# APPLICATION OF A FINITE-ELEMENT MODEL TO OVERLAND FLOW AND CHANNEL FLOW IN ARID AREAS

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## ABSTRACT

A mathematical model to simulate overland and channel flow using the finite element technique was adapted and applied to a small semi-arid rangeland watershed (2,035 acres) in the USDA Walnut Gulch experimental watershed in the Southwestern United States. The Holtan equation was used to estimate precipitation excess, and with the precipitation excess as input, the finite-element technique was used to route overland and channel flow. The program was structured with sufficient flexibility so that effect of land use changes either gradual or sudden, on runoff hydrograph could be estimated. Abstraction losses in the stream channel are accounted for. The simulation model predictions are compared with field data for several storms and the comparisons are satisfactory; however, improvements could be made with additional data on antecedent moisture content and better estimates of abstraction losses. Based on these comparisons, it is felt that the model can be used to estimate runoff hydrographs from ungaged watersheds in semi-arid regions.

## INTRODUCTION

Mathematical models to predict the hydrologic response of a watershed system are very important tools in the management of water resources. It is essential that such models integrate the spatial variability of soils, land use, topography, and rainfall over the watershed and simulate the stream flow hydrograph at the outlet of the watershed for results to be meaningful. Models of this type have been applied to many natural watersheds in humid areas with reasonable success (Ross, 1978 and Judah, et al., 1975). In this paper, such a model has been applied to a watershed in the semi-arid Southwest.

The Holtan equation (Holtan, et al. 1975) was used to estimate precipitation excess, and the transformation from rainfall excess to stream flow was accomplished using the Kinematic wave routing procedure. The watershed was subdivided into a number of overland flow and channel flow elements, and the finite-element technique was used to route the overland and channel flow. For a given rainfall on each element, the runoff was calculated considering soil type and land-use distribution. Abstraction losses in stream channels were accounted for after routing the flow through each channel element, using Smith's equation (Smith, 1972). The model was applied for the first time to a small semi-arid watershed in the USDA Walnut Gulch experimental watershed, Arizona, in the Southwestern United States.

## WATERSHED DESCRIPTION

This current investigation is limited to a 2,035 acres sub-watershed (Watershed No. 63.011) located northeast of Tombstone on the upper part of the Walnut Gulch watershed (Figure 1). Rainfall is recorded by 10 weighing-type recording raingages on/or adjacent to Watershed No. 63.011. Runoff stage is measured with a water level recorder at a flume (No. 11) at the outlet. Vegetation in the study area is dominated by mixed grass and brush cover. There are three major branches in the watershed: north, central and south. Two stock ponds control runoff from central branch (Osborn and Simanton, 1982). Channel characteristics are described as being shallow and steep with an overall slope of 1.2 percent. The channels are dry except for brief periods following thunderstorms, and the decrease in flow volume in the downstream direction is largely due to transmission losses.

## THE MODEL THEORY

The model used in this study was developed by Ross (1978) to simulate flood hydrographs from a watershed system using a finite element numerical approximation to solve kinematic equations of one-dimensional flow in open channel. The model consists of two major components. The first generates precipitation excess from rainfall, and the second routes this excess to the downstream outlet of the watershed.

### Precipitation Excess

The amount of precipitation excess was calculated using the Holtan infiltration equation, Holtan, et al. (1975).

$$f = a \cdot S_a^n + f_c \tag{1}$$

where

$f$ = infiltration rate in inches per hour;
$a$ = a coefficient for indexing the effect of cover conditions;
$S_a$ = unfilled storage space to a restrictive layer, in inches;
$f_c$ = final infiltration rate in inches per hour;
$n$ = the ratio of potential plant available water to the potential gravitional water in the "A" horizon.

The soil moisture accounting model was constructed by Li, et al. (1977). For each time period, $\Delta t$, rainfall intensity was assumed constant, and rainfall was compared to the computed infiltration rate. Runoff is the rainfall less the water that infiltrates during a given time interval, and that is retained as depression storage.

### Overland and Channel Flow Routing

The Saint-Venant equations of continuity and momentum were used for overland and channel flow in the development of the element equations. Derivation of these equations is presented in many texts (e.g. Chow, 1959). Equation of continuity can be expressed as:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} - q = 0 \tag{2}$$

where

$Q$ = discharge in the overland flow plane or channel;
$q$ = lateral flow per unit length (rainfall excess for overland flow plane and overland flow output to the channel);
$A$ = flow area;
$x$ = distance in direction of flow;
$t$ = time.

