QUASI THREE-DIMENSIONAL FINITE ELEMENT MODEL OF THE MADRID BASIN IN SPAIN

Jesus Carrera and Shlomo P. Neuman
Department of Hydrology and Water Resources
University of Arizona, Tucson, AZ 85721

Abstract

Groundwater flow in the Tajo River basin surrounding the city of Madrid is studied with the aid of a quasi three-dimensional model. The model is based on an efficient adaptive explicit-implicit finite element method recently described by Neuman et al. (1982). The top layer is unconfined and interacts with the Tajo River and its tributaries. Reproduction of the existing conditions in the aquifer demonstrates the existence of local and intermediate flow patterns which are superimposed on the regional flow pattern. Such flow patterns could not be identified with a conventional two-dimensional model. The manner in which these patterns are affected by topography and stream configuration is clearly illustrated with the aid of three-dimensional plots constructed from a certain viewing angle. Similar plots are used to illustrate the evolution of drawdown zones due to pumpage at predetermined locations in the aquifer.

Introduction

The Madrid aquifer is a thick, detritic unit extending over the central part of Spain (Fig. 1). It exhibits a marked vertical anisotropy due to layering. As a result of structural and topographic control, the flow pattern in the aquifer forms a complex three-dimensional picture. Attempts to model various aspects of this flow pattern have been reported by López García (1979) and Sahuquillo et al. (1975). A full-scale three-dimensional model is currently being developed by the Geological Service of Spain, based on a modified version of the Prickett and Lonquist (1971) finite difference computer program.

The purpose of this paper is to investigate the feasibility of modeling three-dimensional flow in the Madrid aquifer with the adaptive explicit-implicit quasi three-dimensional finite element model, FLUMPS, recently developed by Neuman et al. (1982). The results of this model compared favorably with those obtained from the Prickett-Lonquist model by the Spanish Geological Service.

Hydrogeology

The detritus of the Madrid basin has been formed from alluvial fans (López Vera, 1979). It consists of lenses rich in sand which appear to be randomly distributed within a clay-rich matrix. The granulometric properties of the aquifer material vary according to their source: The three main facies, Madrid, Toledo, and Guadalajara, derive from the west-central range, east-central range, and Toledo Mountains, respectively. As a result of the large spatial variability of the granulometric properties, the hydraulic parameters of the aquifer are also highly variable.

The largest transmissivities (30-100 m^2/day) are encountered in the Madrid unit of the Madrid facies which extends over the central part of the aquifer. The Tosco unit of the Madrid and Toledo facies, in the western part of the aquifer, has transmissivities between 20 and 50 m^2/day. In the east, transmissivities of 10-30 m^2/day characterize the Alcala and Guadalajara units of the Guadalajara facies. Least permeable is the evaporite unit extending over the southern and south-eastern part of the aquifer (Fig. 1), with transmissivities around 5-10 m^2/day.

On the basis of two-dimensional computer modeling studies in the vertical plane conducted by Llamas (1976) and others, vertical hydraulic conductivities appear to be smaller than horizontal ones by a factor of 100. Recharge appears to be around 200 m^3/km^2/day, or 73 mm/year.

The surface drainage system consists of the Tajo river in the south, and its northern tributaries which are perennial except in very dry years.

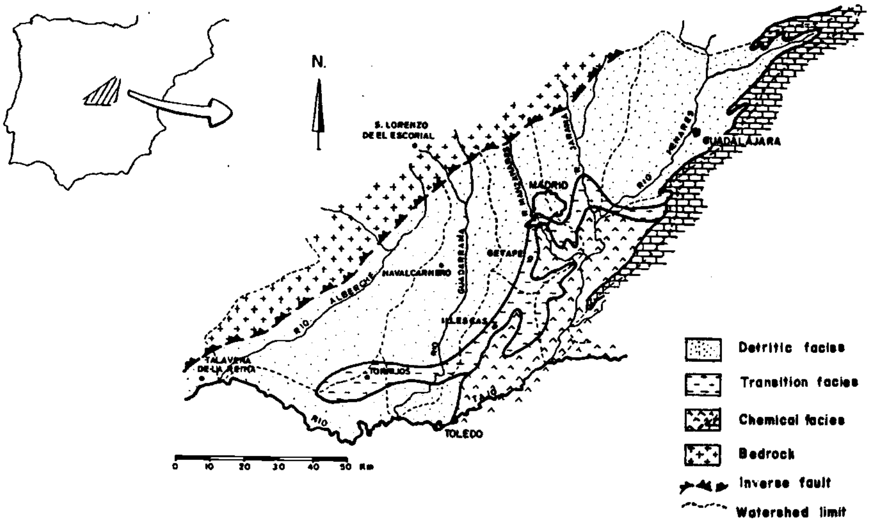

Figure 1. Location and geology of Madrid basin.

