# APPLICATION OF A DOUBLE TRIANGLE UNIT HYDROGRAPH TO A SMALL SEMIARID WATERSHED

by

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## ABSTRACT

Hydrographs of runoff from small watersheds in semiarid regions often have a sharp peak with a relatively short time of rise followed by a slower recession and a tail of low flow. This characteristic shape suggests the possible use of a double triangle unit hydrograph recently introduced to hydrology. The shape of this unit hydrograph is specified by four parameters, which may be estimated by an optimization procedure based on using the sum of absolute deviations or some other suitable criterion as an objective function. Rainfall and runoff data for a number of storm events on a small watershed in the Santa Rita Experimental Range in southeastern Arizona have been analyzed to test the above idea. Double triangle unit hydrographs were fitted to individual storm events. The differences in the shapes of individual unit hydrographs were found to be small so that they could be approximated by a single double triangle unit hydrograph.

## INTRODUCTION

Runoff from small and very small watersheds in semiarid zones is accompanied by substantial infiltration losses in the stream channels. This fact and the usually steep slope of the channels tend to produce sharply peaked runoff hydrographs. The characteristic shape of the hydrographs, which starts from, and ends in a condition of no flow, thus consists of a fairly narrow triangular peak followed by a relatively longer tail of low flow. Within the triangular peak period the time to peak is usually shorter than the recession time.

The characteristic shape of the runoff hydrograph, which is more noticable for short duration storms, suggests that the rainfall-runoff process may be represented by a double triangle unit hydrograph. A single triangle could probably also give fairly good representation of the process but a double triangle offers more flexibility in unit hydrograph shapes. This in turn leads to better agreement between measured and computed flow hydrographs. The added cost of using the double triangle shape is the larger number of parameters handled. However, this does not present any difficulty since all computations are carried out by a computer.

There are some doubts whether the unit hydrograph concept is applicable to the condition prevailing in semiarid watersheds where there are appreciable channel losses and where the area contributing to runoff may not be the entire area of the watershed. Furthermore, the magnitude of the partial area contributing to runoff is probably variable from storm to storm. Nevertheless, the great simplicity of the unit hydrograph approach and the fact that it had been used previously in semiarid watersheds with relatively good results led to the present study.

The purpose of this paper is to present the results obtained in a study of rainfall and runoff records obtained during the 1975 rainy season in one of the Santa Rita Experimental Range watersheds located in southern Arizona. In the study double triangle unit hydrographs were fitted to the storm events, basing the computations on the assumption that the entire area of the watershed contributed to runoff. The individual unit hydrographs best fitted for each storm were derived and the goodness of fit between observed and calculated hydrographs was noted. The possibility of using one unit hydrograph having mean values for its parameters was also investigated.

## WATERSHED AND DATA USED

The Santa Rita Experimental Range covering an area of about 78 square miles is located 30 miles south of Tucson. It was established in 1903 and maintained by the Forest Service, USDA, for the purpose of studying the interrelationships of organisms, attributes, and processes of semi desert ecosystems (Martin and Cable, 1975). Recently a number of small experimental watersheds have been established within the range for the hydrologic study of various aspects of semiarid watersheds. The watersheds are equipped with flow measuring flumes, recording raingages and soil moisture measuring elements, and they are maintained and operated by the Southwest Watershed Research Center, Agricultural Research Service, USDA.

Rainfall and runoff data from one of these watersheds were used in this paper. The data were recorded in the rainy season of July-September 1975, in a watershed identified as Santa Rita Watershed

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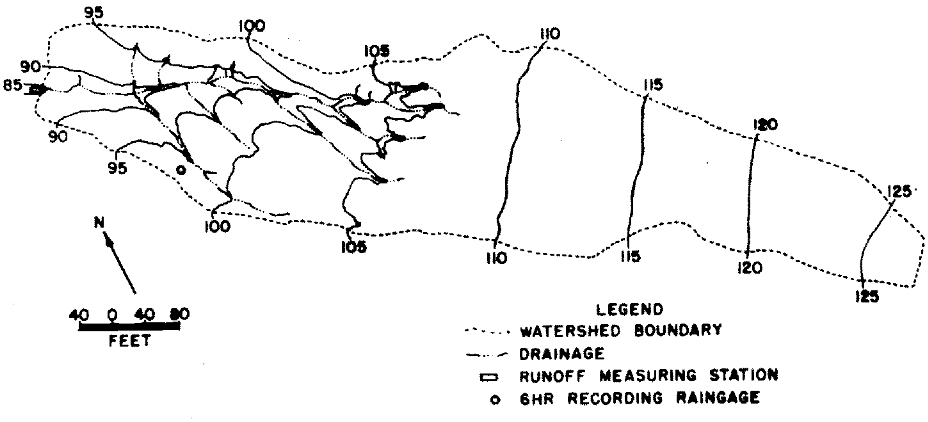


Fig. 1 -- Map of Santa Rita Watershed No. 1 (USDA No. 76.001).

