A MULTIOBJECTIVE APPROACH TO MANAGING A SOUTHERN ARIZONA WATERSHED

by

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ABSTRACT

The case study of an Upper San Pedro River watershed is developed to show how a multiple objective approach to decision-making may be used in watershed management. The effects of various land treatments and management practices on water runoff, sediment, recreation, wildlife levels, and commercial potential of a study area are investigated while observing constraints on available land and capital. The example involves the optimization of five objective functions subject to eighteen constraints. In an iterative manner, the decision-maker proceeds from one noninferior solution to another, comparing sets of land management activities for reaching specified goals, and evaluating trade-offs between individual objective functions. This technique, which involves the formulation of a surrogate objective function and the use of the cutting plane method to solve the general nonlinear problem, hopefully provides a compromise between oversimplified and computationally intractable approaches to multiobjective watershed management.

INTRODUCTION

In many water-related applications of mathematical programming, it is often necessary and desireable to represent the aspirations of the decision maker (DM) in terms of a collection of objective functions, and not just a single objective function. This is certainly the case in the analysis of river basin planning problems and water resource systems in general. In this paper multiobjective theory is applied to a land use management study of the San Pedro River Basin along the Charleston Watershed, located some 20 miles southeast of Tucson, Arizona.

The main tool in the present multiobjective approach to land use management is a mathematical programming model. The formulation of a programming model allows the decision maker to investigate the physical and economic responses of the system to the various land treatments under consideration, as in Eisel (1972). Much of the activity generated in the past regarding modeling and optimization has been limited to linear programming models and understandably so, given the convenience of the straight-forward simplex algorithm (Dantzig, 1963). However, the management of wildland is complicated by the nonlinearity associated with the cost of application of the land treatments, in that the set-up cost is initially high. Increases in water runoff and sediment will also be high right after the land treatment is applied and will level out to quasi-steady state values thereafter. A realistic consideration of the ecosystem will also have to allow for the stochastic nature of many parameters (e.g., animal species, land treatment costs, and so forth). Effective manipulation of these parameters calls for linear programming under uncertainty (Dantzig, 1963) or chance-constrained programming (Charnes and Cooper, 1963). In this study of the San Pedro River Basin, the mathematical model is restricted to a deterministic nonlinear programming model. To deal with the non-linearity, either in the constraints or objective functions, use will be made of the cutting plane method (Kelly, 1960; Griffith and Stewart, 1961; Monarchi, 1972).

THE MANAGERIAL MODEL AND ITS ELEMENTS

MULTIOBJECTIVE ANALYSIS

The adoption of a vector of objective functions in watershed management introduces new judgemental and computational dimensions in the fields of modeling, and mathematical programming. The major, underlying theme behind most strategies, however, is that of systematically eliminating large number of feasible solutions from further contention for the 'best' solution. Two approaches to the multiobjective problem are various weighting techniques and the constraint method (Geoffrion, 1967; Benayoun et al., 1971; Haimes, 1970; Cohon and Marks, 1973). An example of the latter approach is the sequential SEMOPS technique of Monarchi (1972), where the DM uses a surrogate objective function and modified ε-constraint method to arrive at a satisfactum solution. The present paper has benefited greatly from such approach and method of solution.

In the weighting technique, the solution of a multiobjective linear problem, i.e., the generation of the non-inferior set, proceeds first by transforming the vector-valued objective function into a scalar-valued function. The solution of the transformed problem gives a point in the non-inferior set.

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The parameters used in the transformation are then varied systematically to yield further points in the non-inferior set. However, as Cohon and Marks (1973) point out, the weighting technique may fail when the non-inferior set is not convex. Furthermore, computational-considerations favor the ε-constraint method (Haimes, 1970) since parameterization on the right-hand side of the constraints is preferible to parametrically varying weighted coefficients in the objective function. Along this line, a recent contribution is that of the surrogate worth trade-off (SWT) method (Haimes and Hall, 1974). Some aspects of its development are criticized in the appendix in the hope that it gains further attention, as it may offer some interesting possibilities.

Returning to the example, a guideline for wildland management is the multiple use policy that prescribes management for '... outdoor recreation, range, timber, watershed, wildlife, and fish purposes' (U.S. Congress, 1960). Other elements of the multiple use philosophy require '... permanent good for the whole people ...' and '... the greatest good for the greatest number in the long run' (U.S. Congress, 1960). This analysis of the San Pedro River Basin in the Charleston watershed (Figure 1) is concerned with the extent of application of land treatments for purposes of increasing water runoff, commercial and recreational benefits, and still maintain certain goal values on sediment and wildlife levels in the area, while operating with specified capital and land constraints. These management goals need to be translated into mathematical objective functions and constraints to specify our non-linear programming model.

