OPTIMAL UTILIZATION OF PLAYA LAKE WATER IN IRRIGATION

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INTRODUCTION

Under present conditions of increasing demands on water and an overall concern for optimum utilization and conservation of this basic resource, there still remain large quantities of water which have not been so allocated. Two quantities standout rather dramatically to, particularily, residents of the Southwest. These two are: the outflow from major rivers in the South (Mississippi) and Northwest (Columbia) regions and the water stored in the playa lakes primarily in the plains or midwest region.

Playa lakes are known to exist in many regions of the world. All have the same basic physiographic and climatic features. Most occur where lands are relatively flat and there is an absence of well developed surface drainage nets. Also most are found in regions where the climate would be categorized as either arid or semi-arid. Many definitions have been rendered for a playa lake with one being "the sandy, salty, or mudcaked flat floor of a desert basin having interior drainage, usually occupied by a shallow lake during or after prolonged heavy rains. - Webster."

In the High Plains of Texas, there exist approximately 20,000 such shallow lakes or playas [*Schwiesow*, 1965]. The size of these playas vary in size from small shallow depressions with minimal storage potential to very large deep basins providing storage for hundreds of acre feet of water. These playas are filled by surface runoff from the highly erratic precipitation patterns common to the High Plains area.

The volume of potential storage in these playa lakes is an estimated 2.5-3 million acre feet. The occurrence of attaining this volume of storage is a function of precipitation and the antecedent moisture on the watershed but the probability of filling the lakes once a year is relatively high. The probability of filling two or more times is moderately high during years of normal precipitation. Periods of highest probability exist during the spring (May and June) and the late summer (August and September).

At present waters collected and stored in the playa lakes on the High Plains of Texas present a peculiar problem associated with beneficial use. Natural recharge through deep percolation is very low with only 10% being infiltrated. The reason for this minimal amount is the extremely low infiltration rate through the heavy, predominantly clay soils in the bottom of the playas. Rates increase when the lake fill level is high as peripheral infiltration rates are considerably higher. The remaining 90% is generally lost to evaporation except for the small amounts used for irrigation or artificial recharge [*Clyma*, 1964].

As indicated, the present system (or lack of system) of playa lake operation results mainly in evaporation and a limited amount of beneficial uses. There are three possible beneficial uses associated with this storage

- irrigation, artificial recharge, and recreation. Each of these uses posses unique problems which may result in their non-selection. Timing of use is important to all because losses to evaporation are high due to climate and an unfavorable depth-area relationship.

The High Plains is presently being subjected to a problem common to most areas dependent primarily upon an underground water supply - a declining water table. The water source for the area is the Ogallala formation, a vast unconsolidated, hydrologically isolated aquifer. Any augmentation of the source through recharge or development of a surface source will delay and possibly preclude the arrival of an extremely critical situation.

A potential surface source exists within the playa lakes. Since approximately 5.5 million acres [*New,* 1970] are annually irrigated within the area, the use of a large percentage of potential storage within the playas would materially decrease the draft on the underground source. Several problems exist within this source; some of which are water quality, loss to evaporation, questionable depth-area relations and the stochastic nature of the source. If the waters are to be used for irrigation, the undesirable water quality (sediment content) is not a real problem.

The questionable depth-area relationship evolves around the characteristically shallow depths of the playa lakes. This results in the large surface area which proves undesirable. In addition to increasing the inundated lands, the large surface area increases an already high evaporation potential. Also difficulty in pumping from the playa exists because of the shallow depths. This problem can be alleviated in part by the simple excavation of a sump or other depressional area from which pumping could occur.

Losses due to evaporation and the stochastic nature of the source are problems which cannot be totally overcome because of the very nature of the problem. Both are related to climate and to date man has not been able to successfully manipulate climate. Measures can be taken to minimize the magnitude of the loss to evaporation through immediate utilization of the water or by modifying the area-depth relationship of the playa. Unfortunately both measures present unique problems. Immediate utilization for irrigation is questionable as the soil moisture content will be high and additional water applications to the land will not be highly beneficial. Modification of depth-area relations will require extensive earth movements to attain a small but deep storage reservoir or playa. Under present economic conditions, this would be infeasible.

The unpredictability of the occurrence of water in the playa may be the limiting factor in the utilization of playa water for irrigation. An irrigation enterprise must possess sufficient flexibility to utilize playas as an alternate source of water rather than total dependency upon playa water. In order to attain this flexibility, additional expenditures would be required. Many irrigators are reluctant, in view of tightening economic constraints, to adopt this flexibility. Only those with a marginal primary source are willing to take the chance.

