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A SIMPLE MACROECONOMIC MODEL OF TOGO.
THE UNIVERSITY OF ARIZONA, PH.D., 1979
A SIMPLE MACROECONOMIC MODEL OF TOGO

by

Lubin Kobla Doe

A Dissertation Submitted to the Faculty of the DEPARTMENT OF ECONOMICS In Partial Fulfillment of the Requirements For the Degree of DOCTOR OF PHILOSOPHY In the Graduate College THE UNIVERSITY OF ARIZONA

1979
I hereby recommend that this dissertation prepared under my direction by Lubin Kohla Doe
entitled A Simple Macroeconomic Model of Togo
be accepted as fulfilling the dissertation requirement for the Degree of Doctor of Philosophy.

[Signatures and dates]

As members of the Final Examination Committee, we certify that we have read this dissertation and agree that it may be presented for final defense.

[Signatures and dates]

Final approval and acceptance of this dissertation is contingent on the candidate's adequate performance and defense thereof at the final oral examination.
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SIGNED: [Signature]

[Name] Doe
To my parents, Gladys Massan
and Martin Kodzo, to my cousin
John Kpeglo and to the Kufo and
Ahoshi families.
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In the course of my long years of study and learning in Togo, the Ivory Coast, France and in the United States, a large number of people have had a significant influence upon my education. To all of these people, I owe a debt of gratitude. What follows is an inadequate attempt on my part to thank all those whose help has made this particular work come to fruition.

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ABSTRACT

This research deals with a country called Togo; its main objectives are: (1) to develop a static model which permits to identify the determinants of the price of nontraded consumption goods and consequently those of the consumer price index, the nominal per-capita gross national product and the nominal per-capita balance of payments; (2) to study the determinants of the producer price of the agricultural goods exported by the Office des Produits Agricoles du Togo (OPAT); (3) to formulate a dynamic model which permits to examine the steady state properties of the capital labor ratio, the real per-capita government debt and the real per-capita stock of foreign reserves; (4) to estimate the static model, using Togo's data.

Some IS-LM results are maintained in the static model. For instance, an increase in the money stock or a decrease in the lump sum tax will raise the price of the nontraded good. It is also shown that this price will increase if the producer price paid by OPAT increases.

In a dynamic framework, it is shown that the proportion of the export price of crops and consequently the producer price is negatively related to the inventory of
commodities and is positively related to the shadow cost of this inventory. Therefore, any exogenous change in the inventory level and/or in its cost should be taken into account in determining the producer price which OPAT must pay to the farmers.

The results of the dynamic model of the entire economy indicate that, at the steady state, an expansionary fiscal policy will increase the per-capita government debt, the capital labor ratio and the price of the nontraded good. The effect of an expansionary fiscal policy on the steady state value of the per-capita stock of foreign reserves is negative if the policy consists of increasing the government spending on the imported consumption good while it is ambiguous if the policy consists of decreasing the lump sum tax. A contractionary monetary policy which results in an increase in the interest rate on time deposits, at the steady state, will not affect the per-capita government debt while it will raise the capital labor ratio and the price of the nontraded good. Such a policy will have an ambiguous effect on the steady state level of the stock of foreign reserves. The qualitative effects of an increase in the producer price on the endogenous variables are similar to those of an increase in the interest rate on time deposits.

The static model is estimated for the period 1966-77, using ordinary least squares technique. The price index for clothing (a proxy for the price of the imported
consumption good) is found to be the single statistically most significant determinant of the price index for food (a proxy for the price of the nontraded consumption good) and of the consumer price index. The money stock is the major determinant of the nominal gross national product while the price index for exports (a proxy for the producer price) is the single most important determinant of the basic balance (a proxy for the balance of payments).

The empirical results have been used to forecast the endogenous variables beyond the sample period. The projections of the price index for food, the consumer price index and the gross national product seem plausible.

The plan of this study is as follows. First, the major Togolese institutions which are relevant to this research are presented. Second, the most appropriate literature is reviewed. Third, the static model is presented. Fourth, the model which deals with the determination of the producer price is presented, followed by the dynamic analysis of the entire economy. Fifth, the empirical results obtained from estimating the static model are discussed. The study concludes with some projections of the price of the nontraded good, the consumer price index, the nominal gross national product and the nominal balance of payments.
CHAPTER 1
THE INSTITUTIONS

Since my arrival in this country a few years ago, my conversations with Americans of different backgrounds and milieu, when I meet them for the first time, almost invariably follow the same pattern:

"You have an accent; where are you from?" they ask. "Togo", I reply. "Hum, I never heard of that, where is it ..., in Central America, South America, Asia or is it near Pakistan?" "Africa, West Africa", I say. "Where in West Africa, near Uganda, Nigeria?"

After such a dialogue, sometimes the same interlocutor again asks the name of the country, thinking that Togo is the name of the capital or of my home town.

As the questions above reflect, the country of Togo is not familiar to many persons. For this reason, the following sections will attempt to provide the reader with a background solid enough to permit the understanding of the economic relationships which will be explored in the models of this thesis. The first section gives a brief description of the geography and the physical attributes while the second section deals with the economic structure of the country.
The Physical Environment

The Geography

The name Togo derives from Togoville, a small town located on the shore of a lake of the same name. Togo, in Ewe, means "at the edge of the water". The country lies within a rectangle bounded by latitude 1°45' (east) and 0°5' (west) and longitude 11°11' (north) and 6°5' (south). To the north is Upper Volta, to the south is the Atlantic Ocean, to the east is Dahomey and to the west is Ghana. The coastline is 50 km long and the length of the rectangle (north-south) is 510 km. The greatest width from east to west measures 110 km. The country covers 56,000 km² (about one fifth the size of Arizona). Most of the land is flat with more than half of the territory at an altitude of less than 201 m. and scarcely one sixth of the country exceeds 396 m.

The Climate--South of the Attakora Chain

In the southern part of the country, below the 8th parallel, the weather is subequatorial. It is characterized by two rainy seasons (March - June, October - November) and two dry seasons (July - September, December - February). The capital town Lomé receives less rainfall than the other parts of the southern zone. In 1975, the annual average rainfall and temperature in Lomé were respectively 72.25 mm and 26.16°C. Toward the center of the territory, and the
Akposso Plateau, the weather is equatorial with a long rainy season (March - July) and a short rainy period (September - October). The annual average rainfall and temperature in the city of Atakpamé in 1975 were 110.75 mm and 24.85°C.

The Climate--North of the Attakora Chain

A tropical climate prevails with a single rainy season which lasts from March to October at Sokodé and from May to October at Mango. The harmattan wind blows during the dry season and is very strong in January and February.

The Population--Demography

The total population was 1.465 million in 1960, 1.837 million in 1970 and 2.255 million in 1976. The 1970 census indicates that 63.6% of the total population is less than 25 years old. In 1963, school-age children (6-13 years) numbered 408,000, 42% of these children were enrolled compared with 39% in 1965. In 1970, 193,110 students were enrolled in elementary schools, of which 68.6% are male; 17,253 students in junior high and high schools, of which 77.3% are male; 667 students in vocational schools, of which 86.8% are male, and 931,817 students were attending various other schools of which 43.7% are male.

The percentage of the economically active population was 45 in 1960, 44 in 1970 and 43 in 1976. In 1972 it was estimated that more than eight-tenth of the population
reside in localities of 200 to 5,000 inhabitants. In 1973, the urban population numbered 324,000 individuals and the rural population was 1,818,400. The population density was 30 per km$^2$ in 1969, 37 in 1970. The capital Lome has a population estimated at 120,000 inhabitants in 1971.

The Population--Religion

Of the population, 33.3% are of the christian faith. The major congregations are Catholics (20.7%) and Protestants (6.1%). The remaining 12% represent religions such as the Jehovah Witnesses, the Celest Christians, and the Quakers. The protestant pastors of the Bremen Mission were the first to settle in Togo in 1847. However, the first christian edifice on the coast was built in 1835 by Venossa de Jesus, a catholic of Brazilian origin. The catholics and protestants maintain primary, high schools and training institutes throughout the country.

Since the missionaries and the colonizers penetrated the country from the south, most education institutions were located in the south at the outset. In the early years, most of the graduates came from these schools and held responsibility positions in the private and public sectors. There were quite noticeable education and economic gaps between the "literate" south and the other parts of the country. A few years after independence (1960) both the government and the churches made major efforts to build
schools in the other parts of the country and tried to reduce the gaps. For a long time, the College St. Joseph (Catholic) and the College Protestant have been the most prestigious high schools in the country.

Of the population, 12.7% are Muslims. They are particularly numerous in Sokodé and Mango. The majority of the remaining 38.8% of the population have traditional religions such as animism. The beliefs of these religions are diverse. Some of the most common beliefs are the worship of ancestors through the intermediary of the dead and the existence of spirits and ghosts. While in Europe and probably in America some hotels do not have a thirteenth floor and some people do not step outside of their home on Friday the thirteenth or do not walk under a ladder, in Togo, it is also fairly widespread to revere ancestral spirit. As in the rest of Africa, initiation groups and secret societies do exist.

The Structure of the Economy

The work in this section is organized along the following lines. First, the supply side of Gross Domestic Product (GDP) is presented. Second, the demand side of GDP and prices are examined. Third, the macroeconomic policy variables are described.
The Supply of the Gross Domestic Product (GDP)

The GDP at constant market price rose from 34.8 billion FCFA\(^1\) in 1963 to 79.3 billion FCFA in 1973. That is at an annual growth rate of 7.3% over the period. It is estimated that the GDP at current market price will reach 141.6 billion FCFA in 1976, an increase of 14.6% over 1. The Republic of Togo joined the currency union called the Union Monetaire Ouest Africaine or UMOA on November 27, 1963. The other current members of this Union created in 1962 are Dahomey, Ivory Coast, Niger, Senegal and Upper Volta. The only legal monetary unit in the member countries is the Franc de la Communauté Financière Africaine (FCFA). This unit has always been pegged to the French Franc (FF) and the current exchange rate is 1 FCFA = .2 FF. The coins and notes are issued and administered by the Central Bank. The identification number on each bill includes a letter symbol which is specific to the country in which the bill is used. The highest level of authority in the UMOA is the Conference of the Heads of State of the six member countries and the second highest body is the Council of Ministers which determine the monetary policy of the Union. Yet, for all practical purposes, the monetary authorities are the Board of Directors of the common Central Bank called the Banque Centrale des États de l' Afrique de l'Ouest or BCEAO, and the government in each country. The BCEAO has one major branch in each country and is managed by a Governor under the supervision of the Council of Ministers, the Board of Directors and the National Committees of Credit (NCC). It controls the implementation of the treaties and statutes and executes policy decisions made by the superior bodies mentioned above. The Board of Directors of the BCEAO is composed of two representatives appointed by each member of the UMOA and France. In each country, the NCC is composed of the Minister of Finance (Chairman), the two directors of the Board of the BCEAO and four persons appointed by the government. Based on the economic conditions of the country, the NCC determines for a 12-month period, the private and public demand for and supply of funds. This determination is sent to the Board of Directors which decides the amount of the Central Bank participation. The NCC then allocates this amount between the various public agencies, the banks and other financial institutions. The Board of Directors fixes also the various interest and discount rates applicable to the Union.
the previous year. Over the period 1960-73, the contribu-
tion of the various sectors of the economy to GDP are in
percent: agriculture 46.4, trade 12.6, manufacturing 11.3,
public administration and defense 7.4, transportation and
communications 6.6, mining 5.4, construction 3.4, electrici-
city, gas and water 2.3 and other branches 4.8. Similar
ratios are not yet available for 1974, 1975 or 1976. When
they become available, they will show higher figures for
mining due to an increase in the export of phosphate,
construction because several hotels are being built, and
trade, with agriculture remaining the major economic activ-
ity of the country. Some of the objectives of this study
are to formulate a framework to explain and estimate the
determinants of the price of the nontraded goods, the
consumer price index, the gross national product and the
balance of payments. Since the Togolese economy is open
and is primarily an agriculture economy, the operations of
the economic agents in the agricultural and trading sectors
will affect substantially the aggregate supply of goods,
prices and foreign reserves. It may then be useful to
present the most important aspect of these two sectors.

Agriculture is a major (often the major) productive
sector in Togo in terms of its contributions to GDP (46.4%),
employment and trade. In 1960, 80% of the economically
active population was involved in agriculture. This ratio
fell to 73% in 1970 and 70% in 1976. All the outputs of
the export crops and 70% of the outputs of food crops are traded. First, the farming techniques and the land ownership are described and second, the major crops are presented.

There are three categories of soils: (1) the rich lands (20% of the territory) include the "terre de barre" in the south (around Tsevié and Kouvé), and the "terre noire" in the center around Ogou, Anie and Mono rivers and the Akposso lands; (2) the relatively fertile lands (40% of the territory) are located in the valleys of Oti, Zio, Hano and Mono Rivers; (3) the poor lands (40% of the territory). Until 1970, the planted area covered only 10% of the territory. The average size of the farms are small (if not minuscule) by American standards. About 30% of the farms have a size less than 1 hectare (ha) while 43% of the plantations are between 1 and 3 ha and only 13% of the farms have more than 5 ha.

The traditional farming does not use mechanical tools, animals, fertilizers, irrigation nor drainage. The main tools used are the hoe, a sharp cutter called "akpatsa" and the ax. One may assume that farming is an extremely hard activity with these types of tools unless some farming operations, like ploughing, are performed in teams. These teams can number up to 20 members. Some farmers use more sophisticated techniques. One such technique known as "culture sur brulis", consists of setting a fire on the unploughed land. Afterwards, the farm is grubbed and the
planted seeds benefit from the potassium-rich ashes. The Kabre farmers who live on the slopes of the northern mountains and hills build terraces of stone on these slopes to reduce the hollowing of the land when it rains. They use also home-made fertilizers and fight insects by spreading ashes over the tobacco plants. The Ouatchi farmers fight the exhaustion of the soil by rotating the crops (maize, cassava, vegetables) and they facilitate the formation of humus by gathering dead leaves at the foot of the growing plants. They use also a planting method called "vernalisation", which consists of putting the seeds of maize in water mixed with ash until they germinate. Then the small plants are removed and planted. The young plants of maize are pollarded which allows them to benefit entirely from the sap and increase in size. To reach the same goal, they thin out the leaves of the bean plants.

The farming techniques used by the Kabre and the Ouatchi farmers described above are not widespread. In general, an extensive type of agriculture is practiced on small scale farms with little equipment and the work is done by hand. No fertilizers are used for all practical purposes. Irrigation is virtually nonexistent. The use of insecticides, selected seeds and special plants began only recently on the cocoa and coffee farms. The land is not often treated as an input which requires maintenance. The above description is a rather gloomy picture of the
production conditions in a sector which makes the foremost contribution to GDP. However, there are emerging signs indicating improved conditions in the future. One of these is the construction of farming and experimental centers in Glidji, Tové and Sotouboua, where students receive a training in modern farming methods.

In the model developed in this thesis, two aggregate agricultural crops are produced. Various factors can affect the outputs of these crops, such as labor, land, mechanical tools, insecticides, fertilizers and weather. The production functions used here depend on two inputs: labor and capital. The capital stock includes the physical means of production (ax, hoe, "akpatsa", etc.) as well as insecticides and fertilizers. Only two inputs are used in order to limit the mathematical computations, to obtain refutable propositions in terms of the signs of comparative static results, and because little is gained by "crowding" the model with variables about which there are no data.

As regards the property rights, typically, the head of the family owns the land and farms it with the members of the family. In some areas, unused land can be made available by the village to any person who wants to farm it, free of charge. The plot must be returned to the village when the beneficiary stops farming it. There is also a type of metayage system called "di bi ma di bi": literally, "eat a little and let me also eat a little".
Under this system, the farm is given to a tenant farmer who grows cocoa or coffee on it. He takes care of the farm until the trees mature. Then the output is split; 1/3 for the tenant and 2/3 for the owner. Sometimes, farmers recruit workers to perform specific tasks. Finally, there is a form of farm ownership which one may call "white collar farming". In this system, civil servants, especially those who are very high political figures, business executives or other well-paid workers in the private sector plus clergymen, purchase large pieces of land and some equipment and hire workers to grow crops or raise cattle. The white collar landlords retain full property rights of the land and the installations. Incidentally, this kind of farm exploitation is common in most African countries. The Ivory Coast being the most well known example.

Two types of agricultural products are grown in Togo: food crops and export crops. The supply conditions of these crops will be briefly described now.

Eighty percent of the cultivated land is devoted to the production of food crops called nontraded goods later on in this study. Four major plants constitute the basis of alimentation of the Togolese population: maize, cassava and yam in the south, cassava and yam in the center and millet and sorgho in the north. The total output of cassava, 92% of which is produced in the south, has fluctuated between a low of 428,000 tons in 1972 and a high of 1,186,000
tons in 1968. The exhaustion of the soils and the absence of fertilizer explain the low yields (about 6.5 tons/ha against 11 tons/ha in Cameroun). The unit price of cassava more than doubled over the period 1967-76, from 13 FCFA/kg to 29 FCFA/kg. The bulk of the output is consumed in the form of paste or a dry cream called "gali".

Yams can be found in many parts of the country but are grown primarily in the district of Atakpame and in the Akposso region. The cuttings are planted between December and April and take 7 to 12 months to mature. The yields vary with the type of yam, the soil quality and the weather. They range between 2.5 tons/ha and 20 tons/ha. The largest output was recorded in 1967 (1,076,000 tons) and the smallest in 1972 (396,000 tons) with the annual average price being 35 FCFA/kg between 1969 and 1976. It is eaten in the form of a paste called "fufu" or it is boiled, baked or fried.

Maize is grown in many parts of the country; yet, it is the predominant crop in the south where the seeds are planted at the beginning of each of the two rainy seasons (April - May, and August - September). The maturity period lasts about four months. The highest annual yield occurred in 1963 (7 quintals/ha) while the lowest was recorded in 1965 (4.3 quintals/ha). Over the period 1969-76, the annual average output was 128,000 tons while the average unit price was 33 FCFA/kg. In the United States, the maize
(or corn) appears on the table in the form of cakes, cookies, oil or as condiments. In Togo, maize is eaten boiled, broiled, or in the form of a paste called "akple".

Millet and sorgho are primarily grown in the districts of Sokode, Lama-Kara and Mango. The early varieties are planted at the beginning of the rainy season (May - June) and harvested three months later. The average yield is about 4 quintals/ha. Here again, the quantity produced has not followed a systematic trend and between 1969 and 1976, the annual average output was 115,000 tons while the average price was 52 FCFA/kg. Millet and sorgho are eaten in the form of pastes. As mentioned, earlier, after the food crops are harvested, farmers keep about 30% of the output for their own consumption and for use as seeds. They sell the remaining 70% mainly to private dealers who make it available to urban dwellers. Since the bulk of the food crops is consumed by urban populations, the tastes and income of these populations will affect the prices received by farmers for their crops. For this reason, later in this study, the demand functions for food and other goods will be derived from a utility maximization process.

Twenty percent of the cultivated land is devoted to the production of export crops. Coconut trees are grown in the south which produces 37% of the total output of copra. There are palm groves all over the country but the thick plantations are located in the south and in the
plateau region. Cotton is planted in the lower central part of the country. Groundnut is also grown all over the country but the major farms are located in the districts of Sokodé, Lama-Kara and Mango. Castor beans are grown primarily in the districts of Atakpamé and Nuatja. Shea nuts and kapok are harvested in the north. Between 1960 and 1970, export of cocoa, coffee, palm nuts, cotton fiber, groundnut and copra represented respectively 28.85%, 21.15%, 6.95%, 4.70%, 1.83% and 1.57% of all exports. Exports of castor beans, sheanuts and kapok were negligible. The unquestionable importance of cocoa and coffee and space limitation lead to a brief examination of the supply and demand conditions of these two crops only.

The coffee plantations (70,000 ha) are located in the triangle Atakpamé - Akposso - Palimé with a slight overlap beyond the Ghanaian border. The varieties include Niaouli, Robusta (predominant in Togo) and Arabica. The Robusta trees mature in three to four years and bear beans for 25 to 40 years. The yields can reach 300 kg/ha on the best lands. The supply of coffee fluctuated widely over the period 1960-76, peaking at 15,000 tons in 1968 and 1971 and bottoming out at 6,000 tons in 1966 due to inadequate rainfall. Over the 17-year period (1960-76), the annual average export price was 222.25 FCFA/kg while the producer price average 84.12 FCFA/kg, or about 37% of the export price. The difference between the export price and the
producer price is attributable to handling costs, salaries and commissions, export taxes, lump sum taxes and profit. Coffee is the second largest crop exported in Togo. The quantities exported fluctuated apparently in line with changes in the supply. Between 1960 and 1976, the annual average export of coffee amounted to 11,182 tons. The agricultural crops marketing agency called the Office des Produits Agricoles du Togo (OPAT) has always set the producer price at a level which guarantees a net profit to the agency. The total profit on coffee was 72 million FCFA in 1964 and 403 million FCFA in 1968, the highest profit to date. The major buyers of Togolese coffee are France and other Common Market countries.

The cocoa plantations (40,000 ha) are located in the warm and humid regions of the coffee triangle and along the Togo-Ghana border. The major harvest period of cocoa begins in October and ends in May, while the harvest of coffee goes from December to August. As with coffee, after the Service du Conditionement officials inspect the beans, the output is sold to accredited traders who truck it to OPAT's warehouses in Lomé. The output of cocoa has always exceeded that of coffee and has fluctuated less than that of coffee. Apart from some economic reasons, such as price, the relative stability of the output of cocoa may be due to the low sensitivity of this crop to weather after the trees mature. The annual average output of cocoa over the period...
1960-76 was 15,094 tons. Togo has not been spared by the instability of world cocoa price and over the period 1960-76, the export and producer prices averaged, respectively, 197.66 FCFA/kg and 85.71 FCFA/kg. Until 1965, cocoa was not only the major crop exported but the major export good of Togo. Between 1965 and 1968, phosphate ranked first, then cocoa shifted back to the number one spot for a few years. As for coffee, OPAT fixes the producer price each year to ensure a comfortable profit. Since its creation in 1964, OPAT's lowest profit on cocoa was 42 million FCFA in 1964, which was due to a 6.5% decline in export price and an 8.0% increase in the producer price relative to 1963. The highest profit on cocoa of 1.211 billion FCFA in 1968, was due to a 40% rise in the quantity exported over the previous year and a 37.4% export price boost combined with a rather small increase in the producer price (14.3%). The major buyers of Togo's cocoa are the Netherlands and the Soviet Union.

The agriculture sector makes the largest contribution to GDP (46.4%). In 1976, it employed 70% of the economically active population. Eighty percent of the cultivated land is devoted to the production of food crops. Seventy percent of these food crops are sold to local nonfarmers. In 1970, 67.4% of total exports were agricultural goods. These figures illustrate clearly the paramount role which agriculture plays in the economy of Togo.
Trade has been the second major contributor to GDP over the period 1960-73: 12.6% after agriculture (46.4%). This preponderant role which trade plays in the economy of Togo may be explained partly by the fact that the entire output of cocoa and coffee are exported and 70% of the food crops and all imported goods are traded. It should also be pointed out that the government has a fairly liberal commercial policy with no trade barriers such as quotas or prohibitive tariffs.

Cocoa, coffee and phosphate are the major exports of Togo. The Office de Produits Agricoles du Togo (OPAT) is a government agency created in 1964 to supervise the buying and export of crops: cocoa, coffee, cotton palm kernel, copra, groundnut, sheanut, castor beans and kapok (groundnut exports have stopped since 1974). The other duties of the agency include financing programs aimed at improving production conditions in the farms including the construction of roads in rural areas, the creation of agriculture research centers and the purchase of stocks in some companies. Above all, OPAT must alleviate the effect of export price fluctuations on farmers' income by setting the producer prices such that farmers' incomes are stable or rise moderately. It is realistic to assume that the agency is a monopsony in the domestic market because of
its exclusive right to export the crops, while it is a perfect competitor abroad because Togo's outputs of cocoa and coffee are negligible compared to those of the world major producers of cocoa (Ghana, Nigeria, Ivory Coast) and coffee (Brazil, Columbia, Ivory Coast, Uganda).

OPAT's economic behavior is one of the subjects of attention in this study. In particular, it is shown later on that, in order to maximize profit, OPAT should buy the entire output of export crops from the farmers. In a dynamic framework, it is also shown how the agency may determine the optimal producer price. The total after-tax profit of the agency fluctuated starting from a low of 165 million FCFA in 1964 to a record high 1.526 billion FCFA in 1974. On several occasions, OPAT lent funds to the government. The time path of these loans and those made to the government by foreign institutions will be examined along with that of some other aggregates in Chapter 5. In 1976, the major clients of Togo were the Common Market (75.6% of the value of all exports), East Europe (9%), Africa (6.8%), other Western Europe countries (6.0%) and other countries (2.6%) which include Japan (1.9%) and the USA (.15%).

The bulk of the import operations is carried out by foreign companies such as SGGC, CFAO, SCOA, which are French companies, and John Holt and UAC which are British. In 1976,
the major imports were vehicles and precision instruments (17% of all imports), machines and equipments (15%), clothing and cotton textiles (13%), food (12%), iron and other metals (11%), fuel and chemicals (10%) and other imports (22%). Over the period 1960-1976, the most important imported good in value was cotton fabrics and threads. The major suppliers of Togo are the Common Market (66% of the value of all imports in 1976), Japan (5%), USA (5%), Canada (5%), East Europe (4%), and Africa (4%).

There are no shopping centers or malls in Togo. Besides the small stores, many of which are owned by the big foreign companies, the retail trade is operated by thousands of individual persons. Some cities have a solid roofed structure which serves as a central market. In general, the unsold goods are packed and returned home in the evening. The crowded and noisy market place of Lomé is famous in West Africa for its display of very beautiful printed fabrics, most of which are imported from Japan, Denmark and the United Kingdom. For some Togolese merchants, the women in particular, selling printed fabrics in the Lomé market is a very prestigious and often lucrative job and many of the wealthiest persons in Lomé are indeed these women retailers of textiles. They are called "Nana Benz" in reference to the Mercedes Benz cars which they like to own.
The Demand for the Gross Domestic Product (GDP)

The GDP is disposed of in the forms of net exports, consumption and investment.

Balance of Trade and Balance of Payments. Between 1960 and 1976, the value of exports and imports rose to an annual average rate of 35%. In 1970, the percentages of exports of cocoa and coffee relative to all exports were 41.7 (up from 38.1 in 1960) and 17.5 (slightly down from 17.7 in 1968). Over the period 1960-70, cocoa, the single largest export, represented 50% of total export. In 1976, the imports of food items, textiles and fuel represented 35% of all imports. Textile is the single most important imported item and is purchased by both urban and rural populations. The price of textile materials will undoubtedly affect the consumer price index and the other aggregates explored in this study; in particular, the price of nontraded goods. The cost of textile materials to consumers will be used later on as a proxy for the price of the imported consumption good in the empirical work.

The terms of trade defined as the ratio of the price index for exported goods to that of imported goods (both expressed in FCFA) ranged over the period 1960-76, from a low of 30.7 in 1972 to a high 164.4 in 1974 (base 1970 = 100).
Over the period 1960-76, the net exports, or balance of trade, has always shown a deficit except in 1974. The current account balance, defined as the sum of the balances of trade, services and transfers showed a deficit in 1965, 1966, 1971, 1972, 1973 and 1975. It indicated a surplus in 1967, 1968, 1969, 1970 and 1974. Over this period, the cumulative deficit was 23.96 billion FCFA while the total surplus was 38.71 billion FCFA. Between 1965 and 1975, the balance of nonmonetary capital which is the sum of net private and government capital flows was in surplus except in 1969, 1970 and 1974. The basic balance which comprises the current account balance and the net nonmonetary capital account showed a deficit in 1965 (.2 billions FCFA, the smallest), in 1972 (1.49 billion FCFA, the largest) and in 1973. The surplus of the current account balance exceeded the deficit of the capital account balance in 1969, 1970 and 1974. The current and the capital account balances were both positive in 1965 and 1968.

The stock of foreign reserves is a component of the monetary base in Togo because there is no sterilization of foreign reserves. Thus it is an important determinant of the economic activity of the country. The obligations of Togo vis-à-vis the West African Monetary Union (of which Togo is a member) requires that the stock of foreign reserves be kept at an adequate level. Needless to say, this stock is
a major decision variable for foreign lenders. It may then be appropriate to attempt to identify the determinants of the net flow of foreign assets, or the balance of payments. This investigation is pursued in the theory developed later on and the basic balance is used as a proxy for the balance of payments in the empirical work.

**Employment, Income, Consumption of the Private Sector and Prices.** It is estimated that the economically active population was 970,000 individuals in 1976 or 43.0% of the total population, slightly down from 44.6% in 1960. In 1976, about 78.5% of the economically active population was working in the agricultural sector (78% in 1969), 16.5% was holding nonsalaried jobs (17% in 1969) and 5% were salaried workers (same ratio in 1969). The public sector is the major employer with 60% of the total salaried workers in 1976. As regards the employment in agriculture, most of the workers are grouped in family units.

Farmers derive their income from selling all their outputs of export crops to OPAT and 70% of the food crops harvested to private dealers. Indeed, after the government announces the trading seasons and the producer prices, the accredited agents of OPAT ask and obtain loans from commercial banks. These funds are used partly to pay farmers for selling
their crops. Afterwards, these agents are paid back by OPAT. The government does not intervene in any significant manner in the markets for food crops. Producers of these crops sell about 70% of their outputs to a multitude of dealers who transport them to the cities.

The nonagricultural sector is composed of independent workers, salaried persons and private companies. The foreign private companies are the primary importers of consumption goods. They operate both as wholesale and retail dealers. They have some control over the selling price of the imported goods because they pass on to their customers any additional cost incurred due to inflation abroad or change in minimum wage rates. The independent workers derive their income from selling their goods and services. Most self-employed persons live in the cities and have a consumption behavior similar to that of the salaried workers. Salaried persons work for the government, the few mining and manufacturing firms and the import companies. Some of the salaried workers and many independent workers (the Nana Benz in particular) own real assets such as farms, real estate and saving accounts which generate various types of returns. So the salaried workers' income may comprise wages and salaries and returns from assets. In the face of inflation, the total income of these asset owners will be affected by capital gains or losses on these assets. The effects of these capital gains
or losses on the prices of nontraded goods and the per-capita real stock of the assets considered in this study will be examined in Chapter 5.

The income of farmers and nonfarmers is spent, used to pay debts and saved. The composition of the consumption budget of the typical salaried worker differs from that of the farmer because the salaried worker devotes a larger proportion of his income to the purchase of food items. Farmers grow most of their food. The major food items which they buy are meat, fish, salt and beverages. Farmers and nonfarmers buy durable goods such as clothing items, construction materials, and tools, and under normal circumstances, the largest part of the spending of farmers is made on manufactured goods, almost all of which are imported. Although these goods are not destroyed by the act of consumption, they are treated in this study as consumption goods first, because there are no data on their stocks and lifespans and second, because the definition of capital good used in this work is a good which can be used to produce another good. Overall, the real spending on consumption goods by the private sector rose at an annual average rate of 6.8%, from 31.2 billion FCFA in 1960 to 65.8 billion FCFA in 1973.

As mentioned earlier, a portion of the personal income is devoted to paying taxes as well as debts owed to
private individuals and to banks. As regards saving, there are no stocks, corporate bonds or government bonds markets in Togo. In general, only urban dwellers hold saving accounts because they have access to the few saving institutions located in the cities. Wealth is held more commonly in the form of money and real assets which might include jewelry, land, real estate and some capital goods. Money is held for transactions and precautionary purposes. The absence of well-structured secondary markets for the real assets limits considerably the existence of a speculative demand for money in Togo. There are virtually no descriptive nor statistical information about the stock of real assets in Togo. Needless to say, the number of mathematical and conceptual difficulties rise fairly sharply with the introduction of new variables, especially those that concern assets which accumulate over time. These factors explain why the only assets held by the private sector in the models presented in Chapters 3 and 5 are money, time deposits and capital. The bulk of the national savings comes from the private sector whose savings reached 8 billion FCFA in 1969, or 93% of national savings. In 1960, 31% of gross domestic investment was financed by local private savers. This ratio fell to 18% in 1973 after the record high of 93% in 1969.
One of the objectives of this study is to attempt to explain and to estimate the consumer price index (CPI). Therefore it may be useful to describe briefly this index here. The data available are primarily based on surveys of consumption patterns conducted in Lomé. However, except for some adjustments, these observations adequately reflect the situation in other cities. For instance, transportation costs tend to make prices of imports higher in the Central and Northern parts of the country than in Lomé. The family units surveyed are African families and the base year is 1963. The CPI rose from 100.0 in 1963 to 198.5 in 1976. The annual average growth rate over this period was 7.047%. The index declined in 1967 because of a substantial decrease in the price of some food items. In recent years, the CPI rose an annual rate of 12.8% in 1974, 18.06% in 1975 and 11.64% in 1976. The major component of the CPI which is food, rose 10.88% in 1974, 25.41% in 1975 and 17.67% in 1976. It is estimated that during the first quarter of 1977, the CPI rose at an annual rate of 30%. The sharp increase in food prices in 1975 and 1976 are partly due to the drought which affected Togo and the Sahel region the preceeding years. In 1973, the outputs of cassava, yam, maize, millet, rice and beans dropped respectively 45.7%, 39.7%, 30.9%, 34.1%, 92.4% and 71.8% with respect to their 1972 levels. The supplies of these crops, except that of beans which
declined, rose at negligible rates over the last few years so that the inflationary pressure due to inadequate supplies remains in the economy. In addition, there is a stronger inflationary pressure arising from the demand side. The latter pressure results from increases in wages granted by the government in 1974, 1975 and 1977 and from increases in producer prices of export crops. The prices of imports have also risen over the years but the increases were sharper in recent years than in the 1960's.

In conclusion, farmers buy essentially imported goods while salaried and independent workers buy both imported and food crops. One of the purposes of the models presented in this thesis is to attempt to identify the determinants of food prices and estimate the food price equation in equilibrium. The prices of some of the imported and nontraded goods are summarized in the CPI which this thesis will also attempt to explain and estimate. The rate of growth of 7.04% of the CPI between 1963 and 1976 is due to the low inflation which prevailed in Togo before 1970. In recent years, the CPI rose at higher rates. Furthermore, by its very construction, the CPI covers only certain goods. It is the belief of this writer that the cost of living is far higher than indicated by the above figures. Indeed, during a stay in Lomé in August 1978, this writer went to the market and found that a bottle of
roasted groundnut and a bottle of red palm oil which used to cost 100 FCFA in late 1973 were priced respectively at 500 FCFA and 800 FCFA. A Kodak 35 mm roll of film priced in some Tucson (USA) stores at less than four dollars cost the equivalent of ten dollars.

Government Revenues and Public Spending. Taxes, government spending and government debts are exogenous variables in the static model presented later on. A brief examination of these variables follows below.

The government derives most of its revenues from import, export and income taxes. The tax rate applied to corporate profit is 12.5% for craftmen, 25.0% for individuals and 37.0% for firms. The tax rate on wages and salaries varies between 2.4 and 7.2% for income in excess of 8,000 but less than 20,000 FCFA/month. A maximum tax rate of 54% may be applied to income in excess of 320,000 FCFA/month. Although the personal income tax and the corporate profit tax are the major direct taxes in Togo, their sum never exceeded the 1.0 billion FCFA level before 1970. The sum of these two taxes grew fairly rapidly since that year to reach 2.98 billion FCFA in 1975. Import and export taxes constitute the major source of revenues to the government. The tax base of imports is the CIF value of the imported goods while that of exports is the FOB value of the agricultural
and mineral goods exported. The maximum tax rate on imports is 60% while that on exports is 42.5%. Over the period 1961-76, export duties rose more than seven fold reaching a high of 2.15 billion FCFA in 1976, while import revenues almost quadrupled to 4.56 billion FCFA in 1976. The model developed in Chapter 3 and estimated in Chapter 6 includes a lump sum tax variable and ad valorem nonprohibitive tax rates on imports and exports.

The government spends part of its revenues on consumption goods. Real government spending on these goods rose at an annual average rate of 8% from 3.0 billion FCFA in 1960 to 7.3 billion FCFA in 1973. Most of these goods are imported manufactured goods. It is assumed in the model presented in later chapters that the government does not buy and consume food crops.

The government spending on investment goods is primarily financed through loans made by OPAT and foreign institutions. Indeed, the overall government savings has been negligible. It never reached 1.0 billion FCFA until 1971. However, there are major government investments in mining and manufacturing firms (brewery, textile mills, oil refinery, cement plant and infrastructure). The gross domestic investment (GDI) of both the private and public sectors nearly quadrupled between 1960 and 1973 and rose
18% in 1974 and 46% in 1975, but dropped 11% in 1976. Over the period 1960-73, 53.7% of GDI was financed from domestic origins. This investment accumulated to become the capital stock and the time path of the capital labor ratio will be examined in the dynamic version of the model presented in this study.

