

THE APPLICATION OF STEP-DRAWDOWN PUMPING TESTS FOR DETERMINING
WELL LOSSES IN CONSOLIDATED ROCK AQUIFERS

by

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ABSTRACT

The concept of a step-drawdown test was first introduced by Jacob, and further modifications in the technique were made by Rorabaugh. Analysis of step-drawdown test data enables the quantification of the components of drawdown due to formation or aquifer loss, and due to well losses in a pumped well. This technique has been used to test approximately 100 wells that were drilled in crystalline and basalt formations in central India. Test data have been analyzed by Rorabaugh's method and by a graphical method, and the results of a number of tests are presented and discussed.

Anomalies in the test analysis often proved helpful for interpreting aquifer irregularities. In general, the well loss constant decreases with an increase in specific capacity and the aquifer loss constant decreases with increasing transmissivity. Significant reductions in specific capacity during a step test occur in wells with high well losses. An attempt is made to quantify the well losses in a consolidated rock well, and a number of practical applications of step-drawdown tests are discussed.

INTRODUCTION

The Evangelical Lutheran Church (E.L.C.) Water Development Project has been involved in ground-water development activities since 1971. To date, over 500 tubewells have been drilled for agricultural, village, industrial, institutional and municipal water supplies. The Project's area of operation is in the Satpura hill region of central India and includes the districts of Seoni, Chhindwara and Betul. Prior to 1971, this area was almost entirely dependent on surface water and shallow open wells (30-40 feet deep) for water supply. The high density of these open wells (0.75/square mile overall) combined with the increased usage of electric pumpsets for irrigation has resulted in the seasonal overpumping of shallow aquifers in many areas. The recent drilling of over 1,000 deep wells in the area has alleviated the problem to a degree.

Approximately 100 pumping tests have been carried out on production wells that were to be equipped with power pumps. Testing procedures generally included a step-drawdown test followed by a constant-rate test. Test data were analyzed by standard analytical methods and for a number of tests reasonable values of hydraulic properties were obtained. The purpose of this paper is to:

1. Discuss the theory of the step-drawdown test and methods of step-test data analysis.
2. Review results of previous studies where step-drawdown tests were applied.
3. Discuss the results of the step-drawdown tests in the study area.
4. Attempt to quantify well losses in hard rock wells.
5. Discuss practical applications of step-drawdown tests.

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AREA

The subject area lies in the central part of the country and is traversed by the Satpura hills. The hilly regions are mainly forested; the remaining area is undulating or flat with an average elevation of around 2,100 feet. The subject districts occupy an area of 11,826 square miles, and contain 5,156 villages and towns, with a total population of 2,993,961. Approximately half of the total area is forested; the remaining half is under intensive cultivation.

GEOLOGY

The geological formations in these districts range in age from Precambrian to Recent. The general rock types are: granite, gneiss, schist, quartzite (Archean), sandstone and shale (Gondwana), and basalt (Cretaceous-Eocene). The majority of wells tested have been in the crystallines and basalts.

Deccan trap, or basalt, is an extrusive igneous rock formed due to the cooling of lave from fissure eruptions. Aquifers occur in the vesicular or weathered amygdaloidal portions of a lava flow, at brecciated or broken-up flow contacts, and in fracture openings.

In the crystallines, aquifers occur in permeable weathered zones, and where bedrock is jointed and fractured. The yield of an individual well is dependent on the thickness and permeability of weathering and on the intensity, interconnection, and areal extent of joints and fractures.

PUMPING TESTS

Approximately 100 pumping tests have been conducted and observation wells were available for only a few of the tests. The majority of wells tested were drilled by air hammer rigs and were from 6.0 to 6.5 inches in diameter. Submersible pumps of 5.75 and 3.75 inches in diameter were used for testing. The discharge pipe was equipped with a standard water meter and water levels in the pumped well were measured by an electric sounder. The testing of each well was conducted in the following manner:

1. A step-drawdown test is conducted for six hours consisting of 3 to 6 steps.
2. Recovery is measured for 12 hours before starting the next phase.
3. A constant-rate test is run for 12 to 24 hours and recovery is measured for the same duration as pumping.

