INITIAL ABSTRACTION AND LOSS IN THE CURVE NUMBER METHOD

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The Curve Number Method

The curve number method is widely used in hydrologic design and environmental impact analysis to generate direct runoff (or more properly, rainfall excess) from a rainstorm. The core of the method is a simple hyperbolic equation

$$Q = (P - Ia)^2 / (P - Ia + S) \quad [1]$$
$$P \geq Ia,\ Q = 0 \text{ otherwise}$$

in which Q is the direct runoff depth, P is the rainfall depth, S is a storage index indicating land conditions, and Ia is the initial abstraction, or the rainfall prior to the onset of direct runoff. When all the above quantities are expressed in inches, then curve number, CN, is a transformation of S via

$$CN = 1000/(10 + S). \quad [2]$$

An important simplification in the development of the method is that initial abstraction, Ia, is 0.2 of the storage coefficient S, or

$$Ia = 0.2S. \quad [3]$$

Substituting this in the place of Ia in equation 1 and simplifying yields the current widely used form of the equation

$$Q = (P - 0.2S)^2/(P + 0.8S) \quad [4]$$
$$P \geq 0.2S,\ Q = 0 \text{ otherwise.}$$

Initial Abstraction

Demonstration of the 0.2S assertion is found only in the originating NRCS (then SCS) document *National Engineering Handbook, Section 4, Hydrology* (USDA 1972), or NEH4. A single figure in NEH4 (Figure 10.2) without text explanation shows 112 points of Ia vs. S, supposedly taken from real but unidentified small watershed rainfall-runoff data sets. As shall be used later here, Ia was found as the rainfall depth prior to the hydrograph rise, and with total P and Q known, S was determined by direct solution from equation 1.

The original P, Q, and Ia data are not given in NEH4, so for this study the figure points have been scaled from an enlargement of the NEH4 plot. The scaled points, which reproduce NEW4 Figure 10.2, are shown in Figure 1. Statistical description of the scaled data is given in Table 1.

Also, fitting an origin-based linear regression ($Ia = \lambda S$) to the data gives

$$Ia = 0.1110S \quad [5]$$
$$r^2 = -9.7386\%$$
$$SE = 0.4589 \text{ in.}$$

Note that the regression standard error exceeds the standard deviation of Ia, leading to a negative r^2. Thus the original data give essentially no fixed associative relationship. However, the Ia = 0.2S line in Figure 1 does separate the points into two equally sized subsets. This is equivalent to determining the median Ia/S. Thus the commonly used 0.2 is a median measure, assumed to be representative of small agricultural watersheds in the United States.

The adequacy of this value of $Ia/S = \lambda = 0.2$ is the primary topic of this paper. The general value of Ia/S, called lambda (λ; see Chen 1976) has not been previously challenged or studied from larger data sets. The matter is explored here using extensive small watershed rainfall-runoff data and two different methods.

METHODS

Event Analysis

This is the method originally used to affix the ratio Ia/S = 0.2 in NEH4 as described above. First, Ia was determined from the runoff record and the concurrent rainfall record as the depth of storm rain that had fallen when the observed hydrograph began to rise. Then the quantity P – Ia and the observed runoff depth Q are used in equation

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1 to solve for S. Third, knowing Ia and S, the ratio was taken as λ. With this method, each individual storm event gave a value of λ, and the median value was taken to characterize the watershed. Figure 2 is a cumulative frequency plot of event Ia/S for several representative watersheds.

General Model Fitting

The second method simply used observed rainstorm event values of P and Q to fit values of λ and S—via an iterative two-way least squares procedure—in the expression

$$Q = (P - \lambda S)^2/(P + (1 - \lambda)S) \quad [6]$$

$$P \geq S,\ Q = 0 \text{ otherwise.}$$

Here, both the storage index S and the abstraction ratio λ take the roles of fitting coefficients in the general expression of equation 1. With this approach a single value of λ and S is determined for each watershed, along with goodness of fit statistics. Figure 3 gives an event rainfall-runoff plot fitted to equation 2 for a typical situation.

DATA AND RESULTS

Event Analysis

This analysis used applied detailed rainfall-runoff data from 114 small watersheds—mainly agricultural—at 26 sites in the United States. The data set included 2501 station-years and 52,910 rainfall-runoff events. However, in the interests of avoiding bias from small events, only storms with (P – Ia) ≥ 1.00 inch were used, and for statistical considerations, only watersheds with at least 20 such events in the record were used. This reduced the analysis to 86 watersheds, 2163 station-years, and 5501 events. The data were all taken from USDA-ARS electronic sources by breakpoint analysis and reduction programs at the University of Arizona.

