USING AN EROSION EQUATION TO PREDICT SEDIMENT YIELD FROM OVERLAND FLOW SYSTEMS

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ABSTRACT

The erosion process by overland flow was analyzed in its fundamental aspects. A general predictive erosion equation was developed by combining the conditions for conservation of mass of the sediment, boundary hydraulic shear, and the sediment transport formula of Einstein-Brown. The ability of the equation to predict sediment yield from overland flow areas was demonstrated using field data from rainfall simulator plots. Comparison of the results indicated that the erosion equation presented herein can be used to predict sediment yield from unrilled overland flow areas with satisfactory confidence.

INTRODUCTION

The Universal Soil Loss Equation (USLE) is the technology most widely used today for upland erosion prediction (Wischmeier and Smith 1978). The equation was developed as a method to predict long-term average annual soil loss from interrill (sheet erosion) and rill areas. Despite its widespread use and the breadth of experience that it incorporates, the equation is empirically based on data gathered from uniform slope test plots, and does not incorporate the processes of detachment and deposition by overland flow. Several modifications of the USLE have been proposed to overcome its conceptual limitations (Renard et al. 1974; Williams 1975; Onstad and Foster 1975; Renard and Simanton 1990). All these modifications, however, do not eliminate the primary limitations of the USLE to consider detachment by boundary shear flow processes. Since the USLE is not satisfactory in estimating soil erosion, this research proposes:

1) To develop an alternative general predictive erosion equation for overland flow areas by combining the conditions for conservation of mass for the sediment, flow hydraulic shear, and sediment transport relationships.

2) To verify the validity of the equation by applying it without calibration to previously obtained erosion data from rainfall simulator plots on rangelands.

For noncohesive sediment, the amount of soil loss is a function of the ability or power of flowing water to move sediment particles. For cohesive sediment, the energy of raindrop impact plays an important role on soil erosion. Soil erosion on sand and silt textural classes are essentially determined by the flow capacity characteristics. The erosion equation developed in this study applies to these types of soils.

BASIC ASSUMPTIONS

The following assumptions were used in the development of a general erosion equation: (1) overland flow is one-dimensional, steady and gradually varying; (2) the composition of the soil surface layer is homogeneous; (3) the slope gradient is uniform; (4) rilling processes are not significant; (5) sediment is comparatively loose and noncohesive; (6) sediment entrainment is limited only by the sediment transport capacity of the flow; (7) deposition is not significant; and (8) runoff rate (rainfall intensity minus infiltration rate) is constant throughout the overland flow reach under consideration.

GOVERNING EQUATIONS

The conservation of mass for the sediment in one-dimensional, steady and gradually varying flow on a slope surface is:

$$\frac{d\Phi_s}{dx} = P_s \tag{1}$$

where Φ_s is the rate of sediment transport, including suspended sediment in volume of material per unit time and unit width $[L^3/L/T]$, P_s is the fine-sediment entrainment (pickup) rate per unit time and unit area $[L^3/L^2/T]$, and x is the distance in the direction of flow along the slope surface [L].

For comparatively loose and noncohesive sediment, entrainment is a function of transport capacity of the flow that depends on the boundary hydraulic shear. Li et al. (1973) presented an expression for boundary hydraulic shear (τ_o) $[MLT^{-2}/L^2]$ for turbulent flow (Reynolds number between 2000 and 12600) as:

$$\tau_o = \gamma\left[\frac{(k+0.012)q^{7/4}\nu^{1/4}}{8g}\right]^{1/3} S_o^{2/3} \tag{2}$$

in which γ is the fluid specific weight [MLT^{-2}/L^3], k is a constant describing the Darcy-Weisbach friction factor without rainfall, q is the flow rate per unit width [L^2/T], ν is the fluid kinematic viscosity [L^2/T], S_o is surface slope, and g is the acceleration of gravity [L/T^2]. Li et al. (1973) obtained the values of k = 0.24 for flow over smooth surfaces. Komura (1976) suggested the value of k = 0.60 for flow over rough surfaces.

The continuity equation for flowing water at equilibrium on a slope surface is:

$$q = q_o + q_* x \tag{3}$$

where q_o is the inflow rate per unit width in the upper end of slope [L^2/T] and q_* is the lateral inflow rate per unit area of the slope [L/T] defined here as the difference between rainfall intensity and infiltration rate.