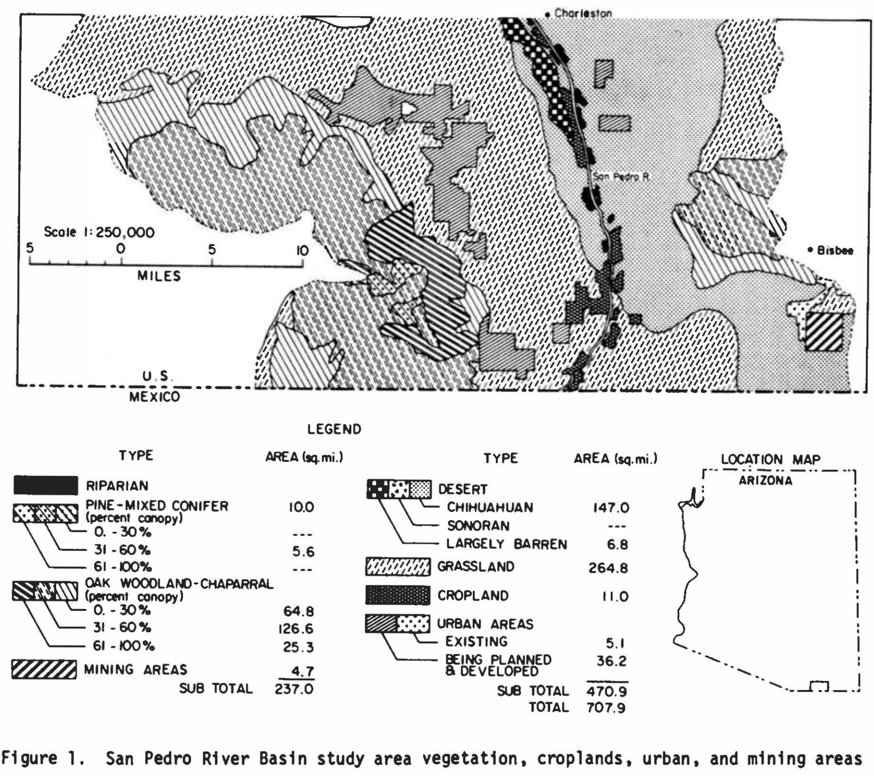

Figure 1. San Pedro River Basin study area vegetation, croplands, urban, and mining areas

VEGETATION TREATMENTS

A map of the Charleston watershed is presented along with a description of the vegetation types in the area in Figure 1. Opportunities for water yield improvement through manipulations of the Riparian and Phreatophite communities along the San Pedro River are difficult to assess, although some estimates are available (Ffolliott and Thorud, 1974). 75% removal of Riparian stands by mechanical means is considered for ultimate conversion to grasslands. Treatment of limited portions of Chihuahuan desert includes 100% removal of vegetation followed by rock-rake and drum roller to compact treated areas (Cluff, 1971). Desert grasslands occupy some 169,000 acres below the Woodland-Chaparral zone of the Huachuca mountains at about 5,500 feet elevation. Limited application of the compacted earth (C.E.) treatment described above is considered. 75% removal of Woodland-Chaparral stands by mechanical means is followed by conversion to grassland cover. Treatment of the Pine-Mixed Conifer areas includes 75% logging of commercial stands along with partial removal of some understory trees and shrubs. Estimates of variations in water runoff, sediment, wildlife level, recreational and commerical benefits have been accumulated for the various combinations of vegetation type and land treatment.

OUTDOOR RECREATION

Some of the methods available for recreation evaluation include gross expenditures method, cost, market value, and willingness to pay method. In the willingness to pay method a multiple regression model is used with willingness-to-pay per household visit as a function of income of the household, frequency of visits to the area, and the length of stay in the area. Demand schedules are then simulated to estimate yearly monetary benefits (Knetsch and Davis, 1966). The user-benefit method appears to be quite realistic in that it begins with the estimation of an economic demand curve expressing demand for the whole recreation experience. Estimates of consumer benefits for hunting, fishing, and general rural outdoor recreation are then developed consistent with that demand curve. The lack of information concerning the quantitative effects of specific land use management techniques on outdoor recreation and environmental benefits, however, complicates the formulation of these elements as objective functions or constraints. The user-day procedure (Ad Hoc Water Resources Council, 1964) for estimating costs and benefits is attractive because of its simplicity and is used in this analysis. Estimates of recreation user-days were then made for areas under the various 'improved' and 'present' land use management practices.

