A MULTIATTRIBUTE APPROACH TO THE RECLAMATION OF STRIPMINED LANDS

by

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ABSTRACT

A multiattribute utility function is used to model preferences on outcomes of alternative reclamation schemes for stripmined lands, using Arizona and Wyoming examples. Each scheme should at least help restore land to its premining value, and is composed of three sets of actions: mining operations, preparations for postmining land use, and mitigating actions. Grazing and runoff augmentation are examples of postmining land use goals, and mitigating actions may be measures to protect the environment like pollution control in runoff or infiltration. Conflicting objectives are involved, including the maintenance of sufficient coal production, the alleviation of detrimental environmental effects, and the minimization of loss. Since the environmental effects are fraught with uncertainty, a multiobjective decision-making scheme under uncertainty is set up to analyze the problem. The decision model ranks alternative reclamation schemes on the basis of the preference function of a group decision maker, each member of which assessing a separate subset of single attribute utility functions.

INTRODUCTION

The purpose of this paper is to present a framework for applying multiattribute utility theory to the ranking of alternative schemes for reclaiming stripmined lands. Stripmining, which is regulated by the federal Surface Mining Control and Reclamation Act of 1977, is not acceptable without actions to reduce negative impacts and actions to restore the lands to at least premining value (Imes and Wali, 1977). Uncertain operating conditions that the decision maker cannot influence are called states of nature, and due to these uncertainties several outcomes of some realized scheme are random. Accordingly, the evaluation of alternatives has to account for the probability distribution of outcomes rather than fixed outcomes. The fact that a stochastic process may produce unwelcome outcomes under a distribution with otherwise acceptable expectation introduces risk into the decision making, and hence the need to account for risk attitudes. The choice of a reclamation scheme may therefore provide a good example of an application of utility theory that accounts for probability distributions of the states of nature as well as for the risk attitude of the decision maker.

The research reported here on defining a multiattribute utility criterion is thought of as part of a comprehensive systems analysis, i.e. the type of trade-off analysis that ideally should be the basis for deciding on mitigating actions and postmining land use. Much effort goes into the environmental impact statements to make it likely that the chosen scheme satisfies the regulations. In the analysis reported here the regulations are not treated as strict constraints because the achievement of a certain attribute or performance level is subject to chance. Operations may also be exempted from regulations on the basis of benefits of granting exemptions.

The paper is organized as follows. In the next section, the problem is defined. Then the multiple objectives are examined more closely, and in the following section the method is presented. Then follows a description of the assessment of utility functions, and finally comes a discussion of the utility theory framework.

THE PROBLEM

A reclamation scheme has to be found that strikes a balance between several objectives. Two fundamental objectives are to provide for sufficient coal supplies, and to afford acceptable protection of the water resources. These objectives are conflicting because a desired level of coal production may have to be traded off against unwelcome environmental impacts. An accepted reclamation scheme is therefore a result of trade-offs between various attributes or criteria related to the multiple objectives. What is needed then is a scalar measure of preferability combining all attributes. Such a measure will allow a ranking of alternative schemes.

An analysis to support decisions concerning reclamation of stripmined lands may be bisected into establishing a decision model and a preference model. The relationship between the two models is illustrated in Figure 1. This paper deals mainly with the preference model. A set A of alternative reclamation schemes are considered for ranking

$$A=(a_1, a_2, \ldots) \quad (1)$$

A scheme a_i consists of three sets of actions; the set M_i of interim land use actions, the set C_i of mitigating actions, and the set R_i of postmining land use preparations. Thus

$$a_i=\{M_i, C_i, R_i\} \quad (2)$$

where the subsets of a_i each have specific actions as members.

