity. General creativity was defined as a competence to generate unconventional figures with overall meaning (Urban & Jellen, 1996).

Mathematical knowledge. Mathematical knowledge was defined as the competence to deeply understand basic mathematical concepts and computation (i.e. use of arithmetic operations - addition, subtraction, multiplication, or division).

Mathematical problem solving performance. Mathematical problem solving performance was defined as a performance of an act to make unknown situations/questions known in the domain of mathematics.

Working memory. Working memory was defined as a brain system that provides temporary storage and manipulation of the information necessary for such complex cognitive tasks as language comprehension, learning, and reasoning (Baddeley, 1992).

Verbal ability. Verbal ability was defined as the cognitive ability to understand and use language.

Fluency. Fluency was defined as the ability to deliver information quickly and accurately.

Measurement of Variables

The Measurement of Mathematical Problem Solving Performance

DISCOVER assessment. The Discovering Intellectual Strengths and Capabilities while Observing Varied Ethnic Responses (DISCOVER) assessment scores were used in this study to measure the participants' problem solving performance in mathematics. The DISCOVER assessment model was developed to identify gifted students from culturally diverse groups by observing the number and the choice of problem solving strategies used by children. The assessment was grounded in the theory of multiple intelligences (Gardner, 1984), the theory of the triarchic mind (Sternberg, 1989), and studies of creativity (Getzels & Csikszentmihalyi, 1976).

The DISCOVER assessment was developed from research involving numerous age groups, cultures, languages, geographic locations, and ethnic identities. Its corresponding design is applicable and accurate in all languages and cultures, virtually eliminating the ethnic, cultural, and linguistic bias prevalent in many current instruments. In the DISCOVER assessment, rather than relying on paper-and-pencil tests, performance is measured over a broad perspective of potential abilities, using a range of materials. Each ability is measured in a way appropriate to its own characteristics, rather than being filtered through another ability. This component is critical to an accurate assessment of abilities and unfortunately is not present in many traditional instruments. Consider, for example, a math exercise that is presented as a word problem in English. A child with a dominant language other than English might struggle with the language component, thus masking his or her true ability to solve the problem. In this case the ability actually being assessed is linguistic, not mathematical, producing potentially skewed or erroneous results. For accuracy, each ability must be assessed (as much as possible) in a manner free from dependency on any other ability.

Problem solving has been a key component of the DISCOVER assessment model. Problem solving was conceptualized in the model based upon the problem classification proposed by Getzels and Csikszentmihalyi (1976). In this context, problems were classified as either closed or open based on the number of alternatives available to the problem solver. For example, a problem was defined as closed if it could be solved in only one way and open if it could be solved in an infinite number of ways.

For the DISCOVER assessment model, problem structure was rated on a scale that ranged from a Type I problem to a Type VI problem (Table 1.1). A Type I problem would be closed whereas a Type VI problem would be completely open-ended. All conceivable problems could fall somewhere on the continuum between the two extremes. Sak and Maker (2005) investigated the construct validity of the problem continuum by determining the relationships among the problem types. They found that correlations between the problem types varied according to the proximity of the types to each other. For instance, the correlation between Type I and Type II problems was .49, between Type I and Type III was .41, and between Type I and Type IV was .39. All correlations were statistically significant at the .01 level, showing the validity of the distinctions between problem types.

Table 1.1 has the problem types in the DISCOVER problem continuum (Maker & Schiever, 2005), a tool used to design assessments and curricula. The six problem types have been displayed from Type I through Type VI, along with how much information was known—how much structure was provided—for both the problem presenter and the problem solver in each Problem Type.

The math section of the assessment included Types I, II, IV, and V problems. Open-ended problem solving performance was assessed using Type IV and V problems whereas closed problem solving performance was assessed using Type I and II problems. The DISCOVER assessment varied in form and implementation according to the age group being assessed. Different forms were developed for Pre-K, K-2, 3-5, 6-8, and 9-12.

Previous studies of the DISCOVER assessment showed high inter-rater reliability ranging from 80% to 100% (Sarouphim, 1999; Griffiths, 1996). Sak and Maker (2003) investigated the predictive validity of DISCOVER, and found that it explained 20% of the variance in Stanford 9 Math scores with p=.007 and 20% of the variance in AIMS Math scores with p=.009. These results provided evidence for the predictive validity of DISCOVER. The results obtained by Sak and Maker (2004) and Maker (2001) showed that moderate correlations existed between the DISCOVER assessment and math achievement ($r= .30$, $p<.01$) and IQ scores ($r=.35$, $p<.01$).

The Measurement of General Intelligence

Raven's Colored Progressive Matrices (RCPM). The RCPM is a norm-referenced test that was designed in 1948 to measure general intelligence of young children aged 5 through 11. The RCPM is a nonverbal group test, typically used in educational settings, and can be administered individually or after the age of 8 in a group format. The test consists of 36 items, grouped into three sets (A, Ab, B) of 12 items in each set. Set A consists of problems in a continuous pattern. Items in sets Ab and B include four parts, three of which are presented and individuals have to choose the correct one from the alternatives. Items in each set increase progressively in perceptual difficulty. The test was designed in spatial form, and it contained structural representations of space, which was used to describe relations between objects in a visual scene, such as the relative location of these objects (Lovett, Forbus & Usher, 2010). Each item in the test had a missing part in a pattern to be completed from the given choices. The correlations between the item difficulties, established separately for different ethnic groups, ranged from .97 to 1.00 (Jensen, 1980). Also the test-retest reliabilities have ranged from .71 to .92 and concurrent validity estimates ranged from .55 to .86 (Sattler, 1988).

As a non-verbal measure, the RCPM has been described by its authors as a "fair measure of ability for individuals from different cultures because it was not influenced by language differences. This helps reduce cultural bias in your employee evaluations – an important benefit in today's multicultural society and global workforce." (Raven's Progressive Matrices™, 2012).

The Measurement of General Creativity

Test for Creative Thinking-Drawing Production (TCT-DP). The TCT-DP (Urban and Jellen, 1996), a tool for measuring general creativity, was designed as a cross-cultural instrument. The TCT-DP test consisted of six figural fragments that stimulate further drawing in a free and open way: a semi-circle, a point, a large right angle, a curved line, a broken line, and a small open square outside the large square frame. The drawing product was evaluated and scored by means of 14 evaluation criteria: continuations, completions, new elements, connections made with a line, connections made to produce a theme, boundary breaking that was fragment dependent, boundary breaking that was fragment independent, perspective, humor, affectivity, unconventionality (sub scores A, B, C, D), and speed (Urban & Jellen, 1996). These fourteen scores were then combined into a total score.

According to Urban and Jellen (1996), the reliability of the TCT-DP was high—from .89 to .97. The authors (Urban & Jellen, 1996) claimed that the validity of the TCT-DP was difficult to evaluate because no comparable instrument existed, and cited studies showing low or no correlations between the TCT-DP and measures of achievement as evidence of its discriminant validity. However, others found correlations ranging from .21 to .41 with the Raven Matrices and the TCT-DP (Urban & Jellen, 1996). The TCT-DP was designed as a nonverbal assessment. The test was field-tested with hundreds of elementary students in 11 countries from diverse backgrounds.