ECOSYSTEM STABILITY

Traditionally, multiobjective analysis in watershed management has dealt with water runoff, sedimentation, commercial benefits, etc.... but has failed to recognize and effectively deal with the animal wildlife aspect of management as an element in the general vector optimization problem. The soils, vegetation cover, and associated animal species are an interdependent complex of the watershed ecosystem, and is considered to be in a quasi-steady state for analysis purposes. Attempting to predict ecosystem response (e.g., adaptation of animal species to treated areas, variations in population size, relative dominance of species, and so forth) to land-use practices is, by any account, a formidable task. One approach calls for the identification of species and population characteristics for each vegetation type in the watershed. Now, whereas it is relatively simple to assess changes in water runoff of sedimentation as a result of a land treatment, it is extremely involved to assess changes in the wildlife of the area. Economic considerations, at least, would not permit a complete account of all the species in a given ecosystem. And if a limited list of species is arbitrarily considered there remains to decide what trophic levels in the food chain are to be ignored in preparing such list. Also, because it is a limited and arbitrary list of species, it will undoubtedly reflect human preference for certain species. A tool capable of recognizing and integrating the many and diverse elements in the ecosystem is general system theory (Wymore, 1967). If watershed management and wildlife conservation activities are to be considered jointly, system theory can provide guidance as to what type of information is needed, and a mathematical vehicle for simulating ecosystem response to land use practices.

MATHEMATICAL FORMULATION

ASSUMPTIONS

The time horizon for this non-linear programming problem is a 30-year period which corresponds to the expected effective life of potential soil treatments. The division of this 30-year period into shorter time increments is necessary because water runoff rates, sedimentation rates and so forth will not remain constant over the entire period. Therefore, the 30-year period is divided into three 10-year subperiods. In the notation to follow, the subscript $i = 1, 2, 3, \ldots, 11$ will identify a land treatment, and $J = 1, 2,$ and 3 will denote decision periods.

Description of variables,

In reference to our river basin shown in Figure 1, the managing agency must decide whether to continue with present management practices or implement new ones, and the extent of application of these. Let

x_{1j} = sq. mi. of <u>grassland</u> with current management practices in decision period J.

x_{2j} = sq. mi. of grassland with compacted earth (CE) treatment (to increase runoff) in period J.

x_{3j} = sq. mi. of <u>Chihuahuan desert</u> with current management practices in period J.

x_{4j} = sq. mi. of Chihuahuan desert converted to grasslands (to increase runoff and grazing) in decision period J.

x_{5j} = sq. mi. of Chihuahuan desert with compacted earth (CE) treatment (to increase runoff) in period J.

x_{6j} = sq. mi. of <u>Riparian stands</u> with current management practices in period J.

x_{7j} = sq. mi. of Riparian stands with 75% removal and conversion to grasslands (to increase runoff and grazing) in period J.

x_{8j} = sq. mi. <u>Oak Woodland</u> with current management practices in period J.

x_{9j} = sq. mi. of Oak Woodland stands with 75% removal and conversion to grasslands (to increase runoff and grazing) in period J.

x_{10j} = sq. mi. of Pine-Mixed Conifer with current management practices in period J.

x_{11j} = sq. mi. of Pine-Mixed Conifer with 75% logging (to increase runoff and economic benefits) in decision period J.

The x_{ij} are decision variables in decision period J for J = 1, 2, and 3.

Objective functions,

The noncommensurable objective functions in our vector optimization problem include the following

water runoff,

$$Z_1 (\underline{X}) = \sum_i \sum_j W_{ij} \ X_{ij} - 1,345 \tag{1}$$

sedimentation,

$$Z_2 (\underline{X}) = \sum_i \sum_j S_{ij} \ X_{ij} - 6,635 \tag{2}$$

animal wildlife unbalance,

$$Z_3 (\underline{X}) = \sum_j \Big[(a_{1j} - a_{2j}) \ X_{2j} + (a_{3j} - a_{4j}) \ X_{4j} + (a_{3j} - a_{5j}) \ X_{5j} \\ (a_{6j} - a_{7j}) \ X_{7j} + (a_{8j} - a_{9j}) \ X_{9j} + (a_{10j} - a_{11j}) \ X_{11j} \Big] \tag{3}$$

recreation,

$$Z_4 (\underline{X}) = \sum_i \sum_j r_{ij} \ X_{ij} - 11,830 \tag{4}$$

commercial,

$$Z_5 (\underline{X}) = \sum_i \sum_j C_{ij} \ X_{ij} - 5,699 \tag{5}$$

where objective functions (1), (4) and (5) are to be maximized, and (2) and (3) are to be minimized. In subsequent studies, the signs of (2) and (3) may be changed, so that a vector maximation problem be obtained. W_{ij} Represents the water runoff (in 1000 acre-foot/sq.mi.) associated with pair i of vegetation type and land treatment throughout period J, S_{ij} is the sediment rate in acre-foot/sq.mi., a_{ij} is the animal biomass in lb-years/sq.mi., r_{ij} is the recreational benefit in \$1000/sq.mi., and C_{ij} stands for commercial benefits, in \$1000/sq.mi. resulting from logging and grazing (after subtraction of cost of logging operations and seeding).

