ANNUAL WATER YIELD USING PRECIPITATION AND TEMPERATURE: GRUNSKY'S EQUATION RECONSIDERED

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The problem of water yield from ungauged watersheds faced by planners looking to enhance local supply on the San Francisco peninsula in the early 1900s was formulated by C.E.Grunsky in 19008. He made of the following paraphrased observation from the limited information at his disposal: 'The percent of annual rain that becomes runoff is equal to the inches of rainfall, up to 50 inches of rain." He was referring to flows (or yields) as depths/time, i.e., inches/yr. His assertion becomes algebra as

$$100Q/P=P \qquad P<50, \qquad [1]$$

and solving for Q,

$$Q = P^2/100 \qquad P\leq 50, \qquad [2a]$$

or,

$$Q = 0.01\,P^2 \qquad P\leq 50, \qquad [2b]$$

The "up to 50 inches" proviso avoided unnatural or outrageous results that Grunsky foresaw beyond that point. First, if P>50 in Equation 1, then an increment of additional rain calculates more than that increment of additional runoff, an intuitively unreasonable outcome and nuance (Fig. 1). In the extreme, note that if P=100, then Q=100, suggesting no losses, or that all rain becomes runoff. Thus, he added: "Above 50 inches, the runoff is the rain less 25 inches" or,

$$Q = P-25 \qquad P\geq 50, \qquad [3]$$

Equations [2] and [3] are the complete 2-part relationship for the conditions found in the central California maritime area near San Francisco. This marvel of simplicity, which we call Grunsky's equation here, might also be called "Gunsky's Rule" or "Grunsky's Rule-of-Thumb." The objective of this study is to revive Grunsky's equations, propose an extended and more generalized statement of Grunsky's equations, and determine Grunsky's α values for California watersheds (17 river basins). Found distinctive of Mediterranean climate and closely associated with average annual temperatures (T), Grunsky's α values are extended to establish a general underlying α(T) hydro-climate relation giving insights to reasonable estimates of annual yields for California watersheds. The water yield model is then applied and validated with watersheds data from southern of France and Portugal.

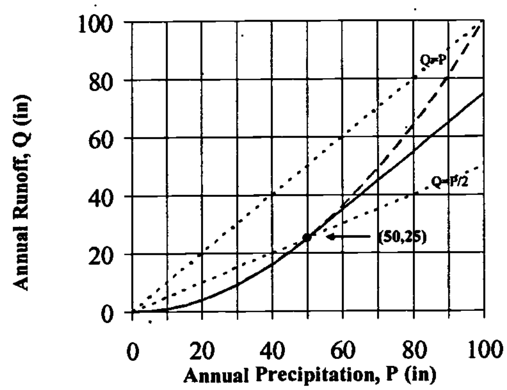

Figure 1. Grunsky's equation for $\alpha = 0.01 in^{-1}$

GENERALIZATION OF GRUNSKY'S EQUATION

Given that the above was established for a specific set of conditions, i.e., the lands and climate found locally in the San Francisco area, an extended statement – actually a hypothesis - of Gunsky's equation can be formulated. Equations 2 and 3 can be re-stated as

$$Q = \alpha P^2 \qquad P<P^* \qquad [4a]$$

$$Q = P - L^* \qquad P>P^* \qquad [4b]$$

where P* and L* play the role of the 50 in/yr (threshold rainfall) and 25 in/yr (constant loss above the 50 inch threshold) for the original conditions. The coefficient α fulfills the role of 1/100 or 0.01, with the dimensions of in^{-1}.

The values for P* and L* can be found at the P where the slope of the runoff function (Equation 4a) becomes 1:1 (Fig. 1). Differentiating and setting equal to 1, and solving for P gives

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$$dQ/dP = 2\alpha P = 1 \quad [5]$$

or,

$$P = P^* = 1/(2\alpha) \quad [6]$$

at P=P*, then

$$Q = Q^* = \alpha P^{*2} = \alpha[1/(2\alpha)]^2 = 1/(4\alpha) \quad [7]$$

and,

$$L^* = P^* - Q^* = 1/(2\alpha) - 1/(4\alpha) = 1/(4\alpha) \quad [8]$$

Thus, for the general case of "Grunsky's equations"

$$Q = \alpha P^2 \qquad P \leq P^*$$

$$Q = P - L^* \qquad P \geq P^*$$

where $P^* = 1/(2\alpha)$ and $L^* = 1/(4\alpha)$. Note that the watershed runoff behavior is defined by the coefficient α. A plot of the general function for various values of α is shown in figure 2. The quadratic part ($Q=\alpha P^2$) pertains for Q<P/2, and the linear part (Q=P-L*) applies for Q>P/2. This results because Q*/P*=1/2. From the original .Grunsky's case, α = 1/100 = 0.01 in^{-1}, P* = 50 in and L* = 25 in.