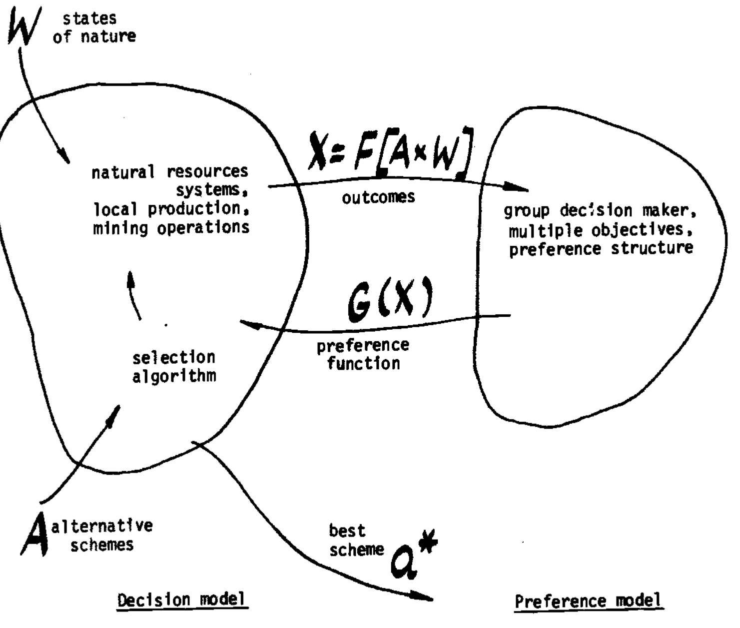

Figure 1 Models dichotomy

Interim land use actions are obviously the surface mining operations, perhaps along with other activities. Examples of mitigating actions are watering and chemical stabilization with respect to dust control during topsoil and overburden handling, utilization of emission control equipment, and disposal systems for solid and liquid wastes (Doyle, 1976). Examples of postmining land use goals are livestock grazing for the Eagle Butte mine in Wyoming (USGS 1977), and water harvesting on the Black Mesa in Arizona (Thames, 1979).

To some degree the surface mining action set M_i is the same for different schemes , which is also the case for the set C_i. There will also be a certain amount of overlapping in that the design of a mining plan, i.e. the choice of mining method, equipment, and sequence of operations is a function of regulations and planned postmining land use. For example the interfingering of surface mining operations and end use preparations is an important optimization problem as shown by Kirk (1978). The ranking of alternative schemes, however, assumes that each alternative $\{M_i, C_i, R_i\}$ has an optimal representation as far as possible before ranking takes place. Another example of an intra-scheme optimization study is given by Brinck & al. (1976) relating grading and furrowing to conditions for livestock grazing. The output of the decision model in Figure 1 is the set X of system outcomes, which are the realized levels of the attributes of the reclamation scheme

$$X=(x_1, x_2, \ldots) \quad (3)$$

where x_1 is the realized level on attribute 1, outcome x_2 is the realized level on attribute or performance criterion 2, etc. Examples of outcomes are: coal production in tonnes per day, area in hectares over which aquifers are destroyed to a specified degree, wildlife habitat in no. of head of certain species, monetary loss, and livestock herd size grazing on the reclaimed lands.

The decision model comprises the model F of the technological and physical relations defining the state transitions. Then X is the set of system states resulting from the transformation of the cartesian product of A and W due to the state transition model F

$$X=F[A \times W] \quad (4)$$

where W is the set of states of nature

$$W=(w_1, w_2, \ldots) \quad (5)$$

Obvious states of nature to be accounted for are random conditions affecting the pollution in runoff and infiltration, like precipitation and infiltration rates on strip mine spoils. But also the enforcement of environmental protection rules in uncertain, and thus a member of the set of states of nature. The preferences of the decision-maker are used to rank the schemes a_i, $i=1,2,\ldots$ These preferences are given by the output of the preference model G as illustrated in Figure 1. G is a preference mapping of outcomes or consequences of actions into the real numbers R (Bertsekas, 1976)

$$G: X \rightarrow R \quad (6)$$

The decision on the best reclamation scheme a* thus has to be reached under uncertainty and by trading off multiple attributes. The task is now to establish the scalar measure of preferability G to find a* among the set A according to a decision rule. This decision rule says that the best alternative a* is that alternative a_i which maximizes the preference function G

$$a^*=a_i, \text{ iff } \; G(a_i)=\max_{a \in A} G(x_1(a), x_2(a), \ldots) \quad (7)$$

In our case G is the multiattribute utility function u

$$u=u(x_1, x_2, \ldots) \quad (8)$$

to be explained in a separate section.

GROUP DECISION MAKER AND MULTIPLE OBJECTIVES

In this section the decision maker is defined as a group decision maker (GDM) comprising several groups, and the objectives and related attributes are looked into more closely. This paper does not treat the problem in its full complexity, hence the sets of attributes, interest groups etc. might have to be expanded for a full case study.

