PLANNING MODELS OF AN IRRIGATED FARM WITH LIMITED WATER

by

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INTRODUCTION

This paper results from the dissertation prepared by the author (Blank, 1975) while a graduate student at Colorado State University. The dissertation presented an irrigation planning model incorporating probabalistic rainfall. The purpose of this paper is to briefly summarize the model and to present some of the results of the work in a form which may be useful to irrigation planners and managers.

A general research problem is to recommend to a farmer having limited resources, which crop or crops he should grow. This decision depends not only on government restrictions and the farmer's limited resources of capital, land, labor, fertilizer and water, but on the farmer's regard for risk. This risk, measured by yearly variation in income, may be caused by such uncontrollable variables as crop prices or climatic events such as hail, frost, or flooding. It may also be caused by lack of precipitation combined with lack of irrigation supply or insufficient delivery capacity to meet the crop demands. The farmer's objective may vary from maximizing his expected return to minimizing the statistical variance of that return.

Solution of the general crop planning problem would require developing a model having inputs from a number of disciplines, including agronomy, economics, statistical hydrology, etc. From the general problem various specific problems can be formulated, based on the particular characteristics of the application site and the purpose the model is expected to fill.

A specific problem dealt with in this study is to develop an irrigation planning model to recommend to a farmer which crops to grow given the following assumptions:

1. The farmer has a set of crops which can successfully be grown with his climatic conditions on his soil type.

2. The single variable input affecting yield is quantity of water. Fertilizer application amounts are assumed independent of amount of water applied. Quality of water from various sources is assumed not to affect yield.

3. The seasonal water supply is limited and the rate of delivery within the season is limited.

4. Precipitation is a random variable contributing significantly to the water supplied to the crops.

5. Other variables are deterministic, taking on their expected values.

6. The farmer's objective is to maximize expected return, providing the variance of this return due to random precipitation remains at some "acceptable" level.

Other researchers have dealt with certain aspects of the foregoing problem. Mobasheri (1968) related seasonal water inputs to yield in a linear program for determining optimal crop acreages. DeLucia (1969), Anderson and Maass (1971, revised 1974) and Young and Bredehoeft (1972) developed multiple crop models in which water inputs in various time periods are related to yield. Hall and others (Parsons, 1970) in a study of irrigation in India also included limited fertilizer, manpower, animal power, storage and transportation. Smith (1973) included random precipitation by formulating a chance constrained linear program, the results of which were used as input to a multiple year simulation.

The current study deals with the multiple time period, multiple crop case and includes random precipitation in the analysis. The main contribution of the study is that it determines an optimal precipitation planning policy (PPP). That is, it determines what percentage of the mean seasonal precipitation amount the farmer should include in his analysis to choose what crops to plant.

Due to availability of data and in an effort to complement a previous study (Young and Bredehoeft, 1972), the application site selected for the current study was a representative farm in the South Platte Valley near Ft. Morgan, Colorado. Under current water availability conditions, the current practice is to irrigate to field capacity at each irrigation. This study looked at a future time when water is limited and the price of water approaches its marginal value. Under these conditions, the farmer's decision, after the crops have been planted, is to optimally allocate his water resource among his mix of crops. It may be desirable under water shortage conditions to irrigate one crop at a critical growth stage while shorting another crop. Another possibility is that it may be more profitable not to irrigate a crop since the increase in return (marginal return) is less than the cost of the water (marginal cost).

In order to make the water allocation decision quantitatively, the farmer must possess a production function for each crop enabling him to evaluate potential yields and returns under different

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water management alternatives. Included in the pre-season crop pattern optimization of this study is a recommended water allocation throughout the season based on random precipitation with other climate variables assuming their expected values.

OVERVIEW OF THE OPTIMIZATION MODEL

The following, which may be classified as a "passive" programming approach, represents the overall optimization strategy.

1. Determine optimal timings and amounts of irrigation for the single crop case as a function of water shortage in any period or combination of periods. This is accomplished by a dynamic program. From these results a production function is obtained relating water applied in each time period to monetary return.

2. Solve a deterministic linear program for each discretized precipitation planning policy. The planned for precipitation amounts are subtracted from the crop water use coefficients in each time period.

3. Test the results of each precipitation planning policy by a simulation program which calculates the expected return and variance under each policy. This program incorporated the linearized production function generated in 1.

4. Continue to vary the planned for precipitation amounts until the optimal precipitation planning policy is determined. The recommended crop acreages are determined by using the optimal precipitation planning policy in the deterministic linear program.

