#100

PROJECT COMPLETION REPORT
OWRT Project No. A-086-ARIZ

Developing a New Deconvolution Technique to Model Rainfall-Runoff in Arid Environments

Agreement No. 14-34-0001-8003

Project Dates: 10/01/77 - 9/30/78

Principal Investigators

S. P. Neuman
Department of Hydrology and Water Resources

S. D. Resnick and R. W. Peebles
Water Resources Research Center

University of Arizona
Tucson, Arizona 85721

Acknowledgment: The work upon which this report is based was supported by funds provided by the State of Arizona and the United States Department of Interior, Office of Water Research and Technology as authorized under the Water Resources Act of 1964.

September, 1982

ABSTRACT

The primary accomplishment of this research has been demonstrating the power of the deconvolution technique developed by Neuman and de Marsily (1976) in dealing with noisy rainfall-runoff records of short duration. Such records are encountered in arid environments where rainfall often occurs in short isolated bursts and the data are measured with a considerable margin of error. Our research work consisted of superimposing known noise on synthetic rainfall-runoff data and examining the ability of the Neuman-de Marsily deconvolution method to estimate the correct impulse response of the system when the data include only a single storm event. Approximately 50 Monte Carlo simulation runs were performed for each of three different noise models considered in our work. The results clearly demonstrated that the deconvolution model leads to reliable estimates and can be used with confidence in the presence of realistic noise levels. In addition to the Monte Carlo simulation tests and their analysis, certain improvements were introduced into the original deconvolution technique. In particular, the original version of the technique required that the hydrologist exercise subjective judgement in choosing the "best" solution for the deconvolution problem from a large number of admissible solutions. Our new method of selecting the "best" result is based on a comparative analysis of residuals and is more reliable than the earlier subjective approach. The improved method has been applied to real as well as synthetic rainfall-runoff data.

INTRODUCTION

The research work under this contract has been conducted by graduate student David B. Dunbar and summarized in his M.S. thesis entitled "Analysis of a Parameter Estimation Technique for Linear Hydrologic Systems Using Monte Carlo Simulation" submitted to the Department of Hydrology and Water Resources, University of Arizona, Tucson, in 1981. The present report is a brief summary of Mr. Dunbar's thesis.

A major accomplishment of this research has been demonstrating the power of the deconvolution technique developed by Neuman and de Marsily (1976) in dealing with noisy rainfall-runoff records of short duration. Such records are often encountered in arid environments where short rainfall bursts are common, and the data are corrupted by significant measurement errors.

The deconvolution technique of Neuman and de Marsily (1976) uses parametric linear programming to determine the instantaneous unit impulse response of a linear system. The system is assumed to be deterministic and represented by the convolution integral

$$y(t) = \int_0^t u(t-\tau)x(\tau)\, d\tau \qquad (1)$$

where $x(\tau)$ is input, $y(t)$ is output, and $u(t-\tau)$ is the unit impulse response of the system. If x is rainfall and y is runoff, then u is the instantaneous unit hydrograph. Deconvolution is the process of identifying the functional relationship between u and time, given a historical record of x and y. The Neuman-de Marsily method focuses on the deconvolution of short records (single or multiple rainfall events) when the data are corrupted by noise. Although our work deals with rainfall-runoff relations, the method is equally applicable to the relations between streamflow and groundwater recharge, or groundwater pumpage and water table decline.

The method consists of representing u by discrete values at fixed time intervals, Δt, and identifying these values by parametric linear programming. The parametric programming aspect of the method arises from the fact that in identifying the ordinates of u, one attempts to minimize two criteria simultaneously: A performance criterion, J_C, measuring the difference between observed and computed outputs, y, and another criterion, J_p, measuring the "plausibility" of the computed functional relationship between u and time. The particular plausibility criterion employed by Neuman and de Marsily measures the degree of smoothness of the estimated impulse response function: The smoother is this function, the smaller is the value of J_p. The choice of such a smoothness criterion is based on the belief that watersheds are strongly damping systems and as such, u should not exhibit high-frequency oscillations.

