WAVEFRONT ANALYSIS AND CALIBRATION FOR COMPUTER GENERATED
HOLOGRAMS

by

Wenrui Cai

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A Dissertation Submitted to the Faculty of the

DEPARTMENT OF OPTICAL SCIENCES

In Partial Fulfillment of the Requirements

For the Degree of

DOCTOR OF PHILOSOPHY

In the Graduate College

THE UNIVERSITY OF ARIZONA

2013
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SIGNED: Wenrui Cai
ACKNOWLEDGEMENTS

I would like to express my sincere gratitude to my advisor, Prof. Jim Burge, for introducing me into the exciting world of optical testing. Working with Jim has been a delightful experience. His guidance and support provide me with an excellent atmosphere for doing research.

I am also grateful to the committee members, Prof. Jim Wyant, Prof. Chunyu Zhao, and Prof. Ping Zhou, for helping me to develop my background in optical engineering with their patient guidance and thought-provoking suggestions.

I would like to thank Buddy Martin and Bob Parks for hiring me to work on meaningful projects, namely supporting the measurement of Giant Magellan Telescope (GMT) and developing software algorithm for point source microscope (PSM). I would also like to thank Todd Horne for helping me develop the mechanical system of the Diffractive Optics Calibrator (DOC).

I also would like to thank my colleagues and friends in Large Optics Fabrication and Testing group (LOFT), College of Optical Sciences and Steward Observatory Mirror Lab (SOML), for their friendship and support.

Finally, I want to contribute my deepest appreciation to my beloved fiancée, Qian Li and my parents, Xiaoqing Zhu and Weiguang Cai for their love and support over all these years. They have been a great influence of my life.
DEDICATION

To my mother and father

Xiaoqing Zhu and Weiguang Cai
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ABSTRACT

Interferometry with computer generated holograms (CGH) has evolved to be a standard technology for optical testing and metrology. By controlling the phase of the diffracted light, CGHs are capable of generating reference wavefronts of any desired shape, which allows using of interferometers for measuring complex aspheric surfaces. Fabrication errors in CGHs, however, cause phase errors in the diffracted wavefront, which directly affects the accuracy and validity of the interferometric measurements. Therefore, CGH fabrication errors must be either calibrated or budgeted.

This dissertation is a continuation and expansion of the analysis and calibration of the wavefront errors caused by CGH in optical testing. I will focus on two types of error: encoding error and etching variation induced errors.

In Topic one, the analysis of wavefront error introduced by encoding the CGH is discussed. The fabrication of CGH by e-beam or laser writing machine specifically requires using polygon segments to approximate the continuously smooth fringe pattern of an ideal CGH. Wavefront phase errors introduced in this process depend on the size of the polygon segments and the shape of the fringes. We propose a method for estimating the wavefront error and its spatial frequency, allowing optimization of the polygon sizes for required measurement accuracy. This method is validated with both computer simulation and direct measurements from an interferometer.

In Topics two, we present a new device, the Diffractive Optics Calibrator (DOC), for measuring etching parameters, such as duty-cycle and etching depth, for CGH. The system scans the CGH
with a collimated laser beam, and collects the far field diffraction pattern with a CCD array. The relative intensities of the various orders of diffraction are used to fit the phase shift from etching and the duty cycle of the binary pattern. The sensitivity of each etching parameter is analyzed and the design choice is provided. The system is capable of measuring variations that cause 1 nm peak-to-valley (P-V) phase errors. System performance is verified by measurements from both a white light interferometer and a phase shift Fizeau interferometer. The device will be used primarily for quality control of the CGHs. As an example, DOC is utilized to measure etching parameters of a CGH. The results can be used to evaluate the fabrication performance and guide future design. DOC is also capable of generating an induced phase error map for calibration. Such calibration is essential for measuring freeform aspheric surfaces with 1 nm root-mean-square (RMS) accuracy.
1. INTRODUCTION

1.1 HISTORY

Invented by D. Gabor [1.1] in 1948, holography employs optical interference to store both wavefront amplitude and phase information onto photographic recording media. This recorded interference pattern is called hologram, which behaves like a diffraction grating. A hologram can reproduce wavefronts with the same amplitude and the same phase function as the original object wavefront.

In 1966, Brown and Lohmann [1.2] first showed that wavefronts could be produced with a binary hologram, which is generated via computer synthesis. This new approach was called computer generated hologram (CGH), which is distinguished from the traditional hologram primarily by its ability to design and optimize a wavefront of a synthetic or virtual object mathematically rather than experimentally. The high degree of flexibility in generating complex wavefronts has made CGH extremely useful. CGHs are applied in different fields including optical data storage, three-dimensional computer display, image processing, interconnection and optical testing.

