FRACTAL MODIFICATION OF TREE-RING CHRONOLOGIES FOR DISCHARGE RECONSTRUCTIONS

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Dendrochronological streamflow reconstructions have been performed since the mid 1930s. Early 1900s streamflow studies (Hardman and Reil 1936; Hawley 1937; Schulman 1945, 1951) generally compared tree-ring records with streamflow, and estimated wet and dry periods for pre-gauged streamflow. To facilitate comparison between precipitation and streamflow, and subsequently between the tree-ring record and streamflow, the series were commonly smoothed using a Hahn filter, a 3 yr moving average with double weight given to the central value. The central questions remain, though, of how much is enough, or how much is too much smoothing to facilitate improved comparisons?

Transformations of raw data (for example, logarithmic or quadratic) can be performed on hydrological or tree-ring data to force a more normal distribution of the data to improve statistical inference (Loaiciga et al. 1993; Cleaveland 2000). Cleaveland and Stahle (1989) examined inhomogeneities in a hydrologic record and found that inhomogeneities increase the error component in streamflow reconstructions. One method to overcome the inhomogeneities uses the ratio of the slopes of the discharge record lines to modify the hydrologic record to improve comparability of the inhomogeneous subsections of the record (Loaiciga et al. 1993). The objective of this study was to use the fractal parameters of the tree-ring and hydrologic records to improve streamflow reconstructions by modifying the tree-ring series with the Hurst values of the tree-ring and hydrologic series.

METHODS

Six pinyon sites were sampled in the Paria River basin (Figure 1). A minimum of 10 trees were sampled within each site, and samples were prepared and mounted according to procedures described by Stokes and Smiley (1996). Cores were cross dated, and ring widths were measured to within ± 0.01 mm. To maximize sample depth across all of the sites, the chronologies were truncated from 1700 to 1998. A standard chronology was created by removing the differential growth trend among trees.

The roughness-length method was used to determine the fractal dimension (Df) and the Hurst exponent (H) of the tree-ring and hydrologic series (Table 1; Malinverno 1990; Kulatilake and Um 1999). A modification of the Barefoot et al. (1974) exponential weighted smoothing technique was used to smooth the tree-ring record, where the ratios of the Hurst exponents are a scaling factor between the tree-ring and hydrologic series. For two-dimensional space, fractal dimensions will range from 0 to 2, with values approaching 2 indicating a greater space-filling capacity, or greater randomness, of that feature (Mandelbrot 1983).

Streamflow discharge records were obtained from USGS gauging station 09382000 located at Lees Ferry, Arizona, for the period from 1924 to 1998. The total streamflow discharge for a year is based on the water year, October 1 through September 30. The water year was partitioned into three subperiods: (1) October 1 through March 31 (Winter 1), (2) October 1 through May 31 (Winter 2), and (3) November 10 to April 17 (Winter 3). The annual and the Winter 2 partitions are the subject of this study.

Multiple linear regression was used to estimate past streamflow. The coefficients of determination were adjusted to account for the loss of degrees of freedom due to the addition of predictors (Weisberg 1985). The validity of each model was determined by examining the estimated model coefficients, the residuals from modeling, the root mean square error (RMSE) of calibration and verification, and the reduction of error (RE) statistic of calibration and verification. Each model was verified using the PRESS statistic (Weisberg 1985).

