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THE EFFECTS OF AGE, IQ, AND INSTRUCTIONAL SEQUENCE ON THE ACQUISITION OF BASIC COUNTING SKILLS

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THE EFFECTS OF AGE, IQ, AND INSTRUCTIONAL SEQUENCE ON THE ACQUISITION OF BASIC COUNTING SKILLS

by

Wayne Charles Piersel

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I hereby recommend that this dissertation prepared under my direction

by Wayne Charles Piersel

entitled The Effects of Age, IQ, and Instructional Sequence on the Acquisition of Basic Counting Skills

be accepted as fulfilling the dissertation requirement for the Degree

of Doctor of Philosophy.

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ABSTRACT

The purpose of this study was to investigate the influence of presentation sequence on the acquisition and retention of selected counting and one-to-one correspondence skills. In addition, the effects of the individual characteristics of intelligence, mental age, chronological age, and pre-instruction knowledge were examined in relation to learning and retention of these skills. Finally, the adequacy of a previously validated learning hierarchy for these skills was explored.

Using a placement test for the skills being investigated in this study, 138 children were selected from 206 children attending five day care centers to participate in this investigation. The children who participated in the study demonstrated a lack of knowledge of the skills targeted for instruction. Six experimental groups representing six instructional orders for the teaching of skills _A_ (rote counting _1_ to _10_), _B_ (counting sets of movable objects to _10_), and _C_ (counting fixed, ordered sets of objects _1_ to _10_). Skill _D_ (counting fixed, unordered sets of objects _1_ to _10_) was taught last in all experimental groups. For each instructional sequence, a specific skill was taught to each child for five days or to mastery if less than five days with the next skill in the sequence for that experimental group being taught.

The instructional package, which was the same for all six experimental groups, consisted of relevant stimulus materials for the skill being taught, Sesame Street Finger Puppets, and teaching procedures which involved (a) the two finger puppets modeling the skill to be learned for the child, (b) the child imitating the modeled skill, (c) providing feedback and praise for correct performance or feedback and guided practice for incorrect performance. To obtain estimates of intelligence and mental age, the Slosson Intelligence Test and the Peabody Picture Vocabulary Test were administered to each child.

Application of analysis of variance for repeated measures did not reveal any significant differences between groups for acquisition or retention of the basic counting skills. In addition, there were significant differences between groups for the acquisition and retention of Skill D. Structural analysis substantiated that positive transfer existed among the four skills. As predicted, the contribution of the individual characteristic variables of intelligence, mental age, chronological age, and pre-instruction knowledge decreased across time. Also, as predicted, increasing amounts of variance were being accounted for by skill mastery within the learning hierarchy with each successive week of the study. There was only a minimal effect of the various instructional orders on the hierarchical structure of the skills. The educational practice of carefully sequencing subordinate sets of skills was not supported by this study. Implications for behavior assessment and therapy were discussed.

CHAPTER 1

INTRODUCTION

The interest in task analysis and learning hierarchies and their applications to educational problems of diagnostic assessment, placement testing, and instructional sequencing has been receiving increased attention in recent years (e.g., Bernard, 1975; Gagne, 1968, 1973, 1977; Gagne and Briggs, 1974; White and Gagne, 1974; Passmore, 1975; Resnick, 1973, 1977; Scandura, 1977; White 1973, 1976). Unfortunately, the empirical research on the validation of learning hierarchies and related issues has not kept pace with this increased interest in their application to instruction (Cotton, Gallagher, and Marshall, 1977; Mayer, 1977; Tennyson, 1972; White, 1973, 1974a, 1976).

There have been several lengthy reviews and critiques of the task analysis literature (Bernard, 1975; Passmore, 1975; Resnick, 1973; Tennyson, 1972), instructional sequencing (Posner and Strike, 1976; Scandura, 1977), and methodological inadequacies have been noted (Cotton et al., 1977; Bart, 1974; Bart and Kruz, 1973; White, 1973, 1974b). Taken as a whole, these works have delineated the theoretical, methodological, and educational problems that exist in this area of scientific inquiry.

The assumption that learning hierarchies can be directly applied to problems of instruction and curriculum development has been questioned. Indeed, Scandura (1977) writing from a structural process systems approach to "instructional science" questions whether learning hierarchies and the related issue of instructional sequencing are either desirable or efficient from an educational perspective. He takes the position that an algorithmic approach should be given serious consideration in curriculum development and instructional planning. An algorithm is a procedure which will produce correct results when applied to any problem of a given class of problems (Gerlach, Reiser, and Brecke, 1977). Soulsby (1975) also criticizes the application of learning hierarchy technology to the derivation of instructional sequences questioning the assumption that the ability to profit from instruction can only be exhibited by those learners who have previously acquired the relevant subordinate or prerequisite skills. Both Scandura (1977) and Soulsby (1975) make the point that some learners appear to skip prerequisite skills in demonstrating mastery of the terminal skill while other learners who can demonstrate mastery of the terminal skill appear to have forgotten selected subordinate skills that lead to terminal skill mastery. However, Scandura (1977) cites only one study (Gagne and Bassler, 1963) in support of the forgetting of subordinate skills and cites no literature to support his contention that some individuals appear to skip prerequisite skills in achieving terminal skill mastery. Gagne and Briggs (1974) note that the basic reason for sequencing of instruction is that all learning cannot occur at one point in time and must, therefore, be implemented so as to occur in a series of steps across time.

In a discussion and analysis of competent instruction, Glaser (1963, 1976) describes four basic components of a psychology of instruction. The first component pertains to the analysis of competent performance. Of central concern to this component is the use of task analysis to derive an analytic description of what is to be learned. Secondly, the initial state of the learner must be described so that instruction can proceed to develop competent performance. The learner's skills and capabilities as they pertain to the material to be mastered must be assessed. Thirdly, the educational environment must be arranged to facilitate the acquisition of competent performance. Procedures, techniques, and materials must be brought to bear in the instructional process. Finally, effects of the instructional program must be assessed periodically during the course of learning and following the completion of the routine to evaluate the learner's progress in mastering the material and to assess the learner's acquisition in terms of generalization and transfer. The technology of task analysis and the utilization of learning hierarchies play a crucial role in all four components of Glaser's (1976) instructional model.

The conceptualization of "learning hierarchies" and the concept of "intellectual skills" can be largely attributed to Gagne and associates (Gagne and Brown, 1961; Gagne et al., 1962; Gagne and Staff, 1965; Gagne, 1962, 1968, 1973, 1974, 1977). Although the technology of "task analysis" had been utilized by military (Miller, 1962) and industrial personnel (cf. Bernard, 1975) for development of job descriptions and related job training programs, Gagne seems to be the major contributor in developing the educational and curriculum implications of task analysis to derive potential learning hierarchies.

According to Gagne (1968), the psychological development of an individual may be conceptualized as the result of the cumulative effects of learning. Cumulative learning implies that the acquisition of any subordinate skill may mediate transfer to a next higher-order, superordinate skill. Behavioral researchers such as Staats (1971), Bijou (1970), and Bijou and Baer (1978) similarly espouse a theory of development in their reference to "prior learning (reinforcement) history" as a partial explanation for an individual's current behavior reportoires (capabilities). Gagne assumes that, with the exception of the most basic tasks, it is impossible for an individual to acquire a new skill unless that individual is in possession of all the skills upon which the new skill is dependent (Gagne, 1962, 1968). This is the "prerequisite-skills hypothesis" which states that learning hierarchies consist of intellectual skills arranged in a hierarchical order such that each subordinate skill is prerequisite to the skill(s) immediately above it in the hierarchy. The prerequisite-skills hypothesis assumes that each subordinate skill in the learning hierarchy is necessary for the performance of the superordinate skills immediately above it. While possession of all relevant subordinate skills does not guarantee superordinate skill mastery, failure to learn subordinate skills is assumed to inhibit mastery of superordinate skills.

A second major hypothesis related to the learning hierarchy model is referred to as the "positive-transfer hypothesis" (Gagne, 1962, 1968, 1973; White and Gagne, 1974). The transfer-hypothesis asserts that prerequisite skills will mediate transfer to the superordinate skills to which they are related. Hence, if one skill is prerequisite to another, mastery of the prerequisite skill will facilitate learning the other higher skill. Most of the research investigating learning hierarchies has examined the prerequisite-skill hypothesis and has not examined the positive-transfer hypothesis (Cotton et al., 1977; White, 1973, 1974a,c).

Gagne's writings have stimulated a considerable quantity of research relative to the development of curriculum content hierarchies and related sequences of instruction. One curriculum area that is ideally suited for the development of learning hierarchies and for evaluating their relevance to the instructional process is the area of mathematics. The basic arithmetic skills of counting and one-to-one correspondence are typically recognized as basic skills that pre-school and kindergarten children must acquire to master subsequent quantification and arithmetic skills (Suydam and Weaver, 1975). While there exists some information describing the hierarchical ordering of these skills (Bergan, Horan, and Groff, 1976; Resnick, 1977; Resnick, Wang, and Kaplin, 1970, 1973; Wang, 1973), there is virtually no information available on the optimal instructional order of these basic skills. There is also no information available to answer the question as to whether altering the instructional order will alter the hierarchical structure of these intellectual skills. The effect of instructional order of a sequential set of skills on terminal skill acquisition-transfer has not been investigated.

Most probably the reason for this lack of relevant research stems from the fact that the literature on learning hierarchies is just beginning to emerge, the number of subjects and time required to conduct such a project is prodigious, and the procedures suggested for the studying of positive-transfer have not been practical to utilize except when the validation involves a very small number of learning tasks (Bergan, in press; Cotton et al., 1977; White, 1973, 1974c). Hence, most of the learning hierarchy validation studies have been psychometric in nature and have primarily been concerned with testing the prerequisite-skills hypothesis. The typical strategy (i.e., Gagne, 1962; Gagne and Staff, 1965; Resnick, 1973; Resnick et al., 1973; Wang, 1973) has been to administer a number of items measuring a series of skills identified through task analysis and ordered in a hypothesized manner to a large number of subjects. The pass-fail relations between pairs of skills in a subordinate-superordinate relation would be analyzed using some statistical procedure (e.g., Multiple Scalogram Analysis, White and Clark Test of Inclusion, Ordering Theoretic Method) to assess the adequacy of the hypothesized hierarchy. The term "Psychometric Validation" has been applied to this set of procedures. Since this research paradigm does not employ direct instructional intervention, the psychometric assessment is measuring prior learning. Resnick (1973, 1977) has stated that psychometric validation studies of learning hierarchies are only suggestive with respect to transfer properties of the hierarchy and appropriate instructional sequences. Indeed, it is an assumption that a learning hierarchy validated psychometrically represents a "natural" learning hierarchy or that "natural" learning hierarchies even exist.

There are a number of factors other than subordinate skill mastery which may influence the acquisition of skills in a learning sequence. The manner in which a learner is taught is among the more important of these factors. To date most studies designed to investigate the validity of learning hierarchies have invariably utilized an auto-instructional programmed format. This format requires learners to respond to items (frames) presented in a textbook format until they have completed all items in the unit. They are then tested for mastery of the skills in the hierarchy (White and Gagne, 1974). Validation studies involving adults and peers as models directing instruction in a manner similar to what is likely to occur in a classroom is lacking. Additionally, the effect of having adult instructors manipulate the presentation sequence of a hierarchically sequenced set of skills has not been examined with the study by Uprichard (1970) being the exception.

The recent emergence of the structural equation models (e.g., Bergan, in press; Duncan, 1975; Goodman, 1972) afford a convenient method for studying the positive-transfer assumption. One study that utilized path analysis in examining a psychometrically validated hierarchy of skills for counting and one-to-one correspondence (Bergan et al., 1976) derived a learning hierarchy which closely resembled the previously derived psychometric hierarchy (Resnick et al., 1973). Additionally, Bergan et al. (1976) was able to demonstrate that structural analysis models not only could be used to confirm hierarchical relationships, but that structural equations could also be useful in reflecting the influence of other variables on the acquisition of superordinate skills. However, to date no studies have been reported which take into account the influences of alternative teaching-instructional sequences on the acquisition of skills in psychometrically validated learning hierarchies.

The influence of individual learner characteristic variables such as chronological age (CA), intelligence quotient (IQ), mental age (MA), amount of relevant knowledge (PRET), and entry skill levels have typically not been directly investigated as they relate to the acquisition of hierarchically ordered skills. Walbesser and Eisenberg (1972) speculate that the literature indirectly suggests that the lower the ability of the learner, the greater the number of subordinate behaviors necessary to achieve positive transfer to the terminal behavior. One study that did examine the effect of an exogenous variable (IQ) on skill acquisition in a mathematics hierarchy (Bergan, Karp, and Neuman, 1979) found that IQ was an important determinant for initial skill acquisition but became a non-significant variable for terminal skill acquisition. Acquisition of previous hierarchy skills became the significant determinant.

This concern for individual characteristics and how they interact with an experimental variable has been well articulated by Cronbach (1957) and Snow (1977) in their discussion of the aptitude-treatment-interaction model. Gagne (1962) has also suggested that individual characteristics of learners could be expected to affect acquisition of component skills in hierarchical learning. Gagne's comments focused on ability measures. He viewed abilities as highly generalized intellectual skills which served to mediate positive transfer in a given skill hierarchy along with the acquisition of subordinate skills.

In relation to this view of the effects of individual learner characteristics (abilities) on hierarchical learning, Gagne advanced the theory that the relation between abilities and learning would either remain constant or increase as instruction became less than maximally effective. Briggs (1968) argued in a similar manner in reviewing the relations among abilities, prior learning, and sequencing of instruction. Williams (1972) has also noted that intelligence measures are generalized learning sets and are, therefore, good predictors of school learning. He also suggests that intelligence should be subject to change through school learning. This assertion would be especially plausible to the extent that intelligence tests have been designed to reflect characteristics of the subject (learner) that are relevant to his/her performance on a wide variety of school related situations and tasks.

To assess the effects of various instructional orders on the acquisition of a psychometrically validated hierarchy and to examine the influence of various individual characteristic variables, one would need to experimentally expose groups of subjects to differing instructional orders. In the case of young children, one would have to conduct the instruction individually or in small groups utilizing a model to present the task to be learned. The utilization of self-instructional programs would not be appropriate for preschool children.

