A PROPOSED MODEL FOR FLOOD ROUTING IN ABSTRACTING EPHEMERAL CHANNELS [1/]

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INTRODUCTION

In much of southern Arizona, almost all runoff from semiarid rangeland watersheds results from intense highly variable thunderstorm rainfall [Osborn and Reynolds (1963)]. Compounding the problems of predicting runoff resulting from such rainfall is the presence of broad alluvium-filled channels that abstract large quantities of runoff [Babcock and Cushing (1942), Keppel (1960), Keppel and Renard (1962), Allis, Dragoun, and Sharp (1964), Renard and Keppel (1966), Qashu and Buol (1967), Wallace and Renard (1967), Burkham (1970a, 1970b), and Lane, Diskin, and Renard (1971)]. These abstractions or transmission losses play an important role in diminishing streamflow, in supporting riparian vegetation, and in providing natural water recharge to local aquifers and the regional groundwater [Renard (1970)].

Ephemeral streams in arid or semiarid zones introduce an added difficulty in flood routing because of the large transmission losses [Keppel and Renard (1962)]. An example of the influence of transmission

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losses on hydrographs in ephemeral streams is shown in Figure 1. These hydrographs represent the inflow and outflow on a 4.1-mile reach of channel on the Walnut Gulch Experimental Watershed. Notice that the maximum discharge rate is reduced some 50% while the volume of flow is reduced about 35% in traveling the 4.1 miles in the absence of tributary inflow. While this is intended as a typical example, the reduction in volume is not always less than the reduction in peak discharge. In this study, flood movement and transmission losses are represented by a system using storage in the channel reach as a state variable which determines loss rates.

FLOOD ROUTING PROCEDURES

The term flood routing refers to the determination of a flood wave at a downstream location when the flood wave at some upstream location is known. While there is an extensive body of literature on flood routing, a few primary sources are Gilcrest (1950); Linsley, Kohler, and Paulhus (1949, 1958); SCS Engineering Handbook (1957); and Chow (1959, 1964). In very irregular and rough natural channels, the changes in the flood wave or hydrograph can be significant. Chow (1959) differentiates between hydraulic and hydrologic flood routing as follows: "The hydraulic method of flood routing is distinguished from the hydrologic method by the fact that the hydraulic method is based on the solution of the basic differential equations for unsteady flow in open channels whereas the hydrologic method makes no direct use of these equations but approximates in some sense to their solutions." Throughout the remainder of this paper, the term flood routing will be restricted to the simplified hydrologic method of flood routing.