GROUP DECISION MAKER

The opening of a coal field for stripmining affects many groups of people as well as the nation at large. Typical interest groups are mining companies and public utilities, regulatory agencies, people supporting local postmining production activities, and environmental groups. The group decision maker (GDM) will be composed of representatives from interest groups, and decision analysts as well as other experts. Following Krzysztofowicz (1978), the GDM divides into subgroups related to specific fields of expertise or responsibilities. The GDM accepts the single attribute utility functions derived by the subgroups as the GDM's own.

In the multiattribute acceptance of a reclamation scheme the nation is represented by the regulatory agencies (Hipel & al., 1974, 1976). They enforce rules that are the results of ongoing trade-offs between national objectives like low inflation, independence from oil imports, and reduced impacts on humans and nature. The reasons for formalizing the decision analysts' position as a subgroup within the GDM are twofold. It is often impossible to complete an analysis without judgement on the part of the analysts, not least in the choice of analytical tools. Judgements in turn may unintentionally imply preferences for outcomes. Hence the decision makers ought to be made aware of this in a formal manner reminding them of the need that they understand the assumptions along with the conclusions. Secondly, the model development, data compilation, and sensitivity analyses the analysts carry out all build up systems insight beyond presentation of quantified answers to questions posed. Their inclusion as decision maker makes explicit their responsibilities for making it all available.

The next step is to describe the objectives which the reclamation shall serve. The interest groups and the GDM were discussed first for the reason that only those objectives are sure to be included for which there are people prepared to stand up.

THE OBJECTIVES

The alternative reclamation schemes have the following objectives:

1) to produce sufficient amount of coal

2) to effect a postmining land use reaching at least premining value

3) to alleviate pressures on the environment

4) to minimize costs and losses

Keeney & Raiffa (1976) develop the objectives' hierarchy to the point of a one-to-one relationship between a lowest level of objectives and attributes. This is not done here, instead the next step within the framework of a cost-effectiveness analysis is applied, which is to transform the objectives into specifications (Duckstein and Kisiel, 1977). These in turn lead to the definition of attributes.

THE SPECIFICATIONS

Specifications in their full description are the translations of objectives into technological, economic, social, and environmental sub-objectives. Here also standards and regulations are included.

To objective 1) then correspons the following specifications or sub-objectives

1) minimize reductions from desired production goals

2) do at least as well as required by mining health and safety standards

to objective 2)

3) maximize the profit of the postmining land use

4) maximize the protection of already established local production

objective 3)

5) landscape as esthetically as possible

6) maximize the protection of renewable and non-renewable resources other than coal

7) meet the air, water, and reclamation standards as well as possible

and to objective 4)

8) maximize mining productivity

9) minimize reclamation cost

Spec. 9), i.e. the cost of reclamation is kept separate from the profit of local production activities resulting from the reclamation, spec. 3). This is done because reclamation costs including preparations for the local end use are tagged on to the coal price whereas the proceeds from the postmining land use go elsewhere. The profitability of the mining is not specified explicitly because mining profits is a function of national priorities and regulations in energy matters. But this study does not treat national trade-offs, and thus, profit is accordingly not specified as an objective. But given that the productivity is maximized (spec. 8), the profit is maximal for any coal price level.

THE ATTRIBUTES

The attributes are now simply the subobjectives or specifications of last section stripped of the imperatives maximize, minimize, etc. Also seeminglystrict regulations are listed as attributes. When strict regulations relate to uncertain outcomes the strictness becomes muted, but alternatives, although flexible, should be kept as close as possible to the regulations.

The resulting attributes derived from the specifications are

x_1 coal production
x_2 attainment of mining health and safety standards
x_3 profit of postmining land use

x_4 level of already established local production
x_5 esthetics of the landscaping
x_6 non-destruction of renewable and non-renewable resources not to be mined
x_7 air quality
x_8 water quality
x_9 attainment of reclamation standards
x_{10} mining productivity
x_{11} reclamation cost

INTER-SCHEME RANKING AND INTRA-SCHEME OPTIMIZATION

If a reclamation scheme contains actions or control variables that only bear upon a single objective like profit, they can be jointly optimized w.r. to maximum profit separately from any multiattribute ranking or optimization. But when two or **more** conflicting objectives are in force for some or all actions, their optimization will have to be handled by a multiattribute preference procedure. Brinck & al. (1976) optimize schemes with essentially two types of actions; livestock species for grazing, and furrowing graded spoil piles. The optimization problem was to combine the two actions to achieve the single objective of maximizing expected profit. The action to let livestock on the reclaimed range, however, is also the most prominent attribute subject to strong subjective preferences. Thus if cattle grazing should turn out to have a smaller expected profit than sheep grazing, a trade-off based on a biattribute preference function would be appropriate.