There are a variety of instructional methods and procedures that could be employed to instruct young children in basic mathematics skills. One method that has been clearly demonstrated to be effective in teaching young children utilizes modeling and feedback as an instructional technique (e.g., Bandura, 1969, 1971; Bergan et al., 1979; Flanders, 1968; Zimmerman and Rosenthal, 1974; Zimmerman and Kleefeld, 1977). The use of an instructional paradigm consisting of modeling, practice, feedback, and guided practice as a teaching package with young children has received strong support (Jeske, 1978; Karp, 1978; Swanson, 1976). Bandura (1977) notes that modeling not only indicates to the observer the behavior that is to be learned but also transmits feedback to the observer on how the response can be acquired. This feedback may be accomplished by physical demonstration, pictorial representation, or verbal description. Kulhavy (1977) identifies feedback as any of a variety of procedures that are utilized to convey to a learner the correctness or incorrectness of an instructional response. The present study utilized an instructional package consisting of modeling, practice, feedback, guided practice, and praise for all experimental conditions.

Past research has failed to investigate the effect of the presentation order for a hierarchically organized set of skills on subordinate and superordinate skill acquisition. If a learner is instructed in an order differing from the hierarchical order, will this alternative instructional order alter the hierarchical structure? The relation of learning hierarchy validated psychometrically to a learning hierarchy validated experimentally remains unclear. To date, according to Cotton et al. (1977), there have been no adequate tests of the positive-transfer hypothesis. The role of individual characteristics including IQ, CA, MA, and pre-instructional knowledge on the acquisition of hierarchical skills remains unclear. These limitations of the research to date appear to be due to the type of validation procedures chosen, practical limitation of time, subject availability, size of the learning hierarchy, and from failures to examine the diverse literature in learning and instruction. The recent research in structural analysis has provided a means to examine the positive-transfer hypothesis and to also examine the influence of exogenous variables such as individual learner characteristics on skill acquisition in a learning hierarchy.

Purposes of the Study

1. The effect of altering the instructional order of a hierarchically organized set of basic counting skills has not been adequately investigated. All possible presentation orders of the three counting skills will be taught to examine the differential effects, if any, on learning and retention of these hierarchically organized skills.
2. The effect of instructional sequence on the mastery of a terminal skill in a hierarchically organized set of counting skills will be investigated. The terminal skill will be taught last in all instructional sequences to examine the differential effect, if any, on terminal skill acquisition.
3. The effects of individual learner characteristics of intelligence, chronological age, mental age, and pre-instruction knowledge will be examined across time on the mastery of the basic counting skills. The influence of subordinate skills on superordinate skill mastery will also be examined to investigate the positive-transfer hypothesis.
4. The effect of sequences of instruction for a set of intellectual skills on the hierarchical organization of these skills is unclear. The prerequisite-skills hypothesis will be examined to determine if the instructional order alters the previously validated hierarchical structure.

CHAPTER 2

REVIEW OF THE LITERATURE

Due to the complexity and diversity of the relevant literature in the areas of task analysis, learning hierarchies, instructional sequencing, mathematics curriculum, relation of individual characteristics to learning, and teaching techniques, this review discusses only those areas and literature that are directly relevant to the purposes of the present investigation.

This review will initially critique the literature related to establishing the existence of learning hierarchies in subject matter areas such as mathematics. Gagne's research will be presented followed by a discussion of the literature on the models and validation techniques applied to learning hierarchies. Issues will be elucidated, terms clarified, and assumptions of the learning hierarchy reviewed. The literature utilizing the auto-instructional format will be critiqued with emphasis on studies examining the sequencing variable.

The next major area of the review will deal with the empirical literature on learning hierarchies, sequencing of skills, and direct instruction with children. Emphasis will be placed on research with mathematics as the curriculum content area.

The relation of individual learner characteristics of intelligence, chronological age, mental age and prior knowledge will be reviewed. The difficulties identifying relevant research in this area will necessitate reviewing more general studies of intelligence and measures of learning not directly related to learning hierarchies or school learning.

The final section of the review will discuss the literature on modeling, practice, and feedback as it pertains to children's learning. The focus will be on building a rationale for the instructional package used in the study and which is described in Chapter 3.

Empirical Literature Related to the Existence of Learning Hierarchies

Gagne's Research and Writings

Gagne and his associates' research (Gagne, 1962; Gagne and Bassler, 1963; Gagne and Brown, 1961; Gagne et al., 1962; Gagne and Paradise, 1961; Gagne and Staff, 1965) and theoretical-discussion articles and texts (Gagne, 1968, 1973, 1974, 1977; Gagne and Briggs, 1974) have provided much of the pioneering work in the application of task analysis technology to educational problems. Gagne has also been primarily responsible for the development and validation of learning hierarchies within the educational setting. His work has focused on the content area of mathematics and geometry with less attention being devoted to science and social science.

In an initial investigation, Gagne and Paradise (1961) used a programmed instruction format to teach 118 seventh and eighth graders a 22 item hierarchy which culminated in the solving of linear equations. They were also interested in examining the influence of various related and unrelated skills that learners bring to an instructional situation which could potentially affect the efficiency of learning. Subsequent to the learners having completed the auto-instructional program, these individuals were tested for mastery of the terminal skill, mastery of the subordinate skills, and were tested for transfer by being asked to solve a novel set of linear equations.

For each connection or pairing between two skills in the hierarchy, the learners were divided into four groups. The first group consisted of those who passed both the higher skill and the lower skill for that connection. The finding, that both the subordinate and superordinate skills were mastered, was interpreted as support for positive transfer. The second group was composed of those who passed neither the higher nor the lower skill. This was interpreted as supporting the prerequisite skill hypothesis. The third group was composed of those who passed the higher skill and failed the lower skill in the pair. This group needed to be quite small or nonexistent in order for the prerequisite skill hypothesis to be supported. The fourth group, consisting of those who failed the higher skill but passed the lower skill, was viewed as indicating a lack of positive transfer.

Using post-test data, Gagne and Paradise (1961) derived an index called "proportion positive transfer" which yields values ranging from 0.00 to 1.00. Theoretically, the higher the value, the more valid the hierarchical relations between the subordinate and superordinate skills in the learning hierarchy. For various connections in the hierarchy, they obtained proportion positive transfer coefficients ranging from .91 to 1.00.

In addition to validation of the hypothesized learning hierarchy, Gagne and Paradise (1961) examined the relation of several measures of achievement to learning rate and mastery in the hierarchy. As predicted, the correlation of relevant basic abilities of computations, symbol recognition, and integration were moderately high (.54 to .68) with learning rate, number of sets mastered, and performance on the transfer test. The relation of irrelevant basic abilities (vocabulary knowledge) was generally low (.12 to .22). The correlation between relevant basic abilities decreased as the learners progressed through the hierarchy while the correlations of irrelevant basic abilities remained constant and at a low level. The correlation of general basic abilities with learning rates while seemingly contradictory with the observed decrease of correlations with skill mastery is explained by reasoning "that increasing numbers of individuals effectively drop out as learning proceeds; thus the variance which remains becomes more clearly that of relevant basic abilities" (Gagne and Paradise, 1961, p. 13).

Gagne and Paradise (1961) effectively argue that basic abilities which are relevant to learning sets in the hierarchy should initially mediate positive transfer to them. This in turn should be measurable as an increase in the rate of learning. In progressing through the hierarchy, correlations between these basic abilities and rate of attainment of skills should decrease, since such relations come to increasingly depend on mastery-transfer of immediately subordinate skills. Gagne and Paradise (1961) state that general intelligence should correlate moderately with learning rate throughout the learning hierarchy and should show no change in correlation at various levels in the hierarchy since intelligence is viewed as mediating general rather than specific transfer. This is also true for irrelevant basic abilities. If a program were perfectly effective, then the detection of individual differences (variability) would not be measurable. These explanations have not been adequately tested (cf., Bergan et al., 1979).

In a second major investigation employing a 14 item learning hierarchy resulting in the addition of integers, Gagne et al. (1962) had 136 seventh graders complete a self-instructional program. The learners were tested on achievement of the 12 subordinate skills and on mastery of the two terminal skills. In an improvement over the Gagne and Paradise (1961) study, between two and nine items were used to assess mastery of each of the skills in the hierarchy. The authors noted that for each of the 12 skills, the learners made either zero or perfect scores. Using the index of proportion positive transfer, they confirmed the existence of the initially hypothesized hierarchy. The authors noted the need for a statistical test for hierarchical dependencies as they did obtain instances of subjects passing the higher and failing the lower skill. Gagne and Paradise (1961) outlined plausible reasons for obtained exceptions to the prerequisite skill relations of (a) errors in measurement, (b) delay in testing, and (c) errors in the construction of the hierarchy. White (1973, p. 363) noted that "the nature of hierarchy research tends to produce a proportion of invalid connections in postulated hierarchies, because investigations can only lead to rejection of connection, not to the identification of previously overlooked ones."

In one of the few studies to investigate the effect of instruction on identified skill deficiencies, Gagne (1962), after assessing each learners' skills in a learning hierarchy, taught the missing skills to the subject and then again assessed their mastery of the skills in the hierarchy. Gagne demonstrated that none of the subjects lacking sub-skills were able to achieve mastery of the terminal skill of finding the sum of _N_ terms. After teaching each of the subjects to mastery on the missing skills, all subjects were capable of demonstrating mastery of the terminal skill. The small sample size (N=7), failure to control for testing effects, and lack of appropriate control groups limits interpretation of these results. This study was noticeably different in that Gagne taught all of the subordinate skills that each subject was lacking mastery of and was then able to show that following mastery of these subordinate skills, each individual learner was able to master the terminal skill. Unfortunately, the implications for diagnostic and placement assessment for educational problems were not developed. Additional investigation of the idea of using learning hierarchies for assessment and educational remediation remain largely unresearched.

In an attempt to deal with the problem of defining mastery of a skill, Gagne and Staff (1965) assessed mastery of skills in a 19 element hierarchy by developing three items for each skill that differed in degree of similarity to the teaching items. Although they were able to substantiate the existence of the learning hierarchy, the item variation format may have resulted in the problem of a differing domain. This results in the items that purport to measure the same skill requiring differing capabilities to answer suggesting differing skills (Hively, Patterson, and Page, 1968).

Another issue that was not adequately dealt with in the early learning hierarchy research is the issue of continuing to demonstrate mastery of terminal skills in a learning hierarchy while demonstrating forgetting of previously mastered subordinate skills. Gagne and Bassler (1963) followed up the students used in the Gagne and Staff (1965) and found that while performance on the terminal skill in the learning hierarchy remained high, performance on subordinate skills had significantly decreased. This finding would seem, if replicated, to question the existence of learning hierarchies as initially defined. Scandura (1977) cites the forgetting of subordinate skills and the skipping of prerequisite skills in hypothesized learning hierarchies as sufficient evidence for questioning the assumptions that learning hierarchies exist (i.e., prerequisite-skills hypothesis and positive-transfer hypothesis).

Summary

While this early series of studies has had great heuristic value in stimulating numerous subsequent studies and related discussion, recent investigators (Cotton et al., 1977; Resnick, 1977; White, 1973, 1974a,b) have criticized hierarchy research on a number of issues and methodological shortcomings. The imprecise specification of hierarchy elements, small sample size in some of the studies, failure to take into account errors in measurement, use of one element to assess mastery of a skill, use of questionable statistical techniques, and delay of testing skill mastery until the end of the learning program represent some of the major limitations. Additionally, Gagne and his colleagues did not vary instructional order, take into account the learner's familiarity with the subject matter and content, nor perform a test of the transfer hypothesis. The issue of how a hierarchically organized subject matter should be presented to learners was not addressed. Although Gagne (1962) did demonstrate that instruction in missing skills appeared to facilitate mastery of the terminal skill, he did not specifically examine various instructional orders to see if the hierarchical order would represent the maximally effective instructional sequence.

Learning Hierarchies and Skill-Order Presentation in Auto-Instructional Learning Programs

In addition to the attention being devoted to the development of learning hierarchies and the search for empirical support for their validity, the careful sequencing of instructional stimuli (skills) has become an important educational question. Although Scandura (1977) and Mayer (1977) question whether the relation of learning hierarchies to sequencing of instruction is an educationally sound practice, the literature remains unclear. Indeed, Gagne (1968, 1973) writes that learning hierarchies while indicating all of the prerequisite skills that an individual must possess to master a terminal skill, and specifically cautions against directly transferring this hierarchical arrangement into an instructional order. Scandura (1977) cites the example of individuals skipping prerequisites during instruction while subsequently demonstrating mastery of the skipped skill as well as the terminal skill as a caution against letting learning hierarchies dictate instructional order and content.

In an early study examining the effects on learning in a scrambled versus logical sequence utilizing an auto-instructional program, Roe, Case, and Roe (1962) found no difference between the two groups of college freshmen. They utilized a 71 item program on elementary probability. They reported data indicating that scrambling the item sequence had no effect on time required for learning, error score during learning, score on criterion test administered at the end of the learning program, nor on amount of time required to complete the post test. The authors concluded that for short instructional programs the presentation sequence may be relatively unimportant with sequencing being a function of information content of the items, length of the program, and individual learner characteristics. They also presented descriptive statistics based on mathematical aptitude and performance on the four measures employed in the study. There is an indication that familiarity (aptitude) might have been a factor in performance in the instructional program.

Considering the many variables which can affect learning besides the presentation order of items, Levin and Baker (1963) suggested that program content might be a factor in determining whether or not presentation sequence would affect learning. Using a sample of second grade students, they failed to obtain significant differences between logical and scrambled sequences in a self-instructional program in geometry. This study also examined the individual characteristic of IQ and reported significant correlations with learning rate and post test scores. Failure to perform multiple regression analysis and small *N* limit interpretation of the data. In addition, an examination of the correlation of IQ and measures of learning across time and inclusion of additional instructional orders would have greatly contributed to the importance of the study. Holland (1967) points out that failures to obtain differences between logical and scrambled instructional sequences may be due to the fact that auto-instructional program items are not highly interdependent; hence, no learning hierarchy exists. Holland concludes that what was needed was not more studies of logical versus scrambled instructional orders, but rather studies examining the, "when," "how much," and "in what way" questions regarding item sequencing in learning programs. Levin and Baker (1963) did not indicate whether the learning program used in their study was intended to form a learning hierarchy.

