A Hierarchical Model for Arizona's Water Resources

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Abstract

Arizona's water resources system consists primarily of four active management areas (Tucson, Phoenix, Pinal and Prescott), the Central Arizona Project, and the Salt River Project. The problem of water allocation among user categories involves pumping from aquifers and diversions of surface flows. In systems less complex than Arizona, allocation policies may appear obvious. In this case, however, a two-level hierarchical management model is proposed to control water allocation to users: the active management areas as a lower echelon, and the Arizona Department of Water Resources at the higher level. A system theoretic approach combined with recent developments in the decentralized control theory are proposed to be included in the model. A significant characteristic of the proposed model is the ability to consider possible interactions among the active management areas as a result of policy decisions at the State level. A dynamic optimization model based on a state space formulation with total energy required as the objective function is solved for each of the subsystems. Detailed information thus generated at the regional level is then appropriately aggregated for statewide decision making. An iterative algorithm is suggested.

Introduction

Management of water resources at the State level is becoming increasingly more complex so that systems theoretical concepts are very useful in analyzing the issues involved (Major and Lenton, 1979). The application of these concepts to water resource systems analysis and planning has been a matter of considerable interest in recent years (Loucks et al., 1981) leading to a variety of mathematical models. In many instances, state-space models (in contrast with decision-space models) seem to be particularly useful because of the possibility of defining clearly the state of the system (e.g., groundwater levels, volume of water stored in a surface reservoir). Thus a number of models exist for groundwater aquifers (Burt, 1976) as well as for surface water resources (Maidment and Chow, 1981). At the same time, input-output models are being developed for water resources management (Hendricks and Dehaan, 1981), as well as models of large-scale systems using decomposition techniques (for example, see Haines, 1977). Lastly, state and dynamic optimization formulations exist for water resources management problems. However, dynamic optimization (Buras, 1972) seems to be more accepted by water management agencies because of the dynamic nature of most variables identified in a large number of water resources systems. Typical optimization criteria for dynamic models are energy requirements, needed storage capacity, water shortages or water spills.

The study of socio-economic frameworks, whether at the national or at state level, often require the construction of complex, large-scale models. However, the use of large-scale models in a planning exercise of whatever scope is still an infrequent event and there are serious reservations regarding their possible use in this context (Dantzig 1981). One such reservation arises from the observation that planning and management models are often designed to achieve a higher rate of performance in resource use, without much consideration of the behavior and reaction of various decision makers at low levels of the hierarchy, who are the actual users of the resource. It appears, therefore, desirable to attempt, at least, a reformulation of large-scale planning models so as to represent regional problems at an adequate level of detail, and linking these sub-systems in an effective manner with the broader decision-making framework.

The Problem

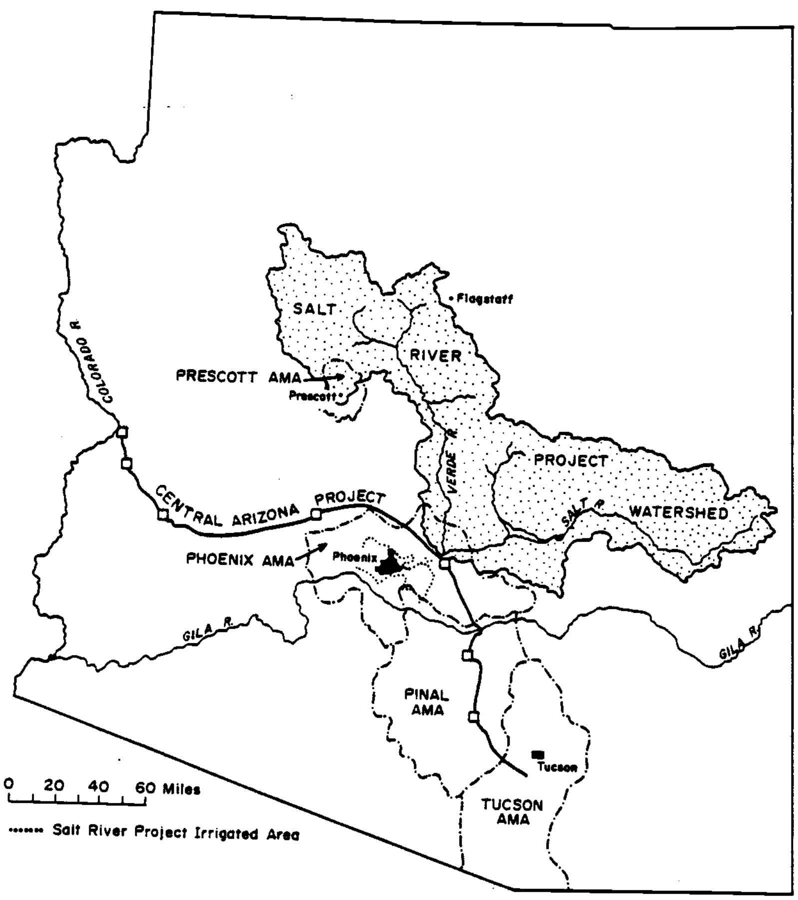
Arizona's water resources present a picture of considerable complexity. The area of the State, about 113,900 square miles (291,600 sq. km), makes it the sixth largest in the U.S. The average precipitation is approximately 13" (330 mm), with about half the State receiving less than 10" per year (250 mm). Of an estimated average of 80 million acre feet (Maf) of precipitation falling over Arizona each year (98.7 billion cubic meters), more than 95 percent is lost to evaporation and plant transpiration (Arizona State Water Plan, 1975). The remaining 4 Maf/yr (4.9 Bcm/yr) represent the yearly runoff, both surface and subsurface. It is estimated that the yearly groundwater recharge amounts to 0.3 Maf (370 million cubic meters) and the dependable surface flows do not exceed 2.5 Maf (3.1 Bcm).

