A DETERMINISTIC MODEL FOR SEMI-ARID CATCHMENTS

by

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INTRODUCTION

Semi-arid environments exhibit certain hydrologic characteristics which must be taken into consideration if the behavior of catchments in these areas is to be effectively modeled. For example, convective storms cause most of the annual surface runoff from semi-arid catchments. These storms occur as high intensity and short duration thunderstorms during the summer months. They are highly localized so that only a small portion of the catchment actually contributes flow to the storm hydrograph. Also streams in semi-arid catchments are ephemeral with flow occurring only about 1% of the time. Transmission loss, defined by Keppel and Renard (1962) as abstractions from surface flow during flow periods, occurs in these streams and may represent a substantial percentage of runoff.

Efforts have been made by several investigators to take the above features into account in predicting flow from these catchments. Such efforts, however, were concentrated on the stream channel itself rather than on the entire contributing area and were based on input-output type analyses. In particular Lane (1972) proposed a general storage equation for flow routing in ephemeral channels that considers transmission losses.

The objective in this study is to develop a simple synthetic catchment model that reflects the above mentioned features of the semi-arid environment and for which (i) the simplifying assumptions do not preclude the inclusion of the important components of the runoff process, (ii) parameters of the equations representing the component processes have physical interpretation and are obtainable from basin characteristics so that the model may be applicable to ungaged sites.

To this end we apply a reductionist approach in which the entire catchment (including the channel reaches) is subdivided into a finite number of meshes. The various components of the runoff phenomenon are delineated within each mesh as independent functions of the catchment. Simplified forms of the hydrodynamic equations of flow are used to route flow generated from each mesh to obtain a complete hydrograph at the outlet point. The observation, Duckstein et al. (1972) that runoff from individual events are independent permits us to develop the model on an event simulation basis.

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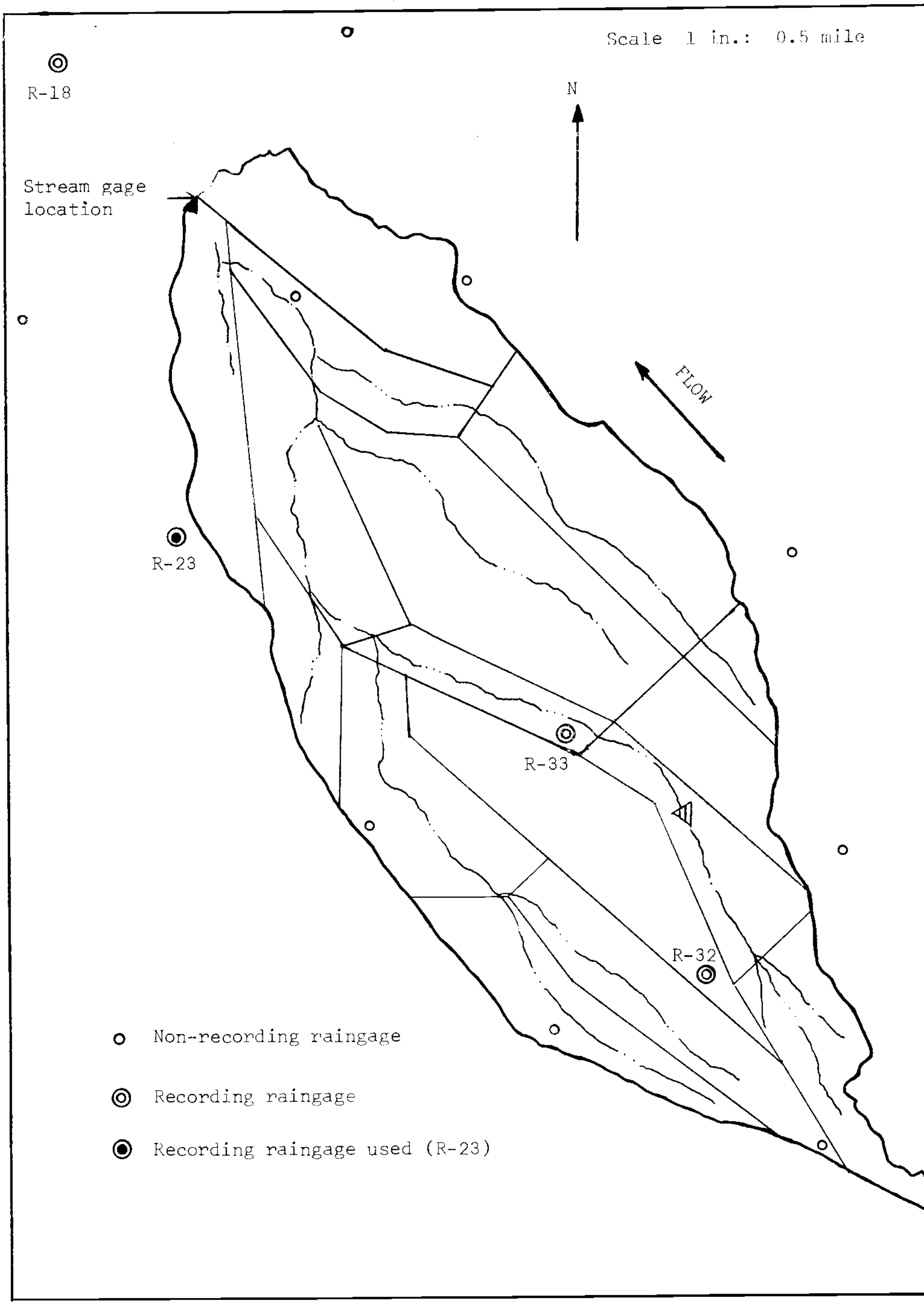