Wodke et al. (1967) in a well designed series of studies, hypothesized that the effects of instructional sequencing on learning depended on the characteristics of the subject matter being taught and on the individual differences among the various learners. They predicted that if the subject matter was, in fact, a learning hierarchy, then scrambling the items would affect learning. They also hypothesized that familiarity with the content and ability of the learners would interact with mastery of the learning program and the sequencing variable. They developed two programs with one involving the teaching of number bases and the other teaching a sequence on the anatomy of the human ear. They failed to find any differences between the scrambled and logical instructional orders for either the anatomy or the mathematical programs. They did find evidence of an aptitude by treatment interaction based on ability (pretest knowledge measures) for both programs. They did not present data substantiating the existence of learning hierarchies.

In a more recent investigation of sequencing, anxiety, IQ, and familiarity with instructional content, Tobias (1973) investigated the hypothesis that frame sequence would be important for novel but not for familiar materials. Assigning 120 college students to either a random or logical sequence for a familiar or unfamiliar program content on heart disease, Tobias (1973) found scrambling to have a strong effect for the unfamiliar content area accounting for 31% of the variance. Predicted attribute by treatment interactions were not found. The findings of an effect for familiarity, with novel material, lends further support to the findings of Brown (1970) and Tobias and Weiner (1963). The inconsistent findings of Wodke et al. (1967) on the interpretation of sequence and familiarity could potentially be explained by the fact that the subjects in their study had been previously exposed to programmed learning. Additionally, low pretest scores alone may not have been sufficient to judge degree of program content familiarity.

While most of the studies on sequencing within a programmed instruction format have employed logical and scrambled orders, Niedermeyer, Brown, and Sulzen (1969) compared three versions (logical, scrambled, and reversed orders) using a guided discovery program on number series originally used by Gagne and Brown (1961). No significant differences were obtained among the three instructional groups for time to complete study and post test scores. The logical sequence group made significantly fewer errors and was the only experimental group to differ significantly from the control group on a ten item test of introductory concepts and on a ten item transfer test of problem solving skills. There was no IQ by sequence interaction; the high IQ group performed significantly better in all three experimental groups. The authors concluded that for short instructional programs sequence of items is probably not a critical variable. Although the authors employed a learning program that had been previously validated to represent a learning hierarchy (Gagne and Brown, 1961), Niedermeyer et al. (1969) did not present any data to substantiate that the skills did represent a learning hierarchy. They did not examine the relation of IQ with changes in learning across time. Spencer and Briggs (1972) also employed logical, scrambled, and reversed instructional orders for a programmed lesson on algebra. Even though the skills had been demonstrated to represent a learning hierarchy, they found no significant differences between the hierarchical, scrambled, and reverse orders on post test hierarchy mastery. An additional finding did suggest that culturally-economically disadvantaged groups receiving the random presentation did best on the retention test.

Using a program to teach mastery of a fraction hierarchy with a terminal skill of the addition of rational numbers with like denominators, Phillips and Kane (1973) examined five instructional sequences. The instructional sequences were derived from item difficulty, Guttman Scalogram Analysis, Pattern Analysis, Correlational Analysis, and the AAAS Approach (American Association for the Advancement of Science). In addition, the 11 subtasks were also ordered according to the usual textbook sequence and were arranged in random order to provide a control group. Each of the 11 items comprised a lesson with differing numbers of frames being written for each lesson. The students were given two pre-tests to select only those students who lack the skills to be taught in the experiment and to exclude students who did not possess sufficient entering skills. The results did not show any significant differences between the seven instructional sequences for achievement in the learning programs, retention, or transfer. Their transfer procedure consisted of administering a test on the day following completion of the program composed of ten problems analogous to the items used to measure mastery of the terminal skill. Their measure of transfer was a measure of generalization across time and stimuli. The logical sequence group did require significantly less time to complete the learning program. Their careful control of pre-instruction knowledge could have limited the obtained results. Tobias (1973) suggests that familiarity may be a significant variable in sequence effect. In this study, the 163 elementary students had approximately similar entry level skills for fraction knowledge.

Making a distinction between terminal objectives at the end of a course of instruction and terminal skills at the end of a sub-unit, Miller (1969) had samples of high school students work through a 96 frame program on matrix arithmetic. The students were either exposed to a condition where the overall order was disrupted or where the overall order of units was maintained but the order within each sub-unit was disrupted. In addition, one-half of the students in each of the four experimental conditions received as the first page of their learning program an outline specifying the definitions and operations to be covered in the learning program. The results indicated that the students who were exposed to learning programs where the macro-order (logical order of the units) was preserved did significantly better than students who were exposed to scrambled presentation of the units. This "prior information variable (provision of a summary sheet at the start of the program)" had no significant effect on learning. The results seem to indicate that the presentation order of frames does not effect learning as long as the order of concepts is preserved. Hamilton (1964) reached a similar conclusion in a study examining logical versus random order of item presentation. These studies would seem to suggest that the ordering of major instructional objectives may be important, but that the instructional order of subordinate skills within an individual unit may not be critical. Miller's (1969) attempt to examine the familiarity variable did not result in any effect. However, giving a learner a list of objectives immediately prior to the beginning of the experiment may not be sufficiently thorough to differentiate subjects on the degree of familiarity with the material.

Briggs and Naylor (1962) and Naylor and Briggs (1963) in their work on task complexity and task organization as it relates to the part-whole training controversy, concluded that, for a task of high complexity and inherent organization, the whole procedure would be most efficient. However, for a task that is relatively unorganized and relatively complex, the part method would be most efficient. The issue of instructional sequencing and its relation to learning hierarchies can be viewed as a variant of the part-whole controversy (Cunningham, 1971). Some of the apparently contradictory findings could be due to applying instructional sequencing and the part method to material more suited to the whole training approach. The focus on careful sequencing of skills and items would be more beneficial for complex, unorganized tasks.

Summary

Niedermeyer (1968) concludes that little evidence exists for the current emphasis given to the careful, precise sequencing of frames and skills within the programmed instruction format. He makes a distinction between "learning sequences" and "instructional sequences" relating the issue to a "means versus ends" question. The sequencing of frames in an auto-instructional program is a means question, while what is to be taught is an ends question. What is to be taught is a much more important question. Tobias (1973) found that only two of eleven studies reviewed on the issue of sequencing provided any support for logical over random orders in program presentation. Briggs and Naylor (1962) provide data that suggest task complexity and organization are important components in sequencing of items.

The literature critiqued in this section did not employ validated learning hierarchies in the examination of the sequencing issue. While this limits the direct relevance to the issue of the need to sequence instruction according to hierarchical order, the sequencing variable has received only minimal support. Individual characteristics of general ability, subject matter familiarity, and pre-instruction knowledge have been shown to influence performance in programmed learning tasks. Miller's (1969) study also suggests that the ordering of major learning units (objectives) was important with the ordering of skills within a learning unit less critical. Much of the cited research (Cunningham, 1971; Niedermeyer, 1968; Mayer, 1977; Miller, 1969) may have been dealing with learning within a single unit rather than investigating learning and instructional sequencing across several terminal objectives.

Models, Issues, and Methods for Validation of Learning Hierarchies

The evidence for the existence of learning hierarchies and their relation to the sequencing of instructional objectives will ultimately be decided empirically. Task analysis, an informal-logical procedure, is generally employed as a starting point to generate a set of skills-tasks that can be ultimately ordered into a learning hierarchy. Lacking a technology for task analysis, Gagne (1962, p. 356) asked the following question: "What kind of capability would an individual have to possess if he were able to perform this task successfully, were we to give him only instructions?" This procedure is systematically repeated until the task is broken down into simpler subordinate skills. The skills must then be ordered for the establishment of a learning hierarchy.

Presently, there are two basically distinctive approaches to the validation of learning hierarchies. One approach, typified by the work of Gagne and others, has come to be referred to as the "psychometric approach." Although there are many variants to this approach, it essentially involves the assumption that a person will not pass an item representing a skill higher in the hierarchy and fail any items representing skills lower in the learning hierarchy. This assumes that the assessment of performance on the skills is not separated temporally to permit any training to occur that could influence performance on one or both skills. This approach usually involves giving a test containing a series of items representing various skills for a hypothesized sequential order to a large number of individuals. The examination of the pass-fail patterns permits the refinement and validation, and reordering of the items and skills they represent into a formal learning hierarchy. There are a number of statistical models and related procedures available for analyzing the data and confirming the hierarchical ordering of the skills. Airasian (1971), Airasian and Bart (1973, 1975), Bart (1974), Bergan (in press), Cotton et al. (1977), Kruz and Bart (1974), White (1974a), and White and Clark (1973) have all critically reviewed existing statistical models and related research and have presented alternatives to deal with the noted psychometric limitations. The procedure described above is generally employed to confirm the "prerequisite-skills hypothesis" which states that a learner cannot fail an item lower in the hierarchy while demonstrating mastery of an item higher in the hierarchy. This approach is, essentially, ordering skills based on a subordinate-superordinate relation.

The second approach to the validation of a learning hierarchy has been referred to as the experimental approach or the "transfer experiment." This paradigm also has several variants including attempting to train a higher level skill with the learner missing one or more prerequisite skills or presenting various skill sequences to groups of learners to ascertain hierarchical effects on learning, retention, and transfer (generalization). While many researchers (Cotton et al., 1977; Resnick, 1973, 1977; Gagne, 1973; White and Gagne, 1974) have stated that the positive-transfer experiment is the more definitive test of a learning hierarchy, this positive-transfer experiment according to Cotton et al. (1977) has not been conducted. Cotton et al. (1977), Linke (1975) and White (1973, 1974a,b,c) have aptly criticized the current "state of the art" in learning hierarchy research for failure to experimentally validate learning hierarchies using the transfer experiment.

In his summary and critique of the empirical literature on learning hierarchies, White (1973, 1974b) identified five problem areas consisting of (a) small sample size, (b) use of loosely defined elements-skills, (c) use of only one question per skill, (d) delays in testing for mastery of skills until the end of the learning program, and (e) failures to take into account errors of measurement. White (1973) additionally observes that exceptions may occur to hypothesized prerequisite skill relations because of (a) errors of measurements, (b) random occurrence of forgetting of skills learned while working through a learning program, (c) errors in construction of the hierarchy, and (d) theoretical failure of the learning hierarchy model.

Gagne (1968, 1973, 1977) has observed that many investigations that have failed to confirm the learning hierarchy model or that have encountered problems with specific hierarchies may have knowledge-information items rather than intellectual skills. The critical components of a learning hierarchy (Gagne, 1973, pp. 21-22):

> (1) They are descriptions of successively achievable intellectual skills, each of which is stated as a performance class (e.g., "Given columns of two-placed numbers and the instruction to add, solves to find the sum"). Each specific skill thus described is placed in a hierarchy in such a way that the skills subordinate to a given skill are hypothesized (or at some point empirically demonstrated) to contribute substantially to the learning of the given skill, in the sense of exhibiting positive transfer to it.
>
> (2) Learning hierarchies provide descriptions only of intellectual skills, not verbal information, cognitive strategies, motivational factors, or performance sets. All of these latter variables are acknowledged to contribute to the learning of any of the intellectual skills described in the hierarchy, but the intention is not to describe them specifically.

(3) In essence, then what each box of the hierarchy describes are only those prerequisite skills that must be recalled at the moment of learning so that this critical internal component of the total stimulus situation will be present.

(4) In theoretical terms, a hierarchy is not intended to be a description of an entire instructional sequence. It indicates to the designer of instruction: What ever else is present in the learning situation (verbal instructions, sets, etc.), these particular prerequisite skills must be available to the learner. This usually means they are recalled by the learner at the time the new learning occurs. Accordingly, a reasonable sequence of instruction can be designed to ensure that these subordinate skills are successively mastered. There is nothing fundamentally immutable about such a sequence, in a theoretical sense; it is basically only a matter of convenience, and some hierarchies (or instructional sequences based upon them) may indeed be found to work better than others, on the average. The theoretical requirement is only that the stated prerequisite skills for any given skill must be available to the learner as a part of the stimulus situation at the time learning occurs.

The distinction between "intellectual skills" and "knowledge (information)" is critical, especially in light of the fact that several studies which purported to investigate learning hierarchies were, in fact, dealing with verbalized knowledge (Gagne, 1973). An "intellectual skill" is a learned capacity that constitutes "knowing how" as contrasted with information which consists of "knowing about" something. The various types of intellectual skills (discriminations, concepts, rules, and higher order rules) are learned capabilities which enable the learner to do various things by means of symbolic representation (Gagne, 1974). Gagne (1974) states that intellectual skills are learned by acquiring prerequisite skills, while knowledge is best learned in a meaningful context (see Ausubel, 1968; Johnson, 1975, for a discussion of learning and meaning).

White (1974a,c) offers a nine stage procedure for validating learning hierarchies:

Stage 1--Define in behavioral terms the element that is to be the pinnacle of the hierarchy.

Stage 2--Derive the hierarchy by asking Gagne's question. What must the learner be able to do in order to learn this new element given only instruction of each element in turn, from the pinnacle element downwards.

Stage 3--Check the reasonableness of the postulated hierarchy with experienced teachers and subject matter experts.

Stage 4--Invent possible divisions of the elements of the hierarchy, so that very precise definitions are obtained.

Stage 5--Carry out an investigation of whether the invented divisions do in fact represent different skills.

Stage 6--Write a learning program for the elements, embedding in it test questions for the elements.

Stage 7--Have at least 150 subjects, suitably chosen, work through the program, answering questions as they come to them.

Stage 8--Analyze the results to see whether any of the postulated connections between elements should be rejected.

Stage 9--Remove all rejected connections from the hierarchy.

Implementing the nine-step procedure outlined above, White (1974b) examined several previously researched hierarchies to demonstrate implementation of his model. He discovered that most rejected connections consisted of verbalized knowledge. He also found that learning hierarchies producing large numbers of invalid connections had not been initially checked by experienced judges in the content areas.