The Measurement of Mathematical Knowledge and Reading Ability

Iowa Tests of Basic Skills (ITBS). The ITBS was a norm-referenced nationally standardized test providing a comprehensive measurement of growth in word analysis, vocabulary, listening, reading, methods of study, the mechanics of writing, and mathematics. The mathematics section included subtests of problem solving, math concepts, and computation. The

ITBS ‘math concepts’ score was used as a measure of the participants’ mathematical knowledge in this study. The ITBS ‘reading’ score was used as a measure of the participants’ reading ability in this study. Test–retest stability coefficients over a one-year time interval were measured in the .70 to .90 range and internal consistency and alternate forms reliability coefficients were in the .80s and .90s (Linn & Miller, 2005). The ITBS was used at Schools A and C.

The test publishers stated that they minimized bias through item analysis by expert reviews and quantitative analyses of test results from tryout studies conducted with students from diverse backgrounds. However, the ITBS assessment was critiqued by educators and researchers because of its verbal structure. Some educators claimed that the verbal complexity of the items in the ITBS might be a potential problem for students from diverse backgrounds (Jamal, 2007).

Comprehensive Tests of Basic Skills/4 (CTBS/4). The CTBS/4 was designed to measure achievement in reading, language, spelling, social studies, study skills, mathematics, and science. The mathematics section included subtests of ‘concepts and arithmetic’, and ‘computation’. The CTBS/4 ‘concepts and arithmetic’ score was be used as a measure of the participants’ mathematical knowledge in this study. The reading section included subtests of vocabulary and comprehension. The CTBS ‘reading comprehension’ score was used as a measure of the participants’ reading ability in this study. Reliability coefficients (KR-20) for the levels used by the sample ranged from .88 to .94 (Shepard, 1985). The CTBS/4 was used at Schools B and D.

The Measurement of Working Memory

Structure-of-Intellect (SOI) Learning Abilities Test. The SOI learning abilities test (Meeker & Meeker, 1976) was designed to measure discrete cognitive abilities based on Guilford's structure of intellect (SI) theory, in which intelligence was viewed as being comprised of operations, contents, and products. The SOI test has been available in two alternate forms, A and B. Each form could be individually or group administered to students in grades 2-12. The test included 26 subtests and each subtest measured each of Guilford's 26 intellectual abilities (Guilford, 1967). For example, the first subtest, CFU, measured "cognition of figural units". Along with general cognitive assessment, the SOI has been used widely to diagnose learning disabilities, prescribe educational interventions, profile strengths and weaknesses, identify reasons for underachievement, match cognitive style and curriculum material, and screen for gifted students. The test-retest reliabilities of the SOI have ranged from .00 to .74 (Coffman, 1985). For this study, the SOI total memory score was used as a measure of the participants' working memory. The total memory score was calculated as the sum of four subtests: memory of figural units (MFU), memory of symbolic implications (MSI), memory of symbolic systems (MSS), and memory of symbolic units (MSU).

The SOI test was described as an effective instrument in identifying gifted students from minority backgrounds (Meeker, 1978). Roid (1985) claimed that the SOI test was an ideal assessment for culturally and linguistically diverse students because of its predominant nonverbal and figural structure.

The Measurement of Verbal, Spatial, and Quantitative Abilities

Developing Cognitive Abilities Test (DCAT). The DCAT (Beggs & Mouw, 1980) was a group administered test designed as a measure of learning characteristics and abilities that contribute to academic performance of students in grades 1-12 (Wick, Beggs, & Mouw, 1980). The test had three categories: verbal, quantitative, and spatial. Each category of the DCAT was comprised of 27 items, for a total of 81 test items. Internal consistency coefficients of the test ranged from .70 to .96, with the majority in the mid .80s (Wick, 1990). The DCAT's verbal, spatial, and quantitative scores were used as measures of verbal, spatial, and quantitative abilities respectively. The DCAT also has been used as a screening measure for identifying potentially gifted students (Wick, 1990). One of the important distinguishing characteristics of the DCAT is the link between specific items and Bloom's (1956) cognitive taxonomy (Canivez, 2000).

Canivez and Konold (2001) assessed the fairness of the Developing Cognitive Abilities Test (DCAT) across gender, race/ethnicity, and socioeconomic dimensions. They concluded that the DCAT provided a generally unbiased assessment of cognitive abilities across race/ethnicity, gender, and SES.

Data Collection

The data related to general intelligence, mathematical knowledge, working memory, verbal ability, spatial ability, quantitative ability, reading ability, and mathematical problem solving were collected at the end of the spring semester of grade 3 in a regular classroom setting. The assessments were administered on different days. The data related to general creativity were collected almost a year later than the data related to other cognitive abilities.

Test Administration

The DISCOVER Assessment was administered by classroom teachers or DISCOVER team members in classroom settings. Students were given a blank sheet and a worksheet containing math problems. Classroom teachers and DISCOVER research team members gave students clear instructions, worked sample problems with them, and made certain all understood the tasks. Students were given as much time as they needed to complete the assessments and were encouraged to use the blank paper they were given to work out problems and solutions.

Test Scoring

The DISCOVER mathematics assessment included four parts. In the first part of the assessment, students solved Type I problems. Students were asked to compute one correct answer for each problem in this part. In the second part of the assessment, students solved Type II problems. In this part, students were asked to solve magic squares through placing numbers and applying math operations. In the third part of the assessment, students solved Type IV problems. Students were asked to write true addition or subtraction problems by using only the three numbers given in the problem. In the last part, students solved Type V problems. Students were asked to write as many problems as possible that had a certain number as the answer. The answer was given in the problem.

Qualified researchers and graduate assistants scored students' solutions to the problems. Students' solutions were scored in two categories: open-ended and closed. Students were given one point for each correct answer. The scores from part I and II were summed as the student's closed problem solving performance score. The sum of scores from part IV and V was used as the open-ended problem solving performance score.

Data Analysis

To answer the research questions, the author employed statistical procedures described below using SPSS. First, preliminary analyses were conducted in the form of a missing data analysis for the data set. These analyses were conducted to explore systematic errors in the data set resulting from missing data or errant unrepresentative results due to observations with excessive influence.

To answer Research Question 1, Pearson correlations were performed to determine the relationship among intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, quantitative ability, and problem solving performance in open-ended and closed mathematics problems. To answer Research Question 2, a one-way between-group multivariate analysis of variance (MANOVA) was performed to examine differences between boys and girls. MANOVA was an ideal analysis technique to explore whether the mean differences between the groups on the combination of dependent variables were likely to have occurred by chances (Pallant, 2010; Tabachnick & Fidell, 2007).

To answer Research Questions 3, 4, 5, and 6, multiple regression analyses were performed to predict students' problem solving performance. To employ multiple regression analyses, intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability constituted independent variables. For Research Question 3, the author used closed problem solving performance scores as the dependent variable, and for Research Question 5, open-ended problem solving performance scores were used as the dependent variable.

For Research Questions 3, 4, 5, and 6, structural equation modeling (SEM) could be preferred as a powerful data analysis technique (Bagozzi and Fornell, 1982) rather than multiple regression analysis. However, the design and structure of the study were not appropriate for the use of SEM. For example, a central assumption in SEM is that the relationship between the observed variables and their constructs, and between one construct and another, is linear. SEM has no established tools for handling variations from this assumption, unlike linear regression that has established and proven remedial data transformational methods for handling data that have nonlinear relationships (Gefen, Straub, and Boudreau, 2000). Linear regression also can deal with multicollinearity (violations of the assumed independence of predictor variables), outliers, heteroscedasticity (unequal variance among the measurement items), and polynomial relationships, such as: $Y = b_0 + b_1X + b_2X_2$ (Hair et al., 1998; Neter et al., 1990). In this study, a moderate multicollinearity was expected to occur among the independent variables as suggested in the review of research. SEM has no tools to identify, let alone handle, these violations of the major distribution assumptions.