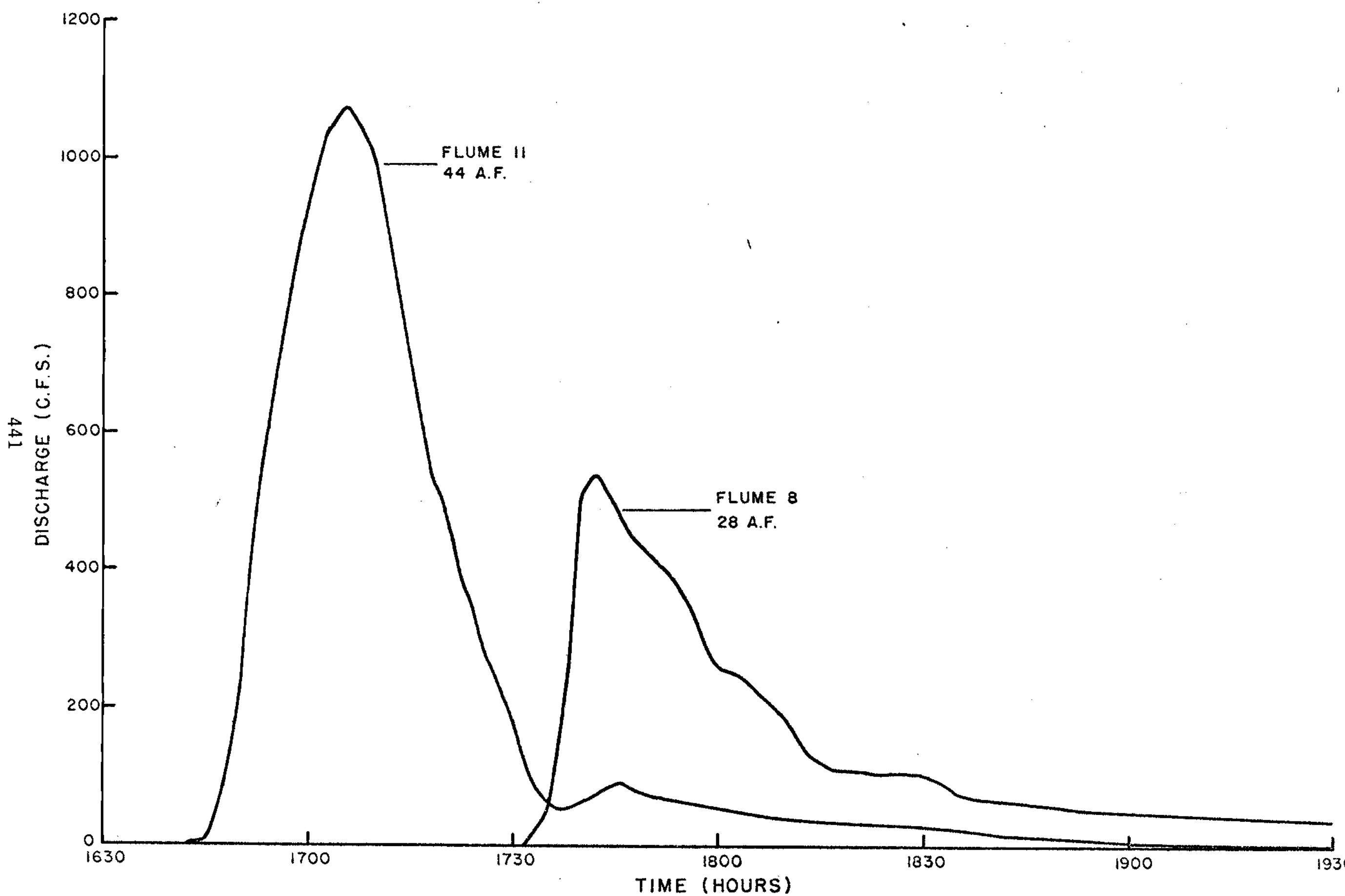

Figure 1. Hydrographs for reach 11-8, event of 300766.

In natural channels with little or no tributary inflow or outflow and no losses or abstractions, the inflow, outflow, and storage are related by the continuity equation

$$\dot{S} + Q = P \tag{1}$$

where:

$\dot{S}$ is the time derivative of storage,

Q is the outflow from the reach, and

P is the inflow to the channel reach.

In addition, P, Q, and S are related to the depth of flow, Chow (1959). If X is defined as a weighting factor, then the storage within a reach is given by

$$S = XS_i + (1-X)S_o \tag{2}$$

where S_i and S_o are storages determined by the depths at the upstream and downstream points of the reach. Since S_i and S_o are determined from the inflow and outflow, respectively,

$$S = K[XP^n + (1-X)Q^n] \tag{3}$$

where the variabiles are as defined above, and K and n are constants. In rectangular channels n = 0.6, and in natural channels n is assumed equal to unity [Chow (1959)]. The popular Muskingum method is a form of Equation 3 where n = 1.0 [McCarthy (1938)].

Kulandaiswamy, Krishnaswami, and Ramalingam (1967) list three storage equations commonly used in flood routing:

$$S = KQ \tag{4}$$

$$S = K\ [XP + (1 - X)Q] \tag{5}$$

$$S = KQ^n \tag{6}$$

Equation 4 is the so-called linear reservoir model, Equation 5 is the Muskingum method, and Equation 6, unlike the other two, can be nonlinear if n is not equal to 1.0. The above authors then proposed a more general storage equation derived by Kulandaiswamy (1964) relating the derivatives of the inflow and outflow to storage. In a brief but elegant paper, Diskin (1967) derived the general solution of the Muskingum flood routing equation using Laplace transforms. Other than specifying the parameters for applying the Muskingum method, which can be found in several handbooks, the theoretical development of the method was completed by Diskin (1967).

GENERAL FORM OF THE MODEL

The proposed model is also a simplification of the known process and is a storage model. In this study we added transmission losses so that the continuity equation becomes

$$\dot{S} + L + Q = P \tag{7}$$

where:

$\dot{S}$ is the time derivative of the storage,

L is the transmission loss rate,

Q is outflow from the reach, and

P is the inflow to the reach.

All variables are functions of time.

If Equation 7 is rewritten with L and Q as functions of the storage, we have

$$\dot{S} + F(S) + G(S) = P \tag{8}$$

with

$$L = F(S) \tag{9}$$

and

$$Q = G(S) \tag{10}$$

where the variables are as defined previously. Equation 8 is a general representation of a storage routing model, and can be linear or nonlinear depending on the forms of F and G.