White also noted that many of the validation models utilized procedures that could only remove incorrect connection while being incapable of discovering previously overlooked connections. The interpretation of results of learning hierarchy investigations have been quite subjective. The inclusion of knowledge items along with intellectual skill items would prevent that particular study from being an adequate test of the learning hierarchy model.

White's procedures outlined above generally substantiates Gagne's theory that hierarchies do, in fact, represent patterns of prerequisite intellectual skills leading to the terminal skill or skills inherent in a specific task (Gagne, 1962, 1977). White (1974c) has written that this implies that instructional sequences for subject matter containing intellectual skills should be more effective when they are based on a validated learning hierarchy. Gagne did not directly investigate the relation of instructional sequencing of a hierarchical organized task and has stated that, "I am not sure that a learning hierarchy is supposed to represent a presentation sequence for instruction in an entirely uncomplicated way" (Gagne, 1973, p. 3).

Learning hierarchies would seem to have promising potential in the realm of curriculum for diagnostic and placement assessment (Resnick, 1977; Resnick and Ford, 1978; Resnick et al., 1973). Once a task analysis has been completed and initial validation studies have been conducted, it would be a relatively simple and direct task to construct tests of the elements when they have been clearly defined. The skills would have been ordered which would permit initial placement and subsequent diagnosis to be conducted. Learning hierarchies and their hierarchical tests would also afford a convenient monitoring system to clarify changes in learning across time for purposes of changing the learning program.

Bergan et al. (1976) contend that the model described above and related validation techniques have significant limitations and find the learning hierarchy model quite inadequate. As an alternative, they suggest the use of structural models developed by mathematical sociologists based on Sewall Wright's (1921, 1960) work on path analysis as an alternative to the existing model for representing causal relations among intellectual skills. This would allow for relations among variables to be accounted for in a more complete fashion. Studies by Bergan et al. (1976) and Bergan et al. (1979) utilized multiple regression procedures to conduct structural analysis to determine the direct and indirect effects of individual characteristics and mastery subordinate skills on hierarchical skill performance. Another limiting feature of existing hierarchy models is its inability to describe relations among more than two variables. That is, a skill may be classified as either prerequisite or not prerequisite to another skill.

Bergan (in press) has proposed that the relation among intellectual skills be described in terms of a structural behavioral model. Such a model can be represented by diagrammatic form using arrows to depict relations among variables. Simultaneous equations are employed to describe the hypothesized causal relations. Reports by Bergan et al. (1976), Bergan et al. (1979), Jeske (1978) and Karp (1978) represent empirical studies utilizing structural equations based on regression analysis to examine learning hierarchies and the influence of individual characteristic variables on skill acquisition.

Learning Hierarchies, Instructional Sequencing, and Direct Instruction with Children

As previously mentioned, the majority of the literature dealing with task analysis, learning hierarchies, and instructional sequencing have involved a programmed instruction format and the administration of tests, and have been carried out with subjects eight years of age and older. Cotton et al. (1977) and White (1973, 1974a,b,c, 1976) have succinctly detailed the inherent difficulties in conducting large scale validation studies using young subjects and direct instructional strategies. This has resulted in most learning hierarchy studies being psychometric in nature in that the subjects are tested for mastery of the hypothesized skills with or without having worked through a learning program. The pass-fail relations are then examined to determine the validity of the *a priori* ordering of the skills. While providing an adequate test of the prerequisite skill hypothesis, this approach does not permit a test of the positive-transfer hypothesis. In addition, the examination of the relation of the learning hierarchy order to the optimal instructional sequence is rarely conducted.

Cotton et al. (1977), Gagne (1973), and White (1973) have all emphasized the need to test the positive-transfer hypothesis to establish the validity of a particular learning hierarchy. However, studies taking a psychometrically validated learning hierarchy and teaching the skills from the hierarchy in all possible orders has yet to be satisfactorily conducted.

The research reviewed in this section is limited to studies pertaining to direct instruction with children, psychometric validation studies with young children, or literature involving the subject matter area of mathematics.

Psychometric Validation Studies of Mathematics Curriculum

In what has become the prototypical procedure for investigating hypothesized learning hierarchies, Wang, Resnick, and Boozer (1971) examined the performance of 78 kindergarten children on a battery of basic arithmetic tasks designed to assess knowledge of counting, one-to-one correspondence, and numeration. Using Scalogram Analysis, they found support for the hierarchical ordering of skills within each of the three learning units. In addition, they also found an ordering for the three learning units. The new psychometric ordering was slightly different from the *a priori* identified order for three of seven tasks involving numerals 0 to 5 and also for tasks involving numerals 6 to 10. They also found orderings of skills for the unit dealing with 0 to 5 and for the unit 6 to 10 differed for counting and one-to-one correspondence. The data for this investigation were collected by administering the items for each skill to children individually in a test-like format. While this procedure examined the prerequisite skill hypothesis, no test of the positive-transfer hypothesis could be made.

In their conclusions, Wang et al. (1971) suggest that assessment of *a priori* learning is a valid technique to examine the order in which children acquire skills since none of the children in the study had received formal instruction. Hence, they state that the learning hierarchy established by their study could be viewed as reflecting the natural sequence of acquisition rather than as an articifical sequence imposed by formal schooling. These conclusions are highly questionable since the influence of television and parental instruction were not controlled. Wang et al. (1971) cautions that sequences of behavior based on scaling data do not necessarily directly translate into optimal sequences of instruction. This study made no attempt to examine the contribution of individual characteristics.

In an extension of the Wang et al. (1971) study, Wang (1973) performed another psychometric validation study of the three units from the 1971 study along with three additional units within a basic mathematics curriculum. A series of items for each of the skills within each of the learning units was administered to subsamples from a group of 150 preschool, kindergarten, and first grade children. Wang (1973) was attempting to generate empirical evidence for the cumulative dependencies of the individual skills within each learning unit as well as generating evidence for the interdependence of the six learning units which themselves formed a learning hierarchy. Wang was able to obtain reproducibility coefficients ranging from .934 to .993 from scalogram analysis for each of the six learning units and for the ordering of the six learning units. Resnick et al. (1973) provide an extensive discussion of task analysis and curriculum design for an early mathematics curriculum which includes the mathematics units studied in the Wang et al. (1971) and Wang (1973) studies.

In a much needed improvement in the investigation of learning hierarchies, Bergan et al. (1976) examined the hierarchical ordering of six items comprising unit II of the mathematics curriculum (Resnick et al. (1973). Unit II, which consisted of counting and one-to-one correspondence to 10, consists of six skills which have been psychometrically ordered by scalogram analysis (Wang et al., 1971; Wang, 1973). Bergan et al. (1976), noting that the item order presentation during instruction or testing represents a potential confounding, taught all six skills individually to each child daily for ten days. The order of items was randomized with modeling, practice, and feedback comprising the instructional package. Additionally, another important feature of the Bergan et al. (1976) study was the pretesting of all subjects with only subjects who demonstrated a lack of mastery of the skills being taught included in the study. In effect, this study controlled for prior knowledge and familiarity with the content under investigation.

Utilizing a structural analysis approach (Duncan, 1975), Bergan et al. (1976) derived a resultant learning hierarchy that closely resembled the hierarchy earlier devised by Wang (1973). Their data did suggest that, contrary to the learning hierarchy model, acquisition of subordinate skills facilitated superordinate skill learning without being prerequisite to the superordinate skill. Structural analysis also permitted examination of the positive-transfer hypothesis and was capable of examining the effects of variables outside the learning hierarchy model. In particular, the influence of such exogenous variables as individual characteristics on the acquisition of skills within the learning hierarchy were studied. Bergan et al. (1976) did not examine the relation of the hierarchical order to an optimal instructional order.

In a study that examined the relation of a learning hierarchy to irstructional orders, Caruso and Resnick (1972) employed component analysis procedures and derived a potential learning hierarchy of double classification skills. Employing a matrix classification task, the authors instructed a sample of 26 kindergarten children in either the optimal or reverse sequence. The children taught in the hypothesized sequence learned the complex-terminal task more quickly and subjects who mastered the terminal task demonstrated the greatest amount of transfer to another task similar in task requirements. In analyzing the learning data, Caruso and Resnick (1972) found that no subject mastered superordinate skills without having also mastered all subordinate skills. They were also able to demonstrate positive-transfer in that subjects who were initially trained on level one demonstrated greater mastery of level two than did subjects who were initially trained on level two and then trained on level one skills. Similarly, subjects initially trained at level two and then on level three skills demonstrated greater mastery of level three than did subjects initially trained on level three skills and then on level two skills. The authors also administered a double classification task that was not part of the three level hierarchy and found that substantial transfer was exhibited by subjects who had successfully mastered the most complex task of the hierarchy. This transfer was independent of instructional order.

Although this study is an improvement over previous research, they did not perform statistical tests for the transfer part of the study. Additionally, small sample size, investigating only two of six possible instructional orders, and experimental selection bias limit interpretation of the results. In particular, as the study progressed, less able subjects (poor learners) were effectively screened out of the study resulting in more able children, who completed the study, compared with the less able children who only completed portions of the study.

In another attempt to examine transfer relations using a matrix classification task, Resnick, Siegel, and Kresh (1971) trained 27 kindergarten children on tasks of inferring and placing to determine if they were hierarchically related. Subjects who learned the simpler task (placing) first made fewer errors in trials to criterion on the more complex task than did subjects who received the reverse order. While instances of subjects mastering the more complex task first when it was initially taught appears to contradict the learning hierarchy hypothesis, most subjects who mastered the more complex task also demonstrated mastery of the simpler (subordinate) task even though they were never specifically taught the subordinate task. Lack of measures during the learning process and failure to conduct statistical tests limit the value of this study. This study is suggestive that learning hierarchies do exist and that they may be representative of optimal instructional orders. The small sample size may have resulted in insufficient power to detect a significant difference that may have existed.

In a study that was conducted to determine the most efficient sequence of instruction for teaching three set relations ("equivalence," "greater than," and "less than"), the optimal sequence was evaluated by Uprichard (1970). Using a sample of 32 preschoolers assigned to eight conditions, he examined the effectiveness of each of six possible instructional orders and had two control groups with one receiving only weekly criterion tests and the post test and the other control group receiving the post test. His results suggested that the instructional sequence of "equivalence," "greater than," and "less than" is the optimal instructional order. He also examined positive-transfer and found that the optimal instructional order also resulted in the highest score on the transfer test. During the instructional part of the study, the author periodically assessed each child's knowledge on all three skills to examine the prerequisite hypothesis and found very few instances where the subjects mastered "greater than" or "less than" without already having mastered "equivalence." This prerequisite skill relation was independent of the instructional order.

Uprichard's (1970) study would generally seem to be supportive of the existence of learning hierarchies providing evidence for both the prerequisite-skill hypothesis and the positive-transfer hypothesis. It also suggests that the optimal instructional sequence was the same as the hierarchical order. This study, however, has several limitations including, the small sample size (N=32; four per condition), stopping the experiment when one group obtained criterion performance, failure to control for the amount of time a given group was exposed to instruction, failure to specify and monitor instructional procedures, and inability to employ inferential statistics. Additionally, no test was performed to demonstrate the existence of a learning hierarchy.

Summary

The literature investigating the existence of learning hierarchies and instructional application with young children is limited. The primary subject matter areas has been mathematic skills. The majority of the research has been psychometric in nature assessing the child's mastery of skills based on prior learning (Wang et al., 1971, Wang, 1973). While the prerequisite skill hypothesis has received some attention, the positive-transfer hypothesis has been examined primarily by Uprichard (1970). Cotton et al. (1977) has suggested that a test of this hypothesis is critical and that a satisfactory experiment is yet to be conducted.

With the exception of the Uprichard (1970) study and the study by Caruso and Resnick (1972), little attention has been devoted to effect of order of presentation of hierarchical organized skills on learning and positive transfer. The Uprichard and Caruso and Resnick studies do suggest that instructional orders approximating the hierarchical order are more efficient and that irregardless of the order in which the skills are taught, the prerequisite skill hypothesis remains valid.

None of the studies examined the influence of individual characteristics on skill learning, and none of the studies did an adequate test of retention of learned skills.

Empirical Literature Related to the Influence of IQ on Learning of Academic Tasks

While the interest in the relation of general ability to various indices of learning has waxed and waned since Binet's development of an assessment instrument that has come to be called an "intelligence test," the literature is surprisingly sparse regarding the relation, if any, of intelligence measures to measures of actual school learning (Briggs, 1968; Lavin, 1965; Sewell and Severson, 1974; Yeager and Lindvall, 1967; Zeaman and House, 1967). Mercer (1977) stated that she was unable to find a single study that actually investigated the effects of IQ on actual measures of learning in school settings. Stevenson (1972) in his well known work on children's learning does not cite any literature examining the performance of children in educational settings or that investigate the relation of IQ to types of learning that occur in classroom settings. He and his colleagues (Stevenson et al., 1968; Stevenson et al., 1976) examine the interrelations of a variety of measures including teachers ratings, IQ, learning tasks, tests of academic achievement, etc., and their predictive utility. However, they do not include any measures of classroom based learning. Staats (1971) in his chapter on hierarchical learning and the cumulative acquisition of personality makes reference to the potential usefulness of the construct of intelligence in predicting how well a child will learn when confronted by a new task. However, Staats provides no empirical support for his views. Stevenson (1972) also fails to provide support for the view that intelligence is a relevant measure in school settings.

Among the difficulties encountered in the literature on measures of general ability and its relation to school learning measures are confusion over what the term IQ means and what instrument to employ to assess this construct, and lack of agreement on what constitutes adequate, relevant measures of school learning. Resnick and Glaser (1977) employing what they refer to as the "layman's" definition of intelligence define IQ as the ability to learn, implying that this means the ability to learn important things from one's environment. "Intelligence is precisely the ability to acquire new abilities under less than optimal environmental conditions, conditions where the appropriate solution routines are not directly prompted or specifically taught" (Resnick and

Glaser, 1977, p. 228). Intelligence becomes "the ability to acquire new behaviors in the absence of direct or complete instruction, and this ability involves processes that can facilitate the transition from simpler to more complex cognitive performance" (Resnick and Glaser, 1977, p. 227). When children are exposed equally to instruction and differences in learning occur, it is precisely this difference in amount learned under approximately equivalent conditions of exposure that makes intelligence tests viable as predictive instruments.

