ESTIMATING TRANSMISSION LOSSES IN EPHEMERAL STREAM CHANNELS

by

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ABSTRACT

Procedures have been developed to estimate transmission loss volumes in abstracting (losing) ephemeral streams. A two-parameter linear regression equation relates outflow volume for a channel reach to the volume of inflow. A simplified two-parameter differential equation describes the transmission loss rate as a function of length and width of the wetted channel. Linkage relationships between the regression and differential equation parameters allow parameter (and thus, transmission loss) estimation for channels of arbitrary length and width. The procedure was applied using data from 10 channel reaches. Maximum loss rates were observed on Walnut Gulch, Arizona, and minimum loss rates were observed on Elm Fork of the Trinity River, Texas. All other data were between limits established at these two locations. Examples illustrate typical applications and show step-by-step procedures required to use the proposed method. Results and interpretations were summarized, and needs for additional research were specified.

INTRODUCTION

In much of the southwestern United States, watersheds are characterized as semiarid with broad alluvium-filled channels that abstract large quantities of streamflow (Babcock and Cushing, 1941; Burkham, 1970a, 1970b; and Renard, 1970). These abstractions or transmission losses are important because streamflow is lost as the flood wave travels downstream, and thus, runoff volumes are reduced. Although these abstractions are referred to as losses, they are an important part of the water balance. They diminish streamflow, support riparian vegetation, and recharge local aquifers and regional groundwater (Renard, 1970). Therefore, procedures are needed to estimate outflow volume, and, from that, transmission losses in abstracting streams.

Various procedures have been developed to estimate transmission losses in ephemeral streams. These procedures range from simple regression equations to estimate outflow volumes (Lane, Diskin, and Renard, 1971) to simplified differential equations for loss rate as a function of channel length (Jordan, 1977). Contrasted with these simplified procedures dealing only with the volume of losses are the procedures used to route hydrographs through a losing channel. Lane (1972) proposed a storage-routing model as a cascade of leaky reservoirs, and Wu (1972) used the leaky reservoir concept in modeling surface irrigation. Smith (1972) used the kinematic wave model to route hydrographs in channels of ephemeral streams. Smith demonstrated that the wave front becomes steeper due to shock formation, and that the hydrograph peak decreases in a downstream direction due to infiltration. Peebles (1975) modeled flow recession in ephemeral streams as the discharge from a single leaky reservoir with loss rates proportional to the depth of flow and storage volume proportional to the square of the depth.

Therefore, there is a range of complexity in procedures for estimating transmission loss volumes and rates in ephemeral stream channels. In general, the simplified procedures require less information about physical features of the channels but are less general in the application. The more complex procedures may be more physically based, but they require correspondingly more data, and more complex computations.

The purpose of this study was to develop a simplified procedure for estimating the volume of outflow, and from that the transmission losses, given an inflow volume at a point upstream. We also sought to develop procedures for estimating flow volume at any point along the stream between the inflow and outflow stations (Lane and Staff of the Southwest Rangeland Watershed Research Center, 1979).

Since the study concerns streams where water is abstracted, the outflow volume must be less than the inflow volume. We assumed that for a particular channel reach,infiltration rates and other properties were uniform with channel length and width, so that the relationship between inflow and outflow volumes is unique, given the same antecedent conditions.

We sought a simplified procedure with a minimum number of parameters and with reasonable bounds or limits on the estimates of transmission losses. Although all physical characteristics affecting transmission losses could not be explicitly incorporated in the equations, we sought a procedure that would directly account for channel size (length and width) and, thus, facilitate transfer of results from gaged to ungaged channel reaches. Finally, we sought a procedure that would be accurate for the average or representative conditions for a particular channel reach. With these criteria, we expected to reproduce trends over a range of data. Relatively larger errors are expected for very small or very large inflows, or for events occurring under extreme conditions of antecedent moisture.

ASSUMPTIONS AND LIMITATIONS

For a given channel reach, we assumed infiltration and other channel properties were uniform with distance along the reach and with width across a channel cross-section. We assumed a unique relationship between inflow and outflow volume under given antecedent conditions. However, the procedure did not account for sediment concentration in the streamflow, temperature effects, seasonal trends, differences in peak flow rate, or hydrograph shape for the same inflow volume. Therefore, the procedure was designed to compute outflow volume and, from that, transmission loss volume, and did not compute flow rates or account for flow duration. Outflow and transmission loss rates were defined as functions of distance, not functions of time. Finally, we assumed a threshold volume or initial abstraction, and then a linear relation (above the threshold) between inflow volume and outflow volume. The assumption of a threshold volume made the outflow-inflow relation nonlinear in the systems theory sense, and outflow and loss rates were shown to be nonlinear functions of distance.