A cumulative frequency plot of median Ia/S for the 86 watersheds described above is given in Figure 4. The data and analysis results for the 86 watersheds are summarized in Table 2. It should be noted that the median values found for λ were distinctly less than the book value of 0.20. In fact, of the 86 watersheds, only 5 showed values greater than 0.20. From this experience, a value of about 0.05 seems more appropriate.

In Table 2, which covers 86 watersheds, N_{yr} is the number of years in the individual watershed record, N is the number of rainfall runoff events, and N_1 is the number with P – Ia ≥ 1.00 inch. CN is the curve number found (in other studies not covered here) by the asymptotic method (Hawkins 1993) using ordered data pairs and Ia = 0.2S.

General Model Fitting

Requiring only a sufficient array of event rainfall and direct runoff depths, this method leads to least squares values of S and λ. Data sets from four sources were used:

- Hewlett: This information was supplied to the senior author in 1984 by Dr. John Hewlett of the University of Georgia, but was drawn to a large extent from U.S. Forest Service sources. The data were used as the basis for a prior paper on small watershed hydrology (Hewlett et al. 1984). Seventeen watersheds were used, mostly forested, located in the eastern United States.

Table 1. Properties of Ia and S from NEH4 Figure 10.2 (N = 112).

Variable	Minimum	Mean	Median	Std Dev	Skewness	Maximum
Ia (in)	0.01	0.4770	0.05	0.4381	1.2819	1.90
S (in)	0.19	2.6253	2.03	3.1711	3.1595	20.00
Ia/S	0.1025	0.2740	0.2378	0.2788	3.3346	2.20

Table 2. Event analysis data summary and median Ia/S results.

	DA	CN	N_{yr}	N	N_1	λ
	Ac	–	#	#	#	–
Minimum	0.65	58.24	5	100	20	0.0005
Mean	813.62	80.51	24.5	561.9	64.0	0.0607
Median	18.90	80.51	23	387	53	0.0380
Maximum	27469	91.06	53	2677	244	0.2907
Totals			2163	48323	5501	

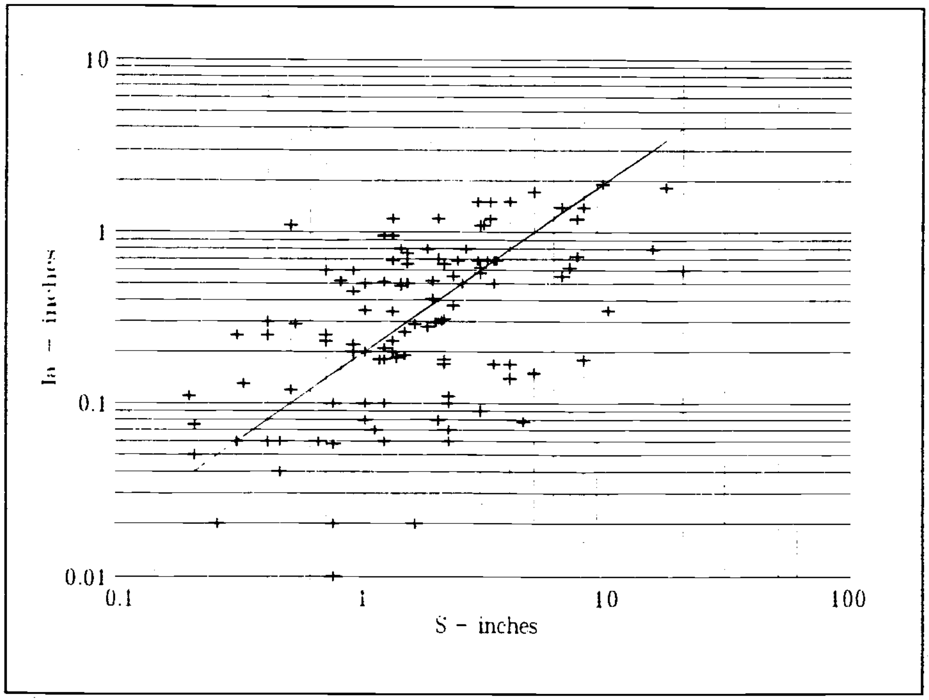

Figure 1. Relationship of Ia to S from NEH4, Figure 10-2. The shown line is Ia = 0.2S.