SINGLE CROP OPTIMIZATION

In order to establish the multiple crop model, data from the single crop case are required. Various authors have developed models which determine optimal times and amounts of irrigation for a single crop when the seasonal supply is limited. Hall and Butcher (1968) developed a three state variable dynamic program for determining optimal application times and amounts for a single crop having a limited irrigation supply. Minhas et al. (1974) adopted an ET prediction model, developed a multiplicative yield relation for wheat, and solved an optimization problem of maximizing yield while meeting a seasonal water availability constraint. DeLucia (1969) and Dudley, et al. (1971) formulated dynamic programming models incorporating random precipitation.

For this study, a model is required which determines optimal irrigation timings and amounts when the capacity of the delivery system is limited and the single crop competes with other crops for this limited water supply. Thus, for a given water shortage in any time period and any possible combination of such shortages, the model must determine the optimal irrigation policy and the forecast yield and return. The optimal policy is thus determined as a function of the shortage in each of the time periods over the season.

The single crop optimization is formulated as a dynamic program having state variables soil moisture and state of the plant. For each level of water shortage and each combination of shortages in various time periods, the dynamic program allocates the available irrigation water in order to maximize the return for the crop. Included in the return function are terms representing the variable cost of water, the gross return of the crop and a fixed cost per irrigation. The program thus simulates the situation when the crop competes for water with other crops.

MULTIPLE CROP OPTIMIZATION

The analysis of the single crop case does not include random precipitation, but rather only specific values of possible shortages of water in each time period. In order to include precipitation in the multiple crop case, continuous functions relating water inputs to return are required. These functions are obtained from the results of the single crop optimization. In general, linear functions used to approximate non-linear relations are adequate only over a restricted range. In this study, linear functions were obtained which appeared to be adequately representative of all the crops and shortages encountered in the Ft. Morgan situation. These functions are intended to be most accurate for a range from the mean precipitation amount (when adequate water is available to irrigate all crops fully), down to a seasonal precipitation shortage from the mean of 8 cm. This shortage occurs less than 10% of the time. For greater shortages, additional data points could be generated and possibly non-linear functions fitted.

A linear program was formulated to select crop acreages which maximize return for a given precipitation level. Effective precipitation amounts are subtracted from the water use coefficients. Constraints include seasonal availability of water, capacity constraints on quantity of water delivered in each time period, land availability, and maximum and minimum acreage constraints dictated by local market conditions.

THE PRECIPITATION PLANNING POLICY

The approach of this study for handling random precipitation is to introduce a concept referred to as the "precipitation planning policy." A farmer adopting a 10% precipitation planning policy would plan for 10% of the mean precipitation amount to supplement his irrigation supply in each time period and would allocate his fixed seasonal irrigation supply during the season according to the precipitation to date and a forecast of 10% of mean precipitation for the remainder of the season.

Under a 0% precipitation planning policy, rainfall would be neglected and, with irrigation water limited, a "conservative" mix of crops would be selected, one whose water requirements would always be met, but whose return would presumably be relatively low. Alternatively, under a 100% precipitation planning policy, the farmer would plan for the mean precipitation event and would select a crop mix consisting of higher valued and higher water using crops, but could face reduced yield when less than mean precipitation amounts occurred.

GROWTH MODELS

All irrigation optimization models implicitly require as input a simulation model to predict return, or equivalently yield, from a crop under any given irrigation and precipitation regime. A simple, accepted input-output relation between water inputs and yield would be useful; however, in actuality other variables affect yield. In this study an intermediate variable was introduced to link inputs to outputs. This variable should assist in the transferability of the model.

Photosynthesis, the mechanism by which plants manufacture carbohydrates, may be viewed as the process which results in the final yield. Photosynthesis occurs when carbon dioxide, water and sunlight are all available to the plant. For most crops, including corn, carbon dioxide enters the plant through open stomata, while water vapor leaves through these stomata at a rate determined by atmospheric conditions and moisture availability. Water vapor that the plant loses through the open stomata makes up the major portion of the total water transpired by the plant. When the stomata close, the flow of both carbon dioxide into the plant and water vapor out of the plant are restricted.

Transpiration may thus be used as a proxy measure of plant growth. By maximizing transpiration for a given set of atmospheric conditions, photosynthesis is also maximized since when transpiration occurs the stomata are open and carbon dioxide is also entering the plant at the maximum rate for the given atmospheric conditions. Advancing one step further, it can be postulated that lack of transpiration, hence reduced carbon dioxide availability and reduced photosynthesis, would have greater effects on the final yield in some growth stages than in others.