DEFINITIONS AND UNITS

Inflow volume, P, is the volume of inflow (acre-feet) at the upstream end of the channel reach, and outflow volume, Q, is the volume of outflow (acre-feet) at the downstream end of the channel reach. Transmission loss volume, P-Q, is the volume of losses (acre-feet) in the reach. The reach length, x, is the length of the channel (miles) between the upstream or inflow station and the downstream or outflow station. Channel width, w, is the average width of the channel (feet) for the reach. Ideally, w is the average width of channel wetted by the flood wave. In actual practice, the average width is the width of the channel between channel banks before "out-of-bank" flow occurs. Bank full discharge or average channel forming discharge can be used to estimate average channel width. Threshold volume, P_o, is the inflow flow volume (acre-feet) required before outflow begins at the downstream station. Threshold volume can be interpreted as an initial abstraction or loss before outflow begins.

DEVELOPMENT

Two simple methods of analysis were used. The first is a linear regression procedure and the second is a simple differential equation expressing the rate of change in outflow volume with distance.

LINEAR REGRESSION PROCEDURE

In this procedure, the volume of outflow is assumed proportional to the volume of inflow (Lane, Diskin, and Renard, 1971):

$$Q = \begin{cases} 0 & , P \leq P_o \\ a + bP & , P > P_o \end{cases} \tag{1}$$

where

Q = outflow volume, acre-ft

P = inflow volume, acre-ft

P_o = threshold inflow volume, initial abstractions, acre-ft

a = intercept, acre-ft, and

b = slope.

We assumed that for an abstracting channel $a \leq 0.0$ and $0.0 \leq b \leq 1.0$ so that the threshold volume is

$$P_o = -a/b \tag{2}$$

If there are n pairs of (P_i, Q_i) data for a reach, then linear regression or least squares analysis can be used to derive estimates of a and b in Eq. 1.

The main disadvantage of the regression procedure described above is that the parameters a and b are unique to the particular reach and data set analyzed. That is, for a given channel reach of length x and width w, if we have values of $a = a(x, w)$ and $b = b(x, w)$, what are the values of a and b for different values of x and w? The traditional approach is to gage a large number of streams, and then try to relate a and b to channel properties, including x and w, to develop "regional" regression equations for a and b. The disadvantages of this procedure are: (1) observed data are required for a large number of channel reaches; (2) with small data sets, spurious correlations are common, and (3) arbitrary limits may be required so that the regional regression equations meet the constraints on a and b.

The proposed alternative to the traditional approach is to construct a model directly incorporating x and w into the outflow-inflow (outflow as a function of inflow) equations. We followed this procedure using a differential equation to describe changes in outflow volume as a function of the rate of change of the inflow.

SIMPLIFIED DIFFERENTIAL EQUATION

Jordan (1977) proposed a simplified relation between inflow volume, P, and losses in a channel reach of length x. He suggested that

$$\frac{dP(x)}{dx} = -kP(x). \tag{3}$$

However, introducing a threshold (Lane, Ferreira, and Shirley, 1979) into Eq. 3 yields:

$$\frac{dP(x)}{dx} = -c - kP(x). \tag{4}$$

A particular solution to Eq. 4 that meets the original assumptions and constraints is

$$Q(x) = (P - P_o(x))e^{-kx} \tag{5}$$

where Q(x) is the outflow volume a distance x downstream from the inflow station, and P is the specified inflow volume. With this notation, Q(x) and $P_o(x)$ are functions of distance, but P is inflow at a fixed location. The threshold in Eq. 4 is $c = -ke^{-kx} P_o(x)$. As in Eq. 1, we can consider $P_o(x)$ as a threshold and write the solution as

$$Q(x) = \begin{cases} 0 & , P \leq P_o(x) \\ (P - P_o(x))e^{-kx} & , P > P_o(x). \end{cases} \tag{6}$$

As Lane et al. (1979) showed, $P_o(x)$ is a function of x, and thus it incorporates distance within the outflow-inflow equation. Assumptions or constraints on Eq. 6 are that $P_o(x) \geq 0$ and $k \geq 0$, so that outflow is less than, or equal to, the inflow.