Constraint set,

The above objective functions are to be optimized under specified land and capital constraints. Land Constraints:

$$x_{1j} + x_{2j} \le 264.8 \tag{6}$$

$$x_{3j} + x_{4j} + x_{5j} \le 147.0 \tag{7}$$

$$x_{6j} + x_{7j} \le 10.0 \tag{8}$$

$$x_{8j} + x_{9j} \le 126.6 \tag{9}$$

$$x_{10j} + x_{11j} \le 5.6 \tag{10}$$

for J = 1, 2, and 3.
Capital Constraints:

$$d_{2j} (115\text{-}39 \ \ln X_{2j}) \ X_{2j} + d_{4j} X_{4j} + d_{5j} (115\text{-}39 \ \ln X_{5j}) X_{5j} \\ + d_{7j} X_{7j} + d_{9j} X_{9j} + d_{11j} X_{11j} \le D_j \tag{11}$$

for J = 1, 2, and 3. The function h_j (X) in inequality (11), $h_j (X) \le D_j$, is a convex function as can be verified by noting that the Hessian Matrix of h_j (X) is positive - semidefinite for achieveable values of x_{ij}. The parameter d_{ij} represents the cost of land treatment (in \$1000/sq.mi.) associated with pair i of vegetation type and land treatment of the beginning of period J, and D_j is the capital available for period J.

SOLUTION PROCEDURE

CUTTING PLANT METHOD

Mathematically, the non-linear programming problem may be stated as follows:

minimize: $Z(\underline{X})$

subject to: $g_k(\underline{X})$ -0 (12)

where $X \varepsilon E^n$ and Z and the g's are differentiable convex functions. The basic concept behind the cutting-plane algorithm is that of replacing the original convex set (e.g., the feasible region) $R \subset E^n$ by a polytope $P \supset R$, to which the simplex method can be applied in a sequential manner (Kelly, 1960). Towards that end the original set of constraints is replaced by a set of half-spaces.

Some computer programs have successfully applied this technique and demonstrated it in the literature (Griffith and Stewart, 1961; Monarchi, 1972).

Development of the trade algorithm :

1. A domain $D_1 \varepsilon E^n$ of admissible solutions is given, defined by the set of constraints

$$g_k(\underline{X}) \leq 0,$$
$$X_d \geq 0, \quad \underline{X} = (X_1, X_2, \ldots, X_n) \tag{13}$$

for k = 1, 2, ..., 1, and d = 1, 2, ..., n.

2. There are five objective functions $z_1(\underline{X})$, $z_2(\underline{x})$, ..., $z_5(\underline{x})$, as defined in (1) through (5) above. Form the total objective function $C_1(\underline{x})$

$$C_1(\underline{X}) = \frac{Z_1 - Z_{1MIN}}{Z_1^* - Z_{1MIN}} - \frac{Z_2 - Z_{2MIN}}{Z_2^* - Z_{2MIN}} - \frac{Z_3 - Z_{3MIN}}{Z_3^* - Z_{3MIN}} + \ldots + \frac{Z_5 - Z_{5MIN}}{Z_5^* - Z_{5MIN}} \tag{14}$$

where

z_i^* = optimum value of the i-th objective function,

z_{iMIN} = minimum value of the i-th objective function.

3. Carry out successively the optimization with respect to each individual objective. The maxima (or minima) obtained for each of the objectives form a vector U_1:

$$U_1 = \left| \begin{array}{l} \max z_1(\underline{x}) \\ \underline{x} \; \varepsilon \; D_1 \\ \min Z_2(\underline{x}) \\ \underline{x} \; \varepsilon \; D_1 \\ \vdots \\ \max z_5(\underline{x}) \\ \underline{x} \; \varepsilon \; D_1 \end{array} \right| \tag{15}$$

4. Carry out the optimization with respect to the total objective function $C_1(x)$. The resulting solution vector x_1 is then used to generate vectors W_1 and V_1:

$$W_1 = \left| \begin{array}{l} z_1\ (\underline{x}_1) \\ Z_2\ (\underline{x}_1) \\ \vdots \\ z_5\ (\underline{x}_1) \end{array} \right| , \qquad V_1 = \left| \begin{array}{l} G_1\ (\underline{x}_1) \\ G_2\ (\underline{x}_1) \\ \vdots \\ G_5\ (\underline{x}_1) \end{array} \right| \tag{16}$$

where $G_i\ (x_1) = \dfrac{Z_i(\underline{X}_1) - Z_{iMIN.}}{Z_i^* - Z_{iMIN.}}$;

the decision maker now poses the following question: "Have all the objective functions $z_i\ (\underline{x}_1)$ satisfactory values?" To answer this question the decision maker makes use of the elements in vector W_1. In the case of an affirmative answer the multiobjective problem has been solved and vector W_1 represents a desired solution. Vector V_1, then, also represents a desired solution but in terms of goal percentages achieved.

