CONCEPTUAL AND NUMERICAL MODELING OF ICE IN A
GLOBAL CLIMATE FRAMEWORK

by
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Signed: _______________________
DARIN COMEAU
DEDICATION

Dedicated to my parents, John Comeau and Bonnie Cameron.
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Ice is both an important indicator, and agent, of climate change. In this work we consider conceptual and numerical models of ice in the global climate system on two ends of the climate modeling spectrum. On the simple end of the spectrum, we introduce a low-dimensional global climate model to investigate the role of oceanic heat transport on ice cover, particularly in the initiation of global ice cover, known as Snowball Earth events. We find that oceanic heat transport is effective at keeping the ice margin at high latitudes, and neglecting to include oceanic heat transport can lead to drastically different climate states. On the complex end of the climate modeling spectrum, we implement an iceberg parameterization in the Los Alamos National Laboratory’s sea ice model CICE. Novel to our approach is we model icebergs in two frameworks - as Lagrangian particles, and as an Eulerian field. We allow icebergs to interact dynamically with the surrounding sea ice, and the modeled iceberg thermodynamics allow them to melt as they drift, serving as vehicles of freshwater injection into the ocean from land ice sheets. We focus on Antarctic icebergs, which tend to be larger than those found in the Arctic and are more likely to encounter substantial sea ice pack.
Chapter 1

Introduction

Ice is both an important indicator, and agent, of climate change. Ice cover greatly affects planetary albedo, the percent of incoming solar radiation (insolation) that is reflected from the Earth’s surface, which in turn plays a significant role in our global heat budget. The ice-albedo feedback is a simple example of a non-linear, positive feedback in the climate system. As ice cover retreats, more ocean is exposed that has a lower albedo, and thus absorbs more insolation. This warms the planet, encouraging further ice loss, creating a positive feedback. This is of concern presently (e.g. [LZ05], [HBT06], [CPGS08], [Hol12]), particularly as 2007 and 2012 were years of record low summer Arctic sea ice extent. The ice-albedo feedback can also work in the other direction - as ice cover grows, the planetary albedo is raised, less energy from the sun is absorbed by the Earth, the Earth cools, further encouraging ice production. In the extreme limit, this would lead to complete ice cover on the Earth, a climate state known as "Snowball Earth" first proposed by Kirschvink in [Kir92] after geologic evidence suggested glaciation in the tropics. The question of whether the Earth has ever been in such a state has been highly controversial ([Lub02]), and has led to a number of alternative hypothesis (e.g. [HCBP00], [AP10], [AVK11]).

Ice is also a vehicle for freshwater redistribution, altering the composition of the Earth’s oceans. Icebergs move freshwater from land ice sheets, glaciers, or ice shelves and inject meltwater into the ocean as they drift and deteriorate. This is the dominant way in which freshwater leaves the Antarctic ice sheet, and there has been recent interest to improve iceberg modeling (e.g. [SBN06], [JDF+09], [MA10]), particularly after recent giant calving events, such as the B-15 event of March 2000, the largest ever recorded at approximately the size of Jamaica. The Greenland ice sheet, the other
substantial source of icebergs, is also experiencing melting ([CWT06], [Mot07]). In July 2012, NASA had estimated that approximately 97% of the ice sheet’s surface was experiencing melting. These trends, together with increased computing and modeling capabilities, have led to greater recent interest in the role of icebergs on the oceans and sea level rise.

Climate modeling spans a broad spectrum. At one end, there are simple models, which are conceptual low dimensional models that are highly idealized. These simple models can be useful in providing insight into coarse global climate, or in determining the role of particular processes on the climate system. For example, energy balance models, first introduced by Budyko in [Bud69], capture the global heat budget through a simple energy balance with the sun’s radiation, and these models are still useful today, as demonstrated by Rose and Marshall in [RM09]. As another example, Stommel introduced box models for modeling ocean circulation in [Sto61], where the ocean is represented simply as two boxes, each well mixed and representing the high and low latitude ocean, and provides insight on the role of temperature and salinity in density driven ocean circulation. Climate modelling approaches increase in complexity, through intermediate complexity models that increase dimension, spatial resolution, and processes modeled. State of the art fully coupled general circulation models (GCM) lie on the other end of the climate modelling spectrum, which can model global climate on grids of order 1° by 1° or less, as well as multiple vertical levels for the ocean and atmosphere, and may include components for sea ice, land ice, and land, each of which may be coupled to each other. These models are sometimes the result of decades of development, representing the work of many scientists. An example of this class of models is the Community Earth System Model (CESM) of the National Center for Atmospheric Research (NCAR), and it is these class of models that form the basis for the International Panel on Climate Change’s (IPCC) Assessment Reports, which are used as the foundation to guide global climate policy.

In this dissertation we consider conceptual and numerical models of ice in the
global climate system on two ends of the climate modeling spectrum. In Chapter 2 we develop a simple low dimensional climate model to investigate the role of ice and oceanic heat transport, with particular emphasis on the Snowball Earth state. In Chapter 3 we move to the other end of the climate modeling spectrum and implement an iceberg parameterization in the Los Alamos National Laboratory’s sea ice model CICE (Community Ice CodE), which is itself a component of the larger general circulation model CESM. The key results from each modelling approach are summarized in Chapter 4.
Chapter 2

A Simple Climate Model with Dynamic Ocean & Ice Components

2.1 Background

Since the discovery of geologic evidence suggesting glaciation in the tropics in occurred at least twice in the Neoproterozoic, at roughly 710 and 635 million years ago (Ma) ([Kir92], [HS02]), there has been substantial debate about whether the Earth was ever in a completely ice covered state, known as "Snowball Earth" ([Lub02]). A global ice covered Snowball state is consistent with energy balance climate models of the type first proposed by Budyko in [Bud69], though this does not provide insight into the climate processes important in Snowball state initiation, apart from a runaway ice-albedo feedback, or deglaciation, which seems difficult to explain with such a high planetary albedo. These Snowball events are thought to have lasted several million years, greatly weakening the hydrologic cycle, and raising the question of how life such as algae could have survived during this period ([McK00]). In trying to explain these signs of apparent tropical glaciation in the context of global climate dynamics, alternative hypotheses have been proposed that leave some portion of the ocean free of ice, or covered only in thin ice. These model scenarios are known as 'Slushball Earth' of Hyde in [HCP00], the 'Jormungand' state of Abbot et al. in [AVK11], and the 'Mudball' state of Abbot and Pierrehumbert in [AP10]. In the Mudball scenario, dust is transported through flowing ice to the tropics decreasing albedo, and thus encouraging deglaciation.

In the above studies the ocean played a role insofar as radiation balances, greenhouse gas budgets, and albedo estimates were concerned. Poulsen et al. ([PPJ01]), counts among the first studies to examine the role played by dynamic oceanic heat...
transport on the Snowball scenario. Using the fully coupled ocean-atmosphere Fast Ocean-Atmosphere Model (FOAM), initialized to Neoproterozoic parameter values to facilitate Snowball conditions, they found that global ice cover conditions were produced when using a mixed-layer ocean model that parameterized heat transport through diffusion. However, in fully coupled experiments with the ocean component, the ice margin would retreat to high latitudes. A more recent study by Voigt et al. in [VAPM11] uses the state of the art atmosphere-ocean general circulation model (GCM) ECHAM5/MPI-OM to study the Snowball Earth scenario, the most sophisticated model to be applied to this purpose. They implement a Marinoan (∼635 Ma) land mask in their coupled GCM simulations to take into account the low-latitude continental land mass of the Neopreoterozoic era, as well as lower insolation, relative to Pre-industrial era levels to take into account a weaker and younger sun. In addition to ocean dynamics, their study also included sea ice dynamics, and interactive clouds. All three were previously found to be essential for Snowball initiation (see [PPJ01], [PJ04], [LWJE03], [LWE07]). Voigt, et al., were able to both achieve Snowball initiation, and prevent Snowball initiation in the same setting through a doubling of carbon dioxide levels. Stability analysis of an energy balance model analog, based on the model of Heinemann, et al. in [HJM09], indicates an insolation bifurcation point for Snowball Earth in the Marinoan setting of about 95% to 96% of pre-industrial levels, which agrees with their computational results. In their experiments that resulted in partial ice cover, the ice margin was around 30° to 40° latitude, with the maximum stable sea ice extent of 55% of ocean cover observed in their experiments, evidence that does not support a Slushball or Jormungand scenario.

In [GP03], Goodman and Pierrehumbert first considered viscous flow effects of thick sea ice in the Snowball Earth setting. When considering thick ice, as may be present in a global ice scenario, the fluid dynamics of the material must also be considered in addition to its thermodynamics, as the ice will exhibit plastic deformation under its own weight, causing flow. This same framework was used in [AP10] and
to transport dust to low latitudes in the 'Mudball' scenario. Their model runs outside of a GCM, using FOAM output for forcing data, and does not have an active ocean component, or parameterization for oceanic heat transport. They use the term 'sea glacier' to describe their modeled ice, to distinguish from present day sea ice, which only grows to thicknesses on the order of meters, as well as land ice or ice shelves protruding from ice sheets. Their sea glacier is formed on the ocean primarily from ocean water, yet achieves thicknesses of a land ice sheet without the land/ice interface. They are able to achieve partial glaciation and a Snowball state through changes in the atmospheric forcing (surface temperature and precipitation minus evaporation). They find that the additional viscous flow term is highly effective at allowing the ice margin to penetrate regions of melting, encouraging Snowball Earth initiation.

Atmospheric dynamics and cloud cover undoubtedly play a large role in such climate systems, as demonstrated by Voigt et al. in [VAPM11]. It is, however, difficult to discern what role is played by oceanic transport in these coupled runs due to the necessary inclusion of the complex dynamics of the atmosphere and cloud distribution. This motivated us to consider a simpler model. We aim to extend the framework laid out by Goodman and Pierrehumbert in [GP03] to include ocean heat transport effects. Realistic oceanic transport undoubtedly leads to highly non-uniform heat distributions, with likely local consequences on the global Snowball scenario. However, the focus of our inquiry is to determine if there are global consequences from oceanic transport. We will thus limit our inquiry to the hemispheric balances, and zonally average to a one-dimensional domain, ignoring land masses. This chapter is organized as follows: in Section 2.2 we describe the model, in Section 2.3 we analyze the results of the model, including sensitivity to unconstrained model parameters, and discussion of results and comparison to other studies is in Section 2.5.
2.2 Model Description

Our one-dimensional climate model consists of a box ocean model with transport, similar to that first proposed by Stommel in [Sto61], coupled to a dynamic model for sea ice coverage. The model also includes radiative effects through energy balances of incoming solar radiation, outgoing long-wave radiation, and albedo effects, similar to energy balance models first proposed by Budyko in [Bud69]. The model presumes zonally-averaged dynamics, and as such, depends only on colatitude $\theta$ and time $t$. The ocean and ice components are coupled, and the ice-albedo feedback is present through different albedos for ocean and ice.

