RANKING ALTERNATIVE PLANS FOR THE SANTA CRUZ RIVER BASIN BY Q-ANALYSIS

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ABSTRACT

This paper introduces an intuitive, multicriterion decision making aid utilizing Q-analysis, a technique based in algebraic topology and set theory. This aid ranks twenty-five alternative plans for the water resource management and flood control of the Santa Cruz River Basin. These twenty-five plans have been described in terms of thirteen weighted criteria. Q-analysis is used to investigate a series of binary matrices formed over a range of threshold levels (TLs), indicating different levels at which the plans satisfy the criteria. A computer package performs both Q-analysis and slicing over the TL range. A short discussion concerning additional information that can be drawn from the multicriterion Q-analysis has also been included.

INTRODUCTION

The purpose of this paper is (1) to rank alternative development plans for the Santa Cruz River Basin via multicriterion polyhedral dynamics (MCPD), and (2) to assess whether or not results from MCPD and other multicriterion decision making (MCDM) aids are comparable.

The Santa Cruz River Basin encompasses the semi-arid region in which Tucson is located. This area is subject to a growing population that levies heavier and heavier demands on flood protection, recreational opportunities, and water supply. A single objective approach has been the standard means of solving this type of problem in the past. The main disadvantage of such an approach is that each objective is treated separately from the others (McAniff et al., 1980). A "cost-effectiveness" formulation places the problem into an MCDM perspective and facilitates an assessment of impacts of the alternatives with respect to all the objectives (Popovich et al., 1973; David and Duckstein, 1976; Gershon et al., 1980). A version of this approach is presented in the methodology section with emphasis on use of MCPD as a ranking technique.

MCPD is an application of polyhedral dynamics, also known as Q-analysis, a method based on algebraic topology. An interactive computer package, named the Polyhedral Pack (Pfaff et al., 1981), which has been designed for general applications of polyhedral dynamics, is utilized for this analysis. Q-analysis has previously been proposed as an MCDM aid (Kempf et al., 1979; Armijo et al., 1979; Armijo and Pfaff, 1980; Shea, 1981), and is only one of many tools that can be applied. Just a few of the other techniques include the surrogate worth trade-off method, multi-attribute utility theory (MAUT) (Keeney and Raiffa, 1976), multi-objective simplex (Benayoun et al., 1972), ELECTRE I and II (Benayoun et al., 1966; Roy, 1971; Gershon et al., 1980, 1981), and compromise programming.

A stepwise presentation of the methodology is followed by a description of the case study and an analysis of the results. Then a brief discussion is given and conclusions are drawn.

METHODOLOGY

A detailed explanation of the methodology is available in Armijo and Pfaff (1980), so that only a summary of the application steps is given below:

1. Define the problem in terms of a decision impact matrix
2. Transform the decision impact matrix (data preparation)
3. Perform Q-analysis on the transformed decision matrix, D'
   a. Slice D' into a series of binary incidence matrices using the set of slicing parameters
      i. Define the slicing parameter increment
      ii. Define the maximum slicing parameter (the maximum slicing parameter can either be defined by the decision maker (DM) or automatically set to the maximum value of D')

        iii. Define the set of slicing parameters using the above increment and maximum value
    b. Analyze each binary incidence matrix as a separate complex
        i. Investigate the primal simplicial complex
        ii. Investigate the conjugate simplicial complex
    c. Perform a global analysis of all the complexes
4. Consult the DM and perform sensitivity analyses if desired

Steps 2 and 3, which correspond to the analysis stage of the cost effectiveness approach, can be performed by the Polyhedral Pack.

In the next section, these steps are applied to the Santa Cruz River Basin example.