The reasoning that P_o is a function of x is illustrated in Fig. 1A. For a particular channel of width w_1, an inflow volume P_1, may travel a distance x_1 before it is all lost in the channel. During the time this flood wave travels the distance x_1, it wets a channel area of A_1, resulting in a total loss of $P_o(x_1, w_1) = P_1$. For a larger inflow, P_2, the flood wave may travel a greater distance, x_2, and wet an area, A_2, before it is lost as infiltration into the channel. The total loss is $P_o(x_2, w_1) = P_2$. If it is assumed that the velocity of the wave front is constant and that $x_2 = 2x_1$ and $A_2 = 2A_1$, then the time required for the wave front to reach a distance $x_2(T_2)$ is twice the time required to reach a distance $x_1(T_1)$. With these assumptions, $P_o(x_2,w_1)$ must be greater than $2P_o(x_1, w_1)$, because for x_2 the area (A_1) is wetted for a period $T_1 + T_1$, while the area between x_1 and x_2 is also A_1, and it is wetted for a period $T_2 - T_1$, or T_1, also. If total infiltration is assumed to be proportional to the opportunity time for infiltration to occur (T_1 or T_2), then the total infiltration or loss, P_o is proportional to TA. That is,

$$P_o(x_1, w_1) \propto T_1A_1 \tag{7}$$

and

$$P_o(x_2, w_1) \propto (T_1 + T_1)A_1 + (T_2 - T_1)A_1 \tag{8}$$

which is

$$P_o(x_2, w_1) \propto 2T_1A_1 + T_1A_1 = 3T_1A_1 \tag{9}$$

and meets the assertion that $P_o(x_2, w_1) > 2P_o(x_1,w_1)$, although $x_2 = 2x_1$.

The reasoning that P_o is also a function of w is illustrated in Fig. 1B, and by the logic discussed above. That is, for the same length x_1, the area wetted can also be doubled by doubling the width. If we again assume $T_2 = 2T_1$, then it can be shown (as in Eqs. 7-9) that $P_o(x_1, w_2) > 2P_o(x_1,w_1)$, and P_o becomes a function of both x and w.

These gross assumptions are made as illustrations. However, based on this reasoning and for simplicity, we assume that the losses are a function of x and w,

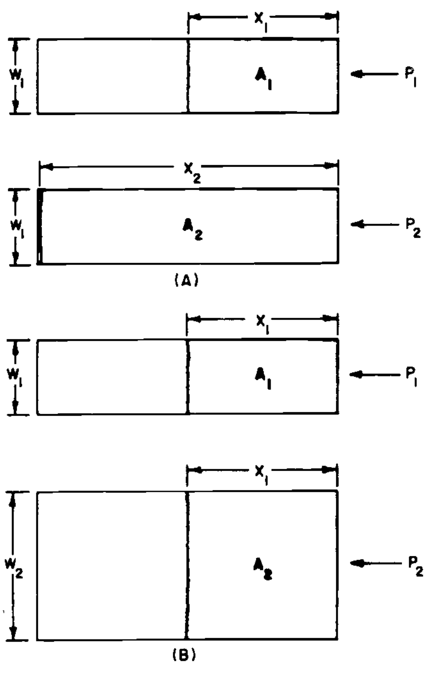

Figure 1. Illustration of increases in channel alluvium wetted with (A) increasing reach length or (B) increasing channel width.

and moreover, symmetric in x and w. Based on this assumption, the simplified model (hereafter the exponential or differential equation model) corresponding to Eq. 6 is:

$$Q(x, w) = \begin{cases} 0 & , P \le P_0(x, w) \\ [P - P_0(x, w)]e^{-k(x, w)}, & P > P_0(x, w). \end{cases} \quad (10)$$

The major disadvantage to the differential equation approach (Eq. 10) is in estimating and interpreting the parameters. We linked the differential equation model to the regression model to use the powerful least squares procedure for estimating parameters.

LINKAGE BETWEEN REGRESSION AND DIFFERENTIAL EQUATION PARAMETERS

The lower portion of Eq. 10 can be expanded as

$$Q(x, w) = -P_0(x, w)e^{-k(x, w)} + Pe^{-k(x, w)}. \quad (11)$$

For fixed x and w, the first term of Eq. 11 is a constant, and the second term depends only on P. Comparing Eq. 11 with the lower portion of Eq. 1, we can write

$$a(x, w) = -P_0(x,w)e^{-k(x, w)} \quad (12)$$

and

$$b(x, w) = e^{-k(x, w)}. \quad (13)$$

Solving Eq. 12 for $P_0(x, w)$, we have

$$P_0(x, w) = \frac{-a(x, w)}{e^{-k(x, w)}} \quad (14)$$

which by Eq. 13 is

$$P_0(x, w) = \frac{-a(x, w)}{b(x, w)}. \quad (15)$$

For fixed values of $x = x_1$ and $w = w_1$, Lane et al. (1979) showed that P_0 for arbitrary x and w is

$$P_0(x, w) = \frac{a(x_1, w_1)}{[1 - b(x_1, w_1)]} [1-e^{+k(x, w)}]. \quad (16)$$

Equations 12-16 provided a means of generalizing a, b, and P_0 for any x and w if k(x, w) could be written as a function of x and w. The final assumption required for linkage was that k(x, w) is the linear function of x and w

$$k(x, w) = xwk \quad (17)$$

where k is a constant for a given channel reach.