2.2.1 Ocean Model

The ocean component of our model follows a traditional box model (e.g. [Sto61], [HLS92]), and the setup similar to the one proposed by Griffies and Tziperman in [GT95] and Kurtze, Restrepo et al. in [KRD10]. Four boxes are used to represent the ocean in one hemisphere, from pole to equator, with each box representing a zonal average across longitude. Referring to Figure 2.1, we define: 'Box 1' as the tropical surface ocean box, 'Box 2' its the polar counterpart; 'Box 3' lies below Box 2 and 'Box 4' below Box 1. The depth of the upper boxes is $d_u$, the depth of the lower boxes is $d_l$, with $d_u \ll d_l$. Boxes 2 and 3 extend from the pole at colatitude $\theta = 0$ to a fixed boundary at $\theta = \zeta$. Boxes 1 and 4 extend from $\theta = \zeta$ to the equator at $\theta = 90^\circ$. We choose the colatitude boundary $\zeta$ to be $45^\circ$ simply because we are interested in investigating global climate regimes of near global or global ice cover, so ice cover will be present in both surface boxes and thus we do not need particular focus on the polar region. The dynamic ice margin is at $\eta(t)$, the ice cover has thickness is given by $h(\theta, t)$, and poleward meridional velocity given by $v(\theta, t)$. Model parameter values, including geometric quantities pertaining to the box structure, are given in Table 2.1.
Figure 2.1. Hemispheric four-box arrangement. Box 1 is the surface tropical ocean box of depth $d_u$ with salinity $S_1$ and temperature $T_1$, zonally averaged. Similarly Box 2 is the surface polar ocean box, with $\zeta$ as the boundary between the surface boxes, and Boxes 3 & 4 are the deep ocean boxes of depth $d_l$. Ice cover sits atop the surface boxes with height $h$ and the ice margin at $\eta$. The arrows between the boxes represent density driven circulation $f$. 
Each box has a (well-mixed) temperature $T_j(t)$ and salinity $S_j(t)$, $j = 1, \ldots, 4$, which determine the density of each box by a linear equation of state

$$
\rho_j(T_j, S_j) = \rho_0 [1 + \beta_S(S_j - S_0) - \beta_T(T_j - T_0)]
$$

Here $\rho_0$ is a reference density corresponding to a reference temperature and salinity $T_0$, $S_0$. $\beta_S$ and $\beta_T$ are the expansion coefficients associated with salinity and temperature. As in [GT95] we take $\rho_0 = 1027$ kg/m$^3$, corresponding to $T_0 = 273^\circ$ K and $S_0 = 35$ psu (practical salinity unit, roughly 35 grams of salt per kilogram of water). The density-driven flow between the boxes is denoted by $f$, where we adopt the convention that $f < 0$ is surface poleward flow (from Box 1 to Box 2). As in [GT95], the (buoyancy driven) transport rate, in Sverdrups (1 Sv = $10^6$ m$^3$/s), is

$$
f = k \left[ \frac{d}{dt}(\rho_1 - \rho_2) + (\rho_4 - \rho_3) \right],
$$

where $k = 8 \cdot 10^4$ Sv/$\rho_0$ is the hydraulic constant which governs the strength of the density driven flow.

The equations for each box’s salinity and temperature will depend on the direction of the mean meridional flow. For $f > 0$ (flow from Box 2 to Box 1, surface flow from pole to equator), the salinity (transport) equations are as follows:

$$
\begin{align*}
V_1 \frac{dS_1}{dt} &= f(S_2 - S_1) + S_1 \int_{\xi}^{\max \zeta, \eta} M(\theta) r_E^2 \sin \theta d\theta, \\
V_2 \frac{dS_2}{dt} &= f(S_3 - S_2) + S_2 \int_{0}^{\min \zeta, \eta} M(\theta) r_E^2 \sin \theta d\theta, \\
V_3 \frac{dS_3}{dt} &= f(S_4 - S_3), \\
V_4 \frac{dS_4}{dt} &= f(S_1 - S_4).
\end{align*}
$$

Here $V_j$ is the box volume of the $j^{th}$ box, and $M(\theta, t)$ is the total production/melting rate of ice, with $M > 0$ corresponding to ice production and $M < 0$ corresponding to melting, in units m/s, described in Section 2.2.3. The terms involving the circulation rate $f$ simply correspond to fluxes across the box boundaries.
We assume that ice sits atop the surface ocean boxes, and that the volume of ice is much less than the total ocean volume, so that ocean box volumes $V_j$ are constant. An important assumption is that ice that is formed is freshwater ice, and as such rejects brine into the ocean. The integral term represents the change in salinity due to net freshwater, added/removed through ice melting/production. The bounds of integration represent the portion of each box covered in ice, and $r_E$ is the Earth’s radius for the surface area element in spherical coordinates. The ice component of the model serves as a saline forcing on the ocean box model component.

Similarly, for $f < 0$ (flow from Box 1 to Box 2, surface flow from equator to pole), the salinity equations are:

$$
\begin{align*}
V_1 \frac{dS_1}{dt} &= |f|(S_4 - S_1) + S_1 \int_{\xi}^{\max \zeta, \eta} M(\theta)r_E^2 \sin \theta d\theta, \\
V_2 \frac{dS_2}{dt} &= |f|(S_1 - S_2) + S_2 \int_{0}^{\min \zeta, \eta} M(\theta)r_E^2 \sin \theta d\theta, \\
V_3 \frac{dS_3}{dt} &= |f|(S_2 - S_3), \\
V_4 \frac{dS_4}{dt} &= |f|(S_3 - S_4).
\end{align*}
\tag{2.2.4}
$$

We opt to use energy equations for the temperature, rather than transport models. This is a significant departure to traditional box models of long-time ocean flows. In the surface boxes, these are circulation, radiative balance between insolation and outgoing long wave radiation, and melting/freezing of ice. Corresponding to $f > 0$, the energy equations are:
\[ c_w V_1 \frac{d(\rho_1 T_1)}{dt} = c_w f(\rho_2 T_2 - \rho_1 T_1) \]
\[ + \int_{\max \zeta, \eta}^{\pi/2} \left( (1 - \alpha_w)F_s(\theta) - \varepsilon \sigma T_1^4 \right) r_E^2 \sin \theta d\theta \]
\[ - \frac{\kappa_w}{D}(T_1 - T_f) \int_{\zeta}^{\max \zeta, \eta} r_E^2 \sin \theta d\theta, \]  

(2.2.5)

\[ c_w V_2 \frac{d(\rho_2 T_2)}{dt} = c_w f(\rho_3 T_3 - \rho_2 T_2) \]
\[ + (1 - \alpha_w) \int_{\min \zeta, \eta}^{\zeta} \left( (1 - \alpha_w)F_s(\theta) - \varepsilon \sigma T_2^4 \right) r_E^2 \sin \theta d\theta \]
\[ - \frac{\kappa_w}{D}(T_2 - T_f) \int_{0}^{\min \zeta, \eta} r_E^2 \sin \theta d\theta, \]  

(2.2.6)

\[ V_3 \frac{d(\rho_3 T_3)}{dt} = f(\rho_4 T_4 - \rho_3 T_3), \]  

(2.2.7)

\[ V_4 \frac{d(\rho_4 T_4)}{dt} = f(\rho_1 T_1 - \rho_4 T_4), \]  

(2.2.8)

where \( c_w \) is the ocean water heat capacity. The first line (2.2.5, 2.2.6) accounts for net energy accumulation due to fluxes across box boundaries. The second line represents the radiative balance, in a similar form as appears in energy balance models dating back to Budyko ([Bud69]). Here \( \alpha_w \) is the ocean water albedo, and \( F_s(\theta) \) is insolation in units W/m^2, for which we use the parameterization of McGehee and Lehman in [ML12], which includes dependence on the Earth’s orbital parameters:

\[ F_s(\theta) = \frac{342.95}{\sqrt{1 - e^2}} \frac{2}{\pi^2} \int_{0}^{2\pi} \left[ 1 - (\cos \theta \sin \beta \cos \gamma - \sin \theta \cos \beta)^2 \right]^{1/2} d\gamma. \]  

(2.2.9)

Here \( e \) is eccentricity of the Earth’s orbit (presently at 0.0167), \( \beta \) is the obliquity (presently at 23.5°), and \( \gamma \) is longitude. The significance of the dependence on orbital parameters will be discussed in Section 2.4.2. Figure 2.2 shows current insolation as a function of colatitude for present orbital parameters.

In the integral in (2.2.5, 2.2.6), insolation is balanced by outgoing long wave radiation, where we follow the assumption that the Earth is a black body and radiates according to the Stefan-Boltzmann law. Here \( \varepsilon \) is the effective emissivity parameter.
and $\sigma$ is the Stefan-Boltzmann constant. We use the full Stefan-Boltzmann law, and we note this is a departure from the typical linearized Stefan-Boltzmann law used in some energy balance models (e.g. [Bud69], [ML12]), though Heinnemann et al. and Voigt et al. also use this full $\varepsilon \sigma T^4$ form for outgoing longwave radiation. The effective emissivity $\varepsilon < 1$ is the ratio of outgoing longwave radiation emitted at the top of the atmosphere and that emitted at the Earth’s surface, and therefore represents the greenhouse effect. Thus atmospheric effects are distilled into this single parameter, which we will use as our control between climate states. The bounds of integration in (2.2.5, 2.2.6) are over the exposed ocean portion of the box.

The last terms in (2.2.5, 2.2.6) represent the effect of ice cover on the ocean box temperature. For the region of the box that is covered by ice, we assume there is heat flow between the bottom of the ice, at freezing $T_f$, to the rest of the ocean box, at average temperature $T_1$, over some vertical length scale $D$. We call $D$ the depth of the thermal boundary layer between ocean and ice, and assume the temperature profile over this boundary layer is linear. We note that $D$ is an unconstrained parameter.
which we set to $D = 1m$, and explore the model’s sensitivity to this parameter in Section 2.4. Here $\kappa_w$ is the sea water thermal conductivity, which we take to be constant, and we neglect dependence on salinity and temperature. The temperature equations for the deep ocean boxes, (2.2.7, 2.2.8) only have terms for the transport associated with fluxes across the boxes.