However, the groundwater which accumulated in the Arizona aquifers over a period of several millennia represents a considerable resource. Within the top 700 ft (about 200 m) below land surface, the groundwater is estimated to amount to 794.55 Maf (980 Bcm); the next 500 ft (about 150 m) contains an estimated quantity of 407.25 Maf (502 Bcm). The total of 1.201 Maf (over 1,480 Bcm) represents a substantial water resource which, if managed judiciously, could last several centuries.

Current water utilization in Arizona amounts to about 4.8 Maf/yr. (5.9 Bcm), 89% of which is used in irrigated agriculture, 7% by municipalities and industries, 3% by mines and the energy sector, and about 1% to maintain fish and wildlife. There is a strong inclination by the State water managers to improve the efficiency of water use in all the economic sectors of the State (Arizona State Water Plan, 1977, 1978).

The economic development of Arizona relies heavily on groundwater; yet significant surface water supplies were developed early in this century by the Salt River Project, and another major project--the Central Arizona Project--is currently under construction for the diversion of about 1.2 Maf/yr. (1,480 Bcm/yr) from the Colorado River. Nevertheless, aquifers will remain major suppliers of water and their rational exploitation is an issue of major concern, as evidenced by the promulgation in 1980 of the Arizona Groundwater Management Act. Under this law, the Arizona Department of Water Resources was established, as well as four Active Management Areas: Tucson, Phoenix, Prescott, and Pinal (Arizona Department of Water Resources, 1981).

A map showing the major elements of the Arizona Water Resources systems appears in Figure 1.



ELEMENTS OF THE ARIZONA WATER RESOURCES SYSTEM

Figure 1

The major components of the State water system can be divided into three groups:

- The Salt River Project (SRP);
- The Active Management Areas (AMA's);
- The Central Arizona Project (CAP).

These groups also represent three organizations connected rather loosely: (i) the Salt River Users' Association (established in 1903); (ii) The Central Arizona Water Conservation District which was established to operate the CAP; (iii) Arizona Department of Water Resources (ADWR). Now, the decision-making process related to the operation of AMA's takes place at several levels within the ADWR. For the purpose of this analysis, a two-level hierarchy of decision-making is considered, as shown in Figure 2.

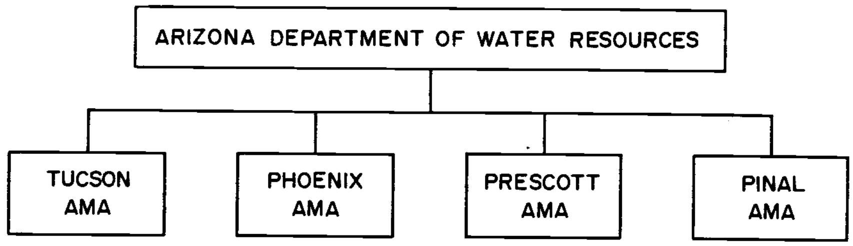


Figure 2

Management of Groundwater Resources in Arizona

This simplified scheme, which reflects the bulk of the water used in Arizona, could be extended to also include the CAP (Yeh et al., 1980) and the SRP. The model proposed in the next section is within the framework of dynamic optimization, to which recent developments in decentralized control theory (Siljak, 1978) could be applied. The problem can now be formulated as follows: What are the alternative policies available to water managers in Arizona, both at the central (State) and at the regional (AMA) levels, given the scarcity of the resource, the possible interactions between components, and the stochastic nature of the demand?