The combined ordinary and investment budgets of the central and local governments and the special accounts was in surplus only in 1961 (102 million FCFA), 1968 (427 million FCFA), and in 1969 (610 million FCFA). The smallest deficit was 210 million FCFA in 1960, while the largest was 4.44 billion FCFA in 1976. The major lender to the government for the purpose of financing the budget deficit has been and remains OPAT and by 1967, it had already lent a cumulative amount of 1.545 billion FCFA to the government. In 1976, almost the entire after tax profit of 459 million FCFA made by the agency went to the government budget. The government made an active use of its credit line with the Central Bank only in 1975 when it borrowed 2.108 billion FCFA to finance part of the budget deficit. This loan was repaid in subsequent years.

As of December 1, 1969, the outstanding public debt was 10.53 billion FCFA of which 14.6% was owed to OPAT, 70.0% to the Kreditanstalt, whose funds were used to finance the port construction, 14.0% to France and 1.4% to
others. The interest payment alone to foreigners represented 45.17% of the service payments on foreign debt. At the beginning of 1976, the outstanding foreign debt was 24.17 billion FCFA (up 31.5% from the previous year). The servicing of the total debt was 3.6% of all ordinary outlays in 1966, 9.1% in 1968 and 8.7% in 1970. The proportion of ordinary spending devoted to servicing foreign debt alone was 10.4% in 1975. These figures illustrate the preponderant role which OPAT and foreign lenders play in financing government outlays. The bankruptcy of the Republic of Congo-Leopoldville which defaulted on several occasions vis-à-vis her foreign creditors is a clear indication that to ignore the trend of the government debt can create very serious economic difficulties, if not paralyze the entire economy of a nation. These factors explain the attempts made in Chapter 5 to examine the time path of the per-capita government debt at the steady state.

In summary, when one combines the tax levied on export crops, the profit tax paid by OPAT and the various loans made by OPAT to finance the budget deficit, it becomes clear that OPAT is the largest source of revenues to the government. The government uses part of these tax revenues to pay civil servants who in turn pay income taxes. All these taxes which are levied on the private sector will
affect the disposable income of the members of this sector. As a result, when the quantities demanded of various goods and assets change, *ceteris paribus*, their prices also will change and in particular, it will be shown that the price of food items will be affected. Finally, it has been mentioned that the government does not rely on money creation to finance its investment programs but on loans from OPAT and foreign organizations. It would be inappropriate to ignore the time path of the per-capita government debt. Therefore, this time path will be examined along with the time paths of the capital labor ratio and the per-capita stock of foreign reserves. To end this institutional description, a brief examination of the macroeconomic tools available in Togo, in particular, the monetary tools is appropriate.

The Macroeconomic Policy Instruments

The purpose of this thesis is not to analyze in detail the government sector or the monetary sector of the Togolese economy, but to formulate a model which incorporates aggregate variables pertaining to these sectors and which is simple enough to lend itself to some empirical investigation. The choice of this line of attack is based upon several factors, some of which are the mathematical difficulties, the ambiguity of almost all comparative static and dynamic signs
and the lack of descriptive and statistical information to sustain a detailed sectorial analysis. In particular, all of my attempts to know whether the government has a coordinated fiscal policy or makes a coherent active use of the monetary and fiscal tools at its disposal have been unsuccessful. The Central Bank also has several monetary instruments but it uses only one -- the discount of custom duty bills -- actively. The Central Bank's publications offer no explanation for the preference for this tool over the others. However, it may be useful to summarize briefly the macroeconomic instruments available to the government and the Central Bank first, because a few of them appear in the model discussed later and second, because they might be used effectively in the future.

The Fiscal Policy Instruments. As in most countries, the Togo government has the power to change the tax rates and tax bases and/or to revise upward or downward its outlays on general, economic, social and other services plus its outlays on investment programs in order to achieve certain economic goals such as price stability or target inflation or employment levels. Note that the marketing agency OPAT does pay export taxes and corporate profit taxes. In addition, it lends funds to the government. To date, it has not been determined clearly whether or not the principal and interest on these loans are repaid. If these
debts are not repaid then they may be associated with a lump sum tax levied on the agency and ultimately on the farmers. If they are repaid then clearly they are loans. I decided to treat these debts as loans because all the documents (such as Survey of African Economies, p. 654) dealing with the operations of OPAT and the government make a clear distinction between OPAT tax payments and OPAT loans to the government. Therefore, in this work, the fiscal tools remain government spendings and taxes while government borrowings from OPAT constitute a debt to be repaid. All government spendings and taxes will be treated as exogenous variables in the static model, while the per-capita government spending on investment goods, the per-capita government tax and the per-capita government debt, which is composed of OPAT and foreign loans, will be treated as endogenous or target variables in the dynamic model of Chapter 5.

The Monetary Policy Instruments. Monetary authorities\(^2\) can use their discretionary power to induce contractions or expansions in the money stock in order to reach certain

\(^2\) The government in each country of the UMOA is a major monetary authority for several reasons. First, because it has two representatives at the Board of Directors of the BCEAO. Second, because it appoints all the members of the National Credit Committee. Third, because it has an overdraft facility at the Central Bank which it may choose to use or not to use in a discretionary manner. So clearly, the Board of Directors of the BCEAO and individual governments are the monetary authorities.
economic goals (i.e., price, employment and output targets). The Board of Directors of the Central Bank sets all the interest rates and discount rates or their major components except the interest rates paid on saving deposits held with the post office financial agencies. In general, these rates are determined partly on the basis of similar rates practiced in foreign countries and their levels are fixed so as to reduce or avoid speculation. The major monetary tools used by the Central Bank, or the BCEAO, to control domestic credit are the discount rate and quantitative and qualitative restrictions on the credit instruments presented at the discount window. By lowering the discount rate or releasing some of the restrictions, the BCEAO can expand, ceteris paribus, the monetary base and thus the money stock by acquiring credit instruments. The most common of these instruments is the custom duty bills which are issued by private and public corporations. In 1975 and 1976, the discount rate was 8.0% (up from 5.5% in 1974) while in 1976, the average interest rate on short-term and long-term time deposits at commercial banks were respectively 3.43% (up from 1.75% since 1965) and 5.53% (up from 4.00% since 1965). In 1975 and 1976, the average interest rate on loans to the public was 6.75% (up from 3.7% since 1965).

To further control liquidity, commercial banks are required to observe a "minimum liquidity ratio" between their short term assets and short term liabilities and a
"minimum solvency ratio" relating their equity to their credit commitments. The Central Bank may also impose a reserve requirement ratio on commercial banks. To date, this tool has not yet been used.

Note also that the government has the power to borrow up to 20% of the previous year fiscal revenues. The government's discretionary decision to use this overdraft facility will increase, ceteris paribus, the domestic credit and thus expand the money stock. As mentioned earlier, between 1970 and 1976, the government used this credit line only once, in 1975, and the loan was repaid the following year. Thus, for all practical purposes it is assumed in this thesis that only the Central Bank can create money.

One final observation is that the money stock is the product of the money multiplier which is treated as an exogenous variable in this study, and the monetary base. The monetary base which is the sum of the domestic credit and the stock of foreign reserves is treated as an exogenous variable in the static model and as an endogenous variable in the dynamic model. The BCEAO does not sterilize the foreign reserves of the members of the monetary union but it includes them in the monetary base of each country. The ratio of domestic credit to the stock of foreign reserves for the Union has been about 1.82 between 1962 and 1970. This ratio rose to about 2.50 in recent years. Therefore,
it will be assumed that there is a known ratio between the
domestic credit and foreign reserves. This ratio or co-
efficient whose size indicates the degree of sterilization
may be treated as policy variable available to the Central
Bank to be used to induce monetary expansions or contractions.
This coefficient is included in the dynamic model.

In summary, the macroeconomic fiscal and monetary
variables which will be used in this study are taxes, govern-
ment spending, domestic credit and the ratio of domestic
credit to foreign reserves.

Summary

The relevant economic background for the models to
be developed later has been described in this chapter. The
economy of Togo is primarily agriculture oriented. The sales
of the good and export crops generate income to their
producers, various middlemen and to the government in the
form of taxes and OPAT loans which are used as part of the
revenues to pay civil servants. All of these economic agents
spend part of their income on imported manufactured goods,
a large part of which are consumption goods. Another portion
of the income of the private wealthowners is held in the form
of money created by a multinational Central Bank. The
behavior of the producers of the agricultural products and
that of the buyers will affect the price of food and thus
the CPI and the GNP. The behavior of these groups, through their demand for money, will also affect the balance of payments. OPAT is a major institution in terms of the amount of the financial resources which it channels from the farmers to the government. Given such an institutional framework, it is possible that some of the effects of macroeconomic exogenous variables such as import price, taxes, and domestic credit, on the price of nontraded goods, the CPI, the per-capita GNP, the per-capita balance of payments, the capital labor ratio and the per-capita government debt may not be the same as those identified for advanced economics. The purpose of this thesis is to explore these effects. Chapter 2 begins with a more detailed description of the problems to be studied and proceeds with a review of the literature.
CHAPTER 2

THE PROBLEMS AND REVIEW OF THE LITERATURE

The Importance of the Agricultural Sector in Togo

In 1967, (a) 580,000 individuals of which 235,800 were males, that is about 75% of the economically active population, were involved in agriculture; (b) 1,571,760 persons or 90% of the total population derived income from agriculture; (c) agriculture alone contributed 37.4% to Gross Domestic Product (GDP); (d) the export of agricultural crops generated 54.1% of all foreign reserves.

Over the period 1960-73, agriculture contributed 46.4% to GDP, followed by trade at 12.6% and manufacturing at 11.3%. In 1976, 70% of the economically active population was involved in agriculture while 72% of the total population derived its income from agriculture. In the same year, exports of agricultural commodities which were primarily cocoa and coffee, amounted to 9.198 billion FCFA or 37% of all exports.

Clearly, these figures show that the agricultural sector is the sector which employs and generates income to
most Togolese. It is the sector which contributes most to GDP and generates the largest part of foreign reserves. There is no separate data on its contribution to the government budget, but one can argue that it is the ultimate source of revenues to the government for the following reasons. First, the government derives most of its financial resources from import and export tariffs. The export of agricultural crops generates these tariff revenues, part of which is used to pay civil servants. These civil servants and the farmers buy the imported goods and ultimately bear the burden of the import tariffs. Second, the government levies corporate profit tax on OPAT, thereby passing the ultimate burden of this tax again to the farmers. Finally, the government borrows funds from OPAT to finance various programs. All these factors suggest that, in last analysis, the agricultural sector is the major contributor to government revenues with the rural sector being the major economic sector in Togo in terms of employment and revenues generated.

**Statement of the Problems**

All the economic materials published about Togo are descriptive. In these materials and in various unpublished documents, some of the problems often alluded to are inflation and how to increase the net amount of foreign reserves.
Regarding inflation, consumers tend to blame the "greedy" import companies which make considerable profits by charging very high prices. Some of these companies argue that their purchase prices and the import taxes have risen such that to stay in business, they must pass their additional costs onto their customers. Inflation hurts the government in several ways. It reduces its real purchasing power and upsets the government's estimates of costs of various economic and social programs so that cost overruns become common. Inflation also puts pressure on the government to raise wages and salaries of the public sector and to set minimum wage rates to be observed by the private sector. Concerning this inflation issue, one observation needs to be made about the nature of the goods involved. The prices of imported goods are often singled out as the major contributor to the rise in the cost of living index in Togo. However, it should be noted that 70% of the food crops produced is traded and consumed locally and the prices of these goods are included in the consumer price index (CPI). Therefore, changes in the CPI may not be due entirely to changes in the prices of imported goods but may also be affected by changes in the prices of nontraded goods.

Concerning the balance of payments, one of the criteria used by the Board of Directors of the Central Bank which sets the amount of domestic credit for each country is
the level of the stock of foreign reserves of that country. Since foreign reserves are not sterilized, the level of the economic activity of Togo will depend upon the level of net foreign assets which is a component of the monetary base. In addition, in its economic development plans, the government sets goals such as a target ratio of coverage of imports by exports. In Togo, like in most underdeveloped countries, the capacity to import primarily depends on the amount of foreign reserves generated from exports. Thus, the balance of payments in general, and the balance of trade in particular, cannot be ignored.

This thesis deals with some aspects of inflation and foreign reserves for the following reasons. First, some unpublished economic documents about Togo indicate that inflation and the net flow of foreign assets are subjects of concern in Togo. Second, inflation has serious economic implications such as the reduction of real purchasing power and cost overruns. When public opinion polls in the USA reveal that inflation is currently regarded as the major economic problem in a country where the per-capita nominal GNP was 7,890 US $ in 1976, the extreme gravity of the problem cannot be ignored in Togo where the per-capita nominal GNP was 260 US $ in 1976. Third, an attempt to influence the rate of price change should be preceded by
an attempt, however crude it might be, to explain it in a fairly consistent framework. That is if a government institution in Togo wants to affect the upward price trend, the first step should be to identify the major determinants of prices. Fourth, the net stock of foreign assets determines the money stock in Togo. In addition, the fact that Togo belongs to a monetary union which pools its foreign 

3. The statutes of the UMOA require each member country to hand over to the BCEAO, her stock of international means of payments resulting from her foreign transactions. The BCEAO in turn will deposit these financial assets in an operation account opened with the French Treasury in Paris. The only exceptions to the rule are: (1) the amount of foreign reserves needed for ordinary operations of the commercial and financial institutions in the member countries; (2) the stock of foreign reserves needed to meet the member country's obligations vis-a-vis the IMF; (3) the stock of foreign reserves (limited), which the Board of Directors of the BCEAO decides to hold in current accounts opened with the Bank of International Settlements or with other Central Banks. The French Treasury pays a short term interest rate on the credit balance of this account. Each member country has access to this pool of funds through the BCEAO. In case the combined usage of these reserves results in their depletion such that the operation account shows a deficit, the following measures must be taken. The BCEAO must: (a) repatriate any foreign exchange it might be holding with financial institutions outside the French area, (b) urge member countries to use their SDR holdings at the IMF to obtain foreign exchange, (c) call in foreign exchange held by private and public institutions of the member of the Union. In the latter case, the BCEAO may first request the surrender of foreign reserves from the countries who used more of these reserves than they brought in. If despite all these measures, the operation account is still negative, then the BCEAO can use the overdraft facility stipulated in the cooperation agreement between France and the UMOA countries. Of course, it will be charged an interest fee.
reserves does not mean that Togo or any other member can run balance of payments deficits indefinitely. Corrective contradictory measures, exist to prevent such a situation from occurring. Togo's government no more than any other member country's government would want to engage in deliberate actions which would result in a contraction of GNP and other aggregates.

In summary, the importance of the agricultural sector in Togo's economy, the preponderant role which OPAT plays, the relatively high inflation in progress, the importance of the foreign reserves and the lack of macroeconomic analytical study of the Togolese economy constitute the major factors which led this writer to this work.

Consider a small open economy in which a pure export good A and a nontraded consumption good B are produced while a consumption good C and a capital good K are imported. The major economic agents in this economy are the government, OPAT, the Central Bank and the private sector which is composed of firms, individual producers and consumers. The purpose of this study is to formulate within such a framework three simple models. First a static model is developed to identify, in equilibrium, the determinants of the price of the nontraded good \( P_B^e \), those of the consumer price index \( P^e \), the per-capita nominal GNP \( y^e \) and the per-capita net flow of foreign reserves \( bp^e \). The equations
of these endogenous variables \( (P_B^e, P^e, y^e, bp^e) \) will be estimated using Togo's data. Second, a model is formulated to explain how OPAT might determine the producer price \( (P_A) \) in a dynamic framework. Third, a dynamic model is developed to explore, at the steady state, the determinants of variables such as the price of the nontraded good, the capital labor ratio, the real per-capita stock of government debt and the real per-capita stock of foreign reserves.

Review of the Literature

Generalities

The group of countries called underdeveloped countries do not constitute a homogenous category. They differ by population size (India 640 million, vs. Togo 2.3 million, in 1976), geographical area (India 3,288,000 km\(^2\), vs. Togo 56,000 km\(^2\)), structure of their outputs (India produces a wide variety of manufactured goods while Togo has few industries), financial wealth (Saudi Arabia vs. Togo) natural resources (Congo-Leopoldville vs. Togo). The World Bank classified underdeveloped countries in three income groups based on their 1976 GNP per-capita expressed in 1976 US dollars. Some of the low income countries (per-capita GNP less than $260) are Kenya, Congo-Leopoldville, Pakistan, Haiti, while the middle income group (per-capita GNP between $260 and $1,340) includes countries such as Angola, Ghana,
Thailand, Chile and Togo, which barely jumped the border line at $260. The third group is the capital surplus oil exporters (Saudi Arabia, Lybia and Kuwait).

Published descriptive economic literature on most of these countries is rare and published analytical work on the countries in question is even rarer. In fact, I found no analytical work on any country in the middle income group whose economic structure might bear some resemblance with that of Togo. Thus it becomes evident that a review of a literature which deals with macroeconomic models formulated for countries such as Togo is an impossible task. I have not been able to locate a substantial body of analytical macroeconomic literature on countries which belong to the other income groups. The published analytical literature on these countries deals with single issues such as the estimation of supply functions for agricultural commodities [Stern (1965), Olayide (1972), Blandford (1973), Labys (1973, 1974)]; or the estimation of saving functions [review article of Mikesell and Zinser (1973)] or the estimation of demand for and supply of money functions [Bhattacharya (1974), etc.]. In all these models, the basic functions and often the functional forms are postulated and a few manipulations are made to obtain the expressions to estimate. These models are accordingly of the very partial equilibrium type and very few linkages (if any at all) to other sectors
or markets are indicated or dealt with. As one moves away from the underdeveloped world to venture into the developed world, it becomes apparent that the published economic literature is much more abundant, yet, it remains mostly a set of scattered models. Though these models involve, in general, more markets and show more ingenuity and skill in the formulations and estimations, they appear unrelated although they deal with the same basic issues. For instance, although the models of Tsiang (1961) and Dornbush (1973) deal with money in an open economy, it is difficult to compare their assumptions, some of which are not explicitly stated, and their results. This thesis does not intend to develop a common framework for various individual models. The observation is only intended to indicate that even in the economic literature of developed countries, it is hard to find published articles and books which regroup in a consistent manner, under the same roof, most of the features of the models developed in this study.

These features are: (1) a production sector in which a pure export good A and a nontraded consumption good B are produced; (2) a consumption sector composed of the government which consumes an imported good C and private individuals who consume goods C and B; (3) a government sector which levies taxes and tariffs, borrows from OPAT and foreign countries and spends; (4) a domestic private
sector which produces goods A and B, buys goods B and C as well as an imported capital good K, pays a lump sum tax and saves; (5) a banking sector which creates money and other financial instruments such as a saving account; (6) a marketing agency which exports good A, makes a profit and lends part of it to the government; (7) a foreign sector which produces goods C and K, uses part of them and exports the remaining, and imports good A.

Before we examine in more detail, the behaviors of the economic agents involved in these various sectors, let us review and critique some models found in the literature.

The Models

There is a vast literature on the pure theory of international trade [Heckscher (1919), Ohlin (1933), Harberler (1936), Samuelson (1939, 1948, 1967), Rybczynski (1955), Jones (1956-1957, 1961), Takayama (1972), Komiya (1967), M.C. Kemp (1969), to name a few].

The models presented in this thesis contain a nontraded good and Komiya (1967) is one of the earliest writers who introduced such a good into a model. A nontraded good or service is a good or service which is not traded internationally because of its very nature, haircuts for instance, or for reasons such as differences in tastes, prohibitive transportation costs, etc. The major characteristic of this good is that its market must clear while the national
market for traded goods need not clear. Komiya (1967, p. 132) built upon the Heckscher-Ohlin-Samuelson (called HOS by J. Bhagwati) models. HOS models essentially describe a static barter trade world in which two goods are produced with constant returns to scale production functions using capital and labor which are in fixed total supplies and which are mobile between sectors and immobile between countries. Komiya improved the HOS framework by introducing a nontraded good and an import tariff. Treating the price of the nontraded good as an endogenous variable and the prices of the exported and imported goods as given, the author found that, in equilibrium, an increase in the price of imports or exports has an ambiguous affect on the price of the nontraded good. If the capital labor ratio in the import sector exceeds that of the export sector which in turn exceeds that of the nontraded good sector, then: (1) an increase in the price of imports will reduce the price of nontraded goods and (2) an increase in the price of exports will raise the price of nontraded goods [Komiya, 1967, p. 135].

The volume of money and trade literature is also impressive [Hargerger (1950), Meade (1951), Alexander (1952), Johnson (1958), Hahn (1959), Mundell (1960, 1968), Tsiang (1961), Kemp (1962), Ott and Ott (1968), Negishi (1968), Dornbush (1973), Mussa (1976), Anderson and Takayama (1977), Kuska (1978), and others]. After "succeeding" in integrating the monetary sector in open economy models, the money and
trade literature branches into what may be called "comparative static results" models and "policy" models. The comparative-static oriented models emphasize the effect of some specific exogenous variables (exchange rate, tariffs, transportation costs, government spendings, taxes, etc.) on some endogenous variables (often the balance of trade of the balance of payments). The policy-oriented models which are usually based on comparative static results, focus on the policy mixes (i.e., various combinations of fiscal and monetary instruments) which may permit to reach certain macroeconomic objectives such as balance of payments equilibrium, price stability, full employment [Mundell (1962), Jones (1968), Ott and Ott (1968), etc.].

The models of this thesis are primarily oriented toward comparative statics and dynamics. Most of the money and trade literature are of the comparative static type and they deal primarily with the effect of devaluation on the balance of trade or on the balance of payments. The effect of a change in the exchange rate on the current account balance or on the balance of payments is not the subject of this thesis partly because all the relevant data on Togo's international transactions are reported in FCFA. The subjects discussed in the static model of this work include the effects of exogenous variables such as the prices of exported and imported goods, taxes and occasionally tariffs, the capital labor ratio and the money stock on the equilibrium
price of nontraded goods, the equilibrium consumer price index, the equilibrium per-capita GNP, and the equilibrium balance of payments. Although the examination of the effect of a change in the exchange rate on the balance of payments is not the subject of this thesis, such a change will affect the price of the exported good A, the prices of the imported goods C and K and the domestic currency equivalent of the stock of foreign reserves. These prices and the foreign reserves are exogenous variables in the static model which will be presented in the next chapter, thus, it is appropriate to review briefly the literature on the balance of payments.

The balance of payments or its component, the balance of trade, has preoccupied economists from time immemorial. One of the earliest traces of the concept of balance of trade appeared in the economic thought known as mercantilism which dominated Europe from the 16th century to the 18th century. "The core of mercantilism, of course, is the doctrine that a favorable balance of trade is desirable because it is somehow productive of national prosperity" [Blaug, 1978, p. 11]. The general proposition is that a country's wealth is increased through the accumulation of gold and silver in the treasures of the Prince. The policies suggested to bring about a favorable balance of trade vis-à-vis every trading partner include the regulation of foreign trade, the promotion of export industries and the imposition of tariffs on imported finished goods.
Thomas Mun (1952) was the first writer to point out that a favorable balance of trade with every individual country was unnecessary as long as total exports exceeded total imports and that the initial amount of gold exported to pay for inputs resulted in exports proceeds from the sale of the finished goods which are far above initial outlays.

Mun alluded to the relationship between prices and the money stock composed of gold and silver primarily. But perhaps, the sharpest exposition of this relationship appeared in the balance of trade adjustment mechanism generally attributed to David Hume (1955) called the specie flow mechanism. Hume contended that a policy of trade surplus was self-defeating. For instance, assume that England has a balance of trade surplus with France and a fixed exchange rate prevails between the two countries. This surplus means that there will be an inflow of bullion, (i.e., specie) from France into England and this inflow will result in an increase in the demand for goods and their prices, assuming constant supplies. The higher prices in England will induce English consumers to switch to cheaper imported goods, so that imports will rise and create a balance of trade deficit in England. There will be an outflow of gold and silver from England to pay for the French goods. A similar process will occur in France which now has a balance of trade surplus. Several other writers such as Adam Smith, the "god" of free markets advocates, attacked the mercantilist doctrine.
Other authors such as Keynes ["Notes on Mercantilism" in the General Theory, 1936] came to the defense of the mercantilists.

In the post-Keynesian period, the questions that most writers addressed themselves to were no longer whether a given country could or should attempt to have a favorable balance of payments vis-à-vis each of her trading partners but what would be the effects of various exogenous variables (i.e., exchange rate, tariff, etc.) on the balance of trade and/or on the balance of payments. Broadly speaking, three schools of thought may be identified in the money and trade literature which are concerned with the balance of trade or the balance of payments: the absorption approach, the Keynesian (or elasticity) approach and the monetary approach.

Alexander (1952), who is an absorption approach advocate, asserts that the real balance of trade is equal to the real GNP less absorption which is composed of the total real spending on consumption and investment goods, and that a devaluation will improve the balance of trade if it increases real GNP by more than it increases absorption.

The major characteristics of the Keynesian school are as follows. First, it is assumed that there exists "unemployment and wage-price rigidity in domestic markets" (Whitman, 1975, p. 507). It must be pointed out that Tsiang (1961, p. 407) who is one of the representatives of this school assumed full employment in one version of his
model. Second, elasticity models contain a very limited number of goods and assets. Three, and more importantly, they assume that the goods and money markets clear under fixed exchange rate. Kuska pointed out that this assumption and some others make "almost all the models in the Keynesian balance-of-payments literature suffer from internal contradictions and deficiencies which makes them unsuitable for balance of payments theories" (Kuska, 1978, p. 659). Four, elasticity models stress the good markets and focus on the balance of trade. The general procedure of these models is to identify the determinants of imports and exports within a demand and supply framework and to substitute the imports and exports functions into the balance of trade expression and obtain comparative static results which are then expressed in the form of elasticities.

One of the earliest contributions to the theory of devaluation in an analytical framework was made by Hahn (1959). Hahn's model differs from the typical Keynesian model by the fact that the author used the concept of balance of payments instead of that of the balance of trade and Hahn's results are not expressed in the form of elasticities. However, by assuming that the goods markets clear and more importantly that the excess demand for money [i.e., function E₁ in expression (4.5) on p. 115 of his paper] is zero,
Hahn's model has a major feature of standard Keynesian models. Hahn's two-country model includes two traded goods which are perfect substitutes so that under free trade, the purchasing power parity relation holds and there is one currency per country. Each country holds its own currency which is the numeraire. One purpose of the model is to examine the effect of the exchange rate on the balance of payments. Hahn assumed that the production possibility frontier is concave and the utility function in each country depends on the quantities of the goods demanded and on the real quantity of money held. In this framework, Hahn found that "assuming the goods markets to be in equilibrium both before and after changes in the rate of exchange, the balance of payments of country one will change in the same direction as the price of currency two in terms of currency one changes provided all goods and currencies are gross substitutes" (1959, p. 117).

In a two-country, one aggregate good and two-asset model (money and bonds) which includes spending identities, demand functions, import and price functions, a money market equilibrium condition and a balance of trade, Tsiang (1961) found that, in general, a devaluation of the home currency has an ambiguous effect on the balance of trade (p. 400). When full employment prevails, a devaluation improves the balance of trade only if the sum of the elasticity of demand
for imports in both countries exceeds one. In a two-country, two-good model, Harberger (1950) found a similar result.

The monetary approach to devaluation differs from the elasticity approach mainly on two grounds. First, the elasticity theorists focus on the flow demand for and flow supply of goods and services (i.e., on the balance of trade) partly because of their belief that "whatever effects particular policies might have on the other accounts in the balance of payments, the impact on the goods and services account would be dominant" (Whitman, 1975, p. 492). The monetarists claim that a deficit (surplus) in the balance of trade may be exceeded or offset by a surplus (deficit) in the balance of capital or each of the two balances may show a deficit or a surplus, so that the emphasis must be placed on the net amount of the two balances [see D. Kemp (1975), Whitman (1975), Mussa (1976)]. Second, unlike the Keynesians, most monetary models do not assume that the money market clears so that in these models the balance of payments is not in equilibrium.

One of the earliest contributions to the balance-of-payments theory using the monetary approach was made by M.C. Kemp (1962). Some of the purposes of Kemp's model was to examine the effects of devaluation on the prices of imports and exports and on the balance of payments. Kemp's framework consists of two countries trading freely two perfectly substitutable goods and each country holding its own currency.
The numeraire in each country is the domestic currency, yet, the price of good one is used to deflate all the nominal variables in order to obtain real variables. Kemp assumed that the excess demand functions depend on the price ratio and the real money stock and that there is a stabilization fund which pegs the exchange rate and agrees "to buy and sell the two national currencies at a fixed rate of exchange" (1962, p. 317). In Kemp's model, the money stock increases or decreases as the balance of payments is positive or negative and the private excess demand for currency in each country needs not be equal to zero (p. 318). Treating the nominal prices of good one and of good two, the money stocks at home and abroad and the balance of payments as endogenous variables and the exchange rate as an exogenous variable, the author showed that if the two goods and money are gross substitutes, then a devaluation will raise the prices of exports and of imports and improve the balance of payments (p. 319 and p. 326). These comparative static results are ambiguous if the gross substitution assumption is not made. Perhaps, the most serious criticism of Kemp's model is that many feedbacks are ignored because the demand and supply functions are not derived, but they are postulated and they include only two arguments.

Dornbush (1973) deviated from preceding monetary models by introducing a nontraded good. The author purports to show that within his framework, first, there is a positive
relationship between the balance of payments and the relative price of nontraded goods, and second, a devaluation lowers the relative price of nontraded goods. In Dornbush's model, there are two large countries but the home country is the center of attention. In each country, there are two goods (a traded and a nontraded good), one asset (money), no bonds. The supply functions of the goods are assumed to depend on the price ratio only while the demand functions depend upon the price ratio and the total real spending.

The initial nominal money stock is given but monetary authorities can change it in order "to maintain a pegged exchange rate" (1973, p. 872). A Cambridge-type-interest inelastic demand for money function is assumed to depend upon nominal income alone. These fairly restrictive assumptions about the arguments of the demand and supply functions made by Dornbush considerably reduce the number of feedbacks and largely explain most of the unambiguous comparative static results obtained by the author. Note also that in Dornbush's model, the traded goods are perfect substitutes and there is free trade so that the purchasing power parity relation holds. Finally, Dornbush assumed that the balance of payments is "proportional to the stock excess demand for money" (p. 876). Dornbush did not separate formally the endogenous variables from the exogenous one and treating the money price of nontraded good relative to the money price of the traded good \( q \) as an endogenous variable, and the balance of payments \( B \) as
an exogenous variable, the author showed that when the non-traded good market clears, the partial derivative \( \frac{\partial q^e}{\partial B} \) is positive. Based on the definition of the price ratio \( q \), Dornbush's result means that an increase in the net flow of foreign reserves will raise the price of nontraded goods, *ceteris paribus*. Using the fact that \( \frac{\partial q^e}{\partial B} \) is positive, Dornbush showed that, in equilibrium, a devaluation lowers the relative price of the nontraded good.

Mussa (1976) expanded along Kemp's line by introducing an extra asset (bonds) and an import tariff. One of the purposes of Mussa's model is to show that an import tariff may improve or deteriorate the balance of payments in the long run. In a two-country, two-good framework, Mussa assumed that wealthowners hold money and bonds and that an ad valorem import tariff is imposed by the smaller country with the proceeds returned to consumers. The export good is the numeraire. By assumption, the supply functions for goods depend on the price ratio alone while the demand functions for goods depend upon the price ratio and income which itself depends on the price ratio. The demand for money depends upon the price ratio, income and the interest rate on bonds. In line with several authors such as Kemp (1962), Mussa assumed that the stock of money changes only "as a result of balance of payments surpluses and deficits" [Mussa, 1976, p. 200]. That is the domestic credit and the money multiplier are constant. Focusing on the smallest country and
treating the money stock as an endogenous variable, Mussa found that an increase in the import tariff rate \( t \), has an ambiguous effect on the money stock because \( t \) has an ambiguous effect on income \( y \) and on the price ratio \( q \) (p. 203). Since a change in the money stock over time is equal to the balance of payments, this means an increase in the import tariff may improve or deteriorate the balance of payments.

In an attempt to provide a unified framework for various scattered results, Anderson and Takayama (1977), abbreviated as AT, developed a two-country model in which there are two goods which are freely traded and one currency per country. Good two is the numeraire in each country. One of the objectives of this model is to show that, in the short run, an increase in the money stock may increase or reduce the general price index. The supply functions of goods are assumed to depend upon the price ratio alone while the demand functions depend upon the price ratio and real spending. The balance of payments is written implicitly as a function of the price ratio and the real money stock. In several monetary approach-type models [Mundell (1960, 1968), Johnson (1953), Kemp (1962), Mussa (1976) and others], the balance of payments is equated to the time derivative of the money stock. This means implicitly that the stock demand for money is constant. AT's model constitutes a significant contribution to the literature of the balance of payments theory because in this model prices are endogenous and the
balance of payments can be affected by changes in the money stock and prices. A similar approach will be used in this thesis. Another interesting feature of AT's model is that it contains a price index $P$ which depends on the nominal prices of the two traded goods. There will be a similar index in the models developed in this study. Treating the nominal prices of the traded goods, and thus the price index $P$, as endogenous variables and the money stock as an exogenous variable, the authors found that, in general, a change in the money stock has an ambiguous effect on the price index. Yet, they found that if goods and money are gross substitutes, then an increase in the money stock will increase the equilibrium price index [AT, 1977, p. 352].

In summary, the money and trade theories associated with the Keynesians focused on the balance of trade and assumed that the money market clears while those associated with the monetary approach consider the entire balance of payments, and do not assume in general that this balance is equal to zero. Some of the monetary approach models examined the effect of devaluation on the price of goods and on the balance of payments or the effect of a change in the money stock on prices. These questions will be explored in the next chapter. To end this presentation of the literature, some money and growth models will now be presented.

The literature on money and growth is abundant [Tobin (1955, 1965, 1968), Sidrauski (1967), Stein (1969,
1970), Burmeister and Dobell (1970), Takahashi (1971), Foley and Sidrauskis (1971), Allen (1972), Turnovsky (1977) and others]. Most of these models deal with a closed economy and there are two broad categories of money and growth models: the Neoclassical group [Tobin (1955), Patinkin (1965), Foley and Sidrauski (1971) and others] and the Keynes-Wicksell group [Rose (1966), Stein (1970), Nagatani (1969) among others].

Both types of models, in general, have the following common points: (1) there is no government sector; (2) there is a single aggregate production function using labor and capital; (3) the price of the aggregate good is used as the deflator; (4) perfect foresight and occasionally adaptive expectations is assumed; (5) the rate of growth of the nominal money stock is an exogenous variable while the capital labor ratio or its inverse and the real money stock are endogenous variables.

The major difference between the two categories of model is the speed of adjustment in the various markets. Neoclassical theorists assume that all markets clear instantaneously (i.e., infinite speed of adjustment) while Keynes-Wicksell (KW for short) theorists assume that the speed of adjustment in the various markets is finite so that prices will have to change to clear the markets when they are in disequilibrium. To explain how a possible sustained disequilibrium in the good markets (i.e., planned investment \( \neq \))
planned saving) might arise, KW theorists suggest that saving and investment decisions are made independently. They indicate that firms determine their investment function from maximizing an expected stream of profits while households determine their consumption and saving functions by maximizing utility and there is no reason, a priori, to expect that the demand for and supply of investment goods based on these two behaviors will coincide. It is the role of market forces (i.e., prices) to make these behaviors consistent. It is perhaps fair to say that most growth models are of the neoclassical type. Furthermore, the speed of adjustment in some fairly popular KW-type models such as Nagatani (1969) have little or no effect at all on the steady state values of the endogenous variables. In this thesis, an infinite speed of adjustment will be assumed. Some neoclassical and KW models will be reviewed briefly now.

One of the earliest contributions to growth models was made by Tobin (1955). Tobin complained that contemporary growth models used linear production functions which permitted no input substitution. He set out to develop a dynamic model which allowed input substitution and which would permit to examine the effect of an increase in the capital stock on the price level. Tobin's model includes one aggregate good produced with a constant returns to scale production function using labor and capital, and money. The money price of the good is used to deflate the nominal variables. The markets
for the inputs, the aggregate good and money are assumed to clear instantaneously. Within this neoclassical framework, treating the price of the good as an endogenous variable and the capital stock as an exogenous variable, Tobin showed that, in general, the relationship between the price of the good and the aggregate capital stock is ambiguous. Yet, this relationship is negative if the system of dynamic equations is stable while it is positive if the system is unstable.

