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PREDICTION OF SEEPAGE THROUGH CLAY SOIL LININGS IN REAL ESTATE LAKES

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OFFICE OF WATER RESEARCH AND TECHNOLOGY PROJECT A-055-ARIZ

PREDICTION OF SEEPAGE THROUGH CLAY SOIL LININGS IN REAL ESTATE LAKES

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The University of Arizona

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ABSTRACT

The rapid expansion in the development of real estate lakes in the Southwest has produced a somewhat haphazard use of clay soils or clays in attempts to seal these lakes against seepage losses. This situation is further aggravated by the fact that very little basic information exists at present on the equilibrium and movement of water in a swelling clay soil, which is the type of natural lining material of direct relevance to seepage control. This report presents new results in the theory of swelling clay soils, including a description of the equilibrium moisture profile and the steady flow of water in a submerged, saturated, natural clay soil liner. The theory then is applied to develop an equation for the rate of seepage (the rate of lowering of the water surface) through a swelling liner in a real estate lake of simple trapezoidal configuration. This equation is compared to the standard results for the seepage rate, as calculated on the classical theory of water flow through non-swelling soils, and is applied to estimate the seepage rate from an experimental reservoir studied by Rollins and Dylla. The principal conclusions are: (a) that the major effect of swelling in the liner, except for very shallow lakes, is to cancel the contribution of gravity to the seepage rate, (b) that the most important factor determining seepage loss is likely to be the soil water tension in the pervious soil surrounding the lake and liner, and (c) that the seepage equation can provide a useful estimate of the rate of loss when the important geometric and soil water parameters for the lake, the liner, and the surrounding soil are available.

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1. Introduction

The development of small, shallow lakes as central features of industrial parks or residential subdivisions has become increasingly widespread throughout the nation in recent years. Even the semi-arid regions of Arizona and Southern California, where the problems of design and management of these developments are probably the most serious, are witness to a rapidly-growing tendency to employ artificial bodies of water as integral parts of the planning of new residential areas. The reasons for this activity - water-oriented recreation, aesthetic enhancement, and the improvement of storm runoff - are nearly self-evident, particularly in view of the now general public interest in the preservation of open space to break the monotony of urban sprawl.

Typically the real estate lake, as these water bodies are usually called, comprises between 5 and 50 acres of water surface, although extremes of less than one acre to more than several hundred acres can be found. They are surrounded commonly by single-family dwellings or by mixtures of single- and multiple-family units. Their construction is usually through the impoundment of streams or through excavation and subsequent filling with ground and surface waters. Once completed, the real estate lake, under proper management, is not allowed to dry out; however, seasonal variations in water level within anticipated limits are likely, especially in arid regions.

A significant number of standard, important hydrologic problems is associated with the design and management of the real estate lake (Rickert and Spieker, 1972). A major one in Arizona is the control of water losses due to seepage (Boyer and Cluff, 1972). This problem has been met typically in a variety of ways, among them (a) the placing of a physical barrier over the lake bed, such as a soil or clay blanket of low permeability, a plastic or rubber membrane, and a concrete or asphaltic sealant; (b) the use of chemical compounds, such as resinous polymers, to create a hydrophobic condition in the bed material, and (c) the application of salts such as sodium carbonate to enhance the swelling characteristics and, therefore, to decrease the permeability of the natural lake bed (Cedergren, 1967, p. 444 ff; Myers, 1969; Boyer and Cluff, 1972). In the first-mentioned method, the blanket material may be a natural soil made "impermeable" through the addition of sodium to its exchange complex or it may be a highly-swelling pure clay such as Wyoming bentonite. This latter material can be one of the more advantageous economically to employ in a problem of seepage control, if its application is planned and executed carefully and the chemical properties of the lake water are given due attention (Shen, 1959; Rollins and Dylla, 1970; Boyer and Cluff, 1972). However, it is possible that its effective use heretofore may have been precluded somewhat by the fact that very little systematic information has been available with regard to its hydrologic properties, in particular, the nature of water equilibrium and steady flow through it when it is saturated. It is expected that to have at hand a more accurate description of these phenomena than has been available will only serve to enhance the attractive features these lining materials already may possess.

Until quite recently the behavior of water in a highly-swelling clay soil was assumed to be described accurately enough by the well-known physical theory of moisture equilibrium and movement in non-swelling soil (e.g., the Buckingham equation for equilibrium under gravity, Darcy's law involving the usual pressure and gravity components of hydraulic head, etc.).

However, Philip (1969a, 1969b, 1969c, 1971) has indicated in a series of pioneering articles, wherein he took the initial steps toward establishing a theoretical foundation for the hydrology of swelling soils, that the various soil water phenomena may differ significantly, even spectacularly, in a swelling porous medium from what they are in a non-swelling medium. The extension of Philip's work to provide more general thermodynamic derivations of his fluid-mechanical results was initiated by Groenevelt and Bolt (1972) and by Sposito (1972, 1973, 1975). In these very recent papers some of Philip's results are established rigorously while others, which already had been questioned in the literature (Youngs and Towner, 1970; Collis-George and Bridge, 1973), are shown to be incorrect. These latter results, in fact, are just those concerning the generalization of the Buckingham equation and the steady flow of water. Thus, despite the qualitative success of the earliest work on the problem, the seepage of water through a swelling soil or clay cannot be said to have been understood thoroughly and the theoretical hydrology of this type of porous medium must be regarded as only in the first stages of development.

In this report the seepage of water through a swelling liner material is considered in detail. Section 2 begins with the derivation of the generalization of the Buckingham equation, which describes the equilibrium vertical distribution of soil moisture. This work is followed by the presentation of a new integral equation for the moisture profile which is directly applicable to the seepage problem. In all cases the equations found reduce to the appropriate expressions for non-swelling media when the terms involving swelling are set equal to zero. In Sec. 3 the steady flow of water through a swelling clay or soil liner is considered and a general equation for such flow is derived. Finally, in Sec. 4, the theoretical results are applied to the prediction of seepage through a swelling liner in a lake of simple, but realistic, geometry. A practical equation which gives the seepage rate as the equivalent rate of fall of the water surface is presented. Those readers not concerned with the detail of the theory of water in swelling media may wish to pass directly to Sec. 4 and its applicable results. A summary and conclusions are given in Sec. 5, and an example of a numerical calculation is presented in the Appendix.