Soil loss (E_s) in weight of material per unit time and unit width [$MLT^{-2}/L/T$] can be estimated as:

$$E_s = (1/L)\ \gamma_s \int_0^L d\Phi = (1/L)\ \gamma_s \int_0^L P_s dx \tag{4}$$

where γ_s is the sediment specific weight [MLT^{-2}/L^3].

To determine E_s from Eq. 4, P_s must be obtained from Eq. 1 with Φ_s being determined from an equation for sediment transport. The sediment transport formula of Einstein-Brown (Vanoni 1975) was used to compute Φ_s. This is:

$$\Phi_s = 40(1/\psi)^3 F_1[(S-1)gd_s^3]^{1/2} \tag{5}$$

where

$$1/\psi = \frac{\tau_o}{\gamma(S-1)d_s} \tag{6}$$

$$F_1 = [(2/3)+36\nu^2/(S-1)gd_s^3]^{1/2} - [36\nu^2/(S-1)gd_s^3]^{1/2} \qquad (7)$$

in which ρ is the fluid specific gravity [M/L^3], S is the particle density, and d_s is the representative particle size and is usually taken as the median size d_{50} [L]. The quantity, F_1, appears in the Rubey formula for fall velocity (Vanoni 1975). Note that $1/\psi$ is the same as the dimensionless shear stress, $\tau_{\bullet}$, introduced by Shields.

DEVELOPMENT OF A GENERAL EROSION EQUATION

The derivation of a soil loss equation consists in substituting Eqs. 2 and 3 in Eq. 5 and taking its derivative with respect to x to find P_s (Eq. 1). A general erosion equation is found by substituting the resulting expression for P_s into Eq. 4 and integrating with respect to x. For $q_o = 0$, the resulting equation is:

$$E_s = \frac{CF_{\bullet}q_{\bullet}^{1.75}L^{0.75}S_o^{\;2}}{(d_s)^{1.5}} \qquad (8)$$

where

$$F_{\bullet} = [(2/3)+(0.0016/d_s^3)]^{1/2} - (0.0016/d_s^3)^{1/2} \qquad (9)$$

in which $C = 3.52\text{x}10^5$, E_s in Kgf/m^2.s, L in m, d_s in mm and $q_{\bullet}$ in m/s. In the derivation of Eq. 8 the following values were used: $k = 0.6$, $\gamma = 1000$ kgf/m^3, $\gamma_s = 1300$ kgf/m^3, $g = 9.8$ m/s^2, $S = 2.65$ and $\nu = 8.5\text{x}10^{-7}$ m^2/s. Eq. 8 is valid for bare soil conditions with high contents of silt and sand. For other types of soil and surface cover conditions this equation should be used with caution. Eq. 8 should be multiplied by C_v (ratio of bare-soil area to total area) and C_e (soil erodibility coefficient) to include the effects of surface cover and soil resistance to soil erosion, respectively.

EVALUATION OF THE EROSION EQUATION

Rainfall simulator plot data from the USDA-ARS in Tucson, Arizona, were used to verify the suitability of Eq. 8 to predict sediment yield on overland flow areas.

Rangeland field data collection for the Water Erosion Prediction Project - WEPP (Lane and Nearing 1989) was a two year project that began in the Spring of 1987. Soils at the selected sites are of the orders Mollisols, Artisols, Entisols and Inceptisols. Moisture regimes are ustic, xeric, and aridic. Surface textures range from loamy sand to clay and many of the soils have appreciable amounts of coarse fragments. The rainfall simulator plots are 3.1 x 10.7 m, and have slopes of 3-12%. The rainfall simulator used in the WEPP rangeland field experiments was developed in 1965 by Swanson and is described in detail by Simanton et al. (1991). Plot treatments consisted of natural, clipped (canopy clipped to 20 mm height and clippings removed) and bare (canopy clipped to ground surface with clipping and all soil surface cover removed with minimum disturbance to the soil surface). The bare soil plots were used in this study. Table 1 summarizes the rangeland soil plot data information.

The inflow rate in the upper end of the plots for each rainfall application was zero. Mean particle size d_s or effective particle size was computed from (Lane and Nearing 1989):

$$d_s = \exp[\Sigma f_i \log(d_i)] \tag{10}$$

summed over the clay, silt and sand particle size classes, where d_i and f_i are the mean diameter and the fraction of the particle class, respectively. Tables 2 and 3 show experimental conditions and the measured and computed sediment yield on the rangeland field experiments for years 1987 and 1988, respectively. The measured sediment yield rates were obtained from experimental results for a 30-min application following 24 hours later an initial run (wet run). Computed sediment yields were obtained from Eq. 8.