Nagatani (1969) suggested a framework in which the actual rate of price change and the ratio of real money stock to capital were endogenously determined as a function of the rate of growth of the nominal money stock. In this closed economy model, there is a labor market, a capital market, a good market and a money market and the price of the aggregate good is used as the deflator. The structural equations of the model, that is equations (1), (2), (3) and (7) in Nagatani's paper reflect the KW assumption of limited speed of adjustment so that prices and the wage have to change in order to clear the relevant markets if they are in disequilibrium. However, the solution to the short run version of the model clearly indicates that the money market must clear [equation (11), p. 194] and that the expected rate of price change must be equal to the actual rate of price change [equation (12), p. 194]. Similarly, in the dynamic model the accumulation equation for the real money
stock and the rate of price change are obtained assuming that the labor market, the capital good market and the money market adjust instantaneously (p. 200). These assumptions are clearly neoclassical in nature. It is within this framework that Nagatani found that, at the steady state, an increase in the rate of growth of the nominal money stock has an ambiguous effect on the real money stock while it increases the actual rate of price change [expressions (42) and (43) in Nagatani, 1969, p. 202].

Stein (1970) generalized Nagatani's framework by introducing an extra financial asset (bonds). Stein's model includes one consumption good, one capital good, money and bonds and a labor market. Depending upon the assumption concerning the speed of adjustment in the various markets, the author showed that, at the steady state, in the neoclassical case, an increase in the growth rate of the nominal money stock decreases the labor to capital ratio while this comparative dynamic result is ambiguous in the KW framework (p. 104).

The essentially closed-economy model developed by Foley and Sidrauski (1971), abbreviated as FS, has a static version with no accumulation equation and a dynamic version; however, only the latter version is presented here. The purpose of the model is to examine how fiscal and monetary tools may influence the share of the output devoted to investment in the economy. The basic idea is that the amount
of investment good purchased depends on its price and on the price of consumption goods. In addition, fiscal and monetary policies affect these prices so that once the price of the investment good is determined (in terms of fiscal, monetary and other variables), the share of output devoted to investment purposes can be determined. The model includes one consumption good, one investment good, money, bonds and capital. The private sector produces, consumes, saves and pays taxes while the government sector levies taxes, consumers, makes transfer payments to the private sector and administers the monetary and fiscal policies. Most of these operations and several others are present in the models of this thesis. The analysis is carried out in per-capita terms and the consumption good is the numeraire. However, unlike previous models, FS treated money as any other good which has a price expressed in terms of the consumption good, so that although there is money in the system, there is only one numeraire: the consumption good. FS's dynamic model includes three static equations and two dynamic equations.

The authors assumed that the real stock demand functions for money, bonds and capital depend upon wealth, income and the rates of return on each of the three assets. These demand functions are equated with the supplies and the system of three static equations is reduced to a system of two static equations. The third static equation is obtained from the equilibrium condition in the consumption good market
where it is assumed that the demand function is positively related to disposable income in a linear form. The per-capita capital stock accumulation equation is obtained by differentiating the capital labor ratio and substituting the investment function derived in conjunction with the supply function of the consumption good. The government debt accumulation equation is obtained from the definitions of the per-capita government debt and the government deficit.

The basic equations which describe FS's dynamic model are:

\[ \frac{dP_k}{dt} = P_k (gP_m, k, \pi_m, x) \quad (2.1) \]

\[ i = i (gP_m, k, \pi_m, x) \quad (2.2) \]

\[ \beta(P_k, k, m, g, e) = 0 \quad (2.3) \]

\[ \frac{d}{dt} k = q_I (P_k, k) - nk \quad (2.4) \]

and

\[ \frac{d}{dt} g = d - ng \quad (2.5) \]

where the dot "o" over a variable denotes its derivative with respect to time,

- \( P_k \) = unit price of the investment good in terms of the consumption good C
- \( P_m \) = unit price of money in terms of good C
\[ g = \text{per-capita stock of government debt} \]

\[ k = \text{capital labor ratio} \]

\[ \pi_m = \text{inflation (i.e., } \pi_m < 0) \text{ or deflation (} \pi_m > 0 \text{)} \]

\[ x = \text{debt-to-money ratio} \]

\[ i = \text{nominal interest rate on bond} \]

\[ e = \text{per-capita government consumption of good } C \]

\[ q_I = \text{investment supply function} \]

\[ n = \text{rate of growth of population} \]

\[ d = \text{per-capita government deficit} \]

\[ \beta \text{ denotes an implicit function.} \]

Equations (2.1) and (2.2) describe the equilibrium condition for the assets markets, equation (2.3) describes the equilibrium condition for the consumption good market while equations (2.4) and (2.5) are the accumulation equations of the capital labor ratio and of the government debt. The signs of the partial derivatives which are shown above each variable derive from the assumptions concerning the supply and demand functions (in particular, the assets are assumed to be gross substitutes) and from Walras Law.

The above system is composed of five equations and nine unknowns, \( g, k, P_k, P_m, \pi_m, i, d, e \) and \( x \). The original model includes a tenth variable: the rate of change in the price of investment \( \pi_k \). This variable is deleted here because it has been set to zero in most of the exercises.
Among the nine variables indicated above, the government policy tools are x, d and e. Since there are more variables than equations, the above system is closed by choosing "arbitrary paths for the government policy tools and the other variables will be forced through equilibrium conditions and the restrictions on the paths of g, k \( \pi_m \) and \( \pi_k \) to follow consistent trajectories" (FS, 1971, p. 99).

In the context of monetary policy, the government may decide to pursue the following goals: a target inflation resulting in a target price (i.e., \( \pi_m = \bar{\pi}_m \) = given value and thus \( P_m = \bar{P}_m \) = given value) and deficit (i.e., \( e = \bar{e} \) = given value and \( d = \bar{d} \) = given value). In this case, the variables to be determined are \( g, k, P_k^e, i \) and \( x \) and the solution procedure is the following. The static equations (2.1) and (2.2) which describe the assets markets are passive in the sense that they determine \( x \) and \( i \) once the other variables are known. The equilibrium price of the investment good \( P_k^e \) is determined in the consumption good market, that is by equation (2.3). First, solve implicitly (2.3) for:

\[
P_k^e = (k, g, \pi_m, e). \tag{2.6}
\]

Then substitute (2.6) into (2.4) so that, at the steady state, the motion of the economy is described in a \( (g,k) \) space by the phase diagram of:
\[ k^* = q_{1} e (k, \ g, \ \tau_{m}, \ e) - nk = 0, \quad (2.7) \]

\[ g^* = d - ng = 0. \quad (2.8) \]

Once the steady state values of \( g = g^* \) and \( k = k^* \) are obtained from (2.7) and (2.8), substitute \( g^* \) and \( k^* \) into (2.6) to obtain \( P_k^* \) and proceed in a similar fashion to obtain \( x^* \) from (2.1) and \( i^* \) from (2.2). In this context of monetary policy, FS found that, a decrease in government deficit or in government spending, *ceteris paribus*, raises the steady state stock of capital (FS, 1971, p. 112 and p. 115). Similarly, it can be shown that a decrease in the price of money (i.e., inflation occurs), *ceteris paribus*, will increase the steady state levels of the capital labor ratio, the per-capita government debt and the equilibrium price of capital.

Concerning fiscal policy, the government may aim at a target price of money and at target debt-to-money ratio. In addition, it may vary its spending \((e)\) and net taxes by an equal amount in order to leave the deficit \((d)\) unchanged or it may change the size of the deficit by varying both spending and taxes or by varying either spending or taxes. Note that the net tax variable is included in the definition of the deficit term \(d\) [see FS, 1971, equation 4-6, p. 56]. FS showed that if the government decides to vary taxes and thus the deficit \((d)\) in order to achieve
predetermined levels of the price of money ($P_m$) and its rate of change ($\pi_m$), the debt-to-money ratio ($\bar{x}$) and spending ($\bar{e}$), then a decrease in $\bar{e}$, ceteris paribus, will increase the steady state levels of the per-capita government debt and the capital labor ratio (p. 133).

Recall that the ultimate purpose of FS's model is to examine how fiscal and monetary tools can be used to affect the amount of output which is invested relative to the amount which is consumed. At the steady state, the following expressions hold:

\[ P^*_k = \gamma (k^*, g^*, \pi_m, e), \]  

\[ q^*_I (P_k^*, k^*) - nk^* = 0 \]  

where (2.9) and (2.10) are obtained by substituting the steady state values of $g$ and $k$ into (2.6) and (2.7).

Expression (2.10) may be written as:

\[ q^*_I (P_k^*, k^*) = nk^*. \]  

The net effect of a change in $k^*$ on the supply of investment good is obtained by differentiating (2.10a) with respect to $k^*$ taking into account expression (2.9). The result is:
\[ \frac{\partial q_I^*}{\partial P_k^*} \left( \frac{\partial P_k^*}{\partial k^*} \right) + \left( \frac{\partial q_I^*}{\partial k^*} \right) = n. \] (2.11)

The left hand side of (2.11) represents the net effect of a change in \( k^* \) on the supply of investment good. At the steady state, this net effect is positive because it is equal to a positive quantity \( n \). This result means that, starting from the steady state, a government fiscal or monetary policy action which results in an increase in the steady state capital labor ratio will increase the share of output devoted to investment purposes. That is, if total output is \( q^* = q_C^* (P_k^*, k^*) + P_k^* q_I^* (P_k^*, k^*) \) where \( q_C^* \) and \( P_k^* q_I^* \) are respectively the share of output devoted to consumption and investment, the quantity \( \partial (P_k^* q_I^*) / \partial k^* \) is positive.

Critique and Evaluation

One general criticism which can be levied against the models presented above, especially the static models [except Komiya (1967) and Kemp (1962)], and against several other models found in the literature is that many key assumptions are not explicitly stated\(^4\). Typical examples are the models of Alexander (1952) whose deficiencies will be presented below and Mundell (1960, 1962). As regards

\[^4\text{In the literature reviewed above, this deficiency is more commonly encountered in the static models than in the dynamic models. Furthermore, the above criticism applies more to the static and dynamic models published in academic journals than to those published in books.}\]
the few assumptions which are stated, often little explanation is provided to support them. Under these circumstances, it becomes very difficult to evaluate the results obtained in the light of the assumptions. Another general weakness of the models of the literature is that their scope is fairly limited. Indeed, the number of goods considered rarely exceeds two. Komiya (1967) is one of the few authors whose model contains three goods. Similarly, the number of assets in the models vary between zero and two. As regards the basic relationships such as the supply and demand functions, which describe the behavior of the various economic agents, in general, they are postulated. Among the static models reviewed above, only that of Komiya (1967) contains a set of explicitly derived supply functions and those of Hahn (1959) and of Anderson and Takayama (1977) contain demand functions which are derived from utility functions. The limited number of goods and assets which are considered in the models of the literature and the fact that the behavioral relationships are postulated reduce considerably the number of linkages between sectors and explain the unambiguous comparative static and dynamic results obtained. It should also be mentioned that, except

5. In the literature reviewed above, this deficiency also is more commonly encountered in the static models than in the dynamic models. Furthermore, the above criticism applies more to the static and dynamic models published in academic journals than to those published in books.
in a few cases such as Foley and Sidrauski (1971) and Kuska (1978), Walras Law is ignored or is not at least explicitly written, so that it becomes almost impossible to check whether the subsumed signs of the various partial derivatives are consistent with this law. The implication is that the internal consistency of the models and consequently the validity of the results obtained are difficult to establish.

Let us turn now specifically to some of the models presented above.

The framework of Alexander (1952) contains several obscure points. First, there is only one good and no explicit money market. The author mentioned that this single good may be thought of as a vector of goods and without any further comment, Alexander proceeded with the concept of one good and one price. But clearly, one cannot talk meaningfully about the price of a good unless there is money or another good or asset which can be used as the numeraire. Second, it is hard to understand how one can assume away the money market and proceed to study the effect of cash balances and money illusion, etc., on the balance of trade as Alexander did. Third, if there is a money market or other markets behind the scene, nothing is said about the situation (equilibrium or disequilibrium) which might prevail in these markets. Four, Alexander objected that the elasticity theorists used partial elasticities (or
derivatives) while total elasticities (or derivatives) were the most adequate approach. But his exposition which is mostly verbal leaves unclear how the partial elasticity (or derivative) deficiency can be avoided if one attempts to formulate a mathematically rigorous absorption model.

The models of Hahn (1959) and Tsiang (1961) both belong to the elasticity theory category, and the general criticisms mentioned above (inexplicit assumptions, limited number of goods and assets, etc.) apply to them. In addition and more importantly, the authors assumed that all markets clear, in particular the money market. In an open economy, the balance of payments, which is the same as the balance of trade in the absence of capital movements, is equal to the difference between the flow demand and the flow supply of money in the home country. This relationship comes from Walras Law and it holds whether the money stock is an exogenous variable or an endogenous variable. The balance of payments is then equal to zero only if all the markets, in particular the money market, clear. Under this condition, "it becomes nonsensical to investigate the effects on either economy's balance of payments of variations in the exogenous variables" [Kuska, 1978, p. 661].

Dornbush's 1973 paper and several other articles dealing with the balance of payments authored by Dornbush are significantly short of supportive explanations of the assumptions and claims made. Why is an interest inelastic
Cambridge-type demand-for-money function assumed [equation (1) in Dornbush, 1973, p. 872]? Where would the terms $\pi$ and $\pi^*$, which represent the speed of adjustment in the home and foreign countries money markets in equations (5) on p. 872 come from if the Walras Law of the models were written? Why does the term $\eta_2$ represent the "compensated elasticity of demand for the home goods" (p. 377). What is the relationship, if any, between the terms hoarding, saving, balance of payments and money used in an interchangeable manner by the author or implied in his writings with respect to standard definitions? Dornbush's article provides no information which can permit to answer these questions and several others.

The major weaknesses of the models presented above are: (1) the major assumptions are not clearly stated; (2) the models include few goods, assets and economic agents and often the basic functions are postulated so that numerous feedbacks are ignored; (3) in general, Walras Law is ignored and this may result in an internal inconsistency of the models in question.

Despite all these deficiencies, the above models have contributed to the advancement of the art in various respects. The model of Hahn (1959) was a significant contribution to the literature from an analytical point of view. In addition, it is based on derived demand functions for goods and money. The elasticity models such as those
of Harberger (1950) and Tsiang (1961) constitute an improvement over some absorption models such as the Alexander model because they include more markets, their assumptions and derivations are a little more explicit and their results are expressed in widely accepted economic terms (i.e., elasticities). The contribution of Kemp (1962) is important, first because the author stated explicitly the underlying assumptions, second because the entire balance of payments is considered, not only the balance of trade, and third because it is not assumed that the money market clears. The main contribution of Dornbush (1973) to the advancement of the money-and-trade literature is through the introduction of a nontraded good while Mussa (1976) can be credited for incorporating an extra asset (bonds) and import tariff into a theory of the balance of payments. Finally, Anderson and Takayama (1977) accomplished the important task of building a house big enough, meaning a framework general enough, to accommodate several homeless people (i.e., seemingly unrelated models).

In the next chapter, a static model will be developed and an effort will be made to state all the key assumptions and explain why they are made. The model will include: (1) a private sector which produces a pure export good A and a nontraded consumption good B, consumes good B and an imported good C, pays taxes, saves in the form of money, time deposits and capital and borrows from banks;
(2) a government sector which levies taxes and tariffs, borrows money from OPAT and from foreign countries, and buys good C and the capital good K; (3) a banking sector which creates money and other financial assets; (4) the marketing agency OPAT which exports good A, makes a profit and lends part of it to the government; (5) a foreign country which produces goods C and K and imports good A. In this model, the various budget constraints will be stated explicitly as well as the Walras Laws, the supply and the demand functions will be derived and numerous feedbacks will be taken into account. None of the models reviewed above contains all the features of the static model of this thesis regrouped under the same roof. However, some of the results obtained in the literature are maintained here. For instance, it will be shown that, as in Komiya (1967), if the capital intensity in the export sector exceeds the capital intensity in the nontraded good sector, then an increase in the price of exports will raise the price of the nontraded good. Similarly, as in Dornbush (1973), it will be shown that there is a positive relationship between the balance of payments and the price of the nontraded good.

As regards the money and growth models presented, the general criticisms mentioned above (unstated key assumptions, limited scope, etc.) apply also to them, though to a smaller extent than it does to the static models. In
addition, these dynamic models are closed as opposed to open economy models. Therefore, foreign repercussions are ignored, in particular no reference is made to the balance of payments nor to the accumulation of foreign reserves.

Despite these weaknesses, the contributions of Tobin (1955), Nagatani (1969), Stein (1970), Foley and Sidrauski (1971) or FS for short, and several others to the advancement of the money and growth literature are significant. Nagatani can be credited with developing a model in which inflation and the real stock of money are treated as endogenous variables. Stein's model is perhaps richer than several preceding models in the literature because it takes into account the effect of inflation/deflation on disposable income [expression (5) in Stein, 1970, p. 83], on the demand for money [expression (7), p. 88], and on investment [expression (32), p. 100]. FS's model is a neoclassical model because it assumes that all markets clear instantaneously. However, it departs from previous models in several respects. First, the reader is led through the model gradually, starting from the foundation (i.e., production sector, consumption sector, etc.) into the roofed house (methodology and results). Second, most of the key assumptions are clearly stated and the major identity constraints are indicated and taken into account to obtain several signs. Third, the model contains more goods and assets
than previous models and the interrelations between the various markets are taken into account.

Among the dynamic models reviewed above, that of FS is the one which has more common features with the dynamic model presented in Chapter 5 of this thesis. One, both models focus on the determinants of the price of a good: an investment good in FS and a nontraded good in the model of this study. Two, they contain an explicit production sector and takes into account numerous feedbacks. Three, both models assume that all markets clear instantaneously, except for the money market in the model of this thesis. However there are some major differences. First, in the model developed in this work, there is an important institution (OPAT) whose behavior affects several economic agents and sectors, in particular it is a source of funds to the government and its operations will affect the investment function of the government and consequently the accumulation equation of the capital labor ratio. There is no such institution in FS. Second, the type and number of assets differ and there will be more markets interactions in the model of this thesis than in FS. Finally, the economy described in this work is open while FS's economy is primarily closed. The implication is that, in addition to the accumulation equations for the capital labor ratio and the per-capita government debt as in FS, there will be an
accumulation equation for the per-capita stock of foreign reserves in the model of this study.

Let us turn now to the models of this thesis. Firstly, the static model is presented. Second, the dynamic model of the determination of the producer price by OPAT is examined. Thirdly, the dynamic model of the entire economy is described. Fourthly, the empirical results obtained from estimating the static model are discussed. Finally, some projections which are based on the equations estimated are presented.
CHAPTER 3

A SIMPLE STATIC MODEL

The purpose of this chapter is to formulate a static model to explain the price of the nontraded good, the consumer price index, the gross national product and the balance of payments. The institutions whose behavior this model attempts to explore have been presented in Chapter 1. These descriptions will not be repeated in detail here. First, the model is described, followed by its solution.

The Model

The Budget Constraints and the World Walras Law

There are four goods and five assets in the model. The goods are: a pure export good A, a domestic nontraded consumption good B, an imported consumption good C and an imported capital good K and the assets are money M, time deposits TD, bank loans BL, OPAT loans OL, and foreign loans FL.

The agents in the home country which is the center of attention include private individuals and firms, a banking sector, a marketing agency and the government. The rest of the world is represented by a foreign country. The operations of the various economic agents can be summarized in their
budget constraints which are written in aggregate terms, using money as the numeraire, as follows:

(1) The Budget Constraint of the Private Sector.

Private individuals and firms produce, consume, save and pay taxes. In addition, they borrow from banks by issuing a bank loans instrument and they hold their savings in the form of money, a time deposits instrument issued by the banks and real capital. Their budget constraint may be written as:

\[ P^A A + P^B B + rTD + \Delta BL^S = P^B B + (1+t_e)e P^*_C D^*_C P \]

\[ + e P^*_K D^*_K P + T + \Delta L + \Delta TD^d + r^*_B BL \]

(3.1)

where

- \( P^A, P^B \) = unit price of goods A and B in the home currency
- \( P^*_C, P^*_K \) = unit price of goods C and K in the foreign currency
- \( A, B \) = quantity of goods A and B supplied by the private sector in the home country
- \( e \) = exchange rate = home currency/foreign currency, (decrease in \( e \) = devaluation of the home currency)
- \( TD, BL \) = stock of time deposits and bank loans in the home currency
\( r, r_{BL} \) = nominal interest rates on time deposits (TD) and on bank loans (BL)

\( D_B, D_{CP}, D_{KP} \) = quantity of consumption goods B and C and capital good K purchased by the private sector in the home country

\( t_C \) = nonprohibitive ad valorem import tariff on C

\( T \) = personal lump sum tax

\( L \) = private demand for money.

The symbol "Δ" refers to the change in the value of the variable between two consecutive periods and the superscript "s" and "d" refer to "supplied" and demanded".

(2) The Budget Constraint of the Banking Sector.

The Central and commercial banks create domestic credit (DC), issue a time deposits instrument (TD), and buy a loan asset (BL). The government also can increase the domestic credit level by using its overdraft facility. For simplicity, a single variable DC is used to represent the domestic credit created by banks and the government and this variable appears in the banking sector's budget constraint alone. Note also that since there is no sterilization of foreign reserves, these reserves are added to the domestic credit to constitute the monetary base and at this stage, for notation purposes, no differentiation is made between the domestic credit, the
foreign reserves, the money multiplier and the money stock. The term money $M$ stands for money stock in the following budget constraint:

$$\Delta M + r_{BL} BL + \Delta TD^S = \Delta BL^d + rTD + \pi_{bank} \quad (3.2)$$

where $\pi_{bank}$ is the profit made by banks.

(3) The Budget Constraint of OPAT. OPAT's officials told this writer that the agency buys the entire output of the export crops from farmers and it is shown in Appendix A that this behavior is consistent with the monopsonistic position of the corporation in the local market for good A. With the proceeds from the export of the agricultural products, the marketing agency pays producers, taxes, other costs and buys a government issued loans instrument (OL). Its budget constraint is:

$$eP_A^*D_A^* + iOL = P_A^*A + P_A^*tA_A^* + OC_A^*DA^* + \pi_A^*DA^* + \Delta OL^d \quad (3.3)$$

where

- $P_A^*$ = unit price of good A received by OPAT expressed in the foreign currency
- $OL$ = stock of OPAT loan expressed in the home currency
- $i$ = nominal interest rate paid by the government on OPAT loans
- $D_A^*$ = quantity of good A exported by OPAT
\( t_A = \) nonprohibitive ad valorem export tariff levied on good A

\( OC_A = \) other costs incurred by OPAT per unit of good A exported such as insurance, transportation costs, etc.

\( \pi_A = \) residual profit made by OPAT per unit of good A exported.

4) The Budget Constraint of the Government. The home government levies a lump sum tax \( T \), an import tariff on good C: \( eP_C t_C \)/unit; an export tariff on good A: \( P_A t_A \)/unit; and it borrows funds by selling debt instruments which it issues to OPAT (\( OL^S \)), and to the foreign country (\( FL^S \)). There will be an additional debt instrument in case the government makes use of its overdraft facility at the Central Bank, but this extra instrument is ignored here because over the 17-year period, the government used this credit line only once (in 1975).

In Togo the government returns no tariff revenues to the private sector directly and the public revenues are used to buy primarily imported consumption and capital goods and to repay the principal and interests on loans, so that the government budget constraint may be written as:
\[ \pi_A D_A^* + eP_C^* D_C^* + \pi_T + \Delta G^S + e\Delta L^S + \Delta eFL^S + \pi_A D_A^* = eP_C^* D_C^* \]

\[ + eP_K^* D_K^* + iOL^* + r^* eFL \]

where

\[ D_{CG}, D_{KG} = \text{quantity of goods } C \text{ and } K \text{ purchased by the government} \]

\[ r^* = \text{nominal interest rate on foreign loans} \]

\[ FL = \text{stock of foreign loans expressed in the foreign currency.} \]

(5) **The Budget Constraint of the Foreign Country.**

The foreign country (i.e., the rest of the world) exports part of her outputs of goods C and K, imports good A and buys a foreign loans instrument (eFL) issued by the home country, so that the budget constraint of the foreign country may be written as:

\[ eP_C^* C^* + eP_K^* K^* + e\Delta M^* + r^* eFL + OCA^* D_A^* + \pi_{bank} = eP_A^* D_A^* + eP_C^* D_C^* + eP_K^* D_K^* + e\Delta L^* + e\Delta FL^d \]

(3.5)

where the star "\(^*\)" refers to variables determined in the foreign country, and,

\[ C^*, K^* = \text{total output of goods } C \text{ and } K \]

produced by the foreign country.
\( L^*, M^* \) = demand for and supply of money in the foreign country expressed in foreign currency

\( D_C^*, D_K^* \) = quantity of goods C and K purchased by the foreign country.

The term which stands for other costs incurred by OPAT (i.e., \( \Omega_C \), \( \Omega_A \)) appears in this budget constraint because part of these costs include transportation, insurance and commissions paid to foreign accredited companies which operate in Togo, and to OPAT agents abroad. The banks' profit term \( \pi_{\text{bank}} \) is included in (3.5) because the largest commercial banks which operate in Togo are totally or partly owned by foreign interests (French and American, primarily). Note also that in the five budget constraints, there are no variables which stand for the repayment of the principals on the various loans. This is because these repayments are accounted for by assumption in the definitions of the amount of loans instruments demanded.

All the budget constraints are formulated in terms of the home currency and the quantities are expressed in aggregate terms. Later on the model will be reformulated in per-capita terms.
(6) The World Walras Law. Expressions (3.1) through (3.5) which impose constraints on the behavior of the various economic agents in the model can be used to obtain the following World Walras Law:

\[ \begin{align*}
&P_A(D_A^* - A) + P_B(D_B - B) + eP_C^*(D_{CP} + D_{CG} + D_C^* - C^*) \\
&+ eP_K^*(D_{KF} + D_{KG} + D_K^* - K^*) + (\Delta L - \Delta M) + (\Delta O_L^d - \Delta O_L^s) \\
&+ (\Delta T_D^d - \Delta T_D^s) + (\Delta B_L^d - \Delta B_L^s) + e(\Delta F_L^d - \Delta F_L^s) \\
&+ e(\Delta L^* - \Delta M^*) = 0. 
\end{align*} \]

Expression (3.6) indicates that in Togo, the private sector, the banking sector, OPAT and the government as well as the economic units in the foreign country must behave in such a manner that the sum of the excess demand for/supply of all the goods and assets is identically zero.

Since money is the numeraire at home and abroad, there are nine independent prices to determine: \( P_A, P_B, P_C^*, P_K^*, r, r_{BL}^*, r^*, i \) and \( e \). It will be argued in the next subsection that all these prices are given in Togo except the price of the nontraded good.
The Institutional Implications, the Assumptions and the Basic Model

The institutional description of Togo presented in Chapter 1 leads to some useful simplifications about the prices on various markets.

(1) The Price for the Pure Export Good A. Togo is a small exporter of cocoa relative to Ghana and Nigeria and a small exporter of coffee relative to Latin American countries and Uganda so that OPAT cannot affect significantly the foreign price of good A which is $P_A^*$. In addition, OPAT determines the producer price $P_A$ which ensures a net profit to the agency. In this static analysis $P_A$ is taken as given while in Chapter 4, it will be shown how $P_A$ may be determined assuming OPAT maximizes profit over time.

(2) The Prices of the Imported Goods C and K. The small country assumption is appealed to in order to postulate that the price of the imported capital good K and that of the imported consumption good C are given to Togo. Indeed, given the data on the quantities of goods imported and on the composition of these imports, it is hard to pinpoint a single good or group of goods whose demand by Togo is big enough to affect the world price of that good or bundle of goods. Thus, it is reasonable to assume that the variables $P_C^*$ and $P_K^*$ are given.
(3) The Interest Rates on Time Deposits, Bank Loans, Foreign Loans, OPAT Loans and the Exchange Rate. It was mentioned in Chapter 1 that the nominal deposit and lending rates practiced by commercial banks in Togo are determined by the Board of Directors of the Central Bank which is common to the six members of the West African Monetary Union. It was also indicated that the monetary unit of this Union which is the FCFA has always been pegged to the French Franc (France is the major trading partner of the six-member countries). Therefore, the nominal interest rates on time deposits and on bank loans as well as the exchange rate can be taken as given. Concerning the interest rate on foreign loans, it is realistic to assume that Togo cannot significantly affect the cost of funds in international financial markets, so that one can consider the interest rate on foreign loans as being exogenously determined. As regards the interest rate on loans made by OPAT to the government, my attempts to know how this interest rate is determined have been unsuccessful. It is possible that this interest rate is not market-determined because OPAT is a public agency and in order to proceed with this study, it is assumed that the interest rate on OPAT loans is given.

In summary, based on the preceding considerations, the price of the pure export good, the prices of the imported goods C and K, the interest rates on time deposits, bank
loans, foreign loans, OPAT loans and the exchange rate can be treated as given while the price of the nontraded good remains to be determined.

(4) The Basic Model. The purpose of this subparagraph is to present the skeleton of the model which will be used later on to determine the price of the nontraded good. From the budget constraints of the private sector (3.1), the banking sector (3.2), OPAT (3.3) and the government (3.4), it follows that the Walras Law for Togo may be written as:

\[ P_B (D_B - B) + \phi + (\Delta L - \Delta M) = 0 \]  \hspace{1cm} (3.7)

where \( \phi = e_P^* (D_{CP} + D_{CG}) + e_P^* (D_{KP} + D_{KG}) + (\Delta TD^d - \Delta TD^s) \)

\[ + (\Delta BL^d - \Delta BL^s) + (\Delta OL^d - \Delta OL^s) + \pi_{\text{bank}} \]

\[ + r^* e_{FL} - e_P^* D_A^* + O_C A_D^* - e_{\Delta FL}^s. \]

As it was mentioned in Chapter 2, the major characteristic of a nontraded good is that its market must always clear, that is:

\[ D_B - B = 0. \]  \hspace{1cm} (3.8)
Under the small country assumption and given the role of the Central Bank and the government in setting the interest rates, equation (3.8) alone will determine the equilibrium price of the nontraded good. It follows from (3.7) and (3.8) that:

\[ \phi \geq 0 \text{ iff } (\Delta L - \Delta M) \leq 0. \] (3.9)

As it is well known, under fixed exchange rate, the balance of payments, which is equal to the excess flow demand for/supply of money \((\Delta L - \Delta M)\) or alternatively which is equal to the negative of \(\phi\), is not in equilibrium in general. That is in (3.9), in general, \((\Delta L - \Delta M)\) or alternatively \(\phi\) is not equal to zero. From this, it follows that either one of the two conditions in (3.9) can be used to determine the level of the balance of payments when the market for the nontraded good clears. To this end, we select the second condition:

\[ BP = \Delta L - \Delta M \] (3.10)

where \(BP\) stands for balance of payments and can be positive or negative.

Expressions (3.8) and (3.10) constitute the skeleton of the static model. In order to write them in per-capita terms, let us define the following variables: \(bp = BP/N\),
\[ \lambda = L/N, \quad m = M/N, \quad d_B = D_B/N, \quad b = B/N \quad \text{and} \quad n = \Delta N/N \]

where it is assumed that the population \( N \) is also time dependent. It follows from these definitions that:

\[ \Delta L/N = \Delta \lambda + n \lambda \quad \text{and} \quad \Delta M/N = \Delta m - mn \quad (3.11) \]

Given (3.11), the balance of payments expression (3.10) can be rewritten in per-capita terms as:

\[ \text{bp} = \Delta \lambda - \Delta m + n(\lambda - m). \quad (3.12) \]

The average annual growth rate of Togo's population between 1970 and 1975 was 2.6%. We shall assume that, in the short run, this growth rate is negligible so that the per-capita balance of payments will be written as:

\[ \text{bp} = \Delta \lambda - \Delta m. \quad (3.13) \]

As regards the equilibrium condition in the market for the nontraded good, given the above definitions of the per-capita demand for and supply of good \( B \), this equilibrium condition (3.8) can be rewritten as:

\[ d_B - b = 0. \quad (3.14) \]

Expressions (3.13) and (3.14) constitute the skeleton of the static model written in per-capita terms. The per-capita demand for and supply of the nontraded good
denoted respectively by \( d_B \) and \( b \) and the per-capita demand for money \( l \) depend on the price of the nontraded good and on several exogenous variables. But as mentioned earlier, given our small country assumption, etc., the price of the nontraded good \( P_B \) is determined in the market for the nontraded good alone, and the next step is to specify the per-capita demand for and supply of good B. Once the price of the nontraded good is determined, one can obtain the equilibrium expressions for the consumer price index, the nominal gross national product and the nominal balance of payments.

The Determinants of the Price of the Nontraded Good and Some Comparative Static Results

The Determinants of the Price of the Nontraded Good

The purpose of the subsequent paragraphs is to identify the determinants of the demand for and the supply of good B (i.e., \( d_B \) and \( b \)), since in view of (3.14), the forms of these are crucial in determining \( P_B^e \). To this end, we shall examine now the production and the consumption behaviors of the private sector.

(1) The Supply Function of Good B. The pure export good A and the nontraded good B are produced in the home country and the supply functions of these two goods are derived from a system which includes production functions, input constraints and efficiency conditions (see Appendix B).
These supply functions may be written in per-capita terms as:

\[ a = a(P_A, P_B, k), \quad b = b(P_A, P_B, k) \tag{3.15} \]

where \( a \) and \( b \) are the per-capita output of goods A and B and \( k \) is the capital labor ratio. To obtain the signs of the partials with respect to \( k \), it is assumed that the export sector is more capital intensive than the nontraded good sector. This assumption is very realistic for Togo, where almost all the relatively modern mechanical tools and the fertilizers as well as insecticides are used in the production of the export crops.

The supply function of B in (3.15) will be substituted into (3.14) while both expressions in (3.15) will be used to derive the demand function for good B: \( d_B \).

(2) The Demand Function for Good B. Individuals in the private sector consume goods B and C, hold capital K, money L and time deposits \( TD^d \), that is, they have three ways of storing wealth: K, L, TD. However, for conceptual and mathematical reasons and in order to obtain some refutable results, it will be assumed that the private sector consumes goods B and C and holds only one asset: money. Thus the budget constraint of consumers may be written in aggregate terms as:
\[ P_A + P_B = P_D + (1 + t_c)e^P_{C CP} + T + \Delta L \quad (3.16) \]

where \( \Delta L = L_t - L_{t-1} \).

Assume that initial stock equilibrium prevails in the money market and that the money stock is fairly constant from one period to the next, so that one can write:

\[ L_{t-1} = M_{t-1} = M_t \quad (3.16a) \]

While this is a rather extreme assumption, since \( \Delta L = L_t - L_{t-1} \), substituting (3.16a) into \( \Delta L \) allows us to obtain the simplification:

\[ \Delta L = L_t - M_t. \quad (3.16b) \]

Hence, (3.16a) is assumed for technical convenience alone.

Substitute (3.16b) into (3.16) and rewrite the budget constraint of consumers in per-capita terms as:

\[ P_A a + P_B b + m = P_B d_B + P_C d_{CP} + t + \lambda \]

where \( a = A/N, \ b = B/N, \ m = M/N, \ d_B = D_B/N, \ d_{CP} = D_{CP}/N, \ t = T/N \) and \( \lambda = L/N \).
Hahn (1959) and some other writers introduced money in the utility function along with goods. A similar approach will be used in this model and it will be assumed that the private sector maximizes the utility function:

$$U = U(d_B, d_{CP}, \ell) \text{ over } d_B, d_{CP}, \ell$$

subject to $P_A a + P_B b + m = P_B d_B + P_{CP} d_{CP} + t + \ell$

where the per-capita outputs $a$ and $b$ are given by (3.15) and $P_{CP} = (1+t_C)e^{P_C}$. It is shown in Appendix E that, in addition to standard assumptions about the utility functions (i.e., twice differentiable, positive but diminishing marginal utilities, etc.), if one assumes that the behavior of Togolese private consumers conforms to a utility function which is separable in the goods and money as Anderson and Takayama (1977) and several other authors assumed, then the demand functions, i.e., the solution to the above problem, may be written in per-capita terms as:

$$d_B = d_B(P_A, P_B, P_{CP}, m, t, k) \quad (3.17)$$

$$d_{CP} = d_{CP}(P_A, P_B, P_{CP}, m, t, k) \quad (3.18)$$

$$\ell = \ell(P_A, P_B, P_{CP}, m, t, k) \quad (3.19)$$
Note that the partials of $d_B$ and $\ell$ with respect to $P_C$ are ambiguous. It is common in the literature to assume that goods and assets are gross substitutes [Hahn (1959), Kemp (1962), Komiya (1967), Foley and Sidrauski (1971), and others] and if such an assumption is made then an increase in the domestic price of imports will raise the demand for the nontraded good and that of money.