UNIT CHANNEL

A unit channel is defined as a uniform channel reach of unit length and unit width. The procedure used in this study was to derive parameters (k, b, a, and P_0) for a unit channel from the corresponding parameters for a channel of fixed length and width. The relationships described by Eqs. 12-17 were then used to derive parameters for any arbitrary x and w. Since the parameters for a unit channel do not involve x or w, they have two advantages. First, they may be characteristic values describing transmission losses for a particular reach. In general, the parameters involve x and w, so mean parameter values cannot be computed for a basin or region from derived parameter estimates for several channel reaches in the basin or region. Second, since the unit channel parameters are independent of x and w, these unit parameters can be averaged (if the infiltration characteristics are fairly uniform) to derive basin or regional parameter values.

The equations to compute unit channel parameters, given parameters for a channel of length x and width w, are summarized in Table 1. Table 2 summarizes the equations for computing parameters for a channel with arbitrary length and width given parameters for a unit channel. The notation used for k and the other parameters in Tables 1 and 2 is that k(x, w) is for length x, and width w, k(1, w) = k(w), k(x, 1) = k(x), and k(1, 1) = k. Notice that the equations are symmetric in x and w so that unit channel parameters in Table 1 can be determined by first eliminating x and then w (as shown), or by first eliminating w and then x (not shown). The same is true in Table 2; we can first introduce w, and then x (as shown), or first introduce x, and then w (not shown).

ANALYSIS AND RESULTS

Data for a number of channel reaches in Arizona, Kansas-Nebraska, and Texas were analyzed using the procedures described above. Data used in this study are summarized in Table 3. The Walnut Gulch data are from our observations; the Queen Creek data are from Babcock and Cushing (1941); the Trinity River data are from Texas Board of Water Engineers (1960), and the Kansas-Nebraska data are from Jordan (1977).

Table 1. Summary of equations for estimating outflow equation parameters for a unit length and unit width channel reach given parameters for a reach of length x and average width w.

Parameter	Values for reach of length x and width w	Values for reach of unit length and width w	Values for reach of unit length and unit width
Decay factor	$*k(x,w)$	$**k(w) = -\frac{\ln b(x,w)}{x}$	$**k = \frac{k(w)}{w}$
Regression slope	$b(x,w)$	$b(w) = e^{-k(w)}$	$b = e^{-k}$
Regression intercept	$a(x,w)$	$a(w) = \frac{[1 - b(w)]a(x,w)}{[1 - b(x,w)]}$	$a = \frac{(1 - b)\ a(w)}{[1 - b(w)]}$ $a = \frac{(1 - b)\ a(x,w)}{[1 - b(x,w)]}$
Threshold volume	$P_0(x,w)$	$P_o(w) = \frac{a}{1-b}\ (1-e^{+k(w)})$ $P_o(w) = \frac{-a(w)}{b(w)}$	$P_o = \frac{a}{1-b}\ (1-e^{+k})$ $P_o = \frac{-a}{b}$

* Basic linkage, $k(x,w) = -\ln b(x,w)$.

**Basic assumptions on changes in k with length and width are $k(x,w) = xk(w)$ and $k(w) = wk$ where k is the decay factor for a unit length and width channel, x is length, and w is width.

Table 2. Summary of equations for estimating outflow equation parameters for a channel of width w and length x, given parameters for a reach of unit length and width.

Parameter	Values for reach of unit length and unit width	Values for reach of width w and unit length	Values for reach of width w and length x
Decay factor	k	$k(w) = wk$	$k(x,w) = xk(w)$ $k(x,w) = xwk$
Regression slope	b	$b(w) = e^{-k(w)}$ $b(w) = e^{-wk}$	$b(x,w) = e^{-k(w)x}$ $b(x,w) = e^{-xwk}$
Regression intercept	a	$a(w) = \frac{[1 - b(w)]a}{(1-b)}$	$a(x,w) = \frac{[1 - b(x,w)]a(w)}{[1-b(w)]}$ $a(x,w) = \frac{[1 - b(x,w)]a}{(1-b)}$
Threshold volume	P_o	$P_o(w) = \frac{a}{1-b}\ (1-e^{+wk})$ $*P_o(w) = \frac{-a(w)}{b(w)}$	$P_o(x,w) = \frac{a(w)}{[1-b(w)]}(1-e^{+wxk})$ $P_o(x,w) = \frac{-a(x,w)}{b(x,w)}$

*This form of the equation for P_0 should be used to minimize round-off errors.