CHAPTER IV: RESULTS

The purpose of this study was to investigate the influences of cognitive abilities on the mathematical problem solving performance of students. This chapter included findings for the research questions in the order that was introduced in the first chapter. First, preliminary analyses were conducted to explore systematic errors in the data set resulting from missing data or errant unrepresentative results due to observations with excessive influence. In addition, descriptive statistics on the variables were obtained and the author assured that none of the assumptions made for statistical tests were violated.

Data Screening and Preliminary Analysis

Missing Data

Exploring systematic errors in the data set resulting from missing data is an important task to consider before analyzing data. According to Tabachnick and Fidell (2007, p.63), if 5% or less data points are missing in a random pattern from a large data set, "the problems are less serious and almost any procedure for handling missing values yields similar results." In this study, only 4.6% of data points were missing, which can be ignored. One procedure suggested by Tabachnick and Fidell (2007) for handling data analysis with few missing values was simply to drop any cases with missing data. Therefore, the author removed 4 participants with missing data from the study and the final participants were 67 students.

Controlling for Assumptions

Multiple regression analysis was the main statistical method used in this study to answer the majority of the research questions. Use of multiple regression analyses indicated that several assumptions were needed to be satisfied (Tabachnick and Fidell, 2007, p.123). The author controlled for the following assumptions.

Outliers. Multiple regression analysis is sensitive to outliers and checking for extreme scores should be part of the initial data screening process (Pallant, 2010). Tabachnick and Fidell (2007, p. 128) described outliers as those “with standardized residual values above about 3.3 or less than –3.3.” Pallant (2010, p. 127) recommended that “outliers can either be deleted from the data set or, alternatively, given a score for that variable that is high but not too different from the remaining cluster of scores.” In this study, no outliers were found in the dependent variables. Few outliers were found in the independent variables (3 outliers from MPSP in closed problems and 2 outliers from MPSP in open-ended problems; however, none of the outliers were removed from the data set. First of all, removing outliers from a sample size of 67 would diminish the strength of the study. Furthermore, the outliers (such as extreme performance in solving problems) provided important information for such a study exploring the cognitive characteristics because they were rare to find.

Multicollinearity. Multicollinearity occurs when the independent variables are highly correlated. Multiple regression analysis was a more powerful data analysis technique to deal with multicollinearity (Bagozzi and Fornell, 1982) than other techniques (e.g. SEM). However, Pallant (2010) recommends checking for multicollinearity before analyzing data. In this study, independent variables were correlated in a range between -0.316 and 0.676 and a moderate multicollinearity was observed among independent variables.

Normality. The assumption of normality was referred to an aspect of the distribution of scores in which the residuals should be normally distributed about the predicted dependent variable scores (Tabachnick and Fidell, 2007). The author assessed the normality of independent and dependent variables by examining for skewness (the symmetry of a distribution) and kurtosis (the clustering of scores toward the center of a distribution) values (Table 4.1).

Table 4.1

Skewness and Kurtosis Values for Measures

	N	Skewness		Kurtosis	
Measure		Statistic	SE	Statistic	SE
General Creativity	67	0.219	0.293	-0.369	0.578
Reading Ability (CTBS)	33	-0.396	0.409	0.254	0.798
Reading Ability (ITBS)	34	0.377	0.403	-0.706	0.788
Mathematical Knowledge (CTBS)	33	0.298	0.409	0.526	0.798
Mathematical Knowledge (ITBS)	34	0.119	0.403	0.175	0.788
General Intelligence	67	-0.021	0.293	-0.059	0.578
Verbal Ability	67	0.210	0.293	0.049	0.578
Quantitative Ability	67	-0.031	0.293	0.196	0.578
Spatial Ability	67	-0.054	0.293	-0.107	0.578
MPSP in Closed Problems	67	0.172	0.293	0.343	0.578
MPSP in Open-ended Problems	67	0.100	0.293	0.443	0.578
Working Memory	67	0.138	0.293	-0.270	0.578

Note. MPSP = mathematical problem solving performance.

A perfectly normal distribution has a skewness and kurtosis value of zero. Some statisticians recommend a threshold of ± 0.5 as indicative of departures from normality (e.g., Runyon, Coleman, & Pittenger, 2000), whereas others prefer ± 1.00 for skewness, kurtosis, or both (e.g., George & Mallery, 2003; Morgan, Griego, & Gloeckner, 2001). The statistics presented in Table 4.1 for skewness and kurtosis values showed an acceptable normality for all variables employed in this study.

Descriptive Statistics

Descriptive statistics for the cognitive variables and mathematical problem solving performance in closed and open-ended problems have been presented in Table 4.2.

Table 4.2

Descriptive Statistics for Variables

Measure	N	Min.	Max.	M	SD
General Creativity	67	9	43	24.75	9.52
General Intelligence	67	16	50	31.85	7.23
Verbal Ability	67	231	561	377.22	69.98
Quantitative Ability	67	213	488	354.58	53.47
Spatial Ability	67	290	552	399.51	53.23
MPSP in Closed Problems	67	1	39	11.75	6.76
MPSP in Open-ended Problems	67	0	35	8.88	8.10
Working Memory	67	9	35	21.36	6.01
Reading Ability (CTBS)	33	486	730	643.58	62.45
Reading Ability (ITBS)	34	62	117	84.68	14.52
Mathematical Knowledge (CTBS)	33	613	744	654.24	30.88
Mathematical Knowledge (ITBS)	34	72	131	92.62	14.10

Note. MPSP = mathematical problem solving performance.

Research Question 1

To what extent are the cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) and mathematical problem solving performance in open-ended and closed problems related to each other?

The primary goal of this study was to investigate the relationship between dependent and independent variables. As a preliminary step, calculating the basic correlation coefficients between the dependent variables and independent variables was valuable to understand how the variables were related to each other. To answer Research Question 1, the Pearson product-moment correlation coefficients among all the variables were calculated. These coefficients are presented in Table 4.3.

Two different mathematical knowledge and reading ability measures were presented as shown in Table 4.3. At schools A and C, the Iowa Test of Basic Skills (ITBS) was used to assess students' mathematical knowledge and reading ability whereas the Comprehensive Test of Basic Skills (CTBS) was used at schools B and D. For this reason, three correlation analyses were employed separately and the correlation coefficients that were obtained from these analyses have been listed in Table 4.3. For the first analysis, correlation coefficients among the variables including general intelligence, general creativity, verbal ability, quantitative ability, spatial ability, MPSP in closed problems, and MPSP in open-ended problems were explored. For the second analysis, mathematical knowledge and reading ability scores that were obtained from the ITBS were correlated with other variables. For the third analysis, mathematical knowledge and reading ability scores that were obtained from the CTBS were correlated with other variables. Finally all of the correlation coefficients have been listed in Table 4.3.

Table 4.3

Intercorrelations among cognitive variables and mathematical problem solving performance

Variables	1	2	3	4	5	6	7	8	9	10	11	12
1. General Creativity	1.000											
2. General Intelligence	0.435**	1.000										
3. Verbal Ability	0.289*	0.408**	1.000									
4. Quantitative Ability	0.358**	0.443**	0.532**	1.000								
5. Spatial Ability	0.384**	0.485**	0.259*	0.388**	1.000							
6. MPSP in Closed Problems	0.245*	0.419**	0.483**	0.513**	0.263*	1.000						
7. MPSP in Open-ended Problems	0.394**	0.291*	0.452**	0.310*	0.328**	0.300*	1.000					
8. Working Memory	-0.054	0.043	0.021	0.010	0.037	0.167	-0.069	1.000				
9. Reading Ability (ITBS)	0.183	0.047	0.282	0.281	0.167	0.280	-0.316	0.426*	1.000			
10. Reading Ability (CTBS)	0.064	0.247	0.622**	0.291	0.242	0.327	0.286	-0.091	N/A	1.000	-	
11. Mathematical Knowledge (ITBS)	0.393*	0.198	0.333	0.391*	0.472**	0.361*	0.100	0.244	0.457**	N/A	1.000	
12. Mathematical Knowledge (CTBS)	0.106	0.204	0.408*	0.446**	0.303	0.676**	0.401*	-0.061	N/A	0.526**	N/A	1.000

Note. MPSP = mathematical problem solving performance.