To obtain the equilibrium price of the nontraded good, substitute the supply function and the demand functions for good B given by (3.15) and (3.17) into (3.14) to obtain:

$$d_B(P_A, P_B, P_C, m, t, k) - b(P_A, P_B, k) = 0. \quad (3.14a)$$

This is one equation in one unknown $P_B$ whose solution is:

$$P_B^e = P_B^e(P_A, P_C, m, t, k). \quad (3.20)$$

OPAT's data show that from the sale revenue $P_A^*$ per unit of crop exported, the agency pays $P_A/\text{unit}$ to farmers, $t_A P_A/\text{unit}$ to the government as export duties, $OC_A/\text{unit}$ to various middlemen and makes a profit $\pi_A/\text{unit}$. Since the export price $P_A^*$ is given, no generality is lost by assuming that $(OC_A + \pi_A) = 0$ so that the unit producer price can be written:
Substitute \((l+t_c)eP_C^* = P_C\) and (3.21) into (3.20) to get:

\[ P_B^e = P_B^e(P_A^*, P_C^*, e, t_A, t_C, t, m, k) \]  (3.22)

where all the arguments of the function are exogenous.

Komiya (1967) has shown that an increase in the export price will raise the equilibrium price of nontraded goods if the import sector is more capital intensive than the export sector which in turn is more capital intensive than the nontraded good sector. This result is maintained in the model of this study, that is, \(P_B^e\) and \(P_A^*\) are positively related. Why? An increase in the export price \(P_A^*\) will lead to an increase in the producer price \(P_A\) which in turn results in a higher demand for the nontraded good \(B\) while the supply of this good decreases. The increasing demand for \(B\) associated with the falling supply of this good \(B\) will create an excess demand for \(B\) and for this market to clear, the price of the nontraded good must rise. Note also that in this model, in general, a devaluation (i.e., an decrease in \(e\)) has an ambiguous effect on the price of the nontraded good because it exerts a positive effect on \(P_B^e\) through the producer price \(P_A\) while it exerts an ambiguous effect on \(P_B^e\) through the import price \(P_C\). However, if the nontraded consumption good \(B\) and the imported consumption good \(C\) are gross
substitutes, then a devaluation of the FCFA will tend to decrease the price of the nontraded good. Similarly, as shown by Mussa (1976), in general, an import tariff has an ambiguous effect on prices (i.e., $\partial P_B^e/\partial t_C \geq 0$) while a higher export tariff reduces the equilibrium price of the nontraded good unambiguously (i.e., $\partial P_B^e/\partial t_A < 0$). Yet, if the gross substitution assumption is made, then a higher import tariff will tend to increase the price of the nontraded good because under this assumption an increase in the import price $P_C$ will increase the demand for the nontraded good. Finally, note that in the standard textbook aggregate demand -- aggregate supply framework, an expansionary monetary or fiscal policy increases the price of the good. These results are maintained in the model presented here, that is $\partial P_A^e/\partial m > 0$ and $\partial P_B^e/\partial t < 0$.

The Comparative Static Results

Once expression (3.22) is arrived at, various comparative static results can be computed and the effects of the arguments of the price function (3.22) on the equilibrium values of endogenous variables such as the demand functions for goods B, C and money, the supply functions for A and B, the utility function, the consumer price index, the per-capita GNP and the net flow of foreign reserves can be traced. For instance, an increase in the world price of the pure export good will raise the utility level of
consumers. Indeed it can be shown that \( \frac{\partial U}{\partial P_A} \) is positive so that it may not be a bad idea to ask Nestle or General Foods to pay higher prices for the cocoa and coffee bought from OPAT. Similarly, one can show that an increase of the lump sum tax or of the export tariff will reduce the welfare level of the private community. In the pure theory of international trade, the same qualitative effect on welfare occurs if a small country imposes an ad valorem import or export tariff of the same magnitude. This symmetry is not maintained here because, in general, an increase in the domestic price of the imported good \( P_C = (1 + t_C)P_C^* \) has an ambiguous effect on the demand for the nontraded good and for money while it reduces the demand for the imported good and consequently the net effect of the increase in \( P_C \) on the utility level is ambiguous. As a result, an increase in the import tariff may increase or decrease the utility level. In this thesis, we shall focus our attention on the effects of the exogenous variables on the consumer price index, the nominal GNP and the balance of payments.

(1) The Equilibrium Consumer Price Index. The home country consumes the nontraded good B and the imported good C and the prices of these goods determine the consumer price index CPI. It is also reasonable to assume that the price of goods B and C are positively related to the CPI denoted
here by \( P \). When the market for the nontraded good clears, the CPI may be written as:

\[
P^e = P^e(P_B^e, (1 + t_C) eP_C^*)
\]

(3.23)

where \( P_B^e \) is expression (3.22). Substitute (3.22) into (3.23) to get:

\[
P^e = P^e(P_A^*, P_C^*, e, t, t_A, t_C, m, k).
\]

(3.24)

The net effect of the quantity \( P_C = (1 + t_C)eP_C^* \) on the CPI may be decomposed into two parts: a direct effect which is equal to \( \partial P^e/\partial P_C \) and an indirect effect which is equal to \( \partial P^e/\partial P_B^e)(\partial P_B^e/\partial P_C^*). \) Indeed, from (3.23) and (3.22) one can write:

\[
\partial P^e/\partial P_C = (\partial P^e/\partial P_C) + (\partial P^e/\partial P_B^e)(\partial P_B^e/\partial P_C^*). \]

(3.25)

Thus the effects of the foreign price of good \( C, P_C^* \), the exchange rate \( e \) and the import tariff \( t \) on \( P^e \) are ambiguous because the direct effect is positive while the indirect effect is ambiguous. Yet, if the nontraded consumption good \( B \) and the imported consumption good \( C \) are gross substitutes, then \( \partial P_B^e/\partial P_C^* \) is positive and (3.24) becomes:

\[
P^e = P^e(P_A^*, P_C^*, e, t, t_A, t_C, m, k)
\]

(3.26)
This means that if B and C are gross substitutes then inflation abroad (i.e., \( P^* \) increases) or a revaluation (i.e., \( e_C \) increases) or a higher import tariff (i.e., \( t_C \) rises) will raise the equilibrium consumer price index as indicated by (3.26). Of course, this result will be maintained even if the gross substitution assumption is not made but the absolute value of the direct effect exceeds that of the indirect effect in (3.25).

Anderson and Takayama (1977, equation 32, p. 352) showed that, in general, the effect of an increase in the money stock on the price index is ambiguous while it is positive if the goods and money are gross substitutes. In the model presented in this chapter, an increase in the money stock always increases the price of the nontraded good and thus the equilibrium CPI. The difference between the result obtained here and the result of Anderson and Takayama, abbreviated as AT, is due to the fact that the model of this thesis pertains to a small country while AT's model deals with two large countries. The equilibrium level of the consumer price index is one of the equations which will be estimated. The data used as proxies for the price of the pure export good A, the price of the imported good C and the CPI are expressed in the form of indices, so that the CPI relationship to be estimated is the one obtained by substituting expression (3.20) and (3.21) into (3.23) to obtain:
(2) **Equilibrium Nominal Per-Capita Gross National Product.** When the nontraded good market clears, the nominal per-capita GNP may be written as:

\[
y^e = P^a e(P^A, P^e_B, k) + P^e_B e(P^A, P^e_B, k) \tag{3.28}
\]

where \(P^e_B\) is (3.22).

Recall from Appendix C that \(\partial y/\partial P^A\), \(\partial y/\partial P^e_B\) and \(\partial y/\partial k\) are positive, so that one can write (3.28) as:

\[
y^e = y^e(P^A, P^e_B, k). \tag{3.29}
\]

Substitute (3.22) into (3.29) to get:

\[
y^e = y^e(P^*_A, P^*_C, e, t, t, t, m, k). \tag{3.30}
\]

Note that some IS-LM results are maintained in this broader framework. For instance, an expansionary monetary policy raises the equilibrium level of nominal GNP. This is because an increase in the money stock raises the demand for nontraded good while the supply of this good is unaffected which leads to an increase in the price of the
nontraded good. This higher price of good B induces an increase in the supply of the nontraded good and a decrease in the supply of the export good. But as shown in Appendix C, the positive effect of the higher price of the nontraded good on nominal income via the supply of the nontraded good exceeds the negative effect of the higher price of the nontraded good on income via the supply of the export good, so that the net result is an increase in the nominal income. This scenario is only an attempt to explain the effect of monetary policy on GNP in a "snapshot", frictionless manner as it is usually assumed in the IS-LM model. It is likely that the Togolese economy is not a friction-free economy and the adjustment may not be as smooth and swift as describes above. Another IS-LM result which is confirmed is that an expansionary fiscal policy also increases nominal income. Indeed, a decrease in the lump sum tax t or in the export tariff $t_A$ tend to create an excess demand for the nontraded good. The price of this good rises and presumably it induces an increase in income in the production sector. Concerning the price of the pure export good, an increase in the producer price of cocoa exerts a net upward effect on nominal income and this higher price tends to raise the demand for the nontraded good. This creates an excess demand for the nontraded good which is worsened by a reduction in the supply of this good stemming from the higher price of cocoa. To clear the market for the nontraded
good, the price of this good must rise in order to curtail the demand and to induce more production so that the market for the nontraded good moves toward equilibrium. During this process, the output of the pure export good tends to increase due to the initial increase in the price of cocoa and to decrease due to the subsequent higher price of the nontraded good. The output of the nontraded good tends to rise due to the higher price of maize and to decrease due to the initial increase in the price of cocoa. However, as shown in Appendix C, the net result is that the upward effect exceeds the downward effect so that an increase in the producer price paid by OPAT will raise ultimately the nominal income in equilibrium. Finally, note that an increase in the foreign price of imports or of the import tariff may increase or decrease the equilibrium level of nominal income. These relationships are positive if goods B and C are gross substitutes.

There are no data on all the individual arguments of expression (3.30), so that the equation to be estimated is obtained by substituting (3.20) and (3.21) into (3.29) in order to get:

\[ y^e = y^e \left( \frac{e_P^A}{1+t_A}, (1+t_C)^* e_P^C, m, t, k \right). \]  

(3.34)
The Equilibrium Net Flow of Foreign Reserves.

The net amount of foreign reserves is equal to the difference between the flow demand for and the flow supply of money, that is:

$$bp = \Delta\ell - \Delta m.$$  \hspace{1cm} (3.13)

Recall that the per-capita demand for money function derived from the utility maximization process is:

$$\ell^e = \ell^e(P_A, P_B^e, P_C, t, m, k)$$  \hspace{1cm} (3.19)

where $P_B^e$ is given by (3.20).

Assuming that the arguments of $P_B^e$ are time dependent, substitute (3.20) and (3.21) into (3.19) and the resultant expression into (3.13) and rewrite the balance of payments expression in first difference form as:

$$bp^e = bp^e(\Delta(eP_A^*/1+t_A^*), \Delta(1+t_C^*)eP_C^*, \Delta m, \Delta t, \Delta k)$$  \hspace{1cm} (3.35)

Expression (3.35) indicates that when the market for the nontraded good clears, an increase in the spread of the producer price of the pure export good will raise the net flow of foreign reserves while an increase in the net amount of the lump sum tax will reduce this flow. Expression (3.35) indicates also that an expansion of the net
flow of domestic credit will generate more foreign reserves if this expansion increases the flow demand for money by more than unity. This is so because, from (3.13) and (3.19) one can obtain the following expression:

$$\frac{\partial p_e}{\partial m} = \frac{\partial \ell_e}{\partial m} + \left(\frac{\partial \ell_e}{\partial P_B^e}\right)\left(\frac{\partial P_B^e}{\partial m}\right) - 1.$$ 

Deardorff (1977, Model 5) found that an expression similar to the above expression $\frac{\partial p_e}{\partial m}$ is unambiguously negative, only if the Marshall-Lerner condition is met. As regards the impact of the price of the imported good on the balance of payments, in general, an increase in the spread of this price may reduce or increase the net flow of foreign reserves. In particular, as in Mussa (1976), a higher import tariff has an ambiguous effect on the balance of payments. This effect is positive if goods B and C and money are gross substitutes. Concerning the effect of devaluation on the balance of payments, Kemp (1962) and other authors have shown that a devaluation of the home currency improves the balance of payments of the home country if the gross substitution assumption is made. This result is maintained in the model of this study under the same assumption, that is $\frac{\partial p_e}{\partial \ell_e}$ is negative if goods B, C and money are gross substitutes.

In this chapter it has been assumed that the private sector produces a pure export good A and a nontraded consumption good B. It was also assumed that the members of this
sector maximize a utility function (whose arguments are the
two goods B and C and money holdings) subject to
a budget constraint. The supply function of good B and the
demand function for this good derived from the subsumed
behavior were substituted into the equilibrium condition
in the market for the nontraded good to obtain the equili-

trium price of the nontraded good \( P_B^e \) which in turn led to
the equilibrium consumer price index \( P^e \), the equilibrium
gross national product \( y^e \), and the equilibrium balance of
payments \( bp^e \). These expressions which will be estimated
in Chapter 6 are:

\[
P_B^e = P_B^e \left( \frac{eP_A^*}{(1+t_A)} \right), (1+t_C)eP_C^*, t, m, k) \quad (3.20)
\]

\[
P^e = P^e \left( \frac{eP_A^*}{(1+t_A)} \right), (1+t_C)eP_C^*, t, m, k) \quad (3.27)
\]

\[
y^e = y^e \left( \frac{eP_A^*}{(1+t_A)} \right), (1+t_C)eP_C^*, t, m, k) \quad (3.34)
\]

\[
bp^e = bp^e \left( \Delta \left( \frac{eP_A^*}{(1+t_A)} \right), \Delta (1+t_C)eP_C^*, \Delta t, \Delta m, \Delta k \right). \quad (3.35)
\]

In the empirical work, it will be assumed that goods
B and C are gross substitutes so that the partial deriva-
tives of the endogenous variables \( P_B^e, P^e \) and \( y^e \) with
respect to the quantity \( P_C = (1+t_C)eP_C^* \) are positive in
equations (3.20), (3.27) and (3.34). As to the effects of
an increase in $\Delta P_C$ or $\Delta m$ on $bp^e$, these comparative static results will be left as empirical questions.

In the next chapter, the behavior of OPAT will receive the main attention and the question asked is how this agency may determine the producer price which it pays to the farmers.
CHAPTER 4

THE DETERMINATION OF THE PRODUCER PRICE
OF THE PURE EXPORT GOOD BY OPAT

The government agency OPAT has the exclusive right
to export nine cash crops, among which cocoa and coffee are
by far the most important commodities in terms of their
contribution to total profit.

OPAT determines the prices to be paid to the pro-
ducers of these commodities based on the level of inventory
of crops, on various costs, on expectations about future
world prices of cocoa and coffee, etc. These prices are
suggested to the government which generally agrees and makes
them official. In the determination of these prices, the
unwritten objective of OPAT is to maximize total profit.
The actual method used by OPAT to arrive at the figures
suggested to the government is unpublished. It appears
from the conversations of this writer with OPAT's officials
that the price-makers use no techniques derived from an
intertemporal profit maximization behavior under constraints.
Thus, it seems appropriate to attempt to formulate a simple
dynamic model which will permit to study an optimal pricing
scheme. In this model, the producer price \( P_A \) is no longer
constant as it is in the static model but it is a variable
proportion \( \alpha \) of the export price \( P^*_A \).
The model is presented in the first section while the phase diagrams of the system are examined in the second section.

**A Model for the Determination of the Producer Price**

First, the assumptions are listed and second, the functional relationships which describe the model are presented.

Before proceeding any further, let us make a few observations. The first remark is that since the supply functions for A and B are expressed in per-capita terms, all the other relevant variables (such as total cost and total profit) must also be deflated by the population in order to conserve the same dimension. The second observation is that in this chapter, the export price $P_A^*$, the price of the nontraded good $P_B$ and the capital-labor ratio $k$ are treated as exogenous variables. This simplification permits to focus on the behavior of OPAT. In the next chapter, the price of the nontraded good and the capital-labor ratio can change over time. The third remark is that in this chapter, all the time dependent variables are assumed to be continuous variables.
The Assumptions

(1) The Constraints Functions.

The producer price $P_A$ -- As mentioned above, the relationship between the producer price $P_A$ and the export price $eP_A^*$ is:

$$ P_A = \alpha(t) P_A^*, \ 0 < \alpha < 1. \quad (4.1) $$

The proportion $\alpha$ is between 0 and 1 because OPAT cannot acquire the output of good A free of charge nor can the agency pay a producer price as high as the world price which it receives under normal circumstances.

The per-capita output of good A -- It has been shown in Appendix B that in a two-sector production model, the per-capita outputs of goods A and B may be written as:

$$ a = a(P_A, P_B, k) \ and \ b = b(P_A, P_B, k). \quad (4.2) $$

It is generally assumed that the supply functions are homogenous of degree zero of prices. This assumption is made here. Furthermore, in order to obtain an interior solution, it is assumed that the production possibility curve is concave. This assumption imposes some restrictions on the functional forms of the supply functions. For instance, the Euler condition and the concavity condition of the transformation curve may be met or may be violated if
the per-capita supply function of A is convex while that of B is concave. It is implicitly assumed in this model that the supply curve of good A is linear while the supply function for good B is concave.

The inventory constraint -- In the previous chapter, it was assumed implicitly that OPAT holds no inventories. However, although the goods exported are perishable if they are held beyond a certain period of time, and despite the fact that OPAT cannot significantly affect the world prices of these goods, it is conceivable that OPAT decides not to sell the entire output of crops at the going price, by its own volition (perhaps in an anticipation of future higher prices), or in accordance with other foreign sellers. The implication will be that OPAT will have to take into account the existence of these inventories of unexported commodities in determining the producer prices to pay to farmers, in particular it is possible that the agency may be able to improve its profit position by reducing or increasing its inventories. The inventory constraint faced by the agency may be written as:

\[ s = a - d_A^* - \delta s, \quad s(0) = s_0 \geq 0, \quad s(t) \geq 0 \quad (4.3) \]

where \( s(t) = \) per-capita stock of good A at time t,

\( \delta = \) proportionate rate of deterioration of \( s(t) \), \( 0 < \delta = \) constant \( < 1 \),

\( d_A^* = \) quantity of good A exported by OPAT.

The dot "o" refers to time derivative.
The cost function -- First, OPAT incurs costs associated with the handling of good A from the farms to the warehouses located at the port of Lomé. These costs include the cost of carrying the crops from the buying posts to the nearest collection towns, the storage cost in these towns and the transportation cost to the warehouses. Second, OPAT must pay the shipping fees associated with delivering the commodities to the foreign buyers. Third, OPAT must pay the costs of storage, guarding and insurance of the inventories of unsold crops which are carried on from one period to the next. Finally, OPAT must pay salaries and taxes.

My conversations with some officials of OPAT indicate that no theoretical nor empirical work has been done to determine the total cost structure of the agency - that is no work has been done in order to identify the major determinants of the total cost function and the best functional form which might describe this cost. However, these conversations revealed that some of the major determinants of this function could be the size of the harvest of the crops (i.e., output a), the transportation and insurance costs which vary with the quantities exported (i.e., \( d_A^* \)), inventory related costs and salaries. As mentioned earlier, the functional forms which the individual costs and the total cost might take are unknown. To proceed with this study, it is assumed that the per-capita cost c of OPAT written in implicit form has the following properties:
Expression (4.4) indicates that the per-capita cost of OPAT increases as the agency buys more output of good A from the farmers or as it holds a larger inventory. This cost also rises when the agency pays transportation, insurance and other costs associated with the export of the crops. Furthermore, it is assumed that these marginal costs increases as the supply of A or the inventory level rises. The per-capita cost of OPAT is assumed to increase at an increasing rate when the inventory rises for reasons such as inadequate storage facilities.

(2) The Profit Function. OPAT sells each period the quantity $d^*_A$ of good A to foreign countries and receives $eP^*_A$ per unit where $P^*_A$ is expressed in the foreign currency and $e$ is the exchange rate. The agency buys the amount $a$ of good A from farmers and pays them $P_A$ per unit. This price $P_A$ is a time-dependent proportion $\alpha(t)$ of the world price $eP^*_A$ ($t =$ time). The purpose of this model is to determine $\alpha(t)$ and thus $P_A$. The agency incurs various costs whose components are mentioned above and its inter-temporal per-capita profit function to be maximized may be written as:
\[ \pi = \int_{0}^{\infty} \left( e^{P_A} d_A - P_A a - c \right) e^{-\rho t} dt \]

where \( c \) is the per-capita cost and \( \rho \) is the discount rate.

Note that the profit expression in the parenthesis is the same as OPAT's budget constraint (3.3) written in per-capita terms and assuming away the export tax term and OPAT loans.

In the above profit function, \( \pi, P_A, a \) and \( c \) are endogenous while \( e^{P_A}, d_A \) and \( \rho \) are exogenous.

The Model

(1) The Hamiltonian Functions. The profit maximization problem may be written as:

\[
\text{maximize } \pi_t = \int_{0}^{\infty} \left( e^{P_A} d_A - P_A a - c \right) e^{-\rho t} dt \text{ over } \alpha, s, P_A, a, \text{ and } c \text{ subject to the following budget constraints:}
\]

\[ P_A = \alpha(t)e^{P_A}, \quad 0 \leq \alpha \leq 1, \quad (4.1) \]
\[ a = a(P_A, P_B, k) \quad (4.2) \]
\[ s = a - d_A - \delta s, \quad s(0) = s_0, \quad s(t) > 0, \quad (4.3) \]
\[ c = c(a, s, d_A), \quad c_a > 0, \quad c_s > 0, \quad c_{d_A} > 0, \quad (4.4) \]

\[ c_{aa} > 0, \quad c_{as} > 0, \quad c_{ss} > 0. \]
where \( a(t) \) is the control variable and \( s(t) \) is the state variable and all the variables are assumed to be nonnegative. It is assumed that the above problem has an interior solution (i.e., \( 0 < a < 1 \)) so that the constraint \( 0 \leq a \leq 1 \) is ignored.

Substituting the constraints (4.1), (4.2) and (4.3) into the objective function and subsuming \( P_A, a \) and \( c \), the Hamiltonian function may be written as:

\[
H = e P_A^* d_A^* - \alpha e P_A^* a(\alpha e P_A^*, P_B, k) - c[a(\alpha e P_A^*, P_B, k), s, d_A^*] \\
+ \lambda [a(\alpha e P_A^*, P_B, k) - d_A^* - \delta s].
\]  

(4.5)

where \( \lambda \) is the costate variable.

The necessary conditions for the pair \( s^e(t), \lambda^e(t) \) to be a solution to the above problem are:

\[
\frac{\partial H}{\partial \alpha} = 0 \quad (4.6a)
\]
\[
\rho \lambda + \lambda = -\frac{\partial H}{\partial s} \quad (4.7a)
\]
\[
\frac{\partial}{\partial s} = \frac{\partial H}{\partial \lambda} \quad (4.8a)
\]

\[
\lim_{t \to \infty} e^{-\rho t} \lambda(t) \geq 0, \lim_{t \to \infty} e^{-\rho t} \lambda(t) s(t) = 0 \quad (4.9)
\]

where the Hamiltonian is evaluated at the optimum position.
(2) The Determinants of \( \alpha \) and the Accumulation Equations. Let us focus on conditions (4.6a), (4.7a) and (4.8a).

The \( \alpha \) "function" -- From expressions (4.5a) and (4.6a), it follows that:

\[
\frac{\partial H}{\partial \alpha} = -\alpha p_A^* \alpha - \alpha e p_A^* \alpha - c a \alpha + \lambda \alpha = 0. \quad (4.6b)
\]

Given (4.6b), condition (4.6a) can be written in implicit form as:

\[
J(\alpha, \lambda, s; e p_A^*, P_B, k) = 0 \quad (4.6c)
\]

where the signs of the various arguments come from the assumptions made about the supply function for good A and the cost function of OPAT.

Expression (4.6c) may be inverted to obtain the following function for the proportion \( \alpha \) of the export price \( e p_A^* \) paid to the farmers by OPAT:

\[
\alpha = \alpha(\lambda, s; e p_A^*, P_B, k). \quad (4.6d)
\]

Expression (4.6d) is the \( \alpha \) "function". It indicates that if the inventory of crops rises following say an abundant harvest,
ceteris paribus, then OPAT should reduce the proportion of the export price paid to farmers in order to maximize profit. That is, the producer price $P_A$ should not be fixed independently of the level of unsold commodities. Expression (4.6d) indicates also that if the aggregate capital stock is unchanged while a substantial number of Togolese emigrate so that the capital labor ratio rises, then ceteris paribus, OPAT should reduce the producer price paid to farmers by reducing $\alpha$. As regards the effect of an increase in the export price $eP_A^*$ on the proportion $\alpha$, in general, this effect is ambiguous. This is so because in expression (4.6c), the sign of the partial $\partial J/\partial eP_A^*$ is given by:

$$J^* = [a + eP_A^*a_\alpha] + [a_\alpha eP_A (aeP_A^* + c_\alpha - \lambda)$$

$$+ a (\alpha + c_{aeP_A^*})]$$ (4.10)

where $a = \partial a/\partial \alpha$, $c = \partial c/\partial a$, $a = \partial a/\partial eP_A^*$ and $c_{aeP_A^*} = \partial c/\partial eP_A^*$. Expression (4.10) is obtained from (4.6b), noting that the producer price $P_A$, the per-capita output of $A$, which is denoted by $a$, and the cost function $c$ are given respectively by (4.1), (4.2) and (4.4). In expression (4.10),
the terms in the first bracket represents the change in farmers' "marginal revenue" due to a change in the export price while the terms in the second bracket represents the change in the marginal amount of all other costs including the cost of holding the inventory incurred by OPAT due to a change in the export price. The first bracket is unambiguously positive while the second bracket may be positive or negative. Indeed, given our assumption that the supply function of good A is linear and homogenous of degree zero in $P_A$ and $P_B$ and the assumptions about OPAT's cost function, it can be shown that $a^* > 0$ and $c^* < 0$. Furthermore, it follows from (4.6b) that the quantity $(ae_{P_A}^* + c_a - \lambda)$ is negative so that the second bracket expression in (4.10) has an ambiguous sign. Following an increase in the export price, ceteris paribus, if the change in the marginal payments made by OPAT to farmers exceeds (is smaller than) the change in the marginal amount of other costs incurred by the agency, then the derivative $\partial J/\partial e_{P_A}^*$ will be positive (negative), and consequently OPAT should reduce (increase) the proportion $\alpha$ of the export price paid to the farmers in order to maximize profit. The qualitative effect of a change in the price of the nontraded good on the proportion $\alpha$ is related to the sign of the following expression obtained from (4.2), (4.4), and (4.6b):

$$J_{P_B} = e_{P_A}^* a_{P_B} + [a_{P_B} (ae_{P_A}^* + c_a - \lambda) + a_{\alpha} c_{a_{P_B}}]$$  \hspace{1cm} (4.11)
where \( \frac{\partial a}{\partial P_B} = \frac{3a}{3P_B} \) and \( \frac{\partial a}{\partial P_B} = \frac{\partial a}{\partial P_B} \). Given the assumptions concerning the supply function for good \( A \) and OPAT cost function, it can be shown that \( \frac{\partial a}{\partial P_B} < 0, \frac{\partial \alpha}{\partial P_B} < 0, \frac{\partial a}{\partial P_B} < 0 \) and as before, \( \alpha > 0 \) and \( (\alpha e^A + c_a - \lambda) < 0 \), so that the first term in (4.11) which is equal to the change in the marginal revenue of farmers due to a change in \( P_B \) is negative while the second bracket term which represents the change in the amount of other costs incurred by OPAT may be positive or negative. Clearly, if following an increase in the price of the nontraded good \( P_B \), the second bracket term in (4.11) is negative, then \( \frac{\partial J}{\partial P_B} \) will be negative and thus \( \frac{\partial a}{\partial P_B} \) will be positive, so that OPAT should increase the proportion \( \alpha \) of the export price paid to the farmers. Of course, if the second bracket term in (4.11) is positive, then no conclusion can be drawn, a priori, about the effect of a change in \( P_B \) on \( \alpha \). It is important to keep in mind that the effects of changes in the export price \( e^A \), the price of the nontraded good \( P_B \) and the capital labor ratio \( k \) on the proportion \( \alpha \) mentioned above are explored ceteris paribus. Hence, these results should not be taken too strictly because as it will become apparent shortly, changes in such exogenous variables while they will not affect the stock of crops \( s \), at a given point in time, will change the instantaneous value of the shadow price \( \lambda \) which in turn will affect the proportion \( \alpha \).
The perfect foresight function -- This function can be obtained by differentiating the Hamiltonian (4.5) with respect to $s$ and substituting this derivative into condition (4.7a). The result is:

$$\lambda = V(\lambda, s, \alpha; e^P_A, P_B, k, \rho, \delta) \tag{4.7b}$$

where the signs of the arguments of the implicit function $V$ depend on the assumptions made about the cost function.

The inventory constraint -- Differentiate the Hamiltonian (4.5) with respect to $\lambda$ and substitute in (4.3a) to obtain:

$$s = a(\alpha e^P_A, P_B, k) - d_A - \delta s. \tag{4.8b}$$

Rewrite (4.8b) in implicit form as:

$$s = W(s, \alpha; e^P_A, P_B, k, d_A, \delta). \tag{4.8c}$$

The dynamic model for the determination of the producer price is then described by the following three expressions:

$$\alpha = \alpha(\lambda, s; e^P_A, P_B, k) \tag{4.6d}$$

$$\lambda = V(\lambda, s, \alpha; e^P_A, P_B, k, \rho, \delta) \tag{4.7b}$$
Methodology and Phase Diagrams

The Methodology

In the above system, there are three variables \( \lambda, s \) and \( \alpha \) whose time paths must be determined. The optimal level of \( \alpha \) is not determined independently of those of \( s \) and \( \lambda \) because \( s \) and \( \lambda \) are arguments of the \( \alpha \) function in (4.6d). Therefore the procedure to obtain a solution to the system consists of substituting equation (4.6d) into expressions (4.7b) and (4.8c) and thus examining the time paths of \( s \) and \( \lambda \).

Substitute (4.6d) in (4.7b) to obtain:

\[
\dot{\lambda} = \lambda(\lambda, s; e^\text{P}_A, P_B, k, \rho, \delta) \tag{4.12a}
\]

and substitute (4.6d) into (4.8c) to get:

\[
\dot{s} = s(\lambda, s; e^\text{P}_A, P_B, k, \rho, \delta). \tag{4.13a}
\]

If a steady state exists, it will be described by the following system:

\[
\dot{\lambda} = \lambda(\lambda, s; e^\text{P}_A, P_B, k, \rho, \delta) = 0 \tag{4.12b}
\]
and
\[ s = s(\lambda, s; eP_A^*, P_B, k, d_A^*, \delta) = 0. \tag{4.13b} \]

In what follows, it is assumed that there is a steady state.

**The Phase Diagrams**

Expressions (4.12b) and (4.13b) describe the paths of the shadow cost and the inventory of good A at the steady state. These expressions indicate that, in \((s, \lambda)\) space, the slope of the \(s = 0\) curve is positive while that of the \(\lambda = 0\) curve is ambiguous. This latter slope is ambiguous because as indicated by (4.6d) and (4.7b), an increase in the inventory exerts a direct positive effect on \(\lambda\) and an indirect negative effect on \(\lambda\) through \(\alpha\). Figure 1 represents the case in which the \(\lambda = 0\) is negatively sloped. In Figure 2, the \(\lambda = 0\) curve is positively sloped but this slope is assumed to be less steep than the slope of the \(s = 0\) locus while in Figure 3, the \(\lambda = 0\) curve has a steeper slope. The case in which for some ranges, \(\lambda = 0\) is negatively sloped while for some others it is positively sloped is not examined here.

As indicated by expression (4.13b), the inventory of crops increases when \(\lambda\) increases, meaning that \(s\) is positive to the right of the \(s = 0\) curve and is negative to the left of this curve. Similarly, the \(\lambda = 0\) locus divides the same space into two distinct regions where \(\lambda\) is negative to the left and is positive to the right.
Figure 1. A Phase Diagram for the Producer Price Problem when the $\lambda = 0$ Curve is Negatively Sloped.
Figure 2. A Phase Diagram for the Producer Price Problem when the Slope of the $\lambda = 0$ Curve is Positive but is Less Steep than the Slope of the $s = 0$ Curve.

Figure 3. A Phase Diagram for the Producer Price Problem when the Slope of the $\lambda = 0$ Curve is Positive but is Steeper than the Slope of the $s = 0$ Curve.
The direction of motion of the state and costate variable indicate that, in each case, there are two stable branches denoted by (SB) among the various paths that OPAT pricing policy can take. Given the underlying assumptions, if OPAT's behavior results in the agency operating in any but the two regions in which there are stable branches, over time, no optimum will be reached. That is there would be no inventory-shadow cost combination which would permit the agency to determine the optimal proportion of the export price to pay to farmers in order to maximize profit. More interesting cases can be examined when OPAT operates on one of the "stable" branches. Indeed, for instance, assume that the economic conditions described by Figure 1 are applicable to OPAT. If the agency operates anywhere but on the stable branches, the inventory-shadow cost combination which corresponds to that position would be nonoptimal because of the transversality conditions (4.9). For example, assume that OPAT is operating in the region containing the stable branch denoted by (SB₀) and that the current stock of crops is s^G. The only shadow cost which can permit the agency to maximize profit at that inventory level is λ^G. If OPAT chooses a smaller shadow cost, say λ^D, over time, starting from point D, the inventory will rise until the s = 0 curve is reached and then it will fall. During the same process the shadow cost will fall from its previous level λ^D. As indicated by
(4.6d), the gradual increase of the stock of commodities and the continuous decrease of the shadow cost will make OPAT reduce the proportion $\alpha$ of the export price which it pays to the farmers. This proportion $\alpha$ will get smaller and smaller when the minimum stock level is reached while the shadow cost $\lambda$ continues to decrease and the corresponding producer price will not be optimal. Since the agency must pay a positive price to the farmers, it cannot continue to choose a smaller and smaller shadow cost $\lambda$. This means to reverse the decreasing trend of the proportions $\alpha$ and thus of the producer price $P_A$, OPAT will have to select higher and higher shadow costs and move gradually toward an optimum producer price. If the agency missed the optimum position $G$ and selected a shadow cost, say $\lambda^F$ which is greater than the optimal shadow cost $\lambda^G$, then starting from point $F$, in general, both the inventory of crops and the shadow cost will increase indefinitely and the proportion $\alpha$ of the export price paid and consequently the producer price may increase or decrease over time. In the special case in which the stock level increases only very moderately while the shadow cost rises indefinitely, the proportion $\alpha$ and thus the producer price $P_A$ will also tend to rise indefinitely. The continuous increase in the proportion $\alpha$ and in the producer price $P_A$ in the event of a selection of a shadow cost which is above the optimal shadow cost $\lambda^G$ will also be nonoptimal because the agency must pay a producer price such that $\alpha$ is less than
unity and also because of the transversality conditions. So that, for a given level of inventory, the agency will reduce the shadow cost gradually and thus reduce \( \alpha \) and \( P^A \) and move toward the optimal position \( G \). To this position corresponds an optimal pair \( (\lambda^G, s^G) \) which can be substituted into (4.6d) to obtain the optimal proportion \( \alpha^G \) of the export price to be paid to the farmers. The resultant producer price \( P^G_A \) can be obtained by substituting \( \alpha^G \) into (4.1) to get \( P^G_A = \alpha^G eP^*_A \). This producer price \( P^G_A \) is the only one which will ensure a maximum per-capita profit to the agency, given the inventory level \( s^G \). Any other producer price would not lead to the highest profit, given \( s^G \). The reasoning would be the same if OPAT were operating in the region containing the stable branch (SB\(_1\)) instead of operating in the region which contains (SB\(_0\)). The main difference between the two regions is that if OPAT does not operate on any one of the stable branches, then, over time, the inventory of crops will remain unchanged or will increase in the (SB\(_0\)) region while it will remain constant or it will fall in the (SB\(_1\)) region. Once OPAT is on either stable branch or stays on it, it will eventually reach the steady state position E as time passes and the pair \( (\lambda^e, s^e) \) will be substituted into (4.6d) to obtain \( \alpha^e \) which in turn will be substituted into (4.1) to obtain the steady state producer price \( P^e_A = \alpha^e eP^*_A \).
As indicated by expression (4.12b) and (4.13b), an increase in the value of any of the exogenous variables $eP_A^*, P_B, k, p, \delta$ or $d_A^*$ will shift one or both curves and thus will affect the profit maximization values of $\alpha$ and $P_A$. In particular, an increase in the foreign demand for the exported good (i.e., $d_A^*$ rises) will shift the $s = 0$ locus and the stable branches to the right. If we focus again on Figure 1, this increase in the foreign demand for good A will reduce the steady state level of the inventory of crops and will raise that of the shadow cost. From expression (4.6d) it can be seen that the decrease in inventories coupled with the rise in the shadow cost will raise the steady state proportion of the export price which should be paid to farmers. The net result will be an improvement in farmers' income, ceteris paribus, as they receive a higher producer price. Note that if OPAT's operations conditions are described by Figure 2 or Figure 3, then an increase in the foreign demand for good A will raise the steady state levels of both the inventory of commodities and their shadow cost, and consequently will exert an ambiguous effect on the proportion $\alpha$ and on the producer price $P_A$. Then clearly, in the situation described by Figure 1, at the steady state, farmers will benefit from an increase in the foreign demand for the good which they produce while if the conditions
described in Figures 2 and 3 prevail, they may or may not receive a higher producer price. An increase in the discount rate used by OPAT to evaluate the present value of its profits will shift the $\lambda = 0$ locus to the left while the $s = 0$ curve remains unchanged. The stable branches also will shift leftward. The effect of this rise in the discount rate is a fall in the steady state values of the inventory and the shadow cost in the situation described by Figures 1 and 3 while the steady state levels of these variables will increase in the event of Figure 2. The net result of the rise of $\rho$ on the proportion $\alpha$ and on the producer price $P_A$ will thus be ambiguous, in general, as it can be seen in (4.6d). Similarly, no clear-cut results can be obtained if the export price $eP_A^*$, the price of the nontraded good $P_B$, the capital labor ratio $k$ or the rate of deterioration of the inventory changes, ceteris paribus. For example, if the partial $\partial \alpha / \partial eP_A^* > 0$ in (4.6d), then $\partial \lambda / \partial eP_A^* > 0$ and $\partial s / \partial eP_A^* > 0$ and thus an increase in $eP_A^*$ shifts both the $\lambda = 0$ and the $s = 0$ curves to the left. The result is that at the steady state, if OPAT's operations are described by Figure 1, then the rise in $eP_A^*$ will increase or decrease the stock of commodities and will reduce the optimal shadow cost and thus exert an ambiguous effect on the proportion $\alpha$ and the producer price $P_A$. Note, however, that if the increase in the export
price $eP_A^*$ results in a net increase in $a$, farmers will benefit both from the larger share of the export price paid by OPAT and from the initial increase in the export price, and OPAT will thus be maximizing profit by retaining a smaller proportion of a larger "pie" (i.e., a larger export price $eP_A^*$).