Figure 1. ATTERBURY WATERSHED SHOWING VARIABLE MESH STRUCTURE AND LOCATION OF GAGES.

A portion of the model we describe below, hereafter referred to as the Arizona model is based in part on the Purdue watershed model as reported by Huggins and Monke (1966).

The Purdue model is a deterministic event based model which determines the volume and rate of runoff , given rainfall assumed uniformly distributed over small catchments - an assumption that does not hold true for convective storms. To approximate to a distributed system the catchment was subdivided into a finite number of square meshes. The smaller the mesh size the better the approximation to a distributed system. However, the computation time and consequently the cost of running the program is increased considerably. Also, the model considers whole meshes only and since the catchment boundary will usually not coincide with the sides of the outermost meshes, a net addition or removal of a mesh or part thereof, may lead to substantial errors in the simulation.

MODEL DESCRIPTION

For the Arizona model, the catchment is subdivided into a finite number of meshes wherein each mesh may be of any size and shape and may consist of any number of sides (limited to seven in the program). The mesh boundaries are delineated on the basis of the catchment topography, soils and other attributes such that the physical characteristics of the catchment within each mesh are approximately uniform. The extremes of the sides and the centroid of each mesh are identified by their coordinates referenced from a given datum. A routine that computes both the centroid and the area of each mesh by the planimeter method is incorporated. A map of the Atterbury watershed [W-2], located southeast of Tucson, Arizona, with a variable mesh shape structure is shown in Figure 1.

RAINFALL INPUT

The model considers that a single raingage exists within or in the vicinity of the catchment at a fixed location. To obtain a spatial and temporal distribution of rainfall over the catchment it is assumed that, i) the runoff producing storm is unicellular for a convective storm, i.e., only one storm cell exists in the mesoscale area, ii) the location of the storm center (point of maximum intensity) is random and is not restricted to within the basin, iii) the storm is stationary. Rainfall data may exist in the form of intensity values or as rainfall total for the event. The latter case may occur when the available data is from a non-recording gage or has been obtained by regression for an ungaged site. To make use of the rainfall total to simulate hydrographs from the type of model we are concerned with here, an empirical time distribution of rainfall intensities for the locality of interest must be available.

A routine that determines rainfall amounts at the centroid of each mesh given rainfall in the gage was incorporated in the model. Given the amount over the mesh this routine computes the vector of rainfall intensities, which are, for each time interval, assumed to be uniform over the mesh. Four options are included to provide alternatives in spatial and temporal distribution of rainfall combinations. A description of the options follow:

(a) When the rainfall total only is available and a uniform spatial distribution is assumed, temporal distribution of rainfall intensities is obtained from the curves due to Fogel (1972). These empirical curves were derived from an analysis of storm events happening over the Tucson basin and were given as the cumulative % of storm rainfall depth as a function of the cumulative % of storm duration. See Figure 2. The curves were stored in a discretesized form in the program.