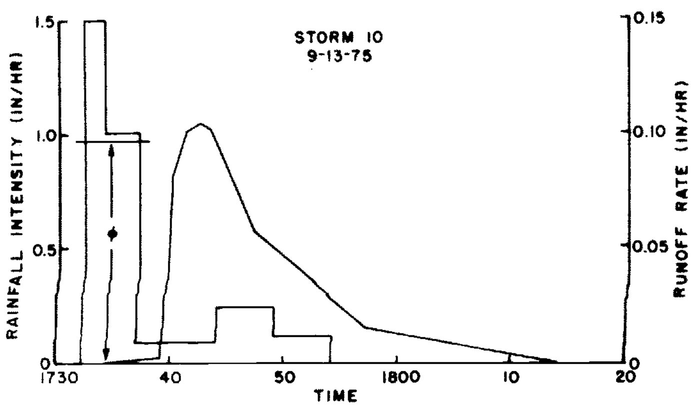


Fig. 2 -- Rainfall and runoff records for storm of Sept. 13, 1975 (No. 10).

No. 1 (USDA No. 76.001). The watershed covers an area of 4.02 acres (175,000 square feet). It has a roughly rectangular shape approximately 1100 feet long and 160 feet wide (Fig. 1). A well defined main channel extends through the lower half of the watershed over a length of 510 feet and ends in a head cut. A number of secondary channels drain the lower half (drainage density 0.012 ft/ft$^2$) but the upper half has no well defined channels in it. The slope of the main channel in the lower half is 3.6%. The average land slope in the lower half is 8.1% and 5.2% on the right bank and on the left bank of the main channel respectively. The average land slope in the upper half of the watershed is 3.4%.

The 1975 rainy season produced a total number of 10 runoff events on watershed 76.001 during the period of July to September. The first two, on July 7 and 10, occurred before the installation of the equipment and went unrecorded. The remaining events were recorded and they are listed in Table 1. For the purpose of this paper a storm event has been defined as one in which a distinct peak is produced. In two cases (August 12 and September 1) double peaked hydrographs were observed. Conforming to the definition adopted, each of these was separated into two single peaked hydrographs and was listed separately in Table 1. The number of storm events thus available for the present study is 10 as listed in the table.

| Storm No. | Date | Total Rainfall | | | Runoff Hydrograph | | Rainfall Excess | | |
|---|---|---|---|---|---|---|---|---|---|
| | | Duration (min) | Depth (in) | Max Intensity (in/hr) | Volume (in) | Peak Flow (in/hr) | $\phi$ Index (in/hr) | Duration (min) | Max Intensity (in/hr) |
| 1 | 7.12.75 | 32 | 0.73 | 4.8 | 0.169 | 1.15 | 2.34 | 8 | 2.46 |
| 2 | 7.24.75 | 63 | 0.34 | 1.6 | 0.006 | 0.04 | 1.48 | 3 | 0.12 |
| 3 | 7.27.75 | 12 | 0.23 | 2.1 | 0.018 | 0.18 | 1.55 | 2 | 0.54 |
| 4 | 8.8.75 | 37 | 0.51 | 4.0 | 0.075 | 0.72 | 2.50 | 3 | 1.50 |
| 5 | 8.12.75 | 45 (storms 5–6) | 0.87 (storms 5–6) | 3.3 | 0.069 | 0.59 | 1.67 | 4 | 1.63 |
| 6 | 8.12.75 | | | 4.5 | 0.202 | 1.12 | 1.39 | 9 | 3.15 |
| 7 | 9.1.75 | 154 (storms 7–8) | 0.66 (storms 7–8) | 1.2 | 0.018 | 0.10 | 0.85 | 3 | 0.35 |
| 8 | 9.1.75 | | | 0.8 | 0.007 | 0.04 | 0.66 | 3 | 0.14 |
| 9 | 9.13.75 | 13 | 0.24 | 2.1 | 0.039 | 0.32 | 1.52 | 4 | 0.57 |
| 10 | 9.13.75 | 22 | 0.14 | 1.5 | 0.019 | 0.11 | 0.97 | 5 | 0.53 |

Table 1. Rainfall and runoff data for the ten storms used.

