A NEW SENSITIVITY ANALYSIS FRAMEWORK FOR MODEL EVALUATION AND IMPROVEMENT USING A CASE STUDY OF THE RANGELAND HYDROLOGY MODEL

H. Wei, M. A. Nearing, and J. J. Stone[1]

Many numerical models involve utilization of a large number of input parameters, which often results in complex interactions between inputs and algorithms within the model. In all models it is generally desirable to understand the relationships between the sensitivity of outputs on input parameter values and how these relationships affect model predictions. This is important not only for gaining a better understanding of the model behavior, but also for detecting model deficiencies and unreasonable responses induced by the high level of model complexity and the high number of model input factors.

Sensitivity analysis (SA) is a method widely used to ascertain simulation model response to changes in its input factors. In practice, SA is not only applied to examine the importance of input parameters but is also considered as an important element of the model development process. SA helps to elucidate the impact of different model structures, to prepare for model parameterization, and to direct research priorities by focusing on the parameters that contribute greatest to uncertainty in the model response (Saltelli and Campolongo 2000; Breshears et al. 1992).

The objective of this paper is to provide a new local sensitivity analysis framework that can be used to effectively show the interdependencies of sensitivity to multiple model inputs, and which can be used in the model development process to help identify undesirable or illogical model responses. We used an algorithm similar to the Morris framework, but a different local sensitivity index to build a localized sensitivity matrix for a model. The sensitivity matrix was further analyzed to make SA more effective as an aid in the model development process. We illustrate the use of this framework to (1) examine the distribution of the sensitivity to each parameter, and thus to list and classify the importance of input parameters; (2) decompose the dependency of model response on input parameter values and understand the parameter interactions, using correlations and regressions; and (3) reveal the nonlinear relationships, thresholds, and potential weaknesses or problems of model structure.

We applied our new method to the Rangeland Hydrology and Erosion Model (RHEM), which was developed from the Water Erosion Prediction Project model (Flanagan and Nearing 1995; Nearing et al. 1989; Laflen et al. 1997). Several local SA tests have been conducted on WEPP (Baffaut et al. 1997; Tiscareno-Lopez et al. 1994). These previous studies were based on site-specific data and parameter values, and the results could not be extrapolated to other locations. This paper, taking the erosion predictions in RHEM as an example, not only highlights the local sensitivities but also describes how to investigate the interactions between RHEM parameters and how to identify unusual RHEM behavior. Results from this study will be helpful in improving the understanding of the model behavior and parameter interactions in RHEM, and also in improving the integrity of the model predictions.

METHODOLOGY

Sensitivity Equation

The local sensitivity index in this paper is quantified by equation 1:

$$Si_{(x^0)} = \frac{\left(Y\left(x_1^0,\ldots,x_1^0+\partial x_i,\ldots,x_1^0\right)-Y\left(x^0\right)\right)/Y\left(x^0\right)}{\partial x_i / x_1^0} \quad (1)$$

where $Si_{(x^0)}$ is the sensitivity of output Y to the input factor x_i at the point of $x^0\left(x_1^0,\ldots,x_i^0,\ldots,x_1^0\right)$. $Si_{(x^0)}$ is a non-dimensional, localized index that represents the normalized response of output to the increase of the input value x_i. The absolute

[1]USDA-ARS Southwest Watershed Research Center, Tucson

magnitude of $Si_{(x^o)}$ indicates the degree of sensitivity of Y to x_i at the point of x^0. A positive (negative) S_i indicates the positive (negative) relationship between Y and x_i; that is, the increase of x_i will cause the increase (decrease) in Y. The percentage of $\partial x_i / x_i^0$ is expected to be small enough to ensure the S_i to be representative at the point x^0. One of the merits of equation 1 is that if $\partial x_i / x_i^0$ remains a constant percentage, the value of $Si_{(x^o)}$ can be used to compare the sensitivity to an input variable at its different magnitudes. It can also be used to compare the sensitivity of output to different individual input factors, for example the sensitivity of Y to x_i and x_j at the point of x^0.