(b) When the rainfall total only is available and rainfall is assumed to be distributed according to an area-depth formula, the rainfall total, R_g, is used to compute the amount R at an assumed storm center using an iterative search of the Newton Raphson method. The area-depth formula used in the model was developed by Fogel and Duckstein (1969) and is given below as equation (1).

$$R_g = R \exp(-\pi z^2 b) \quad (1)$$

and

$$b = \alpha \exp(-\beta R) \qquad 0.75 \leq R \leq 5.0 \quad (2)$$

where R_g is the rainfall total recorded in the gage, in inches, at a distance z miles from the storm center and α and β are constants with numerical values of 0.27 and 0.67 obtained by regression. Thus, given R, the rainfall amount $R^{(i)}$ over the centroid of a mesh i (identified by its coordinate vector,(X) is then computed. The time distribution of rainfall over each mesh is computed using the curves obtained by Fogel (1972).

(c) For the case when rainfall intensities are available and a uniform spatial distribution is assumed, the recorded intensities at the gage are used for all meshes.

(d) When the rainfall intensities are available and an area-depth formula is adopted, the rainfall total is first computed by summing the product of the intensities and the corresponding time intervals. The amount at an assumed storm center and subsequently at mesh centroids is computed as in (b). The intensities over each mesh are computed in direct proportion to that over the gage.

RUNOFF SIMULATION

The mesh is used as a basis for describing the fundamental form of the model. The components of the runoff phenomenon identified and subsequently used in the model are the interception, depression storage and the infiltration processes. Evaporation is assumed to be negligible. The mathematical relationships used to represent the individual processes are discussed below.

Interception. Following Merriam (1972) an asymptotic negative exponential build-up of interception storage is assumed to describe the interception process prior to the satisfaction of the interception storage capacity. During this interval only the effective mesh area (mesh area minus the area covered by the intercepting surface) receives rainfall. The entire mesh surface receives

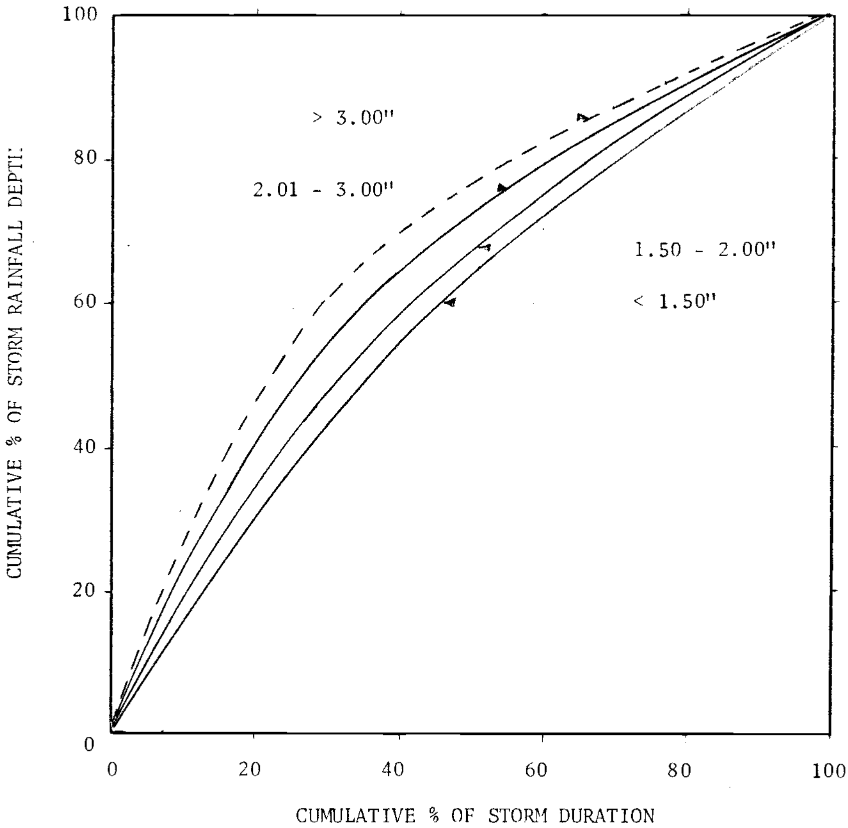

FIGURE 2. EMPIRICAL TIME DISTRIBUTION OF CONVECTIVE STORMS IN THE TUCSON AREA, ARIZONA. (AFTER FOGEL, 1972)