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

As seen in Table 4.3, mathematical problem solving performance (MPSP) in closed problems was correlated with several cognitive abilities including mathematical knowledge (CTBS), $r = 0.676$, $p < .001$; quantitative ability, $r = 0.513$, $p < .001$; verbal ability, $r = 0.483$, $p < .001$; and general intelligence, $r = 0.419$, $p < .001$. Also MPSP in closed problems was correlated with certain cognitive abilities including general creativity; $r = 0.245$, $p = .046$ and spatial ability, $r = 0.263$, $p = .032$. However, reading ability and working memory were not correlated with MPSP in closed problems at a significant level.

Mathematical problem solving performance (MPSP) in open-ended problems was correlated with several cognitive abilities including verbal ability, $r = 0.452$, $p < .001$, general creativity, $r = 0.394$, $p = .001$, and spatial ability, $r = 0.328$, $p = .007$. Also MPSP in open-ended problems was correlated with some cognitive abilities including mathematical knowledge (CTBS), $r = 0.401$, $p = .021$ and quantitative ability, $r = 0.310$, $p = .011$. However, no significant correlations were found between MPSP in open- problems and working memory, $r = 0.167$, $p = .176$. Reading ability was not correlated with MPSP in open-problems at a significant level.

General creativity, general intelligence, verbal ability, quantitative ability, spatial ability, MPSP in closed problems, and MPSP in open-ended problem were the variables correlated with each other significantly. However, working memory was not correlated with any dependent or independent variable.

Research Question 2

To what extent are boys and girls similar or different in mathematical problem solving performance in open-ended and closed problems?

In Table 4.4, the means and standard deviations are presented for boys and girls in measures of cognitive abilities and mathematical problem solving performance. Reading ability and mathematical knowledge scores were grouped in two categories as seen in Table 4.4, because two different tests, ITBS and CTBS, were used to assess each of them.

Table 4.4

Comparison between Boys and Girls in Measure of Variables

Measures	Girls (N=31)		Boys (N=36)		Total (N=67)	
	M	SD	M	SD	M	SD
General Creativity	23.968	7.825	25.417	10.843	24.746	9.524
General Intelligence	32.968	5.683	30.889	8.294	31.851	7.228
Verbal Ability	389.935	58.496	366.278	77.689	377.224	69.980
Quantitative Ability	349.097	44.083	359.306	60.630	354.582	53.470
Spatial Ability	393.355	43.635	404.806	60.404	399.507	53.230
MPSP in Closed Problems	11.355	4.223	12.097	8.409	11.754	6.764
MPSP in Open-ended Problems	9.548	8.667	8.306	7.649	8.881	8.097
Working Memory	21.387	5.993	21.333	6.109	21.358	6.009
	(N=20)		(N=13)		(N=33)	
Reading Ability (CTBS)	643.500	54.714	643.692	75.227	643.576	62.447
Mathematical Knowledge (CTBS)	649.350	28.839	661.769	33.544	654.242	30.883
	(N=11)		(N=23)		(N=34)	
Reading Ability (ITBS)	86.818	14.324	83.652	14.816	84.676	14.518
Mathematical Knowledge (ITBS)	94.364	13.336	91.783	14.666	92.618	14.099

Note. MPSP = mathematical problem solving performance.

As seen in Table 4.4 boys had higher mean scores in closed problem solving performance than girls (for boys *M* = 12.097 and for girls *M* = 11.355), whereas girls had higher mean scores than boys in open-ended problem solving performance (for boys *M* = 9.548 and for girls *M* =. 8.306). In addition, boys had higher mean scores than girls in general creativity, quantitative ability, and spatial ability whereas girls had higher mean scores than boys in general intelligence, verbal ability, and working memory (Table 4.4).

To answer Research Question 2, a one-way between-group multivariate analysis of variance (MANOVA) was performed. The dependent variables included general intelligence, general creativity, working memory, mathematical knowledge, reading ability, mathematical problem solving performance, verbal ability, quantitative ability, and spatial ability. The independent variable was gender. Preliminary assumption testing was conducted to check for normality, linearity, univariate and multivariate outliers, homogeneity of variance covariance matrices, and multicollinearity, with no violations noted.

No significant difference was found between males and females on the combined dependent variables, F (12, 54) = 1.459, p = .208; *Pillai's Trace* = .164; *partial eta squared* = .164. When the results for the dependent variables were considered separately, no differences reached statistical significance, using a Bonferroni adjusted alpha level of .004 (Table 4.5). This result showed no significant differences between girls and boys in cognitive abilities including general intelligence, general creativity, working memory, mathematical knowledge, reading ability, mathematical problem solving performance, verbal ability, quantitative ability, and spatial ability.

Table 4.5

Gender Differences in Problem Solving Performance and Cognitive Abilities

Measures	Mean Square	F	Sig.
General Creativity	34.969	0.382	0.539
Reading Ability (CTBS)	0.291	0.000	0.993
Reading Ability (ITBS)	74.587	0.347	0.560
Mathematical Knowledge (CTBS)	1215.203	1.285	0.266
Mathematical Knowledge (ITBS)	49.571	0.244	0.625
General Intelligence	71.984	1.386	0.243
Verbal Ability	9322.549	1.930	0.169
Quantitative Ability	1735.950	0.604	0.440
Spatial Ability	2184.011	0.768	0.384
Working Memory	0.048	0.001	0.971
MPSP in Closed Problems	9.180	0.198	0.658
MPSP in Open-ended Problems	25.728	0.389	0.535

Note. MPSP = mathematical problem solving performance.

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).

Research Question 3

To what extent do the cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) predict mathematical problem solving performance in closed problems?

Hierarchical multiple regression was employed to assess the ability of cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) to predict mathematical problem solving performance in closed problems, after controlling for the influence of gender. Preliminary analyses were conducted to ensure no violation of the assumptions of normality, linearity, multicollinearity, and homoscedasticity. As seen in Table 4.6, 4.7, and 4.8, regression models have been presented in two categories: ITBS and CTBS. At schools A and C, the ITBS was used to assess students' mathematical knowledge and reading ability and the CTBS was used at schools B and D.

Table 4.6

Changes in Explained Variance for Performance in Closed Problems

	Measures of mathematical knowledge and reading ability					
	ITBS			CTBS		
Model	SS	df	F	SS	df	F
1[a]	0.689	1	0.032	196.818	1	4.078
2[b]	352.192	9	2.821*	1097.994	9	4.717**

Note. SS = sum of squares.
a Predictors: (Constant), Gender
b Predictors: (Constant), Gender, reading ability, general intelligence, math knowledge, verbal ability, quantitative ability, general creativity, spatial ability, working memory

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).

As seen in Table 4.7, gender was entered into Model 1, and accounted for 0.1% (ITBS) and 11.6% (CTBS) of the variance in mathematical problem solving performance (MPSP) in closed problems. After entry of cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) into Model 2, the total variance explained by the model as a whole was significant, $R^2 = 0.514$, $F\ (8, 24) = 2.821$, $p = .02$ (ITBS) and $R^2 = 0.649$, $F\ (8, 23) = 4.717$, $p < .001$ (CTBS).