Using Kinematic wave approximation reduces the momentum equation to the form

$$S = S_F \tag{3}$$

where

$S$ = bed slope;
$S_F$ = friction slope

The use of the Kinematic approach is satisfactory in this study because the channel slope is steep and the discharge rate does not change rapidly. Further, Woolhiser and Liggett (1967) stated that the Kinematic wave approximation gives accurate results when the dimensionless parameter K is greater than 10

$$K = L\, gS/v^2 \tag{4}$$

$L$ = length of flow plane;
$S$ = bed slope;
$v$ = uniform flow velocity;
$g$ = acceleration of gravity.

A number of investigators have considered finite element solutions for general and one-dimensional idealization (e.g. Judah et al. 1975; Ross, 1978). Galerkin's weighted residual method was used to minimize the residuals of equation (2), obtaining the following equation describing one-dimensional flow for one element:

$$\frac{1}{\Delta t}\begin{bmatrix}2/3 & 1/3\\ 1/3 & 2/3\end{bmatrix}\begin{Bmatrix}A_1\\ A_2\end{Bmatrix}_{t+\Delta t} - \frac{1}{\Delta t}\begin{bmatrix}2/3 & 1/3\\ 1/3 & 2/3\end{bmatrix}\begin{Bmatrix}A_1\\ A_2\end{Bmatrix}_{t} + \frac{Q_2(t)-Q_1(t)}{\ell}\begin{Bmatrix}1\\ 1\end{Bmatrix} = q\begin{Bmatrix}1\\ 1\end{Bmatrix} \tag{5}$$

where

$A$ = area at the node;
$Q$ = discharge at the node;
$t$ = time;
$\Delta t$ = routing time increment;
$\ell$ = element length;
$q$ = lateral flow into the element.

This equation is applied to all elements to create the global matrix, and its size depends on the number of nodes in the overland flow element or channel flow element. Applying the initial and boundary conditions to the global matrix, the solution will be obtained for $A\ (t + \Delta t)$ for all the nodes by using the marching process (Desai, 1979).

Then the Manning equation will be used to obtain Q $(t + \Delta t)$ at each node.

## MODEL DEVELOPMENT

A procedure was described by Li, et al. (1977) to subdivide the watershed into units that have a single soil type and land-use combination and identified them as hydrologic response units (HRU's). Rainfall excess was assumed to be uniformly distributed over the area of an HRU. To develop an HRU map, a soils map and a land-use map must be prepared. For watershed 63.011, the soil map was taken from a general soil survey map for Walnut Gulch watershed (Gelderman, 1970). The land-use map was constructed from a general vegetation map for the watershed (Renard, 1970). The HRU map was constructed by overlaying the soil and land-use map (Figure 2). Another discretizing procedure must be done to reflect significant changes in physical characteristics affecting flow, such as slope and roughness. In our study using a contour map, the watershed is divided into five subsheds, each with its stream channels and their accompanying drainage areas. The drainage area can be further subdivided into overland flow elements (Figure 3). Also, the channels are divided into elements where the lateral flow from an overland flow strip is entering the channel element uniformly along its length from both sides.

According to the discretization procedure discussed earlier, a flow element will usually contain more than one HRU. The rainfall excess from each HRU within one element can be weighed according to its area, to provide a single rainfall excess sequence for the entire element. Once the weighted rainfall excess is determined for each element, it can be routed through each overland flow strip for each time step, and the value of flow at the last node of the strip will be an input to the channel, where the flow can be routed again along the channel element.

### Channel Transmission Losses

Large transmission losses from the channel bed in arid and semi-arid areas present a special problem in flood routing. With the knowledge of losses along each section of a stream during passage of flood wave, flood routing procedures can be modified to take this loss into account.

The point infiltration function chosen for this study is that used by Smith (1972),

$$U = K\,(t-t_o)^{-b} + Uo \qquad (6)$$

where

$U$ = point infiltration velocity in inches per hour;
$Uo$ = is the long term saturated infiltration rate in inches per hour;
$t$ = time from the beginning of flow, in minutes;
$\left.\begin{matrix} K \\ b \end{matrix}\right\}$ = soil specific constants.