rainfall only after the interception storage capacity is satisfied. Merriam's equation is given below as:

$$S^{(1)} = [Sc^{(1)} - S^{(1)}][1 - \exp(-u/Sc^{(1)}] \quad (3)$$

where $S^{(1)}$ = rate of build-up, in/hr; $Sc^{(1)}$ = interception storage capacity, inches; $S^{(1)}$ = cumulative interception and u = rate of rainfall over the mesh, in/hr.

Depression storage. Huggins and Monke (1966) expressed the volume of depression storage as a function of the depth of water within the mesh. A storage-depth relationship was obtained and given in a dimensionless form as $Sc^{(2)}/S^{(2)} = f(hc/h)$ with $S^{(2)}$ = Ah; where $Sc^{(2)}$ = depression storage capacity for the mesh, hc = corresponding depth of water in storage, h and A are the current depth and surface area of water in storage. Based on the results of field tests, the above functional relationship was approximated with an equation of the power type, given as equation (4) below:

$$S^{(2)} = aW^{b} \quad (4)$$

where a and b are constants, $S^{(2)} = Sc^{(2)}/S^{(2)}$ and W = hc/h. Equation (4) is used to represent the depression storage process.

Infiltration. Holtan's (1961) equation as modified by Huggins and Monke (1966) is used to describe the infiltration process. The modified form is given below as equation (5):

$$f = f_s + m\left[\frac{Sc^{(3)} - F}{P_T}\right]^{n} \quad (5)$$

where f = infiltration rate, in/hr; f_s = steady state infiltration rate, in/hr; $Sc^{(3)}$ = storage capacity of soil above the impeding stratum (total porosity minus antecedent soil moisture), inches; F = cumulative infiltration, in inches and P_T = total porosity of the soil above the impeding stratum, inches; m and n are constants.

The following procedure was used by Huggins and Monke (1966) in the application of equation (5): (a) when the moisture content in the control volume is less than field capacity, the drainage rate is taken to be zero; (b) the soil within the control volume is assumed to be completely saturated when f becomes constant at f_s; (c) the drainage rate D, is assumed equal to the infiltration rate when the rate becomes constant and (d) when the water content is between field capacity and saturation, D is computed according to the relation:

$$D = fc\left[1 - \frac{\theta u}{\theta g}\right]^{3} \quad (6)$$

where θu = unsaturated pore volume, inches; and θg = maximum volume of gravitational water, inches; i.e., total porosity minus field capacity.

Overland flow. The kinematic wave approximation is used to describe the unsteady flow over the meshes which act as overland flow surfaces. Wave properties are given by a stage-discharge relation and the equation of continuity.

The stage-discharge relation used is the Manning friction formula, equation (7) below:

$$v = kd^m \tag{7}$$

and

$$K = 1.486\sqrt{So}/N \tag{8}$$

where v = local velocity, fps; d = flow depth, ft; So = slope of ground surface, N = Manning resistance coefficient and m = constant = 2/3. The continuity equation, for wide channels, is given by:

$$\frac{\partial d}{\partial t} + \frac{\partial (vd)}{\partial x} = q(x,t) \tag{9}$$

where x and t are the space-time coordinates in ft. and seconds, respectively; q(x,t) is inflow per unit area. Substituting for v in equation (9) and differentiating we have

$$\frac{\partial d}{\partial t} + K(m+1)d^m \frac{\partial d}{\partial x} = q\ (x,t) \tag{10}$$

Equation (10) is the kinematic wave equation used and is solved numerically in the finite difference form:

$$I_1 + I_2 - O_1 + \frac{2S_1^{(4)}}{\Delta t} = O_2 + \frac{2S_2^{(4)}}{\Delta t} \tag{11}$$

where $\Delta t = t_2 - t_1$; subscripts 1 and 2 refer respectively to values at the beginning and at the end of a time increment Δt and $S^{(4)}$ is surface storage after satisfying depression storage.