Gagne (1962, 1968) has argued that intellectual development is the result of the cumulative effects of learning. This proposition is highly similar to the model promulgated by Staats (1971) who suggests that a child's intelligence at any given time consists of the various reportoires of skills that the child has learned. How a child will learn a skill is not governed by internal-organic-mentalistic qualities, but is governed by the basic behavioral reportoires that he brings to the task. The views of Gagne, Staats, and Resnick and Glaser would then suggest that tests of intelligence are, in fact, little more than a content referenced test sampling a broad range of prerequisite skills. Without relying on a construct of intelligence, the predictive capability of a test of intelligence can be explained on the basis that it is a test of general prerequisite skills. Indeed, the predictive capability of tests of intelligence for a variety of learning tasks remains moderately high (Henderson et al., 1973; Sewell and Severson, 1974; Stevenson et al., 1968, 1976; Zigler and Trickett, 1978). One would expect that in a study instructing young children in an academic task such as counting or one-to-one correspondence, that measures of intelligence would correlate moderately with the placement test and with initial learning. In one study that did examine the relation of intelligence (WISC-R), Sewell and Severson (1974) found that intelligence related moderately to sight vocabulary acquisition, paired associate learning, and post achievement test scores for reading and math for a sample of 60 first grade Black students.

Gagne (Gagne and Paradise, 1961; Gagne, 1962; Gagne and Staff 1965) theorized that if learning programs were of perfect effectiveness, everyone would pass all the component elements and related tasks in the hierarchy resulting in the variance being reduced to zero with the resulting correlations on tests of various abilities with competencies in the learning hierarchy being zero. However, to the extent learning programs and instructional procedures are not maximally effective, the probability of mastering various skills will be increasingly dependent on individual characteristics such as basic abilities and previous learning. Given less than perfect instructional conditions, the individual's probability of mastering each skill will be increased to the extent of his scores on various measures of basic ability (IQ).

It appears that only one study (Bergan et al., 1979) has been conducted to directly test the hypothesis that intelligence will increase or decrease in predictive utility based on adequacy of instruction. Bergan et al. (1979) examined the influences of IQ and prerequisite skill knowledge on the acquisition of a terminal skill of having a learner identify which of four pictures contained a stated number of objects. In addition, this study also varied instructional strategies using parents as teachers. Using structural analysis, the authors were able to demonstrate that IQ had a direct influence on mastery of the prerequisite skill, but had no direct influence on acquisition of the terminal skill. Chronological age also had greater influence on acquisition of the subordinate skill. Prerequisite skill mastery and the experimental variable of training variation had the great effect of terminal skill mastery. This study also demonstrated that observation of a model and practice with feedback was superior to merely observing a model describe the tasks without the observer engaging in practice or receiving feedback. The authors conclude that with respect to cognitive skills related to early mathematics learning, mastery of terminal skills is dependent on how learners are taught and on previous mastery of prerequisite skills. The importance of individual characteristics on terminal skill acquisition is reflected through subordinate skill learning rather than directly influencing superordinate skill mastery.

Examining the effect of item scrambling of the third unit of a four unit self-instructional program in geometry, Levin and Baker (1963) found no differences between experimental and control groups with a sample of 36 second graders on measures of acquisition, retention, and transfer. The relation of IQ to pretest, acquisition and post scores were all significant. The results suggest that high IQ children tended to do better in the logical ordering group while low IQ children tended to perform better in the scrambled instructional order. The best predictor of performance on the post-test was pre-test performance.

In an exploratory investigation of the relation of IQ to various measures of school learning, Yeager and Lindvall (1967) note that there is a paucity of studies examining the concept of rate of learning and even fewer studies comparing individual characteristics such as IQ to actual learning measures. Using a large sample of children in grades three to six, the authors correlated measures of IQ to five measures of learning within an individually prescribed instruction approach to teaching mathematics. IQ was found to relate moderately to initial placement tests for each of the math units (range from .39 to .81), and related significantly to one of eight units for time to complete the unit. IQ did not significantly correlate with the number of units completed during the academic year and failed to correlate significantly with amount of material left to complete in each unit.

It should be noted that the curriculum materials in reading and mathematics were part of a series of studies on validation of learning hierarchies and were therefore the subject of precise sequencing. The authors concluded that rate of learning is not a general characteristic of learners and is more influenced by the specific learning task. The predictive utility of IQ and its contribution to learning would also seem to be questionable in this study. These results are in agreement with the findings of Bergan et al. (1979) and Gagne (1962). When students are interacting with effective instructional procedures and content, the contribution of this individual characteristic is dramatically reduced. The correlation of IQ with the initial placement test further supports this contention. IQ correlates well with prior learning, but not with appropriate directed instruction.

In a study by Skanes et al. (1974), the investigators examined the ability to learn to solve letter series problems within low and high transfer practice conditions. They also studied the influence of pretest scores and IQ on ability to learn the letter series problems using a sample of 2,097 students in grades five through nine. Their IQ measures were obtained from the Otis Quick Scoring Mental Ability Test and the Ravens Progressive Matrices. Using regression analysis, the authors found that Raven's IQ was positively related to ability to transfer. The study did not reveal similar results for the Otis IQ. They also found that the use of a pretest had differential influence on subsequent performance in the learning task. A pretest effect was obtained using Otis IQ with high Otis IQ students benefiting while low Otis IQ students experienced lowered performance in the learning task. No similar effect was obtained for the Raven's IQ. The authors conclude that IQ is an important variable to consider in investigations involving learning and that the type of instruments utilized to obtain IQ scores are also in need of critical examination.

Summary

The literature reviewed in this section appears to support the hypothesis that intelligence is moderately related to achievement test scores for reading and math. The correlations hold up across ages, grades, and ethnic groups although the correlations decrease in magnitude for ethnic minorities. Only four studies (viz., Bergan et al., 1979; Levin and Baker, 1963; Yeager and Lindvall, 1967; Sewell and Severson, 1974: and Skanes et al, 1974) examine the variable of intelligence in conjunction with measures of actual learning that resembles activities found within a classroom context. Only two of the studies (Bergan et al., 1979; Levin and Baker, 1963) employed learning activities involving a sequencing variable. The data seem to suggest that general measures of ability and previous learning correlate with placement, initial learning in an instructional program, but that given optimal educational programming, these antecedent-individual characteristic variables decrease in importance while mastery learning within the educational task increases in importance. The study by Yeager and Lindvall (1967) was an especially good demonstration of the relation of IQ to entry level in an individually prescribed instructional program with learning rate being generally not related to IQ.

One would predict that in a study examining the influence of IQ and the related score of MA with performance in a hierarchical organized curriculum such as mathematics, that the relation of IQ and MA while having strong initial relation to performance, would show a gradual decrease in correlation with measures of learning provided the instruction and curriculum content were optimal.

Evidence for the Efficacy of Modeling, Feedback, and Guided Practice as Instructional Techniques

The efficacy of modeling procedures in altering the behavior of children and adults has received extensive documentation in both analog and field studies. The social learning theory developed by Bandura (1969, 1971, 1977) has been utilized to explain the acquisition of a variety of behaviors in children. Bandura (1969) posits four sets of processes in the modeling paradigm consisting of attentional, retentional, reproduction, and motivational processes. For vicarious learning to occur the observer must attend to and retain the behavior that is modeled. For the behavior to be performed by the observer, the observer must be motivated and capable of reproducing the observed response. Incentive conditions, observer characteristics, and properties of the modeling cues and of the model may all be considered attention controlling variables. The retentional processes are aided by coding and the rehearsal of the modeled stimuli. Motor reproduction processes are concerned with the observer's actual reproduction of the modeled activity. The final component of the observational paradigm is the motivational process. This component determines whether or not an observer who has acquired and retained a modeled behavior will, in fact, reproduce the modeled behavior. Such factors as societal sanctions and anticipated reinforcement control the probability of the response.

Modeling has been of demonstrated utility in teaching children language development (Zimmerman and Rosenthal, 1974), moral judgment (Bandura and McDonald, 1963), equivalence (Rosenthal and Kellogg, 1973), conservation (Henderson, Swanson, and Zimmerman, 1975), seriation (Jeske, 1978; Swanson, 1976), creativity (Zimmerman and Dialessi, 1973), fractions (Karp, 1978), and divergent responding (Belcher, 1975). Zimmerman and Rosenthal (1974) provide an extensive review and discussion of the empirical literature on modeling. These and other studies have adequately substantiated that modeling is an effective educational strategy.

Jeske (1978) found that modeling plus performance (practice) plus feedback had a significant influence on the acquisition of seriation skills in preschool children. His data suggest that modeling plus performance and modeling plus performance plus feedback were both superior to modeling along when teaching young children seriation tasks. Ronning (1977) utilized modeling to teach a formal operations task to first-, third-, and fifth-graders. While the author found that performance on the task of identifying the correct number from the population of numbers 1 to 100 increased with age, he also found that the question asking strategy to arrive at the correct number could be significantly improved via modeling. Another source of support for using modeling with young children to teach quantification skills comes from a study by Henderson et al. (1975). Utilizing a televised modeling format, they were able to successfully teach young Papago Indian children seriation responses. This study demonstrates that modeling need not be an actual adult but can be a televised model.

In a study examining the effects of training teachers of five year olds in the use of modeling, Zimmerman and Kleefeld (1977) trained kindergarten teachers to teach seriation tasks within a modeling format. Their data indicated that teachers trained in modeling techniques were more effective in teaching seriation and made greater use of modeling than the untrained group who were not effective in teaching seriation. Teachers use of specific modeling behaviors correlated with success in teaching seriation.

Bandura (1977) points out that one of the major influences of modeling is to transmit feedback to observers on how responses can be acquired. The information may be communicated by physical demonstration, pictorial demonstration, or verbal description. Feedback as described by Kulhavy (1977) refers to any of numerous procedures that are used to tell a learner if an instructional response is correct or incorrect. Modeling can serve to both communicate the desired response to an observer and to inform the observer as to the correctness of his modeled response.

Karp (1978) examined the effects of behavioral (demonstration), symbolic (verbal), and a combination of behavioral and symbolic modeling on a group of 72 children using a task requiring the acquisition of four skills in a fraction hierarchy. Her results suggested that behavioral modeling and behavioral plus symbolic modeling were both superior to symbolic modeling alone and to a control group that was given no feedback following their responses.

Summary

Based on the current literature on instruction, modeling would appear to be a critical component of any effective teaching procedure. Additionally, the research by Jeske (1978), Swanson (1976), and Karp (1978) would seem to strongly indicate that an instructional package consisting of modeling, practice, feedback with guided practice and praise would seem to maximize the effectiveness of the learning environment. The need to maximize the attention of the observer on the modeled task has been aptly documented by Bandura (1969). The use of Sesame Street Finger Puppets has proved to be highly effective (Bergan et al., 1976).

CHAPTER 3

METHOD

Subjects

One hundred and thirty eight children from five day care centers in Tucson, Arizona were selected to participate in the study. These children were selected from a larger sample of 206 children on the basis of pretesting which revealed that they possessed none of the arithmetic skills targeted for investigation and obtained a score of five or less on the pretest. Of the 138 children who began this study, 108 children finished, including 53 girls and 55 boys ranging in age from 30 months to 58 months. The children came from middle and lower class areas of the community. Seven of the children were Black, two were Vietnamese, 18 were Mexican American, and 81 were Anglo in ethnic background.

Of the 30 children who did not complete the study, six were dropped because of failure to cooperate, five children experienced irregular attendance at their respective day care center, and 19 children dropped out of their day care centers during the course of the study. Parental permission was obtained on all children who participated in this study.

Experimenters

The experimenters consisted of 31 undergraduate students at The University of Arizona enrolled in introductory psychology or education courses. They ranged in age from 18 to 36 years. All individuals serving as experimenters were volunteers who had the option of receiving independent study credit. None of the experimenters received any financial remuneration.

Each experimenter underwent two hours of simulated training in groups of seven or eight at the University and one hour of actual practice in the day care center where they would later be conducting the experiment. The simulated training involved examining a complete set of teaching and recording materials, receiving a verbal explanation, observing the author model the teaching and data recording procedures, and practicing with a fellow experimenter as a subject. The training at the day care center consisted of the author modeling the procedures with a child, having the experimenter conduct the teaching and data recording procedures, and providing corrective feedback. The children used during training were not included as part of the sample. Experimenters were randomly assigned to day care centers and worked with an approximately equal number of children in each experimental condition. During the course of the study, each experimenter was observed twice weekly by the director of the research or his assistant to ensure adherence to specified procedures and to guard against experimenter drift.

Materials

The materials consisted of Sesame Street Finger Puppets, plastic chips of red, white, and blue colors, and 13 stimulus cards containing animal, insect, and toy figures. During an instructional session with an individual child, three puppets, up to ten chips of the same color, and selected stimulus cards, were utilized depending on the task and experimental phase.

Tasks

Four sequential learning tasks comprising a portion of the counting and one-to-one correspondence unit dealing with numerals from _1_ to _10_ taken from Unit II of a hierarchically sequenced introductory mathematics curriculum (Resnick et al., 1970, 1973) were used. Skill _A_ required the child to recite the numerals in order from _1_ to _10_. Skill _B_ required the child to count a set of movable objects of a stated quantity moving the objects out of the set as they were counted. Skill _C_ required the child to count a fixed, ordered set of objects on a card, touching each object as it was counted. Skill _D_ required the child to count a fixed, unordered set of objects, touching each object as it was counted. There were six teaching trials for each skill and seven generalization trials for each skill. Appendix A provides an example of the instructions for each of the skills and a list of the tasks.

Procedures

A pretest for the four skills (Wang, 1973) was administered to each child to determine eligibility for participation in this study. The 138 children, who obtained a score of five or less on the test and were unable to recite the numerals from 1 to 10, were randomly assigned to one of six experimental groups. There were 23 children assigned to each experimental condition. Children in each group received instruction in each of the four skills with the order of skill instruction being determined by their group membership. The instructional procedures were conducted individually with each child being taught daily for eight to twelve minutes at the day care center that the child attended.