5. Otherwise, select the objective function $Z_i(\underline{x}_1)$ with the most satisfactory goal value achieved, $G_i(\underline{x}_1)$, and specify a value ε_i such that $z_i(\underline{x}_1) \geq \varepsilon_i$ (If the i-th goal is to be maximized). For the actual value of ε_i the decision maker makes use of the information contained in vectors U_1 and W_1.

6. Define the new solution space D_2 with the constraints

$$
\begin{aligned}
g_k(\underline{x}) &\leq 0, \\
X_d &\geq 0. \\
Z_i(\underline{x}) - \varepsilon_i &\geq 0,
\end{aligned}
\tag{17}
$$

and repeat steps (2), (3) and (4) to generate a total objective function $C_2(x)$ and vectors U_2, W_2, and V_2. This time, however, $C_2(x)$ will contain one term less since one objective function now forms part of the new constraint set D_2. Proceed to step (5) until a satisfactory vector $\underline{\varepsilon} = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_5)$ is achieved.

EXAMPLE SOLUTION AND RESULTS

The study area is the Charleston watershed, Figure 1, located some 20 miles southeast of Tucson, Arizona. Five objective functions and eighteen constraints (linear and non-linear) on land, capital, and extent of treatment make up the non-linear model used in the analysis. Associated with each land treatment and vegetation type there is a collection of data parameters representing water runoff, sediment, wildlife, recreation, and commercial levels over three 10-year periods. These tabulated parameters are shown on Table (1). Obtaining the necessary economic and physical data proved difficult and in some instances they simply were not available for the area in question. It was then resorted to tabulated data for watershed areas with similar physical characteristics (Branson and Owen, 1970; Bartlett, 1974; Cluff, 1971). A computer program was then used to generate the results shown on Table (2).

TABLE 1 - PHYSICAL DATA PARAMETERS

PARAMETER	PERIOD 1 J=1	PERIOD 2 J=2	PERIOD 3 J=3	PARAMETER	PERIOD 1	PERIOD 2	PERIOD 3
w_{1J}	.64	.64	.64	r_{1J}	3.0	6.0	9.0
w_{2J}	13.44	8.96	4.48	r_{2J}	.0	.0	.0
w_{3J}	.64	.64	.64	r_{3J}	1.5	3.0	4.5
w_{4J}	3.84	2.56	1.28	r_{4J}	18.0	15.0	9.0
w_{5J}	6.72	4.48	2.24	r_{5J}	.0	.0	.0
w_{6J}	1.28	1.28	1.28	r_{6J}	15.0	30.0	45.0
w_{7J}	38.40	25.60	12.80	r_{7J}	27.0	22.5	13.5
w_{8J}	1.28	1.28	1.28	r_{8J}	6.0	12.0	18.0
w_{9J}	5.76	3.84	1.92	r_{9J}	18.0	15.0	9.0
w_{10J}	1.92	1.92	1.92	r_{10J}	9.0	18.0	27.0
w_{11J}	9.60	6.40	3.20	r_{11J}	45.0	37.5	22.5
s_{1J}	5.0	5.0	5.0	c_{1J}	7.2	5.04	2.88
s_{2J}	35.0	20.0	10.0	c_{2J}	.0	.0	.0
s_{3J}	1.0	1.0	1.0	c_{3J}	1.44	1.44	1.44
s_{4J}	32.5	20.0	10.0	c_{4J}	6.72	3.840	.96
s_{5J}	35.0	20.0	10.0	c_{5J}	.0	.0	.0
s_{6J}	10.0	10.0	10.0	c_{6J}	3.6	3.6	3.6
s_{7J}	54.0	36.0	18.0	c_{7J}	15.2	8.0	.80
s_{8J}	5.0	5.0	5.0	c_{8J}	3.6	2.52	1.44
s_{9J}	40.0	30.0	15.0	c_{9J}	63.2	56.0	48.8
s_{10J}	2.0	2.0	2.0	c_{10J}	.0	.0	.0
s_{11J}	24.5	22.5	17.5	c_{11J}	28.00	28.00	28.00
a_{1J}	37422.0	24948.0	12474.0	d_{2J}	.64	.64	.64
a_{2J}	18711.0	12474.0	6237.0	d_{4J}	6.40	6.40	6.40
a_{3J}	53004.0	35336.0	17668.0	d_{5J}	.64	.64	.64
a_{4J}	44892.0	29928.0	14964.0	d_{7J}	6.40	6.40	6.40
a_{5J}	21861.0	14574.0	7287.0	d_{9J}	6.40	6.40	6.40
a_{6J}	35265.0	23510.0	11755.0	d_{11J}	35.20	35.20	35.20
a_{7J}	26460.0	17640.0	8820.0	D (K-\$)	500.00	400.00	300.00
a_{8J}	33360.0	22240.0	11120.0				
a_{9J}	19218.0	12812.0	6406.0				
a_{10J}	27432.0	18288.0	9144.0				
a_{11J}	16848.0	11232.0	5616.0				