K and b are determined from various physical measurements by analysis of input and output of reaches on Walnut Gulch watershed.

## RESULTS AND DISCUSSION

Using the soil profiles and the hydraulic characteristics for the sandy loams of the Hathaway soil series in the study area gives an estimate of the three soil moisture characteristics: S = total storage capacity, G=gravitational water capacity, and AWC = available water capacity (Table 1).

**TABLE 1. Summary of Characteristics of Hydrologic Response Units**

| HRU No | a [1] | AWC [2] | G [3] | S [4] | $f_\epsilon$ [5] (in/hr) | Depth (inch) | [6] Soil Type | [6] Land Use |
|---|---|---|---|---|---|---|---|---|
| 1 | 0.3 | 0.183 | 0.221 | 0.404 | 0.6 | 12 | HaC | Grass |
| 2 | 0.3 | 0.188 | 0.153 | 0.341 | 0.6 | 12.3 | HbB | Grass |
| 3 | 0.3 | 0.187 | 0.167 | 0.354 | 0.6 | 10 | HbC | Grass |
| 4 | 0.35 | 0.187 | 0.167 | 0.354 | 0.6 | 10 | HbC | Brush |
| 5 | 0.3 | 0.179 | 0.222 | 0.401 | 0.6 | 16 | HnC | Grass |
| 6 | 0.35 | 0.179 | 0.222 | 0.401 | 0.6 | 14 | HnC | Brush |

1) Cover Coefficient
2) Plant available water = S - G
3) Gravitational water =Total porosity - moisture percent at a tension of 0.33 atm.
4) Total storage = Total porosity - moisture percent at a tension of 15 atm.
5) Final infiltration rate
6) See Figure 2.

The rainfall data from ten recording gages are assigned to each overland flow element using the Thiessen Polygon method (Figure 3). Figures 4 and 5 give recorded and simulated hydrographs for September 10, 1967 and July 28, 1966, respectively. For September 10, 1967, the peak discharge and time to peak are well simulated, but the volume is less than the recorded volume by 19 percent. For the small scale runoff, July 28, 1966 storm, the second peak does not appear in the predicted hydrograph. The peak and time to peak are reasonably acceptable but the volume is less than the recorded volume by 35 percent. The model can be used to simulate the effect of any change in the land use. For example, the effect of grass seeding and brush clearing was examined in a specific storm and found that for the grass seeding condition a subsequent decrease in the runoff occurred. On the other hand, in the brush clearing case, an increase in the runoff was obtained.

A better estimation of the antecedent moisture condition and transmission losses parameters would enhance the model prediction of the outflow hydrograph. Also, because the model is very sensitive to the depth of soil which is usually taken as root depth (usually "A" horizon) , great care must be taken in choosing the soil depth.

In conclusion, the model was designed to integrate the spatial variability of soils, land use, topography and rainfall hydrograph, and based on the results given from watershed 63.011, flow comparisons are reasonable and would be acceptable for many design applications.