The corresponding equations for $f < 0$ (surface poleward flow) are:

$$c_w V_1 \frac{d(\rho_1 T_1)}{dt} = c_w |f| (\rho_4 T_4 - \rho_1 T_1) + \int_{\max \zeta, \eta}^{\pi/2} ((1 - \alpha_w) F_s(\theta) - \varepsilon \sigma T_1^4) r_E^2 \sin \theta d\theta
- \kappa_w D (T_1 - T_f) \int_{\zeta}^{\max \zeta, \eta} r_E^2 \sin \theta d\theta,$$

$$c_w V_2 \frac{d(\rho_2 T_2)}{dt} = c_w |f| (\rho_1 T_1 - \rho_2 T_2) + \int_{\min \zeta, \eta}^{\zeta} ((1 - \alpha_w) F_s(\theta) - \varepsilon \sigma T_2^4) r_E^2 \sin \theta d\theta
- \kappa_w D (T_2 - T_f) \int_{\min \zeta, \eta}^{\zeta} r_E^2 \sin \theta d\theta,$$

$$V_3 \frac{d(\rho_3 T_3)}{dt} = |f| (\rho_2 T_2 - \rho_3 T_3), \quad (2.2.12)$$

$$V_4 \frac{d(\rho_4 T_4)}{dt} = |f| (\rho_3 T_3 - \rho_4 T_4). \quad (2.2.13)$$

### 2.2.2 Sea Ice Model

We largely follow the 'sea glacier' treatment of Goodman and Pierrehumbert in [GP03] (henceforth GP03) and later Li and Pierrehumbert in [LP11], with a few noted exceptions. The equation for the ice thickness, by conservation of mass, is

$$\frac{\partial h(\theta, t)}{\partial t} + \nabla \cdot [v(\theta, t) h(\theta, t)] = M(\theta, t), \quad (2.2.14)$$

where $h(\theta, t)$ is ice thickness, $v(\theta, t)$ is colatitudinal ice velocity, and $M(\theta, t)$ is the ice melting/production term. That is, the change in ice thickness is due to an advective viscous flow term and a net ice production/melting term. Ice is an example of a
non-Newtonian fluid, which deforms under its own weight by compressing vertically and stretching laterally, causing lateral ice flow. The flow rate, first suggested by Glen in [Gle55], has been empirically found to be a power law relating strain rate \( \dot{\varepsilon} \) and stress \( \tilde{\sigma} \), and is of the form

\[
\dot{\varepsilon} = A \tilde{\sigma}^n,
\]

where \( A, n \) are rheological parameters. Non-Newtonian viscous flow is obtained when \( n \neq 1 \). Letting \( n \) approach \( \infty \), (2.2.15) represents the behavior of a perfectly plastic material, where no deformation occurs until some yield stress is met, at which point rapid deformation occurs. In [Gle55], Glen performed experiments applying compressive stresses to cylindrical polycrystalline ice (with an effort to mimic ice composition of glacial ice), and found an empirical fit to the power law (2.2.15) with \( n \approx 3.2 \), with \( n = 3 \) subsequently becoming the common exponent to use for a Glen’s flow law of glacial ice. This was extended to a floating ice-shelf model by Weertman in [Wee57], and later by Macayeal and Barcilon in [MB88]. From this work, GP03 use a Glen’s flow law to describe ice velocity \( v(\theta, t) \) in terms of rheological parameters:

\[
\nabla \cdot v(\theta, t) = \mu^n h(\theta, t)^n,
\]

where \( n = 3 \) is the Glen’s flow law exponent, and \( \mu \) is a viscosity parameter, given by ([Wee57]):

\[
\mu = \frac{1}{4} \rho_i g \left( 1 - \frac{\rho_i}{\rho_w} \right) \overline{A}^{1/n}.
\]

Here \( \rho_i, \rho_w \) are the densities of ice and water, \( g \) is gravity, and \( \overline{A} \) is the depth-averaged Glen’s flow law parameter (e.g. [San79], [GP03], [LP11]), given by (suppressing dependence on colatitude \( \theta \))

\[
\overline{A} = \frac{1}{h} \int_{-h}^{0} A_0 \exp \left( \frac{-Q}{RT(z)} \right) dz.
\]

\( R = 8.31445 \) J/K/mol is the gas constant, \( A_0, Q \) are parameters split by a temperature boundary (as in [BTW71], [GP03], [LP11]) given in Table 2.1 and \( T(z) \) is the vertical temperature profile through the ice.
The ice surface temperature $T_s(\theta)$ is given by a primary radiative balance, similar to that which appears in our equations (2.2.5/2.2.10, 2.2.6/2.2.11), as well as a term accounting for heat transfer through the ice. The (average annual) ice surface temperature $T_s(\theta)$ is given by the following equation, in units W/m$^2$:

$$c_i \rho_i h(\theta) \frac{dT_s(\theta)}{dt} = F_s(\theta)(1 - \alpha_i) - \varepsilon \sigma T_s(\theta)^4 + \kappa_i \frac{T_f - T_s(\theta)}{h(\theta)},$$

(2.2.19)

where $c_i$ is the specific heat of ice, $\alpha_i$ is the albedo of ice, and $\kappa_i$ is the thermal conductivity of ice. The last term accounts for heat transfer through the ice, assuming a linear temperature profile in the ice from the surface $T_s(\theta)$ to the base at freezing $T_f$.

Our ice melting/accumulation term $M(\theta,t)$ is a departure from the treatment of [GP03], and these differences are highlighted in Section 2.5. Ice melting or production can occur either from heat transferred through the ice from the surface, or from heat transferred through the ocean through an ice ocean thermal boundary layer $D$, and is given by, in units m/s:

$$M(\theta,t) = \frac{\kappa_i}{\rho_i L h(\theta,t)} (T_f - T_s(\theta,t)) - \frac{\kappa_w}{\rho_w LD} (T_{1,2}(\theta,t) - T_f),$$

(2.2.20)

where $L$ is the latent heat of fusion of ice. The first term on the right side accounts for heat transfer through the ice, and balances the equivalent term in equation 2.2.19 to conserve energy. The second term accounts for heat transfer with the ocean through the thermal boundary layer $D$, discussed in (2.2.5/2.2.6), where an equivalent term appears in the energy budget for the ocean box temperatures. Since only average ocean box temperatures are computed by our model, an artificial and arbitrary jump occurs across the box boundary in the melting equation (2.2.20). To avoid this, we regularize the ocean temperature profile with a sine curve, achieving the average temperatures $T_{1,2}$ at the center of each box. An example of this regularized ocean temperature profile is given in Figure 2.3. When $M(\theta,t) > 0$, there is net accumulation of ice, and when $M(\theta,t) < 0$, there is net melting.
In 1-D spherical coordinates in colatitude $\theta$, Equations 2.2.14 and 2.2.16 reduce to:

$$\frac{\partial h(\theta, t)}{\partial t} + \frac{v(\theta, t)}{r_E} \frac{\partial h(\theta, t)}{\partial \theta} = M(\theta, t) - \mu^n h(\theta, t)^{n+1},$$  \hspace{1cm} (2.2.21)

$$\frac{\partial v(\theta, t)}{\partial \theta} + v(\theta, t) \cot \theta = r_E \mu^n h(\theta, t)^n.$$  \hspace{1cm} (2.2.22)

2.2.3 Model Setup

The parameters for the problem are listed in Table 2.1. The ocean component is run on a decadal time step, with the ice dynamics sub-cycled on a monthly time step. For each ocean time step, we solve the set of differential equations for box temperatures and salinities (Equations 2.2.4, 2.2.5, 2.2.10, and 2.2.6, 2.2.11), and surface temperature (Equation 2.2.19) using a simple forward Euler method. At each ice time step, we solve (2.2.21) using a second-order upwind scheme after solving for the velocity in (2.2.22). We discretize our longitudinal domain with 100 points, and set $v = 0$ for
Table 2.1. Physical parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Units</th>
<th>Value</th>
</tr>
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<tbody>
<tr>
<td>hemispheric extent</td>
<td>ℓ</td>
<td>θ</td>
<td>π/2</td>
</tr>
<tr>
<td>extent of Box 2 and 3</td>
<td>ζ</td>
<td>θ</td>
<td>π/4</td>
</tr>
<tr>
<td>depth of Box 1 and 2</td>
<td>d_u</td>
<td>m</td>
<td>200</td>
</tr>
<tr>
<td>depth of Box 3 and 4</td>
<td>d_l</td>
<td>m</td>
<td>3000</td>
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</tr>
<tr>
<td>volume of Box 2</td>
<td>V_2</td>
<td>m³</td>
<td>1.17 \times 10^{16}</td>
</tr>
<tr>
<td>volume of Box 3</td>
<td>V_3</td>
<td>m³</td>
<td>1.76 \times 10^{17}</td>
</tr>
<tr>
<td>volume of Box 4</td>
<td>V_4</td>
<td>m³</td>
<td>4.25 \times 10^{17}</td>
</tr>
<tr>
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<td>kgcdot m^{-3}</td>
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</tr>
<tr>
<td>reference salinity</td>
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<td>psu</td>
<td>35</td>
</tr>
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<td>283</td>
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<td>sea ice albedo</td>
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<td>ice/ocean boundary layer*</td>
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</tr>
<tr>
<td>ice viscosity parameter</td>
<td>A_0</td>
<td>Pa^{-3}·s^{-1}</td>
<td>3.61 \cdot 10^{-13} \ T &lt; 263.15;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1.734 \cdot 10^{3} \ T &gt; 263.15.</td>
</tr>
<tr>
<td>ice viscosity parameter</td>
<td>Q</td>
<td>J·mol^{-1}</td>
<td>60 \cdot 10^3 \ T &lt; 263.15;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>139 \cdot 10^3 \ T &gt; 263.15.</td>
</tr>
<tr>
<td>Glen’s flow law exponent</td>
<td>n</td>
<td>-</td>
<td>3</td>
</tr>
</tbody>
</table>

In this equation (2.2.21) the for boundary condition in the ice profile evolution equation. In the event that ice reaches the equator, we follow the treatment of GP03 to bring the velocity at the equator to zero through a balance of a back-pressure term from the force of ice colliding with ice from the other hemisphere (assuming symmetry of Northern and Southern hemispheres). They found the appropriate version of Equation 2.2.21 in this case is

\[
\frac{\partial v(\theta, t)}{\partial \theta} + v(\theta, t) \cot \theta = r_E \mu^n \left( h(\theta, t) - \frac{b}{h(\theta, t)} \right)^n, \tag{2.2.23}
\]
where the back-pressure constant $b$ satisfies
\[
v(\theta = 0^\circ) = r_E \int_0^{2\pi} \mu_0 \left( h(\theta, t) - \frac{b}{h(\theta, t)} \right) \sin \theta d\theta,
\]
which we solve by Newton’s method.