Step-test data are analyzed by Rorabaugh's method (1953), and by a graphical method. Constant-rate tests are analyzed by the Cooper-Jacob (1946) modified, non-leaky artesian formula. As testing work progressed and a number of tests were analyzed, anomalies in the test data proved helpful for interpreting aquifer irregularities. For a number of tests, hydraulic properties were obtained and were generally found to be reasonable. After drawdowns were corrected for well losses, values of corrected specific capacity for 12 hours of pumping and transmissivity were compared to theoretical plots of Q/s versus T .

The validity of analytical methods for the evaluation of pumping tests in consolidated rock aquifers has long been discussed and debated. The derivation of the basic equations governing ground-water flow are dependent on a number of assumptions, viz., the aquifer is intergranular, homogeneous, isotropic, infinite in areal extent, of uniform thickness, and for confined aquifers the flow is both radial and laminar. When one considers the mode of occurrence of groundwater in consolidated rock wells, that is, through joints and fractures, it is difficult to visualize homogeneity, isotropy and radial laminar flow, except possibly in a brecciated or highly fractured media.

Eagon and Johe (1972), in discussing the occurrence and movement of water in carbonate rocks, have noted that hydraulic characteristics seem to be inconsistent in the vicinity of the borehole and this may be of some importance initially in a pumping test, but as the cone of depression becomes larger and covers a representative area of the aquifer these irregularities assume less importance. And the larger the area considered, the more nearly some carbonate aquifers effectively assume the hydraulic characteristics of a homogeneous media.

In effect, when the cone encompasses a representative area of the aquifer, the resultant drawdown in the well represents the sum total effect of the hydraulic characteristics of the aquifer in the area encompassed by the cone, including any irregularities, etc. It seems reasonable that, in a crystalline or basalt aquifer where there are a number of fractures or brecciated zones connected on an areal basis, a representative sample of the aquifer could assume the characteristics of a homogeneous media.

REVIEW OF LITERATURE

JACOB'S METHOD

Jacob (1947) introduced a concept of a multiple step-drawdown test with the objective of determining well losses and the effective radius of a well. Jacob noted that drawdown in an artesian well has two components, the first component termed "aquifer or formation loss" arises from the "resistance" of the formation. It is proportional to discharge, Q , and increases with time as the cone of influence expands. The second component, termed "well loss," represents the loss of head that accompanies the flow through a well screen, and in the casing to the pump intake. Well loss is proportional to the square of the discharge and is independent of time. The following equation was defined:

$$s_w = DQ + CQ^2 \quad (1)$$

where s_w = the total drawdown in the well;

DQ = the aquifer or formation loss;

CQ^2 = the well loss;

B = the aquifer loss constant. It represents the total resistance of the formation from the well face out to the radius of influence. Its units are sec/ft^2 or ft/gpm , and it increases with the log of time; and

C = the well loss constant. Its units are sec^2/ft^5 or ft/gpm^2 . C is constant with time.

Jacob further defined the "effective radius," r_w , which is the distance, measured radially from the axis of the well, to a point outside the well where the theoretical drawdown based on the logarithmic head distribution equals the actual drawdown just outside the screen. Thus, the total drawdown in a well can be expressed as:

$$s_w = \frac{Q}{4\pi T} \left[\ln \frac{4tT}{r_w^2 S} - 0.5772 \right] + CQ^2 \quad (2)$$

and

$$B = \frac{1}{4\pi T} \left[\ln \frac{4tT}{r_w^2 S} - 0.5772 \right] \quad (3)$$

where Q = the discharge in cfs;

T = the transmissivity in ft^2/sec ;

t = the time in seconds;

S = storativity; and

r_w = the effective well radius.

The following equation was developed for computing the well loss from step-drawdown test data:

$$C = \frac{\frac{\Delta s_i}{\Delta Q_i} - \frac{\Delta s_{i-1}}{\Delta Q_{i-1}}}{\frac{\Delta Q_i}{\Delta Q_{i-1}} + \Delta Q_i} \quad (4)$$

where Δs_i = the incremental drawdown for step i for a fixed time, t_i ; and Δs_{i-1} is the incremental drawdown for step $i-1$.

ΔQ_i = the incremental increase in discharge from step $i-1$ to step i ; and ΔQ_{i-1} is the same from step $i-2$ to step $i-1$.

Note, when using equation (4), that $t_i = t_{i-1} = t_{i-2}$, etc. That is, the time at which Δs_i is determined for each step must be equal.