The Model

The formulation of the problem indicates its complexity, yet any operating decision must consider the reliability of system's performance. A basic approach to the reliability problem in water storage systems is mentioned by Sniedovich (1979). A simple concept related to the reliability of dynamic systems states (Siljak, 1978) that

> "A dynamic system composed of interconnected subsystems is reliable if all subsystems are self-sufficient and their interdependence is properly limited."

Here "interdependence" is interpreted as interaction between subsystems, such as those between aquifers and surface streams (Illangasekare and Morel-Seytoux, 1982). In the case of the Arizona Water Resources, the interactions between the subsystems are shown in Figure 3. An important detail in this diagram is that aquifers are main sources of water for each AMA. The AMA's are connected with ADWR by a two-way link: the AMA's transmit to ADWR information regarding their respective state (including demand to be satisfied), and ADWR may issue guidelines for groundwater abstraction.

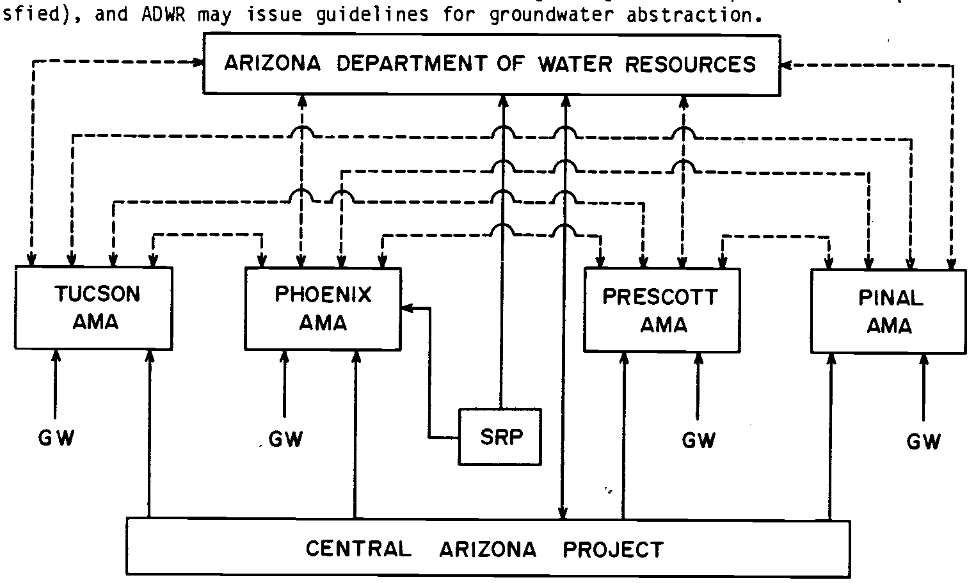


Figure 3

Schematic representation of Arizona Water Resources System

Colorado River water will be supplied to all four AMA's, either directly (Tucson, Phoenix, Pinal) or indirectly by an exchange arrangement via SRP (Prescott). A two-way connection is assumed to exist between CAP and ADWR: information regarding flows diverted from Colorado to CAP is transmitted continuously to ADWR; ADWR could modify CAP allocations to AMA's

The SRP supplies water--both from surface diversions and from aquifers--only to the Phoenix AMA, which overlaps much of the SRP irrigated area. The connection with ADWR is mostly indirectional: water-related information flows from SRA to ADWR.

The scheme shown in Figure 3 is modeled using a deterministic formulation. Let the vector X_i

$$x_i = (x_{i1}, x_{i2}, x_{i3})^T , i = 1,2,3,4, \tag{1}$$

represent the state of subsystem i, where i=1 is associated with Tucson AMA, i=2 with Phoenix AMA, i=3 with Prescott AMA, and i=4 with Pinal AMA. Here x_{i1} is the average depth to water table, x_{i2} is the availability of CAP water, and x_{i3} is the availability of SRP water. Clearly

$$\begin{aligned} x_1 &= (x_{11}, x_{12}, 0)^T, \\ x_2 &= (x_{21}, x_{22}, x_{23})^T, \\ x_3 &= (x_{31}, x_{32}, 0)^T, \\ x_4 &= (x_{41}, x_{42}, 0)^T. \end{aligned} \tag{2}$$

Let

$$B_i = (b_{i1}, b_{i2}, b_{i3})^T, i = 1,2,3,4 \tag{3}$$

be the net "recharge" of the water supply in subsystem i. Here b_{i1} is groundwater recharge; b_{i2} is the diversion from the Colorado river to CAP theoretically allocated to the i-th AMA; and b_{i3} is the SRP target allocation to the Phoenix AMA (i=2).