Since J_c and J_p are noncommensurate, one is faced with a multicriterion or vector optimization problem of the kind

$$\underset{\underline{x}}{\text{v min}} \quad \underline{J}(\underline{x}) \tag{2}$$

subject to

$$\underline{g}(\underline{x}) = 0 \tag{3}$$

where $\underline{x}$ is a vector of real nonnegative decision variables including the ordinates of u, $\underline{J}(\underline{x})$ is the vector (J_c, J_p) of real-valued objective functions, and $\underline{g}(\underline{x})$ is a vector of real-valued constraint functions, including the non-negativity condition on each component of $\underline{x}$. The symbol vmin indicates that minimization is to be performed in a vector sense, i.e., the problem is to find all feasible points $\underline{x}$ in the decision space that are noninferior: A noninferior point is one at which decreasing J_c will automatically cause an increase in J_p, or vice versa. Fig. 1 shows qualitatively how J_c varies with J_p within the noninferior domain of decision variables when the Neuman-de Marsily method is used. Clearly, the solution

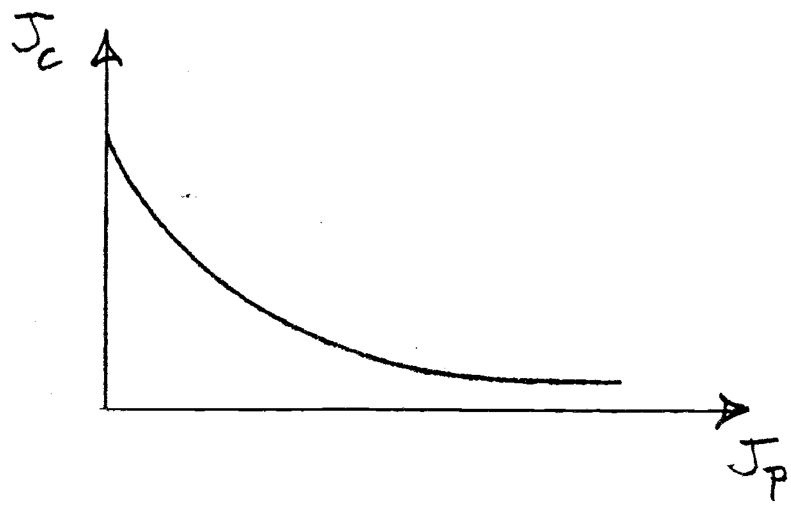

Fig. 1. Bicriterion noninferior curve of J_c versus J_p.

is nonunique, and one cannot find a single optimum impulse response function in this manner.

The noninferior set and bicriterion curve can be generated by using one of two general approaches: The ε-constraint approach, and the Lagrange multiplier approach (e.g., Neuman and Krzysztofowicz, 1977). Neuman and de Marsily (1976) as well as Dunbar (1981) use the first approach and implement it by means of parametric linear programming. In principle, it should be a relatively simple matter to implement the Lagrange multiplier approach with the aid of parametric quadratic programming as discussed in connection with the inverse problem of aquifer hydrology by Neuman (1976) and Neuman and Kafri (1976).

Since a noninferior solution is not unique, a decision must be made as to which solution in the noninferior set is "best." Neuman and de Marsily (1976) rely on the hydrologist's subjective judgement to choose the point on the J_c versus J_p bicriterion curve (see Fig. 1) that represents the preferred impulse response function. This judgement must be based on the recognition that as J_c decreases and J_p increases, the model output is closer to the observed output (i.e., the model performs better), but the resulting impulse response is less "plausible" (therefore, the model may perform poorly with different future inputs). On the other hand, as J_c increases and J_p decreases, n may become more plausible, but model performance deteriorates. The approach of Neuman and de Marsily calls for a judicial trade-off between these two situations.

The present work accomplishes two objectives: (1) It frees the Neuman-de Marsily deconvolution method from the high degree of subjectivity involved in selecting the preferred impulse response function among all the noninferior alternatives, and (2) it demonstrates through Monte Carlo simulations that the selected impulse response function is indeed best in that it has a smaller estimation error variance than all other alternatives. The Monte Carlo approach also provides a means of quantifying the uncertainty associated with the deconvolution model.