CASE STUDY OF THE SANTA CRUZ RIVER BASIN

OVERVIEW

As in many semi-arid communities, water supplied by imports and natural recharge fall short of demand, while flooding remains an impending threat. In 1975, the depletion-supply ratio was 3.3 to 1, the three primary users being agriculture (41%), municipalities (29%), and mining (27%). A projected decline in agricultural use over the next 30 years will be overshadowed by urban and mining sectors, especially since population in the greater Tucson area is growing rapidly. The water resources manager must therefore grapple with the conflicting goals of providing sufficient water to competing users and controlling floods (Gershon et al., 1980).

The problem has been formulated through interaction with Tucson Urban Study - Corps of Engineers group (Corps of Engineers, 1978). Twenty-five alternative plans have been generated by pairing five flood-control actions with each of five water supply actions. These alternatives have been evaluated against thirteen qualitative and quantitative criteria and then ranked by the multicriterion technique ELECTRE in Gershon et al., (1980). Solutions have also been offered via compromise programming and MAUT (McAniff et al., 1980; Duckstein et al., 1981). This problem, with solutions obtained by applying established MCDM techniques is now taken as an example for testing MCPD. It should be stressed, however, that the purpose of this paper is only to present briefly the methodology and highlight comparability of MCPD's results with those of other MCDM techniques. Other related aspects, such as model choice, are handled in McAniff et al. (1980), Duckstein et al. (1981), Gershon et al. (1981), and are beyond the scope of this paper.

Next, after defining the problem and discussing data preparation, MCPD is applied and the results are compared to those obtained from other MCDM techniques.

PROBLEM DEFINITION

In this section, alternative plans, criteria, and weights are briefly defined, leading to an impact matrix D (Gershon et al., 1980).

Alternative plans or systems as generated through consultation with the Tucson Urban Study - Corps of Engineers group (Corps of Engineers, 1978), five actions for flood control and five actions for water supply have been defined. Table 1 gives the agreed-upon combinations yielding the 25 alternative plans used in this analysis.

Table 1. Alternative Plans

ALTERNATIVES	LEVEE CONSTRUCTION	FLOOD CONTROL CHANNELIZATION	RESERVOIRS AND DAMS	FLOOD PLAIN MANAGEMENT	NO ACTION
Waste water reclamation	1	2	3	4	5
Groundwater development	6	7	8	9	10
Central Arizona Project	11	12	13	14	15
Conservation and education	16	17	18	19	20
No action	21	22	23	24	25

CRITERIA

Denoting by (Q) the quantified criteria, thirteen criteria used herein are: aquifer level (Q), urban water quality, agricultural water quality, expected flood losses (Q), expected flood frequency (Q), preservation of designated areas, effect on wildlife and vegetation, implementation costs (Q), operation and maintenance costs (Q), natural resources, preservation of existing facilities, and creation of new opportunity.

WEIGHTS

Two sets of weights are used in the analysis:

{W1} = {all 10} on (0,10)
{W2} = {9,3,3,4,5,5,5,2,2,2,2,1.5, 1.5} on (0,10)

The weights in {W2} correspond respectively to the previously defined criteria.

DECISION IMPACT MATRIX

The decision impact matrix D is found in Table 2, and is defined such that each entry represents the rating of project j with respect to criterion i.

Table 2. Impact Matrix D

CRITERIA	W	1	2	3	4	5	6	7	8	9	10	11	12
Aquifer Level ft/yr	9	2.7	1.6	2.0	2.4	3.5	2.7	1.6	2.0	2.4	3.5	2.7	1.6
Water Quality Urban	3	e	a	d	a	b	e	a	d	a	b	e	a
Water Quality Agric.	3	a	b	b	b	b	a	b	b	b	b	a	b
Expect. Flood Losses	4	7.72	7.72	0	19.45	26.33	7.72	7.72	0	19.45	26.33	7.72	7.72
Expect. Freq. per yr	5	.01	.01	.003	.02	.04	.01	.01	.003	.02	.04	.01	.01
Pres. Desig. Areas	5	d	d	c	a	a	d	d	c	a	a	d	d
Effect on Wild. Veg.	5	c	b	d	a	a	d	c	e	c	c	c	c
Implem. Costs $M	2	12.7	16.8	12.3	1.9	0.2	32.5	36.6	32.1	21.8	20.	28.1	32.2
O & M Costs $M	2	37.6	37.8	38.2	37.2	37.0	2.6	2.8	3.2	2.2	2.0	2.2	2.4
Indirect Costs	2	c	c	b	d	c	d	d	d	e	e	b	b
Natural Resource	2	c	c	c	b	a	d	d	d	c	c	e	e
Pres. of Exist. Fac.	1.5	c	b	c	b	b	c	b	c	a	a	c	b
Creation New Oppr.	1.5	b	b	a	a	c	d	d	b	c	e	d	d