Table 1. Some estimated regional values of the coefficient α

Region	α (mm^{-1})	α (in^{-1})
Great Plains, Texas, Florida, Southwest Desert	<0.000197	<0.005
East Central states and West Coast	0.000197-0.000394	0.005-0.010
Great Lakes, New England, Northern Appalachians	0.000394-0.000787	0.010-0.020
Rocky and Sierra Nevada Mountains	>0.000787	>0.020

Source: Sellers (1969)

Values of α From Specific Rainfall-Runoff Data

Knowledge of local data and hydrologic behavior is the hydrologist's stock-in-trade. Thus, insights to reasonable - and unreasonable - estimates annual yields can be gotten from specific P:Q data. The process is simple: with known P and Q, presumed to valid and representative, equations [4a] and [4b] can be solved for α.

$$\alpha = Q/P^2 \qquad Q<P/2 \quad [9a]$$

$$\alpha = 1/[4(P-Q)] \qquad Q>P/2 \quad [9b]$$

where P and Q are mean values (in/yr). However, the two Grunsky equations represent one continuous relationship, so if fitting with multiple data points exists on both sides of P*, then both of the equations can be applied. Hopefully the same value of α would prevail above P* and below P*.

Values of α From Literature

Grunsky's original application was straightforward and local, and gave no values for α beyond the implied value of 0.01: Indeed, he did not generalize. In fact, in 1915 he published in the ASCE Transactions a second/revised approximation of a standard California "α", no, he didn't call it that, of about 0.008. This was given as a graph with no data points shown. However, Sellers (1969) gave estimated regional reference values of coefficient α from gauged watershed data under a different nomenclature, and are shown in Table 1.

This table is almost the sole available reference listing of α in textbook form. To this must be added "Central Coastal California, from Grunsky's founding works, with α=0.010. While these are in general alignment with data-derived values, Sellers does not give sources for the above. Most α determinations from precipitation-runoff data vary from ca 0.0025 to 0.0130, and vary distinctively with general geography and climate, but show a clustering tendency within regions. Another attempt to provide table values based on the evaluation of results, is given by Gifford et al (1975). Joel Justin, a contemporary of Grunsky, and also concerned with similar matters, studied streams in the northeastern U.S. and published in 1914 (Justin, 1914) his "k" values (our "α" here) for watersheds in New York, New Jersey, West Virginia, Connecticut, and southern Canada. Perhaps the most comprehensive effort has been the recent nation-wide work of Vogel, Wilson, and Daly (1999) data for constants and exponents for unit hydrologic budgets, with long-term data from 18 regions in the US, and which used not only P, but temperature (°F) and drainage area (A) for inputs as well in multiple regressions in logarithmic form. While their work used metric units (mm/yr, km^2, with output in m^3/sec), their regression results can be simplified, converted to English units and expressed in consistent hydrologic budget terms, making it comparable to Grunsky's equation form. When the above exponential formulations for Q are set equal to αP^2 an implied "α" is gotten by back-calculating (Table 2). Insofar as the above Vogel *et al* equations include not only the precipitation (not confined to P^2), but the temperature, drainage area as well, the "α" calculations shown above are for an arbitrary reference case of area, A=10m^2, temperature, T=60F, and rainfall, P=40in/yr. For example, for the California case, α calculates to 0.0087in^{-1}. It might be noted that even with these subjective choices that influence outcomes, this result is close to Grunsky's value of α=0.01, and even closer to his 1915 chart presentation of about 0.009.

Furthermore, the precipitation exponent of 1.999 is essentially identical to Grunsky's value of 2. The heavy role of temperature in fixing "α" should also be noted and will be treated subsequently. An analysis of annual runoff from watersheds in the western U.S. by Hawley and McCuen (1982) led to a series of exponential equations similar to those later developed by Vogel *et al.* (1999). A simple study (Hawkins, 1991) of annual precipitation and runoff for 11 ungaged basins in central Arizona gave values of $Q = 0.004019P^2$, $r^2 = 95.08\%$, SE = 0.42 in/yr, seen as fitting the Grunsky precedent.

Table 2. α values derived from Vogel et al. (1999) regional regression model of annual streamflow for the United States.

