An Application of the Almon Polynomial Lag to Residential Water Price Analysis

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A recent article by Agthe and Billings (1980) concerned itself with the application of Koyck form lagged dynamic economic models to measure the price elasticity of demand for residential water consumption. The Koyck form lagged models have a lagged dependent variable, quantity of water consumed, and assume that the greatest adjustment of quantity to price occurs in the first period after the price change and succeeding adjustments are progressively smaller and less important. This study will consider an alternative dynamic economic model in which the independent variable price is assigned a distributed lag. The distributed lag model chosen is the Almon Polynomial lagged model (Almon, 1965) as it is expected that the lagged response to a price change is nonlinear in nature. The results of several potential Almon lag forms are compared to the results of the simple Koyck lagged model found in the Agthe and Billings (1980) study. This Koyck form was by far the best result found in the earlier study.

One of the most intriguing advantages of the Almon polynomial lagged model over the Koyck model is the ability to specify the form and the length of the adjustment to the price changes. Given the fairly long periods of stable price structures for water and similarly priced public utility type goods, the consumer has a long time period to adjust and readjust his position to given price structure in the market place. Therefore, initial overadjustment to price changes with later readjustments is a viable hypothesis for consumer behavior. This behavior can be accounted for in the polynomial lag model but not in the Koyck lag model. The Almon process also reduces the possibility of autocorrelation that is frequently associated with the lagged dependent variable that appears in the Koyck model.

The Models and Variables

This study utilizes the general form of the Almon lag model (Gujarati, 1978) as follows:

$$\begin{aligned} Q &= b_o + b_1 W + b_2 Y + b_3 D \\ &\quad + b_4 P_t + b_5 P_{t-1} + \ldots\ldots + b_1 P_{t-n} \end{aligned}$$

and the Koyck model in the form:

$$Q = b_o + b_1 Q_{t-1} + b_2 W + b_3 Y + b_4 D + b_5 P_t$$

where:

Q = Average water consumption (100 cubic feet) per household for active single residential water connections by month (January 1974 thru November 1977) (Tucson, 1978)

D = The difference between what the typical consumer actually pays for water and what would be paid if all of the water were purchased at the marginal rate. (Dollars)

Y = Personal income per household (dollars per month) (Arizona)

W = Evapotranspiration for Bermuda grass minus rainfall (inches) (Boul, 1963; U.S.)

P = Marginal price facing the average household (cents per 100 cubic feet) (Tucson, 1974, 1975, 1976, 1977)

t = Time period in months

n = Length of the price lag in months

Water consumption per household (Q) is the dependent variable. Households in this study include all single family residences, apartments, condominiums, mobile homes, duplexes, and triplexes served by individual water connections. Multiple units served by a common water meter were excluded from the study, eliminating those households which receive water at a zero marginal price because its cost is included in their rental payments. The inclusion of a large number of such households would result in an underestimation of the price elasticity of demand for water among households which must pay individual water bills.

A correct specification of the demand model requires the use of two price related variables when there are block rates and/or flat rate service charges in the price schedule (Billings and Agthe, 1980; Nordin, 1976; Taylor, 1975). Since the Tucson water rates include increasing block rates and flat rate charges, two variables must be used in the demand model. The first, marginal price (P) or the per unit price in the marginal block, is the price the average consumer would have to pay for additional unit of water each month, based on average household water use and the City of Tucson water rate schedule. The second price variable (D) is the difference between what the consumer actually pays for water and what would be paid if all water were purchased at the marginal price. This variable measures the flat rate charge imposed on the consumer plus the difference between the amount paid under intra-marginal rates (i.e., block rates below the marginal rate) and what would have been paid at the consumer's marginal price.

The use of the price variable (D) provides a measure of the income effect of any changes in the rate schedule which do not alter the marginal price. It also measures income effects arising from marginal price changes which are in addition to those which would be predicted on the basis of the change in marginal price alone. The difference variable measures a pure income effect which might be expected to have the same impact on water use as any other change in income.

