

A RPN PROGRAM FOR THE GENERALIZED PENMAN EQUATION

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Abstract

The Penman potential evapotranspiration model has been expanded and generalized for a wide range of climatic conditions by J. Doorenbos and W. O. Pruitt (1975, FAO Irrig. & Drain. Pap. 24). Their comprehensive model relies heavily on tabulations and graphs to facilitate its application in remote areas where computer facilities are lacking. Future applications of this model have been substantially enhanced by our development of a program that runs on a HP-41C/CV calculator. The RPN program occupies 1778 bytes of memory, and will run on a 41CV, or a 41C with a quad memory module. A printer is also needed. The required climatic data can be input in a variety of forms, and program execution was planned with particular attention to the non-technical user. The entire calculator system (41CV, printer, card reader) cost less than \$750, and provides for very portable use with a high degree of computing power.

Introduction

As agricultural water resources become increasingly depleted the need for efficient utilization of irrigation water increases. Potential evapotranspiration predictions have become a vital aid for optimizing irrigation programming. Although extensive research has been achieved on all levels of predicting potential evapotranspiration (PET), a gap exists in relaying this technology between the scientific and agricultural communities.

In recent years large agricultural interests have begun to employ computers to access irrigation requirements. Their use has proven cost-effective in optimizing water input - crop yield relationships. Unfortunately, this computer-based technology has not yet become reasonably accessible to the small-scale agriculturist. Small calculators provide an alternative, however.

Hand-held programmable calculators have existed for nearly 10 years. Early models were not capable of supporting software proficient in predicting PET. As with computers, programmable calculator design technology has improved a thousand-fold in the past decade. High-level scientific calculations and impressive data storage capabilities are now available in shirt-pocket size units, and presently only a slim distinction exists between programmable calculators and computers. The one indisputable difference is the portability of hand-held programmable calculators.

With the introduction of the HP-41C calculator (Hewlett-Packard Calculator Div., Corvallis, OR) in 1979, extensive PET prediction capabilities have become accessible to the small-scale agricultural community. The 41C is expandable through a wide range of peripheral devices to suit many calculation and data storage needs. Additionally, the 41C is now capable of interfacing with "real-world" analog signals through a new digital multimeter.

The 41C is the controller of an interactive array of peripheral devices collectively referred to as HP-IL (Hewlett-Packard Interface Loop). These peripheral devices include mass storage tape drives, printer-plotters, digital multimeters and relays. As with computers, the calculator communicates with the user in words as well as numbers. Like all Hewlett-Packard calculators, the 41C utilizes a Reverse Polish Notation (RPN) logic system that is extremely "byte" efficient and user friendly.

A RPN program has been developed for the 41C that utilizes the Penman method for determining PET predictions, based upon Doorenbos and Pruitt's (1975) comprehensive version of the Penman method, which they generalized for a wide range of climatic conditions.

The model occupies 1778 bytes of program memory, with an additional data storage requirement totalling 140 bytes. The program requires a HP-41CV calculator, or a HP-41C calculator with a quad memory module. A peripheral printer is also required, and a magnetic card reader or digital cassette drive is helpful.

The program was designed with particular attention towards the non-technical user. It reduces the complicated calculation procedures that were previously required, down to a simple data input procedure. The needed climatic data can be input in a variety of forms. Furthermore, the program is flexible in the type of units of the input data.

The Generalized Penman Model

The Penman combination model for evaporation (Penman, 1948) has been widely used for evapotranspiration estimates. The model is a simplification of the energy exchange processes at the evaporating surface, yet its fundamental basis has contributed to a general acceptance of results obtained over a wide range of conditions. Doorenbos and Pruitt (1975) have generalized the Penman equation to extend its application to an even wider range of environments. In doing so they have compiled a useful assortment of graphical and tabular functions that make the application simpler and less laborious. The RPN program in this paper includes algorithms for these functions, and provides a further reduction in the effort required for application.

The basic model, taken from Doorenbos and Pruitt (1975, page 29), is:

$$ET_o^* = W R_n + (1-W)f(u)(ea-ed) \quad (\text{not adjusted}) \quad (1)$$

radiation aerodynamic
term term

where: ET_o^* is the reference crop evapotranspiration in mm/day (not adjusted);

W is a temperature-related weighting factor;

R_n is net radiation in equivalent evaporation in mm/day;

$f(u)$ is a wind-related function; and

$(ea-ed)$ is the difference between the saturation vapor pressure at mean air temperature and the mean actual vapor pressure of the air, both in mbar.