Region	α (mm^{-1})	α (in^{-1})
New England	0.00043	0.0109
Mid-Atlantic	0.00024	0.0061
South-Atlantic-Gulf	0.00021	0.0052
Great Lakes	0.00026	0.0065
Ohio Valley	0.00024	0.0060
Tennessee Valley	0.00041	0.0103
Upper Mississippi	0.00017	0.0042
Lower Mississippi	0.00021	0.0052
Souris-Red-Rainey	0.00019	0.0047
Missouri River	0.00025	0.0064
Arkansas-Red-White	0.00022	0.0057
Texas-Gulf	0.00050	0.0128
Rio Grande	0.00016	0.0041
Upper Colorado	0.00025	0.0063
Lower Colorado	0.00021	0.0052
Great Basin	0.00019	0.0049
Pacific Northwest	0.00029	0.0073
California	0.00034	0.0087

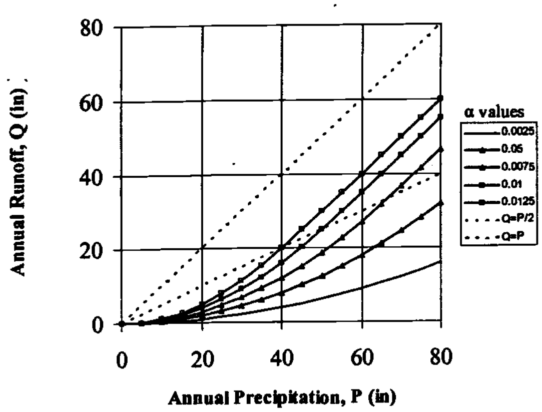

Figure 2. Plot of Grunsky's equation for various α.

GENERALIZED MEDITERRANEAN WATER YIELD MODEL

Mediterranean α values

In a study of geologic sedimentation and fault movement in California, Kolterman and Gorelick (1992) summarized the long-term runoff and climate of coastal streams, from the Smith River in the north to the San Luis Rey River in the south. The digital data was not included in their journal paper, but was scaled from plots provided, and is shown in Table 3. From this values of α were calculated from the P and Q for each the basins. While this gave a wide scatter of α, it was found to be very closely associated with the average annual temperatures via the equation:

$$\alpha\ (\text{in}^{-1}) = 0.01967 - 0.000966T(^{\circ}C) \qquad [10]$$

with $r^2 = 89.922\%$ and SE = 0.00094in^{-1}. The very good fit in equation 10 (Fig. 3) from such a wide range of sites suggests a general underlying hydro-climatic relation that might be extended to broader application.

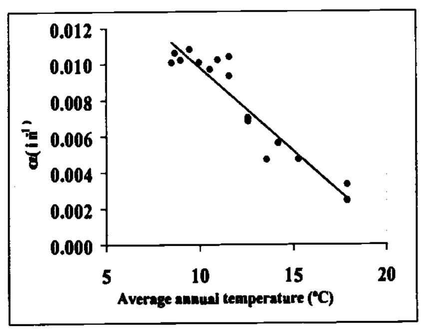

Figure 3. Plot of Grunsky's α vs temperature (California data).

Table 3. Average annual precipitation (P), runoff (Q), temperature (T), and α for California coastal watersheds from Kolterman and Gorelick (1992) scaled data.

River	P (in yr^{-1})	Q (in yr^{-1})	T (°C)	α (in^{-1})
Smith	109.06	84.25	8.51	0.0101
Klamath	84.65	61.02	8.69	0.0106
Redwood Cr	79.13	54.72	9.01	0.0102
Mad	64.57	41.73	9.5	0.01084
Eel	59.06	34.29	10	0.0101
Navarro	52.36	26.65	10.6	0.0097
Russian	45.28	20.91	11	0.0102
Cache Cr	30.91	6.46	12.6	0.0068
Putah Cr	40.94	20.91	12.6	0.0069
Napa	40.94	15.67	11.6	0.0093
Alameda Cr	22.44	3.54	12.6	0.007
Pajaro	18.19	1.84	14.2	0.0056
Salinas	26.06	3.22	13.6	0.0047
San Antonio	25.43	6.73	11.6	0.0104
Santa Clara	18.19	1.54	15.3	0.0047
Sta. Margarita	15.16	0.75	17.9	0.0033
San Luis Rey	17.24	0.71	17.9	0.0024

Local Calibration of α(T) Model From Mediterranean Hydrologic Data

Southern France Réal Collibrier watershed

A limited test of the Mediterranean generality hypothesis is enabled with composited data from four small watersheds at the Collobrier site southern France, presented in Table 4, and described in Andreassian (1992). Using the Collobrier P and Q, α is found to be 0.0089 in^{-1}. Using Equation 10, based on the temperature alone, the α value also calculates to 0.0090 in^{-1}. The French Collobrier data falls on the California trend line (Fig. 4), hinting at general application to Mediterranean climates on a wider basis.

Table 4. Average watershed area, annual precipitation (P), runoff (Q) and temperature (T) for Réal Collobrier in southern France (Andréassian, 1992). The composited data from the 4 small watersheds at the Collibrier site is presented below. α = 0.0089 in^{-1}.