Teaching Package

The instructional procedures involved the use of three Sesame Street Finger Puppets of the child's choosing and appropriate task materials. Two of the finger puppets were used by the experimenter with the third puppet being used by the child when he practiced the task being modeled by the puppets the experimenter used. The teaching procedures which were adapted from Bergan et al. (1976) were as follows: (1) The child's attention is focused on the task that is to be modeled; (2) The task to be learned is stated by the first puppet; (3) The first puppet models the task for the second puppet and the child; (4) The second puppet correctly performs the modeled task and is praised by the first puppet; (5) The first puppet requests the child and his/her puppet to perform the task just like they did: (6) The child receives feedback and praise for correct performance or feedback and guided practice for incorrect performance; (7) The next item for the skill is then taught beginning with step 1; (8) After the six teaching trials have been conducted, the child's learning is assessed with seven different tasks (generalization trials); no modeling or feedback is given during this generalization phase. This sequence completes an individual instructional session for a particular child. Appendix A contains the direction for the instructional procedures for each of the four skills.

Experimental Conditions

Each experimental group represented one of the six possible combinations of the skills A, B, and C with skill D always occurring last in the instructional sequence for all six conditions. For example, condition I consisted of the instructional order A (reciting the numerals to 10), B (counting a set of movable objects), C (counting fixed, ordered sets of objects), and D (counting fixed, unordered sets of objects). Condition II consisted of the skill order A, C, B, D; condition III consisted of the order B, A, C, D; condition IV consisted of the order B, C, A, D; condition V consisted of the order C, A, B, D; and condition VI consisted of the order C, B, A, D. Skill D was always taught during the last week of the experiment of each condition to provide for an empirical test of the transfer-hypothesis. Table 1 provides a layout of the study.

Each child was instructed in each skill until reaching a criterion of two perfect sessions on the generalization trials or for five consecutive days, which ever was shorter.

Table 1. Format for the Conduct of the Study.
Order of Skill Instruction by Group

Skill	Group Order	Week 1	Week - Skill Being Taught Week 2	Week 3	Week 4
1.	A B C D	A	B	C	D
2.	A C B D	A	C	B	D
3.	B A C D	B	A	C	D
4.	B C A D	B	C	A	D
5.	C A B D	C	A	B	D
6.	C B A D	C	B	A	D

Measures

In addition to the pretest mentioned above, the individual characteristics of intelligence and chronological age were also utilized. The measures of intelligence (IQ) and mental age (MA) were obtained from the administration of the Slosson Intelligence Test (SIT) and the Peabody Picture Vocabulary Test (PPVT). The SIT was administered during the second week of the study with the PPVT being administered during the third week of the study. The two measures of intelligence were administered by the research director or his assistant. None of the experimenters had access to the intelligence test data.

Each child's performance on the targeted skill was recorded daily during each instructional session. Appendix B contains an example of a recording sheet used for each week of the study. Additionally, provision was made for systematically assessing (probing) the child's acquisition of the three skills not being taught during that week. The probing procedures followed the suggestions of Horner and Baer (1978) using the multiple probe design. The multiple probe design, which represents a variant of the multiple baseline design was developed to be utilized when the continuous collection of baseline data could be potentially reactive or excessively cumbersome. Studies that involve the repeated teaching of intellectual skills present problems of reactivity since the assessment of skilled performance represents a practice trial. The utilization of this modified multiple baseline design permits a detailed examination of an individual child's performance while minimizing the reactive effects. This design permits one to examine the changes in skill mastery of the untargeted skills as a function of instruction in a particular skill within a learning hierarchy.

The measures of skill acquisition were gathered daily for the skill to be taught for both learning and generalization trials, with probing of the three remaining skills being done on a less frequent basis according to the schedule determined from the multiple probe design. The probes consisted of administering the generalization trials for the skills in question.

CHAPTER 4

RESULTS

The data for the instructional sequencing experiment were analyzed using a 6 (treatment) x 5 (trials) repeated measures analysis of variance for equal n's. Descriptive statistics for each of the four skills by week including two week retention scores are presented in Table 2. Descriptive statistics for skill by group for each week and the two week retention test are presented in Table 3. A summary of the analysis of variance for repeated measures is presented in Table 4.

The results of the analysis of variance did not yield significant results for groups ($F = 2.18$, df 5, 102, $p < .0607$) and for groups x trials ($F = 1.11$, df 6, 408, $p < .3364$). Significant effects were found for trials ($F = 158.2$, df 4, 408, $p < .0001$). Since no significant effects were obtained for groups, post hoc tests for significance were not performed. The instructional order did not have a significant effect on rate of learning nor on the two week retention test.

To examine the effects of order of instruction of the skills on the acquisition of the terminal skill D, another 6 (treatment) x 5 (trials) repeated measures analysis of variance for equal n's was conducted. Descriptive statistics for skill D by group are presented in Table 5 for each week of the study and for the two week retention test. A summary analysis of variance for repeated measures is presented in Table 6.

Table 2. Means and Standard Deviation for Mathematics Skills _A_, _B_, _C_, and _D_ for Weeks 1, 2, 3, 4, and Two Week Retention Test

Skill	Week 1		Week 2		Week 3		Week 4		Retention	
	X	SD	X	SD	X	SD	X	SD	X	SD
A	2.52	3.17	3.76	2.95	4.60	2.52	5.24	2.14	5.44	2.03
B	3.00	2.33	4.06	1.94	4.95	1.79	5.26	1.56	5.52	1.59
C	2.54	2.25	3.77	2.20	4.78	2.07	4.76	1.76	5.07	1.68
D	1.58	1.83	2.27	2.13	2.96	2.17	3.70	2.15	4.31	2.16

Table 3. Means and Standard Deviations for Week 1, Week 2, Week 3, Week 4, and Retention Trials

Group	Week 1		Week 2		Week 3		Week 4		Retention	
	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
1 (A B C D)	8.06	7.17	13.28	5.79	14.78	5.89	17.99	5.20	19.56	5.31
2 (A C B D)	8.50	9.59	11.85	9.69	15.94	8.51	16.06	7.94	17.94	7.65
3 (B A C D)	10.22	9.12	15.77	8.16	18.83	7.88	21.16	7.01	20.61	7.62
4 (B C A D)	12.44	8.63	17.16	8.31	20.56	6.77	22.22	4.89	24.50	4.53
5 (C A B D)	11.39	8.27	13.11	8.23	17.28	7.45	20.72	5.68	22.56	5.41
6 (C B A D)	7.56	8.51	11.61	8.40	13.44	7.64	15.83	7.91	16.99	8.02

Table 4. Instructional Sequence: Analysis of Variance

Source	df	MS	F	p
Between Groups				
Treatment	5	502.49	2.18	.0607
Error	102	229.81		
Within Groups				
Trials	4	1971.79	152.80	.0001
Groups x Trials	20	14.80	1.11	.3364
Error	408	12.90		

Table 5. Means and Standard Deviations for Skill D by Group for Week 1, Week 2, Week 3, Week 4, and Two Week Retention Test

	Week 1		Week 2		Week 3		Week 4		Retention	
Group	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD	$\bar{X}$	SD
1	1.28	1.49	2.00	2.09	2.11	1.68	3.17	1.77	3.89	1.75
2	1.44	2.01	1.89	2.03	2.89	2.22	2.83	2.04	3.72	2.29
3	1.67	1.91	2.89	2.42	3.83	2.52	4.56	2.10	4.56	2.28
4	2.39	2.17	2.89	2.17	4.17	2.09	4.89	1.64	4.72	1.47
5	1.72	1.53	2.22	1.89	2.78	1.80	4.22	1.84	4.72	1.88
6	1.00	1.72	1.72	2.11	2.00	1.97	2.56	2.41	3.28	2.49

Table 6. Skill D: Analysis of Variance

Source	df	MS	F	p
Between Groups				
Treatment	5	45.58	2.918	.0166
Error	102	15.62		
Within Groups				
Trials	4	128.57	113.143	.0000
Groups x Trials	20	2.01	1.760	.0022
Error	408	1.14		

Significant effects were found for groups ($F = 2.918$, df 5, 102, $p < .0166$), for trials ($F = 113.143$, df 4, 408, $p < .0000$), and for groups x trials ($F = 1.760$, df 20, 408, $p < .0022$). Tukey pairwise comparisons indicated no significant differences among groups at the end of weeks one and two for performance on skill D, but did indicate significant differences between groups for weeks three and four and for the two week retention test. Tables 7, 8, and 9 indicate the results for the Tukey Tests (Kirk, 1968; Myers, 1972) between each pair of means.

As Table 7 indicates, groups 3 and 4 were significantly different from groups 6 and 1 with group 4 also differing significantly from groups 2 and 5. Table 8 provides the results of all pairwise comparisons for week four showing that groups 3 and 4 differ significantly from groups 6, 2, and 1. On the two week (Table 9) retention test for skill D, group 4 differs significantly from groups 6, 2, and 1. Groups 5 and 3 also differ significantly from group 6. These results seem to reflect that group 4 which consisted of the skill instructional order of B, C, A, and D was generally more effective in facilitating acquisition of skill D than groups 6, 2, and 1. The results do not lend support for conducting instruction according to prerequisite skill order. Indeed, for this study which utilized basic counting skills, starting with some skill other than rote counting seems to be more effective in facilitating acquisition of skill D (terminal skill) than following the hierarchical order. While there was a significant difference in terms of mastery of the terminal skill, there were no significant differences between groups for mastery of the entire hierarchy.

Table 7. Tukey Test for Skill D for Week 3

	1	5	2	3	4
6	.11	.78	.89	1.83*	2.62*
1		.67	.79	1.72*	2.06*
5			.12	1.05	1.39*
2				.94	1.28*
3					.34

$^{*}p < .05$

Table 8. Tukey Test for Skill D for Week 4

	6	2	5	3	4
1	.39	.66	2.05*	2.36*	2.62*
6		.27	1.66*	2.00*	2.33*
2			1.39*	1.73*	2.06*
5				.64	.67
3					.33

$^{*}p < .05$

Table 9. Tukey Test for Skill D on Two Week Retention Test

	2	1	3	5	4
6	.44	.61	1.48*	1.44*	2.44*
2		.17	.84	1.06	2.06*
1			.67	.83	1.83*
3				.16	1.16
5					1.00

*$p < .05$

A structural analysis was also performed from which a structural equation model was derived (Duncan, 1975). This model describes causal relations among various exogenous (individual characteristics) variables and scores (endogenous variables) for the four counting skills. The data were analyzed for each series of measures (weeks 1, 2, 3, 4, and retention test) to assess changes in the structural equation model across time. Ordinary least squares regression (Kerlinger and Pedhazur, 1973) was utilized to estimate path coefficients to describe the direct effects of exogenous and predetermined variables on dependent variables within the model. In addition, the regression analysis also revealed the amount of variation in performance on dependent variables which was unaccounted for by exogenous and predetermined variables. Four multiple regressions were performed to derive the path diagrams for each week of the study and on the two week retention test. For each week of the study and for the retention test: (1) task A was regressed on age, pretest score, and

Slosson mental age; (2) task B was regressed on task A and CA, PT, and MA; (3) task C was regressed on task B, task A, CA, PT, and MA; and (4) task D was regressed on task C, task B, task A, CA, PT, and MA. In each of the analyses, the exogenous variables (age, mental age, and pretest score), which were presumed to not be predetermined by other variables in the model, were entered into the regression equations first. Each of the skills was entered according to its position in the learning hierarchy previously ordered by multiple scalogram analysis (MSA) (Resnick et al., 1973). Skill A was entered before skill B because skill A occurred before skill B in the MSA hierarchy.

Figures 1 to 5 provide path diagrams which summarize the results of the structural analysis. Only those paths representing coefficients differing significantly from zero are shown in the diagrams. The exogenous variables include age, MA derived from the SIT, and pretest scores on the four skills comprising the hypothesized hierarchy. Arrows represent the direction of causal relations in each of the five models. Path coefficients specifying the magnitude of direct effects of variables within the model are given in the parentheses above the arrows. The square of each of the disturbance terms gives the proportion of variance in each of the dependent variables unaccounted for by model variables. As can be seen in Figures 1 to 5, a substantial amount of variation was unaccounted for in the initial path analysis following one week of the experiment, with the amount of unaccounted for variance being gradually reduced with each successive path diagram. Since only paths with coefficients significantly different from zero are shown, exogenous variables such as IQ, and sex are now shown. The increasing amounts of

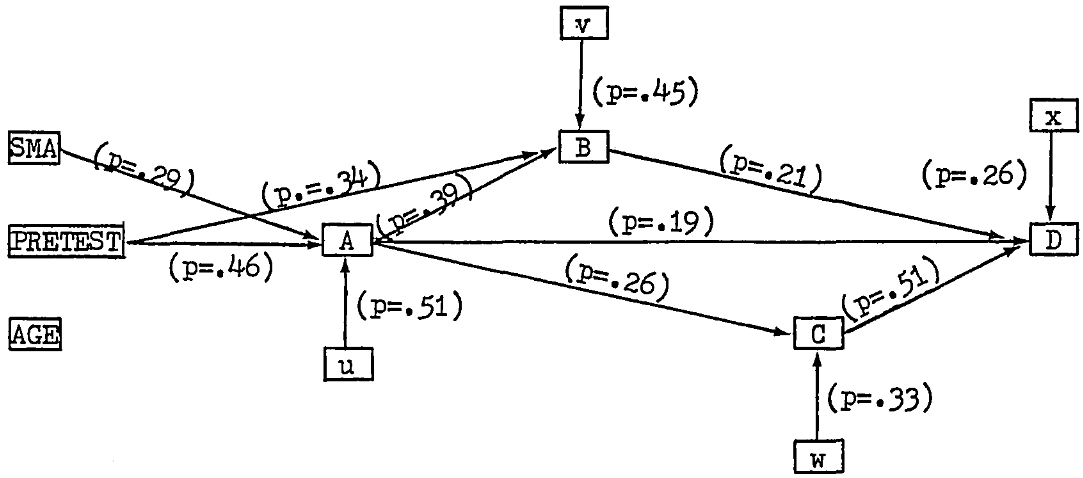

Figure 1. Path Diagram Describing Statistically Significant Relations for Skills A, B, C, D, for Week 1