To find ET_o , the reference crop evapotranspiration, ET_o^* needs to be adjusted for day and night-time weather conditions.

Components of the RPN Program

The computations with this model are often rather tedious, because the wide range of acceptable data may be in many different forms and limits. The programmable calculator reduces this effort significantly. The components of the Penman model will be described separately and the algorithms identified.

Vapor Pressure

The moisture content of the air appears in various forms, and may be expressed in many different sets of units. The model uses millibars for units in calculating the vapor pressure deficit of the air ($ea-ed$), but relative humidity, dewpoint, or dry- and wetbulb temperatures can also be input into the program. The model calculations are for daylong averages of $ea-ed$. In practice, this can be obtained from an average of the maximum and minimum values. Even a single measurement of actual vapor content (ed) can give a reasonable estimate of the mean daily ed for use in the program.

The saturation vapor pressure (es , mb) at mean air temperature ($t_{mean} = (t_{max} - t_{min})/2$) is calculated in the program from Murray's (1967) approximation:

$$ea = A \exp [B t_{mean}/(c + t_{mean})] \quad (2)$$

for $t_{mean} \geq 0^\circ\text{C}$, and where $A = 6.1078$ mb, $B = 17.269/^\circ\text{C}$ and $c = 237.3^\circ\text{C}$.

Observed vapor pressure (ed) data may be available, in which case subtraction yields the desired ($ea-ed$).

Relative Humidity Input. The ed is estimated from mean relative humidity ($RH_{mean} = (RH_{max} + RH_{min})/2$) from the relation $ed = RH \ ea/100$, where ea is calculated from Murray's formula (Equation 2) at t_{mean} .

Example: given $t_{max} = 35^\circ\text{C}$, $t_{min} = 22^\circ\text{C}$, $RH_{max} = 80\%$, $RH_{min} = 30\%$. The program calculates $ea = 38.91$ mb at $t_{mean} = 28.5^\circ\text{C}$, $ed = 21.40$ mb at $RH_{mean} = 55\%$ and $ea-ed = 17.51$ mb.

Dewpoint Input. In this case, the ed is estimated directly from mean dewpoint temperature (t_{dew} , $^\circ\text{C}$) with Murray's approximation (Equation 2), and ea is again calculated with Murray's approximation using t_{mean} .

Example: given $t_{max} = 35^{\circ}\text{C}$, $t_{min} = 22^{\circ}\text{C}$, $t_{dew} = 17.5^{\circ}\text{C}$. The program calculates $e_a = 38.91$ mb at $t_{mean} = 28.50^{\circ}\text{C}$, $e_d = 20.00$ mb at $t_{dew} = 17.5^{\circ}\text{C}$, and $e_a - e_d = 18.91$ mb.

Psychrometric Inputs. The program uses the general form of the aspirated (ventilated) psychrometric equation (see, for example, Fritschen and Gay, 1979) to calculate

$$e_d = e_s - \gamma (t_{dry} - t_{wet}) \quad (3)$$

where $e_s(\text{mb})$ is saturation vapor pressure at $t_{wet} (^{\circ}\text{C})$ from Equation (2), γ is the psychrometric constant ($\gamma = 0.66 P/P_o$, $\text{mb}/^{\circ}\text{C}$, and P/P_o is the ratio of actual atmospheric pressure to that at sea level where $P_o = 1013.25$ mb), and $t_{dry} - t_{wet} (^{\circ}\text{C})$ is the wetbulb depression. Actual atmospheric pressure is not normally available, so the program estimates the ratio P/P_o from the International Standard Atmosphere (List, 1949):

$$P/P_o = ((288 - 0.0065Z)/288)^{5.256} \quad (4)$$

where Z is elevation above sea level, in meters.

Example: given $t_{max} = 35^{\circ}\text{C}$, $t_{min} = 22^{\circ}\text{C}$, and a midmorning observation of $t_{dry} = 25^{\circ}\text{C}$ and $t_{wet} = 20^{\circ}\text{C}$, at an elevation $Z = 0$ m (sea level). The program calculates $e_d = 20.08$ mb, $e_a = 38.91$ mb at $t_{mean} = 28.5^{\circ}\text{C}$, and $e_a - e_d = 18.83$ mb.

Wind Function

The wind function $f(u)$ in Equation 1 expresses the mixing power of the atmosphere, as a function of daily wind travel measured at a height of 2m (U_2 , km/day). The program uses the formula of Doorenbos and Pruitt (1975):

$$f(u) = 0.27 (1 + U_2/100). \quad (5)$$

The program corrects wind measured at other than 2 m height with an empirical approximation:

$$U_2 = UZ / (0.1877 \ln(Z) + 0.87025) \quad (6)$$

where UZ is the wind run (km/day) measured at height Z (m) above the surface.