Multiattribute trade-off procedures do not necessarily contain algorithmic feedback to the optimization of actions. A continuous action variable that interacts with conflicting objectives has to be discretized, and each level of a_i thus defined enters into a combination with other actions to make yet another alternative scheme to be ranked under the multiattribute preference procedure.

THE METHOD

In this chapter the choice of a multiattribute utility function for the preference function G, eq. (6), is explained and its derivation is outlined. Once G. has been estimated, the decision rule for finding the best scheme is given by eq. (7). When outcomes are deterministic, the preference function is called a value function (Keeney & Raiffa, 1976); in the uncertainty case such as treated here, it is called a utility function.

A MULTIATTRIBUTE UTILITY FUNCTION

The scalar index of preferability G is here u, the multiattribute utility function derived on the basis of the von Neumann - Morgenstern utility theory expounded in Raiffa (1970), Keeney and Raiffa (1976), Krzysztofowicz (1978), and several other references. The concept of utility theory and its application and alternative forms of multiattribute utility functions are detailed in the literature and will not be repeated here, but for brief comments, (Gros, 1975; Richard, 1975; Keeney and Wood, 1977; Duckstein & Krzysztofowicz, 1978). Given assumptions about preferential and utility independence certain functional forms of the utility function can be derived. A fundamental concept of multiattribute utility theory is the assumption of utility independence. Its role in multiattribute utility theory is similar to that of probabilistic independence in multivariate probability theory. The existence of utility independence means that there also exist utility functions over attributes individually, i.e. single attribute utility functions in a multiattribute environment. Under the condition of mutual utility independence the multiattribute utility theory leads to the conclusion that a utility function is either multiplicative or additive (Keeney and Raiffa, 1976, p. 288). A multiplicative form for multiple attributes is for $K \neq 0$

$$1+ku(x_1,x_2,..)=\prod_{i=1}^{n}(1+kk_i u_i(x_i)) \qquad (9)$$

where u_i are the single-attribute utility functions for the i'th attribute, $i=1,2,..,n$. k and k_i are scaling constants. The latter can also be called corner utilities (Duckstein and Krzysztofowicz, 1978).

Some authors like Gros, (1975) prefer the phrase "preference function" to "utility function" to avoid confusion with public utilities. Here, preference function has been used as the general name for the mapping defined by G, in eq. (6), and utility function is understood in the von Neumann - Morgenstern meaning.

THE SINGLE ATTRIBUTE UTILITY FUNCTION

The utility function allows ranking of probability distributions of the outcome on the basis of expected utility $\bar{u}$. The outcome whose utility is the expected utility of the outcome - distribution is called the certainty equivalent $\hat{x}$ as the decision maker in his judgement is indifferent between this outcome for certain and the game given by the probability distribution. Thus instead of ranking alternatives by expected outcome $\bar{x}$ they are ranked by expected utility $\bar{u}$

$$\bar{u} = E[u(x)] \qquad (10)$$

If $\hat{x}$ is equal to $\bar{x}$ the decision maker is indifferent to any particular realization of the random x, i.e. he is globaly risk neutral. This is the case when u is a monotonicaly increasing linear function. Other utility functions may possess local non-neutral risk attitudes, even if $\hat{x}$ equals $\bar{x}$. It is also possible to see the ranking in terms of the certainty equivalent $\hat{x}$ being a shifted expectation of x.

The derivation of single-attribute utility functions are often framed in the probabilistic notion of two-pronged lotteries with a most preferred reward and a least preferred , for each attribute (Halter & Dean, 1971; Lavalle, 1970). The two-pronged even-chance lottery spanning the whole stochastic range of the outcome, i.e. with the two extreme outcomes as rewards produces one utility point which defines the indifference between the lottery and the estimated certainty equivalent of the lottery $\hat{x}_\ell$. As the utility function is calibrated by the utility of the most preferred outcome being 1, and the utility of the least preferred outcome x_0 being 0, the utility of said lottery is ½. Because utility functions are unique up to a positive linear transformation, this technique can be continued for each of the new outcome intervals resulting from the mid-utility splitting, linking the interval utility functions together by said property. In this way enough points can be estimated to draw a curve.