As seen in Table 4.7, the cognitive abilities explained 51.3% (ITBS) and 53.3% (CTBS) of the variance in MPSP in closed problems, after controlling for gender responding. When compared with Model 1, Model 2 improved the predictions for MPSP in closed problems significantly, *F change* (8, 23) = 3.168, $p = 0.02$ (ITBS) and *F change* (8, 24) = 4.355, $p = 0.001$ (ITBS).

Table 4.7

Summary of Hierarchical Regression Analysis for Variables Predicting Closed Problem Solving Performance

	Measure of mathematical knowledge and reading ability							
	ITBS				CTBS			
Model	R	R^2	ΔR^2	SEE	R	R^2	ΔR^2	SEE
1[a]	0.032	0.001	0.001	4.625	0.341	0.116	0.116	6.947
2[b]	0.717	0.514	0.513*	3.724	0.805	0.649	0.532**	5.086

Note. SEE = standard error of estimates.
a Predictors: (Constant), Gender
b Predictors: (Constant), Gender, reading ability, general intelligence, math knowledge, verbal ability, quantitative ability, general creativity, spatial ability, working memory

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).

Research Question 4

What cognitive variables are the best predictors of the mathematical problem solving performance in closed problems?

To answer Research Question 4, hierarchical regression analysis was employed. Model 2 included the following set of predictors: intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability. Mathematical problem solving performance (MPSP) in closed problems was used as the dependent variable.

Table 4.8

Hierarchical Analysis of Mathematical Problem Solving Performance in Closed Problems

		Measures of mathematical knowledge and reading ability					
		CTBS			ITBS		
Model	Predictor	SE	β	t	SE	β	t
1	Gender	2.475	0.341	2.019	1.695	0.032	0.180
2	Gender	2.460	0.100	0.594	1.509	0.185	1.174
	General Creativity	0.116	-0.060	-0.420	0.083	-0.052	-0.286
	Reading Ability	0.021	0.009	0.050	0.059	0.314	1.601
	Math Knowledge	0.037	0.488	3.083**	0.065	0.560	2.690*
	General Intelligence	0.176	0.439	2.552*	0.144	0.958	3.857**
	Verbal Ability	0.019	0.122	0.642	0.015	-0.357	-1.906
	Quantitative Ability	0.028	0.260	1.220	0.017	0.099	0.530
	Spatial Ability	0.019	-0.107	-0.701	0.021	-0.553	-2.615*
	Working Memory	0.173	0.210	1.585	0.135	-0.329	-1.748

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).

As found in Research Question 3, Model 2 accounted for 51.3% (ITBS) and 53.3% (CTBS) of the variance in MPSP in closed problems, after controlling for gender responding. As shown in Model 2, when the CTBS was used as a measure of mathematical knowledge and reading ability, students' mathematics knowledge and general intelligence were found to be the only significant predictors of their MPSP in closed problems (Table 4.8).

When the ITBS was used as a measure of mathematical knowledge and reading ability, students' mathematics knowledge, general intelligence, and spatial ability (only for the ITBS category) were found to be the only significant predictors of their MPSP in closed problems (Table 4.8). However, working memory, verbal ability, reading ability, quantitative ability, and general creativity did not explain the variance in MPSP in closed problems at a significant level.

Research Question 5

To what extent do the cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) predict mathematical problem solving performance in open-ended problems?

Hierarchical multiple regression was used to assess the ability of cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) to predict mathematical problem solving performance (MPSP) in open-ended problems, after controlling for the influence of gender. Preliminary analyses were conducted to ensure no violation of the assumptions of normality, linearity, multicollinearity, and homoscedasticity. As seen in Table 4.9, 4.10, and 4.11, regression models have presented in two categories: ITBS and CTBS. At schools A and C, the ITBS was used to assess students' mathematical knowledge and reading ability whereas the CTBS was used at schools B and D.

Table 4.9

Changes in Explained Variance for Performance in Open-ended Problems

	Measure of mathematical knowledge and reading ability					
	ITBS			CTBS		
Model	SS	df	F	SS	df	F
1[a]	76.650	1	1.632	77.036	1	1.027
2[b]	872.187	9	3.287**	1112.702	9	2.404*

Note. SS = sum of squares.
a Predictors: (Constant), Gender
b Predictors: (Constant), Gender, reading ability, general intelligence, math knowledge, verbal ability, quantitative ability, general creativity, spatial ability, working memory

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).

As seen in Table 4.10, gender was entered into Model 1 and accounted for 4.9% (ITBS) and 3.2% (CTBS) of the variance in MPSP in open-ended problems. After entry of cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) into Model 2, the total variance explained by the model as a whole was significant, $R^2 = 0.552$, F (8, 24) = 3.287, $p < .001$ (ITBS) and $R^2 = 0.649$, F (8, 23) = 2.404, $p = .0487$ (CTBS).

In addition, as seen in Table 4.10, the cognitive abilities explained 51.3% (ITBS) and 46.3% (CTBS) of the variance in MPSP in open-ended problems after controlling for gender responding, *F change* (8, 23) = 3.373, $p = 0.01$ (ITBS) and *F change* (8, 24) = 2.386, $p = 0.049$ (CTBS).

Table 4.10

Summary of Hierarchical Regression Analysis for Variables Predicting Open-ended Problem Solving Performance

	Measure of mathematical knowledge and reading ability							
	ITBS				CTBS			
Model	R	R^2	ΔR^2	SEE	R	R^2	ΔR^2	SEE
1[a]	0.220	0.049	0.049	6.854	0.179	0.032	0.032	8.662
2[b]	0.743	0.552	0.504**	5.430	0.680	0.463	0.431*	7.490

Note. SEE = standard error of estimates.
a Predictors: (Constant), Gender
b Predictors: (Constant), Gender, reading ability, general intelligence, math knowledge, verbal ability, quantitative ability, general creativity, spatial ability, working memory

** Correlation is significant at the 0.01 level (2-tailed).
* Correlation is significant at the 0.05 level (2-tailed).

Research Question 6

What cognitive variables are the best predictors of mathematical problem solving performance in open-ended problems?

To answer Research Question 6, hierarchical regression analysis was employed. Model 2 included the following set of predictors: intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability. Mathematical problem solving performance (MPSP) in open-ended problems was used as the dependent variable.

Table 4.11

Hierarchical Analysis of Mathematical Problem Solving Performance in Open-ended Problems

		Measures of mathematical knowledge and reading ability					
		CTBS			ITBS		
Model	Predictor	*B*	*SE*	*t*	*B*	*SE*	*t*
1	Gender	3.127	3.086	1.013	-3.209	2.512	-1.277
2	Gender	-1.639	3.623	-0.452	-2.689	2.2	-1.222
	General Creativity	0.227	0.17	2.829**	0.264	0.122	2.172*
	Reading Ability	-0.014	0.031	-0.459	-0.195	0.086	-0.963
	Mathematical Knowledge	0.073	0.055	1.331	0.043	0.095	0.449
	General Intelligence	-0.648	0.259	-2.505*	0.243	0.21	1.155
	Verbal Ability	0.051	0.028	2.022*	0.011	0.022	2.503*
	Quantitative Ability	0.008	0.042	0.194	-0.012	0.024	-0.505
	Spatial Ability	0.04	0.027	1.486	0.005	0.03	0.178
	Working Memory	-0.091	0.255	-0.357	-0.139	0.197	-0.707

** Correlation is significant at the 0.01 level (2-tailed).

* Correlation is significant at the 0.05 level (2-tailed).