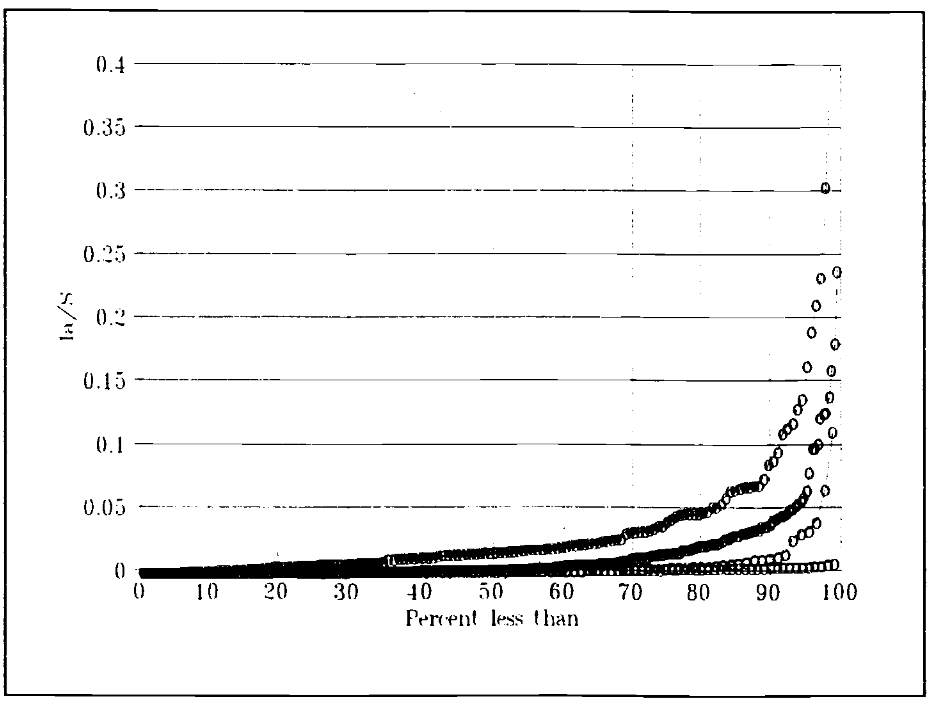

Figure 2. Variety of Ia/S = λ determined by event analysis for four selected watersheds. The four successive plots are, from uppermost to lowest: Watkinsville, GA, W-1, 19.2 ac; Coshocton, OH, 106, 1.56 ac; Riesel TX, W-1, 174 ac; and Treynor IA W-1, 74.5 ac.

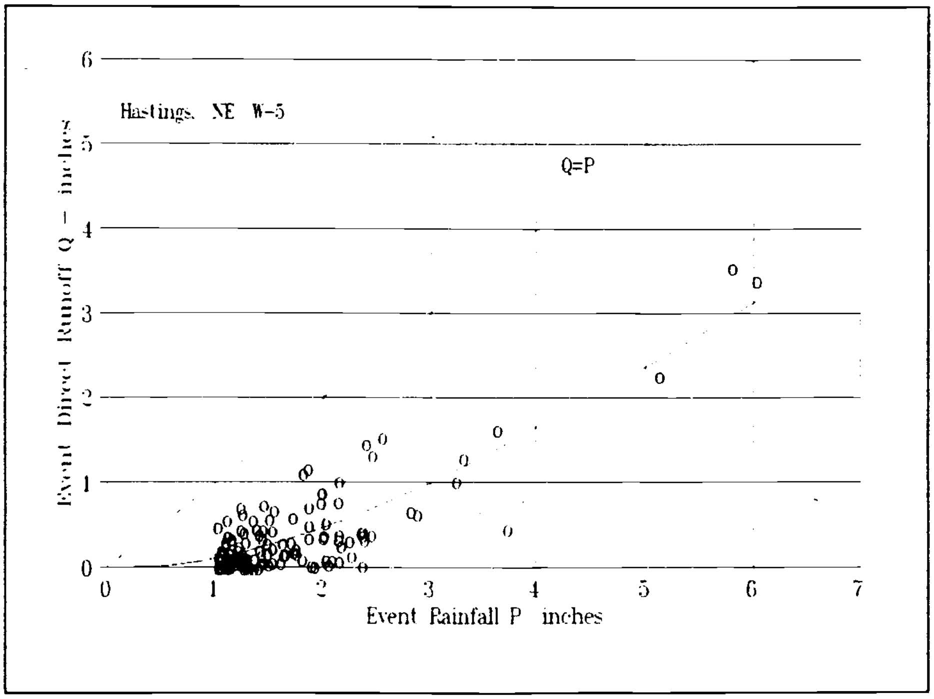

Figure 3. General model fitting to equation 6 for natural data from Hastings, Nebraska watershed W-5 for all P ≥ 1 inch. The drainage area is 411 acres. The fitted equation has r^2 = 72.98%, Se = 0.2808 inch, N = 134, and the fitted values of S = 3.7501 inch and λ = 0.0527. The period of record is from 1939 to 1967.