TABLE 2 MODEL SOLUTION RESULTS

VARIABLE (sq.mi.)	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6	RUN 7	RUN 8
$x_{2,1}$	.00	.00	.00	.00	.00	.00	5.97	.00
$x_{2,2}$	.00	.00	.00	.00	.00	.00	4.64	.00
$x_{2,3}$	.00	.00	.00	.00	.87	.00	.00	.00
$x_{4,1}$	35.22	.00	78.12	.00	.00	.00	.00	.00
$x_{4,2}$	32.00	14.02	62.50	.00	22.61	13.40	5.17	.00
$x_{4,3}$	24.87	.00	6.37	.00	25.68	17.23	34.92	.00
$x_{5,1}$	.00	.00	.00	.00	.00	.00	.00	.00
$x_{5,2}$	.00	.00	.00	.00	.00	.00	.00	.00
$x_{5,3}$	.00	.00	.00	.00	.00	.00	.00	.00
$x_{7,1}$	10.00	.00	.00	10.00	10.00	10.00	10.00	.00
$x_{7,2}$	.00	.00	.00	.00	.00	.00	.00	.00
$x_{7,3}$	.00	.00	.00	.00	.00	.00	.00	.00
$x_{9,1}$	32.90	78.12	.00	.00	.00	.00	.00	7.11
$x_{9,2}$	30.50	48.47	.00	.00	.00	.00	.00	23.36
$x_{9,3}$	22.00	.00	.00	.00	5.01	29.64	11.94	25.68
$x_{11,1}$	.00	.00	.00	.00	.00	.00	.00	.00
$x_{11,2}$	.00	.00	.00	.00	.00	.00	.00	.00
$x_{11,3}$	.00	5.60	.00	.00	.00	.00	.00	1.99
ACHIEVED GOAL LEVELS (PERCENTAGES)								
G1, Runoff	100.0%	40.4	39.9	58.9	67.8	67.0	83.0	10.7
G2, Sediment	14.5	14.0	6.5	93.5	75.0	75.0	75.0	75.0
G3, Wildlife	11.1	.0	40.0	94.6	81.3	79.0	73.6	72.9
G4, Recreation	41.1	.0	100.0	43.7	50.8	40.0	40.0	40.0
G5, Commercial	65.7	100.0	2.7	6.0	4.2	20.9	6.7	41.2

Optimization of the individual objective functions yielded vector U_1:

$$U_1 = \begin{vmatrix} 586.36 \times 10^3 \\ .00 \\ .00 \\ 1200.00 \\ 6627.20 \times 10^3 \end{vmatrix} \quad \begin{matrix} \text{acre-foot, runoff} \\ \text{acre-foot, sediment} \\ \text{lbs-years, wildlife} \\ \text{\$ dollars, recreation} \\ \text{\$ dollars, commercial benefits} \end{matrix}$$

That is, the runoff function alone, for instance, was optimized to yield 586.36 x 10^3 acre-ft. of water over a 30-year period. This value represents additional water over that period that would result from the implementation of the various land treatments. The extent of these land treatments is shown on Table (2), runs 1, 2, and 3. From U_1 we also observe that if the runoff and commercial functions were to be maximized independent of the other three functions undesireable increases in sediment and reduction in wildlife and recreation benefits would result. Optimization with respect to the total objective function in run 4 yields vectors W_1 and V_1:

$$W_1 = \begin{vmatrix} 345.36 \times 10^3 \\ -240.00 \\ -88.05 \times 10^3 \\ -630.00 \times 10^3 \\ 44.00 \times 10^3 \end{vmatrix} \text{, and} \quad V_1 = \begin{vmatrix} .589 \\ .935 \\ .946 \\ .437 \\ .006 \end{vmatrix}$$

and results in the treatment of the Riparian vegetation alone during the first period, e.g., $x_{7,1}$ = 10.0 sq. mi. At this point we see that although G_2 and G_3 have very acceptable levels, they are achieved at the expense of low levels on runoff and commercial benefits. In an effort to arrive at a compromised solution, the next optimization in run 5 leads to constraining the sediment goal level to 0.75 (e.g., ε_2 = -932.86 to yield:

$$W_2 = \begin{vmatrix} 397.55 \times 10^3 \\ -932.86 \\ -309.93 \times 10^3 \\ -398.34 \times 10^3 \\ 278.34 \times 10^3 \end{vmatrix} \text{, and} \quad V_2 = \begin{vmatrix} .678 \\ .750 \\ .813 \\ .508 \\ .042 \end{vmatrix}$$