The data shown in Table 3 are not entirely consistent in that the events represent floods of different magnitude. The Walnut Gulch data are from a series of small to moderate events representing in-bank flow, whereas the Queen Creek data are for relatively larger floods and, no doubt, represent some out-of-bank flow. The Trinity River data represent pumping diversions entirely within the channel banks. Data for the Kansas-Nebraska streams represent floods of unknown magnitude, which may include out-of-bank flow.

The hydrologic data summarized in Table 3 were analyzed using linear regression analysis (Eq. 1) to estimate the parameters $a(L_c, w)$, $b(L_c, w)$, $P_0(L_c,w)$, and $k(L_c,w)$, where L_c is the total length of the channel reach in miles, and w is the average channel width in feet. These parameters are summarized in Table 4. Using the procedures outlined in Table 1, parameters were computed for unit length channels, unit width channels, and unit length and width channels (Table 5).

WALNUT GULCH EXPERIMENTAL WATERSHED, ARIZONA

Data from four of the channel reaches on Walnut Gulch were selected for detailed analysis. The unit channel parameters for these four reaches are shown in Table 5. The surface area of the channel available for infiltration is approximately

$$A_c = xw(5280/43560) \qquad (18)$$

Table 3. Summary of hydrologic data used in analysis of transmission loss data.

Location	Reach identification	Length L_c (mi)	Average width w (ft)	Number of events	Inflow volume Mean (acre-ft)	Inflow volume Standard deviation (acre-ft)	Outflow volume Mean (acre-ft)	Outflow volume Standard deviation (acre-ft)
Walnut Gulch, AZ	11-8	4.1	38.	11	16.5	14.4	8.7	11.4
	8-6	0.9	--	3	13.7	----	11.4	----
	8-1	7.8	--	3	16.3	----	1.62	----
	6-2	2.7	107.	30	75.1	121.6	56.9	101.0
	6-1	6.9	121.	19	48.3	51.7	17.1	26.5
	2-1	4.2	132.	32	49.3	42.7	24.4	31.4
Queen Creek, AZ	Upper to Lower Gaging Station	20.0	277.	10	4283.	5150.	2658.	3368.
Elm Fork of Trinity River, TX	Elm Fork-1	9.6	---	3	454.	----	441.	----
	Elm Fork-2	21.3	---	3	441.	----	424.	----
	Elm Fork-3	30.9	120.	3	454.	----	424.	----
Kansas-Nebraska	Prairie Dog Creek	26.0	17.	5	1890.	1325.	1340.	1218.
	Beaver Creek	39.0	14.	7	2201.	2187.	1265.	1422.
	Sappa Creek	35.0	23.	6	6189.	8897.	3851.	7144.
	Smokey Hills River	47.0	72.	4	1217.	663.	648.	451.

Table 4. Summary of regression model and differential equation model parameters for selected channel reaches.

Location	Reach identification	Reach no.	Length L_c (mi)	Average width w (ft)	Regression intercept $a(L_c,w)$ (acre-ft)	Model slope $b(L_c,w)$	Threshold volume $P_o(L_c,w)$ (acre-ft)	Decay factor $k(L_c,w)$	R^2
Walnut Gulch, AZ	11-8	1	4.1	38.	-4.27	0.789	5.41	.2370	.98
	8-6	2	0.9	--	-0.34	0.860	0.40	.1508	.99
	8-1	3	7.8	--	-2.38	0.245	9.71	1.4065	.84
	6-2	4	2.7	107.	-4.92	0.823	5.98	.1948	.98
	6-1	5	6.9	121.	-5.56	0.469	11.86	.7572	.84
	2-1	6	4.2	132.	-8.77	0.673	13.03	.3960	.84
Queen Creek, AZ	Upper to lower station	7	20.0	277.	-117.2	0.648	180.90	.4339	.98
Elm Fork of Trinity River, TX	Elm Fork - 1	8	9.6	--	-15.0	1.004*	----	----	.99
	Elm Fork - 2	9	21.3	--	+7.6*	0.944	----	----	.99
	Elm Fork - 3	10	30.9	120.	-8.7	0.952	9.14	.0492	.99
Kansas-Nebraska	Prairie Dog Creek (PD)	11	26.0	17.	-353.1	0.896	394.10	.1098	.95
	Beaver [illegible] (BC)	12	39.0	14.	-157.3	0.646	243.50	.4370	[illegible]
	[illegible] (SC)	13	35.0	23.	-1076.3	0.796	1352.10	.228[illegible]	[illegible]
	Smokey Hills River (SH)	14	47.0	72.	-99.1	0.614	161.40	.4878	.81

*Channel reaches where derived regression parameters did not satisfy the constraints.

where A_c is the wetted area in acres. The size or scale characteristic of the channel is the length-width product, xw. Values of xw were related to the k values from Table 5, as shown in Fig. 2. The equation

$$k(x, w) = kxw = 0.000850\ xw \quad (19)$$

is a least squares (through the origin) line fitted to the data points shown in Fig. 2.