Example: given that wind run at 3 m height is 250 km/day. While Equation (6) calculates $U_2 = 232.24$ km/day, the program rounds to 2 decimal places to match Doorenbos and Pruitt's (1975) table, and yields $U_2 = 232.50$ km/day and $f(u) = 0.90$.

Weighting Factor

The weighting factor is Doorenbos and Pruitt's (1975) simplification of variables in Penman's original derivation (i.e., $W = \Delta / (\Delta + \gamma)$, where Δ is slope of saturation vapor curve, de_a/dt ($\text{mb}/^{\circ}\text{C}$) at t_{mean} , and γ is the psychrometric constant which was defined earlier). The weighting factor W depends upon temperature and pressure. Δ is calculated in the program by differentiating Murray's approximation for e_a and t_{mean} , and incorporating the pressure correction for γ . The pressure ratio (P/P_o) is estimated in the program by the International Standard Atmosphere given earlier in Equation (4).

The program calculations are:

$$W = [e_a(B_c)/(c + t)^2] / [(e_a(B_c)/(c + t)^2) + 0.66(288 - 0.0065Z)/288)^{5.256}] \quad (7)$$

where e_a is calculated at $t = t_{mean}$ with Equation 2, the constants B_c are defined in Equation 2, and Z is elevation above sea level in meters.

Example: given $t_{max} = 35^{\circ}\text{C}$, $t_{min} = 22^{\circ}\text{C}$, elevation = 95 m. The program calculates $W = 0.78$, $1 - W = 0.22$. Doorenbos and Pruitt (1975) show $W = 0.77$ for these data; the difference is probably due to rounding.

Net Radiation

The radiation term may lead to some tedious calculations, because the necessary data are often lacking and one must resort to various estimation approaches. If net radiation (R_n) data is available, then the Penman model can be applied quite easily. If not, then estimation procedures usually begin with an evaluation of net solar radiation (R_{ns}), based upon extraterrestrial solar radiation (R_a), estimates of cloud cover from climatological observations, and an estimate of crop albedo. The net longwave exchange (R_{nl}) is then estimated from meteorological characteristics, and the two estimates are combined to yield net radiation:

$$R_n = R_{ns} - R_{nl} \quad (8)$$

Note that R_n is shown as the difference between R_{ns} and R_{nl} in Equation (8), since daily R_{nl} is usually a negative quantity in the crop environment.

Solar Declination. If solar radiation (R_s) data is not available, it can be estimated from extraterrestrial solar radiation (R_a), as a function of latitude and time of year. This requires solar declination (DEC). The program calculates DEC^(D) as follows (adapted from Ball, 1978, p. 253):

$$X = [0.9856(DOY + 0.5)] - [0.2458333((YR \text{ Mod } 4) - 1) - 15.785 + .0095 YR] \quad (9)$$

$$Y = (0.02 \sin 2X + 1.916 \sin X + X) - [(-47/2750)YR + 111.24] \quad (10)$$

$$DEC = \arcsin(\sin Y \sin 23.42) \quad (11)$$

where DOY is day of year (1 to 366) and YR is calendar year (e.g., 1982). The modulo function returns the remainder of YR divided by 4 (i.e., 1, 2, 3, or 0). When YR Mod 4 = 0, the program uses 4 instead of 0 in Equation (9). The program uses an algorithm from Ball (1978, p. 241) to calculate day of year from calendar date.

Example: given the date of July 20, 1972 (DOY = 202). The program calculates DEC = 20.58° at solar noon.

Extraterrestrial Solar Radiation. Given declination (DEC) and latitude (LAT), extraterrestrial solar radiation (R_a , in mm of evaporation) is calculated from known solar geometry as follows:

$$S = \sin \text{LAT} \sin \text{DEC} \quad (12)$$

$$C = \cos \text{LAT} \cos \text{DEC} \quad (13)$$

$$N = (\arcsin(-S/C))/7.5 \quad (14)$$

$$V = N/24 \quad (15)$$

$$R_a = (48/\pi)[VS + C \sin(57.29578V)] \quad (16)$$

Example: at 30°N, on July 20, 1972, $R_a = 16.9$ mm/day.