Another important concept Jacob discussed is the variation of specific capacity (Q/s) with discharge and time. Specific capacity decreases with time and if there are no well losses in a pumped well, then, for any fixed time, the specific capacity is independent of discharge. That is, for a fixed time of pumping in a given well, specific capacities at different rates of discharge will be the same. If, however, well losses represent a portion of the total drawdown in a pumped well, then, for a fixed time, the specific capacities at higher rates of discharge will be less than specific capacities for lower rates of discharge. The higher the well losses, the greater the percent decrease in specific capacity. This is because well losses are equal to CQ^2 .

RORABAUGH'S METHOD

Jacob assumed that the head loss due to turbulent flow is approximately proportional to the square of the velocity. Rorabaugh (1953) suggested that instead of assuming a value of $n = 2$ in the well loss term, it is best to first determine if the flow is laminar or turbulent and for the turbulent flow regime determine a value for n from a graphical solution of equation (6) on log-log paper. Rorabaugh stated that drawdown in an artesian well resulting from the withdrawal of water is made up of head loss resulting from laminar flow in the formation, and head loss resulting from turbulent flow in the zone outside the well, through the well screen, and in the well casing. Two expressions were developed for computing the drawdown, s_w , in a well being pumped at rate Q . The first expression (equation (5)) is applicable for laminar flow where Q is less than some Q_c , which is the critical or transitional Q below which laminar flow prevails. The second expression (equation (6)) is applicable for the turbulent flow regime:

$$Q < Q_c \quad s_w = BQ + C'Q \quad (\text{for laminar flow}) \quad (5)$$

$$Q > Q_c \quad s_w = BQ + CQ^n \quad (\text{for turbulent flow}) \quad (6)$$

He further noted that Jacob's use of $n = 2$ was based on the assumption that the critical or effective radius is constant as discharge varies. It is more probable that at low discharges flow is laminar and as discharge is increased turbulent flow will first occur at the well face, and as discharge is further increased the boundary between laminar and turbulent flow will move outward into the well. Rorabaugh attempts to compensate for this variation in the critical radius with discharge by applying two equations: equation (5) for laminar flow, and equation (6) for turbulent flow. Also,

the application of the exponent n in the term CQ^n compensates partially for the movement of the laminar-turbulent flow interface with an increase in Q .

RESULTS OF PREVIOUS STUDIES USING STEP-DRAWDOWN TESTS

Step-drawdown tests have been used by a number of people in a variety of geological formations in various parts of the world. The results of a few studies are summarized.

NORTHWEST OHIO, U.S.A.

The Northwest Ohio Water Development Plan (Eagon and Johe, 1972) carried out a drilling and testing program in order to evaluate the quantity and quality of the groundwater available in northwest Ohio. Seventy-six wells were drilled and tested in limestone and dolomite rock.

Step-test data were analyzed by three methods, viz., Rorabaugh's graphical method, Bruin and Hudson's (1955) graphical method, and Jacob's method (equation (4)). It was felt that graphical solutions had an advantage over averaged numerical data used in Jacob's method and that the CQ^2 relationship approximated well losses to an acceptable degree for the pumping rates used. The effect of aquifer dewatering on drawdown and on the plot of s_w/Q versus Q was discussed, and it was noted that dewatering caused an increase in the slope of the plot.

It was noted that the well loss constant, C , is related to the number and properties of openings that contribute water to a well and if dewatering occurs, the number of openings is decreased, thus causing an increase in both the value of C and drawdown. This, in turn, causes an increase in the slope of the plot of s_w/Q versus Q . Values of the well loss constant were plotted against values of specific capacity (24 hours) on log-log paper. The plot indicates a general relationship between the two parameters, and the well loss constant is inversely proportional to specific capacity.

ILLINOIS, U.S.A.

The Illinois State Water Survey (Walton and Csallany, 1962; Csallany and Walton, 1963) used the step-drawdown method in many pumping tests in sandstone, dolomite, and limestone wells in Illinois. In most cases, Jacob's method was used to analyze the data. It was noted that the well loss constant, C , increased when water levels were lowered below the top of shallow dolomite aquifers. Step-test data were used to correct observed specific capacities for well losses and values of corrected specific capacity and transmissivities were compared with theoretical plots of Q/s versus T . It was also noted that the effects of well development could be ascertained from step-test results and that a step-test could be used to determine the degree of deterioration of a well after some period of time. Well efficiency was defined to be equal to BQ/s_w . Lastly, high values of C were obtained from wells with low specific capacities and low values of C for wells with high specific capacities. This was attributed to greater turbulence and well losses in low transmissive formations.