No attempt is made in this report to analyze Philip's early work in detail or to discuss the steps in his derivations which are believed to have produced erroneous results, as these considerations were deemed beyond the scope of the project objectives and more appropriate for an article in a technical journal. However, a continual effort is made to compare the results obtained with well-known expressions for soil water behavior and with the accepted equations for seepage through non-swelling lining materials (Bouwer, 1969).

2. The Equilibrium Moisture Profile in a Lining Composed of Swelling Soil or Clay

The variation of moisture content with changes in the vertical coordinate in a porous medium produces the moisture profile. For a salt-free soil which does not swell or exhibit hysteresis in its moisture characteristic, the equilibrium moisture profile is described by the well-known Buckingham equation (Buckingham, 1907). This equation may be written

$$\frac{d\theta}{dz} = \frac{-\rho_w g}{(\partial\tau/\partial\theta)_{T,P}} \tag{1}$$

where θ is the mass of water per unit mass of dry soil, τ is the water potential, ρ_w is the density of pure water in a chosen reference state, g is the acceleration due to gravity, and z is the vertical coordinate, measured positive upward from some reference plane. The partial derivative of the moisture characteristic, $\tau(T,P,\theta)$ is evaluated at a fixed temperature T and externally-applied pressure P. The extension of Eq. (1) to swelling porous media may be accomplished by generalizing its thermodynamic derivation, which was given first by Babcock and Overstreet (1957).

A swelling soil or clay free of dissolved substances may be described completely in terms of the following independent thermodynamic variables (Sposito, 1973):

T = absolute temperature

P = externally-applied pressure

m_w = mass of water

m_s = mass of soil or clay

μ_a = chemical potential of air

The chemical potential of air (the soil or clay is assumed to be in equilibrium with the atmosphere) is chosen rather than the more obvious mass of air because the former variable is much more easily measured and controlled than the latter variable. For example, the laboratory control of μ_a requires only the monitoring of the local barometric pressure.

Given the thermodynamic variables, the gravichemical potential of water in a swelling porous medium can be written down at once (Sposito, 1975). In differential form this is

$$d\mu_{gw} = -\bar{S}_w dT + \bar{V}_w dP + \left(\frac{\partial \mu_w}{\partial m_w}\right)_{T,P,\ldots} dm_w + \left(\frac{\partial \mu_w}{\partial m_s}\right)_{T,P,\ldots} dm_s + \left(\frac{\partial \mu_w}{\partial \mu_a}\right)_{T,P,\ldots} d\mu_a + d\phi \qquad (2)$$

where:

$\mu_{gw} = \mu_w + \phi = \mu_w + gz$ = chemical potential + gravitational potential

$\bar{S}_w = \left(\frac{\partial S}{\partial m_w}\right)_{T,P,\ldots}$ S = total entropy of soil-water-air system

$\bar{V}_w = \left(\frac{\partial V}{\partial m_w}\right)_{T,P,\ldots}$ V = total volume of the system

Equation (2) describes a thermodynamic process involving water in a so-called gravitational phase, which is a volume of soil, water, and air that is small enough to justify the use of point functions such as $\theta(z)$ to describe its properties, but is large enough to justify the use of thermodynamics. The soil profile is a vertical continuum of such gravitational phases. If the temperature and the barometric pressure are held constant and the identity (Sposito, 1975)

$$\left(\frac{\partial \mu_w}{\partial \theta}\right) d\theta = \left(\frac{\partial \mu_w}{\partial m_w}\right)_{m_s} dm_w + \left(\frac{\partial \mu_w}{\partial m_s}\right)_{m_w} dm_s \tag{3}$$

is introduced, Eq. (2) reduces to

$$d\mu_{gw} = \bar{V}_w dP + \left(\frac{\partial \mu_w}{\partial \theta}\right)_{T,P} d\theta + d\phi . \tag{4}$$

The variation in gravichemical potential thus is related to variations in the pressure on a gravitational phase, in its moisture content, and in its locations relative to the water table. The coefficient of the pressure variation, $\bar{V}_w$, is equal to the slope of the swelling curve, which is a graph of the volume of the phase, as it swells reversibly against a constant applied pressure, versus the mass of water absorbed (Sposito, 1973). This point is illustrated in Figure 1. For a non-swelling soil at moisture contents below the value at air entry, $\bar{V}_w = 0$. For a swelling soil, $\bar{V}_w$ varies between 0 and $1/\rho_w$ as θ increases from 0 to its value at air entry, as indicated in the figure.