Procedure

Figure 1 is a flow chart of our methodology. It starts with selecting the input and output parameters to be analyzed. The ranges of each input parameter should then be given to build the parameter space of interest, which could encompass the full realistic range of all input parameters. Then points, x^0, were randomly selected from the parameter space, and sensitivity indices were calculated for each parameter at the selected point. The Latin hypercube sampling (LH) method (McKay et al. 1979) was used for random sampling of points, x^0. At each point selected by LH, the model was executed $(1 + I)$ times. The first run was to calculate and save the output value at the point x^0 with no perturbation, and the next I times were to calculate new output values after increasing each parameter, one at a time, by a predetermined percentage $(\partial x_i / x_i^0)$. The local sensitivity index for each input parameter at the point was then calculated using equation 2 based on the $(1+ I)$ values of the output at this point. The sampling procedure and local sensitivity index calculation were repeated 10,000 times, after which the parameter space was well covered and points were well distributed. At the conclusion of the runs a sensitivity index matrix had been constructed from the results, containing the values of each parameter at each selected point and the local sensitivity of output to each input parameter at each point. The absolute values of the sensitivities were also generated for further analysis. A FORTRAN program was written to connect the model, LH sampling, the sensitivity loops, and the building of the sensitivity matrix.

Model RHEM and Input Parameter Space

We selected 14 input parameters used in the hydrology and erosion components of RHEM for the sensitivity analysis. *Soil loss* (kg/m^2), the amount of soil erosion from the hillslope, was selected as the targeted output variable. The parameter space of interest for this study is the entire applicable space of the RHEM model. Thus the full range of reasonable parameter values for each input variable was used to build a 14-dimensional parameter space. The sources of the ranges came from the recommendations in the WEPP model manual (Flanagan and Nearing 1995) and WEPP database (unpublished data, personal communication; see Table 1).

Table 1 gives the name, range, and descriptions of each input parameter studied. The range of each input parameter was required for the sensitivity study. The increment of each input parameter and the total number of samples were also required for the SA program. The increment was arbitrarily set at 5 percent in this study. A small value of the increment is preferred to make the sensitivity index representative of the exact localized effect, but it must be large enough to avoid rounding errors in the calculations. The total number of points should be determined by considering not only the number of input parameters but also the complexity of the model. We used 10,000 points in this study as a representative sampling of the full input parameter space.

RESULTS

Approximately 50 percent of the total 10,000 events did not generate rainfall excess, which means that runoff and erosion from these events was zero, and approximately 20 percent of the total events yielded runoff of less than 5 mm, which was considered to be too small to be of interest in terms of output. As a result, only the 3180 of the 10,000 simulated events that generated runoff greater than 5 mm were saved in the sensitivity matrix for further analysis.

Localized Sensitivity

Absolute local sensitivity can be used to compare the relative importance of the input factors. Each row of the sensitivity matrix generates a ranking of parameter importance based on the rank of the absolute sensitivity values at each point. However, the importance of a factor varied from point to point. For example, at point 23, the ranking of sensitive parameters was *rain, dur, ke, ns, xip, ki, sln, psd, slp, fe, fr, rsp, kr*, τ_c. At point 30, the ranking was *psd, rain, dur, xip, ke, ki, ns, slp, fe, sln, fr, rsp, kr*, τ_c. Figure 2 gives lists of the four most important factors based on the count of the