Rainfall excess was obtained from the total rainfall hyetograph by a simple $\phi$-index procedure. The values of the $\phi$-index required to satisfy the equality of volumes of input and output are listed in Table 1 for each storm event. Also listed are the duration and maximum intensity of rainfall excesses as obtained by the above separation method. Time discrepancies between the start of the rainfall excess hyetograph and the start of the runoff hydrograph were attributed to clock errors and were disregarded. An example of the raw rainfall and runoff data is shown in Fig. 2. The event shown in storm No. 10 which is a medium sized storm for the range of storms obtained in the 1975 rainy season.

## THE DOUBLE TRIANGLE UNIT HYDROGRAPH

The double triangle unit hydrograph adopted for the present study may be considered to be an extension of the simple triangular unit hydrograph. It is obtained by assuming the watershed response to be composed of two parts, each represented by a triangular hydrograph. The first part is a fast response with a high peak and short time base and the second a slow response having a low peak and a longer time base. The latter may be assumed to start either at the same time the fast response starts (Fig. 3a), or alternatively, at the time corresponding to the peak of the fast response (Fig. 3b). The result obtained in either case by adding the ordinates of the individual hydrograph (Fig. 3c) is a three line polygon, the shape of which depends on the relative proportions of the two hydrographs added.

The use of a double triangle as a model for unit hydrographs of small watersheds has been proposed by Ardis (1972, 1973). Detailed results obtained with this model on a number of watershed are presented

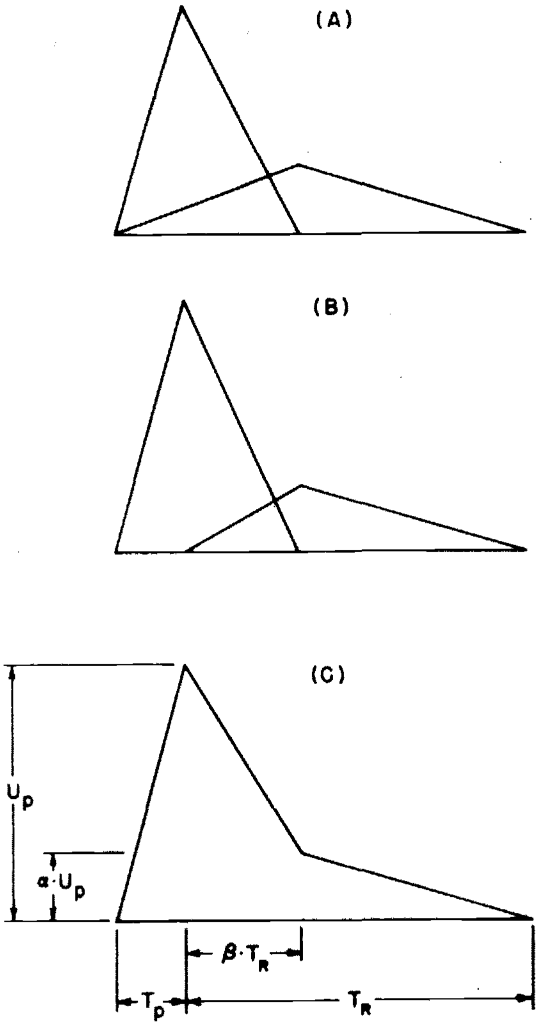


Fig. 3 -- The double triangle unit hydrograph.

in a report published by T.V.A. (1973). The use of a single triangle unit hydrograph has been proposed by the S.C.S. (1957). Examples of such use for agricultural watersheds are given by Ogronsky and Mokus (1964) and for urban watersheds by Jens and McPherson (1964). A design procedure for estimating the parameters of a triangular unit hydrograph is given in a recent paper by Reich and Wolf (1973).