In this chapter, I have attempted to show how OPAT might determine the proportion of the export price to pay to the farmers in order to maximize profit. It has been shown that this proportion $a$ is negatively related to the stock of crops $s$ and is positively related to the shadow cost $\lambda$ which the agency may choose to evaluate this stock and that not all pairs $(\lambda, s)$ will permit the agency to maximize profit. Furthermore, the values of the proportion $a$ and of the corresponding producer price $P_A$ will be affected at the steady state if some exogenous variables such as the foreign demand for good A or the foreign price of this good changes. Failure to adjust $a$ and $P_A$ to their new levels following a change in an exogenous variable will result in nonoptimal pricing decisions. While the model presented in this chapter focuses essentially on OPAT, the model of the next chapter considers the whole economy and it purports to identify, at the steady state, the determinants of the price of the non-traded good, the capital labor ratio, the per-capita stock of government debt and the per-capita stock of foreign reserves.
The purpose of this chapter is to develop a simple dynamic model which would permit to examine at the steady state, the determinants of the price of the nontraded good, the capital labor ratio, the real per-capita government debt and the real per-capita stock of foreign reserves. The study of these variables is important for the following reasons. One, the capacity of Togo or any other country to produce goods and services for domestic consumption or export depends in part on her stocks of inputs, in particular on her stock of labor and capital. Nowadays, one can hardly talk about the development of a country without referring to her industries and to the size of her population. Therefore, it is appropriate to examine the determinants of the capital labor ratio. Two, the Togolese government is a major employer and investor and part of the public investment is financed with loans made by foreign countries and OPAT. It is reasonable to think that foreign institutions will consider the level of the government debt as a major ingredient in their decisions to grant or reject loan applications made by Togo. Furthermore, OPAT is the foremost domestic lender to the government. This agency essentially transfers financial resources from the private
producers of export crops to the government and another aspect of the contribution of Togolese farmers to the financing of government projects may be assessed by examining the determinants of the government debt. Three, Togo's ability to import goods and services and to borrow from foreign institutions is ultimately constrained by her stock of foreign reserves. Therefore, it is important to study the factors which affect this stock of foreign reserves. Four, one of the determinants of the price of the nontraded good is the capital labor ratio and an increase in this ratio may lead to inflation and, assuming that most people would consider inflation as an "evil", it is then appropriate to study closely the effect of a change in the capital labor ratio on the price of the nontraded good over time. The dynamic model is exposed in section one while the solution is presented in section two.

The Model

The Skeleton of the Model

In the static model of Chapter 3, there are four goods and five assets. The four goods are a pure export good A, a nontraded consumption good B, an imported consumption good C and an imported capital good K, while the five assets are money M, time deposits TD, bank loans BL, OPAT loans OL, and foreign loans eFL where e is the exchange rate. The numeraire used in the static analysis is money.
In this dynamic analysis, the numeraire is the imported consumption good C so that all the relevant variables are expressed in real terms (i.e., in terms of C).

In the static model, the prices for goods A, C, K and for foreign loans are assumed to be known because Togo is small relative to the rest of the world. The interest rates on time deposits and on bank loans are determined by the Board of Directors of the BCEAO while the interest rate on OPAT loans is assumed to be determined by the government. The market for the nontraded good B must clear. The excess flow demand for money is equal to the balance of payments. This excess flow may be positive, negative or equal to zero. All these assumptions are maintained in the dynamic model, so that as regards the prices of the goods and assets, only the price of the nontraded good $P_B$ is endogenously determined.

In the static analysis, the functioning of the economy is pictured at an instant of time and for all practical purposes, this instant has no duration, so that no variable may accumulate. In the dynamic model, the time span is long enough to permit accumulation. All the flow goods which are consumed (A, B, C) do not accumulate because they are destroyed by the very act of consumption. Only the investment good accumulates to become the capital stock K over time. So there will be an accumulation equation for the stock of capital. All the assets (M, TD, BL, OL and FL)
accumulate. However, to cut the problem down to a manageable size, it is assumed that the monetary authorities in Togo can use the policy tools at their disposal (interest rates, qualitative and quantitative restrictions, etc.) to induce changes in the stocks of time deposits TD and bank loans BL in such a manner that the net amount of these changes is zero. That is, $\dot{TD} - \dot{BL} = 0$ where the dot "$\dot{}$" refers to time derivative. Let us mention in passing that in this chapter, all the time dependent variables have a continuous dimension.

In view of the preceding explanation, the dynamic model includes the nontraded goods market and three accumulation equations which describe the time path of the three variables: the per-capita stock of capital, the per-capita government debt and the per-capita stock of foreign reserves.

**The Market for the Nontraded Good B**

The equilibrium condition in the market for the nontraded good B is:

$$d_B - b = 0 \quad (5.1)$$

where $d_B$ and $b$ are respectively the real per-capita demand for and supply of good B.
The Capital Labor Ratio Accumulation Expression

Let the capital labor ratio $k$ be defined as:

$$K = \frac{K}{N} \quad (5.2)$$

where $K$ and $N$ are the total stocks of capital and labor (at time $t$).

Let the exogenous rate of growth of labor be:

$$n = \frac{\circ n}{N} \quad (5.3)$$

where $n$ is positive.

Take the logarithm of (5.2) and differentiate it with respect to time to obtain after substitution of (5.3):

$$\frac{\circ k}{K/N} - nk. \quad (5.4)$$

Expression (5.4) represents the capital labor ratio accumulation identity.

The Government Debt Accumulation Expression

The government budget constraint written in per-capita terms using good $C$ as the numeraire is:

$$\left(\frac{\circ P_{m}^{OL^S}}{N} + \frac{\circ P_{m}^{eFL^S}}{N}\right) = d_{CG} + e_{P_k} d_{KG} - P_{m} t \quad (5.5)$$
where

\[ p_m = \text{unit price of money in terms of good C} \]

\[ O_L^S = \text{net nominal stock of debt instruments} \]
\[ \text{sold by the government to OPAT} \]

\[ eFL^S = \text{net nominal stock of debt instruments} \]
\[ \text{sold by the government to foreign countries} \]

\[ p_m O_L^S, p_m eFL^S = \text{amount of } O_L^S \text{ and } eFL^S \text{ in terms of } C \]
\[ (\text{the variable } O_L^S \text{ and } eFL^S \text{ are in aggregate terms}) \]

the dot "o" refers to time derivative,

\[ dCG, dKG = \text{per-capita demand for goods C and K} \]
\[ \text{by the government in physical unit/period} \]

\[ eP^*_K = \text{unit price of the investment good in} \]
\[ \text{terms of C} \]

\[ t = T/N = \text{nominal per-capita lump sum tax net of} \]
\[ \text{interest payments on government debt} \]
\[ \text{and } p_m t = \text{value of } t \text{ in terms of C.} \]

Let us make a few observations about the above budget constraint.

First, note that since the imported good C is the numeraire, all the nominal prices are deflated by the price of good C which is set throughout this chapter at 1. That is, in this chapter, the prices of the exported good eP^*_A,
the nontraded good $P_B$ and of the investment good $eP_K^*$ are deflated by the quantity $P_C = (1 + t_C) eP_C^* = 1$ initially.

The second observation is that the terms $OL^S_s$ and $eFL^S_s$ represent respectively the net nominal stock of debt instruments sold by the government to OPAT and to foreign countries while the terms $P_m^o OL^S_s$ and $P_m^o eFL^S_s$ represent the net real value of the stocks of these debt instruments and what is important to the government and other economic agents in the long run is the change in the real value of the government debt as opposed to the change in the nominal value of this debt. Put another way, what counts is the real burden not the nominal burden of the government decisions to borrow and spend. This is why the revenue side of the budget constraint (5.5) is written as $[(P_m^o OL^S_s) + (P_m^o eFL^S_s)]/N$ as opposed to $[P_m^o (OL^S_s) + P_m^o (eFL^S_s)]/N$. This second expression ignores the capital gains or losses due to a change in $P_m$ on OPAT and foreign loans because the government revenue, as it appears in (5.5), can be rewritten as:

$$[(P_m^o OL^S_s) + (P_m^o eFL^S_s)]/N = [P_m^o (OL^S_s) + P_m^o (eFL^S_s)]/N$$

$$P_m^o (OL^S_s + eFL^S_s)/N$$

where $P_m^o (OL^S_s + eFL^S_s)/N$ represents the capital gains or losses. These capital gains or losses should not be ignored
if one is studying the behavior of the real per-capita
government debt over time.

Third, note also that the budget constraint (5.5) indicates that budgetary deficits are not financed through the creation of money because since 1960, the government made use of its credit line at the Central Bank only once (1975) and it repaid this loan the following years. Therefore, it is assumed in this study that only the banking sector can affect the money stock.

Finally, note that although OPAT is owned by the government, the profit of this agency does not appear in expression (5.5) because OPAT does not surrender its entire net profit to the government under normal circumstances.

The Foreign Reserves Accumulation Equation

As indicated in Chapter 3, the balance of payments or the net flow of foreign reserves resulting from the economic transactions between Togo and the rest of the world is equal to the excess flow demand for/supply of money. In general, this excess flow is not equal to zero. The data shows that Togo's balance of payments as approximated by the sum of the balance of trade (and services and transfers) and the balance of short term and long term capital was in deficit for several years. Therefore, it is assumed here that the money market does not clear. Since the balance of payments is equal to the excess flow demand for/supply of money, it
may be written in real per-capita terms as:

\[
\frac{\circ}{m} \frac{(P_{eRE})}{N} = \frac{\circ}{m} \frac{(P_L)}{N} - \frac{\circ}{m} \frac{(P_M)}{N} \quad (5.6)
\]

where \( \frac{\circ}{m} \frac{(P_{eRE})}{N} \) = per-capita balance of payments or flow amount of foreign reserves in terms of good C,

\( \frac{\circ}{m} \frac{(P_L)}{N} \) = per-capital flow demand for money in terms of C,

\( \frac{\circ}{m} \frac{(P_M)}{N} \) = per-capita flow supply of money in terms of C.

Expression (5.6) is the same as expression (3.13) in the static model except that in (3.13) the variables are discrete and are expressed in nominal terms while in (5.6) the variables are continuous and are expressed in real terms.

Note that as in the government budget constraint (5.5), capital gains and losses due to a change in \( P_m \) are taken into account in expression (5.6).

In summary, the skeleton of this dynamic model is composed of the equilibrium condition in the nontraded good market (5.1), the accumulation expressions for capital (5.4), government debt (5.5) and foreign reserves (5.6).

The purpose of the next subsection is to specify the determinants of the implicit functions which describe the market for good B and the accumulation expressions.
The Specification of the Functions

(1) The Market for the Nontraded Good B. In Appendix B, the per-capita supply function for good B is derived together with that of good A. In this derivation $P_A$ and $P_B$ represent the nominal prices of A and B (i.e., prices of A and B in terms of money). The efficiency conditions in sectors A and B remain unchanged if the unit prices of A and B are defined in terms of C instead of using money as the numeraire. This means the per-capita supply function for B remains:

$$b = b(P_A, P_B, k) \quad (5.7)$$

where $P_A$ and $P_B$ are expressed in terms of C.

Static demand functions also are derived for goods B and C and money in Appendix E. None of these demand functions is used here. Instead all the demand functions used in this chapter are postulated because of the conceptual and mathematical difficulties associated with deriving dynamic demand functions in a framework in which income among other variables is endogenous. It is assumed that the real per-capita demand for good B depends solely on disposable income and this demand function $d_B$ is written as:
where \( y^d \) is the disposable income. Let us turn now to specifying the determinants of \( y^d \).

In the dynamic model, the time span is long enough to permit the price of money in terms of \( C \) to change. This means there may be capital gains or losses on the financial assets held by the consumers and firms (money and time deposit accounts) and on the financial assets (bank loans) issued by these same economic agents. The budget constraint of these agents may then be written in real per-capita terms as:

\[
y - P_t + (r - \pi_m e)P_m \mu = P_B a + d_C + eP_K \pi_m e
\]

\[
+ \frac{P L}{N} + (r_{BL} - \pi_m e)P_m b_l
\]

(5.9)

where

\[
y - P_t + (r - \pi_m e)P_m \mu = \text{real per-capita disposable income } y^d \text{ (by definition)}.
\]

\[
y = \text{per-capita earned income in terms of } C = P_A a + P_B b = y (P_A, P_B, k)
\]

\[
r = \text{nominal interest rate on time deposits}
\]

\[
\pi_m e = \text{expected rate of change in the price of money in terms of } C \text{ (i.e., } \pi = \frac{\sigma}{P_m/P_m e})
\]

\[
\mu = \text{nominal per-capita stock of time deposits}
\]
$P_{mtd} = \text{per-capita time deposits in terms of } C$

$d_B, d_{CP}, d_{KP} = \text{per-capita demand for goods } B, C \text{ and } K$

by the private sector in physical unit/period

$P_B = \text{unit price of good } B \text{ in terms of } C$

$L = \text{nominal aggregate demand for money}$

$P_L = \text{aggregate demand for money in terms of } C$

$r_{BL} = \text{nominal interest rate on bank loans}$

$r_{BL} - \pi^e_m = \text{real interest rate on bank loans}$

$bl = \frac{BL}{N} = \text{nominal per-capita stock of bank loans}$

$P_{mbl} = \text{per-capita stock of } BL \text{ in terms of } C.$

Note that since $P_m$ is equal to the price of money in terms of $C$, the actual rate of change of the price of money is

$\pi_m = \frac{P_m}{P_m} \cdot$ Inflation (deflation) occurs when $\pi_m$ is negative (positive) meaning the price of money in terms of good $C$ falls (rises). Given the above definition, a negative value of $\pi^e_m$ means consumers and firms expected inflation to occur.

From (5.9) it follows that the real disposable income is:

$$y^d = y(P_A, P_B, k) - P_{mt} + (r - \pi^e_m)P_{mtd}$$

or

$$y^d = y^d(P_A, P_B, k, P_{mt}, r, \pi^e_m, P_{mtd}). \quad (5.10)$$
The effect of a change in time deposits $P_{mtd}$ on disposable income is ambiguous because the real rate of return on this asset may be positive or negative or zero (i.e., $r - \pi_{me}^{e} \geq 0$).

Substitute (5.7) and (5.8) into (5.1) and rewrite the equilibrium condition in the market for the nontraded good $B$ as:

$$db(y^d) - b(P_A, P_B, k) = 0. \quad (5.11)$$

Substitute, in turn, (5.10) into (5.11) to obtain:

$$db(P_A, P_B, k, P_{mtd}, r, \pi_{me}^{e}, P_{mtd}) - b(P_A, P_B, k) = 0. \quad (5.12)$$

This equilibrium condition (5.12) is one of the four equations which describe the dynamic model. It differs from the equilibrium condition (3.14a) of the static model by the fact that it shows that in a dynamic setting, the demand for the nontraded good can be affected by the expected rate of change of the price of money $\pi_{me}^{e}$, the interest rate on time deposits $r$ and the real stock of time deposit $P_{mtd}$.

(2) The Capital Labor Ratio Accumulation Equation.

The real per-capita gross investment is the sum of the real per-capita gross private and public investments.

There are essentially two types of theories concerning the nonresidential investment functions: the models which
assume some form of adjustment of the actual capital stock to a desired level of capital stock and the models which do not make such an assumption, [see review articles of Jorgensen and Siebert (1968), and of Jorgensen, Hunter and Nadiri (1970)]. In Jorgensen (1963, 1965), the demand for the investment good is positively related to the output level and is negatively related to the own price. In Foley and Sidrauski (1971), the demand for the capital good is positively related to income and own rate of return and negatively related to the rates of return on the other assets.

To limit the manipulations, the private sector's desired gross investment function used in this study, which is in line with Foley and Sidrauski (1971) is:

\[ d_{KP} = d_{KP}(\rho_k, \rho_m, y^d, eP_K^*) \]  

(5.13)

where

\[ d_{KP} = \text{per-capita gross private demand for investment in physical unit/period} \]

\[ \rho_k, \rho_m = \text{real rates of return on investment and on money} \]

\[ y^d = \text{per-capita disposable income, in terms of C} \]

6. Although expression (5.13) is in line with Foley and Sidrauski (1971), these authors used the arguments which appear in (5.13) to explain the demand for capital goods while they are being used here to explain the demand for investment goods.
\( eP_K^* \) = unit price of investment good in terms of C.

Note that although the nominal interest rate on time deposits is not market determined, the real rate of return on this asset is affected by the expected rate of change of the price of money and this real rate of return \((r - \pi_m^e)\) may also influence the demand for the investment good. The effect of \((r - \pi_m^e)\) is assumed away in equation (5.13) to ease on the notation. If this real rate of return on time deposits were introduced, it would reinforce the negative effect of a change in the real rate of return on money \(\rho_m\) on the demand for the investment good. In expression (5.13), the disposable income is described by (5.10) and the variables which remain to be specified are \(\rho_k\) and \(\rho_m\).

Since the capital good is being used to produce goods A and B, the real rate of return on capital is the marginal physical product in sector A or in sector B after accounting for the price ratio. That is since \(P_A f'(k_A) = P_B g'(k_B)\), the real rate of return on K is:

\[
\rho_k = f'(k_A) = g'(k_B) P_B / P_A
\]

(5.14)
It is shown in Appendix C that $k_A = k_A(P_A, P_B)$ and $k_B = k_B(P_A, P_B)$ as $k_A > k_B$. Substitute $k_A(P_A, P_B)$ in $f'(k_A)$ and differentiate $\rho_k$ in (5.14) with respect to $P_A$ and $P_B$, noting that $f'' < 0$, to get:

$$\rho_k = h(P_A, P_B).$$ \hspace{1cm} (5.15)

Let $i$ and $\rho$ be respectively the nominal and real rate of return (i.e., interest rate) on any financial asset. The Fisher relation stipulates that $i$ and $\rho$ differ by the expected rate of change of the price of money. More precisely the Fisher relation is:

$$i = \rho + \pi^e \hspace{1cm} (5.16)$$

where $\pi^e$ is the expected inflation or deflation. Since the nominal interest rate on money is zero i.e., $i = 0$ in (5.16), the real interest rate on money is minus the expected inflation or deflation. That is, substitute $i = 0$ in expression (5.16) to obtain $\rho = -\pi^e$. Using the previous notation, the real rate of return on money is:

$$\rho_m = -\pi^e \hspace{1cm} (5.17)$$
Substitute (5.10), (5.15), and (5.17) into (5.13) and rewrite the real demand for the investment good by the private sector as:

\[ d_{KP} = d_{KP}(P^A, P^B, \pi^e, k, P, t, r, P, t, eP_K^*) \], (5.18)

In expression (5.18), it is assumed that the positive effect of \( P_B \) on \( d_{KP} \) via disposable income exceeds the negative effect of \( P_B \) on \( d_{KP} \) via the rate of return, presumably because private investors might be able to determine more accurately their income (i.e., their sales revenues) than the rate of return on the investments which they make. Under such a circumstance, the effect of a change in the rate of return (due to, say, an increase in \( P_B \)) on \( d_{KP} \) may be small or negligible relative to the effect of a change in income (due to an increase in \( P_B \)) on \( d_{KP} \), so that the net result is \( \partial d_{KP}/\partial P_B > 0 \).

As regards the demand for the investment good by the government, it was mentioned in Chapter 1 that the government borrows funds from OPAT to finance infrastructure investments (roads, hotels) or to purchase stocks in some companies. For simplicity, it is assumed that the real per-capita gross public investment depends on OPAT loans and on the price of capital. That is:

\[ d_{KG} = d_{KG}(o\xi, eP_K^*) \]. (5.19)
By assumption, $d_{KG}$ is homogenous of degree zero in $\rho_e$ and $e_P^*$ so that this demand function is expressed in real $K$ terms.

The real per-capita gross investment of the private and public sectors combined is then:

$$\frac{o}{K/N} = d_{KP} + d_{KG}. \quad (5.20)$$

Recall that the gross change in the capital labor ratio $k$ is:

$$o \frac{o}{k} = \frac{K/N}{nk}. \quad (5.4)$$

Assuming a depreciation rate $\delta$ of the capital labor ratio, substitute (5.18) and (5.19) into (5.20) and the resultant expression into (5.4) and rewrite the net capital labor ratio accumulation equation as:

$$k = d_{KP}(P_A, P_B, \pi_m, e_k, P_t, r, P_{td}, e_P^*) + d_{KG}(\rho_e, e_P^*) - (n + \delta)k$$

or

$$k = k(P_B, k, \pi_m, e_\rho; P_A, P_t, r, P_{td}, e_P^*, n, \delta). \quad (5.21)$$

This function (5.21) describes the time path of the capital labor ratio $k$.

(3) The Government Debt Accumulation Equation. The borrowings of funds abroad and their repayments may be due to a wide variety of factors: economic considerations (insufficient national saving, favorable interest rate
abroad, low ratio of government revenues to government spendings, narrow markets, ...), political considerations (misspecification of spending priorities, military and other ties with foreign countries, ...), unpredictable events (floods, drought, ...). Instead of speculating about how these events may affect the real net flow of foreign loans, it is assumed that there is a known ratio \( \varepsilon \) between the real stock of foreign loans and the real stock of OPAT loans. That is:

\[
P_m^{eFL} = \varepsilon P_m^{OL} \quad \text{where} \quad \varepsilon > 0.
\] (5.22)

It follows from (5.22) that one can write:

\[
\frac{P_m^{OL}}{N} + \frac{P_m^{eFL}}{N} = (1 + \varepsilon)\frac{P_m^{OL}}{N}.
\] (5.23)

Define the real per-capita OPAT loans as:

\[
o_\lambda = \frac{P_m^{OL}}{N}.
\] (5.24)

Take the logarithm of (5.24) and differentiate it with respect to time to obtain:

\[
\frac{P_m^{OL}}{N} = o_\lambda + no_\lambda.
\] (5.25)

From expressions (5.6), (5.23) and (5.25), it follows that the government debt equation can be written as:
\[(P_m^{OL_S}/N + (P_m^{eFL_S})/N = (1+\varepsilon)(P_m^{OL})/N = (1+\varepsilon)(o_l+nol) = d_{CG}\]

\[+ eP_K^{d_{KG}} - P_m^{t}. \quad (5.26)\]

Expression (5.26) can be rewritten as:

\[o_l = (d_{CG} + eP_K^{d_{KG}} - P_m^{t})/(1+\varepsilon) - nol \quad (5.27)\]

where \(d_{CG}\) is exogenous and \(d_{KG}\) is defined in (5.19).

Substitute (5.19) into (5.27) and rewrite the government debt accumulation equations:

\[o_l = o_l^0 \pm \frac{d_{CG}}{\pm eP_K^{d_{KG}} - P_m^{t}, n, (1+\varepsilon)} \quad (5.28)\]

Equation (5.28) describes the time path of the real net per-capita borrowings by the government from OPAT and from the foreign countries to finance the combined deficits of the ordinary and investment budgets when they occur.

(4) The Foreign Reserve Accumulation Equation.

The balance of payments equation written in terms of good C is:

\[(P_m^{eRE})/N = (P_m^{OL})/N - (P_m^{M})/N. \quad (5.7)\]

The aggregate money stock in terms of good C can be written as the product of the money multiplier and the real monetary base. That is:
\[ P_m M = v(P_m DC + P_m eRE) \]  
\hspace{1cm} (5.29) 

where

\[ P_m M = \text{aggregate money stock in terms of good } C \]

\[ v = \text{money multiplier} \]

\[ P_m DC = \text{aggregate stock of domestic credit in terms of } C \]

\[ P_m eRE = \text{aggregate stock of foreign reserves in terms of } C. \]

The Central Bank of the West African Monetary Union of which Togo is a member, requires that commercial banks maintain a known ratio \( \tau \) between the stock of domestic credit and the stock of foreign reserves. That is:

\[ P_m DC = \tau P_m eRE \]  
\hspace{1cm} (5.30) 

where \( \tau \) is positive and constant.

Note that expression (5.30) indicates that the smaller the value of the ratio \( \tau \), the smaller the amount of foreign reserves which is being incorporated into the domestic credit and thus into the monetary base and into the money supply. This means the smaller (the larger) the ratio \( \tau \), the greater (the smaller) the degree of sterilization of foreign reserves.

Substitute (5.30) into (5.29) to obtain:

\[ P_m M = v(1 + \tau) P_m eRE. \]  
\hspace{1cm} (5.31)
Holding \( v \) and \( \tau \) constant, it follows from (5.31) that:

\[
\frac{(P_m M)}{N} = v(1 + \tau)(P_m eRE)/N. \tag{5.32}
\]

Substitute (5.32) into (5.7) and rewrite the balance of payments equation in real terms as:

\[
[1 + v(1 + \tau)](P_m eRE)/N = (P_m L)/N \tag{5.33}
\]

where \((P_m L)/N\) is the per-capita change in the demand for money by the private sector in real terms. This variable \((P_m L)/N\) appears exactly in the same form in the private sector's budget constraint (5.9) which can be rewritten as:

\[
(P_m L)/N = y^d - (P_B d_B + d_{CP} + eP_K * d_{KP}) - (r_{BL} - \pi^e_m)P_m b_\ell \tag{5.9a}
\]

where \(y^d\), \(d_B\) and \(d_{KP}\) are described respectively by expressions (5.10), (5.8) and (5.13).

The per-capita demand for the nontraded good B is assumed to depend solely on disposable income. That is:

\[
d_B = d_B(y^d). \tag{5.8}
\]

In a similar fashion, it is assumed that the per-capita demand for the imported consumption good C depends on disposable income only. That is:

\[
d_{CP} = d_{CP}(y^d) \tag{5.34}
\]
Substitute (5.8), (5.34) and (5.13) into (5.9a) and rewrite this expression (5.9a) as:

\[
\frac{(P_m L)}{N} = y^d - [P_B d_B (y^d) + d_{CP} (y^d) + e_P K d_{KP} (h(P_A, P_B), \pi_m^e, y^d, e_P K^*)] - (r_{BL} - \pi_m^e) P_m b_l
\] (5.35)

where \(y^d\) is expression (5.10).

Define the implicit function \(\theta\) such that:

\[
\frac{(P_m L)}{N} = \theta(P_A, P_B, \pi_m^e, k, P_m t, P_m td, P_m b_l, r, r_{BL}, e_P K^*)
\] (5.36)

The signs of the derivatives of the implicit function with respect to its arguments come from the signs indicated in expressions (5.10), (5.35), and from noting that the marginal propensity to spend out of disposable income is between 0 and 1.

Substitute (5.36) into (5.33) and rewrite the balance of payments equation as:

\[
\frac{(P_m eRE)}{N} = \theta /[1 + \nu (1 + \tau)]
\] (5.37)

Define the per-capita stock of foreign reserves in real terms as:
ere = \frac{(P_m eRE)}{N}. \quad (5.33)

Take the logarithm of (5.33) and differentiate the logarithmic expression obtained with respect to time to get:

\frac{o}{(P_m eRE)/N} = ere + nere. \quad (5.39)

Substitute (5.39) into (5.37) to obtain:

ere = \theta/[1 + v(l + \tau)] - nere. \quad (5.40)

Substitute for \theta from (5.36) to get:

ere = ere(P_B, k, \pi_m e, ere; P_A, P_m t, P_m t d, P_b l, r, \ldots - \frac{r_B l}{eP_m e}, n, v(1+\tau)). \quad (5.41)

Equation (5.41) describes the time path of the real per-capita stock of foreign reserves.

The Dynamic Model

The dynamic framework for this simplified economy is described by the following equations:
\[ d_B(P_B, k, \pi m, P_m; P_A, r) - b(P_A, P_B, k) = 0 \]  
(5.12)

\[ k = k(P_B, k, \pi m, o\ell, P_m; P_{td}, P_A, eP^*, n, \delta) = 0 \]  
(5.21)

\[ o\ell = o\ell(o\ell, P_m; d_{CG}, eP^*_K, n, e) \]  
(5.23)

\[ e_\text{ere} = e_\text{ere}(P_B, k, \pi m, e_\text{ere}, P_m; P_{td}, P_b; P_A, e^*_K, n, v, \tau) \]  
(5.41)

The exogenous variables and parameters in this system are \( P_A, eP^*_K, r, r_{BL}, \varepsilon, n, e, d_{CG} \) and \( P_C = (1 + t_C) \) \( eP^*_C \) although \( P_C \) does not appear explicitly in the above equations. The producer price \( P_A \) is determined by OPAT while the price of the investment good \( P^*_K \) and the price of the imported consumption good \( P^*_C \) are determined abroad because Togo is small. The exchange rate \( e \) is fixed while the ad valorem tax rate \( t_C \) on good \( C \) is determined by the government. The interest rates on time deposits \( (r) \) and on bank loans \( (r_{BL}) \) and the ratio of domestic credit to foreign reserves are determined by the Central Bank. The rate of growth of population \( n \) is determined in part by biological factors and in part by the government through family planning programs, tax laws, etc. The ratio of foreign loans to
OPAT loans $\epsilon$ and the government spending on the imported consumption good $d_{CG}$ are determined by the government.

The rate of depreciation of the capital labor ratio $\delta$ and the money multiplier $v$ are treated as parameters.

All the remaining variables $P_B$, $\omega$, $\epsilon$, $k$, $\pi_m^e$, $P_m$, $t$, $td$ and $b\lambda$, are time dependent. In particular, the per-capita stock of assets $td$, $b\lambda$ and the per-capita tax $t$ will be equal to preset values only if the rate of change in their aggregate levels is the same as the rate of change in the population. For instance, $td$ will be equal to a predetermined value $td$ only if the nominal aggregate stock of time deposits $TD$ grows (or falls) at the same rate as the population $N$ grows (or falls) because $td = TD/N$ and $N$ changes over time.

In the above system, there are four equations and nine variables which are not yet specified, $P_B$, $\omega$, $\epsilon$, $k$, $\pi_m^e$, $P_m$, $t$, $td$ and $b\lambda$.

Before indicating how the system may be closed let make some observations about the expected rate of price change $\pi_m^e$ and the actual price of money $P_m$. First, there are various theories about how buyers and sellers perceive the relationship between the expected rate of price change $\pi_m^e$ and the actual rate of price change $\pi$. The behavior of these economic units can be associated with the speed of adjustment of $\pi^e$ to $\pi$. This speed ranges from zero (static expectations) to infinity (perfect foresight). The adaptive
expectations assumption lies between these two extremes. The perfect foresight and the adaptive expectations assumptions have received considerable attention in the theoretical and empirical macroeconomic literature [Cagan (1956), Nerlove (1953), Dow and Dicks-Mireaux (1958), summary article of Bodkin, Bond, Reuber and Robinson (1966), Stein (1970), Foley and Sidrauski (1971), Turnovsky (1977)]. In these and other studies, the expected rate of price change is treated as an exogenous variable [Cagan (1956), Nerlove (1953), Foley and Sidrauski (1971), Turnovsky (1977)] or as an endogenous variable [Burmeister and Dobell (1970), Foley and Sidrauski (1971)]. When the adaptive expectations assumption is used, a new flow variable which represents the forecasting error (between actual and expected price changes) is introduced in the models. This means an additional differential equation must be dealt with. Empirically, this assumption poses the problem of finding a proxy for an unobservable variable (the expected rate of price change). The perfect foresight assumption is used in this study for two reasons: (a) "adaptive expectations" accommodate irrational behavior; (b) the primary concern of this writer is to identify the steady state determinants of the price of the nontraded good $P_B$. Therefore, it will be assumed that:

$$\pi_m^e = \pi = \frac{P_0}{P_m}$$

(5.42)
The second observation is that the price of money in terms of good C denoted by $P_m$ and the actual rate of change of this price $\pi$ are not independent in the sense that, to a known value of $P_m$ corresponds a known value of $\pi$ and vice versa [see expression (5.42)]. Third, since the imported good C is the numeraire and the price of that good is exogenously determined in part because of the small country assumption, the price of money in terms of good C is given. For instance, the value or price of 100 FCFA in terms of needles is equal to the number of needles that Togo can obtain in exchange for 100 FCFA and this number is known for the following reasons: (a) Togo is too small to affect the French price of one needle $P_C^*$; (b) there is a fixed exchange rate $e$ between the FCFA and the French franc; (c) the ad valorem import tariff rate $t_C$ is given so that the import price $P_C = (1+t_C)eP_C^*$ is known and thus the number of needles that Togo can obtain in exchange for 100 FCFA is known, in particular, $P_m = 1/P_C$. The implication of these observations is that $P_m$ and $\pi$ are given so that the variables which remained to be determined are $P_B$, $k$, $\alpha$, $e_r$, $t$, $td$ and $b_{l}$. Three of these variables ($t$, $td$, $b_{l}$) will be treated as target variables while the steady state properties of the remaining four variables will be examined. Indeed, the Board of Directors of the Central Bank sets the various interest rates and the National Credit Committee has the discretionary power to impose quantitative and qualitative
restrictions on loans made by banks to the private sector. So it is reasonable to assume that these institutions can use the different policy tools at their disposal (interest rates, qualitative and quantitative allocations) to affect the aggregate levels of time deposits and bank loans such that the per-capita ratios of these assets remain at predetermined levels during a certain period. As regards the tax variable \( t \), during a private meeting with a Togolese government official, this writer was told that the government required from each tax collecting agency to collect predetermined aggregate amounts of taxes. Furthermore, besides setting the tax structure (i.e., tax base and rates), the government can change its spendings on the imported good \( C \) and the ratio of foreign loans to OPAT loans \( \varepsilon \) in order to make its budget constraint be consistent with the preset taxes. Therefore, one may assume that the government can control the rate of growth of the nominal aggregate stock of taxes in a manner which permits to reach the target per-capita tax level. Treating \( t, td, b \) and \( \beta \) as target variables, the variables left to be determined are \( P_B, k, \alpha \) and \( \epsilon \). Finally, note that the per-capita money stock does not appear explicitly any longer in the system as a separate variable because of assumption (5.30) which leads to expression (5.31) which in turn relates the real money stock to the real stock of foreign reserves.
The Solution of the Model

The problems related to the stability of the entire dynamic system and to the signs of the comparative dynamic results are presented in the first subsection. In the second subsection, the size of the dynamic system is reduced by reducing it into several alternative subsystems. The stability and qualitative results of these subsystems are examined.