SELECTION OF PREFERRED IMPULSE RESPONSE FUNCTION BY COMPARATIVE RESIDUAL ANALYSIS

Each point (J_c, J_p) on the bicriterion curve is associated with a given noninferior impulse response function, and a set of residuals representing differences between observed and computed values of the output, y(t). The statistical properties of these residuals vary from point to point along the bicriterion curve. Thus, one might expect that by comparing various statistical properties of the residuals corresponding to different noninferior solutions, one should be able to gain useful insight into the nature of these solutions that might help in choosing the "best" among them.

Neuman and Yakowitz (1979) introduced the concept of comparative residual analysis in connection with the inverse problem of aquifer hydrology. They showed that if a given model is correct and both J_c and J_p are at their optimum values, the statistical properties of the residuals should closely reflect those expected from theoretical considerations. The reverse would also be true: The further J_c and J_p depart from their optimum values, the more should the residual properties deviate from those expected in theory. This implies that, for a given model, one should be able to identify the optimum values of J_c and J_p by analyzing the manner in which the residual properties vary along the bicriterion curve. Of particular interest to our problem is their finding that the degree of positive correlation exhibited bv the residuals should decrease as the solution approaches its optimum value.

The statistic chosen to measure such correlation is the lag-one autocorrelation coefficient given by

$$R_{ac} = \frac{\sum_{n=1}^{N-1} (e_n - e_{n+1})^2}{\sum_{n=1}^{N} (e_n)^2} \tag{4}$$

where N is the number of discrete output values and e_n is the residual between observed and computed outputs at time interval n. Applications to real and synthetic rainfall-runoff data show that R_{ac} often exhibits a sharp and distinct minimum near the optimum impulse response. However, there are cases where R_{ac} is not sufficient to designate a unique preferred solution. Additional statistics that were found useful in helping to identify the preferred solution include significance of regression, coefficient of variation, and residual sum of squares. Other possibilities should be examined in the future.

MONTE CARLO SIMULATIONS

To study the effect of measurement errors on our deconvolution model, and to investigate the above method of selecting a preferred solution, a series of Monte Carlo simulations were performed. The analysis consisted of superimposing random noise on synthetic rainfall-runoff data by a Monte Carlo procedure and applying the deconvolution method to each set of noisy data generated in this manner. Both white and correlated noise were considered. Approximately 50 Monte Carlo simulation runs were performed for each noise model. The results demonstrate that our procedure is capable of closely identifying the particular impulse response function that is associated with the least amount of estimation error among all alternative noninferior solutions.

Readers interested in further details regarding existing methods of rainfall-runoff modelling and deconvolution, the Neuman-de Marsily method, comparative residual analysis, Monte Carlo analysis, specific applications to real and synthetic rainfall-runoff data, and a description of the computer programs used in our study, are urged to consult the thesis of D. B. Dunbar (1981).

PUBLICATIONS RESULTING FROM RESEARCH

Dunbar, D. B., Analysis of a Parameter Estimation Technique for Linear Hydrologic Systems Using Monte Carlo Simulation, M.S. thesis, Dept. Hydrology and Water Resources, Univ. of Arizona, Tucson, 179 pp., 1981.

REFERENCES

Dunbar, D. B., Analysis of a Parameter Estimation Technique for Linear Hydrologic Systems Using Monte Carlo Simulation, M.S. thesis, Dept. Hydrology and Water Resources, Univ. of Arizona, Tucson, 179 pp., 1981.

Neuman, S. P., The inverse problem of groundwater hydrology: An approach based on parametric linear and quadratic programming, Proc. Intern. Seminar on Regional Groundwater Hydrology and Modelling, IBM Sci. Center, Venice, Italy, 210-245, 1976.

Neuman, S. P. and U. Kafri, The AM-FM concept in parameter identification, Advances in Groundwater Hydrology (Zubair A. Saleem, ed.), American Water Resources Assoc., Minneapolis, Minnesota, 83-101, 1976.

Neuman, S. P. and R. Krzystofowicz, An iterative algorithm for interactive multiobjective programming, Adv. Water Resourc., 1(1), 1-14, 1977.

Neuman, S. P. and G. de Marsily, Identification of linear systems response by parametric programming, Water Resour. Res., 12(2), 253-262, 1976.

Neuman, S. P. and S. Yakowitz, A statistical approach to the inverse problem of aquifer hydrology: I. Theory, Water Resour. Res., 15(4), 848-860, 1979.