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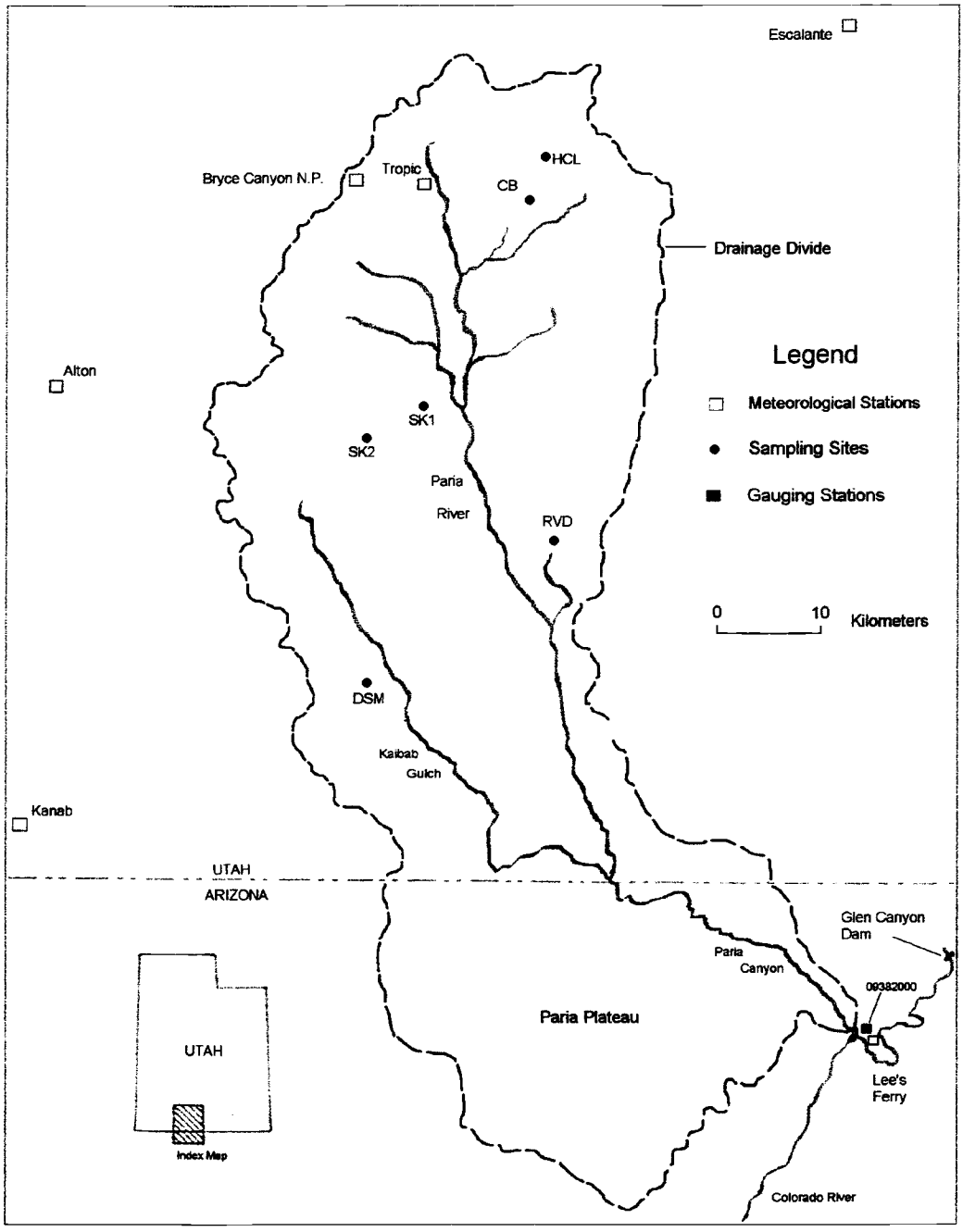

Figure 1. Map of the Paria River basin in southern Utah showing locations of sampling sites, USGS gauging stations, and meteorological stations (modified from Graf et al. 1991). Key to sites: CB = Coal Bench, HCL = Lower Henderson Canyon, DSM = Deer Springs Mesa, RVD = Round Valley Draw, SK1 = Skutumpah Road site 1, and SK2 = Skutumpah Road site 2.

Table 1. Fractal dimensions (Df) and Hurst exponents (H) of the tree-ring hydrologic series and the modified tree-ring series.