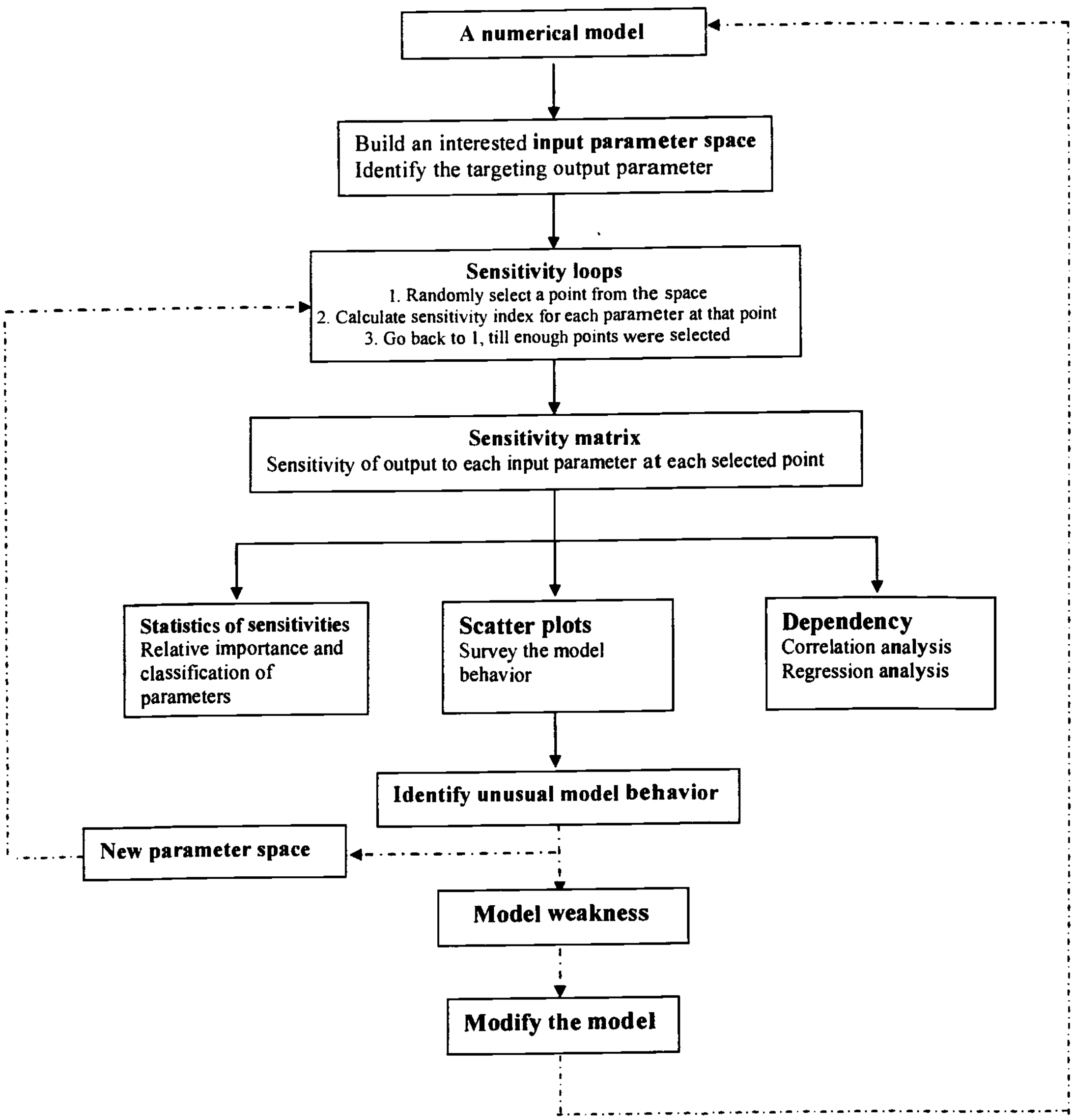

Figure 1. A flow chart of the sensitivity analysis conducted in this paper.