Flood Routing in an Ephemeral Channel

In the usual situation where only the inflow is known and the losses, F(S), and outflow from the channel reach, G(S) are to be determined, the continuity equation is

$$\dot{S} + F(S) + G(S) = P \tag{8}$$

In this analysis it was assumed that

$$L(t) = F(S) = C_1 S^{b_1} \tag{11}$$

and

$$Q(t) = G(S) = C_2 S^{b_2} \tag{12}$$

Then the continuity equation becomes

$$\dot{S} + C_1 S^{b_1} + C_2 S^{b_2} = P \tag{13}$$

The above equation is proposed as a general storage equation for flood routing in ephemeral channels.

In addition to the differential equation, it is sometimes desirable to describe a conceptual model of the system. A general reservoir with abstractions or losses and a siphon outflow is proposed as the basic component of the model. The general model would then be a cascade of such components similar to the Nash cascade of linear reservoirs conceptual model of a watershed [Nash (1957)]. If we define H_0 as a critical value of depth (or storage) in the reservoir, then there are two separate cases or modes of operation: 1) $S < H_0$ so that there are losses but no outflow, and 2) $S \geq H_0$ so that there are both losses and outflow.

Numerical solutions to Equation 13 have been obtained using the Runge-Kutta method. Initial analyses are concentrating on determining the relative sensitivity of the solution to values of the four parameters. Solutions have been obtained for the case with arbitrary input. However, the next step is to determine the analytic solutions so that the components can be cascaded and the general solution obtained.

A Specific Case

In this case it is assumed that the inflow and outflow are known and further that the transmission losses are a linear function of the storage in the channel reach; that is

$$\dot{S} + C_1 S + Q = P \tag{14}$$

With these assumptions, the problem becomes one of determining C_1, the coefficient relating storage and transmission losses. Rewriting Equation 14 and rearranging terms, both sides of the equation can be integrated so that

$$\int_0^t P(t)dt - \int_0^t Q(t)dt = C_1 \int_0^t S(t)dt + S(t) - S(0) \tag{15}$$

where the variables are as defined previously and S(0) is the storage at time zero. In discrete form, assuming $S(0) = 0$, the integrations become summations and dt becomes Δt so that,

$$S_N = \frac{2}{C_1\Delta t + 2}\left[\sum_{i=1}^{N} P_i\Delta t - \sum_{i=1}^{N} Q_i\Delta t - C_1 \sum_{i=1}^{N-1} S_i\Delta t - \frac{C_1\Delta t}{2} S_{N-1}\right] \tag{16}$$

which is an equation in terms of S_N, the storage at time $t = N\Delta t$. SUMP, SUMQ, SUMS, and SUML are defined as the total volumes of inflow, outflow, storage (integral of storage), and losses respectively. An initial approximation to the solution is:

$$\Delta S/\Delta t = P(t) - Q(t) - (SUMP-SUMQ)/TOT \tag{17}$$

$$S_N = \sum_{i=1}^{N} P_i\, \Delta t - \sum_{i=1}^{N} Q_i\Delta t - N\Delta t(SUMP-SUMQ)/TOT \tag{18}$$

$$C_1 = (SUMP - SUMQ)/SUMS \tag{19}$$

$$L_N = C_1 S_N \tag{20}$$

where TOT is the total duration of flow at the point of interest. With this initial solution for S(t) and thus L(t), Equations 16, 18, 19, and 20 are solved by an iterative technique until the successive values of C_1 are arbitrarily close.

The analysis in this example is based on data derived from a 4.1-mile ephemeral channel reach on the Walnut Gulch Experimental Watershed in southeastern Arizona. The channel reach is steep with low banks and an alluvium bed from zero to 11 feet thick. The mean depth to conglomerate is about 6 feet. The hydrographs shown in Figure 1 are measured inflow and outflow to this reach reflecting the losses to the alluvium.