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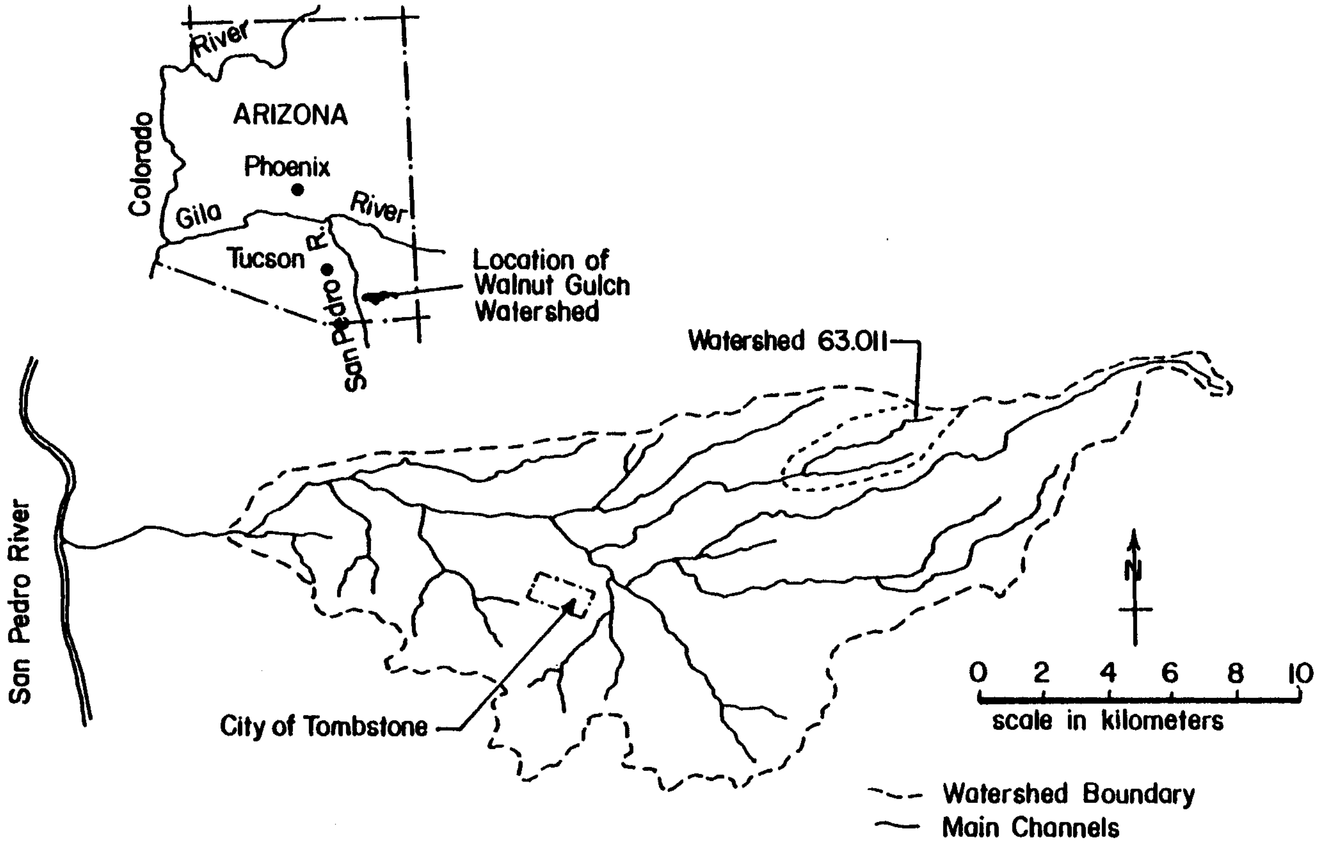