Given the value of xw for a particular reach, Eq. 19 was used to estimate k(x, w), and this, in turn, was used to estimate b(x, w) as

$$b(x, w) = e^{-k(x, w)} \quad (20)$$

and a(x, w) as

$$a(x, w) = \frac{[1 - b(x, w)]a}{(1 - b)} \quad (21)$$

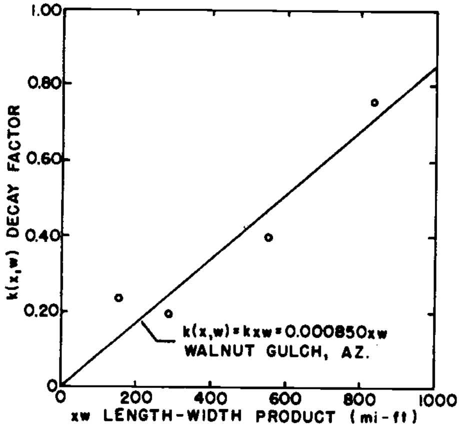

Figure 2. Relation between decay factor, k(x,w), and the channel length-width product for Walnut Gulch, Arizona.

The unit channel regression slope, b, was computed using x = w = 1.0 in Eq. 20, and a was taken as the mean of the a values shown in Table 5. The results of these calculations are shown as the lines in Fig. 3. The points shown in Fig. 3 represent the least squares estimates of a(x, w) and b(x, w), as shown in Table 3. The scatter of the points in Fig. 3 represents residual or unexplained errors in fitting the trend lines. The regression slopes, b(x, w), are a result of the least squares fit of k(x, w), as shown in Fig. 2. However, the a(x, w) trend line resulted from the use of the mean of the a values from Table 5. The scatter shown in Fig. 3A indicated that use of the mean value of a may not result in the optimal fit to the a(x, w) values. At present, however, using the mean value of the individual a values is the recommended procedure.

QUEEN CREEK, ARIZONA

Least squares estimates of $a(L_c, w)$, $h(L_c, w)$, $P_o(L_c, w)$, and $k(L_c, w)$ for Queen Creek are listed in Table 4, and the unit channel parameters are shown in Table 5. For this analysis, data were available for only a single channel reach. Therefore, values of k, b, and a for the single reach were taken as representative for Queen Creek.

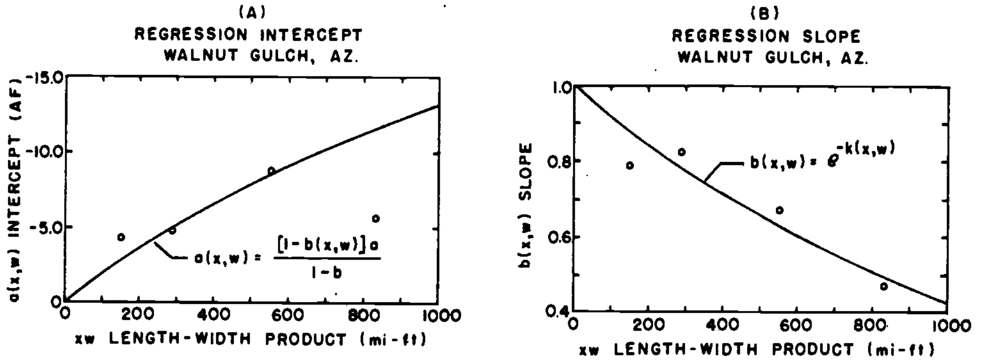

Figure 3. Relations between channel length-width product and (A) regression intercept, and (B) regression slope for Walnut Gulch, Arizona.

ELM FORK OF THE TRINITY RIVER, TEXAS

The least squares estimates of $a(L_c, w)$, $b(L_c, w)$, $P_o(L_c, w)$, and $k(L_c, w)$ for Elm Fork are listed in Table 4, and the unit channel parameters are shown in Table 5. For this analysis, data were available for 3 points on a 30.9-mi reach, and the regression parameters $a(L_c, w)$ and $b(L_c, w)$ met the constraints of $a(L_c, w) \leq 0$ and $0 \leq b(L_c, w) \leq 1$ for the 30.9-mi reach. The value of $k(L_c, w) = 0.0000133$ was about ten times smaller than corresponding values on the other reaches (Table 4).

For the data sets analyzed, the Elm Fork data represent a "minimum loss" case with a relatively small decay factor and correspondingly high regression slope values. For practical flood routing or transmission loss analysis, the loss rates on Elm Fork were negligible, and could be ignored. Nonetheless, the Elm Fork channel reach represented a lower limit case for comparison with other channel reaches.