	Calibration Period (1924–1998)		Modified Tree-Ring Series			
			Annual		Winter 2	
Site	Df	H	Df	H	Df	H
CB	1.855	0.115	0.196	1.804	0.226	1.774
HCL	1.939	0.061	0.210	1.790	0.222	1.778
DSM	1.739	0.261	0.379	1.621	0.423	1.577
RVD	1.909	0.091	0.220	1.780	0.236	1.764
SK1	1.906	0.094	0.329	1.671	0.322	1.678
SK2	1.842	0.158	0.238	1.762	0.204	1.796
Annual discharge	1.802	0.198				
Oct 1–May 31 discharge	1.819	0.181				

RESULTS

The highest adjusted coefficients of determination (R_a^2) show that the Winter 2 partition provides the highest R_a^2 values, with the paired sites CB/HCL providing the highest discharge reconstruction (R_a^2 = 0.59; Table 2). The differences in correlation coefficients are statistically significant at the 95% confidence level (α = 0.05).

After the tree-ring data were adjusted using the Hurst ratios, the Hurst exponent (H) and fractal dimension (Df) of each tree-ring series were recalculated using the roughness-length technique (Table 1). Half of the sites display Hurst values that were closer to the desired Hurst value (0.198) than before the smoothing process, and the other half display more distant values. For the annual calculation, the smoothing process resulted in a close approximation to the annual Hurst exponent (0.196 vs. 0.198) for site CB. Sites HCL, RVD, and SK2 also showed improvements toward the desired Hurst exponent of 0.198. Sites DSM and SK1 showed an exaggerated adjustment beyond the desired Hurst exponent of 0.198.

The Winter 2 calculations show a similar trend in that all sites surpass the desired Hurst exponent of 0.182 as a result of smoothing. Sites CB, HCL, RVD, and SK2 most closely approximate the desired result with Hurst exponents of 0.226, 0.222, 0.236, and 0.204, respectively. The Hurst exponents for sites DSM and SK1 are 0.322 and 0.423, respectively.

The fractal modification of the tree-ring series improved R_a^2 in most cases for the annual series, and decreased R_a^2 values for sites HCL, DSM-RVD, and SK1-SK2. For the Winter 2 reconstructions, only two sites, CB and DSM, show improvement, sites HCL and RVD show a decrease, and sites SK1 and SK2 display no change. The paired sites CB-HCL and DSM-RVD show a decrease, and sites SK1-SK2 show no change (Table 2). The highest R_a^2 values for the annual reconstructions are from sites CB-HCL (0.29). The highest R_a^2 values for Winter 2 reconstructions are also from CB-HCL (0.57).

CONCLUSIONS

This study is the first application of fractal analysis in a dendrochronogical, process-based approach to streamflow reconstruction. The smoothing technique used is based on properties inherent within each series, and provides a more practical

Table 2. Discharge reconstruction summary (R_a^2) for the annual (October 1–September 30) and Winter 2 (October 1–May 31) partition.

	Fractal Series		Original Series	
Site	Annual	Oct–May	Annual	Oct–May
CB	0.23	0.47	0.22	0.46
HCL	0.24	0.49	0.24	0.54
DSM	0.15	0.46	0.11	0.43
RVD	0.16	0.47	0.18	0.48
SK1	0.22	0.43	0.23	0.43
SK2	0.28	0.45	0.25	0.45
CB-HCL	0.29	0.57	0.27	0.59
DSM-RVD	0.16	0.46	0.18	0.52
SK1-SK2	0.24	0.45	0.25	0.45

approach than arbitrary, external smoothing operations, such as 3 or 5 yr running averages. In this study, fractally derived smoothing failed to improve the amount of variance explained in the streamflow reconstructions. In spite of this shortcoming, the fractal analyses provide some interesting concepts:

1. By aligning the fractal parameters of two series with each other, a higher coefficient of determination would be expected. As this was not the case, the fractal-based transformations bring to light inconsistencies within each series not previously addressed, for example, by using ARMA modeling.

2. Fractal examination of tree-ring indices provides insight into the long-term persistence of a time series, and consequently into the stationarity of a series. Fractal dimensions can provide an analytical tool complementary to ARMA modeling.

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