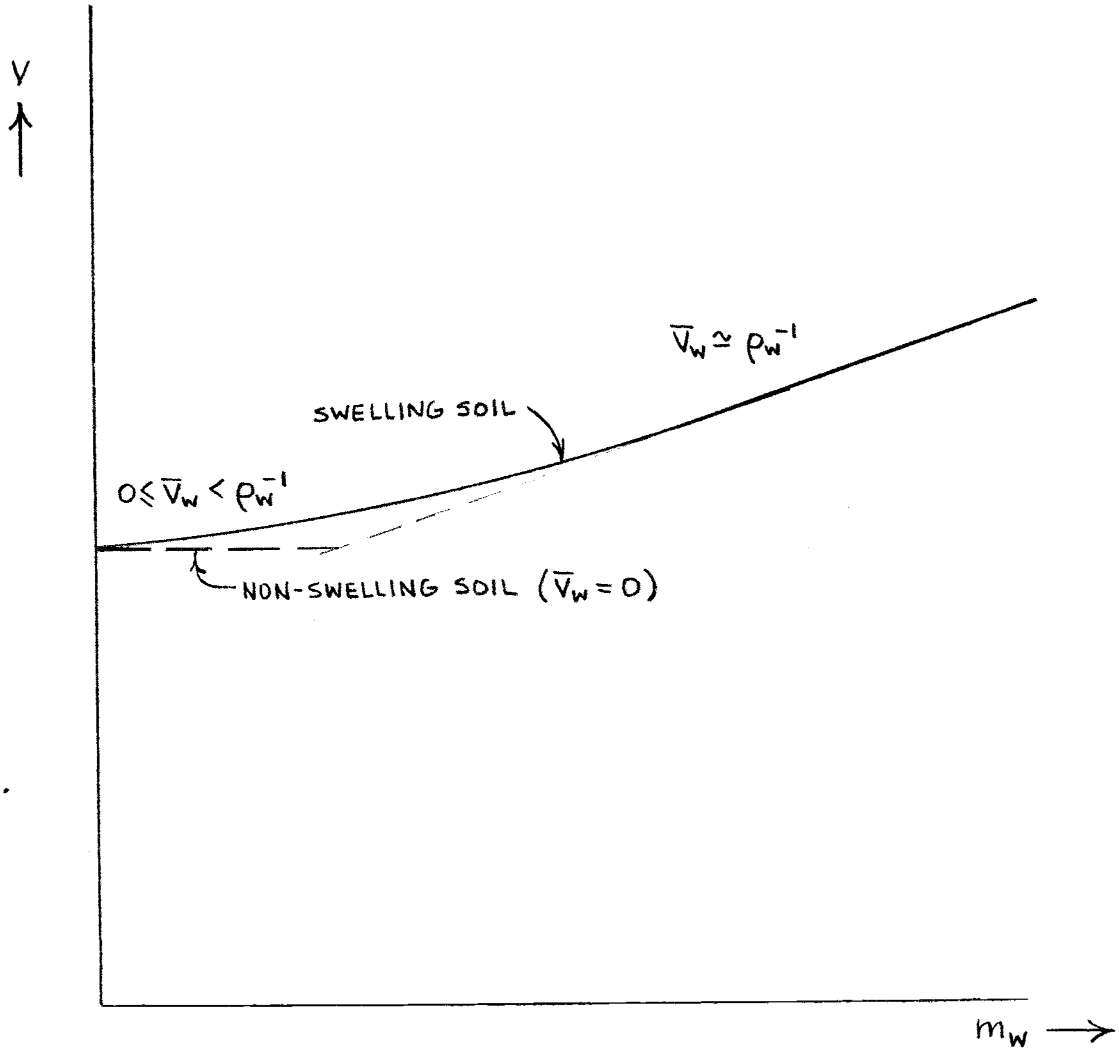

Figure 1. The volume of a structureless, swelling soil as a function of the mass of water absorbed. The dashed line shows the behavior of a non-swelling soil in the unsaturated range.

The condition for thermodynamic equilibrium of the water in a soil is that μ_{gw} have the same value for every gravitational phase: $d\mu_{gw} = 0$. Under this condition Eq. (4) may be rearranged to provide an expression for $d\theta/dz$:

$$\frac{d\theta}{dz} = \frac{(\bar{V}_w \rho_{bw} - 1)}{(\partial\mu_w/\partial\theta)_{T,P}} g \qquad (d\mu_{gw} = 0) \tag{5}$$

where ρ_{bw} is the wet bulk density of the soil and the condition for hydrostatic equilibrium,

$$dP = -\rho_{bw} g \, dz, \tag{6}$$

has been employed. Equation (5) is the required generalization of the Buckingham equation. It reduces to that expression upon introducing the definition $d\tau = \rho_w d\mu_w$ and setting $\bar{V}_w = 0$. Since the partial derivative $(\partial\mu_w/\partial\theta)_{T,P}$ is always observed to be a non-negative quantity, it follows from Eq. (5) that

$$\frac{d\theta}{dz} > 0 \qquad \text{if} \qquad \rho_{bw} > \bar{V}_w^{-1} \tag{7a}$$

$$\frac{d\theta}{dz} = 0 \qquad \text{if} \qquad \rho_{bw} = \bar{V}_w^{-1} \tag{7b}$$

$$\frac{d\theta}{dz} < 0 \qquad \text{if} \qquad \rho_{bw} < \bar{V}_w^{-1} \tag{7c}$$

Therefore, the moisture content at a point in the profile of a swelling soil may increase, remain constant, or decrease with increasing height z above the water table, depending critically upon the relation between the local values of the wet bulk density and the slope of the swelling curve.

(For the unsaturated, non-swelling soil free of entrapped air, $\bar{V}_w^{-1}$ is infinite and condition (7c) is always met.)

An expression for $\theta(z)$ itself may be obtained by integrating Eq. (4). The path of integration is chosen so as to describe mathematically the following thermodynamic process: A gravitational phase, initially in equilibrium at the water table, is isolated from the soil profile and put under atmospheric pressure. It is then allowed to gain water until it has once again come into equilibrium with pure water under the atmosphere at the same elevation. This part of the process will not change the gravichemical potential of the water in the phase. Next the phase is brought into contact with a series of water reservoirs, each at a successively decreasing value of μ_w, until its moisture content has decreased to a certain value θ and the gravichemical potential has decreased numerically by $\Delta\mu_w(\theta)$. Finally, the phase is brought to a new position z above the water table while the pressure on it is increased to the value associated with the water content $\theta(z)$ in the equilibrium moisture profile. This step is accomplished by increasing the pressure on the phase from that of the atmosphere to that of any surface load, through compression, and by permitting the phase to be subject to the overburden pressure due to the wet soil above it. In this way there will be no change in the moisture content of the phase and the gravichemical potential will be returned to the value it had at the beginning of the entire process. The corresponding integrated form of Eq. (4) is, therefore,

$$0 = \int_{P_0}^{P(H)} \bar{V}_w \, dP + g \int_z^H \bar{V}_w \rho_{bw} \, dz' + \Delta\mu_w(\theta(z)) + gz \qquad (8)$$

where H is the height of the soil surface above the water table, P_0 is the atmospheric pressure, and $P(H)-P_0$ is the gauge pressure exerted by any surface load. Equation (8) can be written in a form more suitable for numerical work by introducing the identity

$$z \equiv H - \int_z^H dz'. \tag{9}$$

Then

$$-\Delta\mu_w(\theta(z)) = \int_{P_0}^{P(H)} \bar{V}_w dP + gH + g\int_z^H (\bar{V}_w \rho_{bw} - 1)dz'. \tag{10}$$

Equation (10) is an implicit, non-linear integral equation for the moisture profile $\theta(z)$. Its practical use will depend upon the availability of data on: the moisture characteristic, $-\Delta\mu_w(\theta)$, for a very thin sample of the swelling soil under atmospheric pressure, the slope of the swelling curve, $\bar{V}_w$, as a function of θ and the pressure, and the wet bulk density ρ_{bw} as a function of the same two parameters. A special advantage will be had if these quantities can be expressed as relatively simple functions of θ and the pressure.