ALBERTA, CANADA

The Ground Water Division of Alberta (Lennox, 1966) conducted a number of step-drawdown tests in sedimentary formations of relatively low transmissivity. Lennox found that the common practice of assuming that the slope of the extrapolated drawdown curves for each step are proportional to the pumping rate was erroneous. The correct method is to extrapolate drawdown for each step by projecting the trend encountered toward the end of each step.

Rorabaugh's method proved better than Jacob's for analyzing the step-test data. Half of the eighteen wells tested showed laminar flow conditions throughout the range of pumping rates. Two wells had laminar flow initially and turbulent flow at higher discharges. Seven wells showed turbulent flow for the whole range of discharges. An interesting point with these results, that Hoog (1968) observed, was that, as specific capacity decreases, the maximum pumping rate at which laminar flow exists also decreases, which does not seem to be logical. Lastly, Lennox noted that a prerequisite for a satisfactory test is an adequate range of pumping rates. An increase of threefold was recommended.

IRAN AND WASHINGTON STATE, U.S.A.

Bierschenk (1964) used the graphical method of Bruin and Hudson to analyze forty-seven step-drawdown tests in Iran and Washington State. The wells were in fluvial deposits of Quaternary age with both water table and artesian conditions. In general, the wells were productive.

Bierschenk noted, as did Jacob, that the efficiency of a well is governed largely by the magnitude of well loss and that the efficiency falls off rapidly as discharge is increased. Thus, the efficiency in a well with high transmissivity is offset by well losses to a larger degree than in an aquifer with low transmissivity.

DISCUSSION OF STEP-TEST RESULTS FROM THE SATPURA REGION

Step-drawdown results were analyzed by two methods, viz., Rorabaugh's graphical method and by a graphical solution of the equation $s_w = BQ + CQ^2$. Of the two methods of analysis, the graphical method of plotting s_w/Q versus Q on rectangular paper proved to be the more practical. This method is especially useful where dewatering or boundary conditions occur as these can often be detected by a change in the slope of the plot of s_w/Q versus Q (Figure 1). In the case of dewatering (Figure 1, P.T. 102), the hydraulic characteristics of the aquifer are changed since the number of openings contributing water is decreased by dewatering, resulting in increased turbulence in the aquifer near the well face, and an increase of the value of the well loss coefficient, C . Thus, in the linear plot of s_w/Q versus Q , dewatering results in an increase of the slope.

An aquifer of limited extent (Figure 1, P.T. 77) will have a similar effect on the plot of s_w/Q versus Q , but generally the change in slope is not as pronounced as for the case of full or complete dewatering of an aquifer. If recharge occurs during the test, the slope of the curve will be decreased (Figure 1, P.T. 31).

If Rorabaugh's graphical method were applied to step-test data that contained anomalies, detection of these would be difficult since a trial and error procedure is used to make all the points fall on a straight line when plotted on log-log paper.