Hanks (1974) used a growth index based on transpiration while in this study an index based on evapotranspiration ET is used. Evapotranspiration values are more readily obtainable than transpiration values and the differences in yield prediction would be negligible using a transporation index rather than the ET index.

Two production functions were tested in this study relating ET in various time periods to yield. The functions are the aditive relation

$$\frac{Y}{Y_{max}} = \sum_{i=1}^{n} A_i \left(\frac{ET_i}{ET_{max_i}}\right) \qquad (1)$$

and the multiplicative relation

$$\frac{Y}{Y_{max}} = \prod_{i=1}^{n} \left(\frac{ET_i}{ET_{max_i}}\right)^{\alpha_i} \qquad (2)$$

in which

i = growth stage index
Y = predicted yield
Y_{max} = maximum attainable yield
ET_i = ET in growth stage i
ET_{max_i} = maximum attainable ET in growth stage i
n = number of growth stages
A_i and α_i = parameters fitted to available data.

Both of the foregoing relations were tested and incorporated into the single crop optimization model.

ET PREDICTION MODELS

In order to predict yield for any irrigation regime and with a growth model based on ET, a model which predicts ET is essential. In fact, any model which predicts crop water requirements must keep a soil moisture balance of the form

$$SM_{i+1} = SM_i + IRR_i + P_i - RO_i - D_i - ET_i \qquad (3)$$

in which

i = time index
SM_i = depth of available soil moisture at time i
IRR_i = depth of irrigation at time i
P_i = effective precipitation at time i
RO_i = run-off at time i
D_i = drainage (deep percolation) at time i
ET_i = evapotranspiration at time i; all units in cm.

Run-off and deep percolation may be neglected if the sum of irrigation and precipitation amounts in a time period do not exceed the infiltration rate of the soil, and if they do not bring the soil moisture level above field capacity. Thus, updating soil moisure reduces to establishing an ET prediction model. Jensen and Heerman (1970) and Kincaid and Heerman (1974) employed the Penman equation to predict daily ET. The quantity of data required limits the feasibility of this method for a planning model. For the requirements of this study, a simple model proposed by Yaron, et al. (1972) was adopted. This method predicts ET based on soil moisture and stage of growth. Climate variables are included in the growth stage parameters with the assumption that at the given site the parameters are relatively insensitive to year to year climate variations.

The basic hypothesis of the Yaron model is that the negative of the rate of change of soil moisture is proportional to the available soil moisture. The solution of this differential equation gives

$$w_{o+t} = w_o e^{-b_i t} \tag{4}$$

in which w_o = initial available soil moisture in the i^{th} time period (cm)

w_{o+t} = available soil moisture after t days (cm)

b_i = soil moisture decay constant in the i^{th} time period (dimensionless).

The coefficient b_i has the interpretation

$$b_i = ET_{max} / FC \tag{5}$$

in which ET_{max} = daily ET which occurs at field capacity determined by climate and stage of growth (cm)

FC = available depth of soil moisture at field capacity (a soil property) (cm).

Assuming no precipitation or other source of water, ET after t days is given by

$$ET = w_o - w_{o+t} \tag{6}$$

Combining the ET predicition model with either the additive or the multiplicative production function results in a model to predict yield under any irrigation regime. An advantage of the yield prediction model developed in this study is that it can be directly integrated into an optimization model. Another advantage is that there are few parameters to fit: one parameter for each growth stage for each crop in the ET prediction model, and one for each growth stage for each crop in the production function. A disadvantage in determining these parameters, however, is that results from extensive field trials are required. With experience from a number of trials on different soils and climates, an estimation procedures could be developed.

YIELD PREDICTION RESULTS

Parameters of the ET prediction and the yield prediction models were determined for corn from field trials conducted at the Colorado State University Agronomy Farm. Corn was grown in four blocks with each block receiving a different irrigation schedule. The central sprinkler line was designed to irrigate most heavily along the center of the block with application amounts declining linearly so that outer rows were virtually unirrigated. Soil moisture measurements were made by neutron probe prior to each irrigation at 12 positions across each block with rain gages measuring irrigation and precipitation amounts. Thus, 48 trials were obtained for which various timings and amounts of irrigation water were applied, a soil moisture balance was kept and yields were measured.

Previous results (Danielson and Twyford, 1974) have identified early silking as a critical growth stage for corn. On the basis of this work and the available data, growth stages were defined as presented in Table 1.