Table 2 (Continued)

13	14	15	16	17	18	19	20	21	22	23	24	
2.0	2.4	3.5	2.7	1.6	2.0	2.4	3.5	2.7	1.6	2.0	2.4	2.5
d	a	b	e	a	d	a	b	e	a	d	a	b
b	b	b	a	b	b	b	b	a	b	b	b	b
0	19.45	26.33	7.72	7.72	0	19.45	26.33	7.72	7.72	0	19.45	26.33
.003	.02	.04	.01	.01	.003	.02	.04	.01	.01	.003	.02	.04
c	a	a	d	d	c	a	a	d	d	c	a	a
d	d	b	c	c	d	b	b	c	c	d	b	b
27.6	17.3	15.6	12.6	16.7	12.2	1.8	.01	12.5	16.6	12.1	1.8	0
2.8	1.8	1.6	1.1	1.3	1.7	.6	.5	.6	.8	1.2	.2	0
a	c	b	c	c	b	d	d	c	c	c	e	d
e	d	c	c	b	e	a	a	c	b	c	b	a
c	b	b	c	b	c	a	a	c	b	c	b	b
b	c	e	d	d	b	c	e	d	d	b	c	e

DATA PREPARATION AND ANALYSIS

The Polyhedral Pack transforms matrix D into D' whose values represent the degree to which each alternative plan satisfies the thirteen criteria. Using the set of slicing parameters defined below, a separate analysis for each D' is made.

Using a slicing parameter increment of 5 let the set of slicing parameters be

{5,10,15,...,100} for equal weights, {W1}, and
{5,10,15,...,90} for unequal weights, {W2}.