KANSAS-NEBRASKA STREAM CHANNELS

Data from the 4 channel reaches in Kansas-Nebraska were analyzed to estimate $a(L_c, w)$, $b(L_c, w)$, $P_o(L_c, w)$, and $k(L_c, w)$, as listed in Table 4. The corresponding unit channel parameters are shown in Table 5. There was a relatively large amount of variability in a and k, as shown in the right portion of Table 5.

Table 5. Summary of unit length, unit width, and unit length and width parameters for selected channel reaches.

Location	Reach identification	Unit length parameters			Unit width parameters			Unit length and width parameters			
		$a(w)$	$b(w)$	$P_o(w)$	$a(L_c)$	$b(L_c)$	$P_o(L_c)$	a	b	P_o	k
Walnut Gulch, AZ	11-8	-1.13657	.94384	1.2042	-.12587	.99378	.1267	-.03076	.998480	.0308	.001521
	6-2	-1.93484	.93039	2.0796	-.05059	.99818	.0507	-.01874	.999326	.0187	.000674
	6-1	-1.08819	.89607	1.2144	-.06541	.99376	.0658	-.00950	.999094	.0095	.000907
	2-1	-2.41320	.91002	2.6518	-.08046	.99700	.0807	-.01915	.999286	.0192	.000714
Queen Creek, AZ	Upper to Lower Station	-7.14508	.97854	7.3018	-.52273	.99843	.5236	-.02597	.999922	.0260	.0000783
Trinity River, TX	Elm Fork -3	-.28825	.99841	.2887	-.07427	.99959	.0743	-.002404	.999987	.0024	.0000133
Kansas-Nebraska	Prairie Dog Creek (PC)	-14.30986	.99579	14.3705	-21.86124	.99356	22.0029	-.842008	.999752	.8422	.000248
	Beaver Creek (BC)	-4.95071	.98886	5.0065	-13.65447	.96927	14.0874	-.355480	.999200	.3558	.000800
	Sappa Creek (SC)	-34.28091	.99350	34.5052	-52.07808	.99013	52.5972	-1.493102	.999717	1.4935	.000283
	Smokey Hills River	-2.65060	.98968	2.6782	-1.73337	.99325	1.7451	-.036970	.999856	.0370	.000144

APPLICATION AND EXAMPLES

Application of the procedures can best be seen by considering examples for a typical situation. For the first example, we considered derivation of parameters from observed inflow-outflow data using regression or least squares analysis. In the next example we assumed that these least squares parameters or unit channel parameters were derived and, in turn, were used to derive prediction equations.

EXAMPLE 1. Least Squares Analysis.

Assume the following data in Table 6 are from a channel reach of length L_c = 5.0 mi and average width w = 70 ft, and derive the least squares estimates of $a(L_c, w)$, $b(L_c, w)$, $P_o(L_c, w)$, and $k(L_c, w)$. Using the derived value of $k(L_c, w)$, interpret these transmission loss data relative to the results in Table 4 and Table 5.

Table 6. Hypothetical inflow-outflow data for an ephemeral channel reach

Inflow volume (acre-ft)	10.	20.	25.	15.	100.
Outflow volume (acre-ft)	0.1	6.0	9.0	2.5	75.

Solution: Linear regression analysis of the data in Table 6 produced $a(L_c, w) = -10.38$, and $b(L_c, w) = 0.850$ with $R^2 = .998$. With these values we compute the following:

$$P_o(L_c, w) = -\frac{a(L_c, w)}{b(L_c, w)} = 12.21 \quad (22)$$

$$k(L_c, w) = -\ln[b(L_c, w)] = 0.1625. \quad (23)$$

Therefore, the outflow-inflow equation for the channel reach is

$$Q(L_c, w) = Q(5.0, 70.) = \begin{cases} 0 & , P \leq 12.21 \\ -10.38 + 0.850P & , P > 12.21 . \end{cases} \quad (24)$$

The value of $k(x, w) = kxw = 0.1625$ means that for $xw = 350$, $k = 0.000464$. The value of $k(x, w) = 0.1625$ could be from any of the locations in Table 4 but Elm Fork, and the value of $k = .000464$ could be from Walnut Gulch or Kansas-Nebraska streams (Table 5). Comparison with Fig. 2 shows that this k is smaller than the Walnut Gulch value of 0.000850.

EXAMPLE 2. Estimate Parameters for Arbitrary Channel.

Using the parameter values derived in Example 1 for L_c = 5.0 mi and w = 70 ft, derive the parameters for a channel of arbitrary length x and arbitrary width w.