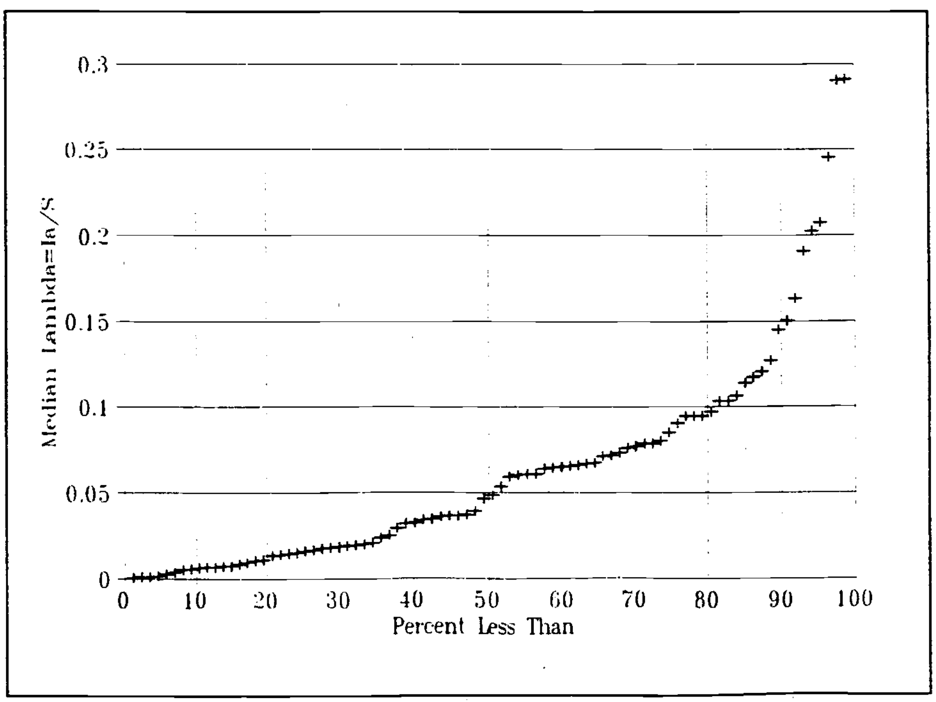

Figure 4. Distribution of median Ia/S = λ for 86 agricultural watersheds as determined by event analysis for events with P – Ia > 1.00 inch. Note the median value of about 0.04.

• Jornada: This data set is from 21 plots on the Jornada Experimental Range, north of Las Cruces, New Mexico. It covers more than 350 runoff events over about 20 years with $P > 1.00$. It was supplied by Dr. T. J. Ward of the University of New Mexico. It was used as the basis for CN determination and effects of cover in an earlier paper (Hawkins and Ward 1998).

• Tucson Urban: This set covers data from seven urban watersheds in the Tucson area in the early 1980s. It has 133 events in 4 years with $P > 1$ inch. It was supplied by the U.S. Geological Survey, Tucson, via the Pima County Flood Control District (Hawkins 1998b).

• ARS: These were 39 agricultural research watersheds in mainly the eastern and midwestern United States. The data were supplied by the USDA, ARS, via the Hydrologic Data Center in Beltsville MD. Almost all were also used in the event analysis determinations of λ.

It should be noted that the 84 watersheds came from four sources, and they cover a gamut of land conditions and types: forests, rain-fed agriculture and rangelands, desert plots, and even urban watersheds. Only storms of $P \geq 1.00$ inch were used, and the general model (i.e. equation 4) was fit to the P:Q data for λ and S by an iterative two-way least squares procedure. A cumulative frequency plot of the results is shown in Figure 5. The results are summarized in Table 3.