### 2.3 Results

We initialize the model in an ice free state, with initial ocean conditions (following the setup of Griffies and Tziperman in [GT95]) of Boxes 1 - 4 set to $T_1 = 283^\circ$ K, $T_2 = 275^\circ$ K, and $T_3 = T_4 = 273^\circ$ K. The initial ocean box salinities are $S_1 = 36.5$ psu, $S_2 = 34.5$ psu, and $S_3 = S_4 = 36$ psu. The effective emissivity parameter $\varepsilon$ is used as a control between climate states - we emphasize that, as we have no atmospheric heat transport or other atmospheric effects in our model other than this parameter to crudely account for greenhouse effects, we do not attribute physical meaning to this parameter value. For example, running the model at $\varepsilon = 0.6$, which is estimated to be a reasonable value for current climate ([VAPM11]), results in an ice-free planet (when initialized with zero ice), and a thermally driven poleward circulation of $\approx 28.25$ Sv. Figure 2.4 shows the ocean circulation evolution through the course of the run, as well as the associated (poleward) heat transport, defined as

\[
H_{\text{ocean}} = c_w \rho_w f (T_1 - T_2).
\]

We note the circulation strength is in line with results referenced in Griffies and Tziperman [GT95] that give an approximate meridional circulation strength of 20 Sv from the coupled ocean-atmosphere model of NOAA’s Geophysical Fluid Dynamics Laboratory (GFDL) model. The salinities of the boxes quickly mix and converge to roughly the same value of 36 psu. The equatorial surface box temperature settles to $T_1 = 288.5^\circ$ K, whereas the other three boxes converge to a common temperature of $T_2 = T_3 = T_4 = 281.6^\circ$ K. Hence we have a strong, thermally dominated poleward
Figure 2.4. Evolution of circulation rate and corresponding oceanic heat transport in an ice free scenario with effective emissivity $\varepsilon = 0.6$.

circulation in this simulation. We also note that turning off ocean circulation (by forcing $f = 0$) in this parameter regime results in partial ice cover, so the presence or neglect of oceanic heat transport will result in vastly different climate states.

2.3.1 Partial Glaciation

Raising the effective emissivity to $\varepsilon = 0.7$, we reach a climate state with a stable small ice cover after running the model 50,000 years to reach steady state. To determine the role of oceanic heat transport, we run the model with the circulation rate $f$ set to zero for comparison against the full model run. Figure 2.5 shows the ice thickness and ice velocity profiles with and without ocean circulation, and we see the ocean circulation is effective at reducing ice thickness as well as pulling the ice margin north. Without the additional heat from the equatorial region moving north, the polar region remains cool, facilitating ice growth. We note the small ice cap is a consistent solution from energy balance models (e.g., Bud69, RM09), which is the
Figure 2.5. Ice thickness and velocity profiles in partial ice cover scenario with effective emissivity $\varepsilon = 0.7$, with and without ocean circulation.

Figure 2.6. Basal melting/accumulation and ice surface temperature in partial ice cover scenario with effective emissivity $\varepsilon = 0.7$.

Ice cover is approximately 140m thick at the pole in the full model run, much thicker than current sea ice cover, and is in line with partial glaciation results from GP03. It is because the ice is this thick that viscous flow effects need to be considered. There is a strong response in the ice velocity, largely due to the thicker ice cover when ocean circulation is not included, resulting in stronger viscous flow. Figure 2.6 shows the steady state ice melting / accumulation rate for ice (for the full model including...
ocean circulation), as well as the ice surface temperature. The net ice accumulation rate approaches 0 towards the ice margin, where also the ice velocity is the highest (approximately 7 m/yr in this simulation). As ice advects beyond this steady state ice margin it is immediately melted by Equation 2.2.20. We also note the ice surface temperature is in line with the air temperature forcing used in the experiments of GP03.

In Figure 2.7 we show the evolution of ocean circulation and the associated (poleward) oceanic heat transport, and note the steady state ocean circulation strength is \( \approx 21.9 \) Sv, which is closer to the numerical results of the GFDL model referenced by Griffies and Tziperman in [GT95] than the ice-free run. As with the ice free runs, the box salinities quickly mix to the same value of approximately 36 psu, while the surface equatorial box temperature settles to \( T_1 \approx 280.08^\circ\) K, and the other boxes mix to \( T_2 \approx T_3 \approx T_4 \approx 274.67^\circ\) K.

**Figure 2.7.** Ocean circulation and heat transport in partial ice cover scenario with effective emissivity \( \varepsilon = 0.7 \).
2.3.2 Global Glaciation

To better represent the Neoproterozoic climate in our model, we lower insolation to 94% of its current value, accounting for a weaker, younger sun, as in Voigt et al. ([VAPMI1]). Further raising the effective emissivity from $\varepsilon = 0.7$ to $\varepsilon = 0.81$, we move from a climate state with a small, stable ice cover to global ice cover, a Snowball Earth. This is the second consistent and stable solution to the energy balance model of Budyko ([Bud69]). With this global ice cover scenario, there is no region of net ice melting, as shown in Figure 2.9. Therefore while our model can initiate a Snowball Earth through runaway ice-albedo feedback, there is no mechanism present to pull the climate out of such a state, and ice will continue to accumulate without bound. However, the ice margin takes roughly 10,000 years to reach the equator from our initialization, and total ice volume greatly levels off after about 40,000 years, so we will study this example in a state of very gradual change after a 50,000 year run.

A mild drawback of our model is that with continual positive net ice production, our assumption of ice being completely salt free ice produces ever increasing and unphysical salinity levels in the ocean boxes. However with global ice production in both surface boxes and circulation present, the system is still in balance as the salinity levels between the boxes remain within 0.7 psu of each other, and runaway salt increase does not affect other aspects of the model.

We note the a change in the circulation as we get a much weaker, saline driven, equator-ward circulation with a strength of approximately 3.78 Sv. The ocean box temperatures largely mix to a common value, with $T_1 \approx 271.19$, $T_2 \approx 271.21$, $T_3 \approx 271.23$, and $T_4 \approx 271.22$. The ice thickness profile in Figure 2.8 shows a peak thickness at the pole of nearly 500m, reducing to about 150m near the equator. The back-pressure term in the boundary condition for ice velocity in (2.2.24) brings the ice velocity to 0 at the equator, the effect of which brings the maximum velocity of $\approx 100$ m/yr northward.
Figure 2.8. Ice thickness and velocity profiles with 94% insolation and effective emissivity $\varepsilon = 0.81$.

Figure 2.9. Ice melting/accumulation and ice surface temperature with 94% insolation and effective emissivity $\varepsilon = 0.81$. 
Examining the model results between these very different climate states of a small, stable ice cap at $\varepsilon = 0.7$ and Snowball Earth at $\varepsilon = 0.81$, we can find a threshold where the model transitions. With an effective emissivity of $\varepsilon = 0.805$, we get a strong response from the ocean circulation. In Figure 2.10 we show the results of a 50,000 year simulation with and without the ocean circulation active. We observe that without oceanic heat transport, we get a Snowball Earth, but with oceanic heat transport, the ice line is held back from the equator.

As with the previous Snowball Earth state, the ocean circulation strength here is considerably weaker than in the small ice cap simulation, 0.51 Sv (poleward). However even this weakened circulation, and thus weakened oceanic heat transport, is still enough to drive the climate into a Snowball state if turned off, demonstrating strong sensitivity in this regime. We note this is consistent with study of classic energy balance models which predict that a large but finite ice cover is an unstable solution ([RM09]). The ocean box salinities settle to $S_1 \approx 35.99, S_2 \approx 36.33, S_3 \approx 36.32$, and $S_4 \approx 36.21$, while the box temperatures settle to $T_1 \approx 274.27, T_2 \approx 271.81, T_3 \approx 271.75$, and $T_4 \approx 271.57$. In Figure 2.11 we see net melting south of approximately 40° colatitude, and the reason ice is able to exist here in steady state is due to viscous
Figure 2.11. Ice melting/accumulation and ice surface temperature in near global ice cover with 94% insolation and effective emissivity $\varepsilon = 0.805$.

2.4 Model Sensitivity to Parameters

2.4.1 Ice-Ocean Thermal Boundary Layer Depth

As mentioned earlier, one unconstrained parameter in the model is the length $D$ of the thermal boundary layer between the ocean and bottom of the ice surface that appears in equations for the surface ocean box temperatures (2.2.5, 2.2.10, 2.2.6, 2.2.11) as well as the ice melting/production equation 2.2.20. The role of $D$ has competing effects in these two equations. For the ocean box temperatures, $D$ appears in the term corresponding to energy loss due to the presence of the ice cover, and thus reduces the ocean temperatures. However this energy loss is balanced in the system by the ice melting/production term, where $D$ appears in a term that encourages melting. There are also other feedbacks in the system, notably the indirect effect of $D$ on the strength of the ocean circulation, and thereby oceanic heat transport, which we have already seen can strongly affect ice cover. Figure 2.12 shows the ice thickness profiles and ocean circulation strengths for our default effective emissivity $\varepsilon = 0.7$ and model
initialization as discussed in Section 2.3, varying with $D = 0.5\text{m}$, $D = 1\text{m}$ (default value), $D = 2\text{m}$, $D = 5\text{m}$, and $D = 10\text{m}$:

![Ice Thickness Profiles](image1)

![Ocean Circulation](image2)

**Figure 2.12.** Ice thickness profiles and ocean circulation strengths with different $D$ parameter values.

For smaller values of $D$ we see higher latitude ice margins, and as $D$ is increased the ice margin pushes further south, down to a full Snowball Earth state for $D = 10\text{m}$. The ocean circulation responds by weakening as $D$ increases, down to a near complete shutdown of 0.1 Sv (poleward) with $D = 5$, and reversing directions and changing circulation regimes to an equator-ward flow of 4.9 Sv with $D = 10$, the case that results in a Snowball Earth state. While certainly quantitative effects to perturbations will be felt in the model, qualitatively our results are not dependent on the choice of $D$, apart from these extreme cases of $D = 5, 10\text{m}$ that result in a shutdown of the ocean circulation for this default parameter setup.

### 2.4.2 Time-Varying Insolation Forcing

Changes in the Earth’s orbit correspond to changes in the strength of insolation, which can have an impact on the Earth’s global climate. These are referred to as the Milankovitch cycles, and consist of three components. Eccentricity refers to the changes in the elliptical orbit of the Earth around the sun (400k and 100k year cycles),
obliquity refers to the changes in the tilt of the rotational axis of the Earth (41k year cycle), and precession refers to the changing orientation of the Earth’s rotational axis (23k year cycle). There have been various studies that link the Milankovitch cycles to the Earth’s paleoclimate record, a famous example being Hays, et al. in [HIS76], which studied a climate record of the past 468,000 years from ocean sediment core samples. In this study, Hays et al. found the relative importance of these cycles on global climate response to be eccentricity having highest effect, followed by obliquity, and precession having the smallest relative effect on climate. In another more recent study by Zachos et al. in [ZPS+01], a longer paleoclimate time series of 4.5 million years was used, and it was found that the obliquity cycle has the strongest effect on global climate, followed by eccentricity and precession.

Figure 2.13. Obliquity parameter for 5Ma, along with associated power spectrum. Peak is at 41kyr cycle.