The shape of the double triangle unit hydrograph is specified by four parameters. A fifth parameter needed for a complete description is obtained from the other four by the condition that the hydrograph enclose a unit area. The five parameters, shown in Fig. 3c, are the time to peak $T_p$, the recession time $T_R$, the peak ordinate $U_p$, the ratio $\alpha$ between the ordinate at break point on the recession and the peak ordinate, and the ratio $\beta$ between the time from the peak to the break point and the total recession time. The four independent parameters adopted herein were $T_p$, $T_R$, $\alpha$ and $\beta$. Using the condition of unit area the fifth parameter $U_p$ is given by

$$U_p = \frac{2}{T_p + T_R\,(\alpha+\beta)} \tag{1}$$

The condition for the recession limb to be concave is given by

$$0 < \alpha + \beta < 1.0 \tag{2}$$

The ordinates of the unit hydrograph for times other than the break points are obtained from the break point ordinates by simple linear interpolations. In the present study the unit hydrograph was taken to be the hydrograph due to a rainfall excess of 1-minute duration. The runoff hydrograph ordinates $Q_{(t)}$ were obtained from the unit hydrograph $U_{(t)}$ and rainfall excess $R_{(t)}$ by a numerical convolution procedure

$$Q_{(t)} = \int_0^t R_{(\tau)}\, U_{(t-\tau)} d\tau = \sum_{i=1}^{t} R_{(i)}\, U_{(t-i+1)} dt. \tag{3}$$

## RESULTS OF ANALYSIS

The optimal set of parameter values for each storm event was obtained by a simple search technique covering specified ranges of possible values for each parameter. These ranges were determined by inspecting the set of data and noting the characteristics of the observed runoff hydrographs and rainfall excess hyetographs. The search procedure adopted consisted of computing the runoff hydrograph corresponding to a given set of parameter values and comparing the computed hydrograph to the observed hydrograph. The result of the search produced the set of parameters values that gave the best agreement between the two hydrographs in terms of some objective function. Three such functions were used, the sum of squared deviations, the sum of absolute deviations, and the absolute difference in the values of the peaks of the two runoff hydrographs.

Each of these objective functions produced a different set of unit hydrographs for the ten storms studied. The unit hydrograph parameters obtained using the sum of absolute deviations as an objective function are listed in Table 2. Using the sum of squared deviations as an objective function produced identical results for six of the storms and for three other storms only the value of one of the parameters was different. The mean values of the parameters for the two objective functions were practically the same. Consequently, only the one set in Table 2 is presented herein. The individual results obtained by the third objective function, absolute peak differences, are also not presented to conserve space, but in Fig. 4, the mean unit hydrograph for this case is compared with the mean unit hydrograph of Table 2. The mean hydrograph was taken to be one for which the parameters have the mean values of the parameters for individual storms. As expected the mean unit hydrograph obtained by matching the peaks of observed and computed runoff hydrographs (Fig. 4b) has a shorter time base, a shorter time to break point and a higher peak. The individual unit hydrographs for each of the 10 storms are shown for comparison in Fig. 5. These are the hydrographs whose parameters are listed in Table 2.

The deviations obtained by using the individual unit hydrographs as well as deviations obtained by using the mean unit hydrograph are presented in Table 2. Listed in the table are the mean of the absolute deviations, the square root of the mean of squared deviations (RMS), the largest absolute deviation, and the absolute value of the difference between peaks of the computed and observed runoff hydrograph. The values are listed in this order both for runoff hydrographs computed using individual unit hydrographs for each storm and for hydrographs computed with the mean unit hydrograph (Fig. 4a), the parameters of which are given at the bottom of Table 2.

The deviations derived from the mean unit hydrograph are of course higher than those obtained by convoluting the individual unit hydrograph with the rainfall excess for each storm. The ratios of the mean deviations for the two cases vary between 1.2 and 2.2 with a mean value of 1.6. The two sets of deviations show however, similar characteristics. Thus the ratios of the RMS deviations to the mean deviations are consistently between 1.4 and 2.1 (mean 1.7) and the ratios of maximum deviations to mean deviations is between 5 and 10 with a mean value of 6.8.

Comparisons between some observed runoff hydrographs and computed hydrographs are shown in Figs. 6 and 7. The curves shown are for a large and a medium runoff event. In each case the observed hydrograph is compared to two computed hydrographs. One was obtained by convoluting the rainfall excess with the individual unit hydrograph for that storm (Fig. 5). The second runoff hydrograph was obtained using the mean unit hydrograph as defined above (Fig. 4a). Further comparison between observed and computed runoff hydrographs is given in Table 3.