Now the runoff goal level has moved up to a more comfortable position, 67.8% but the commercial benefits are still very low, 4.2%. Since recreational and commercial benefits are in conflict with each other (ours is a pareto-optimal solution vector), we compromise recreational benefits to 40% (e.g., $\varepsilon_4 = -718.65 \times 10^3$) in run 6 to yield:

$$W_3 = \begin{vmatrix} 392.86 \times 10^3 \\ -932.86 \\ -348.05 \times 10^3 \\ -718.65 \times 10^3 \\ 1385.10 \times 10^3 \end{vmatrix} \text{, and} \qquad V_3 = \begin{vmatrix} .670 \\ .750 \\ .790 \\ .400 \\ .209 \end{vmatrix}$$

and the commercial benefits increase to 20.9%. So far, the wildlife level has remained comfortably high, and it may be of interest to see how it trades against the other goals. The total objective function of runoff and commercial benefits are then maximized in run 7 with wildlife constrained to 73% (e.g., $\varepsilon_3 = -447.50 \times 10^3$) to yield:

$$W_4 = \begin{vmatrix} 486.67 \times 10^3 \\ -932.86 \\ -447.50 \times 10^3 \\ -718.65 \times 10^3 \\ 444.02 \times 10^3 \end{vmatrix} \text{, and} \qquad V_4 = \begin{vmatrix} .830 \\ .750 \\ .730 \\ .400 \\ .067 \end{vmatrix}$$

Relaxation of the constraints on wildlife unbalance allows for 83% of the maximum runoff. However a quick look at the resource allocation vector $\underline{x}$ in run 7 reveals that this increase has been made possible through the application of the compacted earth (CE) treatment to portions of the grasslands, e.g.,

$$x_{2,1} = 5.97 \text{ sq. mi.}$$
$$x_{2,2} = 4.64 \text{ sq. mi.}$$
$$x_{2,3} = .00 \text{ sq. mi.}$$

where the water runoff yield may be as much as six times higher than, say, that for Chihuahuan desert vegetation. The cost per acre of the CE treatment is also high, and at that level of application (5.974 sq. mi.) the cost is $60.0/acre, approximately, in contrast to $10/acre for the other lower-yield land treatments. But, since the capital was available our optimization model used it within the constraints in the system.

Assumming that the goal levels achieved so far for sediment (75%), wildlife (73%), and recreation (40%) are acceptable, there remains to trade runoff against commercial benefits. Optimization of the commercial objective function alone subject to the above constraints in run 8 yields:

$$W_5 = \begin{vmatrix} 62.74 \times 10^3 \\ -932.86 \\ -447.50 \times 10^3 \\ -718.65 \times 10^3 \\ 2730.40 \times 10^3 \end{vmatrix} \text{, and} \qquad V_5 = \begin{vmatrix} .107 \\ .750 \\ .730 \\ .400 \\ .412 \end{vmatrix}$$

that is, $\underline{x}' = (.830, .067)$ vs. $\underline{x}'' = (.107, .412)$. Further analysis can now yield convex combinations of the two vectors.

SUMMARY

A water resources system was modelled as a multiobjective problem with five objective functions and eighteen constraints to investigate the physical and economic responses of the system to the various land treatments considered. An attempt has been made to provide the DM with a realistic set of objective functions, including those for sediment and wildlife levels, to monitor effectively the impact of his decisions on the environmental quality of the area. The use of a surrogate objective function in the TRADE method enables the DM to generate an initial solution vector without having to generate weighting coefficients for the individual objectives; once this initial solution is generated, it is expressed in terms of goal percentages achieved to assist in the decision-making process. The explicitness of methodology in TRADE will hopefully achieve a compromise between the rudimentary "model-DM" dialog of the constraint method and the large computational requirements of, say, the SWT method.

Experience gained in the use of the cutting plane method (Monarchi, 1972) proved extremely helpful in solving the nonlinear problem. It proved particularly effective in dealing with the logarithmic form of the capital constraint as convergence was quickly obtained to the desired degree of accuracy. Computer runs of 30 seconds per iteration on the CDC 6400 of the University of Arizona Computer Center were typical in the present investigation.