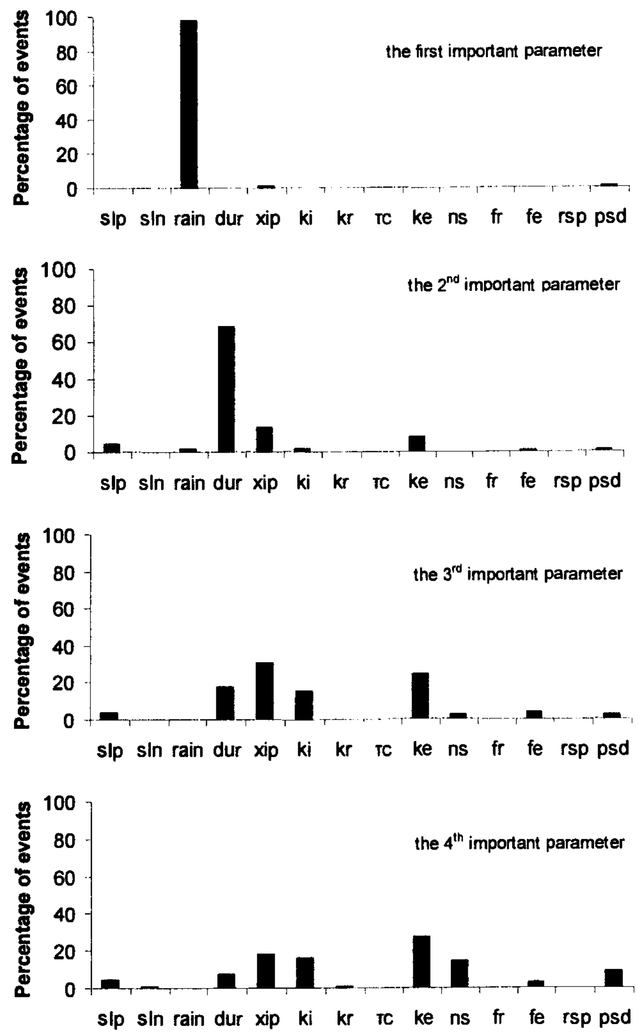

Figure 2. The distribution of the four top-ranked input factors based on the count of events. The first graph shows that *rain* was the most sensitive factor for 97.99% of events, with *psd*, *dur*, *xip*, *slp*, τ_C, and *ke* the most sensitive factors for the remaining 2.01% of the events.

Table 1. Parameters and parameter ranges used in this study.

Parameter	Upper Bound	Lower Bound	Unit	Description
slp	3	30	%	Slope
sln	10	100	m	Slope length
rain	20	120	mm	Rainfall volume
dur	0.5	2	hr	Rainfall duration
xip	1	20	–	Rainfall intensity variable
ki	1000	2000000	$kg*s/m^4$	Interrill erodibility
kr	0.00001	0.004	s/m	Rill erodibility
τ_c	0.0001	7	N/m^2	Critical shear stress
ke	0.8	40	mm/hr	Effective hydraulic conductivity
ns	0.00025	0.7	m	Matric potential
fr	4.07	200	–	Friction factor for runoff
fe	1.11	100	–	Friction factor for erosion
rsp	0.8	5	m	Inter space
psd	7	–1	–	Particle size distribution

events. For 98.0 percent of the total 3180 events, the total rainfall depth, *rain*, was the most important factor, but *psd, dur, xip, slp*, τ_c, and *ke* also showed up as the top-ranked factors, and these variables accounted for the remaining 2.0 percent of the events. Storm duration, *dur*, was the second most sensitive factor for 68.5 percent of the total events; *slp, rain, xip, ki, kr, ke, fe*, and *psd* accounted for the rest of 31.5 percent events. The third and the fourth ranked important factors were more widely distributed among the input variables (Figure 2). The results show that for a complex model in which the input parameters interact with each other, the sensitivity for input factors may vary greatly from point to point in the parameter space.

Figure 3 gives the distribution of each S_i based on absolute values. The parameters are listed by the ranking of their mean sensitivities, the "overall" effect: *rain, dur, xip, ke, ns, ki, psd, slp, sln, fe, fr*, τ_c, *kr*, and *rsp*.