Modeling Approach

Flow in the aquifer is modeled with the adaptive explicit-implicit quasi three-dimensional finite element program, FLUMPS, of Neuman et al. (1982). The quasi three-dimensional aspect of the model requires that the aquifer be subdivided into horizontal layers in which the flow is strictly horizontal. The Madrid aquifer is subdivided into three such layers, as illustrated by the horizontal finite element grids in Fig. 2. Each square in these grids has an area of 25 km^2, and is automatically subdivided by the computer program into two triangles. Flow between the layers is strictly vertical. It is handled by vertical strings of linear finite elements connecting the nodes of the horizontal grids. Ten such strings, each consisting in our case of a single linear element, are shown in Fig. 2.

The entire finite element grid for the Madrid aquifer includes 621 nodes, 285 in layer 1 at the top, and 168 in layers 2 and 3 in the middle and at the bottom, respectively. The grid consists of 981 triangular elements, of which 483 lie in layer 1, and 251 in each of layers 2 and 3. In addition, there are 336 line elements connecting the horizontal grids in the vertical direction.

Layers 2 and 3 have impermeable boundaries on all sides. Layer 1 at the top has an impervious boundary on the north side; the remainder of its boundary follows the outline of streams. Streams also run within the interior of layer 1. All grid points along streams are assigned a mixed (Fourier) boundary condition of the following type: Constant rate of aquifer recharge when the water table lies below the stream bottom, and an aquifer discharge rate proportional to the difference between the computed water table and the water level in the stream when the former lies above the stream bottom. A similar type boundary condition is assigned along ephemeral streams, at spring locations, and over swamp zones or other areas where discharge by evapotranspiration can take place.

The adaptive explicit-implicit aspect of the model consists of solving the finite element equations explicitly at nodes having stability limits in excess of the time step, Δt, and implicitly by LU-decomposition or iteratively at all other nodes (in this work iterations were used). For details

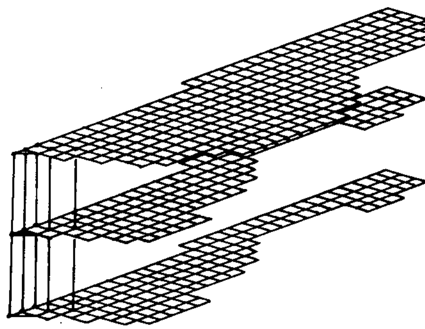

Figure 2. Horizontal finite element grids for layers 1, 2, 3 with a few vertical line elements.

concerning this method of solution, the reader is referred to Neuman et al. (1982). Since explicit solution is much faster than implicit solution, the adaptive scheme leads to considerable savings of computer time. It also leads to improved accuracy because one can use small time steps when hydraulic heads vary most rapidly at relatively little extra computer cost.

In FLUMPS, the size of Δt is varied automatically by the program. At the beginning when derivatives of heads with respect to time are largest, Δt is very small, and most nodes are automatically treated by the program as explicit. As time progresses, the large time derivatives imposed by pumpage dissipate, and Δt increases until at some nodes, Δt may come very close to (or exceed) the stability limits of these nodes. Such nodes are then automatically reclassified as implicit. The stability limit of each node is a function of the mesh geometry and the hydraulic parameters in the immediate vicinity of the node. Since in our case the mesh is uniform but the parameters vary in space, the stability limit varies from node to node. Fig. 3 is a typical graph showing how the number of implicit nodes in our model varies with the number of time steps when Δt is allowed to increase without limit. An unlimited increase in Δt is allowed in all the steady state simulations reported below. In all transient simulations, the maximum Δt is restricted to one year, which is less than the smallest stability limit of any node in the grid. Thus, all the nodes during a transient simulation run remain explicit at all times.

Fig. 4 shows the amount of CPU time on the Cyber 175 computer of the University of Arizona versus the total number of time steps required for each of the 9 steady state and transient runs performed in our study. Despite the respectable size of our quasi three-dimensional grid, the CPU time is small, being less than one minute in 8 out of the 9 runs. When all nodes remain explicit (steady state run 1, transient runs 6-9) CPU time increases linearly with the number of time steps. Points departing from this straight line represent steady state runs in which some, or all of the nodes, become implicit at various stages of the solution process. Departures from the straight line are always in a direction of increased CPU time for a given number of time steps.

Steady State Results

Until the late 1960's, the Madrid aquifer is assumed to be at steady state. The purpose of our steady state simulations was to calibrate the model against recorded data. The final result of the five calibration runs performed gave excellent agreement with the data and with the results obtained by the Spanish Geological Service with the Prickett-Lonquist model (1971). A three-dimensional plot of our results is shown in Fig. 5a.