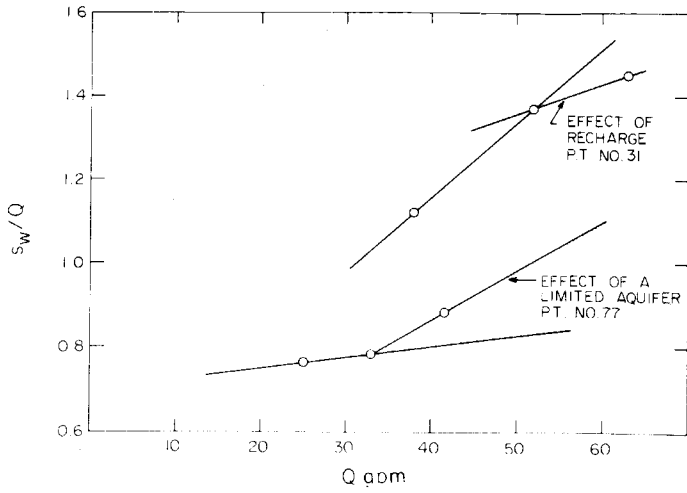
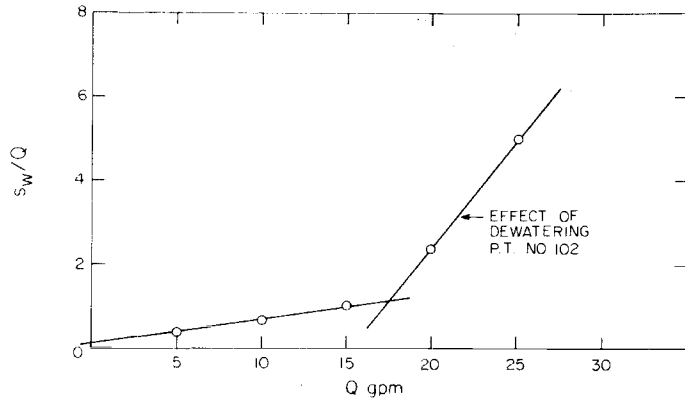
Table 1 contains the results of a number of step-drawdown tests that were conducted in crystalline and basalt wells in the study area. Most of the data in Table 1 are self-explanatory. Columns 4 and 5 contain the aquifer and well loss constants determined from the graphical solution. Columns 6 and 7 contain the constants determined from Rorabaugh's method. In Columns 10 and 11, the aquifer and well loss components are computed for maximum test discharge using B and C from the graphical solution. In Columns 12 and 13, the same is done using B , C and n from Rorabaugh's solution. From this table, it becomes apparent that the proportion of drawdown due to well losses is significant for the majority of wells tested. This factor assumes particular importance for wells that have a low available drawdown.

There appears to be a relationship between percent decrease in specific capacity and well loss. In wells with low well losses, generally the percent reduction in specific capacity is low. In wells with high well losses, the percent reduction in specific capacity is quite significant.

In a few tests, values of the exponent n calculated by Rorabaugh's method were less than 2.0. In some of these solutions with low values of exponent, the value of the well loss constant was quite high, and the proportion of drawdown due to well loss seemed to be unreasonably high.

There does not appear to be any relationship between transmissivity and the proportion of drawdown due to well losses. That is, in low-yielding wells, there are cases of wells with both high and low well losses. The same was observed for wells with high transmissivities. If a significant proportion of well loss occurs in the aquifer adjacent to the well, then it is likely that the quantity of well loss for an individual well could be dependent on the number, orientation and nature of the openings in the area adjacent to the well.

FIGURE 1. PLOTS OF s_w/Q VERSUS Q



In general, step-test data for crystalline wells plotted with less variance than for basalts. The best results were from the highly fractured and productive aquifers in the crystallines.

Figure 2 is a plot of $Q/s_{max} \times Q$ versus C (sec^2/ft^5). Q/s_{max} is the specific capacity at the end of the step test, and Q_{max} is the maximum rate of discharge for the test. From the plot it is apparent that as specific capacity decreases the well loss constant, C , increases. Similar results have been noted by Walton (1962), Hogg (1968), and Eagon and Johne (1972). It is debatable if this is attributed to greater turbulence and well losses in low transmissive formations as Walton has indicated. An inspection of equation (4) indicates that for low specific capacities s/Q will be high, Q will be low and, hence, C will be high.

Figure 3 is a plot of the aquifer loss constant, B (sec/ft^2), versus transmissivity. It can be seen from equation (3) that the aquifer loss constant, B , is proportional to $\ln T/T$ and, thus, should decrease with an increase with T . The data in Figure 3 are quite scattered, but there is a definite pattern of decreasing B with increasing T .

QUANTIFICATION OF WELL LOSSES IN CONSOLIDATED ROCK WELLS

In a well completed in an alluvial formation, well losses are associated with resistance to flow through the well screen and inside the well. In a consolidated rock well, well screens are rarely used; however, well losses represent a significant portion of the total drawdown in both low and high yielding wells.

In a consolidated rock well, well losses are associated with flow inside the well, in the annular space between the pump and borehole, and in the aquifer near the well face. While it may be difficult to quantify the well losses in the aquifer near the well face, the losses due to flow in the well and in the annular space between the pump and the borehole can be determined.

First, head losses are computed for discharges of 50, 100, 150 and 200 gpm in a 6-inch diameter well. The results are in Table 2.