Solar Radiation. Solar radiation (R_s) is estimated from the relation

$$R_s = (0.25 + 0.50 n/N) R_a \quad (17)$$

where n is observed number of hours of actual bright sunshine and N is maximum possible number of bright sunshine hours. The value of N for a given date and latitude is given in Equation (14).

Net solar radiation (R_{ns}) is then obtained by the simple relation

$$R_{ns} = (1 - \alpha)R_s \quad (18)$$

where α is the albedo of the evaporating surface. α is about 0.25 for green crops.

Example: given observed $n = 11.5$ hrs on July 20, 1972, over a crop with albedo of 0.25. The program calculates $R_s = 8.5$ mm/day, $N = 13.67$ hrs, and $R_{ns} = 8.5$ mm/day.

Longwave Radiation. The next longwave radiation (R_{nl} , in mm of evaporation) is obtained in the program as a function of air temperature, vapor pressure and sunshine, using the simple formulation from Doorenbos and Pruitt (1975):

$$R_{nl} = \sigma T_k^4 (0.1 + 0.9 n/N) f(ed) \quad (19)$$

where σ is 2×10^{-9} mm/day^{oK}⁴, T_k is mean daily air temperature in °K ($T_k = t(^{\circ}\text{C}) + 273.16$), n/N is ratio of observed hours to possible hours of bright sunshine, and $f(ed)$ is $0.56 - 0.079/\overline{ed}$ in humid climates, and $0.34 - 0.044/\overline{ed}$ in dry climates.

Example: given $t_{mean} = 28.5$ °C, $ed = 21.4$ mb, $n/N = 11.5/13.67$, and a dry climate. $R_{nl} = 1.94$ mm/day.

Net Radiation. Finally, the desired net radiation is obtained from Equation (8). For this example, $R_{ns} = 8.5$ mm/day, and $R_{nl} = 1.94$ mm/day, so $R_n = 6.6$ mm/day.

Application of the Model

The program output must be adjusted to account for failure of the aerodynamic portion of the Penman model to fully describe the energy exchange processes.

Unadjusted Reference Crop Evapotranspiration

The initial application of the model yields unadjusted reference crop evapotranspiration (ET₀*). This is illustrated with the following example from Doorenbos and Pruitt (1975), portions of which have already been used in this paper. Location is Cairo, at latitude 30°N and elevation 95 m. Data is for July 1972. Solar radiation on the date July 20 is taken to represent the mean for the entire month. Other data for July are: t_{mean} = 28.5°C, RH_{mean} = 55%, sunshine n = 11.5 hr., U₂ = 232 km/day, and α = 0.25.

The program calculates $(1 - W)f(u)(ea - ed) = 3.74$ mm/day, and $W R_n = 5.14$ mm/day, so $ET_0^* = 8.88$ mm/day.

Adjusting for Climatic Characteristics

Doorenbos and Pruitt (1975) surveyed many data and concluded that the unadjusted values need little correction when daytime wind speeds are about twice those during night-time, and RH maximums are > 60%. They identified six other climatic regimes, however, when ET₀* may differ substantially from ET₀ (0.3 ET₀* < ET₀ < 1.2 ET₀*). Their classification follows:

Case #1: ET₀ ~ 1.2 ET₀*. Important in areas with moderate to high radiation (>8 mm/day) during summer months, which consistently have low-humidity winds during much of the day (>4 m/sec) and calm night-time conditions with high night humidity values approaching 100%.

Case #2: ET₀ ~ 1.05 to 1.10 ET₀*. Applies in areas where daytime winds are <4 m/sec and nights are very calm and humid, i.e. RH_{max} > 75%. The daily wind distribution should give a day-night ratio of 3 or more.

Case #3: ET₀ = ET₀*. Daytime wind speeds of about twice those during the night-time are generally found and maximum relative humidity ≥ 60%. No adjustment of Penman ET₀* is required.

Case #4: ET₀ ~ 0.75 to 0.95 ET₀*. ET₀ will be slightly over predicted in areas with moderate to high radiation and with wind speeds < 4 m/sec but about equal during the day and night. Relative humidity during night-time is ≥ 60%.

Case #5: ET₀ ~ 0.65 to 0.80 ET₀* $\frac{1}{2}$. Relatively rare but applies for spring, summer and autumn conditions with moderate to high radiation and when wind speeds during the day and night are between 4 and 8 m/sec with maximum relative humidity < 40%.

Case #6: ET₀ ~ 0.55 to 0.65 ET₀* $\frac{1}{2}$. Also rare, but applies during spring, summer and autumn with moderate to high radiation and when wind speeds during day and night are > 8 m/sec and with relative humidity day and night < 40%.