Define the control and decision vectors p_i

$$p_i = (p_{i1}, p_{i2}, p_{i3})^T, i=1,2,3,4, \tag{4}$$

to be the pumping and delivery rates in subsystem i from groundwater (p_{i1}), from the CAP (p_{i2}) and from SRP (p_{i3}). These control and decision variables are mapped onto the state vectors through function $f_k(\cdot)$, k = 1,2,3.

Given a finite time interval ($0-t_f$), the basic dynamics of the subsystem AMA i are given by the equations

$$x_{ik}(t+1) = x_{ik}(t) - f_k(p_{ik}) + b_{ik}, \quad i=1,2,3,4, \; k = 1,2,3. \tag{5}$$

These equations represent a situation in which the subsystems i are quasi-independent. However, should an AMA require more water from CAP while another AMA may be over-supplied, the ADWR could adjust the control variables p_i so as to maintain the stability of the system. These adjustments generate interactions hij between the subsystems, which have to be taken into account. Thus, the dynamic model of subsystem 1 is

$$x_i(t+1) = x_1(t) - F_1(p_i) + B_1 + h_{12}\cdot x_2(t) + h_{13}x_3(t) + h_{14}\cdot x_4(t). \tag{6}$$

The rate at which the state of subsystem 1 changes is given by

$$\dot{x}_1 = x_1(t+1) - x_1(t) = - F_1(pi) + B_1 + \sum_{j=2}^{4} h_{ij}\cdot x_j(t). \tag{7}$$

In general, the dynamics of subsystem i is expressed by

$$\dot{x}_i = - F(pi) + B_i + \sum_{\substack{j=1 \\ j\neq i}}^{n} h_{ij}\cdot x_j(t). \tag{8}$$

Equation (8) represents the behavior of the subsystems, components of the Arizona water resources system.

The management problem at the statewide decision-making level is formulated in terms of dynamic optimization. A possible optimization criterion is the minimization of an energy cost matrix associated with the control and decision vectors p_i, $E(p_i)$. Thus, the objective function to be optimized is

$$\min_{p_i > 0} J = \int_o^t E(p_i)dt. \tag{9}$$

The minimization problem (9) is subject to the following constraints:

$$\text{(a) demand, } d_i, \int_o^t p_i dt \geq d_i \tag{10}$$

$$\text{(b) minimum "storage" level, } s_i, \quad x_{ij} \geq s_{ij}, \tag{11}$$

$$\text{(c) subsystem components capacities } c_{ij}, \quad p_{ij} \leq c_{ij} \tag{12}$$

Thus the two-level hierarchical model for Arizona's water resources consists of the dynamic optimization (9), subject to the constraints (10), (11) and (12), where the dynamics of the component subsystems are given by equations (8).

Future Work

The conceptual model given by equations (8) through (12) needs to be refined on the basis of actual data. A crucial set of variables which have to be numerically evaluated are the interactions h_{ij}. Details of past performance of groundwater basins currently aggregated into the Active Management Areas should yield the necessary insights in these interactions.

The functions F_i, B_i and E should be formulated explicitly: E is probably a nonlinear function of p_i, while F_i and B_i could be represented adequately by linear relationships. Similarly, the right-hand side of the constraints (10), (11) and (12) need to be evaluated numerically for each subsystem (AMA).

Finally, an efficient computational algorithm will have to be developed to solve the two-level hierarchical model (8) through (12), leading eventually to an interactive computer program which could be used by decision-makers at the State level for managing Arizona's water resources.

Summary

An important issue recently identified in operating large-scale systems is the integration of detailed information generated at a lower (subsystem) hierarchical level in an effective manner for decision-making at a higher (e.g., statewide) level (Buras, 1982). The hierarchical model for the management of Arizona's water resources suggested in this paper represents an initial step in the analysis, understanding, and resolution of this issue.

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