In Table 3 the following symbols apply: N_{ws} is the number of watersheds in the data subset, N is average number of P:Q points, λ is the mean Ia/S, λ_{50} is the median Ia/S, S is the average storage index (inches) in the CN runoff equations found from the least squares fitting, and R^2 is in percent. Also in Table 2 and in Figure 5, Natural Data refers to the P and Q matched and analyzed as they occurred in the natural record. Ordered Data is P and Q matched by rank order, thus matching the return periods of the P and Q. This latter tactic matches the usual design application of the method, and gives more consistent correspondence to external checks.

It should be noted that the natural data give mean results in close agreement to the event analysis—that is, values of λ in the vicinity of 0.05. The median values are uniformly 0. For the ordered data the median values also show results in the same general range. Only with mean values and ordered data is the handbook value of $\lambda = 0.20$ approached.

Applications

Using the above, the effects in routine calculations of direct runoff of using $\lambda = 0.2$ in the place of a truer value can be examined. Some generality and abbreviation can be achieved by using the runoff equation in its dimensionless form. Using equation 6 and dividing by the storage index S gives

$$Q_* = (P_* - \lambda)^2/(P_* + (1-\lambda)) \quad [7]$$
$$P_* \geq \lambda, \; Q_* = 0 \text{ otherwise}$$

where $P_* = P/S$ and $Q_* = Q/S$. Comparisons in Q_* for different values of λ and a range of reasonable P_* values are shown in Figure 6, giving percent deviations in runoff depth from the $\lambda = 0.2$ standard for alternative values of λ.

From Figure 6 it is clear that the greater deviations are at the smaller rainfalls, where smallness is defined relative to S. For larger P/S values the errors converge to 0.

For example, consider the 2 yr, 5 yr, 10 yr, 50 yr, and 100 yr return period for 3 hour design storms in the Tucson area (Zeller 1981), and CNs of 90, 80, 70, 60, and 50. The percent errors in using $\lambda = 0.2$ as compared to using the $\lambda = 0.05$ suggested here are shown in Table 4.

Table 3. General model fitting for λ and S.

			Natural Data				Ordered Data			
Data Set	N_{ws}	N	λ	λ_{50}	S	R^2	λ	λ_{50}	S	R^2
Hewlett	17	235	0.0584	0.0000	20.25	66.73	0.1143	0.0215	15.82	97.71
Jornada	21	17	0.0122	0.0000	14.22	4.91	0.1699	0.0037	5.61	85.52
Tuc/Urb	7	19	0.0000	0.0000	10.77	27.55	0.0545	0.0000	5.58	76.78
ARS/Ag	39	93	0.0755	0.0000	10.42	39.85	0.1572	0.0966	7.99	92.91
All	84	97	0.0499	0.0000	13.39	35.83	0.1432	0.0342	8.78	90.01

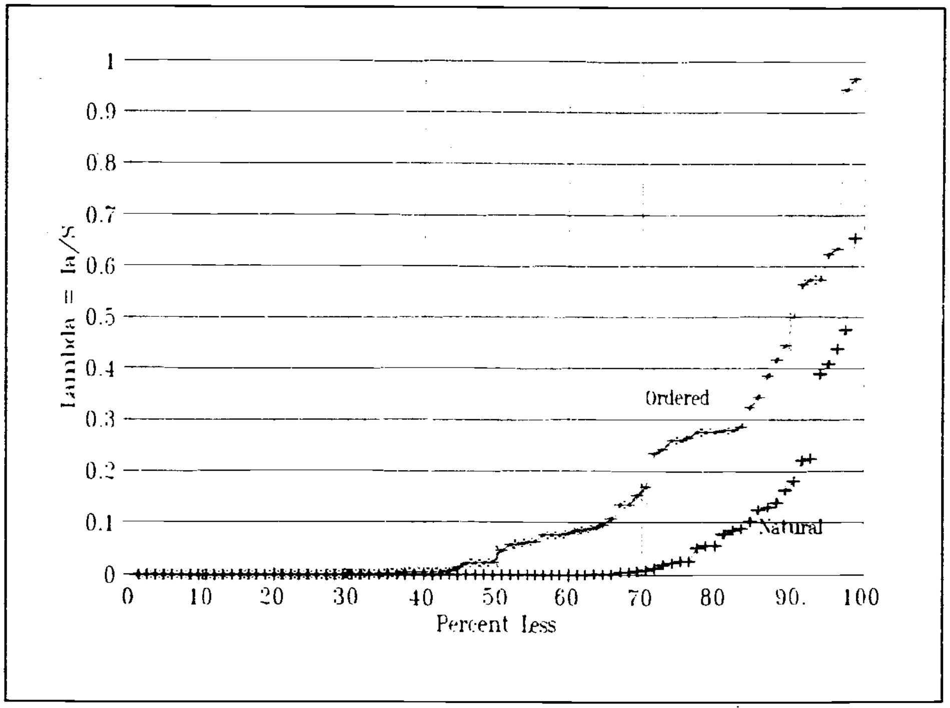

Figure 5. Distribution of least squares determined values of λ = Ia/S for 84 agricultural watersheds, using both ordered and natural data pairs (for events of P > 1 inch).