Case #7: ET₀ ~ 0.30 to 0.35 ET₀* $\frac{1}{2}$. Very rare, but will apply under very strong winds of > 8 m/sec during both day and night, while relative humidity, day and night, is low and < 40%. Case 7 applies only when radiation is low, i.e. during late autumn and winter (R_{ns} < 4 mm/day).

Doorenbos and Pruitt (1975) used graphical adjustments to correct ET₀*. The program uses the linear relationship.

$$ET_0 = ET_0^* (1.629 - 0.21046 C)^{1/2} \quad (20)$$

where ET₀ is adjusted reference crop ET (mm/day), ET₀* is unadjusted ET (mm/day) and C is case number (1 to 7) as identified above.

Example: Doorenbos and Pruitt (1975) conclude that Case #4 characterizes the Cairo climate. Adjustment of ET₀* by Equation (20) yields ET₀ = 7.55 mm/day.

Conclusions

Doorenbos and Pruitt's (1975) generalized version of the Penman equation should reduce the number of anomalous results that occur when the equation is applied in environmental conditions quite different from those of the original site in the English countryside. The generalizations, however, introduce some additional considerations which Doorenbos and Pruitt handle neatly with supplementary graphs and tables. The compression of this supplementary material into appropriate algorithms, and organization into the convenient program described in this paper, will further help bring the Penman equation to bear on practical problems of ET estimation.

$\frac{1}{2}$. These drastic corrections occur when relative humidity is low and conditions are very windy. Such conditions are rare and persist only a few days in most climates (Doorenbos and Pruitt, 1975).

The program fits easily into the HP-41CV handheld calculator. This calculator is powerful, portable and relatively easy to operate. While the program works well for our purpose, it could undoubtedly be changed and improved. The identification and inclusion of the basic algorithms in this paper will make it easy for other users to adapt the Penman model to their special needs.

The current version of Robert Greenberg's Penman program with user's guide can be obtained from the authors for the cost of materials (\$10 includes 11 magnetic cards in "write all" format; make checks to the University of Arizona). It can also be supplied on a micro-cassette for the HP-1L cassette drive. The authors would appreciate comments and suggestions.

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Appendix

The program outputs are listed below for the example problem given in the text.

- a. Executing program PN generates a header and a listing of the inputs required to run the program, and the output to be obtained:
- b. The program prompts for tmax, tmin, RHmax, and RHmin, and then calculates tmean, RHmean, ed and ea-ed:

```
*****
* MODIFIED *
* PENMAN *
* ET MODEL *
*****
```

```
*****
* INPUTS *
```

```
degrees          C.
sause ht.        n.
wind run         kn/day
altitude         n.
radiation        mm./day
```

```
*****
* OUTPUTS *
```

```
E.T.            mm./day
*****
```

```
*****
```

```
VAPOR PRESSURE:
```

```
T*max*= 35.00
T*min*= 22.00
T*mean*= 28.50
ea= 38.91
RH*max*= 88.00%
RH*min*= 38.00%
RH*mean*= 55.00%
ed= 21.40
ea-ed= 17.51
```

- c. The program next prompts for wind travel and height of measurement, and calculates wind speed at 2 m and value of f(u):

```
*****
```

```
WIND FUNCTION:
```

```
u3.00= 250.00
u2= 232.50
f(u)= 0.90
```

- d. The program prompts for altitude, and calculates weighting factors W and $1 - W$ using tmean from before:

WEIGHTING FACTOR:

altitude= 95.00
W= 0.78
1-W= 0.22

- e. The radiation calculations begin with a prompt for observed n, followed by N. Since N is not known in this example, the program will prompt for year, month and day of month. The program outputs day of year, and prompts for latitude. It then calculates N and n/N. It prompts for Ra which if unknown (as in this example) is then calculated and used to estimate solar radiation. The program requests albedo and calculates net shortwave radiation. The longwave calculations begin with a prompt for dry or wet climate designation, followed by calculation of the longwave components, net radiation, and evaluation of Penman ETo* (unadjusted).

NET RADIATION:

n= 11.50
DATE= 7./ 28./ 72.
DAY 282.
S= 28.58
latitude= 38.00
N= 13.67
n/N= 0.84
Ra= 16.98
Rs= 11.33
Albedo= 0.25
Rns= 8.58
DRY Climate
f(ed)= 0.14
f(n/N)= 0.86
f(t)= 16.31
Rnl= 1.91
Rn= 6.59

ETo* = 8.64 mm./day