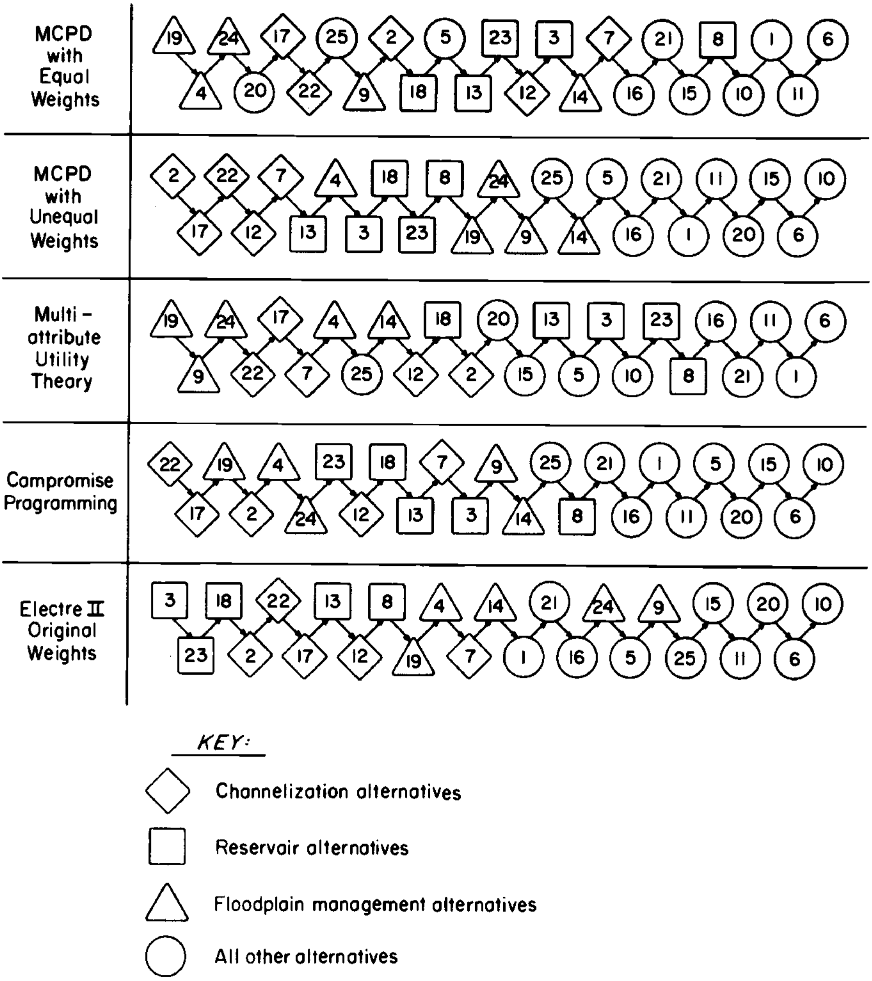
The structure of the problem, that is, the relationship between alternatives and criteria, could be inferred from an internal analysis of each slice; however, no systematic means for condensing information obtained from a series of separate complexes has been developed. Thus, the alternatives are ranked in terms of their expected "centricality" index, which chooses the most desirable alternative as the one satisfying the greatest proportion of criteria.

RANKING ALTERNATIVE PLANS AND COMPARING RESULTS

Comparison of the results of MCPD, ELECTRE, compromise programming, and MAUT are presented in Table 3. All methods lead to a grouping of the alternatives according to flood control actions. In the last three rows in Table 3, that is, ranking by compromise programming, MAUT, and ELECTRE II, the three dominant flood control actions are characterized by dam and reservoir alternatives, flood plain management alternatives, and channelization alternatives. MCPD agrees with this analysis, which Gershon et al. (1981) discuss in greater detail. The approach with equal weights yields top ranking for the alternatives with flood plain management and channelization. In the non-equal weight case, channelization alternatives constitute the dominant group while the alternatives that use dams and reservoirs form the next preferred group. This comparability of results seems to indicate that MCPD may be as acceptable as the more established MCDM techniques.

In addition to this comparability, the two MCPD applications with different weight sets lead to differing results. The method, in this instance, appears to be sensitive to such a substantial change in weights.

Table 3. Results of MCPD and Comparison with Three Other Techniques



DISCUSSION AND CONCLUSIONS

An advantage of MCPD over other MCDM techniques is that it opens the possibility for investigating the problem structure. All other ranking techniques give relatively little rationale to explain those rankings. The structure of a problem, that is, the relationship between alternatives and criteria, in turn, could provide a perspective on the reason behind the rankings. This insight can lead to a systematic elimination of superfluous plans and criteria. In addition, a study of the structure can aid in confirming the correctness of the problem formulation. For instance, if two plans are drastically different from one another with respect to descriptive criteria, then they satisfy almost disjoint subsets of criteria. This condition can easily be detected through a structural analysis.

It should be stressed that the above type of analysis requires further development. The theoretical relationships are well defined, but a systematic approach to a complete interpretation of the results is not yet available. For example, noncomparability between alternatives may be analyzed by counting and identifying which criteria are not satisfied in common. It would seem appropriate to include such a measure of discordance in the ranking procedure. This predicament leaves room for further research in the application and extension of MCPD to new and more complex MCDM problems.

