Distribution of Loss Rates
Implicit in the SCS Runoff Equation

Richard H. Hawkins

Utah State University
Logan, Utah 84322

Introduction

General: It is a practice in applied hydrology, approaching a hallowed tradition, to assume that runoff processes on small watersheds are spatially uniform. The modeling fraternity calls this "lumping", and it is associated with small watersheds almost by definition. Both practitioners and researchers make the uniformity assumption for reasons of ignorance and simplicity. Not much is known of spatial variability of soil, vegetative, and hydrologic properties of landscapes, and that which is known promises confusing complexity if and when realistically applied. Nonetheless, it is also acknowledged that even small watersheds are indeed not uniform, though the consequences of the uniformity assumption are not clearly understood.

As a simple example, consider a small 100 acre watershed producing 1.00 inch of runoff (or 100 acre-inches) from a rainstorm of 2.00 inches. This rainfall excess could have arisen from a variety of conditions: 1) Half the watershed (50 acres) producing 2 inches of excess; 2) The entire watershed yielding 1.00 inch of excess; or 3) Any number of intermediate combinations, such as 80 acres at 1.25 inches, or mixtures such as 20 acres at 2 inches plus 60 acres at 1 inch. Mass accounting requires that the sum of the area-excess products be equal to the basin runoff, or:

$$Q = \sum Q_i a_i / A \qquad (Q_i \leq P) \qquad [1]$$

Where Q_i represents the rainfall excess depth from contributing area a_i, A is the total watershed area, and Q is the net watershed runoff depth.

This paper will offer an organized though untested approach to dealing with this phenomenon in analysis and synthesis of runoff, with accent on its interpretation in the case of the SCS runoff equation.

Background: In a previous paper (3), the writer has developed the notion of distributed effective loss rates, f, on small watersheds, and established the following equivalences. Given that effective loss rates are distributed as g(f), then the rainfall excess rate q for a period of intensity i is shown to be:

$$q = \int_0^i (i-f)g(f)df \qquad [2]$$

The foundation for this is illustrated in Figure 1. By expanding [2], and making distribution and conservation-of-mass interpretation, it can also be shown that

$$G(i) = dq/di \qquad [3]$$

Where G(i) is the value of the cumulative distribution of g(f) at i=f, or (as a net watershed process), the fraction of the watershed yielding rainfall excess at intensity i. The derivative expression in [3] indicates that this is the slope of the rainfall-runoff rate curve at intensity i. Thus, the slope of the rainfall

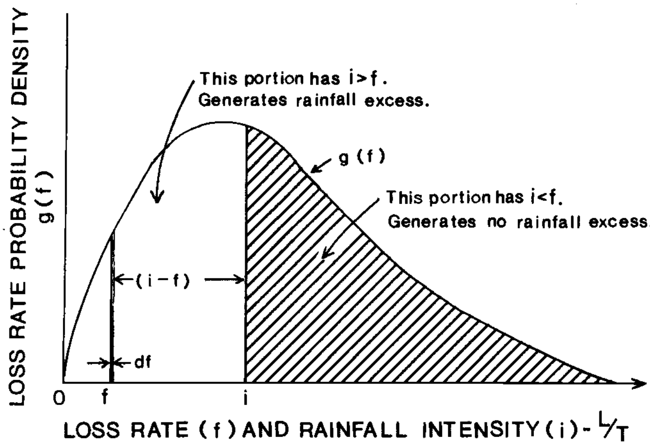

Figure 1. Diagrammatic illustration of probability density for distribution of effective loss rates on a model watershed. While the minimum loss rate is known here as zero, situations in which $f_{min} > 0$ are also possible, as are polymodal distributions, mixtures, and upper unbounded distributions.

function is the fractional contributing area. Equation [3] is differentiated to produce the density function g(), so that:

$$g(i) = d^2q/di^2 \qquad [4]$$

This is the underlying kernel distribution of loss rates. Note that it can be determined through differentiation of the rainfall excess function. This will be explored here for the case of the SCS runoff equation.

When the above reasoning is applied to situations of variable rainfall intensities, net watershed loss rates appear to vary positively with rainfall intensity up to the limit of $i=f_{max}$. Examples of such performances are surprisingly prominent in available data sets from plots and watersheds, thus offering at least indirect support for the general contention of spatially variable loss rates.

<u>SCS Runoff Equation</u>

Background: The SCS equation is the heart of the widely used Curve Number method. The runoff depth function is:

$$Q = (P-.2S)^2/(P+.8S) \qquad P \geq 0.2S \qquad [5]$$

$$Q = 0 \qquad P < 0.2S$$

Where Q is the direct storm runoff depth (inches), P is the storm rainfall depth (inches), and S is an index of basin retention, equal to the maximum possible difference between effective rainfall (P-0.2S) and runoff Q. The land condition index, curve number (CN) is a transformation of S, or:

$$CN = 1000/(10+S) \qquad [6]$$

Equation [5] can be standardized on the parameter S, leading to:

$$Q/S = (P/S-.2)^2/(P/S+.8) \qquad [7]$$

which is a much more convenient form and useful for what is to follow. It is shown in Figure 2.