The technique of finding points on the utility curve by estimating certainty equivalents $\hat{x}_\ell$ of fixed utility lotteries may not work for discrete outcomes.
If outcomes do not exist between the discrete realizations the closest one may have to approximate $\hat{x}_\ell$. A procedure which may be more accurate keeps the lottery over the extreme and realizable outcomes x_0 and x^*, and varies the odds instead. The lottery now will produce x^* with probability p_i and x_0 with probability $1-p_i$. The subject is asked to estimate the probability p_i for which he is indifferent between the lottery and a realizable outcome x_i for certain. Because the cardinal utility functions of the von Niemann-Morgenstern type with which we are working are unique up to a positive linear transformation, and secondly, because the scaling convention for the utility function is the same as for the probabilities p_i, the p_i-value is also a utility and in fact equal to the utility u. Thus the p_i-point is then a point on the u-curve for x_i.

SCALING FACTORS

The general form of the multiattribute utility function based on mutual utility independence is eq. (11), from which eq. (9) is derived for $k \neq 0$,

$$u = \sum_{i=1}^{n} k_i u_i + k \sum_{\substack{i=1 \\ j>i}}^{n} k_i k_j u_i u_j + k^2 \sum_{\substack{j=1 \\ j>i, k>j}}^{n} k_i k_j k_k u_i u_j u_k + \ldots + k^{n-1} k_1 k_2 \ldots k_n u_1 u_2 \ldots u_n \qquad (11)$$

It is seen that for k=0 u is (multi-) linear. The risk neutral form of eq. (11) written for the combination of best outcomes is

$$\sum_{i=1}^{n} k_i = 1 \qquad (12)$$

Thus k = 0 and eq. (12) are both expressions of risk neutrality. For $k \neq 0$ eq. (11) can be reformulated and written for the combination of best outcomes

$$1 + k = \prod_{i=1}^{n} (1 + kk_i) \qquad (13)$$

These expressions are needed for determining numerically the scaling factors k and k_i.

ASSESSMENT

In this chapter the multiattribute utility function is assessed for a case of three attributes related to multiple land use alternatives covering different grazing schemes

x_3 profit to the locals of postmining grazing measured by the range carrying capacity in animal units per hectare and year. Worst outcome is 0 and best outcome is set to 2 AU/ha, year (Brinck et al., 1976)

x_6 wildlife habitat measured in mule deer, from 0 to 400 deer (USGS, 1977)

x_8 ground water quality measured subjectively in % of excellent (Keeney and Wood, 1977)

The procedure follows Keeney and Wood (1977) and can be applied straightforwardto cases with many attributes.

The order is

1) Checks of preferential and utility independence

2) Derivation of the single-attribute utility functions within the subgroups

3) Assessment of the GDM's risk attitude

4) Estimation of the GDM's scaling factors

THE GDM'S QUALITATIVE PREFERENCE STRUCTURE

In eq. (11) the form of the multiattribute utility function was introduced on the assumption of preferential and utility independence. The present assessment does not treat the true case in terms of decision makers and a full spectrum of consequences, and hence proceeds without verifying these assumptions. However, e.g. Keeney and Wood (1977) show a way of doing this.

SINGLE ATTRIBUTE UTILITY FUNCTIONS

The utility functions for x_3, x_6, and x_8 used in this example are shown in Figure 2 a,b, and c respectively. For x_3 a risk prone attitude is explained by unfavorable grazing conditions being manageable, although at a cost. The carrying capacity may come out worse than planned, and water may have to be piped from farther away, but damage is not irreparable. However, to restore wildlife habitat, x_6, takes cooperation from the beasts, and may be impossible or difficult to manage. Therefore the persistent risk averse utility function for the attribute x_6. Ground water, x_8 below a certain quality level ceases to be useable, and the decision maker can then better gamble that the outcome will be good. For low quality levels therefore it shows a risk prone attitude, but changes to a risk averse behavior at higher levels.