Model 2 accounted for 55.2% (ITBS) and 46.3% (CTBS) of the variance in mathematical problem solving performance in open-ended problems, after controlling for gender responding. As shown in Model 2, when the CTBS was used as a measure of mathematical knowledge and reading ability, students' general creativity, verbal ability, and general intelligence were found to be significant predictors of their mathematical problem solving performance in open-ended problems.

When the ITBS was used as a measure of mathematical knowledge and reading ability, students' general creativity ability and verbal ability were found to be significant predictors of their mathematical problem solving performance in open-ended problems (Table 4.11). Gender, working memory, spatial ability, quantitative ability, reading ability, and mathematical knowledge did not contribute significantly to the variance in MPSP in open-ended problems.

CHAPTER V: DISCUSSION

This chapter included a discussion of the findings, limitations of the study, implications, and final thoughts. The purpose of this study was to investigate the influences of cognitive abilities on the mathematical problem solving performance of students.

This study was different from prior studies of mathematical problem solving in that the author measured and analyzed mathematical problem solving performance in two categories (closed and open-ended problems) rather than considering it as a whole. By doing so, the author aimed to explore how the influence of cognitive abilities on problem solving process would vary depending on the types of problems solved.

Discussion of Findings

Relations between Cognitive Variables and Mathematical Problem Solving Performance

The author found that mathematical problem solving performance (MPSP) in closed problems was correlated significantly with cognitive variables including mathematical knowledge, quantitative ability, verbal ability, general intelligence, general creativity, and spatial ability. However, MPSP in closed problems was not correlated significantly with working memory and reading ability.

Similarly MPSP in open-ended problems was correlated significantly with several cognitive abilities including verbal ability, general creativity, general intelligence, spatial ability, mathematical knowledge, and quantitative ability. However, MPSP in open-ended problems was not correlated significantly with working memory and reading ability.

As reported in the review of literature, significant correlations between cognitive abilities and MPSP were expected. However, two cognitive abilities, reading ability and working memory, were correlated with neither MPSP in closed problems nor MPSP in open-ended problems. This finding was not consistent with the findings of previous studies in which working memory and reading ability were found to be significant predictors of mathematical problem-solving performance (e.g., Swanson & Beebe-Frankenberger, 2004). For example, Swanson and Beebe-Frankenberger (2004) found that working memory, fluid intelligence, reading ability, processing speed, and knowledge of algorithms were the only predictors of mathematical problem solving performance of elementary students. One potential explanation for this inconsistency, which is related to the influence of reading ability, could be the differences in the characteristics of the problems that were used to assess students' mathematical problem solving performance. The DISCOVER instrument, which was used in this study to assess students' MPSP, was designed to have minimal verbal content, thus not dependent on reading ability. For example, the problems in the mathematics section of the DISCOVER assessment included only a few words, verbal instructions were given, and those administering the assessment made certain that all students understood what they were expected to do. In contrast to the DISCOVER assessment, Swanson assessed students' mathematical problem solving performance through using word problems. Students who participated in Swanson's study might have needed reading ability of at least a moderate level to comprehend and solve the problems because of the verbal complexity of the long word problems.

Another reason for this inconsistency could be the age differences between the samples of the studies. For example, Swanson's study included children in grades 1, 2, and 3 whereas this study included children only from third grade. Reading ability of children has been found to contribute to student achievement in the earlier years of schooling (Baker et al., 2008) which means that reading ability might be a more significant predictor of mathematical performance for children in the first and second grades than for those in third grade. These two major differences might be possible explanations for the fact that reading ability did not contribute to the variance in mathematical problem solving performance in this study.

In addition to reading ability, working memory was correlated with neither MPSP in closed problems nor MPSP in open-ended problems. This finding is inconsistent with the findings of Swanson (2004) and Iguchi (2008). Both Swanson and Iguchi found that working memory was correlated significantly with mathematical problem solving. Furthermore, in Newell and Simon's (1972) information-processing (IP) theory of learning, the working memory capacity of a person was considered to be a major component influencing his/her problem solving performance. One possible explanation for this inconsistency might be the measurement of working memory. In this study working memory was measured through the Structure-of-Intellect (SOI) Learning Abilities Test, which included four subtests: memory of figural units (MFU), memory of symbolic implications (MSI), memory of symbolic systems (MSS), and memory of symbolic units (MSU). The items on these subtests were designed to measure memorization of figural and symbolic representations. However, the possible solutions for the problems used in this study to measure MPSP did not include the use/memorization of figural or symbolic representations.

When considering the inconsistencies among the studies as a whole, the author recommends that the relationship between cognitive abilities and problem solving performance might possibly vary depending on the content of a problem. For example, reading ability might be correlated significantly with problem solving performance when problems with complex verbal structures are used. On the other hand, reading ability may not be related to problem solving performance when problems with non-verbal structures are used. The author suggests that the content of problems that are used in instruments should be analyzed carefully before using them as a measure of performance. In mathematics, generally the term ‘content’ is used to refer to specific topics and subjects such as (e.g., probability, geometry, and algebra). What I mean by the term, *content of problem,* is different from its use in mathematics. With the term *content of problem,* I refer to a problem’s linguistic complexity, figural and visual structure, and proximity to a problem solver.

Gender Differences in Mathematical Problem Solving Performance

Boys had higher mean scores in closed problem solving performance than girls whereas girls had higher mean scores in open-ended problem solving performance than boys. However, none of these differences were statistically significant. This finding shows that boys and girls are not different in their performance in solving mathematical problems. These findings are consistent with the findings of previous researchers who found no significant gender differences in mathematical problem solving performance of elementary and middle school students (Caplan and Caplan, 2005; Fennema and Tartre, 1985; Hembree, 1992; Hyde et al. 1990, Lindberg et al., 2010).

Another finding was no significant differences in cognitive abilities (including mathematical knowledge, quantitative ability, verbal ability, general intelligence, general creativity, working memory, reading ability and spatial ability) between boys and girls. This finding provides a remarkable explanation for the fact that no significant gender differences existed in mathematical problem solving performance of elementary students. As reported in Chapter 4, cognitive abilities accounted for a substantial variance (ranging from 51.3% to 64.9%) in mathematical problem solving performance. Considering the significant influence of cognitive abilities on mathematical problem solving performance (MPSP), expecting a significant difference between boys and girls in MPSP would not be reasonable.

Predicting Mathematical Performance in Closed Problems

Using a hierarchical regression analysis, the author found that cognitive abilities were significant predictors of MPSP in closed problems as a whole. In Model 1, gender accounted for only 0.1% (ITBS) and 11.6% (CTBS) of the variance in MPSP and it was not a significant predictor of MPSP in closed problems. When cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) were entered into Model 2, the total variance explained by the model as a whole was 51.4% (ITBS) and 64.9% (CTBS). The cognitive abilities explained 51.3% (ITBS) and 53.3% (CTBS) of the variance in MPSP in closed problems, after controlling for gender responding. In Model 2, when CTBS was used as a measure of mathematical knowledge and reading ability, students' mathematics knowledge and general intelligence were the only variables that contributed significant variance to MPSP in closed problems. When ITBS was used as a measure of mathematical knowledge and reading ability, students' mathematics knowledge, general intelligence, and spatial ability were the variables that contributed unique variance to MPSP in closed problems.