The Entire Dynamic System

In what follows, it is assumed that a steady state exists, where the steady state values of the endogenous variables \( P_B, k, \omega \lambda \) and \( e \) are determined by the following equations:

\[
E(P_B, k; \pi, P_{m t}, P_{m d}, P_A, r) = 0 \quad (5.12a)
\]

\[
k = F(P_B, k, \omega \lambda; \pi, P_{m t}, P_{m d}, P_A, r, eP_K, n, \delta) = 0 \quad (5.21a)
\]

\[
\omega \lambda = G(\omega \lambda; P_{m CG}, eP_K, n, \varepsilon) = 0 \quad (5.28a)
\]
where the four equations are expressions (5.12), (5.21), (5.28) and (5.41) written in implicit forms and the letters $E$, $F$, $G$ and $H$ designate unspecified functional forms. In what follows, I will continue to denote the accumulation equations by small letters (i.e., $o$, $ol$, ere) although the relevant functions are not exactly the same as expressions (5.21), (5.28), and (5.41) in order to facilitate their identification. In the above system, $E$, $k$, $ol$ and ere are set to zero because the market for the nontraded good must clear and because the behaviors of the variables which accumulate are being studied at the steady state. Note from (5.12) that in order to obtain the negative sign of the derivative $\partial E/\partial P_B$ indicated in (5.12a), it is assumed that an increase in the price of the nontraded good reduces the excess demand for this good so that $\partial E/\partial P_B$ is negative.

The price of the nontraded good $P_B$ is always determined in the market for this good. That is, equation (5.12a) can be solved to obtain:

$$P_B = P_B(k, \pi, P_m t, P_m t_d, P_A, r).$$  

(5.12b)
Equation (5.12b) in turn can be substituted into (5.21a) and (5.41a) and the accumulation equations can be rewritten as:

\[ k = I(k, \omega_t, \pi, P_m, t, P_m, T, P, r, e_P, \kappa, n, \delta) = 0 \]  

(5.21b)

\[ \omega_t = G(\omega_t; P_m, R_{\omega}, \kappa, n, \varepsilon) = 0 \]  

(5.28a)

\[ \omega_\varepsilon = R(k, \omega_\varepsilon; \pi, P_m, T, P_m, T, P_m, T, P, r, r_{BL}) \]

(5.41b)

In what follows, it is assumed that equations (5.21b), (5.28a), and (5.41b) which describe the dynamic model are linear and the Routh-Hurwitz theorem will be applied to examine the stability of the system or its lack of it. The endogenous variables in this system are \( k, \omega_t \) and \( \omega_\varepsilon \).

Denote by \( Q \) the following matrix:

\[
Q = \begin{bmatrix}
I_k & I_{\omega_t} & I_{\omega_\varepsilon} \\
G_k & G_{\omega_t} & G_{\omega_\varepsilon} \\
R_k & R_{\omega_t} & R_{\omega_\varepsilon}
\end{bmatrix}
\]
where \( I_k = \partial I / \partial k, \ G_{o\ell} = \partial G / \partial o\ell, \) and \( R_{ere} = \partial R / \partial ere \) and where from (5.21b), (5.28a), (5.41b), we may note that \( I_{ere} = G_k = G_{ere} = R_{o\ell} = 0. \) Let \( Q_1 \) be the trace of \( Q, \ Q_2 \) the sum of the second order principal minors and \( Q_3 \) its determinant. Then the necessary and sufficient conditions for the steady state equilibria of the above system to be locally stable are that:

\[
Q_1 = I_k + G_{o\ell} + R_{ere} < 0 \quad (5.43)
\]

\[
Q_3 = I_k G_{o\ell} R_{ere} < 0 \quad (5.44)
\]

\[
Q_1 Q_2 - Q_3 = 2I_k G_{o\ell} R_{ere} + I_k^2 (R_{ere} + G_{o\ell}) + G_{o\ell}^2 (R_{ere} + I_k) + R_{ere}^2 (I_k + G_{o\ell}) < 0. \quad (5.45)
\]

Based on the signs of the partials of functions \( I, \ G, \) and \( R \) with respect to \( k, \ o\ell \) and \( ere \) shown in equations (5.21b), (5.28a) and (5.41b), the entire dynamic system may or may not be locally stable. That is, these signs indicate that \( Q_1 \geq 0, \ Q_3 \geq 0 \) and \( Q_1 Q_2 - Q_3 \geq 0. \)

Since from (5.41b) we may note that \( R_{ere} \) is negative, if \( I_k \) and \( G_{o\ell} \) are both negative, then conditions (5.43)-(5.45) will be met and the entire dynamic model will be locally stable. In view of this, it becomes appropriate to examine the economic factors which can affect the signs of the derivatives \( I_k \) and \( G_{o\ell}. \)
As it can be seen from equations (5.12b), (5.21) and (5.21b), the partial $I_k$ has the same sign as the expression:

$$S = \left( \frac{\partial d_{KP}}{\partial P_B} \right) \left( \frac{\partial P_B}{\partial k} \right) + \left( \frac{\partial d_{KP}}{\partial k} \right) - (n+\delta) \quad (5.46)$$

where $d_{KP}$ is the demand for the investment good by the private sector, $P_B$ is the price of the nontraded good, $n$ is the rate of growth of the population and $\delta$ is the rate of depreciation of the capital labor ratio. The change in the demand for the investment good in response to a change in the price of nontraded good, although positive may be negligible in magnitude because the investment good is bought primarily by the government and foreign companies while the price of the nontraded good is determined in a market dominated by local individual consumers and farmers. The average rate of growth of the Togolese population between 1970 and 1975 was 2.6%, slightly above the 2.3% average for all underdeveloped middle income countries over the same period. There is no information about $\delta$ but it is likely that the value of this parameter will be greater in an underdeveloped country like Togo than in advanced economies because of the relatively poor maintenance of the machines and of factors such as inadequate health care and nutrition. Therefore, if $\frac{\partial d_{KP}}{\partial P_B}$ is negligible and $\frac{\partial d_{KP}}{\partial k}$ is small compared to $(n+\delta)$, expression $S$ in (5.46) may well be negative and thus $I_k$ will be negative. Concerning the derivative $G_{O_k}$, it can be seen from (5.19), (5.27) and (5.28a) that this derivative
has the same sign as:

\[ U = eP_K^* \left( \partial \frac{d_{KG}}{\partial \lambda} \right) - n. \]  \hspace{1cm} (5.47)

It was assumed that the government borrows funds from OPAT and foreign countries to buy machinery to equip factories and to build the communication network, roads, port facilities, etc., and presumably the more funds it can borrow the more investment good it will buy. But it is also conceivable that the government borrows money for purposes other than the purchase of investment goods. For instance, as mentioned in Chapter 1, over the years, OPAT has been the major source of funds to finance government ordinary budget deficits, that is this agency's loans were used to buy consumption goods and to service other government debts. If foreign loans which are earmarked for the purchase of investment goods decline drastically and OPAT loans are used to finance the ordinary budget, then an increase in these loans may not affect significantly the flow of capital goods into the government sector and expression (5.47) and thus the derivative \( G_{\lambda} \) may be negative. Note that expression (6.5) in Foley and Sidrauski (1971, p. 90), which is written as expression (2.5) in the review of the literature in Chapter 2, corresponds to (5.27) in this chapter except that in Foley and Sidrauski, \( d \) does not depend explicitly on \( g \) so that \( \frac{\partial g}{\partial g} = -n < 0 \). Since in Togo, the government borrows funds from OPAT and foreign countries to buy investment goods to
equip factories and build the infrastructure, it may not be totally realistic to assume that $\partial d_{kP}/\partial o_l = 0$. However, in order to obtain the negative sign for $G_{o_l}$ which is required for the model to be dynamically stable and which is in line with Foley and Sidrauski (1971), it is assumed that $\partial d_{kP}/\partial o_l$ is positive but is so small that $U$ in (5.47) and consequently $G_{o_l}$ are negative. Given these signs (i.e., $I_k < 0$, $G_{o_l} < 0$), the entire dynamic system is locally stable and in general, it can be solved implicitly to obtain the steady state expressions for the capital labor ratio $k^e$, the per-capita government debt $o_l^e$ and the per-capita stock of foreign reserves $r_{re}^e$. Afterwards, $k^e$ can be substituted into (5.12b) to obtain the steady state expressions for the price of the nontraded good $P_B^e$. This procedure has been followed but the comparative dynamic results obtained are not reported here because almost all of them are ambiguous due to the ambiguous effects of the changes in the exogenous variables and parameters on the accumulation equations and due to numerous feedbacks. Instead, the above dynamic model of three equations had been reduced to alternative subsystems of two equations and the analysis below focuses on these reduced models. However, before proceeding any further, let us make one observation about the effect of a change in the capital labor ratio on the steady state accumulation equation for the foreign reserves. As indicated in (5.41b), this effect (i.e., the sign of $R_k$) is ambiguous. The stability
of the subsystem which will receive the major attention later in this work requires that $R_k$ be negative. Therefore, it is appropriate to indicate the factors which affect the sign of this derivative. It appears from (5.12b), (5.36) and (5.41b) that $R_k$ has the same sign as:

$$V = \left[ \frac{\partial \theta}{\partial P_B} \frac{\partial P_B}{\partial k} \right] + \frac{\partial \theta}{\partial k}$$  \hspace{1cm} (5.43)$$

where $\theta$ is the per-capita flow demand for money. In expression (5.43), the first bracket represents the indirect effect of a change in the capital labor ratio $k$ on this flow demand for money via the price of the nontraded good $P_B$ while the second bracket represents the direct effect of a change in $k$ on $\theta$ through income. Thus if the indirect effect exceeds (or is smaller than) the direct effect, $V$ and consequently $R_k$ will be negative (positive). One cannot say, a prior, which effect will dominate. In what follows, it is assumed that $R_k$ is negative primarily because this particular sign is required for the subsystem which we shall focus on to be globally stable. Let us turn now to the reduced models where in view of the preceding discussions, it is assumed that the derivatives $I_k', G_{ol}, R_k$ in (5.21b), (5.28a) and (5.41b) are all negative.

The Reduced Dynamic Models

Most of the comparative dynamic results that can be obtained by considering the entire dynamic model described
by equations (5.21b), (5.28a) and (5.41b) under the conditions that \( I_k, G_\ell \) and \( R_k \) are all negative, are ambiguous because several derivatives of the functions \( I, G \) and \( R \) with respect to the exogenous variables have ambiguous signs and because of numerous feedbacks. For this reason, alternative subsystems obtained by reducing the entire dynamic model are considered. These subsystems denoted as Set 1, Set 2 and Set 3 are the following:

Set 1

\[
\begin{align*}
  k &= I(k, o\ell; P_m, P_A, r, n) = 0 \\
  o\ell &= G(o\ell; P_m, d_{CG}, n) = 0
\end{align*}
\] (5.21c)

Set 2

\[
\begin{align*}
  o\ell &= G(o\ell; P_m, d_{CG}, n) = 0 \\
  \text{ere} &= R(\text{ere}; k, P_A, P_m, P_A, r, n, \tau) = 0
\end{align*}
\] (5.28b)

Set 3

\[
\begin{align*}
  k &= I(k; o\ell, P_m, P_A, r, n) = 0 \\
  \text{ere} &= R(k, \text{ere}; P_m, P_A, r, n, \tau) = 0
\end{align*}
\] (5.41d)

The basic equations which compose these sets are the accumulation equations for capital (5.21b), for the government debt (5.28a) and for the foreign reserves (5.41b) with a few less important exogenous variables and parameters.
dropped. It should be mentioned that by assuming that the derivative $I_k$ and $G^e_0$ and $R_k$ are negative, we are assuming that each of the subsystems is globally stable. Set 3 will receive the major attention in this subsection. However, let us examine briefly Set 1 and Set 2.

In Set 1, the endogenous variables are the capital labor ratio and the per-capita government debt while the stock of foreign reserves is held constant. Equation (5.23b) which is closed in $\Omega$ can be solved to obtain the steady state value of the per-capita government debt $\Omega^e$ which in turn can be substituted into equation (5.21c) to obtain the steady state value of the capital labor ratio $k^e$. Finally, $k^e$ can be substituted into (5.12b) to get the steady state price of the nontraded good $P^e_B$. Following this procedure, it can be shown that an expansionary fiscal policy (i.e., decrease in taxes or increase in government spending on the imported consumption good) will raise the steady state levels of $\Omega^e$, $k^e$ and $P^e_B$. As it can be seen from equation (5.41d), the increase in government spending on good C will reduce the steady state level of the per-capita stock of foreign reserves $ere^e$ through $k^e$ while the decrease in taxes will have an ambiguous effect on $ere^e$. And of course, an exogenous increase in $ere^e$ will affect $k^e$.

Set 2 has a very serious defect and this set is mentioned here only to satisfy the curiosity of a reader who may wonder why this subsystem is deleted. Before we indicate
the major deficiency of this set, let us make one observation about how Set 2 is obtained. When the capital labor ratio $k$ is treated as a constant as it is in Set 2, the price of the nontraded good $P_B$ which is determined by (5.12b) becomes known so that the equation which describes the steady state path of the per-capita foreign reserves is obtained by setting equation (5.41) equal to zero. This means the foreign reserves equation (5.41c) is obtained treating $P_B$ as an exogenous variable while in Set 3, the foreign reserves equation (5.41d) is obtained treating $P_B$ as an endogenous variable. The major deficiency of Set 2 is that since $P_B$ is known because $k$ is given, it is possible that the market for the nontraded good will not clear at that known price. So in this Set 3, there is a possibility that the market for the nontraded good $B$ be in disequilibrium.

Our attention will be devoted now to examining Set 3. The dynamic subsystem described by equations (5.21c) and (5.41d) is globally stable because the trace $(I_k^+ + R_{ere})$ is negative and the determinant $(J = I_k^+ R_{ere})$ is positive, given the assumption that $I_k$ is negative. This means starting from any initial position, the direction of motion of the capital labor ratio and of the per-capita stock of foreign reserves will converge toward their steady state values over time. The phase diagram of Figure 4 describes this situation. As regards the comparative dynamic results, equations (5.12b), (5.21c) and (5.41d) can be solved to obtain the following expressions:
Figure 4. A Phase Diagram for the Reduced Dynamic Model (Set 3).
where the steady state price of the nontraded good $P_B^e$ given by (5.51) is obtained by substituting (5.49) into (5.12b).

The above results indicate that an expansionary fiscal policy which results in an increase in the government debt, ceteris paribus, will raise the steady state values of the capital labor ratio and the price of the nontraded good and it will reduce the steady state value of the per-capita stock of foreign reserves. Indeed, assume that the government increases its purchase of the imported consumption good (i.e., $d^Q$ rises), ceteris paribus. As it can be seen from the government budget constraint given by (5.5), the increase in $d^Q$ will have to be matched by an increase in government borrowings from OPAT. This higher level of per-capita government debt $\omega_l$ as indicated by (5.28b) will have a positive effect on the demand for the investment good by the public sector [see (5.19)]. This addition to the capital stock by the government will raise the aggregate capital stock of the economy, so that the capital labor ratio increases over time [see (5.21)]. Since the price of the nontraded good $P_B$ bears a positive relationship with the
capital labor ratio $k$, the increase in $k$ due to the increase in $d_{CG}$ will also increase $P_B$ at the steady state. A similar qualitative result is obtained if the expansionary fiscal policy is initiated through a decrease in the per-capita lump sum tax $t$. Indeed, a decrease in taxes also "forces" the government to borrow more to finance its budget deficit and this results in an increase in the per-capita government debt $o\lambda$. The higher level of government debt will raise the capital labor ratio and thus the price of the nontraded good at the steady state as explained earlier. But in addition, the initial decrease in taxes increases the disposable income of the private sector and thus exerts a positive direct effect on the capital labor ratio and on the price of the nontraded good. So, as regards fiscal policy, there are two observations to be made. First, the IS-LM result which states that an expansionary fiscal policy will raise prices which is confirmed in the static model of this thesis [expression (3.23)], is maintained at the steady state in this dynamic model. Second, an increase in government spending on the consumption good $d_{CG}$ or a decrease in the lump sum tax $t$ has the same qualitative effect on the steady state values of the capital labor ratio $k^e$ and the price of the nontraded good $P_B^e$ but their magnitude may vary because a change in $d_{CG}$ affects $k^e$ and $P_B^e$ only indirectly through the per-capita government debt $o\lambda$ while a change in $t$ affects $k^e$ and $P_B^e$ indirectly through $o\lambda$ and also directly. As
regards the effect of fiscal policy on the steady state level of the per-capita stock of foreign reserves, a rise in the government spending on good C increases the per-capita government debt o\lambda as mentioned above. This increase in o\lambda will raise the steady state value of the capital labor ratio k^e and since there is a negative relationship between k^e and the steady state value of the stock of foreign reserves ere^e, the ultimate effect is that the expansionary fiscal policy (i.e., d^CG rises) reduces ere^e. This result seems plausible intuitively because the consumption good C is imported and the increased purchase of this good, ceteris paribus, will tend to reduce the stock of foreign reserves. When the expansionary fiscal policy is brought about by a decrease in the lump sum tax, t, this fall in t exerts a positive direct effect and an ambiguous indirect effect on the steady state stock of foreign reserves ere^e. Indeed, a fall in t raises the government debt o\lambda which in turn raises the steady state capital labor ratio k^e. This higher capital labor ratio will reduce the steady state value of the per-capita stock of foreign reserves. This is the indirect effect indicated by the positive sign of the following expression: (\partial ere^e/\partial k) + (\partial k/\partial o\lambda)(\partial o\lambda/\partial P^m_t) which is obtained from (5.28b), (5.21c) and (5.41d). But the decrease in t exerts also a direct effect on ere^e which is ambiguous because as it can be seen from (5.12b) and (5.41a), the expression which determines the sign of P^m_t in the foreign reserves accumulation equation
(5.41d) is ambiguous. That is: $\left( \frac{\partial H}{\partial P_B} \right) \left( \frac{\partial P_B}{\partial P_t} \right) + \left( \frac{\partial H}{\partial P_t} \right) \leq 0$. So, an expansionary fiscal policy brought about by an increase in the government purchase of the imported consumption good C will reduce the steady state level of the stock of foreign reserves $ere^e$ while a reduction in the lump sum tax may decrease or increase $ere^e$. Monetary policy can be enacted by changing the interest rate $r$ on time deposits or by changing the ratio $\tau$ of domestic credit to foreign reserves. In this model, an increase in $r$ or in $\tau$ will have no effect on the steady state level of the government debt as indicated by equation (5.28b). A rise in $r$ increases the disposable income of the private sector which in turn raises the demand for the investment good by this sector. As a result, the aggregate capital stock increases and consequently the capital labor ratio $k$ rises at the steady state. The increase in the interest rate on time deposits exerts an indirect positive effect through $k$ and a direct positive effect on the price of the nontraded good as it can be seen in (5.12b) and (5.49). Thus if the Central Bank decides to attract more time deposits by raising the interest rate on this asset, this policy may ultimately result in an increase in the steady state values of the price of the nontraded good and thus of the consumer price index. The effect of an increase in this interest rate $r$ on the steady state level of the per-capita stock of foreign reserves $ere^e$ is ambiguous because of the sign of $\partial ere^e / \partial r$. 
depends on the sign of the expression \( \partial R / \partial r = (\partial H / \partial P_e) + (\partial P_e / \partial r) + (\partial H / \partial r) \geq 0 \) which is obtained from (5.41a), and on some cross terms because \( \partial c / \partial r = [- (\partial I / \partial k)(\partial R / \partial r) + (\partial I / \partial r)(\partial R / \partial k)] / J \) where \( J \) is the determinant of Set 3.

A change in the exogenous ratio \( \tau \) which indicates the degree of sterilization of foreign reserves has no effect on the steady state values of the price of the nontraded good, the capital labor ratio and the per-capita government debt because \( \tau \) has no effect on the basic expressions (5.12), (5.21) and (5.28) which led to the steady state equations.

If the Central Bank decides to sterilize more foreign reserves by reducing \( \tau \), this policy will raise the steady state level of the per-capita stock of foreign reserves.

An increase in the producer price \( P_A \) of the export good \( A \) will have no effect on the steady state level of the per-capita government debt because \( P_A \) does not influence directly the government budget deficit. However, this increase in \( P_A \) will affect the steady state levels of the capital labor ratio \( k_e \), the price of the nontraded good \( P_e \) and the per-capita stock of foreign reserves. Indeed, an increase in the producer price increases the disposable income of the private sector and thus increases its demand for the investment good. This leads to a larger aggregate stock of capital and to a rise in the capital labor ratio at the steady state.
While the increase in the producer price $P_A$ by OPAT exerts only a direct effect upon the capital labor ratio $k^e$, it exerts a direct effect and an indirect effect on the price of the nontraded good $P_B^e$. Indeed, the higher producer price, by increasing disposable income, increases the demand for the nontraded good B. The increase in $P_A$ also reduces the supply of B. The rising demand for B associated with the decreasing supply of this good pushes the price of the nontraded good upward. This is the direct effect of an increase in $P_A$ on $P_B^e$ which was also mentioned in the static model of Chapter 3. However, in addition to this effect, there is an indirect effect which appears because the rise in $P_A$ increases the steady state value of the capital labor ratio which in turn pushes the price of the nontraded good further up. This indirect effect was absent in the static model because the capital labor ratio was held constant. As regards the effect of an increase in the producer price $P_A$ on the steady state value of the per-capita stock of foreign reserves in (5.50), there are several factors which affect the sign of $\frac{\partial \epsilon}{\partial P_A}$. First, the following expression can be obtained from (5.35) and (5.36):

$$\frac{\partial \epsilon}{\partial P_A} = \left[ (\frac{\partial d}{\partial P_A})^+ + eP \right] \xi - \left[ eP \right] \xi \geq 0 \quad (5.52)$$
where

\[ \theta = \text{the per-capita flow demand for money in terms of C} \]
\[ y^d = \text{disposable income in terms of C} \]
\[ \xi = \text{marginal propensity to spend out of disposable income} \]
\[ eP_K^* = \text{unit price of the investment good in terms of C} \]
\[ d_{KP} = \text{private demand for the investment good in physical unit/period} \]
\[ h = \text{real rate of return on investment.} \]

In expression (5.52), the first bracket terms may be called the "income effect" of a change in \( P_A \) on \( \theta \) while the second bracket terms is the "rate of return effect" of a change in \( P_A \) on \( \theta \).

From (5.12b), (5.41a), (5.41b) and (5.41d) one can obtain:

\[ \frac{\partial \text{ere}}{\partial P_A} = \frac{\partial R}{\partial P_A} = \left( \frac{\partial H}{\partial P_B} \right) \left( \frac{\partial P_B}{\partial P_A} \right) + \left( \frac{\partial H}{\partial P_A} \right) > 0 \] (5.53)

where

\[ \text{ere} = \text{flow amount of the per-capita stock of foreign reserves in terms of C} \]
H, R = functional form notations used in equations (5.41a) and (5.41b) to designate the flow amount of foreign reserves ere. The term \( \partial H/\partial P^A \) in (5.53) is equal to \( \partial \theta/\partial P^A \) in (5.52). Finally, one can obtain from Set 3 the following expression:

\[
\frac{\partial \text{ere}^e}{\partial P^A} = \frac{1}{J} \left[ -\left( \frac{\partial I}{\partial K} \right) \left( \frac{\partial R}{\partial P^A} \right) + \left( \frac{\partial I}{\partial P^A} \right) \left( \frac{\partial R}{\partial k} \right) \right]
\]

(5.54)

where J is the determinant of the set. Expression (5.54) shows the effect of a change in the producer price on the steady state value of the per-capita stock of foreign reserves. In this chapter, the producer price \( P^A \) paid by OPAT to the farmers is exogenous and is positively related to the world price \( P^*_A \) of good A. Assume that this world price \( P^*_A \) increases and OPAT raises the producer price \( P^A \), ceteris paribus. The above result (5.54) indicates that if no other exogenous variable or parameter changes, then the increase in \( P^A \) will induce changes in the disposable income and in the rate of return on investment and consequently in the flow demands for investment and for money in such a manner that the per-capita stock of foreign reserves may fall or rise at the steady state. Following an increase in \( P^A \), if the "rate of return effect" exceeds the "income effect" in (5.52), then \( \partial \theta/\partial P^A \) will be negative and consequently \( \partial R/\partial P^A \) will
be negative in (5.53). Under such a circumstance, the initial increase in the producer price will result in a decrease in the steady state value of the per-capita stock of foreign reserves. If the "income effect" exceeds the "rate of return effect" following an increase in $P_A$, such that $\theta / \theta P_A$ is positive, then in general, one cannot say whether the per-capita stock of foreign reserves will increase or decrease at the steady state. Graphically, the effect of a change in an exogenous variable or in a parameter on the steady state values of the capital labor ratio $k^e$ and on the per-capita stock of foreign reserves $e^e$ can be traced through the shift of the $ere = 0$ curve. For instance, since the per-capita government debt $o_0^o$ is a shift parameter in the $k = 0$ locus [see equation (5.12c)], an increase in $o_0^o$ which may result from an increase in the government spending on good C will cause the $k = 0$ locus to shift upward in Figure 4 while the $ere = 0$ curve remains unchanged. The net result is that the steady state value of the capital labor ratio will increase while that of the per-capita stock of foreign reserves will decrease. An increase in the exogenous rate of growth $n$ of the population will shift the $k = 0$ curve downward and will shift the $ere = 0$ curve leftward and the net result is that the steady state value of the capital-labor ratio will fall while that of the per-capita stock of foreign reserves may increase or decrease depending on the relative magnitude of the shift of the curves.
To end this chapter, the static and the dynamic models will be compared briefly. In the static model, no asset accumulates and the "driving force" is the price of the nontraded good $P_B$. The determination of this price leads to the determination of the consumer price index, and of the per-capita gross national product. In the static analysis, since no asset accumulates, the capital stock, the government debt and the money stock are exogenous variables. This assumption is removed in the dynamic model to allow the investment good and the government debt and the foreign reserves to change over time and thus the number of endogenous variables increased from one ($P_B$) to four ($P_B$, the capital labor ratio $k$, the per-capita government debt $\omega$ and the per-capita stock of foreign reserves $\epsilon$). Note also that the static model is formulated in nominal terms while the dynamic model is formulated in real terms. As regards the results, the similarities and the differences between the two models can be examined by comparing the equilibrium price of the nontraded good in the static model given by (3.20) to the steady state price of the nontraded good given by (5.51). These two functions indicate that at the steady state, there is a positive relationship between the price of the nontraded good and the producer price $P_A$ and a negative relationship between the price of the nontraded good and the lump sum tax $t$. But several explanatory variables of the two functions
also differ, in particular, expression (5.51) indicates that the per-capita government debt \( o_i \), the per-capita time deposits, the interest rate on these deposits, the actual rate of change of the price of money in terms of good C and the rate of growth of the population are among the determinants of the price of the nontraded good at the steady state. These variables which are absent in (3.20) appear in (5.51) because they are determinants of the capital labor ratio \( k \). Assume that the static price of the nontraded good corresponds to the steady state level of this price. Then the capital labor ratio \( k \) and the money stock \( m \) which are held constant in (3.23) will no longer remain unchanged if the government engages in an expansionary fiscal policy or the Central Bank changes the interest rate on time deposits, etc. For instance, if the per-capita government debt \( o_i \) increases, say, following an increase in government spending on good C, the steady state value of the capital labor ratio \( k^e \) will rise while that of the per-capita stock of foreign reserves will fall. The increase in \( o_i \) will raise \( P^e_B \) as indicated by (5.51) and thus will increase the consumer price index and the per-capita gross national product at the steady state. In the static model, the increase in government spending on good C will have no effect on the price of the nontraded good because the government debt and the investment good do not accumulate so that the capital labor ratio \( k \) does not increase.
following a rise in $d_{CG}$. Thus the dynamic model gives a more complete description of the determinants of the price of the nontraded good and consequently those of the consumer price index and of the per-capita gross national product than the static model does.

This Chapter 5 concludes the theoretical models presented in this thesis. The next chapter deals with the estimation of the static model.
CHAPTER 6

ESTIMATION OF THE STATIC MODEL

First, the data base and the proxies are presented followed by the discussion of the empirical results obtained from estimating the static model.

The Data and the Proxies

The Sample Size

The empirical work is based on Togo's annual data. Most of the data used are published except for the last few years. The data for the more recent period was obtained through direct contact with persons working with the IMF, the BCEAO and the Direction de la Statistique du Togo. This study covers the period 1966 through 1977. The initial intention was to cover the period 1960 through 1976. This goal was altered due to the lack of price series prior to 1966. Indeed, although the base year for the price indexes used is 1963, for some unknown reasons, there are no published price data for 1964 and 1965 and all attempts to obtain the price data for these two years have failed. Therefore the scope of the empirical investigation is limited by this small sample size. For instance, the results for one of the
equations of the price of the nontraded good indicates that the error terms may be autocorrelated. A first order autocorrelation was assumed, with or without the explanatory variables lagged one year, and this equation was re-estimated. The D\textsubscript{W} test remained inconclusive following these attempts. Higher order autocorrelation or the use of weighted lagged values of the explanatory variables have not been tried because of limited number of degrees of freedom.

The Proxies

Data are needed for the following variables: the price of the pure export good \( P_\text{A} \), the price of the nontraded good \( P_\text{B} \), the price of the imported consumption good \( P_\text{C} \), the consumer price index \( P \), the per-capita domestic credit \( d_c \), the per-capita money stock \( m \), the per-capita lump sum personal tax \( t \), the capital labor ratio \( k \), the per-capita net flow of foreign reserves \( b_p \). The exchange rate has not been isolated because the data on the relevant series are available in FCFA.

The Price of the Pure Export Good \( P_\text{A} \). There are time series on the prices of major agricultural products exported by OPAT and time series on an index of all goods exported by Togo. The overall index has been used in the estimation for the following reasons. First, choosing the original series of cocoa and coffee would require constructing an aggregate
index which involves the prices and quantities of these goods. The data on these quantities and prices are available for harvest seasons which do not coincide with the calendar years. There is probably some measurement error in the data as it is and a conversion of this data from a harvest year basis into a calendar year basis will introduce additional error and it is unclear how all these errors will affect a composite index constructed from the original price-quantity data. Second, since cocoa and coffee had been the major exported goods for several years, their contribution is already reflected in the index of Togo's exports. In addition, although cocoa and coffee are the major export crops, a broad index which reflects the export of other crops also is probably more significant than a limited index which accounts for only cocoa and coffee. Third, some of the exogenous variables in the equations pertain to the entire economy. For instance, the money stock has a national dimension, not a sectorial dimension and using an overall index for exports as a proxy for the price of the pure export good may help to narrow the discrepancy between the dimensions of the variables. The data on the price index for exported goods, base 1970 = 100, come from Statistiques Économiques et Monétaires.
The Price of the Nontraded Good $P_B$ and the Consumer Price Index $P$. As mentioned in Chapter 1, the major food crops produced in Togo are maize, cassava, yam, millet and sorgho. There are time series for the prices and quantities of these crops and a time series for an index of food items which are locally consumed by Togolese. The latter index is also preferred to a composite index which might be constructed from price-quantity series of the food crops for the same reasons as those which led to the use of the price index for all exports as a proxy for the price of the pure export good. The data on the price index for food and the consumer price index are available in base 1963 = 100 and come from the Statistiques Economiques et Monetaires and from the Yearbook of Labor Statistics.

The Price of the Imported Consumption Good $P_C$. There are no price series on individual imported goods. But there is an incomplete series on the price index for fuel and a complete series on the price index for clothing. Both types of goods are among the goods listed in the surveys whose results are used to construct the consumer price index. The price index for clothing is used as a proxy for the price of the imported consumption good. This index should be fairly adequate if one recalls that between 1970 and 1976, cotton fabrics and threads were the most important imported good
(in value) in Togo. The data on the price index for clothing, base 1963 = 100, have been collected from the Statistiques Economiques et Monétaires and from the Yearbook of Labor Statistics.

The Per-Capita Lump Sum Tax $T$. The tax variable which appears in the model of this thesis is a lump sum tax levied on private individuals only as opposed to corporate profit tax and other direct taxes. But the time series for the lump sum tax which is available includes both personal income tax and corporate profit tax. So this combined tax series, which comes from the Statistical Yearbook, has been deflated by the total population series and used in the empirical work. The series on total population come from the Production Yearbook.

The Per-Capita Domestic Credit $dc$ and the Per-Capita Money Stock $m$. The total domestic credit corresponds to line 32 of the International Financial Statistics or IFS tables. The money stock, defined as the sum of the currency outside banks, demand deposits at commercial and development banks plus checking deposits at the post office, corresponds to line 34 of the IFS tables. Both series have been deflated by the total population to obtain their per-capita values.
The Capital Labor Ratio \( k \). There are no data on the capital stock of the agricultural sector nor on the capital stock of the entire economy. There is no information about the initial capital stock which can be used to construct a capital stock series using the data on investment. A time series on the output of electricity generated for domestic and industrial usage has been used as a proxy for the capital stock. This variable was deflated by the total population to obtain the capital labor ratio \( k \). However, this series has been deleted from the sets of explanatory variables used in the empirical work because it appears to be an inadequate proxy for the capital labor ratio and because of the limited sample size.

The Per-Capita Nominal Gross National Product \( y \). The time series on GNP evaluated at current market price is available through 1973 in the World Tables. The data on nominal GNP after 1973 have been obtained from an unpublished source.

The Per-Capita Balance of Payments \( bp \) and the Per-Capita Balance of Trade \( bt \). The proxy that has been used for the balance of payments is the sum of the balance of goods, services and transfers (line 77 a.d. of IFS tables), the net long-term capital flow (line 77 b.d. of IFS tables) and the net short-term capital flow (line 77 c.d. of IFS tables).
The published data are expressed in FCFA prior to 1968 and in US dollars after 1963. The exchange rate reported in the same tables (line ae) has been used in converting the series from dollar into FCFA. The data on the balance of trade come from the *Annuaire Statistique du Togo*.

One final observation: since there is no sterilization of foreign reserves, changes in the balance of payments will affect the flow supply of money because the net stock of foreign reserves is included in the monetary base. The OLS estimation technique used in this work does not yield consistent estimates if the dependent variable in one equation appears as explanatory variable in another equation. The presence of foreign reserves in the dependent variable in the equation for the balance of payments and in the set of explanatory variables as part of the flow supply of money may result in simultaneous equations error. It is in order to remove this source of error that the flow supply of domestic credit is used in place of the flow supply of money as an explanatory variable in the balance of payments equation.

In summary, the definitions of the variables used in this study are:

\[
P = \text{consumer price index (base 1963} = 100)\]

\[
P_A = \text{price index for all exported goods (a proxy for the unit price of the pure export good, base 1970} = 100)\]

\( P_B \) = price index for food (a proxy for the unit price of the nontraded good, base 1963 = 100)

\( P_C \) = price index for clothing (a proxy for the unit price of the imported consumption good, base 1963 = 100)

\( N \) = total population (in millions)

\( T \) = total personal income tax and corporate profit tax (a proxy for personal lump sum tax, in millions of FCFA)

\( DC \) = total nominal domestic credit (in millions of FCFA)

\( M \) = total nominal money stock (in millions of FCFA)

\( Y \) = total nominal GNP (in millions of FCFA)

\( BT \) = total nominal balance of trade (in millions of FCFA)

\( BP \) = total nominal balance of payments (in millions of FCFA).

Thus, all the per-capita variables are expressed in millions FCFA/million of inhabitants and the data on the endogenous variables \( P, P_B, y, bt, bp \) and the data on the exogenous variables \( P_A, P_C, dc, m, t \) are shown in Table 1.
Table 1. Data Used to Estimate Equations 1-11.

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* $P =$ consumer price index (base 1963 = 100)  
* $P_A =$ price index for exports (base 1970 = 100)  
* $P_B =$ price index for food (base 1963 = 100)  
* $P_C =$ price index for clothing (base 1963 = 100)  
* $y =$ per-capita nominal GNP (million FCFA/million inhabitants)  
* $bt =$ per-capita nominal balance of trade (million FCFA/million inhabitants)  
* $bp =$ per-capita nominal balance of payments (million FCFA/million inhabitants)  
* $dc =$ per-capita nominal domestic credit (million FCFA/million inhabitants)  
* $m =$ per-capita nominal money stock (million FCFA/million inhabitants)  
* $t =$ per-capita nominal lump sum tax (million FCFA/million inhabitants)  

Sources: See the section entitled "The Data and the Proxies" in Chapter 6.
The Empirical Results

Generalities

As mentioned earlier, the OLS technique is used to estimate all the equations. All the relevant variables are expressed in nominal terms instead of being expressed in real terms because there is no adequate deflator. No more than three explanatory variables have been used in each equation due to the few degrees of freedom. The tabulated values of the DW ratio assume a minimum sample size of 15 observations but the theoretical values corresponding to this minimum have been used as yardsticks in this study although the maximum number of observations is 12. The functional forms postulated are linear, inverse logarithmic and double logarithmic. A semi-logarithmic form has also been tried, but no regressions using this form are reported. No specific test to detect the presence of multicollinearity has been performed.