The iterative solution to Equation 16 as described above was obtained for 12 inflow-outflow hydrographs on the reach where there was little or no tributary inflow between measuring stations. Figure 2 is a plot of

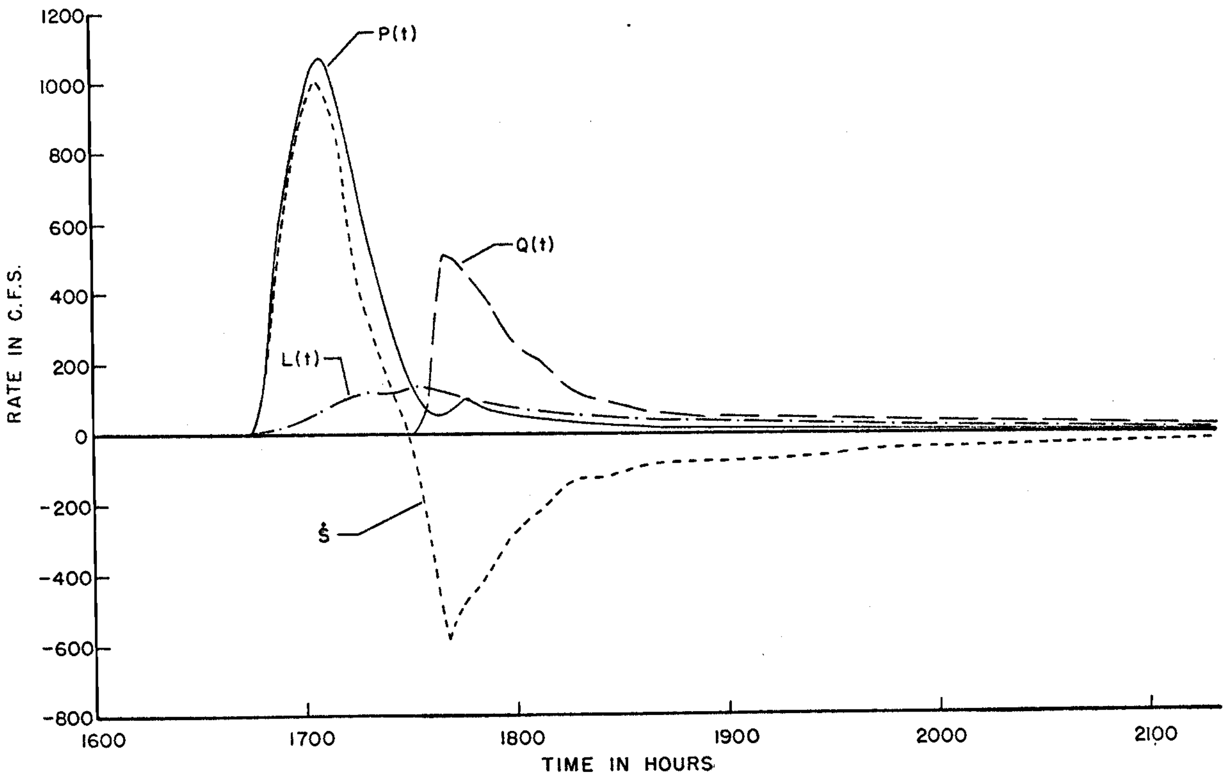

Figure 2. Solution to the differential equation assuming inflow and outflow are known, reach 11-8, event of 300766.

P, Q, L, and $\dot{S}$ for the hydrographs shown in Figure 1. Notice that L is for the entire channel reach. For this event, as in all events, a final value of C_1 was obtained in 20 iterations or less. Table 1 shows hydrograph characteristics and the derived C_1 values for the 12 runoff events used in this study. The relation between C_1 and the peak discharge of the inflow hydrograph is shown in Figure 3. Although the relationship is not strong, C_1 seems to decreased with increasing inflow to the channel reach. The scatter about the regression line in Figure 3 is no doubt due to a variety of sources including the effects of antecedent moisture and measurement errors. If a prediction equation were desired, including these variables would undoubtedly improve the coefficient of determination. However, the decrease in C_1 with increasing inflow can be taken as a measure of the nonlinearity of the system for ephemeral channels.

SUMMARY

A model for flood routing in abstracting ephemeral channels is proposed as a cascade of the general components. Initial analyses have yielded numerical solutions to the equations for the individual components. A linear form of a storage equation with the outflow known has been calibrated to 12 individual runoff events on a 4.1-mile reach of ephemeral channel. Wide variation in the parameters of this linear model with increasing inflow indicates that the assumption of a linear relation between transmission losses and storage is probably incorrect for ephemeral channels. Additional work is in progress to determine the analytical form of the solution for the individual components and for the cascade of components.

Table 1. Hydrograph characteristics and derived C_1 values for 12 runoff events on channel reach 11-8.