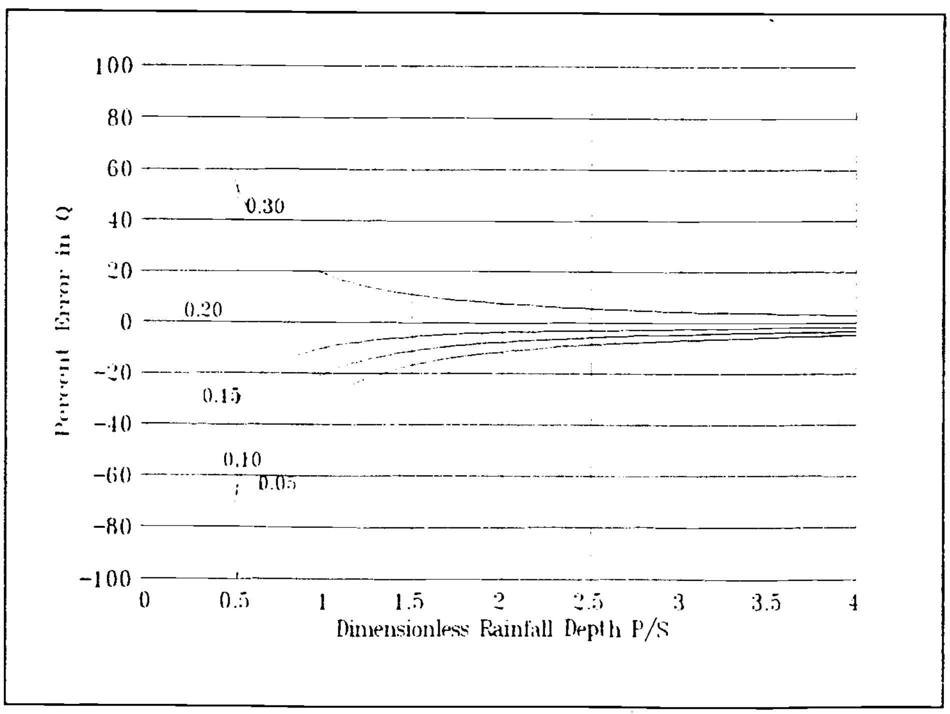

Figure 6. Percent error in the calculation of Q from using λ = 0.20 as compared to alternative values. The y-axis, for example, is 100*(Q(λ = 0.2) – Q(λ = 0.1))/Q(λ = 0.1), and so forth.

Table 4. Percent errors in calculated direct runoff with $\lambda = 0.20$ compared to calculation with $\lambda = 0.05$.

RP yr	P in	Curve Number 50	60	70	80	90
2	1.3	–	–	431	93.7	22.1
5	1.8	–	766	151	51.1	15.1
10	2.3	*	253	111	32.6	10.9
50	3.2	346	111	87.7	21.0	7.1
100	3.6	232	86.8	39.6	17.8	6.1

Notes: All errors above are negative in sign.
– indicates Q = 0 for $\lambda = 0.20$ case.
* indicates that entry is greater than 1000.

Summary and Discussion

Using two different methods and five different data sets, the value of $\lambda = Ia/S$ is consistently found to be less than the book value of 0.20 usually used. Values in the vicinity of 0.05 are more in keeping with this data analysis. The differences implied with current practice are more profound in situations of smaller (i.e., low P/S) rainfalls and lower curve numbers. Relative to using $\lambda = 0.05$, the current assumption of $\lambda = 0.2$ calculates lower runoffs. Differences in equation behavior due to variations in λ become much smaller at higher rainfalls and higher CNs.

These effects should be investigated further. For example, the widely used handbook table CNs are founded on the assumption of Ia = 0.2S. Also, the performance of the many continuous models using soil moisture-driven CNs as a function of the assumed λ is not known, and may be important. Finally, the applied hydrology community should be well aware of these findings when considering the performance and veracity of design and planning models.

Acknowledgments

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