Area (mi^2)	P (in)	Q (in)	T (°C)
3.6	48.5	19.6	11
3.2	44.0	17.0	11
0.6	42.8	19.6	11
0.6	48.4	28.1	11
Composite			
8.0	45.9	21.1	11

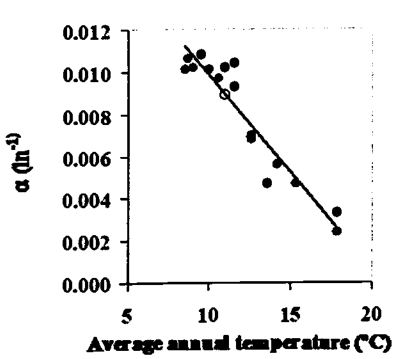

Figure 4. Plot of Grunsky's α vs temperature (California and French data).

Portuguese watersheds

More tests done on data from 12 watersheds (Table 5) located from north to south of Portugal (Instituto da Água, I.P., 2009) validated the generalized application of the water yield model to Mediterranean climates. Computations were made using the water years 1940/41 to 1997/98 for the precipitation-runoff, and time series of more than 30 years for temperature (http://snirth.pt, option Dados Sintetizados>Recursos Hídricos>Climatologia>Boletim de Precipitação). Table 5 presents the average watersheds main characteristics and provides for α values based solely on the analytical Eq. 10 for temperature. Fig. 4 shows the calculated value α on the California and French plot and Fig. 5 adds the plot for the Portuguese watersheds. The predictive power of the model is shown in Fig. 6, where the basins observed Q values are regressed against Q values calculated from the proposed Grunsky's generalized Mediterranean water yield model. The good fit (1:1 line) from such a wide range of sites renders possible extending the model to a broader Mediterranean application.

Table 5. Average characteristics for Portuguese watershed (rivers).

Watershed (Portugal)	Runoff, Q (in/yr)	Avg. Temp, (°C)	Rainfall, P (in/yr)	Alpha, α (in^{-1})
Ave	27.4	13.9	59.9	0.00953
Douro	11.5	13.5	35.7	0.00638
Guadiana	1.7	16.4	23.5	0.00762
Lima	31.3	13.6	70.1	0.00754
Lis	3.9	14.8	33.7	0.00940
Mira	2.3	16.6	26.3	0.00347
Mondego	15.0	14.5	44.6	0.00560
Nabão	13.6	14.5	38.1	0.00379
Sado	1.4	16.1	26.3	0.00489
Tejo	4.8	15.6	31.5	0.00201
Tâmega	26.4	13.2	52.6	0.00304
Zêzere	10.3	14.8	42.9	0.00335

Souce: Instituto da Água, I.P. [Portuguese Water Institute] (2009). (http://snirh.pt/junior/index.php?meunu=2.1

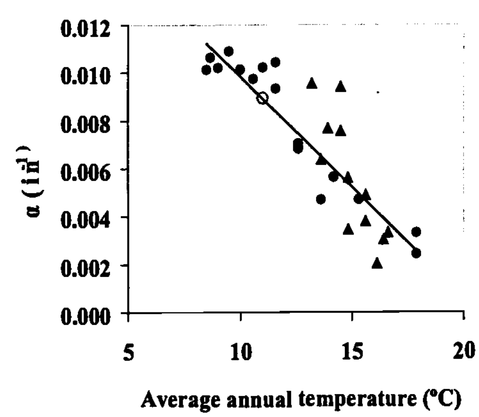

Figure 5. Plot of Grunsky's α vs temperature (California, French and Portuguese data).

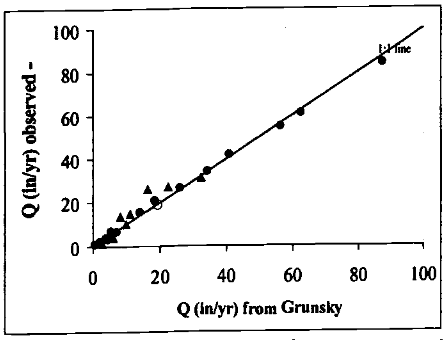

Figure 6. Observed and derived annual water yield using precipitation and temperature (California, French and Portuguese data).

CONCLUSIONS

A link was established in-between the Mediterranean α values and the generalized Grunsky's equations, leading to development of a generalized water yield model for Mediterranean watersheds. Its main advantage, as the Grunsky's approach, relies on its transparency and simplicity, making a convenient basis for local comparisons. By virtue of simple rainfall-runoff relationships, Mediterranean watershed water yields can be assessed. The "Losses" component also conveys meaning in plant geography. The predictive power of the model, and the sought linkage between watershed hydrology and climate were validated using Mediterranean watershed data from France and Portugal.

It should be noted that the model applies to long term averages, not individual years, though some similar trends can be found. In addition, it is best applied to larger watersheds and river basins where the areal idiosyncrasies and geologic happenstances of small watersheds are internalized or averaged out. This smoothing is the rationale for compositing the four Collobrier watersheds in the above discussion. Equally, the model can easily be converted to metric system.

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