Runoff from one mesh serves as part of the inflow into adjacent meshes. Consequently, q(x,t) comprise inflow from adjacent elements and rainfall less abstractions. The flow from a mesh into adjacent meshes is aportioned *a priori* as fractions based on the surface geometry of the mesh. Thus, the fractions are input requirements.

INPUT SPECIFICATIONS

The input deck is divided into two groups. The first group describes the watershed and consists of:

(a) parameters of the equations used to describe the components of the runoff process in the simulation;

(b) mesh data which include mesh slope, soil type, fractions of flow from a mesh into adjacent meshes, coordinates of extremeties and centroid of the mesh.

The second group of input data gives specifications regarding the storm(s) to be simulated. Such data include coordinates of the raingage and storm center location (for convective storm simulation), the type of spatial and time distribution of rainfall input assumed. If, for example, the hyetograph is not available then the rainfall total and duration must be given. In such a case an empirical time distribution of rainfall that is incorporated in the model will be automatically used.

OUTPUT SPECIFICATIONS

Several output options have been included in the program. The options that may be exercised include the print out of:

(a) Watershed description which includes the model parameters, mesh information, e.g., slope and other variables describing the catchment.

(b) Storm description which include the raingage location and type of rainfall input, the storm duration and storm center location (if applicable), the area-depth model used and rainfall intensities.

(c) Simulation results which include a listing of time and discharge ordinates, a plot of the simulated hydrograph, likelihood of the storm producing this hydrograph and the hydrograph descriptors - peak discharge, runoff volume and time lag.

One or more of the major options may be exercised simultaneously. Also, selected options within any major option may be printed out. For example, in exercising option (c), plot of the hydrograph may be omitted.

The next section describes the catchment used in the simulation experiments to validate the model. The sources of data used in the simulation and the results of the simulation runs are discussed.

MODEL VALIDATION

Data from the Atterbury watershed, subwatershed W-2, is used to validate the model. A brief description of the subwatershed and data used in the validation are given below. Results of simulation runs are also given.

WATERSHED DESCRIPTION AND DATA

Subwatershed description. The Atterbury experimental watershed, of which subwatershed W-2 is part, is located about 15 miles southeast of Tucson, Arizona The climate is typically semi-arid with annual rainfall of about 11 inches; about 55% of which falls during the summer months. Rainfall during these months is of the convective type and is responsible for a major portion of the annual runoff.

The subwatershed is uninhabited and has an area of 4.5 sq. mi. with major flow channels that are generally wide with flat grassy beds. Vegetation is sparse and offers little protection to the soil and consequently has little retarding effect on surface flow (Woolhiser, 1959).

Based on the study by Gelderman (1964) the soil series within the catchment are grouped into two types. Type 1 soils are those that are sandy to gravelly loam and the Type 2 soils are those that are silty to clayey loam in texture. The type 1 soils occupy the western and central parts of the catchment and have a zone of lime accumulation that is 6 to 24 inches below the ground surface. A representative soil type is the Mohave sandy loam. The type 2 soils occur at the upper end and the eastern side of the catchment and have loam and sandy loam surface of moderate infiltration rates. Movement of soil water is restricted by the clayey subsoil. A representative soil type is the Laveen loam. Both soil types 1 and 2 are represented in the southern part of the catchment.

Data. The Water Resources Research Center of the University of Arizona maintains a network of recording and non-recording rainfall gages over a 20 sq. mi. area within which lies subwatershed W-2. The center also maintains a volumetrically calibrated tank with a capacity of 15 ac. ft. for the measurement of runoff rates from the subwatershed. The rainfall gages are located on an approximately one mile grid. Several years of data from a plastic raingage network are also available. Based on data from the combined network three convective storms that had only one storm center and one winter type [frontal] storm were selected for simulation. For the convective case, the location of the storm centers was obtained from corresponding isohyetal maps. The rainfall data for all storms simulated were taken from readings of the recording gage, R-23.

Infiltration characteristics of the Mohave sandy loam (soil type 1) and the Laveen loam (soil type 2), taken to be representative of the soil series within the catchment, were obtained by converting the parameters of the time-infiltration curves given by Yost and Gardner (1955) for these soils into those usable in the Holtan infiltration equation as modified by Huggins and Monke (1966).

The antecedent soil moisture prior to each of the storm events simulated was obtained for the soil types in the catchment based on soil moisture measurements at six stations maintained by the Water Resources Research Center.