The data shown in Tables 2 and 3 suggest the order of sediment yield prediction error by Eq. 8. Notice that except for two events in 1987 (Table 2 plot codes F1 and J1) and two events in 1988 (Table 2 plot code H2), 50% of the 36 calculated values of sediment yield were within a 30% error bound. Considering that the entire data set was used for model validation (no parameter fitting), these results serve to establish confidence in Eq. 8 for predicting sediment yield on overland flow areas with soils similar to those used in this study. A larger number of events would probably increase the confidence on model predictions by increasing the percentage of calculated values within the 30% error bounds. It is also almost certain that parameter fitting would reduce prediction errors, since the Einstein-Brown formula was developed based on flume data with well-sorted sediments. Further examination of Tables 2 and 3 indicates that 9 predictions out of 36 (25%) were within the range of 30-60% relative error bounds. Five predictions out of 36 (14%) were within the range of 60-90% relative error bounds. Only 4 predictions out of 36 (11%) were outside the 90% relative error bounds. Again, these results are encouraging considering that an error of up to 400% can result from the application of sediment transport formulae to field conditions (Vanoni 1979). Fig. 1 shows the scatter of the computed versus measured sediment yields about a line of perfect fitting.

CONCLUSIONS

The erosion process by overland flow was analyzed in its fundamental aspects. A general predictive erosion equation was developed by combining the conditions for conservation of mass of the sediment, boundary hydraulic shear, and the sediment transport formula of Einstein-Brown. The ability of the equation to predict sediment yield on overland flow areas was demonstrated using field data obtained from the USDA - Agriculture Research Service in Tucson, Arizona.

The following conclusions can be drawn from this study:

1. A general erosion equation for overland flow areas based on conservation of mass of the sediment, boundary hydraulic shear, and the sediment transport formula can be expressed by Eq. 8.

2. Fifty percent of the 36 predicted values of sediment yield from Eq. 8 were within a 30% error bound. This result serves to establish confidence in Eq. 8 for predicting sediment yield on overland flow areas with soils similar to those used in this study.

3. Although test results are encouraging, the many assumptions involved in the development of Eq. 8 and the complexities of the erosion process on overland flow areas suggest that Eq. 8 should be used with caution on other types of soil and surface cover conditions that differ from those used in this study.

4. Further research is needed to evaluate Eq. 8 at space-time scales that differ from those used in this study.

ACKNOWLEDGMENTS

The writer thanks Roger Simanton and Fatima Lopez of the USDA-ARS Aridland Watershed Research Unit in Tucson, Arizona, for providing all the data necessary for completion of this study.

Table 1. General rangeland soil information

Plot Code	Texture	clay	silt	sand	orgc	cec
				(%)		
A1	Gravelly sandy loam	16.7	14.2	69.1	1.04	13.9
A2	Sandy clay loam	11.2	18.5	70.3	0.85	9.8
D1	Loam	14.4	31.8	53.8	2.28	13.0
D2	Very fine sandy loam	10.5	32.9	56.6	1.27	8.3
E1	Loamy fine sand	3.3	8.7	88.0	0.55	3.4
E2	Loam	13.9	42.4	43.7	1.29	11.6
F1	Loam	15.1	36.0	48.9	3.57	18.5
G1	Silty clay	42.3	53.1	4.6	1.48	19.2
H1	Clay	49.6	37.3	13.1	1.83	36.1
H2	Clay	44.2	33.4	22.4	2.06	31.6
I1	Fine sandy loam	6.4	45.0	48.6	1.15	7.9
J1	Fine sandy loam	9.2	24.3	66.5	0.84	9.0
L1	Clay loam	26.2	42.7	31.1	1.62	30.4

Table 2. Experimental conditions and the measured and computed sediment yield from rainfall simulator studies in 1987