As a first example of the use of Eq. (10), $\theta(z)$ will be computed for saturated Na-bentonite. In this case, because of saturation,

$$\bar{V}_w = \rho_w^{-1} \qquad \rho_{bw} = \rho_c(1+\theta)/[1 + (\rho_c\theta/\rho_w)] \tag{11}$$

where ρ_c is the density of the dry clay particles. Equation (10) becomes, accordingly,

$$-\Delta\mu_w(\theta(z)) = gH + g(S-1)\int_z^H \frac{dz'}{1+S\theta(z')} \tag{12}$$

in the absence of any surface load, where $S=\rho_c/\rho_w$. Now, in the range of relative moisture content $0.5<\theta<8$ the chemical potential of water in Na-Wyoming bentonite at 25^o C under atmospheric pressure may be described by the empirical equation (Sposito, 1973)

$$-\Delta\mu_w(\theta) = 1.75 \times 10^4 g\, \theta^{-2.01} \tag{13}$$

where $\Delta\mu_w$ and g are expressed in CGS units. Equation (13) is a special case of the general empirical relation

$$-\Delta\mu_w(\theta) = gh\theta^{-b} \qquad (h, b, > 0) \tag{14}$$

which has been proposed to describe water in soils (Hillel, 1971, p. 63). The introduction of Eq. (14) and the definition $x\equiv 1/\theta$ brings Eq. (12) into the form

$$x(z) = \left\{\left[H + (S-1)\int_z^H \frac{x(z')}{S+x(z')}\, dz'\right]/h\right\}^{1/b}. \tag{15}$$

This is a non-linear integral equation for $x(z)$ which may be solved by interation on a digital computer, beginning with the "zeroth approximation" $x_0(z) = (H/h)^{1/b}$.

In Table 1 are listed some values of $\theta(z)$ calculated using Eqs. (13) and (15) along with S=2.808 (Low and Anderson, 1958) and H=600 cm. (This value of H was chosen solely to produce a value of $\theta(H)$ in the range of applicability of Eq. (13).) Also shown in the table, for purposes of comparison, are the values of $\theta(z)$ obtained from a single iteration of Eq. (15) and from setting $\bar{V}_w=0$ in Eq. (10) ("non-swelling condition"). It is evident from a look at the numerical data that extensive iteration

Table 1 - The moisture profile in Na-Wyoming bentonite for a water table at depth 600 cm. Also shown are the first estimate of the profile and the profile calculated without accounting for swelling ($\bar{V}_w$=0).

z(cm)	Equilibrium $\theta(z)$	First estimate	"Non-swelling limit"
600	4.96	4.96	4.96
500	4.91	4.91	5.40
400	4.86	4.86	6.01
300	4.82	4.82	6.89
200	4.77	4.78	8.35
100	4.73	4.74	11.60
10	4.69	4.70	34.60

of Eq. (15) was not necessary for producing accurate results and that a marked difference exists between the actual $\theta(z)$ and the values computed under the assumption $\bar{V}_w=0$. The moisture content in the swelling clay is observed to decrease as the water table (z=0) is approached because of the indirect effect of the overburden pressure on the chemical potential of the water. However, since $\bar{V}_w$ has been taken equal to a constant, the effect is not very great in the present case.

As a second example of the application of Eq. (10), the moisture profile may be considered for a saturated, swollen clay liner over which pure water has been ponded to a depth D. In this case the parameter H is equal to -D because the "water table" is congruent with the surface of the ponded water. The values of the vertical coordinate z also are negative. With Eqs. (11) and (14) incorporated, Eq. (10) takes on the form

$$[\theta(z)]^{-b} = \frac{(S-1)}{h}\int_z^{-D}\frac{dz'}{1+S\theta(z')} \qquad (z \leq -D) \tag{16}$$

or, for purposes of computation,

$$x(z) = \left[\frac{(S-1)}{h}\int_z^{-D}\frac{x(z')\,dz'}{S+x(z')}\right]^{1/b} \qquad (z \leq -D) \tag{17}$$

where $x=1/\theta$ once again. The integral equation (17) predicts $x(-D)=0$, i.e., that the water content is infinite at the liner's upper surface, which is also the bottom of the ponded water. This result is due entirely to the mathematical form of Eq. (14), which requires that θ be infinite when $\Delta\mu_w=0$. This point can be seen in detail through an examination of Eq. (8). At the bottom of the ponded water the first and last terms on the RHS of (8) cancel one another. Since the second term on the RHS must be zero when z=-D, Eq. (8) reduces to $\Delta\mu_w=0$ and the conclusion follows.

It is also directly evident from Eq. (17) that x(z)=0 is a <u>solution</u> of the integral equation. This solution, although it is always mathematically possible, will be physically reasonable only so long as

$$z + D + \int_{z}^{-D} \bar{V}_w \rho_{bw} \, dz' \simeq 0 \tag{18}$$

according to Eq. (8). The validity of the condition (18) is assumed either if $\bar{V}_w \rho_{bw} \approx 1$ (See Eq. (7b).) or if the swollen clay liner is sufficiently thin. To make this latter requirement more specific, let

$$\rho_{bw} \simeq \rho_w S(1+\bar{\theta})/(1+s\bar{\theta})$$

where $\bar{\theta}$ is an average moisture content of the liner. Then Eq. (8) may be written

$$[\theta(z)]^{-b} \simeq \frac{|z|-D}{h} \frac{S-1}{1+S\bar{\theta}} .$$

If the condition

$$\frac{|z|-D}{h} << \frac{1+s\bar{\theta}}{S-1} \tag{19}$$

is met, $\theta(z)$ will be effectively "infinite" throughout the liner. In particular, for the case of Na-Wyoming bentonite with $\bar{\theta}$ as low as 0.5, the condition (19) becomes $|z|-D << 2.3\times10^4$ cm, which would be satisfied in every practical case, since the maximum value of $|z|-D$ is the thickness of the liner. Therefore, it may be concluded that <u>the equilibrium moisture content of a saturated, swollen clay liner overlain by ponded water will in all likelihood be constant throughout</u>. This result is fortuitously identical with what would be found for a saturated, <u>non-swollen</u> liner.