The parameterization of insolation in Equation 2.2.9 of [ML12] includes dependence on the orbital parameters corresponding to the two strongest Milankovitch cycles, obliquity and eccentricity. In addition, these values can be calculated using celestial mechanics, and Laskar et al. ([LRJ+04]) has numerical values for these orbital parameters from 250 million years in the past to 20 million years in the future, at 1000 year intervals. We use these obliquity and eccentricity parameter calcula-
Figure 2.14. Eccentricity parameter for 5Ma, along with associated power spectrum. Peaks are at 100kyr and 400kyr cycle.

tions, displayed in Figures 2.13 and 2.14 for the last 5 million years along with the associated power spectrum. Obliquity shows a strong peak in the power spectrum at 41,000 years, and eccentricity shows a peak at 400,000 years, as well as a slightly weaker peak at 100,000 years. Using these variable parameters, we use Equation 2.2.9 to calculate insolation over the last 5 million years, and show the results for a single latitude at 45° N in Figure 2.15. We also calculate its power spectrum, and while we don’t see a strong response from the eccentricity cycle, we do see a peak around 40,000 years, slightly off from the obliquity cycle.

We can then run our model with this time-varying insolation forcing to see if this impacts our results. In Figure 2.16 we show a 250,000 year run with the setup of our stable small ice cover simulation with time-varying insolation. We note while we see fluctuations in global total ice volume and oceanic circulation strength, the change in insolation is not strong enough in the model to move the ice margin. We see similar results, with the global and near global ice cover simulations, notably that the varying insolation is not large enough to initiate Snowball Earth in the near global ice cover simulation, nor enough to pull the model out of a Snowball state once initiated.
Figure 2.15. Average annual insolation at 45°N for 5 million years, and associated power spectrum.

Figure 2.16. Total ice volume and ocean circulation under time varying insolation forcing. The observed cycle corresponds to the roughly 41kyr obliquity cycle present in Figure 2.15.
2.5 Discussion and Conclusions

We have presented a low dimensional climate model consistent with elements of classical low dimensional models. The radiative balance terms similar to those in energy balance models produce states of ice cover consistent with classical energy balance model results of two stable solutions, a small ice cover and a global ice cover, as well as an unstable solution, a large but finite ice extent. The ocean box model produces ocean states in two circulation regimes: a strong, poleward, thermally driven circulation, as well as a weaker, equatorward, saline driven circulation. A summary of these results is given in Table 2.5.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>Effective Emissivity</th>
<th>Circulation</th>
<th>Direction</th>
<th>Density Driver</th>
</tr>
</thead>
<tbody>
<tr>
<td>ice free</td>
<td>$\varepsilon = 0.6$</td>
<td>28.3 Sv</td>
<td>poleward</td>
<td>thermal</td>
</tr>
<tr>
<td>partial ice</td>
<td>$\varepsilon = 0.7$</td>
<td>21.9 Sv</td>
<td>poleward</td>
<td>thermal</td>
</tr>
<tr>
<td>near global ice*</td>
<td>$\varepsilon = 0.805$</td>
<td>0.5 Sv</td>
<td>poleward</td>
<td>thermal</td>
</tr>
<tr>
<td>global ice*</td>
<td>$\varepsilon = 0.81$</td>
<td>3.8 Sv</td>
<td>equatorward</td>
<td>saline</td>
</tr>
</tbody>
</table>

Table 2.2. Summary of results. Asterisk indicates simulations with insolation at 94\% of present day values.

Regarding results from the ice component of the model, we note our near global glaciation case provides results very similar to GP03 in their partial glaciation case, despite some key differences in modeling approaches. Notably, we include dynamic and coupled oceanic heat transport, which GP03 mention as 'perhaps the most significant neglect' of their model. We note our ice melting/production term equation 2.2.20 is similar to the basal melting term of GP03, though we neglect geothermal heat for simplicity, which McKay argues in [McK00] is less than the primary energy balance of sunlight penetrating the ice surface and latent heat from basal freezing by a factor of 50, and deemed negligible in our own experiments (not shown). The model of GP03 also includes a parameterization for precipitation minus evaporation that drives a surface ice melting/production term, which is not present in our model.
as we do not consider atmospheric effects. Another key difference is that rather than taking the ice surface temperature to be a constant forcing based on the air surface temperature forcing, we model this dynamically thus allowing the ice component to respond to the radiative balance. We note the air surface temperature forcing used in the global glaciation experiment of GP03 is in line with our steady state ice surface temperature in the partial glaciation case (Figure 2.6), whereas in our global glaciation simulation we get ice surface temperatures roughly 10 to 30 degrees cooler than this air temperature forcing.

The partial glaciation experiment of GP03 was forced with external forcing parameters consistent with large, but finite ice cover in the GCM simulations of Snowball Earth (see [PPJ01]), which are comparable to our near global glaciation experiment (see Figure 2.10). The ice thicknesses are of the same order of magnitude, with GP03 achieving about 200m near the pole. In our model the thickness reaches about 500m. The ice margins are also comparable, with GP03 reaching an ice margin of just under 70°; our experimental results yield an ice margin situated just below 80° colatitude.

There is a notable difference in the ice velocity however: our computed ice velocities are an order of magnitude less those reported in GP03. One possible reason for this is the viscosity parameter \( \mu \), which GP03 are only required to calculate once due to a static surface temperature forcing, is recalculated in our model in response to the dynamic surface temperature in Equation 2.2.19. Our steady state surface temperature (shown in Figure 2.11) is cooler than the forcing used in GP03 partial glaciation case, and thus our cooler surface temperature creates more viscous ice, slowing down ice flow.

The results of our global glaciation are more in line with that of GP03. While we have global ice cover of roughly 500m at the pole to 150m at the equator, GP03 (later corrected in [LP11] with a bug fix in the back pressure term Equation 2.2.24) give roughly uniform coverage of around 800m. The ice velocities of the two experiments are also roughly in line, with our ice reaching peak velocity of nearly 100 m/year
Figure 2.17. Ice thickness and velocity profiles after reaching equilibrium with $\varepsilon = 0.86$, with and without viscous ice flow effects.

close to the center of the hemisphere (Figure 2.8), and GP03 achieving a top ice velocity of roughly 60 m/year further towards the equator. As we turn off the viscous flow term in our model, we also observe that the ice margin is pulled back from the equator to an ice margin of 58° colatitude, again similar to GP03. This result is consistent with their conclusion that due to the ice thicknesses in this ‘sea glacier’ framework, viscous flow effects are a crucial component, allowing ice to penetrate regions of melting, and ultimately encouraging initiation of Snowball Earth through runaway ice albedo feedback.

We have constructed the model in such a way as to minimize external forcing down to a single insolation field, and have chosen a parameterization that allows us to vary this forcing temporally through key orbital parameters that in turn represent the Milankovitch cycles. While our model does respond in terms of total ice volume to these Milankovitch cycles, obliquity being the most pronounced, the model is not sensitive enough to these forcing changes to exhibit changes in the ice margin.

The main conclusion we reach from this study is that ocean circulation and its associated heat transport does play a vital role in determining the global climate state and ice cover. We have seen in the partial glaciation case that the ocean circulation
severely inhibits ice growth and viscous ice flow, and we have seen in the near global glaciation case that even in that state’s severely weakened ocean circulation, lack of oceanic transport leads to a drastically different Snowball Earth state. This is consistent with the results of Poulsen et al. in [PPJ01], who for the first time used a coupled GCM to simulate Neoproterozoic climate. In [PPJ01], Poulsen et al. also found that they can achieve a Snowball state in model runs with no oceanic heat transport or diffusive oceanic heat transport only, but simulations with fully coupled ocean model and its associated heat transport result in partial ice cover with oscillating sea ice margin.

While atmospheric effects were largely neglected for simplicity and because our focus was on the role of oceanic heat transport in the Snowball Earth setting, including an atmospheric component would be the natural progression of this work. In addition to parameterizing atmospheric heat transport, a parameterization for wind could also be used to model wind driven circulation, a component of ocean circulation not present here.
Chapter 3

An Iceberg Parameterization in CICE

3.1 Background

Icebergs have been of interest to the scientific community for a number of reasons. In the North Atlantic, icebergs pose as dangerous obstacles to ships and off shore structures, and there has been significant focus on hindcasting/forecasting iceberg trajectories (e.g. [Gar85], [Kor89], [Smi93], [BWSJ96], [GBN01], [LH01], [KSSC05]). As icebergs melt, they alter the composition of the ocean, which has been the subject of numerous studies (e.g. [GBN01], [Sav01], [SBN06], [KSSC07]). Interestingly, some of the earliest work in quantifying iceberg deterioration was in the 1970s, with the aim to determine if icebergs could be a viable source of freshwater for human populations in drier climates (e.g. [WC73], [WM78]).

One of the largest icebergs in recorded history, named B15, calved off the Ross ice shelf in March of 2000, and became grounded near the McMurdo station in Antarctica. Its size prevented break up of summer sea ice, disrupting supply ships to the station as well as the neighboring penguin colonies ([ABD06]). We use this as a motivating example to study the dynamic interaction between icebergs and sea ice.

Icebergs in the Antarctic tend to be much larger than their Arctic counterparts, due to the fact that the Antarctic ice sheet is in contact with the ocean over a much larger area than the Greenland ice sheet, which has comparatively few outlet glaciers to inject icebergs into the ocean. Furthermore, Arctic icebergs are typically injected into the ocean where they are quickly brought to open ocean and warmer waters, conditions ripe for deterioration, whereas in the Antarctic, the coastal current may keep icebergs close to the coast where they will interact with sea ice, particularly in the Weddell Gyre, before drifting into the strong Antarctic Circumpolar Current.
(ACC) that has conditions favorable for iceberg deterioration. Icebergs comprise a much higher percent of freshwater mass flux from the Antarctic ice sheet, whereas in the Greenland ice sheet, basal meltwater is the dominant mechanism of freshwater mass flux. It is estimated that the global iceberg calving flux is $\approx 2300 \text{ Gt/yr}$ (1 Gt = $10^{12}$ kg), about 90% of which occurs in the Antarctic [MA10]. Our aim in this study is to incorporate an iceberg implementation into a full GCM, and for the reasons aforementioned our focus will be in the Antarctic.

### 3.2 Model Description

We have implemented an iceberg parameterization in CICE (Community Ice CodE), the Los Alamos National Laboratory’s sea ice model. Icebergs are dynamically and thermodynamically active, and dynamically interact with the surrounding sea ice pack. Ocean forcing is taken from the CCSM3 (Community Climate System Model) 1990 control run (b30.2009) [CBB06], averaged over 20 years into monthly values, including full depth currents. At this point we are not yet running with an active ocean component, so feedbacks from ocean coupling (from iceberg meltwater and the associated latent heat flux) are not present in the following results. Atmospheric forcing is taken from a modified version of the Common Ocean Reference Experiment (CORE) [GBB09], where the 6 hour forcing is interpolated to the sea ice time step (1hr).