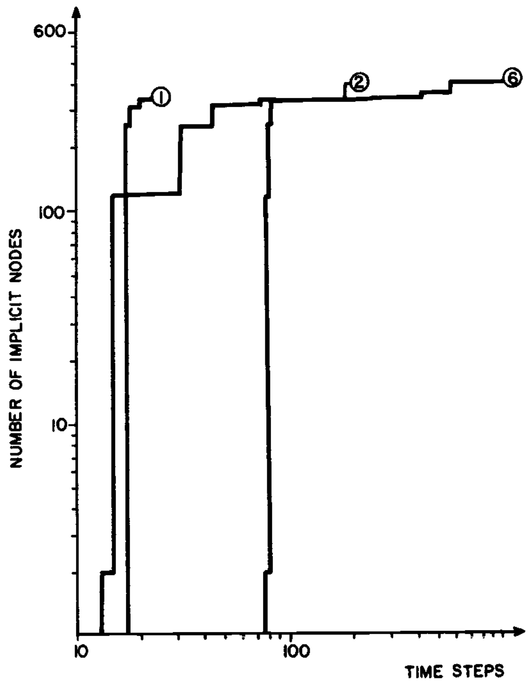

Figure 3. Number of implicit nodes versus time steps for three steady-state runs. Numerals refer to the same runs as those of Figure 4.

The steady state configuration of the water table in layer 1 at the top is a close reflection of ground surface topography. Since there is no pumpage, convex areas represent recharge, and concave areas represent discharge. All streams are gaining water from the aquifer. Of the total recharge, 60 percent leave the aquifer via river nodes, and 40 percent via discharge nodes. Since many of these discharge nodes are close to the streams, much of the corresponding discharge water must reach these streams via small creeks and superficial alluvium.

The water table configuration in layer 1 reflects the existence of local groundwater flow patterns of the kind originally postulated by Toth (1963). The piezometric surface in layer 2 is smoother, exhibiting fluctuations of a smaller frequency and amplitude than the water table in layer 1. These fluctuations can be taken to represent intermediate flow patterns. The piezometric surface of layer 3 is very smooth, sloping gently from east to west, and indicates the presence of regional flow in that same direction.

Transient Results

Transient simulations were performed over periods of 50 years for three different hypothetical pumping regimes. The initial condition in each case is taken to be the steady state. In each case, pumping occurs primarily in layer 1 (to some extent in layer 2) and is heaviest to the northeast, northwest, and southwest of Madrid. A more or less uniform pumping rate similar to the local rate of recharge is imposed on the area east of Madrid. The three pumping regimes differ from each other primarily in the total rate of extraction from the aquifer.

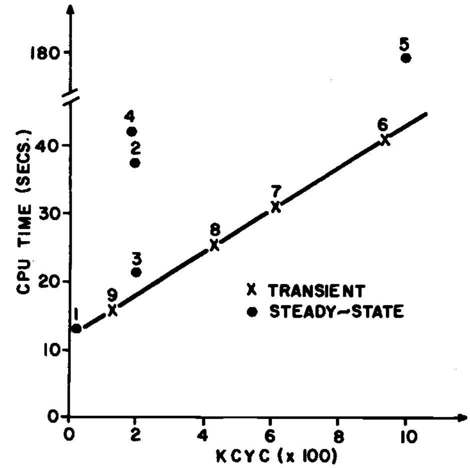

Figure 4. CPU time versus number of time steps (KCYC) for each of nine runs.

Fig. 5b shows the results in all three layers, after fifty years of pumping, at a total rate equal to about 60 percent of total recharge. Maximum drawdown in layer 1 is 130-140 m (clearly, the drawdown near individual wells may be larger). The local flow regime, as reflected by the water table in layer 1, is now substantially different from what it was at steady state. Whereas at steady state the low points along the water table coincided with streams which effectively divided layer 1 into sub-basins, the lows after 50 years of pumpage are different, and there is some transfer of groundwater from one such sub-basin to another. Layers 2 and 3 again show a gradually smoothed replica of the situation in layer 1. However, the amplitude and frequency of the fluctuations in these layers are larger than at steady state, reflecting the effect of pumpage.

As a consequence of pumping, most of the original discharge zones near centers of pumpage are no longer discharging, and some of the stream reaches lose water to the aquifer. After 50 years of pumpage, the total surface discharge is reduced from 100 to 60 percent of total recharge, a rate approximately equal to that of total extraction. This means that about 35 percent of total extraction (equivalent to about 20 percent of total recharge) is derived from storage. If the same rate of pumping continues into the future, the discharge rate will diminish until its sum with the rate of extraction will be equal to the rate of recharge; the system will then be at a new steady state.