The concluding remarks may be summarized as follows: (1) MCPD is a reasonable MCDM aid; (2) MCPD's results are comparable to those of MAUT, ELECTRE, and compromise programming; and (3) MCPD offers the possibility of augmenting rankings with an in-depth investigation of the structural relationship between alternatives and criteria.

ACKNOWLEDGEMENT

This research has been supported in part through funding provided by the Engineering Experiment Station of the University of Arizona.

REFERENCES

Armijo, R., J. Casti and L. Duckstein. 1979. Multicriterion water resources system design by Q-analysis. Paper presented, Joint Nat'l. TIMS/ORSA Mtg., April 30-May 2, New Orleans. Available as paper #78-24, Department of Systems & Industrial Engineering, University of Arizona, Tucson, AZ 85721.

Armijo, R. and R. Pfaff. 1980. Polyhedral structural modeling as an aid in systems planning. Working paper #80-7, Department of Systems & Industrial Engineering, Univ. of Arizona, Tucson, AZ 85721.

Benayoun, R., O. Larichev, J. deMontgolfier and J. Tergny. 1972. Linear programming with multiple objective functions, the method of constraints. Automation and Remote Control, January.

Benayoun, R., B. Roy and B. Sussman. 1966. ELECTRE: Une methode pour guider le choix en presence de points de vue multiples. Direction Scientifique, Note de Travail No. 49, SEMA, Paris.

Corps of Engineers. 1978. Urban Study for the Metropolitan Area of Tucson, Arizona, No. 12192, Tucson, Arizona, December.

David, L. and L. Duckstein. 1976. Multi-criterion ranking of alternative long-range water resources systems. Water Resources Bulletin, 12(4), August, pp. 731-754.

Duckstein, L., M. Gershon and R. McAniff. 1981. Model selection in multiobjective decision making for river basin planning. To appear, Advances in Water Resources.

Gershon, M., R. McAniff and L. Duckstein. 1980. A multi-objective approach to river basin planning. Proceedings, 1980 Mtgs. AZ Sect., AWRA & Hydrology Sect., AZ-Nevada Academy of Sciences, April, Las Vegas, pp. 41-50.

Gershon, M., L. Duckstein and R. McAniff. 1981. Multiobjective river basin planning with qualitative criteria. To appear, Water Resources Research.

Keeney, R. and H. Raiffa. 1976. Decisions with Multiple Objectives: Preferences and Value Tradeoffs. John Wiley & Sons, New York, 569 p.

Kempf, J., L. Duckstein and J. Casti. 1979. Polyhedral dynamics and fuzzy sets as a multi-objective decision-making aid. Paper presented, Joint Nat'l. TIMS/ORSA Mtg., April 30-May 2, New Orleans. Available as paper #78-25, Department of Systems & Industrial Engineering, University of Arizona, Tucson, Arizona 85721.

McAniff, R., L. Duckstein and M. Gershon. 1980. Model selection in multiobjective decision making. Paper presented, Joint Nat'l. TIMS/ORSA Mtg., Nov. 10-12, Colorado Springs. Available as paper #80-23, Department of Systems & Industrial Engineering, University of Arizona, Tucson, AZ 85721.

Pfaff, R., R. Armijo and L. Duckstein. 1981. Polyhedral Pack, a computer program to do Q-analysis. Available from the Department of Systems & Industrial Engineering, Univ. of Arizona, Tucson, AZ 85721.

Popovich, M., L. Duckstein and C. Kisiel. 1973. A cost-effectiveness analysis of municipal refuse disposal systems. J. of the Environmental Eng. Div., ASCE, 99, October, pp. 577-591.

Roy, B. 1971. Problems and methods with multiple objective functions. Mathematical Programming, 1(2), pp. 239-268.

Shea, R. 1981. A handbook of polyhedral dynamics. Master's Report, Department of Systems & Industrial Engineering, University of Arizona, Tucson, AZ 85721.