A numerical measure adopted to describe the goodness of fit is the relative accuracy. This is defined as the mean of the absolute deviations expressed as a percentage of the observed peak flow of the runoff hydrograph. It was found that the relative accuracy obtained with a given method of computation was about the same for all storms. The percentage relative accuracy for runoff hydrographs computed

| Storm No. | Unit Hydrograph Parameters | | | | | Individual Unit Hydrographs | | | | Mean Unit Hydrograph | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | α | β | $T_p$ (min) | $T_R$ (min) | $U_p$ (1/min) | Mean Dev. (in/hr) | RMS Dev. (in/hr) | Max Dev. (in/hr) | Peak Diff. (in/hr) | Mean Dev. (in/hr) | RMS Dev. (in/hr) | Max Dev. (in/hr) | Peak Diff. (in/hr) |
| 1 | 0.05 | 0.40 | 3 | 28 | 0.128 | 0.032 | 0.065 | 0.314 | 0.156 | 0.071 | 0.126 | 0.482 | 0.281 |
| 2 | 0.05 | 0.15 | 6 | 44 | 0.135 | 0.002 | 0.003 | 0.014 | 0.003 | 0.003 | 0.005 | 0.018 | 0.010 |
| 3 | 0.05 | 0.15 | 7 | 44 | 0.127 | 0.006 | 0.011 | 0.048 | 0.048 | 0.009 | 0.016 | 0.074 | 0.062 |
| 4 | 0.05 | 0.15 | 7 | 36 | 0.141 | 0.025 | 0.052 | 0.193 | 0.149 | 0.038 | 0.075 | 0.316 | 0.264 |
| 5 | 0.05 | 0.15 | 6 | 44 | 0.135 | 0.020 | 0.032 | 0.113 | 0.097 | 0.029 | 0.052 | 0.182 | 0.179 |
| 6 | 0.05 | 0.30 | 7 | 32 | 0.101 | 0.028 | 0.045 | 0.182 | 0.141 | 0.048 | 0.078 | 0.234 | 0.068 |
| 7 | 0.05 | 0.30 | 7 | 44 | 0.089 | 0.004 | 0.007 | 0.033 | 0.013 | 0.007 | 0.010 | 0.038 | 0.005 |
| 8 | 0.05 | 0.30 | 7 | 40 | 0.095 | 0.002 | 0.003 | 0.011 | 0.003 | 0.003 | 0.005 | 0.015 | 0.002 |
| 9 | 0.05 | 0.20 | 5 | 44 | 0.125 | 0.017 | 0.032 | 0.139 | 0.063 | 0.020 | 0.039 | 0.154 | 0.089 |
| 10 | 0.10 | 0.35 | 5 | 44 | 0.081 | 0.005 | 0.009 | 0.040 | 0.023 | 0.008 | 0.012 | 0.046 | 0.007 |
| Mean | 0.060 | 0.245 | 6.0 | 40.0 | 0.110 | | | | | | | | |

Table 2. Unit hydrograph parameters and deviations obtained using these unit hydrographs. Based on sum of absolute deviations objective function.

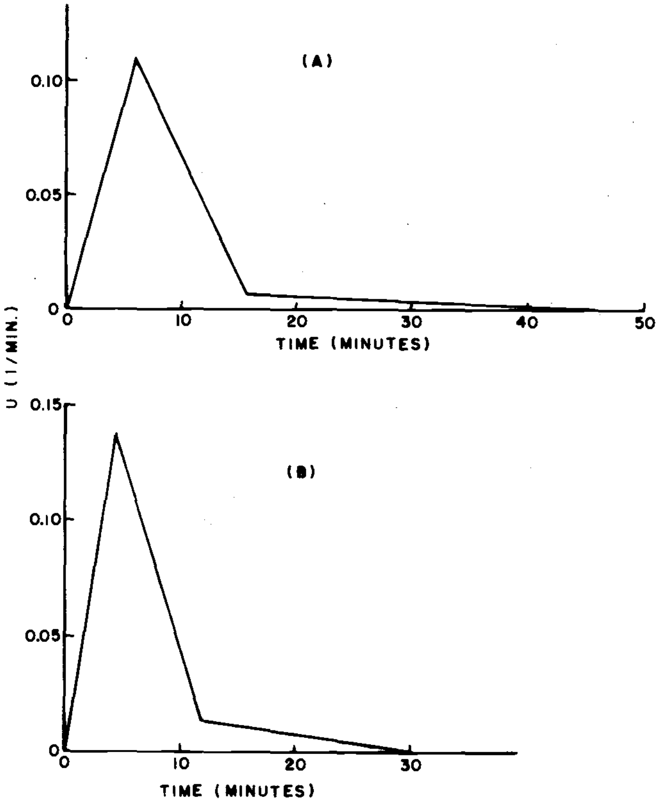


Fig. 4 -- Mean optimal unit hydrographs: (A) based on minimum sum of absolute deviations, (B) based on minimum magnitude of peak difference.