3. Steady Water Flow Through a Lining Composed of Swelling Soil or Clay

The isothermal, steady flow of water through a swelling soil or clay which is free of dissolved substances may be described by a linear flux equation developed in the context of non-equilibrium thermodynamics (Nielsen, et al., 1972, Chapter 3):

$$\vec{J}_w = -\frac{K}{g}\vec{\nabla}\mu_{gw} \tag{20}$$

where J_w is the current density, the volume of flow per unit time per unit area of cross section, and K is the hydraulic conductivity, a function of the moisture content θ. Equation (20) is, of course, a form of Darcy's Law. For vertical flows it reduces to

$$v_z = -\frac{K}{g}\frac{d\mu_{gw}}{dz} \tag{21}$$

where $v_z \equiv J_{wz}$ and $d\mu_{gw}$ is given in Eq. (4). A computationally-useful form of Eq. (21) is

$$0 = g\left(\frac{v_z}{K} + 1\right)dz + \bar{V}_w\,dP + (\partial\mu_w/\partial\theta)_{T,P}\,d\theta \tag{22}$$

which suggests that the constant-flow situation differs from the equilibrium situation in that the gravitational acceleration is augmented by the factor $(1+\frac{v_z}{K})$. An upward flow is thus equivalent to equilibrium in a hypothetical gravitational field of greater strength than g; a downward flow corresponds to a "field" of lesser strength.

As an example of the use of Eq. (22), the steady, vertical flow of water through a swelling soil not subject to a surface load may be considered. With the help of Eq. (6), Eq. (22) may be written in the form

$$\frac{v_z}{K} = \rho_{bw}\bar{V}_w - 1 - \frac{1}{g}\left(\frac{\partial \mu_w}{\partial \theta}\right)_{T,P} \frac{d\theta}{dz} . \tag{23}$$

It is the first term on the RHS of (23) which distinguishes it from the equation which would apply to a non-swelling soil. Because of this term, it is evident that the condition $(\partial\mu_w/\partial\theta)_{T,P} > 0$ is no longer sufficient to ensure that the direction of v_z will be negative when the moisture gradient, $d\theta/dz$, is positive. Indeed, if the product $\rho_b\bar{V}_w$ is large enough, steady flow will be upward in spite of the fact that the moisture content decreases with depth. In this case the effect of the overburden pressure has more than compensated the effects of gravity and moisture content. The pressure has raised the gravichemical potential of the water at the end of the path of transport enough to make $d\mu_{gw}/dz$ a negative quantity.

4. Seepage through a Clay Soil Lining

Because of the many complexities involving man-made lakes in nature, a theoretical description of seepage from these systems must commence with a simplification of the actual soil and boundary conditions. Otherwise a mathematical treatment may become so complicated that it presents an intractable numerical problem or obscures the essential aspects of the seepage process in a cloud of analytical detail. In order to avoid these possibilities and to construct a simple mathematical description, the irregularity of the lake cross section; the effects of precipitation and sedimentation and of biological phenomena; the chemical properties of the impounded water, and heterogeneity in the lining material will be neglected. Accordingly, the results to be obtained should be regarded only as first estimates of the true seepage rate in the field, although they will be accurate and indicative of the essential physical processes involved.

In Figure 2 is shown a diagram and section of a lake of simple geometric configuration whose wetted perimeter has been lined with a relatively thin layer of swelling clay. Besides the assumption of this simplified geometric configuration, the following additional conditions are specified:

(a) The liner is saturated and has a constant moisture content $\bar{\theta}$, in correspondence with the results in Sec. 2. The hydraulic conductivity K_c then will be constant also. The temperature and atmospheric pressure are assumed not to vary.

(b) The underlying soil is pervious and characterized as a whole by a hydraulic conductivity K_s which is much larger than K_c. This soil is in equilibrium with the atmosphere.

(c) The water table remains deep enough for the lake bottom to be above the capillary fringe at all times. Therefore, given condition (b), the soil surrounding the lake bed will be unsaturated. However, because the liner is saturated and is overlain by ponded water, there will be seepage despite the surface tension effects at the liner-soil boundary (Taylor and Ashcroft, 1972, p. 199f). The loss of hydraulic head due to these effects will be neglected (Bouwer, 1964).

Equation (22) now may be applied to compute the steady flow through the bottom of the lake bed. It turns out to be very instructive to do

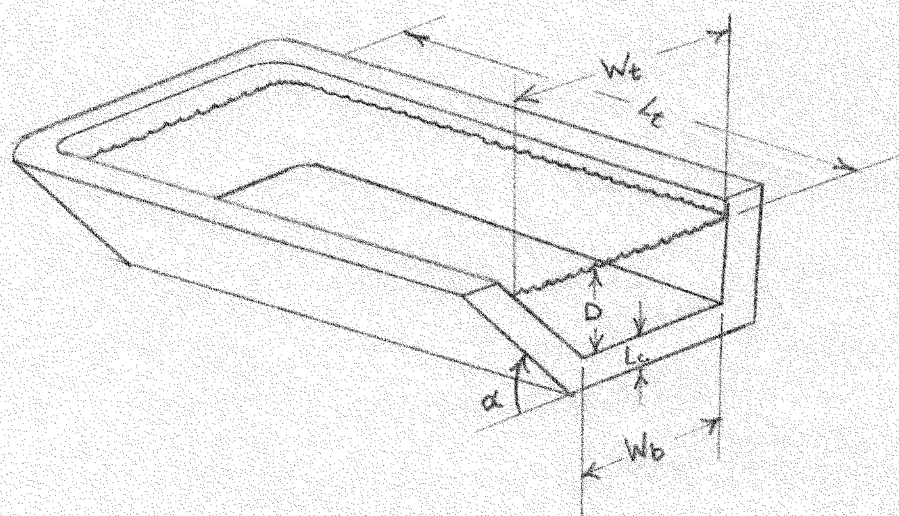

Figure 2. Cross section and diagram of a real estate lake lined with a layer of swelling soil or clay. The thickness of the liner is shown exaggerated for clarity.