ESTIMATION OF SCALING FACTORS

Equations for determining the scaling factors k and k_i are found by considering that eq. (11) for x_i at its best level and all other attributes at their worst, yields

$$u(0,..,0,x^*_i,0..,0)=k_i \qquad (14)$$

This is by the way the reason for calling the k_i corner utilities. By taking two attributes say x_3 and x_6 and keeping the other at its worst level, an indifference relation between two points $(x_3,x_6)'$ and $(x_3, x_6)''$ makes an equation for k, k_3, and k_6 by applying eq. (11) to both sides of this equation.

The simplest way to go may be to decide which of the two attributes is the least important, say x_6, then fix the point $(x_3=0,\ x_6=400)$, and ask for an estimate of the x'_3 value which makes $(x'_3,x_6=0)$ indifferent to the first point. The combination with worst levels supresses k, and the results of two indifference relations framed in this way is

$$k_6 = .1k_3 \quad \text{and} \quad k_8 = .3k_3 \qquad (15)$$

The utilities are read off the estimated utility functions in Fig. 2. With n attributes only n-1 equations can be had in this way. The complementing equation is found by estimating the probability p for which the decision maker is indifferent between a lottery over worst and best combinations, say (x^*_3,x^*_8) and $(x_{o,3},x_{o,8})$, and $(x^*_3,x_{o,8})$ for certain, where p is the probability of getting the best reward. $x_{o,i}$ and x^*_i mean the worst and the best outcome on attribute x_i respectively. Again using eq. (11) for this gamble gives

$$k_3 = p(k_3+k_6+kk_3k_6) \qquad (16)$$

p was estimated to .8. Eq. (16), however, inroduced k for which eq. (13) can be solved

$$1 = k_3+k_6+k_8+k(k_3k_6+k_3k_8+k_6k_8)+k^2k_3k_6k_8 \qquad (17)$$

Eq.s (15), (16), and (17) give the four relations needed. The solution is

$$k_3=.753,\ k_6=.075,\ k_8=.226,\ k=-.221 \qquad (18)$$

Eventually

$$u=.753u_3+.075u_6+.226u_8-.013u_3u_6-.038u_3u_8-.0038u_6u_8+.00063u_3u_6u_8 \quad (19)$$

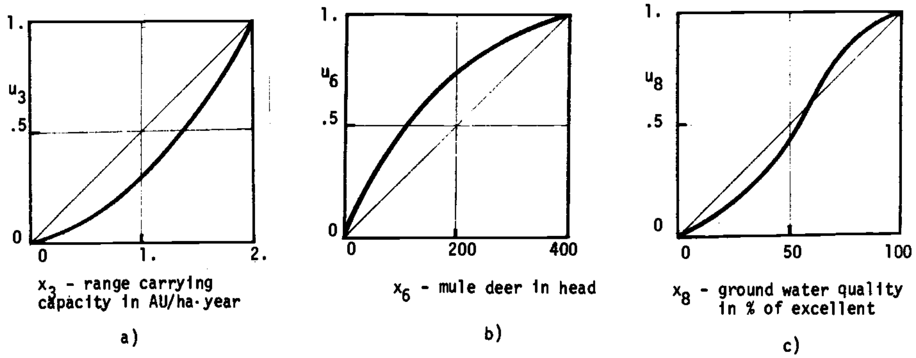

Figure 2 Single attribute utility functions

CONCLUSION AND DISCUSSION

Because of the uncertainties in the states of nature and the risks involved the ranking of reclamation schemes for disturbed lands is a good case for applying the multiattribute utility theory. This paper establishes a framework for a real case application which would have to elaborate the derivation of utility functions and preference structures more carefully than has been done here.

The important step of verifying preferential and utility independence has not been carried out because this analysis does not treat the full case. The true group decision maker and the full spectrum of consequences would have to be present to carry out the independence checks.

Although the decision maker is thought of as a group decision maker nothing has been said about how the group goes about agreeing on scaling factor derivations. For suggestions of how to model alternative group decision makers reference is made to the references cited. The existence of a model for treating the trade-offs between conflicting attributes can be used for sensitivity analyses to assist in solving differences between GDM members.

Finally the dynamic character of the problem has not been touched upon, but it lies close at hand that the reclamation of an area takes place in stages, and calls for framing the development of reclamation plans in a stochastic and dynamic framework.

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