| Storm No. | Observed Hydrograph | | | Optimal Hydrograph | | | Mean Hydrograph | | |
|---|---|---|---|---|---|---|---|---|---|
| | Peak Flow (in/hr) | Time to Peak (min) | Time Base (min) | Peak Flow (in/hr) | Time to Peak (min) | Time Base (min) | Peak Flow (in/hr) | Time to Peak (min) | Time Base (min) |
| 1 | 1.150 | 7 | 47 | 0.999 | 8 | 55 | 0.869 | 10 | 53 |
| 2 | 0.045 | 8 | 22 | 0.044 | 7 | 48 | 0.035 | 7 | 48 |
| 3 | 0.177 | 8 | 27 | 0.136 | 7 | 48 | 0.115 | 7 | 47 |
| 4 | 0.715 | 9 | 39 | 0.566 | 8 | 45 | 0.451 | 7 | 48 |
| 5 | 0.592 | 8 | 38 | 0.495 | 8 | 53 | 0.413 | 9 | 49 |
| 6 | 1.124 | 15 | 65 | 0.983 | 14 | 47 | 1.055 | 13 | 54 |
| 7 | 0.101 | 6 | 30 | 0.092 | 8 | 52 | 0.106 | 7 | 48 |
| 8 | 0.041 | 8 | 23 | 0.038 | 8 | 49 | 0.043 | 7 | 48 |
| 9 | 0.315 | 6 | 35 | 0.252 | 7 | 52 | 0.226 | 8 | 49 |
| 10 | 0.110 | 6 | 36 | 0.096 | 6 | 53 | 0.118 | 7 | 50 |

Table 3. Comparison of characteristics of observed and computed runoff hydrographs.

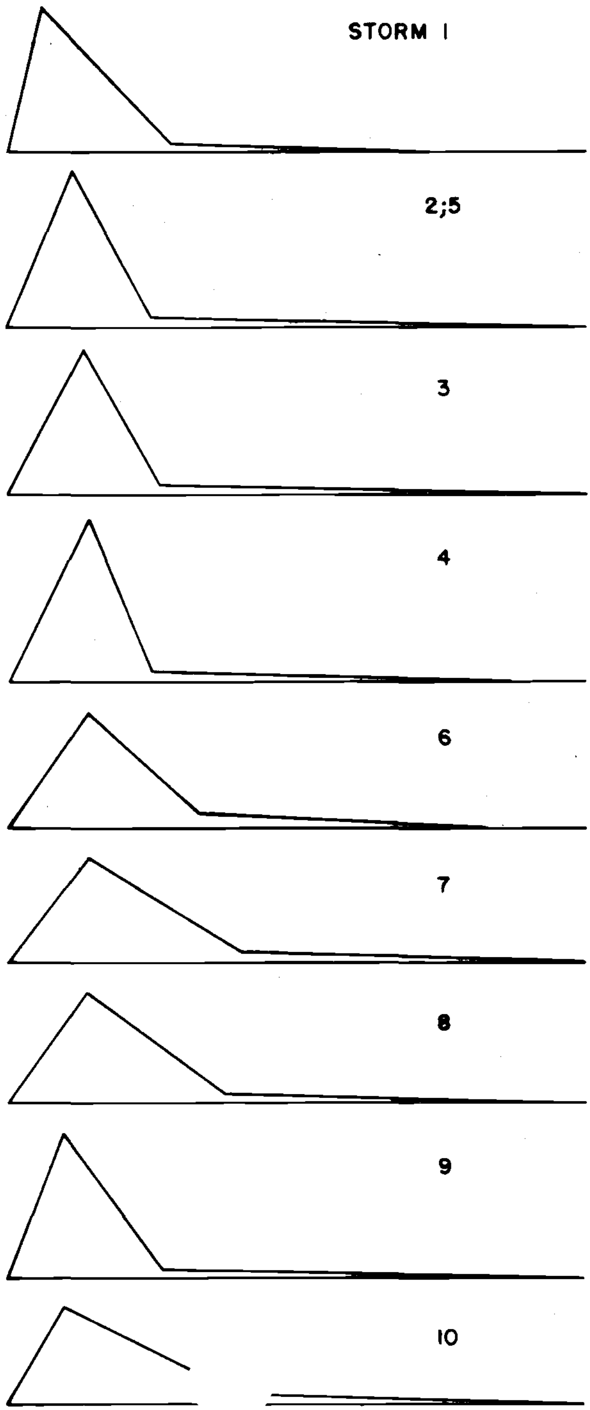


Fig. 5 -- Individual optimal unit hydrographs for the ten storms.

with individual unit hydrographs varied from 2.5% for storm No. 6 to 5.4% for storm No. 9 with a mean value for all storms of 3.8%. The corresponding values for runoff hydrographs reconstructed by the mean unit hydrograph were 3.9% for storm No. 6, 7.3% for storm No. 10, and the mean value for all storms was 6.0%.