Several empirical studies about prices and imported inflation have been published recently [Laidler (1973); extensive general references on inflation in Laidler and Parkin (1975), Spinelli (1976), Kravis and Lipsey (1977), Parkin (1977), and others]. Brunner and Meltzer (1977, 1978) also studied the determinants of prices and of the gross national product using the United States and some European Common Market countries' data. The estimation of the balance of payments also has received a significant attention.
recently [see Zecher, Genberg, Bean and Guitian articles in Frenkel and Johnson (1976), S. Wilford (1977) and S. Wilford and W. Wilford (1978), etc.].

It is difficult, if not meaningless, to draw any conclusion from comparing ratios and coefficients estimated in models whose basic assumptions about the number of goods and assets, the behavior of the various economic agents, the market conditions (i.e., equilibrium or disequilibrium) differ. For instance, the balance of payments equations estimated by Zecher and Wilford is obtained in a straightforward manner from the equilibrium condition in the money market. But, in general, this market does not clear under fixed exchange rate and as pointed out by Kuska (1973) such an assumption has serious implications about the internal consistency and meaningfulness of the models in which the assumption is made. The implication of the difference between the basic assumptions of several models in the literature and those of the static model of this thesis is that the equations which are estimated in the models of the literature differ from those of the static model of this study. In addition, the proxies used, the sample sizes and often the functional forms of the models of the literature differ from those of the model estimated in this thesis. Therefore the data on the dependent variables and on the matrix of explanatory variables and thus the general statistics such as $R^2$, $F$ and DW ratios, the coefficients and the t
ratios of the empirical literature will differ from the data and the results of this study. For these reasons, the general statistics and the coefficients obtained from estimating the model of this study will not be compared with those obtained in the other models found in the literature.

The Results

The Determinants of the Equilibrium Price of the Nontraded Good $P_{B}^{e}$. It was mentioned in Chapter 1 that the prices of food crops rose 11% in 1974, 25.4% in 1975 and 17.7% in 1976. The price of cassava more than doubled between 1975 and 1976, from 13 FCFA/kg to 29 FCFA/kg. The price of red palm oil rose about 140% annually between 1973 and 1978. The model presented in Chapter 3 indicates that the price of food is positively related to the price of the exported good, the price of the imported good and the money stock, while it is negatively related to taxes. The results for estimating this relationship are equations 1, 2 and 3 in Table 2.

In all three regressions, the level of the $R_{s}^{2}$ indicates that at least 95% of the variation in the dependent variable is explained. The F ratios also show that, in each equation, the coefficients taken together are statistically different from zero at the 1% level of significance. Note, however, that the DW ratio falls into the determinant range in equation 1.
Table 2. Empirical Results.

<table>
<thead>
<tr>
<th>Equations and t-ratios</th>
<th>( R^2 )</th>
<th>F Ratio</th>
<th>DW Ratio</th>
</tr>
</thead>
</table>
| **The determinants of the equilibrium price of the nontraded goods**  
\( (P_B^e, 1966-77) \)                                                               |         |         |           |
| 1. \( P_B^e = -13.585 + .086 P_A(-1) + .975P_C + .0011m \) \(-1.431,N\) \(1.777,N\) \(7.146,Y\) \(.836,N\) | \(.986\) | 234.31  | 2.606 \(\text{IN}\) |  
| 2. \( P_B^e = .095P_A(-1) + .303P_C + .0024m \) \(1.374,Y\) \(11.671,Y\) \(2.347,Y\) | \(.984\) | 312.73  | 2.172 \(\text{NA}\) |  
| 3. \( LP_B^e = 3.993 + .000147P_A + .00564P_C + .0000144m \) \(36.682,Y\) \(.234,N\) \(3.857,Y\) \(.812,N\) | \(.951\) | 71.51   | 1.891 \(\text{NA}\) |  
| **The determinants of the equilibrium level of the consumer price index**  
\( (P_e, 1966-77) \)                                                                      |         |         |           |
| 4. \( P_e = 2.605 + .925P_C + .000914m \) \(.372,N\) \(9.638,Y\) \(.935,N\)            | \(.989\) | 495.23  | 2.180 \(\text{NA}\) |  
| 5. \( P_e = 3.246 + .062P_A(-1) + .861P_C + .000934m \) \(.532,N\) \(2.002,Y\) \(9.816,Y\) \(1.081,N\) | \(.992\) | 423.60  | 2.239 \(\text{NA}\) |  
| 6. \( LP_e = 3.805 + .00812P_C + .000027t(-1) \) \(40.245,Y\) \(10.341,Y\) \(-2.402,Y\) | \(.975\) | 195.59  | 2.472 \(\text{NA}\) |  

*Note: The table includes the t-ratios for each coefficient, with significance levels indicated in parentheses.***
Table 2. (continued)

<table>
<thead>
<tr>
<th>Equations and t-ratios</th>
<th>-</th>
<th></th>
<th>F Ratio</th>
<th>DW Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>The determinants of the equilibrium level of nominal gross national product ((y_e^{(1966-77)})).</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7. (y_e = 6694.593 + 163.752P_{(1)} + 2.583m - 1.236t_{(-1)})</td>
<td>.965</td>
<td>92.91</td>
<td>1.579 (NA)</td>
<td></td>
</tr>
<tr>
<td>(1.001,N) (2.266,Y) (5.633,Y) (-2.506Y)</td>
<td></td>
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<tr>
<td>8. (Ly_e = 5.081 + 0.407P_{(1)} + 0.414m - 0.082t)</td>
<td>.993</td>
<td>483.87</td>
<td>1.622 (NA)</td>
<td></td>
</tr>
<tr>
<td>(9.888,Y) (2.610,Y) (13.124,Y) (-3.374,Y)</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td><strong>The determinants of the equilibrium level of the balance of trade ((bt_e^{(1966-77)})).</strong></td>
<td></td>
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<td></td>
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<tr>
<td>9. (bt_e = -1228.255 + 76.343\Delta P_A - 147.114\Delta P_C - 0.604\Delta dc)</td>
<td>.841</td>
<td>18.64</td>
<td>2.309 (NA)</td>
<td></td>
</tr>
<tr>
<td>(-1.467,N) (4.803,Y) (-1.762,N) (-0.999,N)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>The determinants of the equilibrium level of the balance of payments ((bp_e^{(1966-75)})).</strong></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>10. (bp_e = 636.713 + 35.054\Delta P_A - 44.284\Delta P_C - 0.110\Delta dc)</td>
<td>.839</td>
<td>13.16</td>
<td>2.335 (NA)</td>
<td></td>
</tr>
<tr>
<td>(1.235,N) (3.964,Y) (-0.750,N) (-0.234,N)</td>
<td></td>
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<tr>
<td>11. (bp_e = 584.790 + 32.451\Delta P_A - 28.247\Delta P_C - 0.772\Delta t)</td>
<td>.838</td>
<td>13.11</td>
<td>2.145 (NA)</td>
<td></td>
</tr>
<tr>
<td>(1.492,N) (2.696,Y) (-0.361,N) (-0.206,N)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\* \(N = \) coefficient not statistically significant at 5% or 1% levels  
\(Y = \) coefficient is statistically significant at 1% level  
\(NA = \) no autocorrelation of the error terms  
\(INC = \) DW test is inconclusive  
\(\Delta = \) change in the variable between two consecutive periods  
The letter "L" in from a variable means LOGARITHM.
A rise in the price of foreign goods affects the budget constraint of the private sector and thus the private sector's demands for goods B, C and money. It was assumed that an increase in the price of the imported good, ceteris paribus, tends to raise the demand for the nontraded good B. This assumption implies that a rise in the price of the imported consumption good will raise the equilibrium price of the nontraded consumption good. The coefficient of the price of the imported good is positive and is statistically significant at 1% level in all three equations. Hence, if the importers of the consumption goods raise their prices exogenously in response to government policies, or in response to higher supply costs, these price increases will generate higher prices for the locally produced and consumed goods. The coefficients of the price of the exported goods are not statistically significant in equations 1 and 3. However, when the constant term, which is insignificant, is suppressed, the one-year lagged price index for exports becomes statistically different from zero at the 5% level.

Laidler and Parkin (1975, p. 741) quotes Friedman as having written: "Inflation is always and everywhere a monetary phenomenon . . . and can be produced only by a more rapid increase in the quantity of money than in output."
The empirical results obtained in this study do not fully support this claim if it is interpreted as saying that the money stock is the major determinant of prices. Indeed, the coefficient of the money stock is not statistically different from zero in equations 1 and 3, but it is statistically significant at the 5% level when the constant term is deleted from equation 1. This means that, in the short run, the money stock may affect the price of the nontraded good, but the impact is not as strong as Friedman and his followers suggest. The constant term is not statistically significant in equation 1, but it is significant at the 1% level in equation 3 using one-tailed test. A one-tailed test is used because the dependent variable is a price index, and it seems reasonable to expect this coefficient to be positive. A negative constant term in a price equation, as in regression 1, would be difficult to interpret.

Which one of the three equations "best" describe the equilibrium price structure of the nontraded good? A visual inspection of the equations as they stand suggests that equation 1 should be eliminated because of its negative constant term and that equation 3 should be abandoned because it has fewer statistically significant coefficients than equation 2. We are thus left with equation 2 to describe \( P_B^e \). However, other factors such as "how well" each equation forecasts \( P_B^e \) will be taken into account in Chapter 7 to select the equation which "best" describes \( P_B^e \).
The Determinants of the Equilibrium Level of the Consumer Price Index \( P_e \) or \( CPI_e \). The CPI rose at an annual rate of 12.8% in 1974, 13.06% in 1975, 11.64% in 1976 and 30% in the first quarter of 1977. The model developed in this study proposes to explain and estimate the level of this index when the market for the nontraded good clears. The results of the empirical exploration are summarized in equations 4, 5 and 6 of Table 2.

The \( R^2 \) for these equations suggest that the functional forms postulated fit the data fairly well. The F ratios also indicate that the set of regression coefficients are statistically different from zero at the 1% level. The DW ratios indicate that the error terms are not autocorrelated.

It was mentioned in Chapter 1 that cotton fabrics and threads constituted the most important imported bundle of goods over the period 1960-76. When the price of these materials rise, they will increase the \( CPI_e \) by an amount which is statistically significant at the 1% level. This means that, for instance, if the UAC (a subsidiary of Unilever) boasts the prices of its printed fabrics imported from Holland or Japan and this results in a 10% increase in the price index for clothing, the impact on the \( CPI_e \) may be as high as 9.25% (equation 4). It should be noted that the difference between the actual weight in the construction of
the CPI which is 7.7 on a scale of 0 to 100 and the regression coefficients is due to the fact that the CPI is being estimated in equilibrium while its other definitional components are excluded. Since it is the equilibrium expression of the CPI which is being estimated, the findings of this study indicate that the price index for clothing is the major determinant of the CPI and they tend to support the theory of imported inflation.

The model developed in Chapter 3 suggests that there is a positive relationship between the price of a pure export good $P_A$ and the equilibrium price of the nontraded good $P_B^e$ and since the CPI is positively related to $P_B^e$, $P_A$ is also positively related to the CPI. The empirical results of this thesis indicate that the CPI bears a positive and statistically significant relationship at the 5% level to the lagged price index for exports in equation 5. Note that the coefficient of the price index for clothing $P_C$ becomes smaller when $P_A(-1)$ was used as one of the explanatory variables (see equation 5) than when $P_A(-1)$ was excluded (see equation 4). The larger size of the coefficient of $P_C$ in equation 4 seems to indicate that in this equation 4, $P_C$ might have been explaining part of the variation in the CPI which could be attributed to $P_A(-1)$. Changes in the money stock $m$ and in the lump sum tax $t$ affect consumers' budget constraint and in turn
influence their demand for the nontraded good and the price of this good. The ultimate result is that an expansionary monetary or fiscal policy will tend to increase the equilibrium price of the nontraded good and thus the CPI\(e\). The empirical findings reported here confirm the positive effect of an increase in m on CPI\(e\) but the coefficients are statistically negligible and thus Friedman's claim reported above does not seem to be supported by these results either. Equations 4 and 5 were re-estimated using one-year lagged values of the money stock but there was no noticeable improvement in the t ratios. Had the degrees of freedom permitted, these equations would have been re-estimated using weighted lagged values of m. Unlike regression 1 which has a negative constant term, all the constant terms in the CPI\(e\) equations are positive, although that of the linear equation 4 is not statistically different from zero.

As regards the equation that "best" describes the equilibrium level of the CPI, one might be tempted to eliminate equation 4 in favor of equation 5 and/or equation 6 on the grounds that equation 4 contains only one statistically significant coefficient. However, all three equations will be retained and used to forecast the CPI and the equation which gives more plausible projections will be selected.
The Determinants of the Equilibrium Level of the Gross National Product $y^e$ or GNP$^e$. The results obtained from estimating $y^e$ are equations 7 and 8 in Table 2. The $R'^2$ indicate that the linear and the double logarithmic functional forms fit the data very well. The F ratios also show that the sets of coefficients in each equation are statistically different from zero at the 1% level. The DW ratios indicate no autocorrelation in the error terms at the 1% level in both regressions.

The model developed in Chapter 3 shows that the equilibrium GNP, $y^e$, increases as the price of the pure export good $P_A$, the price of the imported good $P_C$ or the money stock $m$ rises, while it falls when the lump sum tax $t$ increases. These qualitative theoretical results are confirmed in the empirical investigation. Indeed, the coefficients of the price index for exports $P_A$ are positive, but are statistically insignificant in the regressions containing the four explanatory variables, $P_A$, $P_C$, $m$ and $t$. However, $P_A$ has been deleted because of limitations on the number of degrees of freedom. The lagged price index for clothing $P_C(-1)$ exerts a qualitatively significant effect on nominal GNP$^e$ at the 5% level, but the elasticity of $y^e$ with respect to $P_C(-1)$ which is equal to .405 in equation 8 seems rather small. The coefficient of the tax variable $t$ is also statistically significant at the 5% level. Based on the size of the t ratios, the current money stock appears
to be the major determinant of the nominal GNP when the market for the nontraded goods and other markets clear. Indeed, the coefficient of m is significant at the 1% level in both equations. In a discussion of the relationship between money supply, price and nominal and real income, Friedman has written: "I regard the description of our position as 'money is all that matters for changes in nominal income and for short run changes in real income' as an exaggeration but one that gives the right flavor of our conclusions" [Friedman, 1970, p. 217]. The empirical impact of m on ye mentioned above lends some support to Friedman's claim that money is perhaps the major determinant of nominal income. It is also interesting to note that in most of his writings, Friedman associates inflation primarily with monetary factors. Tog's data indicate that the price of imported goods, not money, is the major determinant of the equilibrium consumer price index. The constant terms are positive as expected, but only the one in the double logarithmic function is statistically significant at the 1% level.

The Determinants of the Balance of Payments bp

It was shown in the static model that when the market for the non-traded good clears, the following relationship will exist:
The results obtained from estimating this equation are regressions 9, 10 and 11. In regression 9, the dependent variable is the per-capita balance of trade and the period covered is 1966 through 1977 while in regressions 10 and 11, the dependent variable is the per-capita balance of payments and the period covered is 1966 through 1975 because there are no data on capital movements after 1975 and the structure of the data on capital flows permits no plausible extrapolation.

The \( R^2 \)'s for equations 9, 10 and 11 are slightly smaller than those recorded for the previous regressions but they do indicate that at least 88% of the variation in the dependent variables are explained. The F ratios are substantially smaller than those obtained in the equations 1 through 8 but they suggest that the coefficients taken together are statistically nonzero at the 1% level. The DW ratios also indicate that the error terms of the dependent variable are free of autocorrelation at the 1% level.

The empirical results confirm the theoretically positive relationship between the balance of payments (or
the balance of trade) and the change in the price index of exports. Furthermore, the coefficient of $\Delta P_A$ is statistically significant at the 5% level in all three equations. The negative relationship between the balance of payments and the change in the income tax is also confirmed empirically, but the coefficient of $\Delta t$ in equation 11 is statistically insignificant at the 5% level. As regards the theoretically ambiguous relationship between the balance of payments $bp^e$ or the balance of trade $bt^e$ and the change in the price of the imported good $\Delta P_C$, the empirical results indicate that this relationship is negative. Yet, the coefficient of $\Delta P_C$ is statistically insignificant in all three equations using a two-tailed test. Recall that the balance of payments expression is

$$bp^e = \Delta l - \Delta m$$

(3.37)

and the demand for money function is:

$$\lambda^e = \lambda^e (P_A^e, P_B^e, P_C^e, m, t, k)$$

(3.22)

where $P_C = (1 + t_C)eP_C^*$ and $P_B^e$ is (3.23). From these expressions, one can obtain:

$$\frac{\partial bp^e}{\partial \Delta P_C} = \frac{\Delta \lambda}{\partial \Delta P_C} \frac{\partial \lambda^e}{\partial \Delta P_C^*} + \frac{\partial \lambda}{\partial \Delta P_B^e} \frac{\partial P_B^e}{\partial \Delta P_C^*} + \frac{\partial \lambda}{\partial \Delta P_C^e} \frac{\partial P_C^e}{\partial \Delta P_C^*}.$$
In expression (3.39) which represents the net effect of $\Delta P^*_C$ on $bp^e$, it is assumed that the nontraded consumption good B and the imported consumption good C are gross substitutes so that $\partial \Delta P^e_B / \partial \Delta P^*_C > 0$. The ambiguity of the sign of $\partial \Delta x^e / \partial \Delta P^*_C$ is related to the structure of the utility function and the prices of the imported and nontraded goods (see Appendix E). However, one can imagine circumstances under which the quantity $\partial bp^e / \partial \Delta P^*_C$ is negative. Indeed, money is one of the very few liquid assets available to wealthowners in Togo and for farmers and most of the urban population, it is perhaps the only liquid asset. When farmers, self employed and salaried workers have inflationary expectations, as is likely in Togo, they will tend to buy imported goods in anticipation of future higher prices. That is, they will tend to reduce their net cash holdings over time (i.e., $\partial \Delta x^e < 0$) when the spread of the price of foreign goods rises (i.e., $\partial \Delta P^*_C > 0$), so that the quantity $\partial \Delta x^e / \partial \Delta P^*_C$ is negative. In Togo, generally consumers do not hold large and increasing amounts of money for the purpose of buying nontraded goods, and it is possible that the term $\partial \Delta x^e / \partial \Delta P^*_B$ be relatively small. Under such circumstances, the quantity $\partial bp^e / \partial \Delta P^*_C$ will be negative in expression (3.39) because $\partial \Delta x^e / \partial \Delta P^*_C$ is negative while $\partial \Delta x^e / \partial \Delta P^*_B$ is small. Such a scenario would be consistent
with the negative sign of $\Delta P_C$ obtained in all three equations. The theoretical effect of an increase in the flow supply of the domestic credit $\Delta dc$ on the balance of payments $bp^e$ is ambiguous as it can be observed from the following expression:

$$\frac{\partial bp^e}{\partial \Delta dc} = \frac{\partial \Delta \ell^e}{\partial \Delta dc} + \frac{\partial \Delta \ell^e}{\partial \Delta P_B^e} \frac{\partial \Delta P_B^e}{\partial \Delta dc} - 1. \quad (3.40)$$

Togo's data indicate that an increase in $\Delta dc$ will tend to deteriorate the balance of payments, although the coefficients of $\Delta dc$ in equations 9 and 10 are not statistically significant at the 5% level. In the scenario suggested above, it is mentioned that it is possible that $\frac{\partial \Delta \ell^e}{\partial \Delta P_B^e}$ is very small and the presence of such a circumstance will reinforce the possibility that $\frac{\partial bp^e}{\partial \Delta dc}$ is negative. Note that the constant form is negative in the balance of trade equation 9 while it is positive in the balance of payments equations 10 and 11, but this may be due to the fact that the dependent variables which are being explained are different.

The single most important determinant of the balance of trade and of the balance of payments is the price index.

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7. It might be mentioned in passing that in the model used by Zecher [see Frenkel and Johnson (1976)] and Wilford (1977), changes in the flow supply of domestic credit have no effect on the flow demand for money while they affect the flow supply of money and these authors found that there is a negative relationship between the balance of payments and the flow of domestic credit.
of exports. This result is consistent with the fact that the Togolese economy is an export-oriented economy in which essentially the entire outputs of cocoa, coffee and phosphate are exported.

The balance of trade equation has been reported for comparison purposes and also because it may be considered as a first approximation of the balance of payments. A visual inspection of equations 10 and 11 gives no strong clue as to which equation "best" describes the balance of payments.

In the next chapter, the empirical results reported here will be used to make some projections of the price of the nontraded good, the consumer price index, the per-capita gross national product, the per-capita balance of trade and the per-capita balance of payments.
CHAPTER 7

CONCLUSION

The purpose of this thesis has been fourfold: (1) to attempt to develop a static model to explain the determinants of the price of nontraded goods, the consumer price index, the gross national product and the balance of payments; (2) to present a dynamic model which could lead to the determination of the producer price of export crops by OPAT; (3) to develop a dynamic model to obtain, at the steady state, the determinants of the price of nontraded goods, the capital labor ratio, the per-capita government debt and the per-capita net stock of foreign reserves, and (4) to estimate endogenous variables of the static model. To end this work, some projections which are based on the equations estimated in Chapter 6 are now discussed. The first section briefly describes the projected series of the exogenous variables while the second section deals with the projections of the endogenous variables.

The Projections of the Exogenous Variables

These exogenous variables are the price index of exports $p_A$, the price index for clothing $p_C$, the total
domestic credit DC, the total money stock M, the total lump sum tax T and the total population TPOP. The projection period is 1978 through 1985 and the basic procedure to obtain the projected values of the exogenous variables is to apply a certain rate of growth to these variables based on their 1977 values. The rate of change of the export price index $P_A$, the domestic credit DC, the money stock M and the lump sum tax T are obtained in the same manner, so that it will suffice to indicate how the rate of change of $P_A$ is arrived at. First, the rate of change of $P_A$ is computed for each year over the period 1966-77 as $RCP_{Ai} = [P_{Ai} - P_{Ai}(-1)]/P_{Ai}(-1)$ for $i = 1967-1977$ (11 years in total). Second, the quantity $\sum_{i=1}^{10} RCP_{Ai} = .5523$ is computed except for $i = 1974$. The year 1974 is isolated because the phosphate mining company called CTMB was nationalized in 1974 and then the government raised sharply the export price of phosphate. This increase in the price of phosphate affected substantially the price index of exports which rose 154.79% with respect to their 1973 level. The phosphate price has declined ever since and this writer does not anticipate a rise in the export price of phosphate similar to the 1974 increase nor any dramatic increase in the export prices of cocoa and coffee. For this reason, a higher weight of .75 is applied to the sum of the rates of growth of $P_A$ for the period 1966-77 (except 1974) which is equal to 55.23% while
a weight of .25 is applied to the rate of growth of $P_A$ of 154.79% recorded for 1974 alone. The average rate of growth applied to 1977 value of $P_A$ in order to obtain future values is then $(.5523 \times .75 + 1.5479 \times .25)/11 = 7.28\%$. The nationalization of the phosphate company and substantial increases in the export prices of cocoa and coffee in 1976 probably contributed to the sharp rates of growth in 1974 of the money stock (116.95%) and in 1975 of the domestic credit (89.11%) and the direct taxes (444.91%). Following a computation procedure similar to the one described above for $P_A$ and weighting the pre-1974 period more heavily, the annual rates of growth for the total money stock, the total domestic credit and the total direct taxes obtained were respectively 11.33%, 17.24% and 25.77%. The annual rates of change of the price index for clothing were weighted equally to obtain 8.83% while the average annual growth rate of the total population between 1970 and 1975 which is equal to 2.6% was used to forecast the population series.

In summary, it is anticipated that the price index for exports $P_A$, the price index for clothing $P_C$, the total domestic credit $DC$, the total money stock $M$, the total income tax $T$ and the total population $TPOP$ will grow annually at the respective rates of 7.28%, 8.83%, 17.24%, 11.33%, 25.77% and 2.6%. These rates have been applied to the 1977 data to
obtain projections for the period 1973-85 for the export price index $P_A$, the clothing price index $P_C$, the per-capita money stock $m$, the per-capita domestic credit $dc$ and the per-capita lump sum tax $t$. These series are shown in Table 3. They have been used together with the equations estimated to compute the future values of the endogenous variables.

Projections of the Endogenous Variables

These endogenous variables are the equilibrium price of the nontraded good, the equilibrium consumer price index, the equilibrium GNP and the equilibrium net flow of foreign reserves.

Estimates of the Price Index for Food

The projected series of the price index for food, based on equations 1, 2 and 3 of Table 2, are listed in Table 4 respectively as $\hat{P}_{B1}$, $\hat{P}_{B2}$, $\hat{P}_{B3}$. Equations 1 and 2 in Table 2 differ only by the constant term. This constant term is suppressed in equation 2 because it is negative in equation 1 and one should note that the combined positive effect of the price index for exports, the price index for clothing and the money stock exceeds the negative value of the constant term so that $\hat{P}_{B1}$ is positive. The estimates of
Table 3. Projections of the Exogenous Variables.

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<tbody>
<tr>
<td>$\tilde{P}_A$</td>
<td>250.9</td>
<td>269.2</td>
<td>283.8</td>
<td>309.9</td>
<td>332.5</td>
<td>356.8</td>
<td>382.8</td>
<td>410.8</td>
</tr>
<tr>
<td>$\tilde{P}_C$</td>
<td>269.4</td>
<td>293.1</td>
<td>318.9</td>
<td>346.9</td>
<td>377.5</td>
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* The tilde "-" identifies the projections of an exogenous variable.
Table 4. Projections of the Endogenous Variables.

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* The hat "$\hat{\cdot}\$" identifies the projections of an endogenous variable.
the price index for food $\hat{P}_{B3}$ obtained from the inverse logarithmic equation 3 do not seem plausible.

In Chapter 6, in the discussion of the equation which "best" describes the price of the nontraded good, it was suggested that the inverse logarithmic equation 3 be eliminated because it contains only one statistically significant coefficient apart from the constant term. This suggestion is maintained here on the additional ground that the projections $\hat{P}_{B3}$ which are based on this equation are difficult to believe. It was also suggested that equation 1 may be deleted because of the negative constant term and because it contains only one statistically significant coefficient. In view of the projections $\hat{P}_{B1}$ obtained from this equation 1, I am less certain that this equation be abandoned for the reasons mentioned above and I am inclined to think that the projections of the price index for food $\hat{P}_{B1}$ and $\hat{P}_{B2}$ which are based on equations 1 and 2 are plausible.

**Estimates of the Consumer Price Index**

The projections of the consumer price index CPI based on equations 4 and 5 of Table 2 are reported in Table 4 as $\hat{P}_4$ and $\hat{P}_5$. The projections of the consumer price index which are based on equations 4, 5 and 6 in Table 2 are reported in Table 4 as $\hat{P}_4$, $\hat{P}_5$ and $\hat{P}_6$. One may notice that,
as indicated in Table 3, the projected values of the per-capita lump sum tax (\( t \)) are substantially larger than those of the price index for clothing (\( P_c \)). However, the coefficient of the tax variable and that of the price index for clothing as reported in equation 6 of Table 2 are such that the combined positive effect of the price index for clothing and the constant term exceeds the negative effect of the lump sum tax, so that the projections of the consumer price index \( \hat{P}_6 \) are positive.

The projected series \( \hat{P}_4, \hat{P}_5 \) and \( \hat{P}_6 \) indicate an annual growth rate of the consumer price index (CPI) respectively equal to 11.4%, 11.2% and 22.1% over the projection period. A comparison of these rates to the 12.25% annual growth rate of the CPI recorded over the estimation period tends to indicate that the inverse logarithmic function overestimated the CPI for the period 1978-85. It was mentioned in Chapter 6 that one might delete equation 4 in favor of equation 5 and/or 6 because equation 4 contains only one statistically significant coefficient. The fact that equation 6 leads to a projected series \( \hat{P}_6 \) which is a little difficult to believe leads me to revise my earlier suggestion and to consider equations 4 and 5 as the equations which "best" describe the CPI. This means, based on the model of this thesis and on Togo's data, the series \( \hat{P}_4 \) and \( \hat{P}_5 \) may indicate the future trend of the CPI.
Estimates of the Nominal Gross National Product

The projected series of the per-capita nominal GNP obtained from equations 7 and 8 in Table 2 are respectively \( \hat{y}_7 \) and \( \hat{y}_8 \) in Table 4. The combined positive effect of the price index for clothing plus the money stock and the constant term exceeds the negative effect of the tax variable so that the projections \( \hat{y}_7 \) and \( \hat{y}_8 \) are all positive.

A visual inspection of equations 7 and 8 in Table 1 does not permit to isolate the one which "best" explains the per-capita GNP. However, a closer examination of \( \hat{y}_7 \) and \( \hat{y}_8 \) in association with the original data \( y \) in Table 1 leads this writer to believe that the double logarithmic regression 8 underestimated the nominal GNP. For this reason, the projections \( \hat{y}_7 \) which are based on equation 7 are considered to be far more plausible than the projected series \( \hat{y}_8 \).

Estimates of the Balance of Trade and the Balance of Payments

The series \( \hat{b}_t \), \( \hat{b}_p \), \( \hat{b}_p^* \), and \( \hat{b}_p^\dagger \) in Table 4 represent the projections of the per-capita balance of trade and the per-capita balance of payments based on equations 9, 10 and 11 in Table 2. A comparison of the projections \( \hat{b}_t \), \( \hat{b}_p \), \( \hat{b}_p^* \) and \( \hat{b}_p^\dagger \) to the sample data \( b_t \) and \( b_p \) shown in Table 1 reveals noticeable differences between the raw series and the estimated series. Indeed, the actual balance of payments series \( b_p \) has been in deficit 7 years out of 11 while the projected balance of payments series show a deficit
throughout the forecast period. The signs of the raw and projected series on the balance of trade (i.e., surplus/deficit) are quite similar. The size of the deficits indicated by the projections, in particular $\hat{bp}_{11}$, are quite disturbing. The persistent deficits of the projected balance of payments and their disturbing size may be interpreted in various ways. One possible explanation is that the model formulated in this study is seriously deficient and inadequate in terms of being used to estimate the balance of trade and the balance of payments. It would be pretentious on my part to rule out this explanation as a remote possibility. It is also conceivable that the theory and the equations estimated are basically adequate but that the rates of change applied to the exogenous variables are too high or too small so that the forecasts of $P_A$, $P_C$, $m$ and $t$ are biased upward or downward and therefore the projections are seriously incorrect.

A third possibility may be that the theoretical model, the empirical results and the projections of the exogenous variables are adequate, so that the negative trends indicated by the projections $\hat{bt}_9$, $\hat{bp}_{10}$ and $\hat{bp}_{11}$ may occur unless some measures are taken to alter these trends. Let us comment briefly about this third possibility. The country of Togo which this model concerns cannot realistically engage in policies which result in sustained deficits
in the balance of trade and in the balance of payments.
Why? First, because such deficits will seriously reduce
Togo's ability to import goods and services and to obtain
foreign loans and second, because the statutes of the monetary
union to which she belongs do not allow member countries to
place the balance of payments of the union in a deficit
position. If such a situation occurs and Togo is one of
the countries which brought it about, severe austerity
measures will be imposed on Togo by the union in order to
remedy the deficits. For these reasons and assuming that
the above projections are any indication of the trends in
the per-capita balance of trade and in the per-capita
balance of payments, some fundamental changes must occur
to reduce the deficits.

One such change would be to engage in policies which
result in a wider spread of the price index for the exported
goods cocoa, coffee and phosphate. Togo is too small to
affect significantly the world prices of these goods. Yet,
she can engage in policies aimed toward improving the quality
of the goods exported so that she can sell them at prices
higher than those which she will receive otherwise. A
greater spread of the export price index \( \Delta P_A \) will have a
positive effect on the balance of trade and on the balance
of payments, but it should be mentioned also that the higher the price index for exports, the higher will be the price of the nontraded good and the higher will be the consumer price index (see equations 1, 2, 4 and 5 in Table 2). Other possible structural changes which Togo might pursue would be to import consumption goods whose prices do not increase too much over time, or the government should not increase the import tariff rate too much so that \( \Delta P_C \) is relatively small. It must be pointed out that a policy of smaller import tariff rates may be difficult to follow in Togo because the government derives a substantial part of its revenues from import tariffs. Independently of how the small change in the import price \( \Delta P_C \) is brought about, such a change will tend to reduce the balance of payments deficit. A policy designed to maintain import prices constant (i.e., \( \Delta P_C = 0 \)) or to permit them to rise only very moderately will also tend to maintain the price of food and the consumer price index unchanged or to make them rise only moderately. However, such a policy will also tend to leave unchanged or to reduce the equilibrium level of the GNP (see equations 1, 2, 4, 5 and 7 in Table 2).

As regards changes coming from the monetary sector, if the Central Bank decides to decrease substantially the flow
supply of domestic credit between periods, for instance, by acquiring fewer custom duty bills, equations 9 and 10 indicate that such a policy will improve the balance of trade and the balance of payments. This negative effect of $\Delta dc$ on the two balances is generally accepted in the international economics literature. Indeed, it is believed that a contractionary monetary policy tends to reduce the demand for goods and assets relative to their supplies so that their prices decrease. This relationship appears to hold in equations 1, 2, 4 and 5 because the money stock $m$ and the price of the nontraded good $P_B^e$ and the consumer price index $P_e^e$ are positively related. Given the fixed exchange rate, consumers will tend to buy more cheaper home goods instead of their imported substitutes and this change in the pattern of spending reduces the demand for foreign reserves over time.

Finally, as regards the effect of the income tax variable on the balance of payments, this balance may improve if the government reduces the flow of tax revenues levied on workers' income and on corporate profits (see equation 11).

It follows from the above discussion that if the balance of trade equation 9, the balance of payments equations 10 and 11 and the projected values of the exogenous
variables $P_A, P_C, dc, m$ and $t$ are fairly adequate, then some structural changes must occur to reduce the projected deficits of the balance of trade and of the balance of payments. For instance, if these changes result in an increase in the spread of the price index for exports while the price index for clothing, the domestic credit, the money stock and direct taxes remain constant or rise only moderately, then the balance of trade and the balance of payments will improve over time. Yet, it must be noted that under these circumstances, a sharp increase in the price index for exports and moderate increases in the price index for imports and in the money stock will also raise the price of the nontraded good, the consumer price index and GNP while an increase in direct taxes will tend to reduce GNP. This means the same policy may have a favorable effect on some endogenous variables while exerting an unfavorable effect on others. For example, an increase in the spread of the export price index has a favorable effect on the balance of trade and on the balance of payments, but it will also increase the price of the nontraded good and the consumer price index.

In conclusion, let us make a couple of observations. First, the model developed in this thesis has permitted us to identify the determinants of the price of the nontraded good $P^e_B$, the consumer price index $P^e$, the per-capita nominal gross
national product \( y^e \) and the per-capita balance of payments \( bp^e \) and to estimate these functions. The projections of the price index for food \( \hat{P}_{B1} \) and \( \hat{P}_{B2} \), those the consumer price index \( \hat{P}_4 \) and \( \hat{P}_5 \) and the projections \( \hat{y}_7 \) of the per-capita nominal gross national product which are based on some of the estimated functions are fairly plausible. A similar statement cannot be made about the projections of the per-capita balance of payments.

Second, the current study may be expanded along several lines. For instance, although agriculture is the major economic sector in Togo, it is not the only production sector. Trade, mining and manufacturing are playing an increasingly important role and this role should receive more attention. It is possible, if not likely, that the behavior of the economic agents involved in these sectors will affect the domestic prices and the gross national product (GNP) and the balance of payments. Policy makers will be better equipped to make decisions if one can identify the relative effect on prices, GNP and the balance of payments of variables which are specific to the other sectors. Another possible topic for further research may be the behavior of OPAT. Throughout this thesis, it has been assumed implicitly that OPAT knows the world price of the exported good \( A \) with certainty and that perfect foresight prevails. One might maintain the perfect foresight assumption, but now, consider
the world price of good A and consequently the producer price as a random variable with a known distribution function. It has also been assumed that the government repays its debts to OPAT and it may be interesting to study the case in which these debts are not repaid, but are treated by the government as additional taxes levied on OPAT and ultimately on farmers. In the models of this thesis, taxes and the nominal interest rates on time deposits and bank loans have been treated as exogenous or target variables. Perhaps, it might be worthwhile to treat them as endogenous variables because income is endogenous in the models and in general, taxes vary with income, and because the Central Bank takes into consideration the economic conditions in Togo in determining the interest rates. The static model of Chapter 3 is formulated in nominal terms and a possible extension would be to formulate it in real terms and compare the results.