the calculation first as if the liner material did not swell. In this way the technique of computation can be illustrated more simply and a contact can be made in a direct fashion with the well-known results which have been derived previously employing only standard fluid-mechanical methods (Bouwer, 1969, pp. 139-143). For steady flow through the bottom of the lake, the appropriate integrated form of Eq. (22) is (b="bottom"):

$$-g\left(\frac{V_b^{NS}}{K_c}+1\right)\int_{-D}^{-D-L_c} dz = \rho_w^{-1}\int_{P_o+\rho_w g D}^{P_o} dP + \int_{\mu_{sw}}^{\mu_w^o} d\mu_w \qquad (24)$$

where P_o is atmospheric pressure, μ_{sw} is the chemical potential of water in the underlying soil, μ_w^o is the chemical potential of pure water under atmospheric pressure, and

$$d\mu_w \equiv \left(\frac{\partial \mu_w}{\partial \theta}\right)_{T,P} d\theta$$

has been written in the last integral since the pressure remains fixed and P_o during that integration. In the context of the "equivalent gravitational equilibrium" picture, mentioned below Eq. (22), the LHS of Eq. (24) gives the change in "gravitational potential" across the clay lining, while the RHS gives the changes in pressure and in chemical potential of the water (without pressure effects), respectively. When the integrations are carried out, Eq. (24) becomes

$$\frac{V_b^{NS}}{K_c} = -\frac{[D + L_c + (\Delta\mu_{sw}/g)]}{L_c} \qquad (25)$$

where $\Delta\mu_{sw} \equiv \mu_{sw} - \mu_w^o$. Equation (25) is identical with Eq. (20) of Bouwer (1969, p.141), which describes the same seepage process. (Note that Bouwer sets $P \equiv \Delta\mu_{sw}/g$ and otherwise has slight differences in notation.)

For a swelling liner material, seepage through the bottom is described by the following integrated form of Eq. (22):

$$-g\left(\frac{V_b}{K_c}+1\right)\int_{-D}^{-D-L_c} dz = \rho_w^{-1}\int_{P_o+\rho_w gD}^{P_o} dP - \int_{-D}^{-D-L_c} \bar{V}_w \rho_{bw} g\, dz + \int_{\mu_{sw}}^{\mu_w^o} d\mu_w \,. \tag{26}$$

It is clear that the only difference between (26) and (24) is in the integral term

$$-\int_{-D}^{-D-L_c} \bar{V}_w \rho_{bw} g\, dz \equiv -g\tilde{L}_c \tag{27}$$

which represents the overburden pressure on the water in the clay liner. Thus Eq. (26) becomes

$$\frac{V_b}{K_c} = -\frac{[D + L_c - \tilde{L}_c + (\Delta\mu_{sw}/g)]}{L_c} \,. \tag{28}$$

The swelling nature of the liner contributes an additional term $-\tilde{L}_c$ to the hydraulic head in the Darcy expression (28). For the purpose of comparison, Eq. (28) may be written

$$\frac{V_b}{K_c} = \frac{V_b^{NS}}{K_c} + \frac{\tilde{L}_c}{L_c} \,. \tag{29}$$

Since v_b^{NS} is always negative, corresponding to downward flow, and since $\tilde{L}_c$ is always a positive quantity, Eq. (29) shows that <u>the effect of swelling is always to decrease the seepage rate</u>. Note that this effect has <u>nothing</u> to do with the value of the hydraulic conductivity, K_c, which has been assumed to be the same in both Eqs. (25) and (28). The effect is due entirely to the overburden pressure causing an increase in the gravichemical potential and, therefore, a decrease in the absolute value of the flow-producing gradient, $d\mu_{gw}/dz$.

The combination of Eqs. (27) and (11) produces

$$\tilde{L}_c = S \int_D^{D+L_c} \frac{1+\theta(x)}{1+S\theta(x)} dx \tag{30}$$

where $x \equiv -z$. Since $\theta(x)$ is presumed to be constant, this expression reduces at once to

$$\tilde{L}_c = L_c \left[\frac{S(1+\bar{\theta})}{1+S\bar{\theta}} \right] \tag{31}$$

where $\bar{\theta}$ is the constant value of $\theta(x)$. It follows that Eq. (28) can be expressed in the form

$$\frac{V_b}{K_c} = -\frac{D}{L_c} + \frac{(S-1)}{1+S\bar{\theta}} - \frac{\Delta\mu_{sw}}{g L_c} . \tag{32}$$

The first term on the RHS of Eq. (32) represents the gradient of hydraulic head due to the pressure exerted by the water in the lake; the second term represents that due to the combined effect of gravity and overburden pressure across the liner, and the third term represents that due to the water potential in the underlying soil. The first and third terms are readily measured in the field (depth gauge, tensiometer, etc.). The second term can be calculated if $\bar{\theta}$ and the specific gravity of the dry clay particles are known. $\bar{\theta}$ may be estimated directly if the validity of Eq. (14) is assumed: According to the discussion in Sec. 2, the equilibrium value of $\bar{\theta}$ should be effectively "infinite". In that case the second term simply vanishes. Generally $\bar{\theta} \simeq 4$ in a Na-bentonite would not be unreasonable and the second term (with S=2.8) would be expected to be no larger than about 0.25. Therefore, it appears that the primary effect of taking into account the swelling nature of the lining material is the essentially complete cancellation of the gravity contribution to the hydraulic head in Eq. (28); i.e., $\tilde{L}_c \simeq L_c$. This result justifies even further the approximate expression

$$\frac{v_b}{K_c} \simeq -\frac{D}{L_c} - \frac{\Delta\mu_{sw}}{g L_c} \tag{33}$$

which was recommended by Bouwer (1969, p. 142) on the basis of a theoretical development that did not take into consideration the swelling phenomenon.

A practical form of Eq. (32) may be obtained by rewriting v_b in terms of I_{sb}, which was defined by Bouwer (1964) to be the volume rate of downward flow per unit length of lake per unit width of water surface; i.e.,

$$I_s \equiv v\left(\frac{A_s}{A_t}\right) \tag{34}$$

where A_s is the area of the seepage surface (lake bottom, etc.) and A_t is that of the lake surface. Equation (32) then becomes

$$\frac{I_{sb}}{K_c} = \frac{W_b}{W_t}\left[\frac{D}{L_c} - \frac{(S-1)}{1+S\bar{\theta}} + \frac{T}{L_c}\right] \tag{35}$$

where $T \equiv -\Delta\mu_{sw}/g$ is the tension (or suction) head due to water in the underlying soil. The physical interpretation of I_{sb} is that it represents the average rate of fall of the lake surface due to seepage through the bottom.