While there are several types of CGHs such as detour phase hologram [1.3] or Kinoform[1.4], the most common way to produce a CGH for optical testing is the binary synthetic hologram invented by Lee in 1974 [1.5]. This type of CGH is a binary representation of the interferogram, or hologram that would be produced if the reference wavefront were interfered with the object wavefront, which is usually coming from desired aspheric surface [1.6]. Using this method, both
the amplitude and phase information of the wavefront are encoded in the position and the width of the binary fringe patterns.

Early works of CGHs utilized pen plotters to generate the encoded hologram at a larger scale than the final size; then the pattern is photographically reduced to appropriate size [1.7]. Nowadays, thanks to the fast developing semiconductor industry, the state of the art lithography technologies allow CGHs to be directly written by electronic beam or laser at a much higher accuracy (about 15nm RMS in each direction). [1.8, 1.9]

1.2 TEST CONFIGURATIONS OF INTERFEROMETRY WITH CGH.

In the field of optical testing and metrology, CGHs are often used in interferometric systems to produce reference wavefronts. By controlling the wavefront phase of the diffracted light, the application of CGHs allows complex non-spherical surfaces to be measured.

The first application of CGH for interferometric testing of aspheric surface was demonstrated by MacGovern and Wyant in 1971[1.10]. The CGH located inside the interferometer is used in single pass with the CGH common to both test and reference paths. This type of CGH configuration is immune to the wavefront aberration from the CGH substrate and has no hard requirement to diffraction efficiency. However, the aberrations of the interferometer and its accessory optics cannot be easily calibrated. And the interferometer requires new customized set-ups for new testing.
The most common type of configuration is to position the CGH in the test arm outside an interferometer, as shown in Fig. 1. This arrangement allows an easier incorporation of the CGH with commercially available interferometers, where the aberration of the interferometer is calibrated routinely. One shortcoming is the CGH with high diffraction efficiencies are necessary when used in such systems due to the double pass set-up.[1.11]

![Diagram of interferometric test with CGH](image)

Fig. 1.1. Aspheric surface interferometric test with CGH

Several other useful CGH configurations include: reflective CGH used to calibrate null corrector for large telescope primary mirrors [1.12]; measuring optic in transmission in a Mach-zehnder...
interferometer; Fizeau interferometer with spherical reference and CGH correction for measuring large convex aspheres [1.13].

1.3 CGH DESIGN CONSIDERATIONS

As Fercher pointed out [1.14], even though grayscale hologram with continuously varying transmittance has been used in optical testing, binary hologram is advantageous. Firstly, it can be easily encoded and generated by CGH writing machine. Furthermore, the accuracy of the CGH pattern is defined by the positions of the structural line patterns, which can be easily checked and calibrated.

The binary CGH design begins with diffraction order separation. To insure additional orders can be eliminated in the Fourier plane by spatial filtering, wavefront tilt is added to the reference beam. Usually it requires three times the maximum wavefront slope of the surface under test to separate the first and second orders [1.15]. With the prescription of the desired aspheric surface, the CGH phase function is obtained by ray-tracing via optical design software. The design also required the knowledge of CGH substrate dimensions and refractive index, the laser wavelength of the interferometer and certain mechanical constraint of the test set-up.

To align the position of the CGH relative to the interferometer, alignment pattern can be designed as a ring pattern outside the test pattern on the same substrate. It is often used in reflection to generate a spherical wavefront for interference, which is used to guide the alignment in tip, tilt and axial position.
In order to manufacture the CGH, the phase function is approximated into a data format for the CGH writing machine via encoding process. The first topic of the dissertation will discuss in detail the analysis of wavefront error introduced by encoding the CGH.

1.4 CGH FABRICATION ERRORS

There are two basic types of binary CGH: (1) amplitude CGHs, which contain both the phase and the amplitude information of the incident wavefront; (2) phase CGHs, which contain only the phase information and assume constant amplitude of the wavefront generated. Amplitude CGH is usually defined by a thin layer of chrome pattern on glass, with only 10% diffraction efficiency going into first orders. It is commonly used in reflective configurations. Meanwhile, the phase CGH has pattern etched into glass. Its first order diffraction efficiency can reach a maximum of 40% with $\lambda/2$ phase shift between etched and un-etched area.
Fig. 1.2 demonstrates the fabrication process for both types of CGH. They are based on the same processes used to fabricate the photomasks for manufacturing integrated circuits. Each step may introduce errors, which can be categorized into pattern distortion and phase etching variations.
The pattern distortion, also known as CGH pattern position error, results from the deviation of the written fringe pattern from its ideal position. The 1σ uncertainty of 0.012μm is achievable based on current fabrication technique. For a CGH with an average line spacing of 12 μm, the pattern distortion yields only 0.001λ RMS wavefront error for the first diffraction order. This is usually ignored in optical testing [1.16].