As regards the empirical investigation, more attention should be devoted to finding ways to improve the estimation and predictive ability of the balance of payments equations. Perhaps, one should attempt to find more than one proxy for each explanatory variable in these equations and examine the sensitivity of the empirical results to the various proxies. It is also possible that political factors, unpredictable events such as flood or a severe draught may affect the flow demand for/supply of foreign reserves. These phenomena should be taken into account if it is possible to
do so. For policy purposes, the empirical investigation may be extended to cover the estimation of the demand for/supply of the exported good, the nontraded good and money and the price elasticities of these functions. Further research is also needed to determine whether there are some other African countries whose economic structure is quite similar to that of Togo and to which the model of Chapter 3 or its modified version can be applied and estimated. Perhaps, the results from such a study may lead to some meaningful, if not useful, comparisons.
APPENDIX A

DEMAND FOR GOODS A BY OPAT

Under normal circumstances, the government agency OPAT has been observed to purchase the entire output of the pure export crops made available to it by the farmers. The purpose of this appendix is to show that this behavior is consistent with the monopsony profit maximizing position of OPAT. To this end, let:

\[ Q_A = \text{output of goods A made available for sale by farmers} \]
\[ Q^*_o = \text{amount of output A bought by OPAT} \]
\[ P^*_A = \text{world unit price of goods A received by OPAT (in foreign currency)} \]
\[ P_A = \text{producer price of A (i.e., price paid by OPAT to farmers, in home currency)} \]
\[ e = \text{exchange rate} \]

Assume that OPAT pays a proportion \( \alpha \) less than unity of the world price to the farmers (i.e., \( P_A = \alpha e P^*_A \), \( 0 < \alpha < 1 \)). Since OPAT is a monopsonist, the producer price \( P_A \) which it pays will depend on the amount of goods A which it buys (i.e., \( P_A = P_A (Q^*_o) \)). Furthermore, since OPAT's exclusive activity is to market goods A, it is reasonable
to assume that it will always buy a positive amount of the supply of A. The purpose of this appendix is to show that \( Q_A \) is equal to \( Q_A^o \) if OPAT maximizes profit.

The problem may be written as:

\[ \text{maximize } \Gamma = e P_A^* (Q_A^o) Q_A^o - P_A Q_A^o \text{ over } Q_A^o \]

subject to

\[ 0 < Q_A^o < Q_A^o, \ P_A = \alpha e P_A^* (Q_A^o). \]

The first constraint means that OPAT must buy a nonnegative quantity of goods but this amount cannot exceed the available supply. In what follows, we shall assume that the problem has a solution at a positive value of \( Q_A \). Consequently, the constraint \( Q_A^o > 0 \) is ignored and only the constraint \( Q_A^o \leq Q_A \) will be taken into account explicitly.

The Lagrangian may be written as:

\[ Z = e P_A^* (Q_A^o) Q_A^o - \alpha e P_A^* (Q_A^o) Q_A^o + \nu [Q_A - Q_A^o] \]

where \( \nu \) is the Langrange multiplier. Since it is assumed that in equilibrium \( Q_A^o > 0 \), the Kuhn-Tucker conditions are:
\[ \frac{\partial Z}{\partial Q_A} = 0 \]

\[ \frac{\partial Z}{\partial \nu} \geq 0, \quad \nu \geq 0, \quad \nu \frac{\partial Z}{\partial \nu} = 0. \]

Differentiate the Lagrangian to obtain:

\[ \frac{\partial Z}{\partial Q_A} = (1-\alpha) e^{\frac{\partial P_A^*}{\partial Q_A}} Q_A^o + P_A^* - \nu = 0 \]

\[ \frac{\partial Z}{\partial \nu} = Q_A - Q_A^o \geq 0, \quad \nu \geq 0, \quad \nu \frac{\partial Z}{\partial \nu} = 0. \]

The quantity \( \frac{\partial P_A^*}{\partial Q_A} Q_A^o + P_A^* \) is positive because it corresponds to the marginal resource cost. The quantity \( (1-\alpha) \) is positive because \( \alpha \) is less than unity. This means \( \nu \) is positive. Since \( \nu \) is positive, \( \partial Z/\partial \nu \) is equal to zero, meaning \( Q_A = Q_A^o \). That is in order to maximize profit under the above conditions, OPAT should buy the entire output of goods A (i.e., it must operate on the supply curve for A).
APPENDIX B

DERIVATION OF THE SUPPLY FUNCTIONS
FOR GOODS A AND B

The purpose of this appendix is to show that the per-capita supply functions for goods A and B are given by

\[ a = a(P_A, P_B, k) \] and \[ b = b(P_A, P_B, k) \].

To this end, let the production sector be described in aggregate terms as follows:

1. \( A = F(N_A, K_A) \)
2. \( B = G(N_B, K_B) \)
3. \( N_A + N_B = N \)
4. \( K_A + K_B = K \)
5. \( P_A F_N = P_B G_N = w \)
6. \( P_A F_K = P_B G_K = r \)
7. \( \omega = w/r \)

where

- \( A, B \) = total output of goods A and B (in physical units/period)
- \( N_A, N_B \) = amount of labor used to produce goods A and B (unit/period)
- \( K_A, K_B \) = amount of "capital" used to produce goods A and B (unit/period)
- \( N, K \) = total amount of labor and "capital" available in Togo to be used to produce A and B
\( F_N, F_K = \text{marginal physical product of} \\
\text{labor and capital in sector A} \\
G_N, G_K = \text{marginal physical product of labor} \\
\text{and capital in sector B} \\
\)

and \( w, r, \omega \) are respectively the nominal wage, the nominal rate of return on capital and the wage rental ratio.

Expressions (1) and (2) are the two production functions (differentiable, continuous, homogenous of degree one).

Expressions (3) and (4) are the input constraints.

Expressions (5) and (5) are the efficiency conditions.

Let us make a couple of observations. First, the production conditions in the agricultural sector in Togo as described in Chapter 1 indicate that very little modern equipment is used. The word capital is used in a broad sense here and covers the unsophisticated physical tools, insecticides, fertilizers, etc., in a sense, very fertile land may be a theoretical proxy for capital. Second, the input constraints written as strict equalities imply that the aggregate inputs are fully employed. Strict equality instead of inequality is used in part for mathematical reasons and in part because this writer cannot provide a plausible theory for how eventual excess supplies of labor and/or capital may be disposed of when only the agricultural sector is considered.
1. Preliminary exercises: to show that \( \alpha = \eta_A f(k_A) \) and \( \beta = \eta_B g(k_B) \).

The assumption of constant returns to scale means that \( \alpha A = F(\alpha N_A, \alpha K_A) \) and \( \lambda B = G(\lambda N_B, \lambda K_B) \). Let \( \alpha = 1/N_A \) and \( \lambda = 1/N_B \). Then the above expressions become \( A/N_A = F(1, K_A/N_A) = f(k_A) \) and \( B/N_B = G(1, K_B/N_B) = g(k_B) \) where \( k_A = K_A/N_A \) and \( k_B = K_B/N_B \). Differentiate \( F = N f \) and \( G = N_B g \) to obtain the marginal physical products of labor and capital in each sector:

\[
F_K = \partial f/\partial k_A = f', \quad G_K = \partial g/\partial k_B = g', \quad F_N = f - k_A f' \quad \text{and} \quad G_N = g - k_B g'.
\]

Define \( \eta_A \) and \( \eta_B \) as \( \eta_A = N_A/N, \eta_B = N_B/N \). Then using the two input constraints, one can show that

\[
\eta_A = \frac{k - k_B}{k_A - k_B} \quad \text{and} \quad \eta_B = \frac{k_A - k}{k_A - k_B} \quad \text{where} \quad k = K/N.
\]

Define the per-capita outputs as \( \alpha = A/N \) and \( \beta = B/N \). Since \( A/N_A = f \) and \( B/N_B = g \), then \( \alpha = \eta_A f \) and \( \beta = \eta_B g \). That is \( \alpha = \eta_A f(k_A) \) and \( \beta = \eta_B g(k_B) \) where \( \eta_A \) and \( \eta_B \) depends on \( k, k_A \) and \( k_B \) (as mentioned above).

To obtain the above expressions for \( \alpha \) and \( \beta \), the production functions and the input constraints have been used. The next step is to use the efficiency conditions to identify the determinants of \( k_A \) and \( k_B \) and substitute them into the expressions for \( \alpha \) and \( \beta \) to obtain the supply curves.
2. To show that \( k_A = k_A(P_A, P_B) \) and \( k_B = k_B(P_A, P_B) \).

Rewrite the efficiency conditions (5) and (6) in per-capita terms as:

\[
P_A[f(k_A)-k_Af'(k_A)] - P_B[g(k_B)-k_Bg'(k_B)] = + - + - 
\]

and

\[
P_Af'(k_A) - P_Bg'(k_B) = - + + - 
\]

where the signs of the partial derivatives of the implicit functions \( U \) and \( V \) are obtained in a straightforward manner assuming that the production functions exhibit positive but diminishing marginal physical products (i.e., \( f' \) and \( g' \) are positive while \( f'' \) and \( g'' \) are negative). Treating \( k_A \) and \( k_B \) as endogenous variables and \( P_A \) and \( P_B \) as exogenous variables the Jacobian of the above system is:

\[
\theta = U_{k_A} V_{k_B} - U_{k_B} V_{k_A} = P_A P_B f''(k_A-k_B) > 0 \text{ as } k_A > k_B
\]

where \( U_{k_A}, U_{k_B} \) are the partials of \( U \) with respect to \( k_A \) and \( k_B \). Similarly for \( V_{k_A} \) and \( V_{k_B} \).

Use Cramer Rule to obtain:

\[
\frac{\partial k_A}{\partial P_A} = \frac{(-U_{P_A} V_{k_B} + U_{k_B} V_{P_A})}{\theta} > 0 \text{ as } \theta > 0
\]

\[
\frac{\partial k_A}{\partial P_B} = \frac{(-U_{P_B} V_{k_B} + U_{k_B} V_{P_B})}{\theta} < 0 \text{ as } \theta < 0
\]
\[ \frac{\partial k_B}{\partial P_A} = (U_k V_{PA} + U P_{PA} k_A)/\theta > 0 \text{ as } \theta < 0 \]

\[ \frac{\partial k_B}{\partial P_B} = (U_k V_{PB} + U P_{PB} k_A)/\theta < 0 \text{ as } \theta < 0 . \]

Assuming that the export sector is more capital intensive than the nontraded goods sector, the following results are obtained:

\[ k_A = k_A(P_A, P_B) \text{ and } k_B = k_B(P_A, P_B). \]

3. To show that \( a = a(P_A, P_B, k), b = b(P_A^*, P_B, k). \)

a) To show that \( a = a(P_A^*, P_B, k). \)

Recall that \( a = \eta_A f(k_A) \) and \( b = \eta_B g(k_B) \) where

\[ \eta_A = \frac{k_B - k_B}{k_A - k_B} \text{ and } \eta_B = \frac{k_A - k_B}{k_A - k_B}. \]

Using the fact that \( k_A \) and \( k_B \) depend on \( P \), differentiate \( a \) with respect to \( P \) to obtain:

\[ \frac{da}{dP_A} = \frac{\partial \eta_A}{\partial P_A} f + \eta_A f' \frac{\partial k_A}{\partial P_A} - (k_A - k_B) \frac{\partial k_B}{\partial P_A} f - (k_B - k_B) \frac{\partial k_A}{\partial P_A} f + (k_B - k_B) \frac{\partial k_B}{\partial P_A} f \]

\[ = \frac{(k_B - k_B) (k_A - k_B) \frac{\partial k_A}{\partial P_A} f'}{(k_A - k_B)^2} + \frac{(k_B - k_B) (k_A - k_B) \frac{\partial k_B}{\partial P_A} f'}{(k_A - k_B)^2} \]
If \( k_A > k_B \), then \( -k_A < 0 \) and \( k_B > 0 \). Furthermore, the quantity \( (k_A-k_B)f' - f = k_Af' - f - k_Bf' = -w/P_Ak_Bf' < 0 \). Thus, given that \( \partial k_B/\partial P_A < 0 \) and \( \partial k_A/\partial P_A < 0 \), clearly \( da/dP_A > 0 \).

Similarly differentiate \( a \) with respect to \( P_B \) to get:

\[
\frac{da}{dP_B} = \frac{\partial \eta_A}{\partial P_B} f + \eta_A f' \frac{\partial k_A}{\partial P_B}
\]

\[
= \frac{(k_A-k_B)\frac{\partial k_B}{\partial P_B} f + \frac{\partial k_A}{\partial P_B}(k_A-k_B)[(k_A-k_B)f' - f]}{(k_A-k_B)^2} < 0 \quad (B.2)
\]

b) To show that \( b = b(P_A, P_B, k) \).

Differentiate \( b \) with respect to \( P_A \) to get:

\[
\frac{db}{dP_A} = \frac{\partial \eta_B}{\partial P_A} g + \eta_B g' \frac{\partial k_B}{\partial P_A}
\]

\[
= \frac{(k_B-k_A)\frac{\partial k_A}{\partial P_A} g - (k_A-k)g\left(\frac{\partial k_A}{\partial P_A} - \frac{\partial k_B}{\partial P_A}\right)}{(k_A-k_B)^2}
\]
\[
\begin{align*}
&= \frac{(k_A - k_B)(k_A - k)g'^B}{(k_A - k_B)^2} + \frac{\partial k_B}{\partial p_A} \frac{\partial^2 k_B}{\partial p_A^2} g - k_B g' + k_A g' \frac{\partial k_B}{\partial p_A} \frac{\partial^2 k_B}{\partial p_A^2} \frac{\partial g}{\partial k_B} + k_A g' + k_B g' \\
&\leq 0. \quad (B.3)
\end{align*}
\]

c) To show that \( \frac{\partial a}{\partial k} > 0 \), \( \frac{\partial b}{\partial k} \leq 0 \) as \( k_A > k_B \).

Recall that:
\[
a = \eta_A f(k_A) = \frac{k - k_B}{k_A - k_B} f(k_A) \quad \text{and} \quad b = \eta_B g(k_B) = \frac{k_A - k}{k_A - k_B} g(k_B).
\]

Differentiate these expressions with respect to \( k \) to obtain:

\[
\frac{\partial a}{\partial k} = \frac{f}{k_A - k_B} \quad \text{and} \quad \frac{\partial b}{\partial k} = \frac{-g}{k_A - k_B}.
\]

Thus:

\[
\frac{\partial a}{\partial k} > 0, \quad \frac{\partial b}{\partial k} \leq 0 \quad \text{as} \quad k_A > k_B, \quad \text{which may be termed the "Rybczynski conditions".}
\]

4. To show that \( a = a(P_A, P_B, k) \), \( b = b(P_A, P_B, k) \) as \( k_A < k_B \).

Bearing in mind that when \( k_A < k_B \): (1) the Jacobian \( \theta \) is negative so that \( \frac{\partial k_A}{\partial p_A} > 0 \), \( \frac{\partial k_A}{\partial p_B} < 0 \), \( \frac{\partial k_B}{\partial p_A} > 0 \), \( \frac{\partial k_B}{\partial p_B} < 0 \);
(2) \( k-k_A > 0 \) and \( k-k_B < 0 \), one can follow a procedure exactly similar to the one above in order to show that the signs of the partials of \( a \) and \( b \) with respect to \( P_A \) and \( P_B \) are maintained while those with respect to \( k \) are reversed. This means that regardless of the capital intensities, the quantities supplied are positively related to the own price and negatively related to the cross price. Only the effects on these quantities as the capital labor ratio \( k \) changes depend on the capital intensities.
APPENDIX C

EFFECT OF CHANGES IN $P_A$, $P_B$ AND $k$ ON GNP

The objective of this appendix is to show that an increase in the producer price of the pure export goods or an increase in the price of the nontraded goods will increase the nominal GNP while an increase in the capital labor ratio has an ambiguous effect on the nominal GNP.

The notation and results of Appendix B are used here.

1. To show that $\partial y/\partial P_A > 0$.

Write nominal income $y$ as $y = P_A a + P_B b$ where $a = \frac{k-B}{k_A-k_B} f(k_A)$ and $b = \frac{k_A-k}{k_A-k_B} g(k_B)$ and $k_A, k_B$ depend on $P_A$ and $P_B$. The procedure is: (1) to compute the partial derivatives $\partial a/\partial P_A$ and $\partial b/\partial P_A$; (2) to differentiate $y$ with respect to $P_A$ to obtain $\partial y/\partial P_A$ and (3) to substitute the derivatives $\partial a/\partial P_A$ and $\partial b/\partial P_A$ computed in the first step into $\partial y/\partial P_A$ in order to show that $\partial y/\partial P_A > 0$.

Recall the two implicit functions $U(k_A, k_B, P_A, P_B) = 0$ and $V(k_A, k_B, P_A, P_B) = 0$ whose partials are $U_{k_A} = -P_A f''$, $U_{k_B} = P_B g''$, $U_{P_A} = f - k_A f'$, $U_{P_B} = -(g - k_B g')$, $V_{k_A} = P_A f''$, $V_{k_B} = -P_B g''$, $V_{P_A} = f'$, $V_{P_B} = -g'$. The Jacobian of the above system is $\theta = P_A P_B f'' g'' (k_A - k_B)$. Recall also that $P_A (f - k_A f') = P_B (g - k_B g') = w$. 242
The quantities \( \frac{\partial k_A}{\partial P_A} \), \( \frac{\partial k_A}{\partial P_B} \), \( \frac{\partial k_B}{\partial P_A} \), \( \frac{\partial k_B}{\partial P_B} \) have been computed from the implicit functions in Appendix B. Substitute the values of the partials of \( U \) and \( V \) in these quantities to obtain:

\[
\begin{align*}
\frac{\partial k_A}{\partial P_A} &= \frac{w/P_A + k_B f'}{P_A f''}, \\
\frac{\partial k_A}{\partial P_B} &= \frac{-g}{(k_A - k_B) P_A f''}, \\
\frac{\partial k_B}{\partial P_A} &= \frac{f}{(k_A - k_B) P_B g''}, \\
\frac{\partial k_B}{\partial P_B} &= \frac{-w/P_B + k_A g'}{(k_A - k_B) P_B g''}.
\end{align*}
\]

From Appendix B, expression (B.1) is:

\[
\frac{da}{dP_A} = \frac{1}{(k_A - k_B)^2} \left\{ \frac{(k-k_A)}{P_A} \frac{\partial k_B}{\partial P_A} f + \frac{\partial k_A}{\partial P_A} (k-k_B) (P_A f'-f) \right\}
\]

\[
= \frac{1}{(k_A - k_B)^2} \left\{ \frac{(k-k_A)}{(k_A - k_B) P_B g''} f^2 \right. \\
- \frac{(w/P_A + k_B f') (k-k_B) (w/P_A + k_B f')}{(k_A - k_B) P_A f''} \\
\left. \right\}
\]

\[
= \frac{1}{(k_A - k_B)^3 P_A P_B f'' g''} \left\{ (k-k_A) P_A f^2 f'' \\
- (w/P_A + k_B f')^2 (k-k_B) P_B g'' \right\}.
\]
From Appendix B, expression (B.3) is:

\[
\frac{db}{dp_A} = \frac{1}{(k_A - k_B)^2} \left\{ (k-k_B) \frac{\partial k}{\partial p_A} \ g + (k_A - k) \frac{\partial k_B}{\partial p_A} (g - k_B g' + k_A g') \right\}
\]

\[
= \frac{1}{(k_A - k_B)^2} \left\{ \frac{(k-k_B)(w/p_A + k_B f')}{(k_A - k_B)^2 f''} \ g + \frac{(k_A - k)(w/p_B + k_A g')f}{(k_A - k_B)^2 f''} \right\}
\]

\[
= \frac{1}{(k_A - k_B)^3} \left\{ (k-k_B) (w/p_A + k_B f') \ p_B g'' \right\}
\]

\[
\ + (k_A - k) (w/p_B + k_A g') \ p_A f'' \}
\]

Differentiate \( y = p_A a + p_B b \) with respect to \( p_A \) to obtain:

\[
\frac{\partial y}{\partial p_A} = a + p_A \frac{\partial a}{\partial p_A} + p_B \frac{\partial b}{\partial p_A} . \ \text{Substitute for} \ \frac{\partial a}{\partial p_A} \ \text{and} \ \frac{\partial b}{\partial p_A} \ \text{to get:}
\]

\[
\frac{\partial y}{\partial p_A} = a + \frac{1}{(k_A - k_B)^3 p_A p_B f''} \ \left\{ (k-k_A) p_A^2 f'^2 f'' - (k-k_B) (w/p_A + k_B f')^2 \ p_A p_B g'' + (k_A - k) (w/p_B + k_A g') p_A^2 g'' \right\}
\]

\[
\ + (k_A - k) (w/p_B + k_A g') p_A p_B f'' \}
\]
\[
\frac{\partial y}{\partial P_A} = a + \frac{1}{(k_A - k_B)^3 P_A P_B f'' g''} \left\{ (k - k_A) P_A f f' [P_A f - (w/P_B + k_A g') P_B] \\
+ (k - k_B)(w/P_A + k_B f') P_B g'' [P_B g - (w/P_A + k_B f') P_A] \right\}
\]

But \( P_A f - (w/P_A + k_A g') P_B = P_A f - k_A P_B g' - w = P_A (f - k_A f') - w \)
because \( P_B g' = P_A f' \) (efficiency condition in capital market).
Since \( P_A (f - k_A f') = w \) (from efficiency condition in labor
market), the first bracket term is zero.
Similarly \( P_B g'(w/P_A + k_B f') P_A = P_B g-w-k_B P_A f' = 0 \), for
the same reasons.
This means \( \frac{\partial y}{\partial P_A} = a > 0 \).

2. To show that \( \frac{\partial y}{\partial P_B} > 0 \).

From Appendix B, expression (B.2) is:

\[
\frac{da}{dP_B} = \frac{1}{(k_A - k_B)^2} \left\{ (k - k_A) \frac{\partial k_B}{\partial P_B} f + \frac{\partial k_A}{\partial P_B}(k - k_B)(k_A - k_B) f' - f \right\}
\]

\[
= \frac{1}{(k_A - k_B)^2} \left\{ - \frac{(k - k_A)(w/P_B + k_A g')}{(k_A - k_B) P_B g''} f + \frac{(k - k_B)(w/P_A + k_B f')}{(k_A - k_B) P_A f''} g \right\}
\]

\[
= \frac{1}{(k_A - k_B)^3 P_A P_B f'' g''} \left\{ -(k - k_A)(w/P_B + k_A g') P_A f f' \\
+ (k - k_B)(w/P_A + k_B f') P_B g g'' \right\}.
\]
From Appendix B, expression (B.4) is:

\[
\frac{db}{dP_B} = \frac{1}{(k_A-k_B)^2} \left\{ (k-k_B) \frac{\partial k_A}{\partial P_B} g + (k_A-k) \frac{\partial k_B}{\partial P_B} (g-k_B g'+k_A g') \right\}
\]

\[
= -\frac{1}{(k_A-k_B)^3 P_A P_B f'' g''} \left\{ (k-k_B) P_B g^2 g'' + (k_A-k)(w/P_B+k_A g')^2 P_A f'' \right\}.
\]

Now differentiate \( y \) with respect to \( P_B \) to obtain:

\[
\frac{\partial y}{\partial P_B} = P_A \frac{\partial b}{\partial P_B} + b + P_A \frac{\partial b}{\partial P_B}.
\]

Substitute for \( \frac{\partial a}{\partial P_B} \) and \( \frac{\partial b}{\partial P_B} \) to get:

\[
\frac{\partial y}{\partial P_B} = b - \frac{1}{(k_A-k_B)^3 P_A P_B f'' g''} \left\{ (k-k_A) (w/P_B+k_A g') P_A^2 f'' g''
\right.
\]

\[
- (k-k_B) (w/P_A+k_B f') P_A P_B g^2 g'' + (k-k_B) P_B^2 g^2 g''
\]

\[
+ (k_A-k) (w/P_B+k_A g')^2 P_A P_B f'' \right\}
\]

\[
= b - \frac{1}{(k_A-k_B)^3 P_A P_B f'' g''} \left\{ (k-k_A) (w/P_B+k_A g') P_A f'' \left[ P_A f
\right.
\right.
\]

\[
- (w/P_B+k_A g') P_B \right] + (k-k_B) P_B g^2 g'' \left[ P_B g-(w/P_A+k_B f') P_A \right] \right\}.
\]

As shown earlier, the bracket terms are equal to zero so that \( \frac{\partial y}{\partial P_B} = b > 0 \).

This means an increase in the price of the pure export goods \( A \) or in that of the nontraded goods \( B \) unambiguously increases the per-capita nominal income.
3. To show that $\frac{\partial y}{\partial k} \geq 0$

Recall expression (B.5) from Appendix B: $\frac{\partial a}{\partial k} = \frac{1}{k_A - k_B}$ and $\frac{\partial b}{\partial k} = \frac{-g}{k_A - k_B}$. Differentiate $y = P_A a + P_B b$ with respect to $k$ and substitute for $\frac{\partial a}{\partial k}$ and $\frac{\partial b}{\partial k}$ to obtain:

$$\frac{\partial y}{\partial k} = \frac{P_A f - P_B g}{k_A - k_B} \geq 0.$$

The term $f$ is the ratio of the output of goods $A$ to the amount of labor used to produce that output and $g$ is the ratio of the output of goods $B$ to the amount of labor used to produce that output. The data indicate that the average output of the nontraded goods is larger than the average output of the pure export goods. Furthermore, the amount of labor used in the nontraded goods sector is also greater than the amount of labor used in the export sector, so that one cannot say, a priori, whether $f$ is greater than, equal to, or less than $g$. The data also show that the average producer price $P_A$ is substantially higher than the average price $P_B$ of the nontraded goods. Thus, if $f$ and $g$ are approximately equal, the term $P_A f$ will exceed the term $P_B g$. Consequently, given our assumption that sector $A$ is more capital intensive than sector $B$, the term $\frac{\partial y}{\partial k}$ will be positive. In order to proceed with this study, it will be assumed that $P_A f > P_B g$ and thus $\frac{\partial y}{\partial k} > 0$. 
APPENDIX D

THE EXPRESSION \((\frac{d_B}{\partial P_B} - \frac{\partial y}{\partial P_B})\) IS EQUAL TO ZERO

In this appendix, it will be shown that the change in nominal GNP due to a change in the price of the nontraded goods is equal to the quantity of the nontraded goods demanded by consumers.

The notation and results of Appendix C are used here. Recall from Appendix C that:

\[
\frac{\partial y}{\partial P_B} = b - \frac{1}{(k_A - k_B)^3 P_A P_B f'' g''} \left\{ (k-k_B)(w/P_B + k_A g')P_A f'' [P_A f' \right.

- \left. (w/P_B + k_A g')P_B ] + (k-k_B)P_B g'' [P_B g - (w/P_A + k_B f')P_A] \right\}.
\]

So that \(d_B - \frac{y}{P_B} = d_B - b = - \frac{1}{(k_A - k_B)^3 P_A P_B f'' g''} \left\{ (k-k_B)(w/P_B

+ k_A g')P_A f'' [P_A f - (w/P_B + k_A g')P_B] \right.

+ \left. (k-k_B)P_B g'' [P_B g - (w/P_A + k_B f')P_A] \right\}.
\]
The quantity \((d_B - \frac{\partial y}{\partial p_B} = 0)\) because the market for the non-traded goods clears since \(d_B\) and \(b\) represent the per-capita quantities demanded and supplied (i.e., equation (3.14) in the text).

Note that an alternative explanation is that this quantity is zero because as shown in Appendix C the expressions in each bracket is equal to zero.
APPENDIX E

DEMAND FUNCTIONS FOR GOODS B AND C
AND FOR MONEY

The problem here is to maximize \( U = U(d_B, d_{CP}, \ell) \)
over \( d_B, d_{CP} \) and \( \ell \) subject to

\[
P_Aa(P_A, P_B, k) + P_Bb(P_A, P_B, k) + m = P_Bd_B + e(1+t_C)P_C^*d_{CP} + t + \ell,
\]
so as to obtain the demand functions for the nontraded goods
B, the imported consumption goods C and for money. For
convenience, let us use the following notation:

\[
X_1 = d_B, \quad X_2 = d_{CP}, \quad X_3 = \ell, \quad P_1 = P_B, \quad P_2 = e(1+t_C)P_C^*.
\]

\[
y = P_Aa(P_A, P_1, k) + P_1b(P_A, P_1, k) = y(P_A, P_1, k).
\]

The problem may be rewritten as:

maximize \( U = U(X_1, X_2, X_3) \) over \( X_1 > 0, X_2 > 0, X_3 > 0 \)
subject to \( y(P_A, P_1, k) + m = P_1X_1 + P_2X_2 + X_3 + t \).

The following assumptions are made about the utility
function: (1) \( U_1, U_2 \) and \( U_3 \) are all positive; (2) \( U_{11}, U_{22} \)
and \( U_{33} \) are all negative; (3) \( U_{13} = U_{23} = U_{31} = U_{32} = 0 \)
(additive separable utility function); (4) \( (U_{11}U_{22} - U_{12}^2) \)
is positive (i.e., utility function is strictly concave
in \( X_1 \) and \( X_2 \)); (5) \( U_{12} \) is positive.
The Lagrangian may be written as:

\[ Z = U(X_1, X_2, X_3) + \lambda [-P_1X_1 - P_2X_2 - X_3 - t + y(P_A, P_1, k) + m]. \]

**First Order Conditions**

\[ \frac{\partial Z}{\partial X_1} = U_1 - \lambda P_1 = 0 \]
\[ \frac{\partial Z}{\partial X_2} = U_2 - \lambda P_2 = 0 \]
\[ \frac{\partial Z}{\partial X_3} = U_3 - \lambda = 0 \]
\[ \frac{\partial Z}{\partial \lambda} = -P_1X_1 - P_2X_2 - X_3 - t + y(P_A, P_1, k) + m = 0. \]

**Second Order Conditions**

The bordered Hessians are \[ \Omega_1 = -P_1^2 U_{22} + 2P_1P_2U_{12} - P_2^2 U_{11} \] and \[ \Omega_2 = U_{33} \Omega_1 - (U_{11} U_{22} - U_{12}^2). \] The SOC will be met if \( \Omega_1 \) is positive while \( \Omega_2 \) is negative. Since \( U_{11} < 0, U_{22} < 0 \) and \( U_{12} > 0, \Omega_1 > 0 \). Similarly since \( (U_{11} U_{22} - U_{12}^2) > 0, \Omega_1 > 0 \) and \( U_{33} < 0, \Omega_2 < 0 \). So that the sufficient conditions for the utility maximization are met.
To solve for $X_1$, $X_2$ and $X_3$, differentiate the FOC to obtain:

$$
\begin{bmatrix}
U_{11} & U_{12} & 0 & -P_1 \\
U_{21} & U_{22} & 0 & -P_2 \\
0 & 0 & U_{33} & -1 \\
-P_1 & -P_2 & -1 & 0
\end{bmatrix}
\begin{bmatrix}
dx_1 \\
dx_2 \\
dx_3 \\
d\lambda
\end{bmatrix}
=
\begin{bmatrix}
\lambda & 0 & 0 & 0 & 0 & 0 \\
0 & \lambda & 0 & 0 & 0 & 0 \\
(X_1 - \frac{\partial y}{\partial P_1}) & X_2 & -\frac{\partial y}{\partial P_A} & -\frac{\partial y}{\partial k} & 1 & -1
\end{bmatrix}
\begin{bmatrix}
dP_1 \\
dP_2 \\
dP_A \\
dk \\
dt \\
dm
\end{bmatrix}
$$

The Jacobian determinant $J$ of this system is exactly equal to $\Omega_2$ so that $J < 0$.

Using Cramer Rule, the solution to the system may be written as follows:

$$
\frac{dx_1}{dP_1} = \frac{\lambda (-P_2^2 U_{33} - U_{22}) + U_{33} (X_1 - \frac{\partial y}{\partial P_1}) (P_1 U_{22} - P_2 U_{12})}{J} < 0
$$

$$
\frac{dx_1}{dP_2} = \frac{U_{33} [\lambda P_1 P_2 + X_3 (P_1 U_{22} - P_2 U_{12})] + \lambda U_{12}}{J} \geq 0
$$

$$
\frac{dx_1}{dP_A} = - \frac{(\partial y/\partial P_A) (P_1 U_{22} - P_2 U_{12}) U_{33}}{J} > 0
$$
\begin{align*}
\frac{dx_3}{dp_2} &= -\lambda (p_1 u_{12} - p_2 u_{11}) + x_2 (u_{11} u_{22} - u_{12}^2) > 0 \\
\frac{dx_3}{dp_A} &= -\left(\frac{\partial y}{\partial p_A}\right) (u_{11} u_{22} - u_{12}^2) > 0 \\
\frac{dx_3}{dk} &= -\left(\frac{\partial y}{\partial k}\right) (u_{11} u_{22} - u_{12}^2) > 0 \\
\frac{dx_3}{dt} &= \frac{u_{11} u_{22} - u_{12}^2}{J} < 0 \\
\frac{dx_3}{dm} &= -\frac{(u_{11} u_{22} - u_{12}^2)}{J} > 0.
\end{align*}

It can be shown that: (1) \((x_1 - \frac{\partial y}{\partial p_1}) = 0\) (see Appendix D); (2) \(\frac{\partial y}{\partial p_A} > 0\), \(\frac{\partial y}{\partial p_1} > 0\) (see Appendix C). It is assumed that \(\frac{\partial y}{\partial k} > 0\). These results together with the assumptions concerning the utility function permit to obtain the signs indicated above. That is the three demand functions may be written as:

Nontraded goods:
\[ + - + - + \]
\[ d_B = d_B(p_A, p_B, p_2, m, t, k) \]

Imported consumption goods C:
\[ + + - + - - \]
\[ d_{CP} = d_{CP}(p_A, p_B, p_2, m, t, k) \]
Demand for money:
\[ + + \frac{\partial}{\partial P_A} + - + \]
\[ l = l(P_A, P_B, P_C, m, t, k). \]

Substitute \( P_A = e^{P_A^*}/(1+t_A) \) and \( P_2 = (1+t_C)e^{P_C^*} \) in these functions to obtain:

Demand for goods B:
\[ + - \frac{\partial}{\partial P_A} + + - + \]
\[ d_B = d_B(P_A, P_B, P_C^*, e, m, t, t_A, t_C, k) \]

Demand for goods C:
\[ + \frac{\partial}{\partial P_A} - - + - - + \]
\[ d_C = d_C(P_A, P_B, P_C^*, e, m, t, t_A, t_C, k) \]

Demand for money:
\[ + + \frac{\partial}{\partial P_A} + - + \]
\[ = (P_A, P_B, P_C^*, e, m, t, t_A, t_C, k). \]

Observations

1) The signs of the partial of the demand functions with respect to a given argument (for instance \( P_A^* \)) which are maintained in all three functions do not violate Walras Law because there are other goods and assets demanded which are present in the Walras Law of the model but are absent from the above manipulations. If one were to derive the demand functions for all the goods and assets in the initial model, then at least one of the partial derivatives of the
demand functions with respect to, say, \( P_A^* \) must be negative because these partials must sum up to zero.

2) The demand function for goods B and C are indeed negatively sloped (i.e., \( \frac{\partial d_B}{\partial P_B} < 0 \), \( \frac{\partial d_C}{\partial P_C} < 0 \)).

3) As regards the signs of the cross price elasticities of the demand functions for B and C, they are ambiguous. It is assumed in the empirical part of this thesis that the imported good C and the nontraded good B are gross substitutes so that an increase in the price of C will raise the demand for B. This assumption implies that the quantity \( \frac{\partial d_B}{\partial P_2} \) is positive.

4) Note also that a devaluation of the home currency (i.e., \( e \) decreases) does not always result in a decrease in the demand for goods and money. That is:

\[
\frac{\partial d_B}{\partial e} + \frac{\partial d_B}{\partial P_A} \frac{\partial P_A}{\partial e} + \frac{\partial d_B}{\partial P_2} \frac{\partial P_2}{\partial e} > 0
\]

Nontraded goods B:

\[
\frac{\partial d_{CP}}{\partial e} = \frac{\partial d_{CP}}{\partial P_A} \frac{\partial P_A}{\partial e} + \frac{\partial d_{CP}}{\partial P_2} \frac{\partial P_2}{\partial e} > 0
\]

Consumption goods C:

\[
\frac{\partial d_{CP}}{\partial e} = \frac{\partial d_{CP}}{\partial P_A} \frac{\partial P_A}{\partial e} + \frac{\partial d_{CP}}{\partial P_2} \frac{\partial P_2}{\partial e} > 0
\]

Demand for money:

\[
\frac{\partial d}{\partial e} = \frac{\partial d}{\partial P_A} \frac{\partial P_A}{\partial e} + \frac{\partial d}{\partial P_2} \frac{\partial P_2}{\partial e} > 0.
\]
5) While the export tax and the lump sum tax always exert a negative effect on the demand for the goods and money, a higher import tax may increase or reduce the demand for the nontraded goods and money because the partials of these two functions with respect to $P_2$ are ambiguous. Note that the price of the imported consumption goods is denoted in the main texts of this thesis by $P_C$. That is $P_2 = P_C = (1 + t_C) e^{P_*}$. 
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