Plot Code	Precip (mm/h)	q (mm/h)	Dur (min)	Slope	ds (mm)	Cov (Cv)	Sed. Yld (T/ha) Obs	Sim
A1	62.20	50.80	32.10	.102	.061	.731	6.80	8.07
A1	61.00	46.80	24.10	.100	.061	.753	5.22	5.20
A2	56.00	39.30	24.40	.029	.069	.933	0.37	.40
A2	54.60	45.40	30.40	.039	.069	.957	1.26	1.19
D1	56.70	43.10	35.80	.052	.040	.924	2.46	2.25
D1	56.90	34.00	34.20	.049	.040	.859	1.34	1.17
D2	61.50	42.90	28.10	.045	.046	.918	.91	1.30
D2	62.80	42.90	28.10	.044	.046	.959	1.20	1.30
E2	69.00	41.80	28.60	.059	.030	.906	2.75	2.16
E2	54.00	31.90	29.70	.059	.030	.931	1.69	1.44
F1	59.70	34.90	25.60	.112	.034	.896	1.77	5.03
F1	50.80	26.80	21.70	.098	.034	.947	1.29	2.17
G1	61.20	4.70	25.40	.111	.006	.863	.43	.14
G1	59.30	40.80	25.90	.091	.006	.876	3.82	4.33
H1	55.70	41.70	27.20	.078	.007	.727	4.37	2.88
H1	56.00	37.70	26.10	.086	.007	.702	3.16	2.72
H2	53.60	37.10	27.50	.112	.010	.920	3.35	6.19
H2	56.50	37.20	27.10	.116	.010	.882	3.60	6.31
I1	50.60	33.60	27.50	.060	.039	.964	1.49	1.56
I1	54.90	33.70	28.30	.070	.039	.845	1.52	1.92
J1	56.50	32.80	25.80	.066	.063	.736	.52	1.27
J1	52.40	24.30	27.90	.066	.063	.682	.65	.75

Table 3. Experimental conditions and the measured and computed sediment yield from rainfall simulator studies in 1988

Plot	Precip	q	Dur	Slope	ds	Cov	Sed.	Yld (T/ha)
Code	(mm/h)	(mm/h)	(min)		(mm)	(Cv)	Obs	Sim
D1	53.80	39.80	24.90	.052	.040	.852	2.20	1.26
D1	56.70	39.20	22.20	.049	.040	.862	1.60	.98
D2	69.00	48.10	25.70	.045	.046	.781	1.75	1.24
D2	44.90	33.10	21.40	.044	.046	.848	.97	.56
E1	64.50	7.10	26.30	.125	.132	.886	.82	.33
E1	60.50	21.60	28.90	.124	.132	.869	4.70	2.44
E2	60.20	38.70	25.60	.059	.030	.857	2.11	1.60
E2	66.80	46.70	27.50	.059	.030	.810	2.41	2.26
H1	67.80	49.00	28.30	.078	.007	.914	3.19	5.00
H1	64.50	47.10	26.90	.086	.007	.838	5.08	4.94
H2	52.80	37.50	32.00	.112	.010	.871	1.80	6.95
H2	58.40	49.60	26.40	.116	.010	.886	3.73	10.21
L1	66.20	12.50	28.80	.070	.017	.509	.27	.21
L1	60.60	24.90	28.70	.070	.017	.509	.45	.70

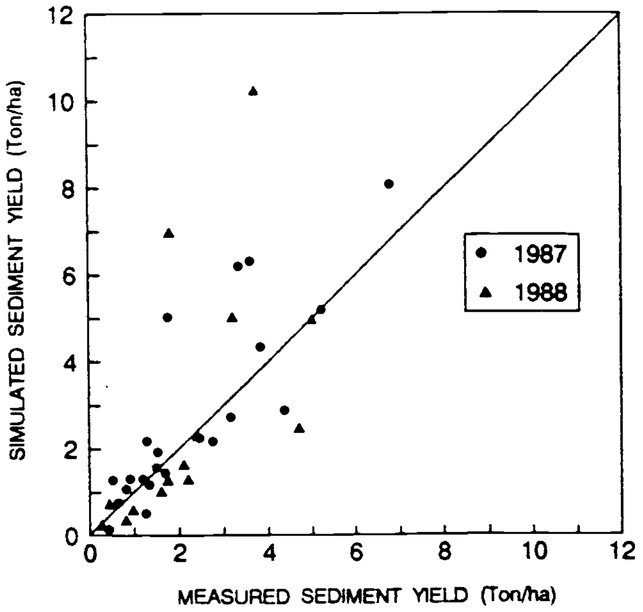

Fig. 1. Simulated versus measured sediment yield from rainfall simulator plots on rangelands.

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