Date of event	Peak Discharge Inflow	Peak Discharge Outflow	Final Value of C_1
	(cfs)	(cfs)	(sec-1)
020863	210.	14.	0.0125
310764	97.	2.	.0122
020864	720.	460.	.0095
050864	360.	140.	.0041
110964	2000.	1900.	.0041
180865	26.	7.	.0162
020965	97.	76.	.0096
300766	1100.	540.	.0054
130867	260.	97.	.0183
250967	70.	18.	.0278
020868	330.	64.	.0153
050868	1100.	430.	.0075

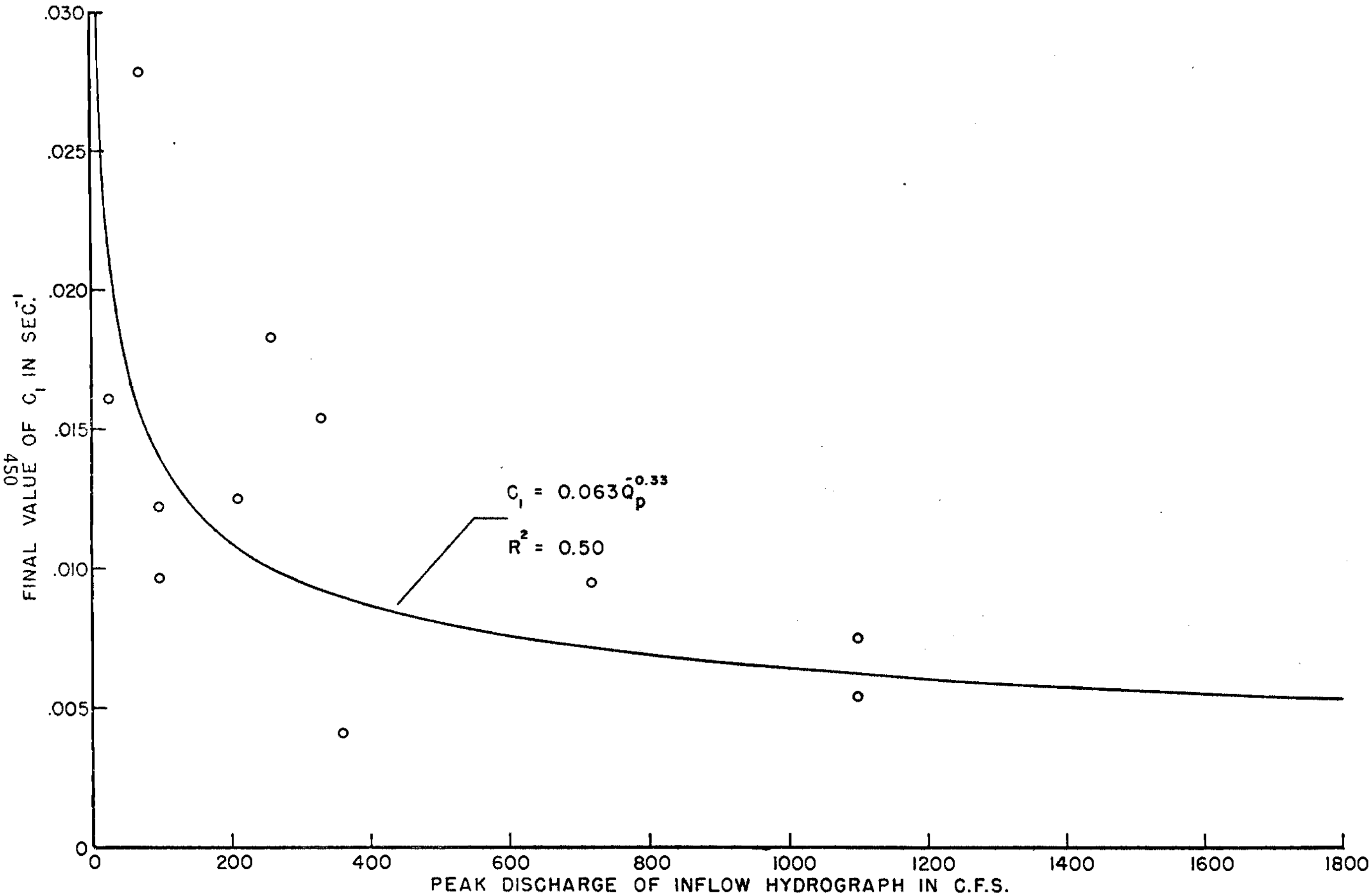

Figure 3. Variation in C_1 with increasing inflow to reach 11-8.

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