Due to the relatively low transmissivity and high storativity of the aquifer, drawdowns are large near pumping centers but the areal extent of the drawdown zones is small. To prevent excessive drawdowns in localized areas, wells and well-fields should be spaced at sufficient distances from each other.

Conclusions

The adaptive explicit-implicit quasi three-dimensional finite element model FLUMPS is well suited for the simulation of steady state and transient three-dimensional flow patterns in the Madrid aquifer. Time steps of one year are treated explicitly with this model, a fact which results in considerable savings of computer time. The model provides a three-dimensional picture of flow in the Madrid basin which clearly illustrates the presence of local and intermediate flow patterns superimposed on the regional pattern. The effect of pumping is shown to be significant but restricted to small areas.

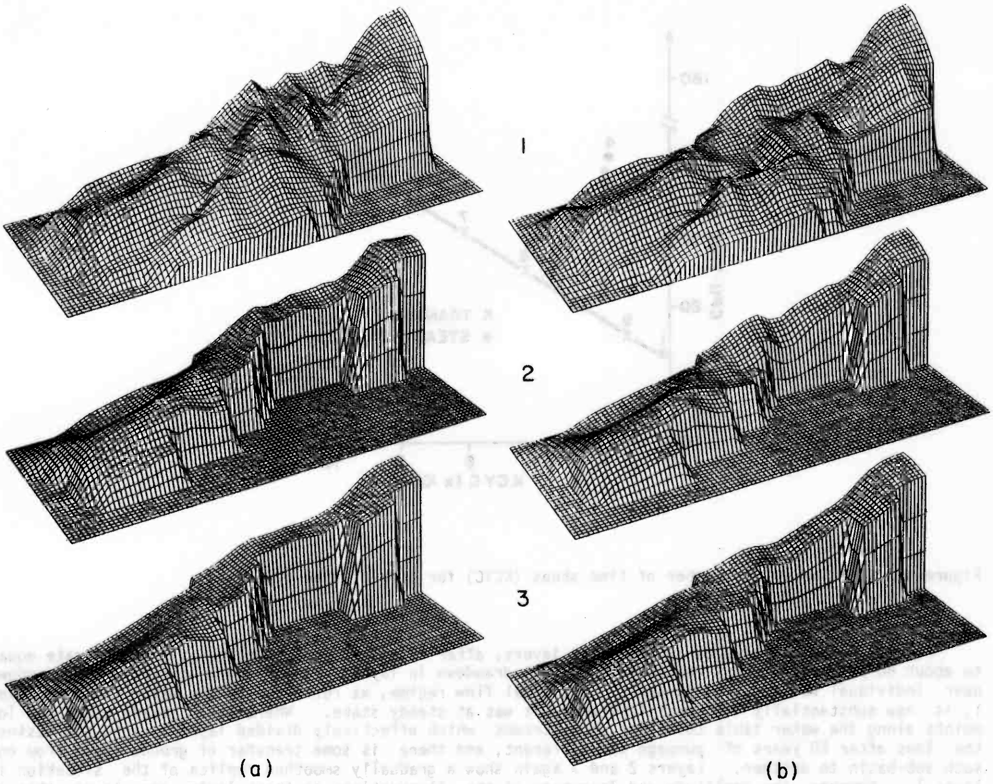

Figure 5. Perspective of piezometric surfaces in layers 1, 2, 3
a. at steady-state
b. after 50 years of pumpage

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References

Llamas, M. R., 1976. La Utilización de Aguas Subterráneas en Madrid. De los Mayrat Musulmanes a los Modelos Digitales. Estudios geologicos. Madrid. 32:121-139.

López García, J., 1979. Historia de la Modelación del Acuifero de la Cuenca del Tajo. Unpublished progress report of cooperative research between S.G.O.P.U. and University of Arizona.

López Vera, F., 1977. Modelo de Sedimentación de los Medios Detríticos de la Fosa de Madrid. Tecniterrae. Madrid. No. 18.

Neuman, S. P., C. Preller and T. N. Narasimhan, 1982. Adaptive Explicit-Implicit Quasi-three-dimensional Finite Element Model of Flow and Subsidence in Multiaquifer Systems. In Press.

Prickett, T. A. and C. G. Lonnquist, 1971. Selected Digital Computer Techniques for Groundwater Resource Evaluation, Illinois State Water Survey Bull. 55, 62 pp.

Sahuquillo, A., B. Martí, B. López-Camacho, A. Balenilla, L. López García, 1975. Modelo Bidimensional para el Estudio de la Componente Vertical del Flujo en un Acuífero de Gran Espesor. II Congreso Iberoamericano de Geologia Economica. Buenos Aires.

Toth, J., 1963. A Theoretical Analysis of Groundwater Flow in Small Drainage Basins. Journal Geophysics Research. 68(16):4796-4812.