Tiscareno-Lopez et al. (1994) conducted a sensitivity analysis on a similar soil erosion model, WEPP, on the USDA-ARS Walnut Gulch Experimental Watershed located near Tombstone, Arizona. The results from their study indicated that rainfall amount was the most sensitive factor on that watershed, followed by *ke*. From our results, *rain* was the first important factor for 98.0 percent of all the events, followed by either *slp, psd, ki*, τ_c, or *ke*, depending on the combination of input values. From Figures 2 and 3, one can see that RHEM is a complex model, whereby the localized sensitivities vary greatly from site to site.

Scatter Plots to Identify Characteristics of Model Behavior

In this section we generate the scatter plots of the sensitivity index S_i at each point over the values of the i^{th} parameter at this point to show how this type of scatter plot can be used to help modelers survey the model response and identify nonlinear relationships, thresholds, and potential model problems.

Figure 4 is a plot of S_{psd} over the corresponding *psd* values. The parameter *psd* is important because it is the only factor that accounts for particle size distribution in this study. Figure 4 is a surprising plot because it reveals an unexpected and undesirable model response around the *psd* value of –3.2. The same sensitivity procedures focused on a narrow region of *psd* (–3.0, –3.4) were processed again and this "closer" look confirmed the inconsistent model behavior. For example, for sediment with a *psd* of –3.31, a 5 percent increase in *psd* could induce 70 percent increase in *soil loss*, which was much more sensitive than those simulations with *psd* outside this region. This is not a reasonable model response for this variable.

Dependencies of Sensitivity Indices on Input Parameter Values

We use regression and correlation analysis in this paper to understand the dependence of the sensitivity for a factor on the input parameter values (Table 2). The coefficient of determination, R^2, of the regression describes the percentage of

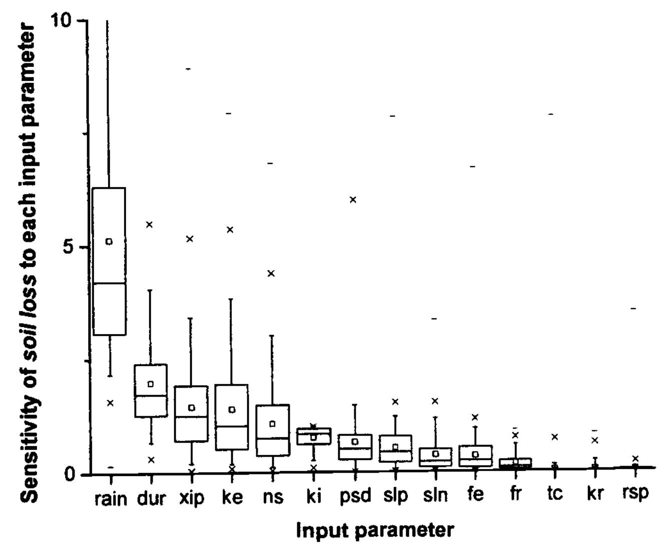

Figure 3. Statistics of absolute sensitivities for each input parameter. The sensitivity on the Y axis only shows values less than 10. The parameters are listed on the X axis by ranking of their mean sensitivities, represented by the smallest boxes. Each S_i is represented as a separate box chart. The box is determined by the 25th and 75th percentiles. The whiskers are determined by the 5th and 95th percentiles. The dash marks are determined by the minimum and maximum values. Descriptions of all input parameters refer to Table 1.