Figure 1. Location and description map of Walnut Gulch Watershed

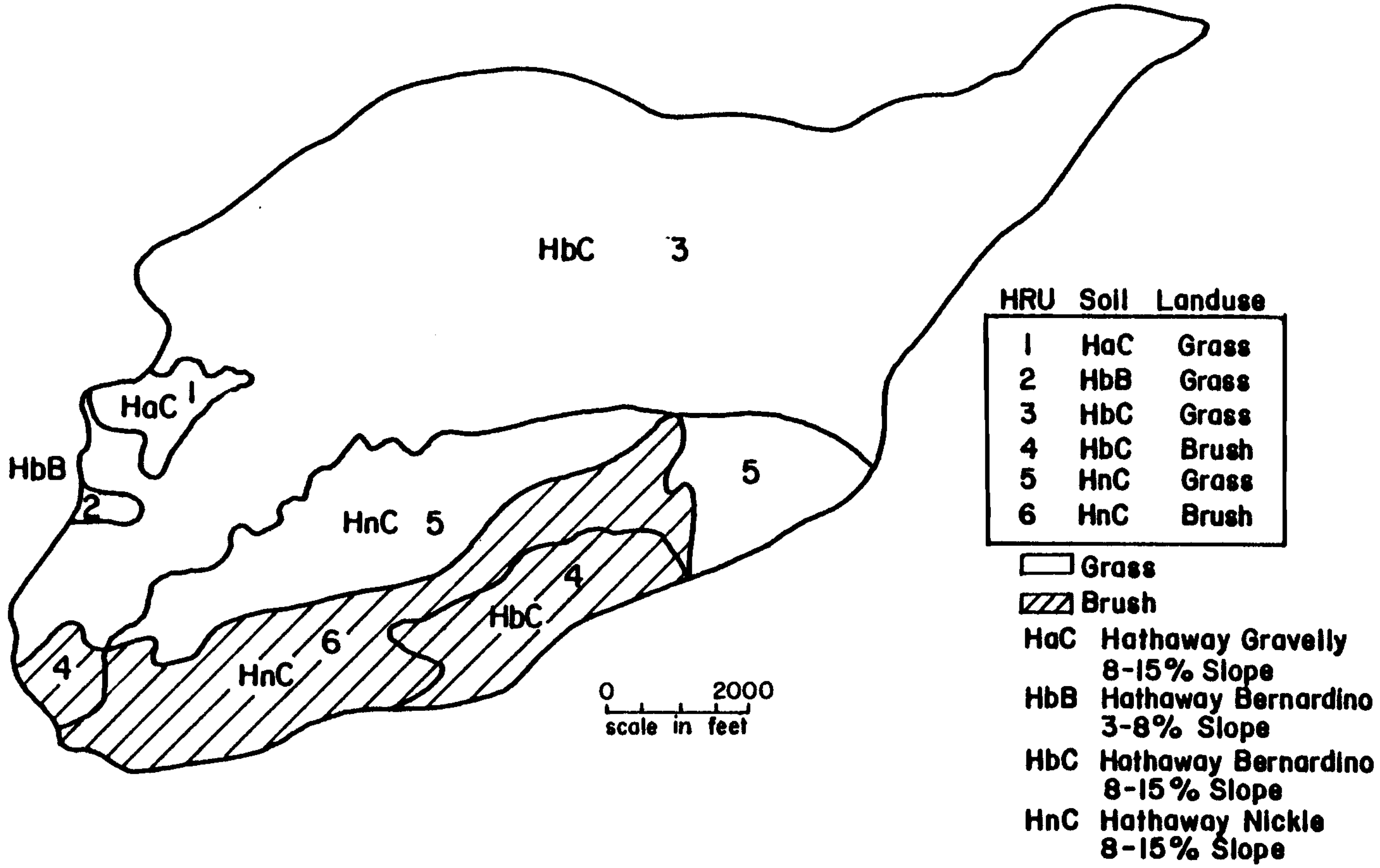

Figure 2. Hydrologic response unit map for