Since this study utilized monthly data, a weather variable (W) to account for variations in sprinkling demand is particularly important. The weather variable incorporates the effects of rainfall and evapotranspiration which influence the amount of water required for lawn, shrub, and tree irrigation. Since Bermuda grass is the primary residential lawn cover in Tucson, its evapotranspiration rate was selected because it is likely that many residents use the appearance of their lawn to judge their irrigation needs.

Personal income per household (Y) was included in the model to account for variations in the economic situation of water consumers during the study period. This variable was found significant in a previous study of residential water demand in Tucson (Billings and Agthe, 1980) and is normally considered an important factor in most demand analysis.

Because of the substantial inflation which occurred during the study period, the model was estimated using real values of the variables. Real values were derived by dividing the price, difference, and income variables by the consumer price index (Board of Governors). Consumers were found to respond to water prices on the basis of real (Price adjusted) values for water rates and income rather than nominal prices in a previous study (Billings and Agthe, 1980).

If one wishes to model the downward overadjustment with later slight upward consumption adjustments, the Almon Lag Model in cubic form, as plotted in Diagram 1, is probably the best model. Of course, a result with an inflection point is also possible. A potential problem with the Almon model is that a lag of at least 4 periods is required when a cubic equation is tested. Since Hogarty and MacKay (1975) argue that most of the adjustment to marginal price changes occurs in the first three months after the price change, there is some possibility of a cubic equation with a minimum lag of 4 months being too long. To allow for this, a second degree equation with a 3 month lag is considered as an alternative.

Results

The similarity of the results (Table 1) for the b values of the Almon lagged variables was gratifying while the accompanying low t-values were less satisfying. The low t values may result from the strong multicolinearity inherent in the Almon procedure. In any event, the consistency of results between equations appears to call for acceptance of the b values in spite of the low t ratios.

The majority of the downward adjustment of water consumption in response to marginal price increases appears to be completed by the end of the 3rd month, a result that agrees with Hogarty and MacKay (1975). While there appears to be a slight upward adjustment in consumption for the fourth and fifth months after the price change, the less stable b values and poor t ratios preclude any definitive statement concerning the exact size of this upward adjustment other than it is likely to be small if it exists. It also appears that the adjustment process is complete by the beginning of the fifth month. These results are in agreement with the earlier result using the Koyck model (Agthe and Billings, 1980). This model also demonstrates the greatest decreases occurring in the first period with successively smaller decreases in water and in later months.

The long run and short run price elasticity of demand coefficients are presented in Table 2. The short run coefficients for the Almon procedure are calculated for each time period by the formula:

$$b_i = \frac{\bar{P}_i}{\bar{Q}_i}$$

and the long run price elasticity of demand coefficients are calculated by the formula:

$$\sum_{i=0}^{4} b_i \frac{\bar{P}}{\bar{Q}}$$

The elasticities were calculated for 5 periods in each case, period t_0 plus 4 lagged periods. The long run results range from .468 to .513. Roughly interpreted, this means that a 1 percent change in price will produce a .5 percent change in water use within 5 periods of the institution of a price change. The Koyck model yielded a long run price elasticity of demand coefficient of .5306. This is slightly larger than the Almon procedure results as the Koyck model does not allow for partial readjustment to a higher consumption level. The Koyck short run price elasticity of demand is -.4164, a coefficient similar to that of the first period Almon procedure values. The price elasticities reported for the Almon process fall well within the range reported in Agthe and Billings (1980).

Conclusion

This study presented the Almon Polynomial Lag as an alternative to the Koyck model to measure consumer response to increased residential water prices because it has the advantage of being able to specify the length and form of the lag. A high amount of multicolinearity inherent in the model precluded obtaining good t ratio results for the most of the lagged prices. Most of the downward adjustment in residential consumption occurs within three months of the price change and any upward readjustment in consumption is likely to be small. Since the upward readjustment produces uncertain statistical results and the price elasticities are about the same for both the polynomial and Koyck results, there is no reason to believe the polynomial results are superior to the Koyck results or that the hypothesized upward adjustment exists.