A program was written to determine best fit coefficients for the ET prediction model described in the previous section. Data from five trials were used to obtain representative values of the coefficients. The program used as input soil moisture levels by layers and incorporated a simple drainage model. A Fibonacci search routine was used to determine optimal values of the coefficients. The results, summed for the entire root zone, are presented in Table 1. The yield prediction coefficients of Eq. 1 and Eq. 2 were determined by a standard linear regression program. The additive equation showed a higher R^2 value and was used in further work. These coefficients are also presented in Table 1.

From the results obtained a simulation model can be formulated to determine percentage of maximum yield for any given irrigation treatment. An example of such a model considering at most four irrigations on specified dates and neglecting precipitation is presented in Table 2 and Table 3. Values of field capacity, ET coefficients, yield coefficients and maximum ET values are those used for the Ft. Collins, Colorado case.

OPTIMIZATION RESULTS

The simulation model discussed in the previous section was included in a dynamic program to maximize return for a representative farm near Ft. Morgan, Colorado, for which economic data were available (Conklin, 1975). Optimal irrigation amounts and timings were determined when water was limited in various time periods. Also, various sensitivity analyses were performed to determine the effects of water cost and labor cost on the optimal irrigation amounts and return. The marginal value of irrigation water for corn (that cost above which the cost of the last unit of water excedes the return from that unit) was determined to be between $10 and $15 per acre foot.

Secondary data sources were used to generate ET prediction and yield prediction models for the other crops (sugar beets, pinto beans, and alfalfa) considered in the study.

For the 280 acre representative farm, the deterministic linear program was formulated as previously described. When precipitation amounts were varied from 100% of the mean to 0%, the results showed return to decrease along an S shaped curve. Acreage of individual crops generally remained constant but the method of corn production varied from full irrigation at the 100% level to non irrigated at the zero precipitation level with other crops generally maintaining full irrigation.

For various precipitation planning policies, with the crop acreages determined by the deterministic linear program, the simulation program was run to determine actual expected return and standard deviation using a 20 year historical record of precipitation. An expectation versus variance curve was generated which tended to show that as the PPP approached 100%, the expected income decreased and standard deviation increased. Near a PPP of 10% the expected return is maximized and the standard deviation is essentially the same as for a PPP of 0%.

CONCLUSIONS

There are many assumptions and limitations of the results of a study such as has been performed. However, the question of how to allocate a limited resource such as water is a vital one and mandates the attempt at such a study. When downstream users rights must be met, the water manager is faced with limiting water diversions, or in the case of the South Platte Valley, of limiting groundwater pumping. One tool of the water manager, in order to predict the economic effects of such a limitation, is the simulation model. This study has demonstrated the development of such a model and has incorporated the simulation model into several optimization models. These models recommend allocation of the water within the season based only on preseason knowledge and select optimal crop acreages. The percentage of mean precipitation was determined which, when included in the crop planning model, results in return being maximized over the long range. This value is not meant to be used as a standard, since its value is determined by many variables, but rather as a warning to researchers who may wish to arbitrarily include the mean precipitation event in their analysis.

Table 1

RESULTS OF ET AND YIELD PREDICTION MODELS FOR CORN BASED ON FIELD TRIALS AT COLORADO STATE UNIVERSITY

Growth Stage	Dates	ET Coefficients	Yield Coefficients
Early Vegetative	May 15 - July 16	.015	.236
Pollination	July 17 - July 23	.019	.159
Maturity	July 24 - Sept. 11	.014	.573

Table 2

ET CALCULATION

1 Growth Stage	2 w_o	3 t	4 b_i	5 w_{o+t}	6 ET	7 IRR	8 w_o'
1	33.2	31	.015	20.82	12.38		
		31	.015				
2		7	.019				
3		25	.014				
		25	.014				

Column 5 is obtained from Eq. 4
Column 6 is Col 2 - Col 5
Column 7 is the irrigation amount
Column 8 is the minimum of (Col 5 + Col 7) and 33.2
Column 2 is Col 8 from the previous row.

Table 3

YIELD CALCULATION

1 Growth Stage	2 ET	3 ET_{max}	4 a_i	5 Term
1		24.89	.236	
2		4.13	.159	
3		19.44	.573	

Col 2 is total from Table 2 Col 6
Col 4 is Col 2 x Col 4 / Col 3 as per Eq. 1
Y/Y_{max} is the sum of Col 5

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