Documentation of the method's development is given in its foundation publication (6) and general subsequent papers (4,7). A critical appraisal of it is given by Chen (1). Despite visible shortcomings, it is the most popular technique of its type in use today, and is applied to a wide variety of situations on an international basis. Its ability to incorporate land use, condition, and site moisture into runoff calculations, its documentation and agency origin, and the lack of suitable alternate techniques have promoted its use.

Assumptions: The distributed loss rate theory requires rainfall and excess rates, not depths (volumes). However, in its original and most robust application, the SCS runoff equation was applied only to daily data, or a time interval of one day. It was used to transform daily rainfall frequency descriptions to a parallel description of daily runoffs. Thus, it is consistent to assume rainfalls and runoffs as one day intensities, or, to be more general, as average intensities for storm durations used. In what is to follow it is necessary to stretch rates to cover assumed or understood storm durations T, such as 24 hours, so that:

$$i = P/T \qquad [8]$$

and

$$q = Q/T \qquad [9]$$

$$f = i-q \qquad [10]$$

As odious as this compromise seems at first glance, it is at least partially justifiable. Equations 8-10 inflict the presumption of a uniform intensity storm, for which there is some literature support, to achieve the SCS runoff equation (5). Additionally, common use of the SCS runoff equation as an infiltration method achieves the same effect by applying it to individual intervals throughout the progress of design storms.

In addition to the above, all the assumptions required of the foundation equations [2] through [4] are necessary. A major item is that the watershed is composed of an undefined large number of independent loss rate cells or contributing smaller runoff units, each operating with a characteristic time constant loss rate, distributed as g(f). The "runoff-runon" process is ignored within these cells.

Curve Number Loss Rate Distribution

Development: The intellectual strategy in linking the SCS equation and the distributed loss rate concept assumes that the former is empirically correct but that the runoff occurs via the distributed processes imagined in the latter, and not as a lumped uniform phenomenon as historically assumed. The equivalence is drawn by simply equating equations [2] and [5]. Therefore, being careful with limits,

$$Q = \int_{.2S}^{P} (P-f)g(f)df = (P-.2S)^2/(P+.8S) \qquad [11]$$

The task is now to solve for g( ). Rather than deal with equation [11] directly, the principles previously discussed and given as equations [2] and [3] may be used. That is, differentiating the right hand of equation [11] (which is equation [5]) with respect to P and simplifying yields:

$$G(P) = dQ/dP = 1-(P/S+.8)^{-2} \qquad P \geq 0.2S \qquad [12]$$

Of concern is the situation when P = f in the above. Thus substituting f for P gives:

$$G(f) = 1-(f/S+.8)^{-2} \qquad f \geq 0.2S \qquad [13]$$

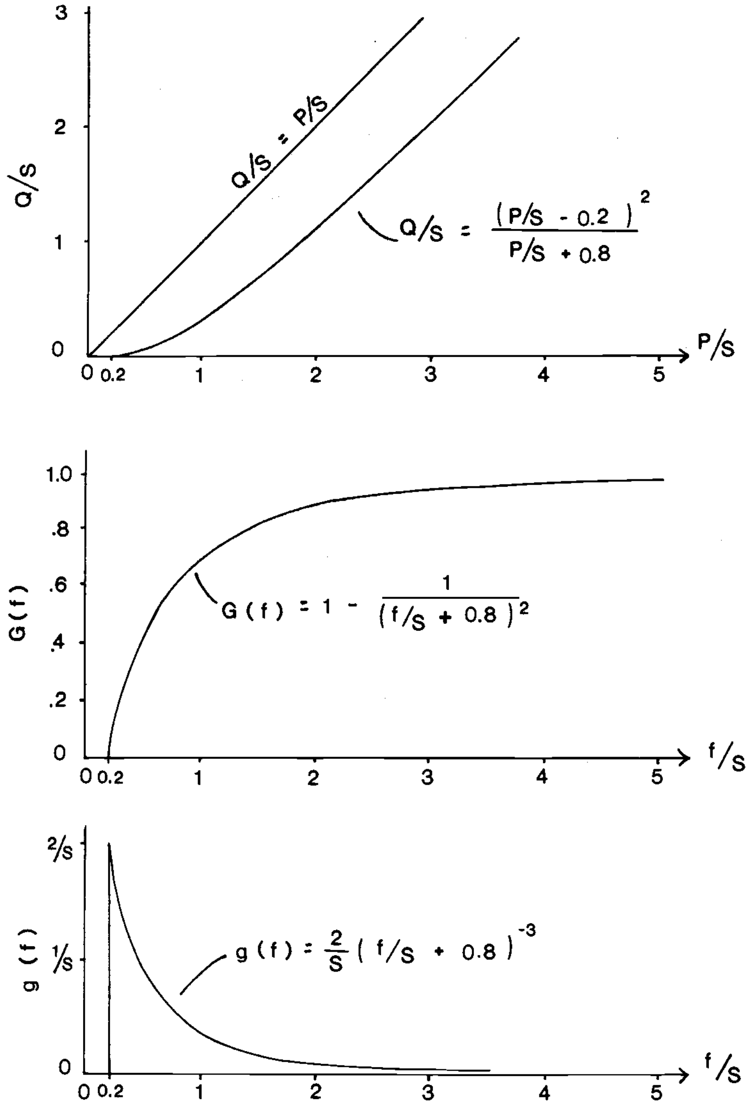

Figure 2. Above: Dimensionless expression of the SCS rainfall-runoff equati standardized on the storage index S. Center: Cumulative distribut of effective loss rates implied by the SCS equation. Bottom: Dens function of loss rates. Note the strong positive skew, and the lo limit of 0.2S. The mean is 1.2S, and the median is 0.6142S.

The density function is obtained by differentiating the above with respect to f, and simplifying, giving:

$$g(f) = (2/S)(f/S+.8)^{-3} \qquad f \geq 0.2S \qquad [14]$$

Equation [14] above is the distribution of storm loss rates f, which when inserted into equation [2] will give equation [5]. It reconciles the two ideas.

Characteristics: Figure 2 shows the original SCS equation, standardized on S, and the simultaneous display of G(f) and g(f), also standardized on the watershed index S. The expected value or mean of the distribution is found by the method of moments as:

$$E(f) = \mu_f = 1.2S \qquad [15]$$

and the median or f_{50} is found to be:

$$f_{50} = (\sqrt{2}-.8)S = 0.6142S \qquad [16]$$

The higher moments, i.e. the variance, skewness, kurtosis, etc. are undefined (they approach infinity). Nonetheless, it exhibits a distinct positive skew. It should be noted that the distribution is valid only for $f \geq 0.2S$, which asserts that the watershed has no loss rates less than 0.2S. This uncomfortable item springs from the original SCS equation's demand for an initial abstraction of 0.2S. Undoubtedly, a more flexible expression relying on a general value of initial abstraction could also be derived.

Application

Contributing Fraction: Given a storm depth and curve number, the inferred contributing fraction or partial area can be calculated. For example a storm of P = 2.00 inches falling on a watershed of CN=80, yields a calculated runoff of 0.56+ inches. Applying equation [13] for f = 2.00 inches calculates $G(f) = 1-(1.6)^{-2} = 0.61$. This may be interpreted as 61% contributing area, or 39% of the watershed with $f \geq 2$ inches, and thus non-contributing. This information would give guidance towards possible land treatment strategies to reduce runoff, erosion, or pollutant pickup.

Loss Rate Equivalences: The relationship in equation [15] can be exploited to provide a translation between mean watershed loss rate and the widely used and handbook-backed Curve Number. Substituting S from equation [6] into equation [15] yields:

$$\mu_f = 1.2S = 1.2(1000/CN-10) \qquad [17]$$

or

$$CN = 1200/(12+\mu_f) \qquad [18]$$

It should be kept in mind that μ_f is the <u>mean</u> watershed loss rate when distributed as given in equation [13]. An identical result using a ø-index has been previously demonstrated (2).

Summary

The SCS rainfall-runoff equation may be united with a spatially varied loss rate via the distribution $g(f) = (2/S)(f/S + 0.8)^{-3}$, which has a mean of 1.2S, a median of 0.6142 S, and undefined higher moments. Use of this notion reconciles the popular concept of partial area contribution with the curve number equation. Interpretations of fractional contributing area may be made for specific conditions.

References

1. Chen, C.L. 1981. Assessment of Soil Conservation Service Model with Runoff Curve Numbers. Presented at International Symposium on Rainfall-Runoff Modeling, Mississippi State University, May 18-21, 1981. (Proceedings in Press)

2. Hawkins, R.H. 1978. Effects of Rainfall Intensity on Runoff Curve Number. In Hydrology and Water Resources of Arizona and the Southwest, Vol. 8, pp. 53-64. (AWRA and AAAS Arizona Meeting, Flagstaff, April 14-15, 1978).

3. Hawkins, R. H. 1981. Interpretation of Source Area Variability in Rainfall-Runoff Relationships. Presented at International Symposium on Rainfall Runoff Modeling, Mississippi State University, May 18-21, 1981. (Proceedings in press).

4. Kent, K. M. 1968. A Method for Estimating Volume and Rate of Runoff in Small Watersheds. U.S.D.A., Soil Conservation Service, CSC-TP-149, 58 pp.

5. Morel-Seytoux, H. J. 1981. Extension of the Soil Conservation Service Rainfall-Runoff Methodology for Ungaged Watersheds Report FHWA/RD-81/060, Federal Highway Administration. 79 pp. (Available from NTIS).

6. U.S. Department of Agriculture, Soil Conservation Service. 1956 (et seq.). National Engineering Handbook, Section 4, Hydrology. USGPO, Washington, D.C.

7. U.S. Department of Agriculture. 1975. Urban Hydrology for Small Watersheds. Tech. Release No. 55.