## THE SINGLE TRIANGLE UNIT HYDROGRAPH

The single triangle unit hydrograph is a special case of the more general unit hydrograph described above obtained when $\alpha + \beta = 1.0$. It is specified by two independent parameters, the time to peak $T_p$ and the recession time $T_R$. The third parameter which is the peak ordinate of the hydrograph $U_p$ is derived from the first two parameters by

$$U_p = \frac{2}{T_p + T_R} \tag{4}$$

Using the same approach to the selection of optimal parameters, single triangle unit hydrographs were also derived for each of the ten storms described above. The values of the parameters and the deviations obtained using the single triangle unit hydrographs are summarized in Table 4.

| Storm No. | Unit Hydrograph Parameters $T_p$ (min) | $T_R$ (min) | $U_p$ (1/min) | Mean Dev. (in/hr) | RMS Dev. (in/hr) | Max Dev. (in/hr) | Peak Diff. (in/hr) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | 15 | 0.125 | 0.050 | 0.112 | 0.594 | 0.130 |
| 2 | 4 | 12 | 0.125 | 0.003 | 0.005 | 0.012 | 0.005 |
| 3 | 6 | 9 | 0.133 | 0.011 | 0.017 | 0.055 | 0.038 |
| 4 | 7 | 6 | 0.154 | 0.033 | 0.055 | 0.195 | 0.095 |
| 5 | 5 | 9 | 0.143 | 0.036 | 0.051 | 0.161 | 0.063 |
| 6 | 6 | 18 | 0.083 | 0.048 | 0.075 | 0.308 | 0.251 |
| 7 | 6 | 15 | 0.095 | 0.004 | 0.008 | 0.033 | 0.007 |
| 8 | 6 | 15 | 0.095 | 0.003 | 0.004 | 0.013 | 0.003 |
| 9 | 3 | 15 | 0.111 | 0.031 | 0.049 | 0.183 | 0.083 |
| 10 | 4 | 18 | 0.091 | 0.008 | 0.013 | 0.058 | 0.012 |

Table 4. Unit hydrograph parameters and deviations for computed and observed runoff hydrograph using a single triangle unit hydrograph.

As expected the agreement between the observed runoff hydrographs and those computed was not as good in this case as it was with the double triangle unit hydrographs. In terms of the relative accuracy defined above, values obtained with individual unit hydrographs varied from 4.1% for storm No. 7 to 9.7% for storm No. 10. The mean relative accuracy for all ten storms was 6.1%, which is about 1.6 times the corresponding value (3.8%) obtained with individual unit hydrographs having a double triangle shape.

Using one mean unit hydrograph for all ten storms gave results that were again poorer in comparison to those obtained with a mean double triangle unit hydrograph. The relative accuracy for this case varied from 6.2% for storm No. 6 to 14.2% for storm No. 8 with a mean relative accuracy of 9.3% which is about 1.5 times the value (6.0%) for the mean double triangle unit hydrograph.

## CONCLUSIONS

The use of the double triangle unit hydrograph for describing rainfall-runoff relations for a small semiarid watershed has produced reasonable to good agreement between observed and computed runoff hydrographs. The mean relative absolute deviation between the two hydrographs was 3.8% when the computed hydrograph was based on individual unit hydrographs and 6.0% when it was derived from a mean unit hydrograph.