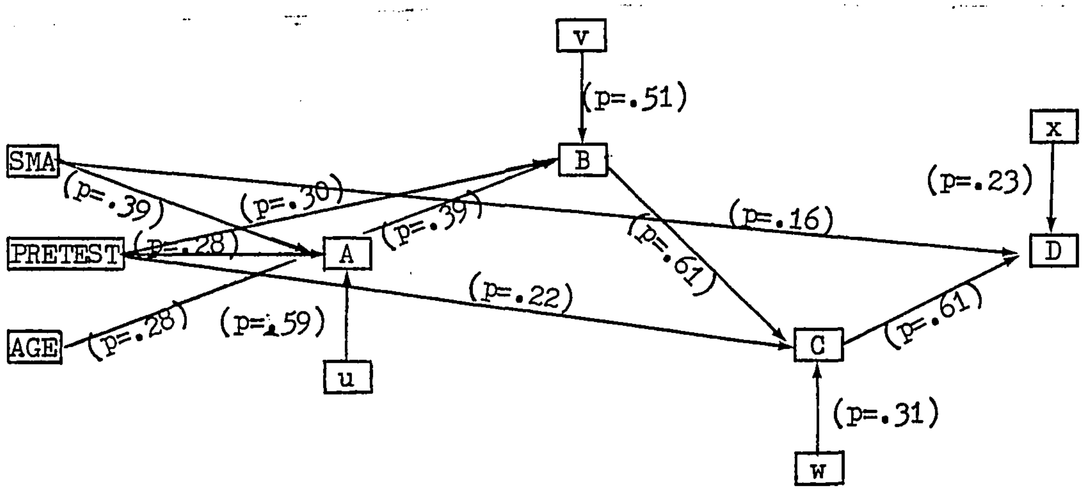

Figure 2. Path Diagram Describing Statistically Significant Relations for Skills A, B, C, D, for Week 2

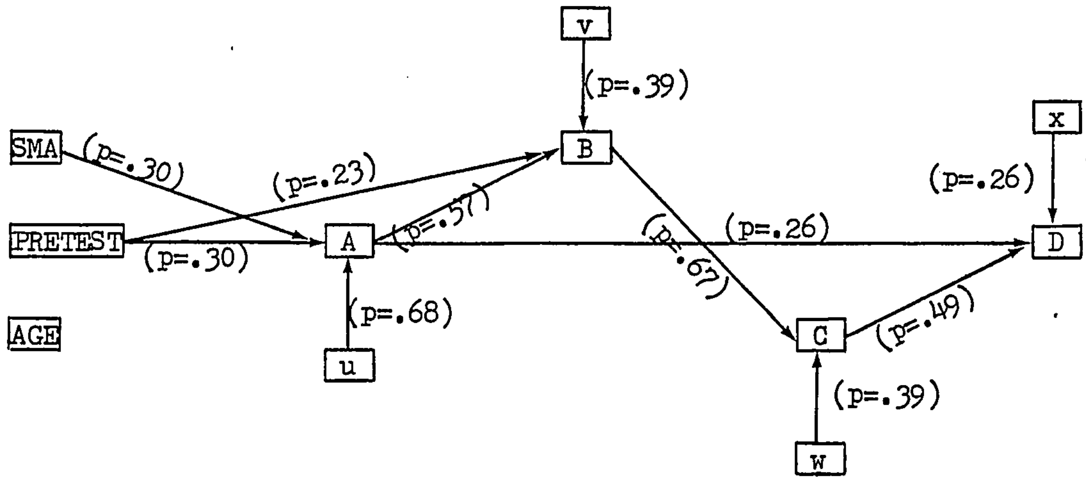

Figure 3. Path Diagram Describing Statistically Significant Relations for Skills A, B, C, D, for Week 3

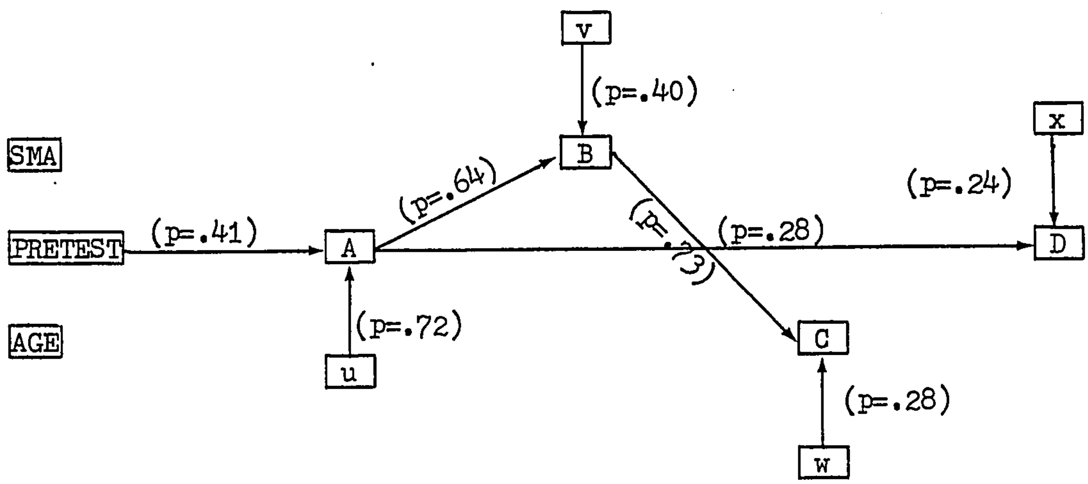

Figure 4. Path Diagram Describing Statistically Significant Relations for Skills A, B, C, D, for Week 4

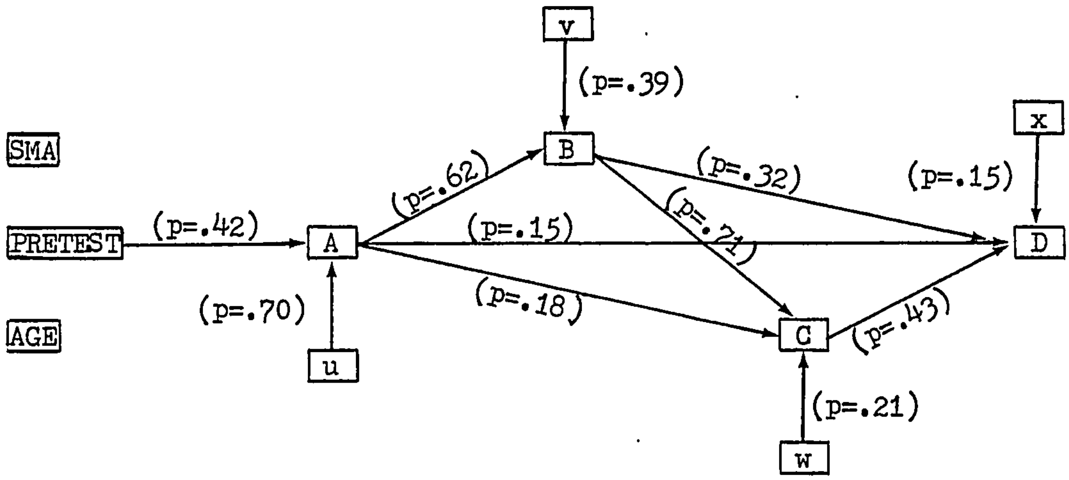

Figure 5. Path Diagram Describing Statistically Significant Relations for Skills A, B, C, D, for the Two Week Retention Test

variance unaccounted for on skill A was the result of only entering individual characteristic variables for regression on skill A. As the study proceeded, increasing amounts of variance were being accounted for by the previous week's skill mastery which is not reflected in the structural equation for skill A in the analysis.

Table 10 presents Pearson Product Moment correlations among selected variables under investigation in the present study. One trend that appears consistently is the decrease in the size of the correlation coefficients between various individual characteristic variables and the mastery scores reported across time. Age, both IQ measures, number of days in study, and pretest score all show decreases across time. The two mental age scores also appear to decrease in correlation with the criterion measures across time but the trend is less clear.

Table 11 represents the pass-fail relations between all possible pairs of skills on the two week retention test. The figures in the last column (pass higher skill only) indicate the validity of the various prerequisite relations. Table 12 is a summary of all disconfirmatory response patterns by experimental by week of the four week experiment. Entries represent observed frequencies where a learner passed the higher skill while failing the lower skill of the pair. There are fewer disconfirmatory responses in the fourth week of the experiment. The prerequisite skills hypothesis would appear to be confirmed for this portion of the learning hierarchy. In spite of alternative instructional orders, skill A was mastered with or before skill B, skill B was mastered with or before skill C, and skill C was mastered with or before skill D. The criterion used to define mastery

Table 10. Pearson Product Moment Correlation Coefficients

	AGE	SIT	SMA	PPVT	PMA	TIM	PRET
SIT	.64						
SMA	-.35	.41					
PPVT	.49	.66	-.17				
PMA	.34	.22	.68	.56			
TIM	.40	.09	-.33	.18	-.13		
PRET	.44	.30	.68	.23	.51	-.34	
WKT1	.48	.22	.64	.14	.48	-.41	.73
WKT2	.51	.17	.65	.18	.55	-.46	.68
WKT3	.38	.13	.58	.14	.47	-.38	.62
WKT4	.41	.08	.54	.11	.37	-.39	.58
WKT5	.41	.08	.54	.14	.42	-.38	.59
RETT	.29	.05	.27	.03	.20	-.14	.22

SIT = Slosson Intelligence Test
SMA = Slosson Mental Age
PPVT = Peabody Picture Vocabulary Test IQ
PMA = Peabody Picture Vocabulary Test MA
TIM = Number of days in study
PRET = Pretest score
WKT1 = Total score on skills at end of week 1
WKT2 = Total score on skills at end of week 2
WKT3 = Total score on skills at end of week 3
WKT4 = Total score on skills at end of week 4
WKT5 = Total score on skills at end of week 5
RETT = Two week retention test

Table 11. Summary Table for Pass-Fail Relations between Pairs of Skills in the Hypothesized Hierarchy. Two Week Retention Test

Skill Pairs	Both Skills Failed	Pass Lower Skill	Pass Both Skills	Pass Higher Skill Only
A - B	15	13	73	4
A - C	23	18	66	2
A - D	12	32	61	3
B - C	20	29	57	2
B - D	15	40	53	0
C - D	30	39	38	1

Table 12. Results of the Examination of the Prerequisite Relationship between Ordered Pairs of Skills by Experimental Group

	Skill Pairs (Disconfirmatory Totals)					
Group	A - B	A - C	A - D	B - C	B - D	C - D
			Week 1			
1	1	0	0	0	0	0
2	0	0	0	1	0	0
3	3	2	0	0	0	0
4	3	2	1	0	0	0
5	2	2	0	1	0	0
6	2	1	0	0	0	0
			Week 2			
1	0	0	0	0	0	0
2	0	1	0	0	0	0
3	1	2	0	0	0	0
4	2	1	0	1	0	0
5	1	0	0	0	0	0
6	0	0	0	0	0	0
			Week 3			
1	1	0	0	0	0	0
2	0	0	0	0	0	0
3	1	1	0	0	0	0
4	2	0	0	0	0	0
5	0	0	0	0	0	0
6	1	0	0	0	0	1
			Week 4			
1	0	0	0	0	0	0
2	0	0	0	0	0	0
3	0	0	0	0	0	0
4	0	0	0	0	0	0
5	0	0	0	0	0	0
6	1	0	0	0	0	0

was correct performance for each of the three items used to assess mastery of skills _B_, _C_, and _D_. Mastery of skill _A_ was defined as correct counting from _1_ to _10_ on two consecutive occasions. These mastery criterion essentially duplicate the criterion used by Wang (1973).

CHAPTER 5

DISCUSSION

The results of this study would seem to question the assumption that the hierarchical order of a set of sequential skills represents the optimal instructional order. These results when viewed with the findings of Miller (1969), Niedermeyer et al. (1969), and Tobias (1973) suggest that learning within a unit of a larger curriculum need not adhere to the hierarchical order. However, this study does not address the additional issue of whether sequencing of larger units is instructionally efficient nor can this study be generalized to other subject matter areas. The data reported by Miller (1969) do suggest that hierarchical order of unit objectives is an important consideration. Cunningham (1971) also suggests that larger, more complex units may be more efficiently mastered when the hierarchical sequence is followed. Naylor and Briggs (1963) indicate that the complexity and inherent organization of the content influence the degree of sequence needed for efficient mastery. The content utilized in the present study was a portion of a larger curriculum with only four of the six skills in the unit being investigated. The present study employed a short sequence that may have been relatively uncomplex. The familiarity variable was partially controlled to the extent that the experimental groups did not differ on pretest scores and were all experiencing similar day care environments. However, regression analysis did demonstrate that pre-instruction knowledge was the most efficient predictor of the individual characteristic variables and the greatest contributor to the structural equations of the exogenous variables. From this perspective the subjects did differ to some extent on the familiarity dimension with regard to the subject matter.

Examining the effects of instructional order on the acquisition of the terminal skill did indicate that a non-hierarchical order (B, C, A, D) was significantly more efficient than the hierarchical order (A, B, C, D) for weeks three and four of the study and on the two week retention test. This analysis can be viewed as a failure of the test of positive-transfer in that the hierarchical instruction order of subordinate skills was significantly less efficient in facilitating mastery of the terminal skill D. Although this finding is somewhat unanticipated, the data do indicate that counting things (skill B) followed by rote counting (skill A) is a more efficient strategy. Scandura (1977) has clearly stated that hierarchical learning may not be beneficial. Briggs and Naylor (1962) have reported that the complexity and organization of the materials being taught were important features in determining the size and number of skills to be presented in an instructional unit. The data from this and other studies would seem to suggest that for sub-units consisting of a small number of skills that order of presentation may be relatively unimportant. There are some studies that suggest that simultaneous presentation of skills may be even more efficient than individual skill presentation. Bergan et al. (1976) was able to teach a number of his subjects to mastery by presenting all six skills of Unit II during each of the ten instructional sessions.