#### 3.2.1 Iceberg Dynamics

Iceberg dynamics are governed by a momentum equation, and Smith in [SB83] introduced the iceberg momentum equation as a force balance. We follow the approach of Lichay and Hellmer in [LH01], which includes a forcing term for sea ice, a term that is commonly absent in modeling studies focused on the Arctic:

$$m \frac{du_b}{dt} = F_a + F_w + F_c + F_{ss} + F_{si}$$  \hspace{1cm} (3.2.1)
where $m$ is iceberg mass, $u_b$ is iceberg velocity, $t$ is time, and $F_*$ are the forcing terms (atmospheric, ocean, Coriolis, sea surface slope, and sea ice). Forcing from the wind and ocean currents, $F_a$ and $F_w$, follow quadratic drag laws:

$$F_a = \rho_a \left( \frac{1}{2} c_a A_{va} + c_{da} A_{ha} \right) |u_a - u_b|(u_a - u_b)$$  \hspace{1cm} (3.2.2)

$$F_w = \rho_w \left( \frac{1}{2} c_w A_{vw} + c_{dw} A_{hw} \right) |u_w - u_b|(u_w - u_b)$$  \hspace{1cm} (3.2.3)

where $\rho_a, \rho_w$ are the densities of air and water, $c_a, c_w, c_{da}, c_{dw}$ are drag coefficients, $A_{va}, A_{vw}$ are vertical surface areas of the iceberg in contact with air and water, $A_{ha}, A_{hw}$ are horizontal surface areas of the iceberg in contact with air and water, and $u_a, u_w$ are the air and ocean velocities. The CCSM ocean data file defines ocean depth at 40 levels, and the used ocean velocity is integrated over the length of the iceberg draft. The Coriolis forcing is given by

$$F_c = -mf k \times u_b$$  \hspace{1cm} (3.2.4)

where $f = 2\Omega \sin \phi$ is the Coriolis parameter, $\Omega$ is Earth’s rotation, $\phi$ is latitude, $k$ is outward normal. For the pressure gradient, or sea surface tilt, force term, we approximate the sea surface tilt $\nabla H$ with a geostrophic approximation, and the term is given by

$$F_{ss} = -mg \nabla H \approx mf k \times u_w$$  \hspace{1cm} (3.2.5)

This sea surface tilt approximation is common for sea ice modelers, and available data for sea surface height was not reliable. The primary force balance in equation 3.2.1 are the Coriolis and pressure gradient forces, which both scale with iceberg mass $m$.

The sea ice term follows that of Lichey and Hellmer ([LH01]), and allows the sea ice to 'capture' the iceberg under sufficiently strong and dense sea ice pack:

$$F_{si} = \left\{ \begin{array}{ll}
0 & \text{if } a_i < 15\
\frac{1}{2} \rho_i c_i A_{visi} |u_i - u_b|(u_i - u_b) & \text{otherwise}
\end{array} \right. \hspace{1cm} (3.2.6)$$

$$u_b = u_i$$
where \( \rho_i \) is density of ice, \( c_i \) is the drag coefficient, \( A_{vsi} \) is the vertical iceberg surface area in contact with sea ice, \( \mathbf{u}_i \) is the sea ice velocity, \( a_i \) is the sea ice concentration, \( P \) is the sea ice strength, and \( P_s \) is a threshold strength. When the sea ice strength and concentration are above thresholds, the sea ice locks in the iceberg and \( \mathbf{u}_b = \mathbf{u}_i \).

For simplicity, geometric quantities are calculated using tabular icebergs with a fixed width:length ratio of 1:1.5. The vertical iceberg lengths in contact with water \( (h_{bw}) \), sea ice \( (h_{bi}) \), and air \( (h_{ba}) \) are given by Archimedes’ Principle:

\[
\begin{align*}
h_{bw} &= \frac{\rho_b h_b + \rho_i h_i}{\rho_w} \\
h_{bi} &= h_i \\
h_{ba} &= h_b - (h_{bi} + h_{bw})
\end{align*}
\]

where \( h_b \) and \( h_i \) are the iceberg thickness and sea ice thickness.

The iceberg momentum equation (3.2.1) is solved numerically using a simple predictor-corrector method consisting of a forward Euler predictor followed by a backward Euler corrector, on a time step of 2 minutes subcycled over the sea ice time step of 1 hour. Icebergs are not permitted to move into a cell that is full with other icebergs, and may become grounded if the draft of the iceberg exceeds the bathymetry, where the bathymetry data used is from the CCSM ocean data file. Density and drag coefficient constants are given in Table 3.1, and we note we use the same drag coefficients as Lichey and Hellmer (LH01).

In addition to the icebergs being dynamically affected by the surrounding sea ice, sea ice can also be dynamically affected by icebergs. So as not to create a positive feedback, where both icebergs and sea ice accelerate each other to numerical instability, we consider separate cases of whether the icebergs and sea ice are moving in similar or opposing directions. When the iceberg and sea ice are moving in opposing directions, defined as when the angle between the velocity vectors is between 90° and 270°, an additional forcing term is added to the sea ice momentum equation, taking
Figure 3.1. Schematic of iceberg / sea ice dynamic interaction in two regimes. (a) When an iceberg is moving against the sea ice, an additional term (Eq. 3.2.7) is added to the sea ice momentum equation to account for this dynamic interaction. (b) When an iceberg is moving in a similar direction as the sea ice, the amount of displaced sea ice (shaded) is ridged behind the iceberg.

the form of a quadratic drag law:

\[ F_b = \frac{1}{2} \rho_i c_i A_b |u_b - u_i| (u_b - u_i) \] (3.2.7)

where \( \rho_i \) is the density of sea ice, \( c_i \) is the drag coefficient, and \( A_b \) is the area of the sea ice/iceberg interface. This forcing term is only applied to nodes where sea ice velocity is pointing into the grid cell with the iceberg (i.e. upstream sea ice flow), as illustrated in Figure 3.1a. As we will see in our simulations, the sea ice pack moves much faster than the icebergs, and in this way the iceberg acts as an obstacle to the faster moving sea ice. When the icebergs and sea ice are moving in similar directions (the angle between their velocity vectors \( \theta \) is between \(-90^\circ \) and \(90^\circ \)), the sea ice that is displaced by the iceberg motion, as calculated in 3.2.8, ridges (mechanically deforms vertically) behind the iceberg, illustrated in Figure 3.1b.

\[ A_d = \frac{W + L}{2} \Delta t |u_b - u_i| \] (3.2.8)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_b )</td>
<td>berg density</td>
<td>900 kg m(^{-3})</td>
</tr>
<tr>
<td>( \rho_i )</td>
<td>sea ice density</td>
<td>900 kg m(^{-3})</td>
</tr>
<tr>
<td>( \rho_w )</td>
<td>ocean density</td>
<td>1025 kg m(^{-3})</td>
</tr>
<tr>
<td>( \Omega )</td>
<td>angular velocity</td>
<td>(7.292 \times 10^{-5}) rad s(^{-1})</td>
</tr>
<tr>
<td>( c_i )</td>
<td>sea ice coefficient of resistance</td>
<td>1</td>
</tr>
<tr>
<td>( c_a )</td>
<td>atmosphere coefficient of resistance</td>
<td>0.4</td>
</tr>
<tr>
<td>( c_w )</td>
<td>ocean coefficient of resistance</td>
<td>0.85</td>
</tr>
<tr>
<td>( c_{da} )</td>
<td>atmosphere drag coefficient</td>
<td>(2.5 \times 10^{-4})</td>
</tr>
<tr>
<td>( c_{dw} )</td>
<td>ocean drag coefficient</td>
<td>(5 \times 10^{-4})</td>
</tr>
<tr>
<td>( P_s )</td>
<td>critical sea ice strength</td>
<td>(1 \times 10^4) N m(^{-1})</td>
</tr>
<tr>
<td>( \Delta t_b )</td>
<td>berg time step</td>
<td>2 minutes</td>
</tr>
<tr>
<td>( \Delta t_i )</td>
<td>sea ice time step</td>
<td>1 hour</td>
</tr>
</tbody>
</table>

Table 3.1. Constants used in the iceberg calculations. All sea ice parameters are set as in [HL08] and [Hun10].

where \( W, L \) are the iceberg dimensions and \( \Delta t \) is the timestep. Additionally, the presence of the iceberg displaces area that cannot be occupied by sea ice (perhaps leading to additional ridging). While combination of equations may not conserve energy, there is no increase in energy, and energy is dissipated though individual equations do conserve momentum.

3.2.2 Iceberg Thermodynamics

Following Gladstone et al. in [GBN01], icebergs melt primarily by three mechanisms: basal melting due to turbulence, lateral melt due to buoyant convection, and erosion due to waves, and we note much of this work is empirical in nature. Basal melting due to turbulence between ocean and berg motion was first estimated by Weeks and Campbell in [WC73] as

\[
M_b = 0.58 |u_w - u_i|^{0.8} \frac{T_s - T_i}{L^{0.2}}
\]

(3.2.9)

where \( L \) is the iceberg length, \( T_s \) is the sea surface temperature, and the iceberg temperature is assumed a constant \( T_i = -4^\circ\)C (from Løset in [Løs93]). The temperature contrast between the ocean and sides of the icebergs leads to lateral melt...
due to buoyant convection along the sides of the iceberg, and was first estimated by El-Tahan et al. in [ETVET87] as

\[ M_l = 7.62 \times 10^{-3} T_w + 1.29 \times 10^{-3} T_w^2 \] (3.2.10)

where \( T_w \) is the ocean water temperature. Wave erosion reduces iceberg mass in a manner that was first estimated by Bigg et al. in [BWSJ97] to be proportional to the sea state \( S_s \) as

\[ M_e = 0.5 S_s \]

This was later extended by Gladstone et al. in [GBN01] to include sea surface temperature dependence as well as sea ice concentration \( a_i \):

\[ M_e = \frac{1}{12} S_s (1 + \cos(a_i^3 \pi)) (T_s + 2) S_s \] (3.2.11)

Following Martin and Adcroft in [MA10], we estimate the sea state by a fit to the Beaufort scale

\[ S_s = \frac{3}{2} \sqrt{|u_a - u_w|} + \frac{1}{10} |u_a - u_w| \]

The mass balance of an iceberg is then given by ([MA10])

\[ \rho_b \frac{d(LWH)}{dt} = \rho_b (-LWM_b - T(L + W)(M_l + M_e)) \] (3.2.12)

We note this erosion term also accounts for loss of iceberg mass through calving from wave erosion, and is thus not entirely a thermodynamic melting equation. As with previous studies (e.g. [BWSJ97], [MA10]), we allow the iceberg to rollover according to the Weeks-Mellor stability criterion ([WM78]):

\[ L < \sqrt{0.92H^2 + 58.32H} \] (3.2.13)

where if this is violated, we switch the width and height dimensions.
3.2.3 Lagrangian Framework

Novel to our modeling approach, we consider icebergs in two different frameworks. The first is as Lagrangian particles, where individual icebergs are tracked as a single, coherent unit by their center of mass, following a particle in cell method ([HE55]). Forcing data is interpolated from grid cell corners to the iceberg’s location within the cell by inverse area weighting factors. The iceberg information is extrapolated to the grid nodes with these same weighting factors. We allow icebergs to overlap grid cells, but cannot enter grid cells that are either too full from other icebergs or sea ice, or if the bathymetry is too low. This framework is useful in modeling particular giant iceberg events (the National Ice Center classifies icebergs longer than 10 nautical miles as 'giant’, where 1 nautical mile = 1.852km), but quickly becomes computationally infeasible to model a realistic iceberg population, estimated to be on the order of tens to hundreds of thousands.