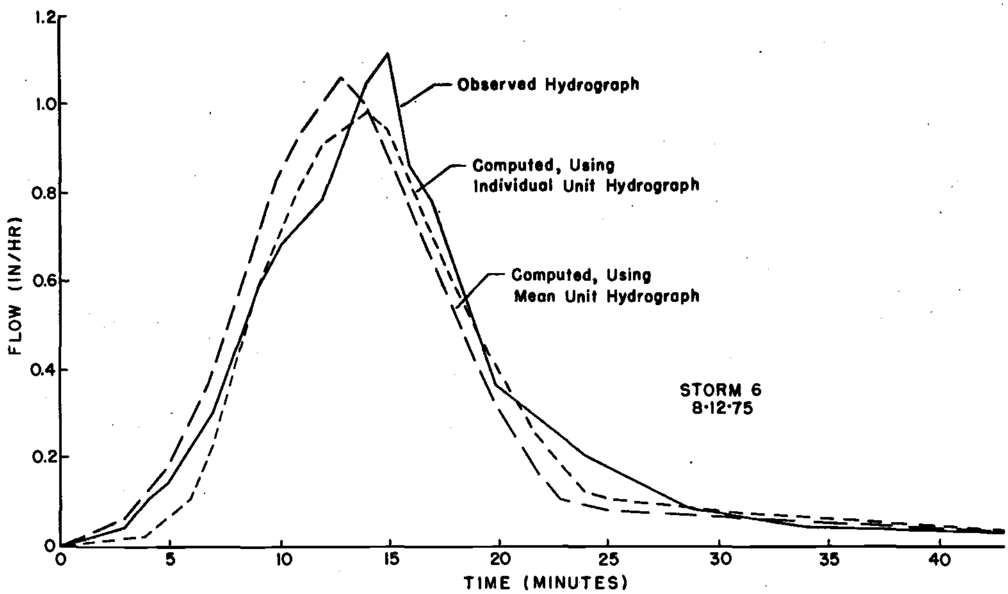


Fig. 6 -- Observed and computed runoff hydrographs for storm of Aug. 12, 1975 (No. 6).

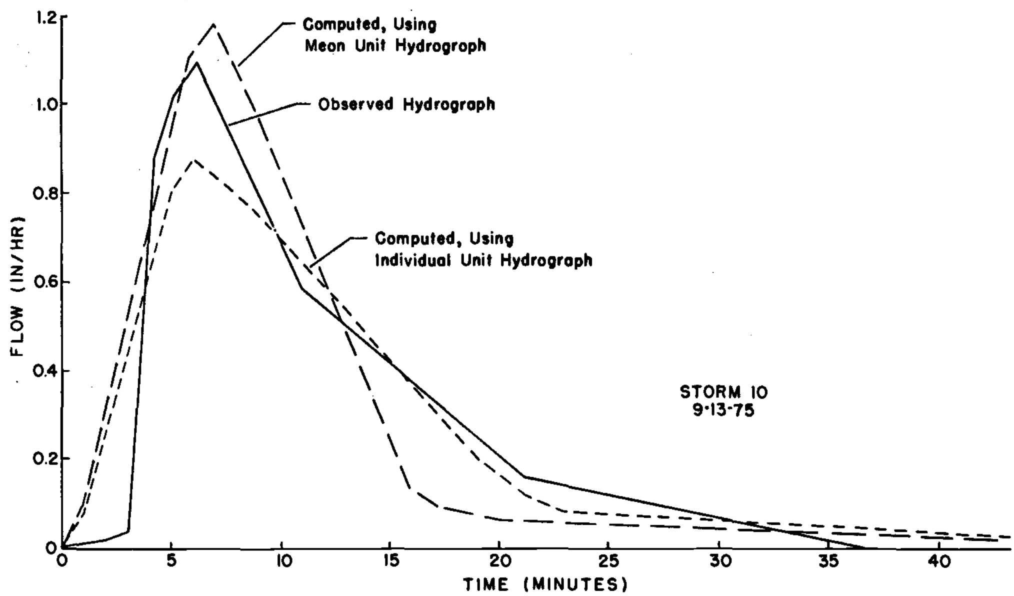


Fig. 7 -- Observed and computed runoff hydrographs for storm of Sept. 13, 1975 (No. 10).

The double triangle unit hydrograph offers an improvement over the single triangle unit hydrograph. The ratio of the mean relative deviations obtained in the two cases was between 1.5 and 1.6. The larger number of parameters for the double triangle, 4 instead of 2 for the single triangle, does not cause any difficulties. The search technique adopted for the evaluation of the optimal parameters is just as simple for 4 parameters as it is for 2 and the numerical convolution program is exactly the same except for the initial computation of the unit hydrograph ordinates.

The changes in the shape of individual unit hydrographs from storm to storm appear to be uncorrelated to storm characteristics. It is probable that antecedent soil moisture conditions influenced the results but information about these conditions was not available. Another possibility is that the area contributing to runoff varied but again quantitative information about the extent of contributing area for individual storms was not available. The effect of partial area contribution forms the subject of another study.

The use of the mean unit hydrograph appears to give a reasonable approximation to the storms studied. In the absence of specific information about the factors that influence the individual unit hydrograph the mean double triangle unit hydrograph is probably the most appropriate unit hydrograph available for predicting runoff hydrographs due to specified excess rainfall hyetographs.

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