Mathematical knowledge and general intelligence are the only variables that contributed significant variance to MPSP in closed problems (for both ITBS and CTBS). Although spatial ability was a significant predictor when ITBS was used as a measure of mathematical knowledge and reading ability, it was not a significant predictor when CTBS was used. In addition, a surprising fact was that none of the other cognitive variables that were found to be correlated significantly with MPSP in closed problems, including general creativity, verbal ability, and quantitative ability were significant predictors in Model 2. One statistical explanation for this finding might be that when the overlapping effects of all other variables are statistically removed, mathematical knowledge and general intelligence might be the only unique contributors to the variance in MPSP in closed problems. However, this statistical explanation brings forth another question: *Why are mathematical knowledge and general intelligence the only variables that contributed significant variance to MPSP in closed problems*?

To me, the answer is related to the structure of problems. According to information processing (IP) theorists, problem solving is associated with three sets of thinking processes: (a) understanding, (b) searching, and (c) implementing solutions. The influence of cognitive abilities on these thinking processes might vary depending on the type of problem. In this study the problems that were used to measure MPSP in closed problem solving performance were Types I, II, and III problems. As explained in Maker's Problem Continuum model, in Type I, II, and III problems, the problem is known to both the presenter and the problem solver. The presenter also knows the correct solution. As Hong (1998) suggested, in solving these kinds of problems, if problem solvers possess appropriate domain-specific knowledge, including basic concepts, facts, and principles of a particular subject matter domain, the learners can solve the problem directly without searching for a solution using various searching strategies. When solvers do not have appropriate knowledge to solve the problem, they are required to use specific strategies to search for a solution (Chi, et al., 1982). Fingar (2012) stated that intelligence can help in creating these strategies by reducing uncertainty and providing insight. Therefore, performance in solving these kinds of problems might be associated with domain-specific knowledge or intelligence.

Predicting Mathematical Performance in Open-ended Problems

Using a hierarchical regression analysis, the author found that cognitive abilities predicted mathematical performance in open-ended problems significantly as a whole. In Model 1, gender accounted for 4.9% (ITBS) and 3.2% (CTBS) of the variance in MPSP in open-ended problems. Gender alone did not predict MPSP in open-ended problems at a significant level. After entry of cognitive variables (intelligence, creativity, memory, knowledge, reading ability, verbal ability, spatial ability, and quantitative ability) into Model 2, the total variance explained by the model as a whole was 55.2% (ITBS) and 64.9% (CTBS). The cognitive abilities explained 51.3% (ITBS) and 46.3% (CTBS) of the variance in mathematical problem solving performance in open-ended problems, after controlling for gender responding. In Model 2, when CTBS was used as a measure of mathematical knowledge and reading ability, students' general creativity, verbal ability, and general intelligence were the only variables that contributed significant variance to MPSP in open-ended problems. When ITBS was used as a measure of mathematical knowledge and reading ability, students' general creativity and verbal ability were found to be the variables that contributed significant variance to MPSP in open-ended problems.

General creativity and verbal ability are the only variables that contributed significant variance to MPSP in open problems (for both ITBS and CTBS). Although general intelligence was a significant predictor when ITBS was used as a measure of mathematical knowledge and reading ability, it was not a significant predictor when CTBS was used. Again this statistical explanation brings forth the same question: *Why are general creativity and verbal ability the only variables that contributed significant variance to MPSP in open-ended problems*? In this study, the problems that were used to measure MPSP in open-ended problem solving performance were Type IV and V problems. As explained in Maker's Problem Continuum model, Type IV and V problems are known to the presenter and the solver, but the problem may be solved in more than one way and the presenter knows the range of solutions. These problems can be solved inductively but they have an accepted range of answers. Because of the structure of these problems, the problem solver's fluency is associated with his/her performance in these types of problems. As Mednick (1962) suggested, fluency is an important component of creativity because more responses to a single prompt results in a higher probability that a problem solver will generate an original idea. Therefore, expecting general creativity to be a significant predictor of open-ended problem solving performance is reasonable.

The relationship between performance in solving open-ended problems and verbal ability might seem complicated to explain. One possible explanation for the fact that verbal ability was found to be a significant predictor of MPSP in open-ended problems might be the relation between fluency and verbal ability. Fluency is described as an aspect of verbal ability (Lamar, Zonderman, & Resnick, 2002). Researchers have found that individuals with high verbal ability perform well on measures of fluency (Ayotte, Potter, Williams, Steffens, & Bosworth, 2009). In addition, as declared before, open-ended problems have multiple potential valid solutions and the problem solver's fluency is associated with his/her performance in these types of problems. Therefore, fluency can be described as a moderator between verbal ability and performance in solving open-ended problems, resulting in verbal ability being found to be a significant predictor of MPSP in open-ended problems.

Another interesting finding was that spatial ability predicted MPSP significantly in neither open-ended nor closed problems. However, spatial ability was described by researchers as an essential factor influencing problem-solving performance (e.g., Hegarty & Kozhevnikov, 1999; Garderen, 2006; Sherman, 1979; Fennema & Tartre, 1985; Lohman, 1979). Furthermore, Sherman (1979) reported that spatial ability was one of the main factors affecting mathematical performance significantly. She also found that individuals with high spatial ability had a wider range of strategies and they were better at determining when to use a particular strategy during problem solving. One possible explanation for the fact that spatial ability did not predict problem solving performance might be that the problems that were used in this study to assess performance in both open-ended and closed problems required the ability to generate, retain, and manipulate abstract visual images at a minimal level. In addition, Fennema and Tartre found that a student's spatial ability did not affect his or her likelihood of solving certain math problems correctly, but the students with high spatial ability were still more likely to be "...able to convert word problems to accurate diagrams, and to use those diagrams to get correct solutions." (1985, p. 193) Therefore, the relationship between spatial ability and mathematical problem solving performance might vary depending on the content and structure of the problems.

Summary of Discussion

When combining all of these findings I propose that two facts are crucial to understand performance in solving mathematical problems. The first is that the relationship between cognitive abilities and problem solving performance may vary depending on the *content* of a problem. For example, reading ability might be a significant predictor of problem solving performance when problems with complex verbal structure are used. On the other hand, reading ability may not be a significant predictor when problems have a non-verbal structure. I conclude that the content of problems that are used in instruments should be analyzed carefully before using them as a measure of performance.

The second is that the relationship between cognitive abilities and problem solving performance may vary depending on the *structure* of a problem. For example, mathematical knowledge and general intelligence were the only significant predictors of performance in solving closed problems whereas general creativity and verbal ability were found to be the only significant predictors of performance in solving open-ended problems. Readers of this study might claim that the impact of problem structure and problem content might intertwine and they might be difficult to distinguish. Although this claim might be true for many problems, these impacts were distinct and evident in this study. The following problems clarify my proposal:

Problem A: Solve this problem: 4 + 6 = ____

Problem B: Write as many problems as possible that have 10 as the answer.

Although these two problems include similar content and concepts, they have different structures. Problem A is defined clearly, has a specific method, and one right answer (closed, Type I). Problem B is defined clearly (all problems written must have an answer of 10), but the methods are not specified, and in fact, are infinite, an infinite number of solutions can be devised (open, Type V). Therefore, I conclude that closed and open-ended problems require different cognitive abilities for reaching successful solutions. These findings are thought-provoking, and can help educators and researchers understand how *structure (type)* and *content* of a problem might influence the relationship between cognitive abilities and problem solving performance.