Table 2

Discharge (gpm)	Head Loss ($h_L/100$ feet)
50	0.08
100	0.34
150	0.72
200	1.24

Next, the head losses in the annular space between the pump and borehole are computed. The standard size of bore for a water well in consolidated rock drilled with an air hammer rig is 4 to 4.5 inches and 6 to 6.5 inches. Pump manufacturers recommend a 3.75-inch diameter pump for the 4 to 4.5 inch diameter bores and a 5.75 inch diameter pump for the 6 to 6.5 inch diameter bore. Head losses are calculated in Table 3 for 6 inch, 6.25 inch and 6.5 inch diameter wells using a 5.75 inch diameter pump for discharges of 50, 100, 150 and 200 gpm. Lengths of submersible pumps range from about 5 feet to 10 feet in the 5.75 diameter class.

FIGURE 3. PLOT OF AQUIFER LOSS COEFFICIENT "B"
VERSUS TRANSMISSIVITY, T.

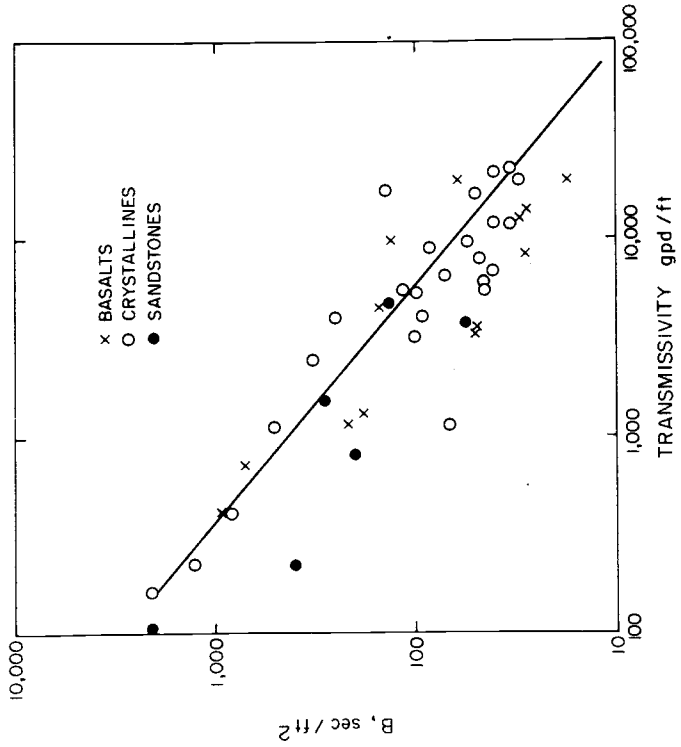


FIGURE 2. $Q_{max} \times Q/s$ AT Q_{max} (gpm)²/11 VERSUS WELL LOSS COEFFICIENT C , $sec^2/11^5$