Solution: First, derive the unit channel parameters using the procedure outlined in Table 1, then derive the parameters for x and w using the procedures outlined in Table 2.

Unit Channel. From equations in Table 1.

$$k = \frac{k(L_c, w)}{L_c w} = \frac{0.162519}{(5)(70)} = 0.000464 \quad (25)$$

$$b = e^{-k} = e^{-.000464} = 0.999536 \quad (26)$$

and

$$a = \frac{(1-b)\, a(L_c, w)}{[1-b(L_c, w)]} = \frac{(1 - 0.999536)(-10.38)}{(1 - .850)} = -0.032125. \quad (27)$$

Arbitrary Channel. From the equations in Table 2.

$$k(x, w) = kxw = 0.000464xw \quad (28)$$

$$b(x, w) = e^{-k(x, w)} = e^{-0.000464xw} \quad (29)$$

and

$$a(x, w) = \frac{[1-b(x, w)]a}{(1-b)} = -68.9849(1-e^{-0.000464xw}). \quad (30)$$

We could evaluate Eqs. 28 - 30 for arbitrary xw to produce graphs as in Figs. 2 and 3. The result would be a graphical procedure to estimate parameters (and thus the outflow-inflow equations) for arbitrary x and w in the hypothetical channel reach.

DISCUSSION AND SUMMARY

Transmission losses are important in determining runoff volumes in streams where runoff is abstracted or lost. A simplified procedure was developed to estimate outflow volumes and, thus, transmission losses in such streams. Transmission loss rates were assumed to vary directly with the surface area wetted by passage of a flood wave through a channel reach. This allowed direct incorporation of channel length and width into parameters of the model. Given parameters for a channel with specified length and width, linkage of the differential equation with the linear regression equation allowed estimation of parameters for channel reaches with arbitrary lengths and widths.

Two examples illustrated a typical application and the power of the simplified procedure to generalize results given a minimum of information on a specific channel reach. The procedure can be easily applied under a variety of circumstances. Moreover, since predicted outflow cannot exceed inflow, the method has reasonable limits.

Additional research is needed to relate model parameters to channel and infiltration characteristics. Since the equations explicitly include channel length and width, scale or size characteristics, except depth or volume of alluvium, are eliminated. Since transmission loss rates are determined by infiltration rates and by opportunity time for infiltration to occur, fruitful areas of additional research should concentrate on physical features of the channels which control infiltration rate and opportunity time. In the first case, particle size data, including median particle size, percent gravel, sand, silt, and clay, should be related to infiltration rate, as should antecedent moisture. Opportunity time should be a function of flow duration data, hydraulic resistance, and channel slope, in addition to infiltration rate. With enough data of this type, it will be possible to estimate model parameters for abstracting channels. In the absence of such analyses, data from Walnut Gulch data might provide upper limits, and data from Elm Fork of the Trinity River might provide lower limits for parameters. Application of the equations with data from these two locations should produce upper and lower limits on expected transmission losses.

List of Symbols

Symbol	Definition	Units
a	Regression intercept in linear rainfall/runoff relationship. Also denoted a(x,w), a(x), or a(w), depending upon which independent variables are held constant.[1]	acre-ft
A_c	Area of channel alluvium wetted by the passage of a flood wave. Also A_1 or A_2.	acre
b	Regression slope in linear rainfall/runoff relationship. Also b(x,w), b(x), or b(w).	dimensionless
c	Arbitrary constant in differential equation.	acre-ft/mi
k	Transmission loss decay factor. Measure of the rate of transmission losses with channel area wetted. Also k(x,w), k(x), or k(w).	$(\text{ft-mi})^{-1}$

L_c	Length of channel reach. L_c is usually the length of a particular reach, while x is a variable distance within the reach.	mi
n	Number of measured (P,Q) data pairs for a reach.	dimensionless
P	Volume of inflow to a channel reach. Also P_1 or P_2.	acre-ft
P_o	Threshold volume or initial abstraction. Volume of inflow required before outflow begins. Also $P_o(x,w)$ or $P_o(x)$.	acre-ft
Q	Volume of outflow from a channel reach. Also Q(x,w), Q(x), or Q(w).	acre-ft
T	Travel time; opportunity time. Time it takes a flood wave to travel a channel reach distance x(T). Also T_1 or T_2.	hr
w	Average width of a channel reach. Also w_1 or w_2.	ft
x	Variable distance in a channel. Also x_1 or x_2.	mi

[1]The notation adopted here, a(x,w), refers to a as a function of x and w; a(x, 1) = a(x); a(1,w) = a(w); and a(1,1) = a. Therefore, a(x,w), b(x,w), etc., refer to values for a channel reach of length x and width w, while a, b, etc., refer to corresponding values for a unit channel (i.e., x = w = 1).

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