This paper assumes that water is available and presents a dynamic programming model useful in determining the optimal utilization of the water collected and stored in playa lakes for irrigation.

REVIEW OF LITERATURE

The problem of optimal utilization of a stochastic water supply is not new to todays water analyst. High speed digital computers and the development of dynamic programming theory has simplified the task of arriving at solutions to this type of problem.

Hall and Buras [1961] presented the application of dynamic programming to the general class of water resource problems to permit optimum development with respect to all benefits. Application of dynamic programming to determine the optimum irrigation practices was utilized by *Hall and Buras* [1961 (Feb)] in presenting a simple graphical procedure for solutions. This procedure permitted the determination of an optimum policy for irrigation of homogeneous lands under conditions of a deficient water supply.

Another application of dynamic programming techniques was by *Hall & Butcher* [1968] in determining the timing of irrigation. The methodology described permitted farmers to determine the time and quantity of irrigation to correspond with critical stages of crop growth. It possesses a limitation in that data relative to critical periods for crops and the magnitude of the adverse effects of associated soil moisture deficiences is lacking for most commercial crops.

Hall and Howell [1970] reported procedures for allocating the degree of assurance of a supply of water rather than taking the water itself as a primary resource. Introduced was the stochastic concept or probability of occurrence of a water supply rather than the presumption of a certainty of supply as used by others. The stochastic feature very nearly approximates the conditions which exist within the playa lake waters.

DEVELOPMENT OF A DYNAMIC PROGRAMMING MODEL

Dynamic programming is a mathematical technique for solving certain types of sequential or time-staged decision processes [*Howard*, 1956; *Roefs*, 1968; *Wagner*, 1969]. In order to use dynamic programming, one must be able to distinguish between system states and decisions. A state variable is one whose value completely specifies the instantaneous situation of the system. The values of the state variables provide the analyst with enough information about the system permitting a decision to be made. A decision is an opportunity to change the state variables at any given stage.

Another prerequisite to the use of dynamic programming is that the principle of optimality cannot be violated. In order to prevent violation, one must be able to identify a value function for a current decision such that it is unrelated now or in the future on any decision except in ways expressed in the value function of the state variable.

Two principle types of dynamic programming are deterministic and stochastic or probabilistic. In deterministic programming, the assumption is that all events are certain and the magnitudes thereof are known. In stochastic dynamic programming, the events and magnitudes are represented by probability distributions.

DETERMINISTIC DYNAMIC PROGRAMMING

A two-state variable deterministic dynamic programming model was developed to determine the optimal utilization of playa lake water for irrigation. The objective of the model was to determine the amount of playa lake water to apply to the land, through conventional irrigation, as well as timing of irrigation to obtain a maximum benefit in terms of crop response.

Let S represent the total amount of storage or water available in the playa lake. The quantity S cannot be considered a fixed quantity as evaporation will decrease this quantity for each subsequent time period. The amount lost to evaporation is a function of the playa surface area and surface area is a function of the amount in storage. The amount in storage is also reduced by the amount of irrigation x, if any, within each time period. Thus the model must be solved for any variable quantity s, where $0 \leq s \leq S$. The quantity remaining in storage in any time period can be calculated using

$$s_t - x_t - e\,[\,f(s_t+s_{t+1})/\,2\,] = s_{t+1} \quad \text{.................................} \quad [\,1\,]$$

where s_t, x_t, $e(f)$ are storage available, amount of irrigation, and evaporation rate per unit area for the average surface area $[\,(s_t+s_{t+1})/\,2\,]$ respectively during the t-th time period. Recharge is assumed to be nonexistant.

The amount of water applied through irrigation is a function of the antecedent soil moisture and is subject to the following constraint

$$\Sigma_{i=1}^{n}\, x_t \leq s_t \quad \text{...} \quad [\,2\,]$$

Realistically, discretizations of irrigation should be compatible with the mode of irrigation in order to eliminate minimal and infeasible amounts. Irrigation efficienc is assumed to be 100%.

Soil moisture content is a function of precipitation, consumptive use, and amount of irrigation. Since the model is deterministic, precipitation was not considered. Moisture available at any time is

$$w_t + x_t - c_t = w_{t+1} \quad \text{.. [3]}$$

where w_t , x_t, and c_t are moisture content, amount of irrigation, and consumptive use during t-th period respectively. Soil moisture content is also subject to the constraint

$$w_p \leq w_t \leq w_f \quad \text{.. [4]}$$

where w_p , w_t , and w_f are moisture contents at wilting point, during t-th period, and at field capacity respectively. Although field capacity is the upper limit on moisture content, it does not necessarily correspond to the moisture content resulting in maximum crop response.