Seepage through the right side of the lake diagrammed in Figure 2 is described by a special case of Eq. (20):

$$v_\perp = -\frac{K_c}{g}\frac{d\mu_{gw}}{ds} \tag{36}$$

where $\vec{ds}$ is an increment of flow path perpendicular to the side. (See Figure 3.) Using the geometric relation $ds = -dz/\cos\alpha$, where α is the angle of inclination of the side relative to the horizontal, Eq. (36) may be written in the integrated form:

$$-g\left(\frac{v_\perp}{K_c} - \cos\alpha\right)\int_{s(z)}^{s(z)+L_c} ds = \rho_w^{-1}\int_{P_o-\rho_w g z}^{P_o} dP - \int_z^{z-L_c\cos\alpha} \bar{V}_w \rho_{bw} g\, dz' + \int_{\mu_{sw}(z)}^{\mu_w^o} d\mu_w \tag{37}$$

which results in

$$\frac{v_\perp}{K_c} = \frac{-z}{L_c} - \frac{(S-1)\cos\alpha}{1+S\bar{\theta}} - \frac{\Delta\mu_{sw}(z)}{gL_c}. \tag{38}$$

(It is to be recalled that $z<0$ below the water surface ("water table") and that a <u>positive</u> value of $v_\perp$ corresponds to seepage through the right side of the lake bed.)

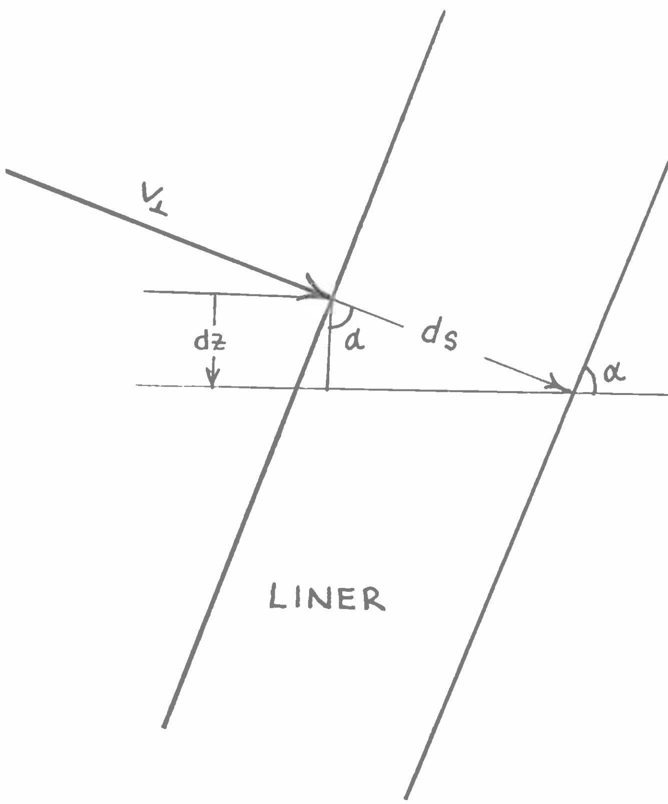

Figure 3. Relation between ds and dz for seepage through the side of the lake.

The flow velocity $v_\perp$, of course, is a function of depth. This depth dependence is a complicating feature which can be eliminated if the assumption is made the K_c and $\bar{\theta}$ and $\Delta\mu_{sw}$ are fixed in value everywhere in the liner and in its surrounding soil, respectively, and if the mean value

$$\bar{v}_\perp \equiv \int_{-D}^{0} v_\perp(z)\,dz \;/\; D \tag{39}$$

is introduced. Thus

$$\frac{\bar{v}_\perp}{K_c} = \frac{1}{2}\frac{D}{L_c} - \frac{(s-1)\cos\alpha}{1+s\bar{\theta}} - \frac{\Delta\mu_{sw}}{gL_c} \tag{40}$$

will describe the averaged steady flow through the right side. The inclusion of an identical expression (except, of course, for its sign) which describes the averaged seepage through the left side and the conversion of $\bar{v}_\perp$ to I_{ss} (s= "side") produces the final result:

$$\frac{I_{ss}}{K_c} = \frac{D/\sin\alpha}{W_t}\left[\frac{D}{L_c} - \frac{2(s-1)\cos\alpha}{1+s\bar{\theta}} + \frac{2T}{L_c}\right]. \tag{41}$$

Water seeping through the end faces of the lake bed should follow Eq. (40) given that the angle of inclination α is the same and that K_c, $\bar{\theta}$, and $\Delta\mu_{sw}$ are again taken constant. Since the ends are trapezoidal and curved their areas are expressed by $\frac{1}{2}$ (W_t' and W_b) D/sinα, where W_t' is measured along the wetted perimeter and so is somewhat larger than W_t, the width of the water surface. For the sake of simplicity, however, $W_t' \approx W_t$ may be used as an approximation and I_{se} (e = "end") may be expressed

$$\frac{I_{se}}{K_c} = \frac{(W_t + W_b)D/\sin\alpha}{2L_t W_t}\left[\frac{D}{L_c} - \frac{2(s-1)\cos\alpha}{1+s\bar{\theta}} + 2\frac{T}{L_c}\right] \tag{42}$$

where L_t is the length of a side of the lake embankment.

The total seepage from the lake finally is written as the sum of Eqs. (35), (41), and (42):

$$\frac{I_s}{K_c} = \frac{1}{W_t}\left\{\frac{D}{L_c}\left[W_b + \frac{D}{\sin\alpha}\left(1 + \frac{W_t + W_b}{2L_t}\right)\right] - \frac{(s-1)}{1+s\bar{\theta}}\left[W_b + 2D\cot\alpha\left(1 + \frac{W_t + W_b}{2L_t}\right)\right] + \frac{T}{L_c}\left[W_b + \frac{2D}{\sin\alpha}\left(1 + \frac{W_t + W_b}{2L_t}\right)\right]\right\}. \tag{43}$$

5. Summary and Conclusion

The principal results of this study are epitomized in Eqs. (5), (10), (16), (22), and (43). They form together a special example of the theory of water equilibrium and steady flow in swelling soil as developed using the methods of equilibrium and irreversible thermodynamics.