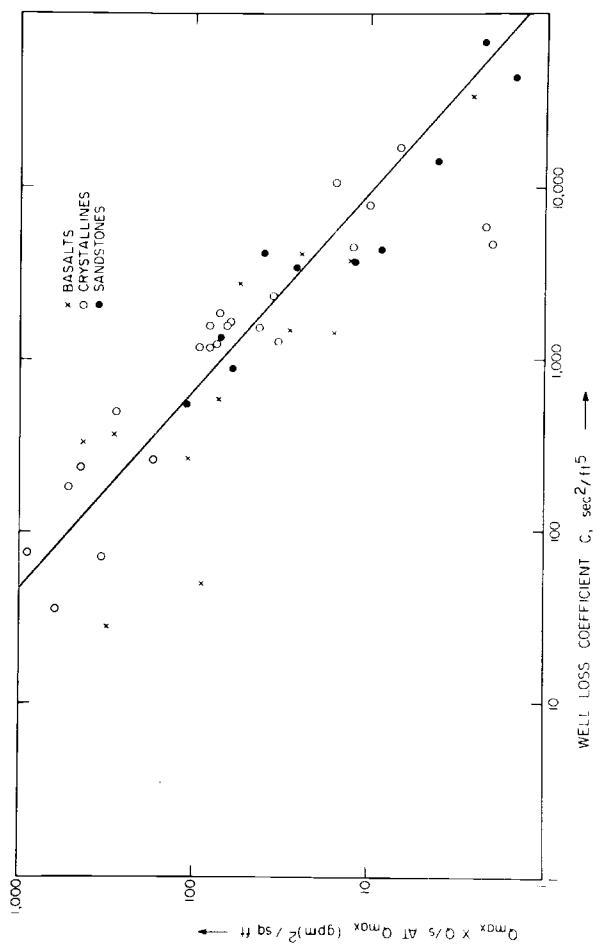


Table 3

Q (gpm)	50	100	150	200
Q (cfs)	0.11	0.22	0.33	0.44
<u>6.50-inch diameter well</u>				
Velocity in the annular space (ft/sec)	2.2	4.4	6.6	8.8
Head loss per foot in the annular space (h_L /ft)	.036	.144	.345	.58
<u>6.25-inch diameter well</u>				
Velocity (ft/sec)	3.4	6.8	10.3	13.75
Head loss (h_L /ft)	.13	.51	1.18	2.11
<u>6.00-inch diameter well</u>				
Velocity (ft/sec)	6.47	12.9	19.4	26.0
Head loss (h_L /ft)	.93	3.72	8.42	15.1

Well losses associated with flow in the well at lower rates of flow are negligible and for discharges around 200 gpm losses are approximately 1.25 ft/100 feet. Losses due to flow through the annular space between the pump and borehole are significant if discharges are high and if the bore is less than 6.25 inches in diameter. It is advisable that 6.5-inch diameter wells be drilled rather than 6-inch diameter wells considering the well losses that occur in the annular space and the increased pumping costs that will result.

Most of the wells tested in this study had diameters of 6.25 inches and above, and maximum discharges were generally less than 100 gpm. Hence, it would seem, for most wells, that a greater proportion of the well loss occurs in the aquifer itself in the vicinity of the borehole.

APPLICATION OF STEP-DRAWDOWN TEST RESULTS

In order to estimate the safe yield of a well, the following parameters must be known: well loss coefficient, C ; critical pumping levels, that is, the depth to productive aquifers; aquifer transmissivity and the rate of drawdown with time; recharge characteristics; and the available drawdown.

Transmissivity and the rate of drawdown with time can be determined from a constant-rate test. If only the pumped well data are available, the Cooper-Jacob modified equation must be used for analysis. From the results of a step-drawdown test, the components of drawdown due to aquifer loss and well loss can be determined as well as values of the aquifer and well loss constants. Thus, calculations of the drawdown due to well loss for any rate of flow can be determined. Recharge characteristics for the study area are fairly straightforward since there are distinct rainy and dry seasons. Critical pumping levels can be determined if the depth to productive aquifers is known, and available drawdown will be the difference between seasonal low pumping levels and critical pumping levels. Lastly, the effects of interference from nearby pumping wells must be taken into account.

If the above parameters are known, the analysis is fairly straightforward if critical pumping levels are above the top of productive aquifers and if there are no boundaries. Constant-rate test drawdowns are corrected for well losses, then the theoretical drawdowns at any rate of flow can be determined since the rate of change of slope is directly proportional to the increase in Q . On the semi-log plot of drawdown versus time, these curves can be extended to the time period of interest and, by adding the well losses for the pumping rate, the actual drawdown in the well at the time of interest can be determined. Using this type of analysis, it can be determined if water levels for a particular pumping rate at a particular time will be above or below critical levels.

If the aquifer is of limited extent, the same method of analysis can be applied, but, in this case, the rate of drawdown with time that occurs due to the limited aquifer must be used rather than the initial rate of drawdown with time. When dealing with wells in consolidated rock aquifers, it is important to realize that fracturing can be localized, resulting in aquifers with a limited extent. It often pays to have a pumping test of sufficient duration to enable the delineation of boundaries.

In aquifers under slightly confined conditions, available drawdown is low; that is, the distance from the static water level to the top of principal water-bearing zones is low. In such wells, dewatering is likely and the estimation of safe yields should be done with caution and care.

One further application of a step-drawdown test is for wells with poor or non-existent drillers' logs, as a step-test often enables the determination of the depth to significant water-bearing zones.

CONCLUSIONS

1. At this point, it seems that the step-drawdown test can be used with fair results in consolidated rock aquifers.
2. The relation of C with Q/S correlated with previous results and B appears to behave according to theory.
3. Well losses comprise a significant portion of the total drawdown in a number of low- and high-yielding wells. It appears that the majority of well losses occur in the aquifer itself.
4. The graphical method of solution is preferable since anomalies in the test data can be helpful for interpreting aquifer irregularities. The CQ^2 relationship gives the best approximation for well losses.
5. The results of a step-test can be quite helpful in the determination of a safe yield for a well. It is also a useful technique for testing wells with poor or non-existent records.

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