watershed 63.011, Walnut Gulch

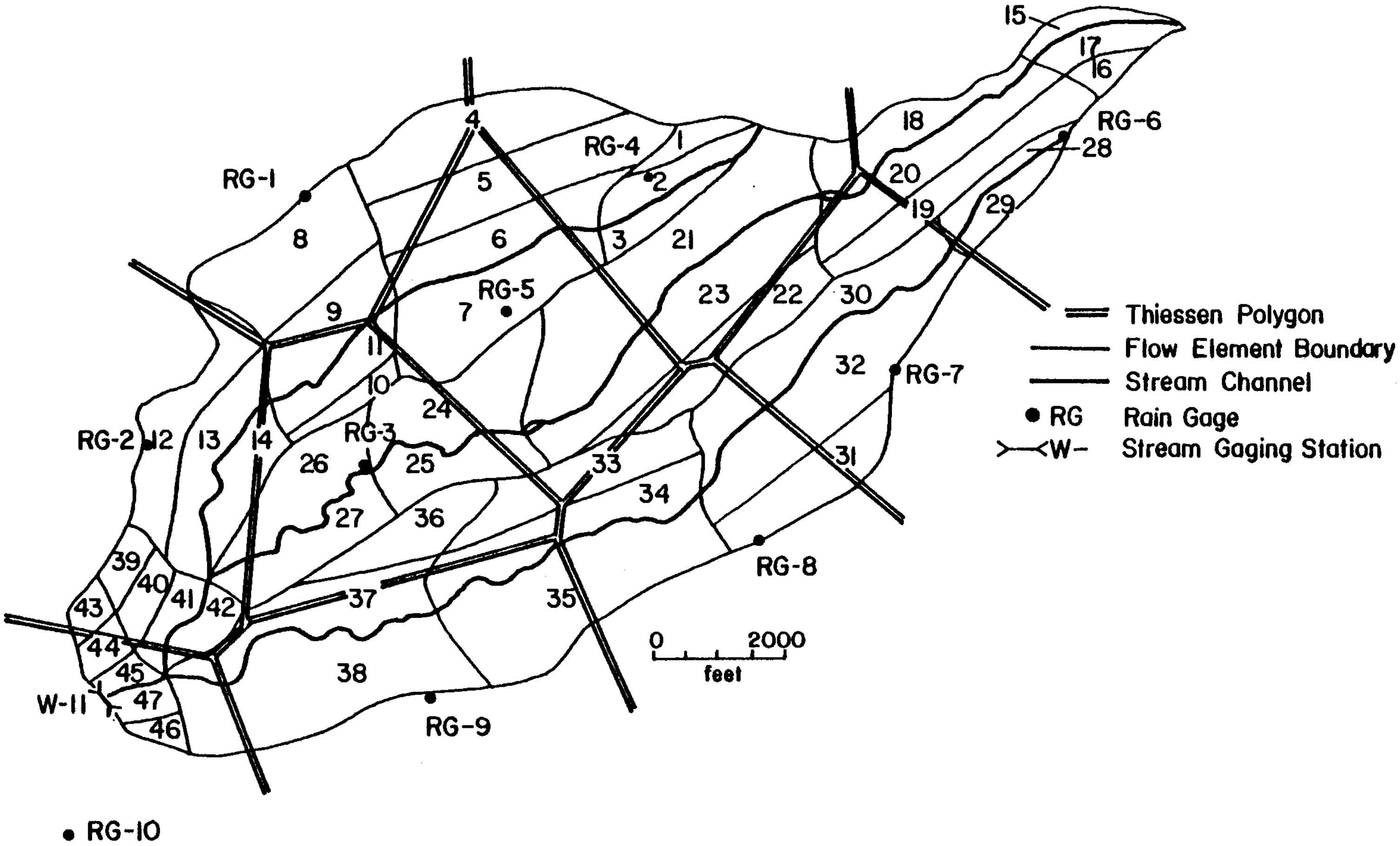

Figure 3. Element grid and Thiessen polygon diagram, watershed 63.011, Walnut Gulch