Woolhiser (1959) observed that the interception due to vegetation (the only intercepting surface) in the catchment under consideration is negligible. Field observations justifies such a conclusion. Consequently, a nominal value for the interception storage capacity, $S_c^{(1)} = 0.001$ was assumed. Judicial estimates were used for model parameters, such as those for the depression storage equation, that could not be estimated for lack of field or experimental data.

RESULTS OF SIMULATION

Three hydrograph descriptors - peak discharge Q, runoff volume V and the time lag T are used to characterize both the simulated and observed hydrographs. A fourth variable, the rainfall at the storm center was used to test the applicability of the Fogel-Duckstein area-depth model to describe thunderstorm rainfall patterns for which it was developed.

Values of the above descriptors are given in Table 1 which shows a generally satisfactory match between observed and simulated variables, particularly, since only initial estimates of the parameters are used. The

Table 1. Hydrograph Descriptors for Simulated Storm Events

Storm Event	Peak Discharge (cfs)		Runoff Volume (ac. ft.)		Time Lag (min)		Rainfall at Storm Center (in)	
	Simulated	Observed	Simulated	Observed	Simulated	Observed	Simulated	Observed
Sept. 13 1962	58.1	50.9	2.1	1.8	23	20	1.04	1.00
Sept. 12 1965	3.63	7.3	0.15	0.55	30	37	1.79	1.25
July 17 1967	270.6	290	20.1	28.5	67	*	1.67	2.08
Feb.** 7 1966	45	38.3	25	10.6	77	*	--	--

* Time distribution of rainfall given in 15-min time increments; exact time of maximum rainfall intensity not available.

** Frontal storm, uniform spatial distribution of rainfall assumed.

validation of the model is continuing with data from other instrumented catchments.

DISCUSSION AND CONCLUSION

An apparent limitation in the application of the model we have described above, for runoff simulation, is the fact that it assumes that the storm is unicellular. However, observed data from a 20 sq. mi. raingage network over the Atterbury Watershed located in southeastern Arizona showed that (Fogel and Duckstein, 1969) of 64 storm events occurring over a 12 year period, 52 of these had one storm center, 9 had two and 3 had three storm centers. From this observation it appears reasonable to assume one storm center particularly for catchments of say, less than 20 sq. mi. in area.

The model is event-based so that there exists the difficulty of providing current and realistic estimates of the parameters in the equations of the model particularly since most estimates are derived from small samples. This should stimulate further research into the nature and effect of the uncertainty in the true values of the parameters on the simulated hydrograph since such simulation results are applied towards economic forecasts.

The above limitations notwithstanding, the model contains certain features that should support its use for simulating flow from semi-arid catchments. The model is versatile and flexible, simulating runoff as it does by considering all the contributing areas of the basin.

With the variable-mesh shape structure the runoff process is better simulated. Since in urban basins street patterns are generally designed to follow the topography, the model may be used to design urban drainage systems that will effectively attenuate peak flow and handle flow volumes.

The model gives the entire hydrograph and the descriptors which may be applied for social and economic forecasts. The time lag may be applied towards flood forecasting by providing the warning time, the peak discharge towards the design of hydraulic structures such as storm outlet structures and street carrying capacity. The simulated runoff volume may be used in estimating required recharge basin capacity and in selecting pump size for artificial recharge purposes.

The flexibility in the model is reflected in the four options of rainfall input type and spatial rainfall distribution model combination. We note that any desired area-depth formula may be easily incorporated particularly if it has been tested for the area of interest.

Lastly and no less important the model may be used as a research tool to investigate, in a sensitivity analysis, the effect of the various sources of uncertainty in input information on the descriptors of the simulated hydrograph. To this end the model has been used, Nnaji et al. (1974), to investigate the effect of uncertainty in the rainfall input on the descriptors of the simulated hydrograph.

CONCLUSION

The model we have described reflects features characteristic of semi-arid environments. It may be used to obtain hydrograph descriptors which are used as design variables given the rainfall pattern and distribution and the model parameters.

Data limitation and the use of judicial estimates of parameters are sources of uncertainty. These uncertainties create uncertainty in the hydrograph descriptors. It is our general objective, in on going research efforts, to quantify the effects of the uncertainties in the input into the model on the descriptors of the hydrograph in an economic sense.

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