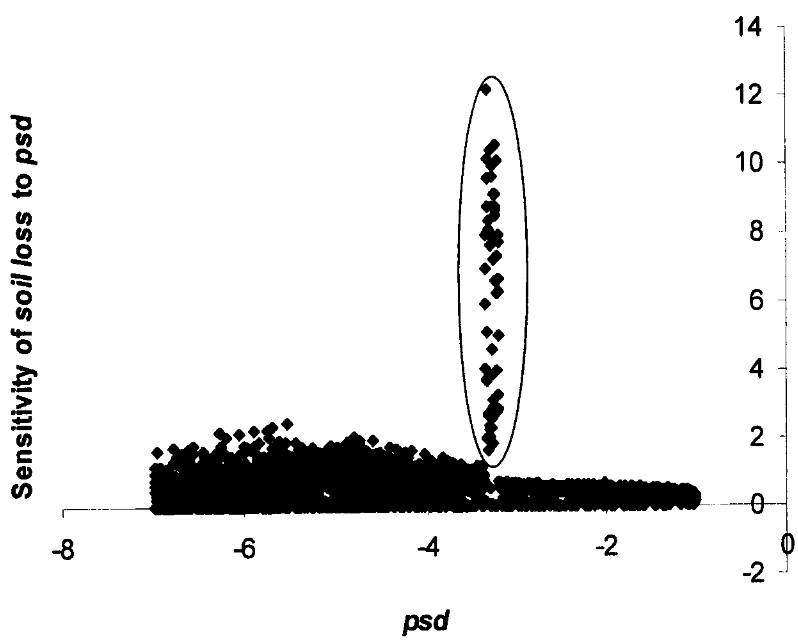

Figure 4. Scatter plot of sensitivity of soil loss to *psd* (S_{psd}) vs. *psd* values. It reveals the incorrect model response when *psd* is close to –3.2.

Table 2. Dependencies of sensitivity indices on the input parameter values.

S_{fr}	S_{sln}	S_{ke}	S_{ki}	S_{rain}	S_{ns}	S_{xip}	S_{fe}	S_{slp}	S_{dur}	S_{kr}	S_{psd}	S_{tc}	S_{rsp}
R^2 of the regression of S_i on the values of the 14 input factors													
0.518	0.511	0.475	0.444	0.432	0.399	0.389	0.376	0.374	0.223	0.220	0.116	0.068	0.024
Correlations coefficient of S_i to each input factors													
kr -0.370	fr -0.346	ke -0.438	fe 0.251	dur -0.177	ns -0.362	dur 0.154	fr -0.214	sln 0.278	xip 0.234	fe -0.087	ki -0.409	kr -0.010	kr 0.065
ki 0.325	sln -0.307	rain 0.335	ki 0.206	slp -0.134	rain 0.249	fr -0.133	sln -0.187	fr 0.259	fe -0.100	ki -0.083	fe -0.370	ki 0.096	rsp -0.044
fe 0.265	slp 0.247	xip 0.324	kr -0.193	fr 0.134	ke -0.193	sln -0.123	slp 0.148	ki -0.219	ki -0.093	slp 0.079	slp 0.352	rsp 0.060	sln 0.035
psd 0.224	ki -0.109	dur -0.282	slp -0.171	sln 0.125	xip 0.180	xip 0.108	xip -0.063	fe -0.185	sln 0.06	τ_c -0.049	psd -0.087	fe-0.059	fr 0.031
sln -0.217	xip -0.102	ns -0.241	psd 0.165	ns -0.103	dur -0.135	ke 0.100	fe -0.056	psd-0.153	slp 0.06	kr 0.046	sln -0.072	slp -0.051	τ_c -0.027
slp -0.172	fe -0.094	fr -0.198	sln -0.16	ke -0.099	fr -0.125	ns 0.089	ki -0.045	kr 0.145	fr 0.05	rsp 0.03	fr -0.071	dur -0.045	rain -0.026
τ_c 0.166	rain 0.086	sln -0.190	τ_c 0.158	rain 0.096	sln -0.110	rain -0.072	rsp 0.038	rain -0.120	kr 0.039	sln 0.015	rain 0.036	xip 0.029	ke 0.024
xip 0.162	kr 0.068	slp 0.084	rain 0.101	xip 0.094	slp 0.057	slp 0.050	rain 0.036	τ_c -0.089	rain -0.036	dur -0.015	rsp -0.034	psd 0.025	ki -0.022
rain 0.145	ke -0.068	τ_c 0.025	rsp -0.098	ki 0.088	rsp 0.021	ki 0.019	τ_c -0.030	ke 0.041	τ_c -0.033	psd 0.013	ns -0.031	rain 0.024	ns 0.021
dur -0.127	ns -0.066	psd 0.024	fr -0.095	fe 0.081	τ_c -0.015	fe 0.015	psd 0.024	xip -0.041	ns 0.032	ke -0.01	ke -0.027	ke -0.020	fe -0.015
fr -0.122	dur 0.043	rsp -0.018	xip 0.089	psd -0.020	kr 0.006	psd 0.012	ke -0.023	ns 0.035	rsp 0.029	ns -0.01	xip -0.006	τ_c - 0.016	psd -0.010
ke -0.060	rsp -0.039	kr -0.017	dur -0.061	τ_c -0.014	psd 0.006	τ_c 0.002	dur 0.014	dur 0.033	psd -0.025	rain -0.008	kr -0.003	ns -0.014	dur 0.005
ns -0.051	psd 0.034	fe 0.012	ke -0.041	kr 0.006	ki 0.003	kr -0.002	ns -0.014	rsp 0.019	ke 0.011	xip 0.004	dur 0.003	sln -0.012	slp -0.005
rsp -0.037	τ_c -0.021	ki -0.001	ns -0.040	rsp 0.003	fe -0.003	rsp 0.001	kr -0.002	slp 0.009	dur -0.010	fr -0.003	τ_c -0.002	fr 0.002	xip 0.004