Table 1 - Best results of the application of the Almon Polynomial Lag to Residential Water Demand in Tucson, Arizona

ALMON POLYNOMIAL LAG FORM

	Degree and Length of LAG		
Variable or Statistic	X^2 4 Mo.	X^3 4 Mo.	Koyck Lag
P_t	-.0964	-.0960	-.0897
t	(-1.91)	(-1.41)	(2.32)
P_{t-1}	-.0347	-.0374	------
t	(-1.404)	(.519)	------
P_{t-2}	-.0006	-.0001	------
t	(-.017)	(.000)	------
P_{t-3}	+.0230	+.0161	------
t	(.00)	(.012)	------
P_{t-4}	+.0064	+.0095	------
t	(.063)	(.037)	------
P_{t-5}	------	------	------
t	------	------	------
Q_{t-1}	------	------	.2154
t	------	------	(2.15)
W	.0053	.0053	-.0042
t	(8.09)	(7.94)	(6.22)
Y	.0436	.0436	-.0418
t	(2.00)	(1.96)	(2.27)
D	-.0061	-.0061	-.0052
t	(-3.46)	(-3.40)	(-3.30)
Constant	-4.589	-4.592	-5.69
t	(-.79)	(-.775)	(-1.15)
Adjusted R^2	.793	.787	.817
F	26.52	22.06	39.50
Durbin-Watson	2.04	2.04	------
h	------	------	1.33

Table 2 - The Short Run and Long Run Price Elasticities of Demand Associated With The Almon Polynomial Lag Results

	Model Chosen and Lag	
Time Period	X^2 4 Mo.	X^3 4 Mo.
t_0	-.4475	-.4456
t-1	-.1597	-.1719
t-2	-.0027	-.0005
t-3	+.1037	+.0726
t-4[1]	+.0285	+.0423
Long Run	-.4675	-.5131

References Cited

Agthe, Donald E. and Bruce R. Billings. 1980. Dynamic models of residential water demand. Water Resources Research. Washington, D.C. 16(3):476-480.

Almon, Shirley. 1965. The distributed lag between capital appropriations and expenditures. Econometrica. 33(1):178-196.

Arizona Department of Economics Security. 1978. Arizona Population and Income Estimates. Monthly unpublished data Phoenix, Arizona.

Billings, R. Bruce and Donald E. Agthe. 1980. Price elasticities for water: A case of increasing block rates. Land Economics. 56(3).

Board of Governors. 1977. Federal Reserve System. Federal Reserve Bulletin. Table A-66 (Consumer Price Index). Washington, D.C.

Boul, S. W. 1963. Calculated actual and potential evapotranspiration in Arizona. University of Arizona, College of Agriculture, Tucson, Arizona. Technical Bulletin 162.

Gujarati, Danodar. 1978. Basic Econometrics. (McGraw-Hill, New York).

Hogarty, Thomas F. and Robert J. MacKay. 1975. The impact of large temporary rate changes on residential water use. Water Resources Research. 11(6):791-794.

Houthakker, Hendrick S. and Lester D. Taylor. 1970. Consumer demand in the United States. 2nd Ed. Harvard University Press.

Howe, Charles W. and F. P. Linaweaver, Jr. 1967. The impact of price on residential water demand and its relation to system design and price structure. Water Resources Research, 3(1):13-32.

Nordin, John A. 1976. A proposed modification of Taylor's demand analysis: Comment. The Bell Journal of Economics. 7(2):719-721.

Taylor, Lester D. 1975. The demand for electricity: A survey. The Bell Journal of Economics. 6(1):74-110.

Tucson, Arizona. January 1, 1974, June 7, 1976 and March 3, 1977. Water Department Rate Schedules. City Ordinance No. 4130, City Ordinance No. 4497 and City Ordinance No. 4626.

Tucson. 1978. Arizona Department of Water Sewers. Water use by Month and customer class. Unpublished data on computer tape. Tucson, Arizona.

U.S. Weather Bureau. 1977. Climatological Data Annual Summary, Arizona. Tables 1, 2, 3 and 4. Asheville, N.C. National Oceanic and Atmospheric Administration.

Young, Robert A. 1973. Price elasticity of demand for municipal water: Case study of Tucson, Arizona. Water Resources Research. 9(4):1068-1072.

Diagram 1

The Expected Consumption and Corresponding b- value Patterns for the x^3 Almon Polynomial Lag.