It has been shown in Sec. 2 that the equilibrium moisture profile in a swelling soil may exhibit an increase, no change, or a decrease in θ (moisture content) with increasing height above a water table. Just which possibility occurs depends sensitively upon the relation between the wet bulk density of the soil at a given point in the profile and the slope of its swelling curve at that point. (See Eqs. (7).)

According to a general integral equation (Eq. (10)) which describes the moisture profile in a salt-free soil, the moisture content in a saturated clay or soil liner for a real estate lake should remain essentially fixed in value with depth. Thus the saturated, swollen clay liner may be treated as a system with constant moisture content. This conclusion, of course, is always true for non-swelling media. It happens to be valid for a typical swelling clay because of the strong clay-water interaction (large h in Eq. (14)) and the generally small thickness of the liner.

Under normal conditions it would be assumed that the lining of a real estate lake remains saturated while the soil beneath it is rather pervious and unsaturated. Given these conditions it is reasonable to estimate seepage through the lining based upon considerations of steady flow. In Sec. 3 it was demonstrated that steady vertical flow in a swelling soil could be upward, zero, or downward when the moisture gradient was positive (moisture content decreases with depth). Again, the actual result will depend upon the relation between the wet bulk density and slope of the swelling curve and the other quantities (pressure head, moisture gradient) driving the flow. For a saturated clay liner it is found that the primary effect of the swelling phenomenon is the cancellation of the gravity head by the overburden pressure on the water flowing. Since it would be expected generally that, except for very shallow lakes, the pressure head due to the water bearing down on the liner is much larger than the gravity head across the liner, the effect of swelling should be minimal in practical cases. (This conclusion does not relate to the fact that the hydraulic conductivity would be smaller in a swelling medium than in a non-swelling medium. It is simply a result of the overburden pressure on water in a swelling soil, which does not exist in a non-swelling soil.) The taking into account of the swelling phenomenon would appear to provide a rigorous justification for neglecting the gravity head in seepage problems involving liners, a practice which has been recommended previously on less precise grounds (Bouwer, 1969, p. 142).

An expression for the seepage (equivalent fall of the lake surface) from a lake with the simple geometry diagramed in Figure 2 appears in Eq. (42). The second term of this equation represents the combined effects of gravity and swelling. As stated above, except for shallow lakes with thick linings, it will be quite accurate to neglect this term and use the remaining expression to estimate the seepage loss. This, of course, would also mean that more elaborate calculations, which take into account different geometries or present flow nets, and which have been based entirely upon the classical theory of non-swelling media (Bouwer, 1969; Cedergren, 1967), could be applied to describe seepage through a swelling clay liner with little or no loss in the accuracy of the estimate.

APPENDIX

A NUMERICAL EXAMPLE OF A SEEPAGE CALCULATION

As a typical example of the application of Eq. (43) to a seepage problem, I_s will be calculated for one of the reservoirs studied by Rollins and Dylla (1970) in their investigation of the effectiveness of several methods of controlling seepage with Na-bentonite clay. The "buried membrane" method which they tested is essentially the same as the arrangement of the clay liner described in this report: a pure layer of bentonite 0.5 in. thick was spread over the reservoir bed and covered with 6 in. of soil. Beneath this bentonite "membrane" was a very permeable, sandy soil. The water table remained between 5 and 6 ft. below the soil surface during the irrigation season and was still lower otherwise. The relevant geometric parameters were:

W_t = 20 ft.	W_b = 5 ft.	L_c = .042 ft.
L_t = 40 ft.	$\tan\alpha$ = .4	D = 3 ft.

The authors did not report the moisture tension in the sandy soil. Given the permeability and the location (Fallon, Nevada) of the soil, it does not seem unreasonable to assume that T was close to the soil moisture tension at the permanent wilting point: $T \approx 15$ atmospheres ≈ 500 ft.

If these data are introduced into Eq. (43), neglecting the small overburden pressure excess due to the 6 in. of soil lying on the clay layer and setting $S=2.8$, $\bar{\theta}=4$, the value of I_s/K_c is found to be

$$\frac{I_s}{K_c} = \underset{\text{pressure}}{55.7} \quad \underset{\text{swelling-gravity}}{-0.2} \quad \underset{\text{tension}}{+15{,}597.4} = 1.565\times10^4.$$

It is obvious that, in this case, the effects of swelling and gravity are negligible and that the moisture tension in the underlying soil determines the seepage rate. Because of the usual shallowness of real estate lakes (D < 10 ft.) and the expectedness dryness of the unsaturated soil surrounding them in semi-arid regions, it may be true that soil moisture tension will generally be the determining factor in seepage.

In order to estimate I_s, the value of K_c must be known. For the situation studied by Rollins and Dylla, K_c is given by

$$\frac{1}{K_c} = \frac{1}{K_{soil}} + \frac{1}{k_{clay}}$$

where K_{soil} is the hydraulic conductivity of the soil lying on top of the bentonite layer. It is expected that $K_{soil} >> K_{clay}$ for this saturated soil and, therefore, that $K_c \simeq K_{clay}$. Values of the hydraulic conductivity of Na-bentonite are very difficult to find in the literature. Shainberg and Caiserman (1971) and Mesri and Olson (1971) reported 1.84×10^{-6} ft/day for Na-Wyoming bentonite saturated with water ($\tilde{\theta}\simeq$) at 25°C. With the help of this figure, the seepage loss is estimated to be I_s = .03 ft/day. Rollins and Dylla measured seepage losses between .006 and .02 ft/day for the reservoir under discussion during the first year of their investigation. (Thereafter losses were larger due to the effects of drying out.) The agreement between the calculated and measured values of I_s is good, given the uncertainty in the estimates of K_c and T, the experimental difficulty in separating reservoir water losses due to evaporation from those due to seepage, and, of course, the fact that seepage in the field is not an isothermal process, as has been assumed in doing the calculation of I_s.

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