The phase etching variations include: duty-cycle variation, etching depth variation and surface roughness error. The effects of all these errors are coupled together. Chang and Burge [1.17] first developed a parametric model to relate the fabrication errors to the wavefront performance. Then Zhou and Burge extended the analysis to optimize the CGH [1.18, 1.19] design parameters to minimize sensitivities to fabrication errors.

The parametric model based on scalar diffraction theory assumes the wavelength of the incident light is much smaller than the grating period (In our case, grating period range from 5 μm to 30 μm with visible light). Fig. 1.2 depicts a cross-sectional view of a binary grating. It is defined by period $S$ and the etching depth $t$. The duty-cycle of the grating is defined as $D = b / S$ where $b$ is width of the etched area. $n$ is the refractive index of the grating. $A_0$ and $A_1$ correspond to the amplitudes of the output wavefront from the un-etched and etched areas of the grating, respectively. The phase step $\phi$ represents the phase difference between these two areas, which equals $2\pi(n-1)t / \lambda$ for a grating used in transmission.
Fig. 1.2. Binary, linear grating profile: $t$ is the etching depth; $S$ is the grating period; $b/S$ is the duty-cycle; and $A_0$ and $A_1$ are the amplitude of the output wavefront from the un-etched and etched areas of the grating, respectively. $\phi$ is the phase step between the two areas.

Based on Fraunhofer diffraction theory, the diffraction efficiency $\eta$ in the far field for different diffraction order $m$ can be derived as:

$$\eta = \begin{cases} 
A_0^2 (1-D)^2 + A_1^2 D^2 + 2A_0 A_1 D (1-D) \cos \phi & m = 0 \\
(A_0^2 + A_1^2 - 2A_0 A_1 \cos \phi) D^2 \sin^2(mD) & m = \pm 1, \pm 2, \ldots
\end{cases} \quad (1.1)$$

The diffracted wavefront phase $\Psi$ in the far field for different diffraction order $m$ can be defined as:

$$\tan \Psi = \begin{cases} 
\frac{A_1 D \sin \phi}{A_0 (1-D) + A_1 D \cos \phi} & m = 0 \\
\frac{A_1 \sin \phi \text{sinc}(mD)}{(-A_0 + A_1 \cos \phi) \text{sinc}(mD)} & m = \pm 1, \pm 2, \ldots
\end{cases} \quad (1.2)$$
Derived from Eq. 1.2, the wavefront phase sensitivity functions, \( \frac{\partial \Psi}{\partial D} \) and \( \frac{\partial \Psi}{\partial \phi} \), are used to estimate the wavefront error due to variation in duty-cycle and phase step.

Besides the CGH pattern fabrication error, the CGH substrate errors also affect the accuracy of the desired wavefront. The transmitted wavefront error of the substrate is the dominant error if it is not calibrated out of the measurement. Even though the substrate error can be calibrated, there are still residual errors such as duty-cycle variation induced phase error and CGH substrate deformation errors [1.16]. Other types of substrate errors are also budgeted, including laser wavelength shift, measurement uncertainty of the thickness, wedge, and refractive index of the substrate.

In summary, a list of the CGH error sources is presented in table 1.1. The current methodologies for measuring each error and the corresponding induced wavefront errors are provided by Zhao and Burge [1.16] at Arizona Optical Metrology LLC. The overall measurement accuracy is 2.6 nm root mean square (RMS). The comment column lists suggestions to improve the methodologies and yield better error estimations. For instance, the etching variation is the largest contributor to wavefront error. The second topic of the dissertation is about the instrumentation that utilized the parametric model to measure etching variations across the CGH pattern and estimates the induced phase error.
Table 1.1. CGH errors and the corresponding Methodologies for verification and calibration