The examination of the prerequisite skill relations among the four tasks selected for study in this investigation indicate a close approximation between the psychometric validation study of Wang (1973) and the study of Bergan et al. (1976). Indeed, Gagne's prerequisite skill hypothesis is strongly supported by the data (Table 12) gathered at the end of the fourth week of the study and on the two week retention test. Only one exception was noted at the end of week four and on the retention test, very few (13) exceptions occurred. This is an especially interesting finding since five of the six instructional groups involved teaching skills out of the hypothesized hierarchical sequence. Irregardless of the instructional order, learners demonstrating mastery of a higher level skill, also demonstrated mastery of the lower level subordinated skill. While they may not have been specifically taught the lower level skill, at the time of superordinate skill mastery, they were able to, nevertheless, to demonstrate mastery of both skills. Hence, for this portion of a basic counting skills hierarchy, mastery of a superordinate skill does imply mastery of prerequisite subordinate skills.

The support for the prerequisite-skills hypothesis lends further support to the behavioral assessment approach. The implication that individuals achieve mastery of skills-objectives by acquiring subordinate skills provides additional justification for criterion referenced testing as opposed to norm-referenced testing. Bijou and Baer (1978) emphasized the importance of initial assessment pinpointing a child's competencies in relation to a particular curriculum content. This provides a basis for determining where to start instruction and also indicates the content of the instruction to be monitored. Learning hierarchies, then, become the means for specifying the content of the curriculum and for deriving the criterion-referenced assessment packages. The monitoring capabilities of learning hierarchies as translated into criterion-referenced tests needs to be further explored.

The behavioral approach to diagnosis and remediation advocated by Bijou and Grimm(1975) involves three interrelated sets of assessments. The first set of assessments provides information about the learner's current repertoires in the areas of concern (e.g., addition of single digit numbers, sums to ten). The second set of assessments provide a continuous description of the learner's progress with regard to the areas of concern. The third set of assessments indicate the skill level attained by the learner at the end of the instructional period.

Learning hierarchies would seem to be ideally suited to provide the skill content on which criterion-referenced tests could be developed. Learning hierarchies would, therefore, be contributing to technology of education alluded to by Bijou and Grimm (1975); a technology necessary for the adequate diagnosis and remediation of educational problems.

The requirement for a continuous description of the learner's progress on the skills under instruction makes the multiple probe design especially attractive. This modification of the multiple baseline design (Horner and Baer, 1978), while not requiring continuous data gathering on all skills, creates a schedule for collecting data on the skills not under intervention. This design permits analysis of an individual's learning patterns to carefully plan changes in teaching procedures and content. It also enables an observer (teacher) to observe changes in related skills not currently being taught. Thus, if a learner acquires prerequisite skills while being instructed directly in a superordinate skill, one can proceed more rapidly through a skill hierarchy towards achievement of the terminal objective.

In the present study children were taught skills that they did not possess in orders differing from the hierarchical ordering for five of the six groups. Yet, the observed relationships between pairs of skills did not differ from the results of the Wang (1973) and Bergan et al. (1976) studies. Criticisms of Wang's approach of testing skill mastery of groups of subjects without employing a learning component do not seem justified on the basis of this data. Although White (1973, 1974a,b,c) presents some evidence in support of his objections of Wang's methodology, there is not sufficient research to determine whether or not effects related to order of acquisition and order of forgetting are likely to render a testing approach to the specification of sequential arrangements among skills invalid. Since a testing approach is more efficient than procedures requiring skill instruction, it is important to determine whether or not sequential ordering among skills can be adequately assessed by a testing strategy (psychometric approach). At the very least, a test-psychometric approach represents a starting point in the validation of a hypothesized learning hierarchy.

An examination of the path coefficients for the effects of the exogenous variables of CA, MA, and pre-instruction knowledge clearly indicate that the role of individual characteristics declined as the study progressed. Indeed, after the end of the third week of the study, only pre-instruction scores contributed a significant amount of variance. The influence of individual characteristics was reduced as one progressed through the skill hierarchy. An examination of the correlation matrix (Table 10) reveals that the relation of CA, MA, and pre-instruction scores maintained moderate significant correlations across time. However, with the entry of skill performance into the equations the relatively-unique contribution of individual characteristics was reduced to a non-significant portion. This indicates that CA, MA, PRET share a substantial amount of variance with skills in the hierarchy. The contribution of CA, MA, and pre-instruction knowledge were confined to skills A and B with most of the contribution being noted on skill A.

These results would seem to support the hypothesis that prerequisite skill performance and entry skill level relate a significant extent to individual characteristics while superordinate skill learning and progress through a learning hierarchy depends largely on the acquisition of prerequisite skills and directed instruction. The influence of individual characteristics measures may possibly represent variation in learner performance that could be attributed to other learned skills which are prerequisite to the skills under investigation, but which have not been adequately assessed. This possibility has important implications with respect to explaining how individual characteristics affect learning and therefore deserves greater study.

Cotton et al. (1977) has identified the performance of the transfer experiment as critical to the validation of learning hierarchies. The data on the effect of acquisition of the terminal skill D provide one form of validation of learning hierarchies. However, that data did not support the application of the hierarchical order as the optimal instructional order.

Examination of the path analysis (Figures 1 to 4) performed over the four weeks of the study indicate substantial positive-transfer from subordinate skills to superordinate skills within the learning hierarchy. The role of individual characteristics gradually declined in importance leaving only the pretest score on the unit under study as a significant contributor to acquisition of skills within the hierarchy. This analysis clearly supports Gagne's (1962) arguments that the contribution of general ability and irrelevant abilities would be nonsignificant under optimal instructional conditions. As the study progressed, mastery of skills in the hierarchy became increasingly dependent on acquisition of prerequisite skills in the hierarchy. Indeed, of the individual characteristics initially investigated, only Slosson Mental Age, chronological age, and pre-instruction knowledge had significant contributions to the path model. As the study progressed, Slosson Mental Age and age dropped out of the equations leaving only pre-instruction knowledge as a variable. By the end of the study, pre-instruction knowledge had a direct influence on only the mastery of skill A.

The implication of this finding when combined with Bergan et al. (1979) seem substantial in suggesting that the focus on instruction needs to be on identified prerequisite skills and that the importance given to norm-referenced measures of general ability is not justified. The influence of these general ability measures aside from predicting entry level skills do not predict progress nor do they prescribe the content of instruction. The use of learning hierarchies in developing instructional units and criterion-referenced assessment instruments based on the learning hierarchy seems well supported.

An examination of Table 10 is illustrative of one potential reason why individuals continue to rely on general ability measures such as IQ for educational programming. The correlations of PPVT IQ and SIT IQ with pretest scores and measures of performance are moderate. The mental age scores are even better predictors. Since these Pearson Product Moment correlations are zero order correlations, they do not represent the unique contribution of IQ or MA to learning. Path analysis does represent the unique effects of the variables on the dependent measures (learning) resulting in the minimal (non-significant) influence of IQ. This is another way of stating that the things that IQ represent (past learning) are of minimal importance within a learning program providing optimal instruction in the relevant skills. Acquisition of subordinate skills within the learning hierarchy becomes the important determinants of progress.

The results of the structural analysis challenge the use of MSA as a procedure for determining the sequential patterning among skills. The observed occurrence of positive-transfer among skills not related in a prerequisite manner is contradictory to the expectations of the learning hierarchy model. The learning hierarchy model assumes that mastery of subordinate skills in a learning hierarchy are prerequisite to the mastery of superordinate skills immediately adjacent to them in the hierarchy. The results of the present study support the finding of the Bergan et al. (1976) study in suggesting that there may be alternative paths to mastery of superordinate skills. The variations in the magnitude of the coefficients associated with the different paths indicate that all alternatives may not be equally efficient in enhancing superordinate skill mastery. These findings raise important issues in regards to the selection of tasks and teaching orders to facilitate superordinate skill mastery. The Wang hierarchy assumes the need to provide instruction in the three skills subordinate to task *D* (skills *A*, *B*, *C*) in order to ensure mastery of skill *D*. The path coefficients clearly suggest that skill *D* may be mastered by teaching skills *A* and *C* and teaching skill *A* only can lead to mastery of *D*. This is not to imply that students who master skill *D* cannot also master skills *B* and *C*, but that to facilitate mastery of skill *D*, it may not be routinely necessary to teach skills *B* and *C*.

Among the possibilities that exist for explanation of why prerequisite skills may be skipped in the mastery of the terminal skill is the use of the modeling, practice, and feedback procedures which may have been sufficiently potent to have made salient all of the skills necessary to acquire skill *D* and the prerequisite simultaneously. From an education viewpoint, there is a need to clarify the necessity of teaching prerequisite skills as separate components and the use of differing instructional strategies as part of the instructional process.

APPENDIX A

TEACHING PROCEDURES

Teaching Procedure for Skill A

The underlined portions are to be read out loud to the child.

(Child's name), I want you to watch and listen to what we are going to do. Place one finger puppet on a finger of each hand. Let the child choose the puppets to be used. Listen to me, Cookie. I'm going to count from ___ to ___. Oscar says the numbers slowly. Oscar says, Your turn, Cookie, you count from ___ to ___ just like I did. Cookie counts slowly from ___ to ___. Very good Cookie, you are a good counter. Now you try it, (child's name). Count from ___ to ___ just like Cookie and I did. If the child counts correctly, Oscar says, Very good, you are a good counter just like Cookie and me, and introduces the next task. If the child counts incorrectly, Oscar says, No (child's name), that is not quite right, let's try it again together, you count with Cookie and me. Go on to the next trial.

Give the teaching trials listed below in the order listed.

Teaching Trials: 123 12345 345 678 678910 1 to 10

Generalization Trials: (Child's name), count from 1 to 10

(For generalization trials, repeat 7 times or stop if child correctly counts to 10 on three consecutive occasions. Give no feedback, do not engage in any instruction.)

Teaching Procedure for Skill B

Oscar spreads ___ chips randomly before the child and says, See how I can count, Cookie. Oscar counts the chips moving them out of the set as he counts. Then Oscar rearranges the set by scrambling the chips and says to Cookie, Let's see you do it Cookie, count these chips and put them over here. (Oscar points to a location outside the original set). Be sure to count as you touch each one. Cookie counts the chips just as Oscar did. Oscar says, Very good Cookie, you are very good at counting things. Oscar again scrambles the chips and says, You do it now (child's name), Count these chips and put them over here (Oscar points to a location outside the original set). If the child counts correctly, Oscar says, That's neat, (child's name), you are a good counter just like Cookie and me. If the child does not count chips correctly, Oscar says, That was not quite right (child's name), Watch me count these chips again, you count them with me. (Have child touch and count the chip with you). Go on to next trial.

	1	2	3	4	5	6	7
Teaching Trials:	2	5	3	8	7	10	
Generalization Trials:	6	5	7	4	9	10	8

Instructions for Generalization: (Child's name) count these chips. If child is correct say, Good, Fine, Okay. If child is not correct, go on to the next generalization trial.

Use the same colored chips for each trial, but vary the color across trials to aid in maintaining interest.

Teaching Procedure for Skill C

Oscar places appropriate card containing a fixed, ordered set of objects before the child and says, See how I can count these Cookie, I will touch each one as I count it. Then Oscar counts all the objects touching each, as he counts it. Then Oscar says, Let's see you do it Cookie, count all of these things. Cookie counts just as Oscar did. Very good Cookie, you counted them just right, just like old Oscar did. Now you do it (child's name), you count these objects, just like Oscar and Cookie, touching each one as you count it. If the child responds correctly, Oscar says, You are as good a counter as Oscar and Cookie. If the child does not count the objects correctly, Oscar says, That's not quite right (child's name), you count these with me. After counting, Good, let's try the next one.

	1	2	3	4	5	6	7
Teaching Trials:	5	8	4	6	9	10	
Generalization Trials:	6	7	9	10	8	4	5

Instructions for Generalization Trials same as for skill B.

Use cards coded TT for teaching trials and cards coded GT for generalization. Cards are coded on back.

Teaching Procedure for Skill D

Oscar places the appropriate card containing a fixed, unordered set of objects in front of the child and says, Watch me Cookie, I am going to count these objects. Oscar then counts each object, touching it as he counts. Then Oscar says to Cookie, Cookie you count these objects, do it just as I did. Cookie carefully counts the objects touching each one as he counts. Oscar says, Very good Cookie, I am so proud of you. Now (child's name), you count these pictures just as Oscar and Cookie did. Touch each one as you count it. If the child responds correctly, Oscar: Very good. If the child does not count correctly, Oscar says, No, that is not quite right, let's count them again together. (Have child count and touch the objects with Oscar). Now, let's try the next one.

	1	2	3	4	5	6	7
Teaching Trials:	3	5	8	10	7	9	
Generalization Trials:	5	8	4	9	10	7	6

Instructions for Generalization Trials the same as for skill B.

Use cards coded TT for teaching trials and use cards coded GT for generalization trials. Code is on back of cards.

APPENDIX B

RECORDING SHEET

GROUP ____

Child's Name: ____________________ Name of School: __________

Week: ________

Days	Date	Skill		Trials 1	2	3	4	5	6	7	Total
1.	________		Teaching								
		____	Generalization								
2.	________		Teaching								
		____	Generalization								
3.	________		Teaching								
		____	Generalization								
		____	Probe								
		____	Probe								
		____	Probe								
4.	________		Teaching								
		____	Generalization								
		____	Probe								
		____	Probe								
		____	Probe								
5.	________		Teaching								
		____	Generalization								
		____	Probe								
		____	Probe								
		____	Probe								

Note: Enter (1) for a correct response to an item; enter a zero (0) for an incorrect response to an item in the appropriate box.

Although spaces for the probes have been provided at the anticipated times, if a child masters the generalization trials on two consecutive days (100%), proceed to give the probes and record in the allotted space noting the date of the probe. Your supervisor will fill in the blanks designating the skills to be taught.

Each child is to be taught each skill for five days, unless the child demonstrates mastery prior to the end of five days. Following mastery of the skill being taught, the experimenter will move to the next skill to be taught for the child.

The probes consist of the generalization trials for each of the skills.

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