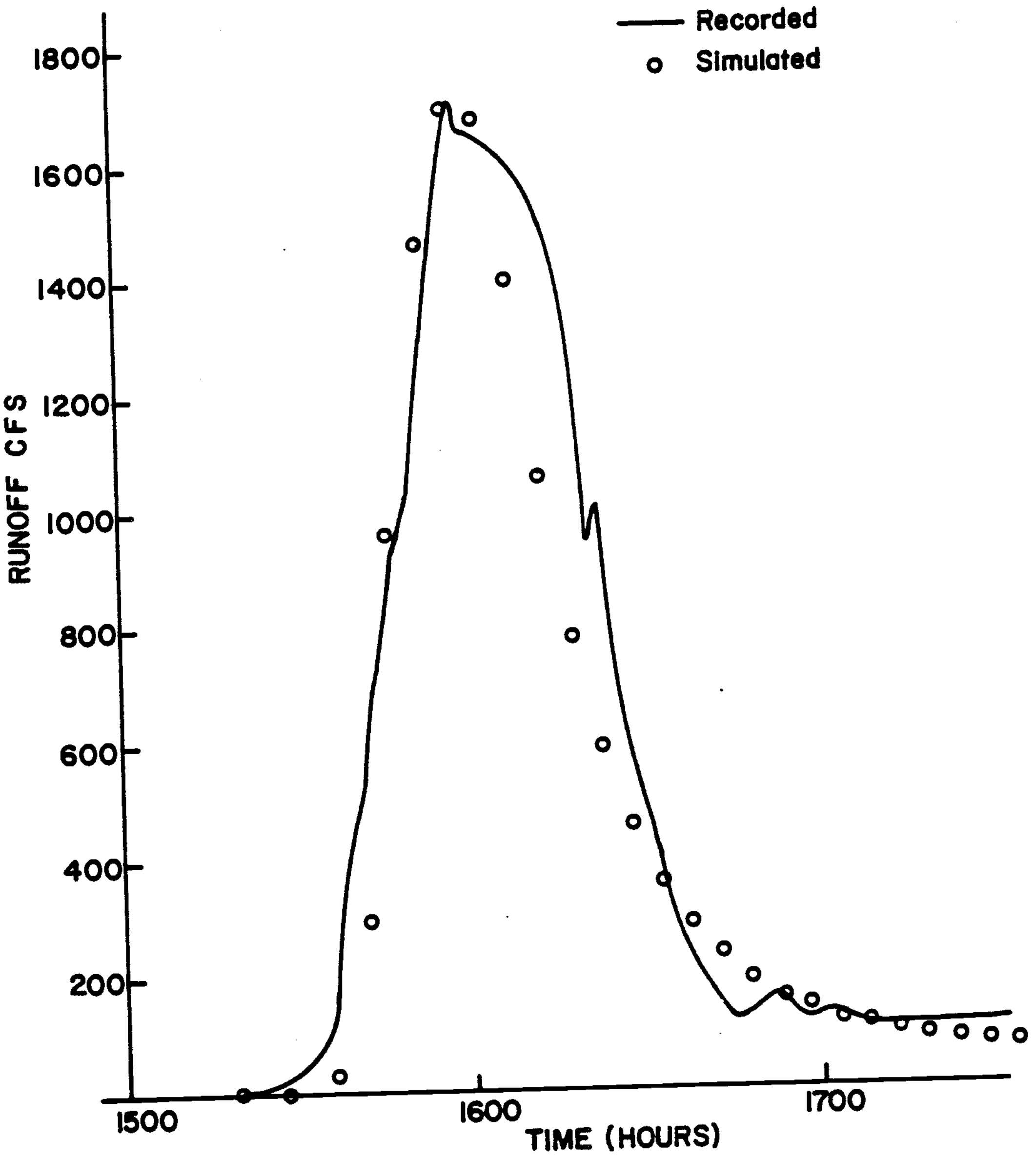

Figure 4. Recorded and simulated hydrographs, September 10, 1967 storm, Walnut Gulch watershed 63.011

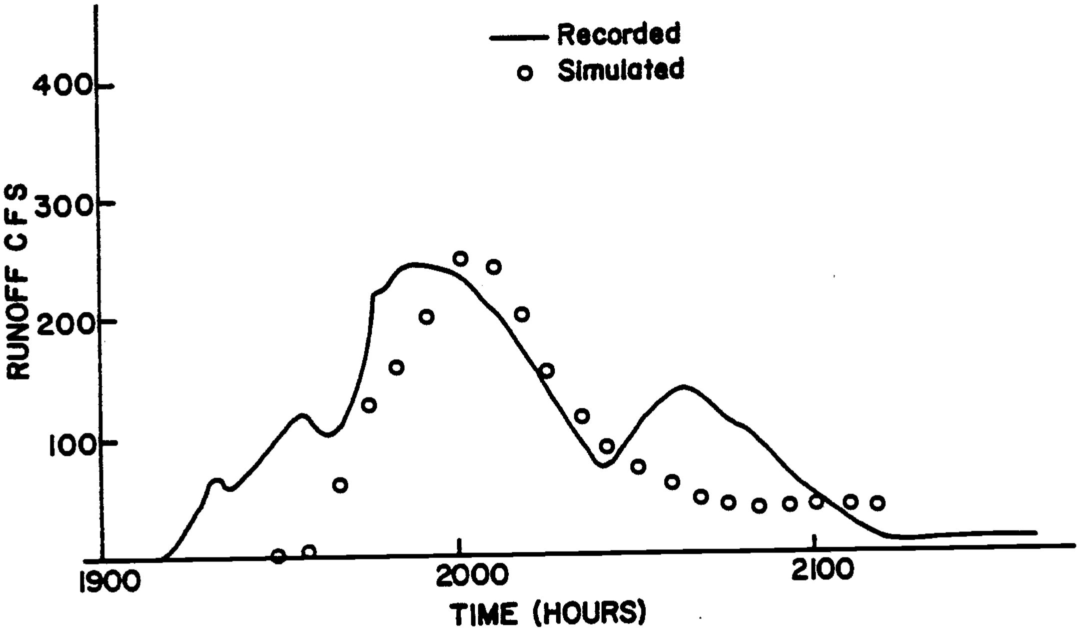

Figure 5. Recorded and simulated hydrographs, July 28, 1966 storm, Walnut Gulch watershed 63.011