Limitations of the Study

This study has several limitations, and these should be considered by readers carefully as they interpret the results. The first limitation is the small sample size, which can contribute to inconsistent findings or findings affected by outliers. The author did not remove the outliers from the data set on purpose because they provided important information and they were rare to find. Different guidelines concerning the number of cases required for multiple regression analysis were suggested by different authors. Stevens (1996) recommended having 10-15 participants for each predictor used in a study for a reliable analysis. In this study the participants included 67 students, which is less than what Steven recommended. In addition, the study sample consisted of only Dine students in grade 3, and therefore, the results might not be generalized to other populations and all grade levels. Future researchers should have a larger sample size with participants from other ethnicities, socio-economic status, and grade levels so that the findings may be generalizable to diverse populations.

A second limitation is the use of two tests (ITBS and CTBS) to measure students' mathematical knowledge and reading ability. This was unavoidable due to the fact that I was required to use the tests normally given in the schools in which the data were collected. Although these tests are seen as being similar, I attempted to eliminate this problem by analyzing the data in two different categories by separating the analysis (one analysis for each test). By doing so, I controlled for the influence resulting from different tests, but for the same reason, I had a smaller sample size in each category. I recommend that future researchers use a single instrument to assess a certain variable to avoid variations resulting from the use of different instruments.

The last limitation of the study is related to data collection. The data from the TCT-DP test that was used to assess general creativity were collected at the end of the spring semester of grade 4 whereas all other data were collected at the end of spring semester of grade 3. The data related to general creativity were collected almost a year later than the data related to other cognitive abilities, which might produce a misleading effect size. Although Brocher (1989) found a high pre- and post-test reliability with a coefficient of $r = 0.81$ after several months, I still have some questions about the potential influence of data collection at different time intervals.

Implications

This study has several important implications for practitioners and researchers in the field of psychology, education, and other related disciplines. The findings of this study provide a better understanding of how the problem solving process works from the information processing perspective.

Implications for Practice

I found that open-ended problems have different processes and components than those of closed problems in mathematics. To develop students' problem-solving skills, mathematics educators and teachers must design proper teaching and learning strategies using methods that correspond to the different characteristics and different nature of problems.

The characteristics of instructional strategies and teaching methods should be in accordance with educational goals. To improve students' mathematical performance in closed problems, educators must focus on enhancing students' mathematical knowledge and they should consider students' intellectual levels. Similarly, to improve students' mathematical performance in open-ended problems, educators must focus on fostering students' creativity and verbal abilities. Underestimating students' cognitive abilities and their interaction with learning processes might possibly result in failures.

Another important implication of the study for educators is related to understanding gender differences in mathematical problem solving performance. In this study, I found no significant differences between girls and boys in any of the cognitive abilities and mathematical problem solving performance of students in elementary school. This finding is in accordance with the findings of other studies that were conducted at the elementary and middle school level. However, gender differences were observed by other researchers during later periods of schooling (high school and college) and boys were found to perform better than girls (Hembree, 1992; Hyde et al., 1990; Lindberg et al., 2010). These findings show that school settings, teaching methods, and other social factors might be promoting the development of these abilities in boys while blocking it in girls. Although a great deal of research has been done on the treatment of boys and girls in the classroom, changes in classrooms are rarely apparent (Kallam, 1996). For example, as Kallam noted, boys still tend to receive more attention and a greater amount of praise, and they are asked more probing questions (Sadker, Sadker, & Long, 1996). Educators and teachers must reconsider their roles as they can provide social equity in the classroom for boys and girls to enable girls to progress at a pace equal to that of boys in the later years of schooling.

Finally, to teachers and other educators, I would say that fostering students' creativity in mathematics classrooms is a major component in learning mathematics. As the findings indicated, creativity contributes to mathematical problem solving performance significantly when the problem has an open-ended structure. Minimal or ineffective use of open-ended problems during instruction has been one common mistake made by teachers in many mathematics classrooms. This common practice does not allow students to use and apply their creativity in mathematics. However, children enjoy creative thinking experiences, and they can learn mathematics while also applying their creative thinking in the use of mathematical principles (Bahar & Maker, 2011). Curriculum designers and educators should produce rich learning settings and materials to address students' creativity. In addition, teachers should create classroom environments in which students can defend their solutions or decisions, and therefore develop their creative thinking.

I believe problem solving should play a key role in education, at all levels, precisely because it is so important in everyday life. Each of us may make hundreds of decisions every day, and the vast majority of these decisions are about how to solve open-ended problems because we are faced with numerous possible variations or alternatives. Deciding what to cook for dinner, for example, typically is an open-ended problem; usually one can see a clear need to do so, yet the method and solution will vary. Likewise is it not imperative for educators to help students develop their problem solving skills, especially to solve complex, open-ended problems? Unfortunately much remains to be done in the standard classroom. Most mathematics problems in school curricula continue to be Problem Types I and II only—drill and practice, find-the-right-answer kinds of approaches. As educators, we cannot expect our students to function in the real world after teaching them twelve-plus years how to solve Problem Types I and II because these Problem Types are virtually non-existent after we graduate.

Implications for Research

The findings of this study are thought-provoking and they have important implications for future researchers as they seek for a better understanding of problem solving processes. One major implication is that researchers should consider the *structure (type)* and *content* of a problem. While reviewing literature for this study, I found that many researchers use high-stakes tests such as the SAT and ACT, state standardized tests, or achievement tests to assess problem solving performance without considering the *structure (type)* and *content* of problems. For some reasons, this practice might be misleading. First of all, not all of these tests are capable of assessing problem solving performance because they are not designed for this purpose. Second, even though they are capable of assessing problem solving performance at some level, different items might have different characteristics and their solution might require different abilities. For example, solving a long word problem might require a higher reading ability than solving another problem that is non-verbal. Similarly, solving a geometry problem with complex shapes might require more spatial ability than solving another problem with less complex shapes. Researchers should be careful when selecting an instrument to assess problem solving performance and if possible they should check the appropriateness of each item in advance.

As reported in Chapter 4, cognitive abilities accounted for a significant variance (ranging from 51.3% to 64.9%) in mathematical problem solving performance. The substantial influence of cognitive abilities on problem solving performance supports the theoretical framework of the study. However, researchers still need to identify the variables that account for the remaining unexplained variance in mathematical problem solving performance. Therefore, I suggest that future researchers should consider the fact that abilities and skills develop in a sociocultural context and they should analyze the influence of demographic variables including ethnicity, socioeconomic status, and cultural variables on problem solving performance.

Finally, future researchers should explore whether the findings related to open-ended and closed problem solving in this study generalize to other kinds of domains. The problems analyzed in this study were from the mathematical domain. Future studies should focus on how cognitive abilities influence problem solving performance in different domains including science, language, social studies, and other disciplines to explore if these findings apply.

Final Thoughts

Problem solving has been a core theme in education for several decades. Educators and policy makers agree on the importance of problem solving skills for school and real life success and they advocate for locating it at the center of education. Despite all these efforts, we, educators, could not succeed to do so. I hope that this study and its findings will be a landmark for further explorations and it will encourage future researchers and educators in their journey to better understand problem solving process. Halmos (1980) once said:

> “I do believe that problems are the heart of mathematics, and I hope that as teachers, in the classroom, in seminars, and in the books and articles we write, we will emphasize

> them more and more, and that we will train our students to be better problem-posers and problem solvers than we are." (p. 524)

I believe the more researchers and educators we have who think like Halmos, the more problem solving will be promoted in schools. Furthermore, as Otten (2010) pointed out, this emphasis on problem solving will not be self-contained only to the domain of mathematics education but also will transfer into society in positive ways by promoting a knowledgeable citizenry and by creating pathways of advancement for students (Hiebert et al., 1996; Schoenfeld, 2007). Therefore, the next generations will have fewer problems to deal with, not just in mathematics classrooms, but also in real life.

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