3.2.4 Eulerian Framework

To combat the computational expense of the Lagrangian framework, we have developed a second modeling framework where the icebergs are modeled as an Eulerian fluid field. Icebergs take up a certain fractional area of each grid cell, rather than having precise location. This field of icebergs is advected in a similar manner as sea ice through an incremental remapping scheme, as described by Lipscomb and Hunke in [LH04]. Iceberg size distribution is commonly estimated as a log normal distribution, and to account for this range of sizes, we distribute iceberg mass across multiple size categories as detailed in Table 3.2. This size distribution is originally in Gladstone et al. [GBN01], and largely based on measurements from ship’s observations, but has also been used for more current studies, such as Martin and Adcroft ([MA10]). While each size category is intended to represent a range of sizes, the representative dimensions listed in Table 3.2 are used for purposes of calculating the melting equations.
<table>
<thead>
<tr>
<th>Category</th>
<th>Length (m)</th>
<th>Height (m)</th>
<th>Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>40</td>
<td>0.24</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>67</td>
<td>0.12</td>
</tr>
<tr>
<td>3</td>
<td>200</td>
<td>133</td>
<td>0.15</td>
</tr>
<tr>
<td>4</td>
<td>350</td>
<td>175</td>
<td>0.18</td>
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<tr>
<td>5</td>
<td>500</td>
<td>250</td>
<td>0.12</td>
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<td>6</td>
<td>700</td>
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<tr>
<td>7</td>
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<td>250</td>
<td>0.03</td>
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<tr>
<td>8</td>
<td>1200</td>
<td>250</td>
<td>0.03</td>
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<td>9</td>
<td>1600</td>
<td>250</td>
<td>0.03</td>
</tr>
<tr>
<td>10</td>
<td>2200</td>
<td>250</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**Table 3.2.** Iceberg size distribution with associated representative dimensions.

**Figure 3.2.** After iceberg melting is calculated, remaining iceberg mass is redistributed across the size categories (rebinning) according to % of mass lost, to reflect shrinking iceberg population. Here $l_i, h_i$ for $i = 1, 2, 3$ are the representative sizes of the icebergs in each category (bin).

discussed in Section 3.2.2 as well as for calculating geometric quantities used in the forcing terms of the iceberg momentum equation. Iceberg mass in each size category is tracked and advected, rather than the particular number of icebergs or individual tracks. As icebergs lose mass through melting and erosion terms, the total iceberg mass is redistributed across the size categories after each melting step, as shown in Figure 3.2.
3.2.5 Iceberg Generation

We are currently running this iceberg parameterization in CICE alone, and thus are not coupled to a land ice component that would provide calving masses. Instead, we divide the estimated 2000 Gt/year Antarctic calving flux into four calving events, equally spaced throughout the year. We then distribute this mass evenly across all Antarctic coastal grid cells for each of the calving events, with the mass in each cell distributed across each of the ten size categories according to the distribution in Table 3.2. While this uniform spatial distribution is certainly an approximation, Martin and Adcroft found in their model that about two thirds of coastal grid cells around Antarctica have an annual calving flux of at least 1 Gt/year. The regions of highest iceberg calving are in the Ross Sea, Amundsen Sea, southwest Weddell Sea, and Davis Sea regions ([MA10]).

3.3 Results

3.3.1 Lagrangian Icebergs

The particular iceberg event we will use to test our model is a giant iceberg named C7, which calved east of the Weddell sea on March 5, 1990. Size and location data were obtained from the National Ice Center over a period of 3 years. Detailed results of this study are presented in Hunke and Comeau ([HC11]), although some important differences should be noted. The iceberg thermodynamics component of the model was not active, as the focus of that study was on the effect of the dynamic interaction between the icebergs and sea ice on the surrounding sea ice, and thermodynamic feedbacks were not part of the study. The geometry of the icebergs was assumed to be cylindrical (as opposed to tabular) due to simplicity in calculating normal forcing faces, though this has negligible impact on our results.

In Figure 3.3 we show the results of four icebergs released on March 5th, 1990
with the observed dimensions of the C7 iceberg event (37km x 18km, 225m height, with a mass of 150 Gt). The model was run for three years, and the available data from the National Ice Center for this iceberg over two years is marked as the blue trajectory. We note there were only 23 data points, resulting in long stretches of linear interpolation in the observational track. In the model simulations, icebergs move in inertial oscillations, and make long strides when the sea ice capturing mechanism is activated in Equation 3.2.6. Due to the sensitive nature of the system, small differences in initial position result in vastly different trajectories. For example, the iceberg labeled 1 becomes grounded early in the model run, and is unable to move (we again note in these simulations, iceberg thermodynamics is not active so the iceberg cannot become free through melting). While our goal is not to be able to hindcast iceberg trajectories with high accuracy, our model trajectories largely followed observed iceberg trajectories in the Weddell Sea, for example as those found in Schodlok et al.’s observational record (SHRF06).
Figure 3.4. Differences between simulations with and without bergs. Thickness differences in cm for (a) May and (b) August 1992. Differences in (d) area (%) and (e) ridged ice mean thickness (cm). The black contour line in (d) is the 90% sea ice area contour.
In Figure 3.4, we show the effect on sea ice thickness and concentration due to the dynamic interaction with icebergs. As shown in (a), (b), anomalies in the sea ice thickness precede icebergs, and are then carried by faster moving sea ice, stretched with flow. The anomalies are largest during winter, and because the perennial sea ice pack in the Antarctic is quite small in extent, these anomalies are largely eliminated during the summer months. When an iceberg is near the ice margin, as in (d) (the black contour line is 90% sea ice area coverage), it is in a position to have the biggest impact on sea ice concentration, as the ice concentrations are high enough so that the interaction cannot be avoided and additional ridging is induced, yet low enough that the iceberg is not captured by the sea ice, in which case there is no dynamic interaction (since then $u_i = u_b$). This would be level ice production, as the iceberg induces areas of open water in its wake. When the iceberg is within higher concentration sea, the dynamic interaction leads to additional ridged ice, as there is not enough open space for level sea ice to form, as illustrated in (e). The total sea ice anomaly from the four icebergs at the end of the three year run is $\approx 24.7 \text{ km}^3$, which is about 0.23% of the Weddell sea ice volume. Further details of the study, including different iceberg initializations and comparison of different dynamic interaction parameterizations, can be found in Hunke and Comeau (HC11).

3.3.2 Eulerian Icebergs

We initialize icebergs by simulating calving events at coastal grid cells in the models. The estimated annual Antarctic calving flux of 2000 Gt/year is divided into 4 equal calving events evenly across the year, and the calving mass is equally distributed among all coastal Antarctic grid cells, where it is then initially divided into each size category according to the distribution listed in Table 3.2.

In Figure 3.5, we show the iceberg concentration (log scale) at the end of a 20 year simulation. There is largely a perpendicular gradient from the coast, and as time
Figure 3.5. Iceberg concentration (log scale) after 20 year simulation, 2000 Gt/year calved evenly across all coastal grid cells, divided into 10 size categories. Iceberg concentration largely follows a perpendicular gradient from the coast, and move clockwise along with the Antarctic Circumpolar Current (ACC).

The system seems to be in a quasi-steady state, and we note the maximal extent of the icebergs is at about 60°S, roughly in line with observations ([GBN01]). An animation of this full simulation can presently be found at:

http://www.math.arizona.edu/~dcomeau/icebergs/iceberg_concentration.gif

In Figure 3.6 we show the iceberg meltwater flux (kg/m²/s, log scale), at the end of the 20 year simulation. Not surprisingly, the spatial distribution of meltwater closely follows that of the iceberg concentration. However the iceberg meltwater exhibits a strong seasonal cycle, whereas the iceberg concentration does not. This is partially due to the fact that the erosion term, which is the dominant melting/deterioration mechanism, is not present during winter months when the sea ice concentration is high enough to block wave action. An animation of this full simulation can presently be found at:
Figure 3.6. Iceberg meltwater flux (kg/m²/s, log scale) after 20 year simulation, 2000 Gt/year calved evenly across all coastal grid cells, divided into 10 size categories. Iceberg meltwater follows a similar spatial pattern to iceberg concentration, though exhibits a strong seasonal cycle in magnitude.

http://www.math.arizona.edu/~dcomeau/icebergs/iceberg_meltwater.gif

In Figure 3.7 we show the time series plot of the iceberg meltwater mass (integrated over the domain and time to give monthly values) and total iceberg mass. While the meltwater exhibits the strong seasonal cycle, the amount melted represents a small enough percent of the total iceberg mass, that there is not a seasonal cycle present in the iceberg concentration. After around 10 years, the annual meltwater slowly approaches 2000 Gt to balance out the annual calving flux, indicating the system is close to an equilibrium state. We note these iceberg concentration and meltwater plots are in broad agreement with the similar modeling studies (e.g. SBN06, JDF+09, MA10).
Figure 3.7. Monthly berg meltwater and total Antarctic berg mass for 20 year simulation, 2000 Gt/year calved evenly across all coastal grid cells, divided into 10 size categories.

Figure 3.8. Dynamic induced sea ice anomaly from comparing a simulation with icebergs to a simulation without icebergs.
3.3.3 Sea Ice

In Figure 3.8, we show a time series plot of the monthly sea ice anomaly and this anomaly as a percent of total sea ice. We note there are strong seasonal cycles in each that are exactly out of phase - the largest anomalies in size occur during the winter months, but make up the largest percent of total sea ice during the summer months of high melting. The total sea ice anomaly oscillates between $5 \cdot 10^3$ and $15 \cdot 10^3$ km$^3$, and comprises around 10% of the total Antarctic sea ice. These results should not be directly compared to the Lagrangian iceberg case, as here we are modeling a realistic regional iceberg volume, as opposed to just 4 single giant icebergs.

![Figure 3.8](image)

**Figure 3.9.** Difference in sea ice thickness (cm) from comparing a simulation with icebergs to a simulation without icebergs. Sea ice anomalies are highly localized, and typically wiped out with summer sea ice retreat.