Maximum crop response generally results from the combined effects of soil tilth, climate, variety, moisture content, etc. The model assumes response is a function of soil moisture only. Let a represent the crop response with values ranging from 0 to 1. Values for a can be synthesized from experience or from actual observations of the crops in question.

Keeping in mind the variable nature of soil moisture, a recursive relationship was developed to determine crop response corresponding to some given irrigation during a given time period. The relationship is defined by

$$f_{t+1}(s,w) = \max_{x_t} [a(w_t, w_{t+1}) \cdot f_t(s,w)] \quad \text{.......................} \quad [5]$$

In examing equation [5], it is important to recall that x_t takes on all values from 0 to s_t in order to increase soil moisture content to w_f or a maximum value of w_t. The production function $[a(w_t, w_{t+1})]$ represents the response for average moisture content during the time periods t and t+1 and the function $[f_t(s,w)]$ represents the response due to all previous values of storage and moisture content.

The solution to equation [5] gives an optimum policy since it specifies the decisions to be made at this stage to obtain a maximum crop response. For each value of s and w, there will be a value of x which will maximize the response for each state of the system.

Enumeration of equation [5] for each succeeding time period, or until water in storage is exhausted, will yield optimum values of x for each period. Analysis of the resulting values will determine the time period during which an irrigation application would yield maximum response.

To demonstrate the feasibility of the model in utilization of playa lake water for irrigation, sample calculations were carried out using assumed data for crop response as a function of moisture content. The model presented herein utilizes synthesized values with response less than 1 at w_f, increasing to 1 at $0.75w_f$ and diminishing to 0 at w_p. The sample calculations were run for 25 stages with each stage equal to one day. Other time discretizations could have been chosen but days were consistent with time units of evaporation and consumptive use.

Irrigation applications were discretized into 0.25 inch increments. Evaporation and consumptive use were taken as constants equal to 0.25 inches per day. Results were readily developed although the actual use of this model would require more specific crop response information and consideration of evaporation and consumptive use as variables rather than constants.

STOCHASTIC DYNAMIC PROGRAMMING

A more representative model of this problem would be a stochastic dynamic programming model. In the stochastic dynamic programming model, precipitation would be treated as a random variable $P_{i,t}$. The subscript i refers to the amount of rainfall and P is the probability thereof. This random variable would be introduced into both of the state transformation equations and would require the use of expectation summations in the recursion equation. Equation [1] would be modified to read

$$s_{i,t} - x_t - e\left[f((s_t+s_{t+1})/2)\right] + f(P_{i,t}) = s_{i,t+1} \quad \ldots\ldots\ldots\ldots\ldots \forall i \qquad [6]$$

for a particular i and where $f(P_{i,t})$ is the inflow resulting from the precipitation i during time period t.

Equation [3] would be modified to read

$$w_{i,t} + x_t - c_t + g(P_{i,t}) = w_{i,t+1} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \forall i \qquad [7]$$

for a particular i where $g(P_{i,t})$ is the increase in soil moisture resulting from a precipitation i during time period t.

The recursion equation would read

$$f_t(s_i,w_i) = \max_{x_t} [a(w_{i,t}\ \sum_i P(w_{i,t+1})w_{i,t+1}) \cdot$$

$$\sum_i P(s_{i,t+1}, w_{i,t+1}) f_{t+1}(s_i,w_i)] \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad [8]$$

SUMMARY

The optimal use of available playa lake water is essential to sound water resources management. Optimal use is not restricted to irrigation but could include artificial recharge and/or recreation. If irrigation is the predominant use, timing of irrigation will be important in order to maximize crop response and utilization of the diminishing supply of playa water.

The deterministic dynamic programming model presented will provide the time and amount of irrigation required to maximize crop response. Two state variables utilized by the model are antecedent soil moisture and amount of water in the playa available for irrigation. Evaporation is the primary factor which will reduce the amount of water available and could become very significant if timing was not considered. The results could be materially refined by utilizing accurate data relative to crop response, evaporation and consumptive use.

A better model is the stochastic dynamic programming model presented which considers the probability of precipitation and resulting filling of the playa lakes. Expectation summations are required in the recursion equation in order to include the probability distributions. The model presented is a first attempt and has not been validated at this time.

It is recognized that neither model includes all of the variables which could possibly be included. Variables such as cost of application of irrigation water, crop diversity, value of crop, time of season, and acreage limitations would materially add to the model. Also incorporation of other uses such as artificial recharge and recreation and their associated benefits would provide a more accurate realization of optimal utilization of the playa lake water.

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