the variance of the sensitivity index that can be explained by the magnitudes of the input parameters. The correlation coefficients of S_i and values of each input parameter reveal the dependence of S_i on each input factor. For example, it can be seen from Table 2 that approximately 50 percent of the variance of S_{fr}, S_{sln}, and S_{ke} could be explained by the magnitudes of the entire input parameter set. The correlation matrix in Table 2 helps to further decompose this dependency. As one can see, S_{fr} is dependent on the factor *kr* and *ki;* thus, the sensitivity of the friction factor of runoff, *fr*, is related to magnitude of the erosion coeffcients.

The coefficients in Table 2 reveal many insights on the relationships between the parameters. For example, S_{ns} and S_{ke} are dependent on the value of *rain*, *dur*, and *xip*. This relationship reflects the fact that the runoff generation in RHEM is controlled by both the rainfall regime (associated with rainfall parameters *rain*, *dur*, and *xip*) and the infiltration regime (associated with hydrologic factors *ke* and *ns*).

Table 2 also shows that there is negative correlation between S_{ke} and *ke*, which indicates that the response of *soil loss* to *ke* is dependent on the magnitude of *ke* itself. The negative correlation coefficient indicates that the higher the *ke*, the more sensitive the *ke*. This relationship makes sense because the high *ke* is often associated with the small amount of runoff and *soil loss*, and the sensitivity of *soil loss* to input factors increases as the *soil loss* value decreases.

CONCLUSION

A sensitivity analysis based on the concept of local sensitivity and Latin hypercube sampling was conducted using the soil erosion component in the model RHEM as a case study. The local sensitivity indices of *soil loss* to 14 input parameters of RHEM at 10,000 points from the full parameter space were obtained and used to build a sensitivity matrix. The sensitivity matrix was analyzed in several ways to draw useful insights on model response and interactions between model parameters: (1) the results highlighted the importance of local sensitivity, which varies from site to site for a complex model such as RHEM. (2) The method was also used to decompose the independency of model response on input parameter values. (3) The method effctively detected model errors.

The method of this paper can be used as an element of the iterative modeling process whereby model response can be surveyed and problems identified and corrected in order to construct a robust model.

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