APPENDIX: THE SURROGATE WORTH TRADE-OFF METHOD AS A POTENTIAL APPROACH TO THE PRESENT PROBLEM

The surrogate worth trade-off (SWT) method (Haimes and Hall, 1974) represents a recent worthwhile effort based on the ε-constraint method to analyze the multiobjective situation. Briefly, it begins by finding the minimum values of each objective function subject to the system of constraints, and proceeds to formulate the multiobjective problem in the ε-constraint form. Use of the generalized Lagrangian function of the system is then made to arrive at the Lagrangian multipliers of form $\lambda_{ij} = -\partial f_i(\underline{X}) / \partial f_j(\underline{X})$ referred to as the trade-off function between the i-th and j-th objective function. Haimes and Hall then suggest the use of a regression model for constructing this function. This computational procedure is a very extensive one, particularly when one realizes that the system has to be solved repeatedly for many values of the constraint ε, and may not always be practical. Should the system be nonlinear, computational requirements may prove very demanding. Also, the regression coefficients may be, in some instances, very sensitive to the ε levels considered. An alternative to consider for a given point in the functional space (i.e., for a level of attainment of $f_i(X^*)$ and $f_j(X^*)$), might be to just consider two straightforward solutions to the system to optimize $f_i(X)$; the first time with $f_j(\underline{X}) \leq \varepsilon$ as a constraint, and the second time with $f_j(\underline{X}) \leq \varepsilon'$, where $\varepsilon' > \varepsilon$. This would entail less than one-tenth of the amount of activity suggested and will reveal the magnitude of λ_{ij} in the immediate vicinity of the decision vector $\underline{X}^*$. Next, the evaluation of a surrogate worth function, W_{ij} is suggested for estimating the desirability of λ_{ij}, where $-10 < W_{ij} \leq 10$. Although W_{ij} is useful in the analysis, it should be pointed out that it's evaluation is highly subjective. Furthermore, in addition to being a function of the relative value of the objective functions, it should probably also be made a function of the level of attainment of these functions (e.g., a function of the absolute values attained for the functions). Otherwise, the optimal decision solution associated with $W_{ij} = 0$ will not be unique, as evidenced in the example provided in that paper.

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REFERENCES CITED

Ad Hoc Water Resources Council. Evaluation standards for primary outdoor recreation benefits, supplement 1 to the President's Water Resources Council policies, standards and procedures in the formulation, evaluation and review of plans for use and development of water and related land resources, Senate Doc. 97, pp. 87th Congress, 2nd Session, June 1964.

Bartlett, E. T. 1974. "A decision-aiding model for planning optimal resource allocation of water basins", Ph.D. diss., University of Arizona, Tucson, Arizona.

Benayoun, R., J. de Montgolfier, J. Tergny and O. Laritchev. 1971. "Linear Programming with Multi-Objective Functions: Step Method (STEM)", Mathematical Programming, Vol. 1, No. 3, pp. 366-375.

Branson, F. A., and J. R. Owen. 1970. "Plant Cover, Runoff, and Sediment Yield Relationships in Mancos Shale in Western Colorado", Water Resour. Res., 6(3), pp. 783-790.

Charnes, A., and W. W. Cooper. 1963. "Deterministic Equivalents for Optimizing and Satisfying under Chance Constraints", Oper. Res., 11 (1), pp. 18-39.

Cohon, J. L., and D. H. Marks. 1973. "Multiobjective Screening Models and Water Resource Investment", Water Resour. Res., 9(4), pp. 826-836.

Cluff, C. B., et al. 1971. "Development of Economic Water Harvest Systems for Increasing Water Supply", Office of Water Resources Research Project No. B-005-ARIZ., The University of Arizona, Tucson, Arizona.

Dantzig, G. B. 1963. "Linear Programming and Extensions", 632 p., Princeton University Press, Princeton, N.J.

Eisel, L. M. 1972. "Watershed Management: A Systems Approach", Water Resour. Res., 8(2), pp. 326-338.

Ffolliott, P. F., and D. B. Thorud. 1974. "Vegetation Management for increased Water Yield in Arizona, Technical Bulletin 215, University of Arizona, Tucson, Arizona.

Griffith, R. E., and R. A. Stewart. 1961. "A Non-Linear Programming technique for the Optimization of Continuous Processing Systems", Management Science, Vol. 7, No. 4, pp. 379-392.

Geoffrion, A. M. 1967. "Solving Bicriterion Mathematical Programs", Oper. Res., 15, pp. 39-54.

Haimes, Y. Y. 1970. "The Integration of System Identification and System Optimization", Ph.D. Diss., Eng. Syst. Dep., School of Eng. and Appl. Sci., University of California, Los Angeles.

Haimes, Y. Y., and W. A. Hall 1974. "Multiobjectives in Water Resource Systems Analysis: The Surrogate Worth Trade-off Method", Water Resour. Res., 10(4), pp. 615-624.

Kelly, J. E. 1960. "The Cutting-Plane Method for Solving Convex Problems", J. Soc. Industr. and Applied Math, Vol. 8, No. 4, pp. 703-712.

Knetsch, J., and R. K. Davis 1966. "Comparative Methods of Recreation Evaluation", in Water Research, edited by A. Kneese and S. Smith, pp. 125-142, John Hopkins Univ. Press.

Monarchi, D. E. 1972. "An Interactive Algorithm for Multiobjective Decision Making", Technical Report 6, Natural Resource Systems Laboratory, Department of Systems and Industrial Engineering, Department of Hydrology and Water Resources, University of Arizona, Tucson, Arizona.

U.S. Congress, hearings before the Subcommittee on Forests of the Committee on Agriculture, U.S. House of Representatives, on HR-10572, National Forests, multiple use and sustained yield, 85th Congress, 2nd. Session, March 1960.

Wymore, A. W. 1967. A Mathematical Theory of Systems Engineering: The Elements, John Wiley and Sons.