In Figure 3.9, we show the results of the dynamic interaction with the Eulerian iceberg run on sea ice thickness. Highly localized areas of increased sea ice production are present, as well as areas of reduced sea ice thickness close to the coast, notably by
the Ronne-Filchner ice shelf near the south Weddell Sea. In Figure 3.10, we show the difference in ice cover (sea ice + icebergs) compared to a model run without icebergs, which has significant impact on albedo in the region. The animations for the full simulation can presently be found at:

http://www.math.arizona.edu/~dcomeau/icebergs/seaice_anomaly.gif
http://www.math.arizona.edu/~dcomeau/icebergs/ice_cover.gif

Figure 3.10. Difference in ice cover \% (icebergs + sea ice) from comparing a simulation with icebergs to a simulation without icebergs. Changes in ice cover will lead to changes in albedo.

3.3.4 Comparing Modeling Frameworks

As we have two frameworks for modeling icebergs, it’s reasonable to examine a fair as possible comparison of the two. In Figure 3.11, we show the results of a 3 year experiment of 4 icebergs through the Lagrangian framework, with the dimensions of the observed C7 event, displayed against the sea ice concentration. In Figure 3.12, we run the same initial iceberg quantities through the Eulerian framework, and plot
Figure 3.11. Iceberg trajectories and sea ice concentration after 3 year simulation of 4 initial C7 size bergs run through Lagrangian framework.

the resulting iceberg concentration. Since the Eulerian framework does not track icebergs as coherent units, the iceberg mass diffuses across the Weddell sea, loosely following the Lagrangian trajectories. As a result of the mass being more spread out, sea ice anomalies are also more spread out than in the Lagrangian framework, and also lessened by approximately an order of magnitude, as shown in Figure 3.13.

In Figure 3.14, we examine the utility of using ten iceberg categories as opposed to grouping all iceberg mass into one category, and using fixed average iceberg dimensions for purposes of calculating forcing terms and melting terms. The resulting spatial distribution in the one category case is noticeably more restrictive than in using all categories, particularly off the coast of East Antarctica and the portion of the ACC outside the Ross sea. This justifies the need for separate iceberg size categories.
Figure 3.12. Iceberg concentration (log scale) after 3 year simulation of 4 initial C7 size bergs run through Eulerian framework. Eulerian icebergs spread out, largely following the trajectories of their Lagrangian counterparts in Figure 3.11.

Figure 3.13. Dynamic induced sea ice anomalies for four giant icebergs through the Lagrangian framework (left) and Eulerian framework (right). The Eulerian framework allows the iceberg mass to spread out, and thus has a smaller effect on the neighboring sea ice than the single, giant coherent units in the Lagrangian framework.
Figure 3.14. Iceberg concentration (log scale) after 3 year simulation, 2000 Gt/yr calved across 1 category (left) vs. 10 categories (right). The difference in spatial distribution warrants the use of multiple size categories.

3.4 Discussion and Conclusions

We have implemented an iceberg parameterization with two frameworks, and use this to study the resulting dynamic effects on sea ice. We emphasize that these feedbacks do not include thermodynamic effects from resulting meltwater injection into the ocean and the associated exchange of latent heat. The global sea ice anomaly from the full Eulerian iceberg run is on the order 10% of total sea ice, and we see roughly 2000 Gt/year of freshwater being injected into the ocean that would otherwise be modeled as coastal runoff. As observational data on total iceberg volume, spatial distribution, and meltwater injection is not readily available, we discuss and compare our results with other modeling studies, although of course there are differences in modeling approaches.

In [SBN06], Silva et al. used data and observations from the National Ice Center (NIC) to create trajectories of giant icebergs. Two melting mechanisms were applied to these icebergs, turbulent basal melting and erosion, to estimate the amount and
spatial distribution of meltwater into the Southern Ocean due to these giant iceberg events. Their erosion term was the same as we used, though their turbulent melting term was a slightly different form. They estimate a freshwater flux for the Southern Ocean to be 50.7 mSv (1mSv = \(10^3 \text{ m}^3/\text{s}\)), whereas our quasi-steady state meltwater of 2000 Gt/year corresponds to approximately 63.4 mSv, so we are in broad agreement.

In \([JDF^{+}09]\), Jongma et al. incorporated iceberg dynamics and thermodynamics into an intermediate complexity climate model ECBilt-CLIO, with coupled ocean and atmosphere components. Their drift component driven by the iceberg momentum equation is a similar form to the one we used, introduced by Smith and Banke in \([SB83]\), and further developed by Smith (\([Smi93]\)) and Bigg et al. (\([BWSJ96]\), \([BWSJ97]\)). The key differences are the lack of a sea ice capturing mechanism, as introduced in Lichey and Hellmer in \([LH01]\), and the presence of a wave radiation force from Smith in \([Smi93]\), which we have neglected here, as it is quite small for tall icebergs (\([HC11]\)). The thermodynamic equations used are almost identical to ours, with the exception of a different parameterization for the sea state \(S_s\) that appears in Equation 3.2.11. Their distribution of meltwater flux (their Figure 2), is in broad agreement with our results, both in spatial distribution and order of magnitude fluxes (note different units). They quantify the resulting sea ice anomalies, though we should emphasize their anomalies are due to thermodynamic feedbacks, as opposed to our results which result strictly from dynamic interactions.

In \([MA10]\), Martin and Adcroft implemented an iceberg parameterization in a full complexity GCM, the CM2G model developed by GFDL, though this model does not have an ice sheet component. Continental run-off is stored in coastal bins and divided among ten iceberg categories, identical to that used in Table 3.2. Their iceberg momentum equation is similar to Jongma et al. (\([JDF^{+}09]\)), and the thermodynamic equations are precisely in the form we use. As they are running with an active ocean component, they run their model over 100 years to get the system into equilibrium. They average an annual calving rate of 2210 Gt/year, close to our prescribed 2000
Gt/year. They exhibit a strong seasonal cycle in calving rates, whereas ours are prescribed until further coupling with CESM (see Section 3.5), but their melting rates are in line with our results (their Figure 1c), as well as their average spatial distribution of meltwater (their Figure 2, not different units). We note that while we generate icebergs by calving mass uniformly across the continent, their iceberg generation is from the land run off model, and thus non-uniform, and while they find high concentration areas of calving (Ross and Amundsen seas, southwest Weddell sea by the Ronne-Filchner ice shelf, and Davis sea region), about two-thirds of their coastal grid cells have a calving flux of over 1 Gt/year. Again we emphasize a key difference in this study compared to ours is that all of their icebergs are treated as Lagrangian particles (or a parcel representing several icebergs), and there is no quantified dynamic interaction with the neighboring sea ice pack.

3.5 Model Limitations and Future Work

We have begun work to couple this iceberg module with the fully coupled Community Earth Systems Model 1.1 (CESM). This coupling will involve the iceberg meltwater and associated latent heat flux being sent to the ocean component, at which point we will be able to observe thermodynamic feedbacks. Currently there is no coupling with the land ice component, and future work would be coupling the calving fluxes from the Antarctic ice sheet to be used for iceberg generation. While we have not implemented this iceberg parameterization in the Arctic for the Greenland ice sheet, this would be a relatively straightforward implementation. We also plan to further develop the iceberg thermodynamics component of the model to move away from empirical equations to an energy conservative form, first by using the existing sea ice thermodynamics. We also envision allowing the smallest icebergs to melt into a sea ice category when small enough.
Chapter 4

Conclusions

We have presented two global climate models with a focus on the role of ice at two extremes of the climate modeling spectrum. In Chapter 2, we introduce a low-dimensional climate model for the purpose of studying the role of oceanic heat transport on the climate state, particularly on ice cover. Building from existing classical simple climate models, specifically an energy balance component similar to Budyko in [Bud69] and a Stommel box model, similar to Stommel in [Sto61] and Griffies and Tziperman in [GT95], as well as more recent work of the sea glacier modeling approach put forth by Goodman and Pierrehumbert in [GP03], we presented a coherent and consistent model with a single parameter, effective emissivity $\varepsilon$, to account for the largely neglected atmospheric effects. One goal of this work was to address a stated neglect of oceanic heat transport by Goodman and Pierrehumbert in their sea glacier model ([GP03]). While we have taken a departure from their approach in some regards, discussed in section 2.5, we still get results largely in line with theirs.

Our model produces climate states varying from ice free, small stable ice cover, large but finite ice cover, to complete global ice cover in a Snowball Earth through the adjustment of the radiative balance through weakening the insolation from present day values to account for a weaker, younger sun in the Neoproterozoic, as well as the emissivity parameter $\varepsilon$, which represents global greenhouse effects. A notable result is that our climate model can move between drastically different states due to the effect of oceanic heat transport. In one particular parameter regime discussed in Section 2.3.2, oceanic heat transport was important enough to keep the ice margin back from the equator, and when this heat transport was turned off, global ice cover occurred. We also investigated the effect of the Milankovitch cycles on our climate model, and
found that while total global ice volume and ocean circulation strength responded to
the Milankovitch cycle corresponding to obliquity, this effect was not strong enough
to actually move the ice margin in any experiment performed.

In chapter 3, we moved to the other end of the climate modeling spectrum towards
complexity, and we presented an implementation of an iceberg parameterization in
the Los Alamos National Laboratory sea ice model CICE. We focus on icebergs in
the Antarctic, which may be much larger than typical Arctic counterparts, and are
more likely to encounter regions of heavy sea ice concentration. We model icebergs
dynamically and thermodynamically, although at present we are not running with
a coupled ocean model, so thermodynamic feedbacks as a result of cooler, fresher
iceberg meltwater injection into the ocean are not present. Novel in our approach is
we quantify a dynamic interaction between the icebergs and sea ice, so that each has
an effect on the other. We also have developed two frameworks for modeling icebergs:
as Lagrangian particles, and as an Eulerian fluid field.

In our Lagrangian framework, we find iceberg trajectories that largely follow ob-
served trajectories in the Weddell Sea. We find that dynamically induced sea ice
anomalies form by sea ice ridging behind the slower moving icebergs, which leads to
areas of open water in front of the iceberg, leading to increased level ice production.
When an iceberg is on the edge of the ice margin, this effect is most pronounced,
whereas icebergs in thicker sea ice concentrations tend to induce more ridged ice
production, as the sea ice is forced to ridge behind the obstacle.

In our Eulerian framework, we are able to simulate a realistic annual calving flux
of icebergs, which we spread across all coastal Antarctic cells. We find a gradient of
iceberg concentration and meltwater injection perpendicular to the coast, stretching
out to approximately 60° S in latitude. The meltwater injection exhibits a strong
seasonal cycle, whereas the iceberg concentration stays quite uniform throughout
the year. These results are consistent with comparable modeling studies. We see
highly localized regions of dynamically induced sea ice anomalies, that largely get
eliminated each year due to the Antarctic sea ice retreat in the summer. We also see a considerable change in total ice cover (sea ice + icebergs), which will have an effect on regional albedo. We are working on implementing this parameterization into the fully coupled CESM, which will allow us to study the thermodynamic feedbacks associated with spatially redistributed freshwater flux from the Antarctic ice sheet.
REFERENCES