<table>
<thead>
<tr>
<th>Error source</th>
<th>Current methodology for budgeting or calibration</th>
<th>Wavefront error (nm RMS)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design</strong></td>
<td>fiting residual</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Estimate RMS wavefront error from maximum fringe departure and averaging effect per measurement pixel.</td>
<td>0.5</td>
<td>need a rigorous analysis with error frequency (topic 1)</td>
</tr>
<tr>
<td><strong>Fabrication</strong></td>
<td>Pattern distortion</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>measure positions of embedded Nikon marks</td>
<td>0.7</td>
<td>limit by the writing machine</td>
</tr>
<tr>
<td></td>
<td>Etching variation</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>measure a few sample points and estimate the induce phase error from a parametric model</td>
<td>2.2</td>
<td>need new device to create an induced phase error map for quality control or calibration (topic 2)</td>
</tr>
<tr>
<td><strong>Substrate error</strong></td>
<td>Figure error</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Calibrate with the transmission wavefront error</td>
<td>1.0</td>
<td>Can be mitigated with high quality substrate</td>
</tr>
<tr>
<td></td>
<td>Laser wavelength</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Refractive index</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>thickness</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>wedge</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>deformation</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td><strong>Alignment error</strong></td>
<td>Use CGH alignment pattern to adjust tip/tilt and axial position</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Root Sum Square</td>
<td>2.6</td>
<td></td>
</tr>
</tbody>
</table>
1.5 MOTIVATIONS FOR THE DISSERTATION

This dissertation addresses effects of both design and fabrication limitations on diffraction wavefront phase generated by CGH. These analysis and calibration methods are essential for measuring freeform aspheric surfaces with 1 nm RMS accuracy.

As listed in table 1.1, the current method to estimate the wavefront error introduced by encoding the CGH is a conservative approximation. It only provides a single phase error value from the averaged parameters of the whole CGH pattern. In the first topic of this dissertation, we present a rigorous analysis with a new parametric model to estimate the wavefront error. This systematic method provides insight to spatial frequency information of the encoding induced phase error for any CGH pattern. In addition, we can generate a 2-dimension induced phase error map and optimize the encoding algorithm accordingly.

As indicated in table 1.1, the wavefront phase error induced by etching variations is the dominant source of error. In the second topic of the dissertation, we present a new device, the diffractive optics calibrator (DOC), for measuring etching variations, such as duty-cycle and etching depth, of a CGH. The DOC can be used to evaluate the fabrication performance and provide feedback to optimize the CGH fabrication process or future CGH design compensations. It can also be used to assess the etching induced errors for CGH quality control or calibration in the optical testing.

The goal of this dissertation is to obtain a thorough understanding of these two specific aspects of the design and fabrication of CGH, respectively. The results of this research provide guidance
to the future design, analysis and fabrication of CGHs in the field of optical testing and interferometry.
2. ANALYSIS OF WAVEFRONT ERROR INTRODUCED BY ENCODING THE
   COMPUTER GENERATED HOLOGRAMS

This section provides a summary of the work and conclusions of the paper found in Appendix A.

A specific measurement consideration about wavefront propagation is included in Appendix B.
2.1 ENCODING RESIDUAL IN THE POLYGON APPROXIMATION

The most common way to generate a CGH pattern for optical testing involves four steps. Firstly, a binary fringe pattern for the desired aspheric wavefront phase function is designed [2.1, 2.2]. Secondly, the smooth pattern is digitized to a geometric pattern layout with adjustable tolerances. Thirdly, the CGH encoding algorithm translates the pattern geometry into a standard exchange data format (e.g. GDSII). Last but not least, this intermediate data file is later converted to a machine specific format (e.g. MEBES, etc.).[2.3]

The CGH fabrication process utilizes technology that was originally developed for semiconductor industry. These e-beam or optical writing machines are optimized for writing x-y features in integrated circuit production. Therefore, the continuous smooth fringes required by the CGH must be encoded into a set of polygon segments that the machines can accommodate. [2.4, 2.5]. The schematic process of segmentation is demonstrated in Fig. 2.1. Smaller polygons yield higher performance, yet require more data. Therefore, for large CGH pattern (>100mm in diameter) with small local periods (about 10μm), the trade-off between the size of the data file and the tolerance on the wavefront error must be considered practically [2.6, 2.7].
Fig. 2.1 The process of segmenting a smooth fringe to polygons.

Any deviation from the ideal fringe causes wavefront phase error in the application of the CGH. We defined this type of phase error as encoding residual, a particular type of error that relates to both the design and fabrication of CGH.

The wavefront phase generated by a CGH is a function of the position of the fringes.[2.7,2.8] Therefore, the deviation from the actual central line of the fringe is used to evaluate the accuracy of the polygon segment approximation of a smooth fringe. The amount of wavefront phase errors produced by the CGH pattern position error can be expressed as

\[
\Delta \phi(x, y) = -m \lambda \frac{\sigma(x, y)}{d(x, y)},
\]

(2.1)
where $\sigma$ is the projection of the fringe position error in the direction perpendicular to the pattern; $d$ is the local period of local fringe; $m$ is the diffraction order; and $\lambda$ is the wavelength. The resulted wavefront phase errors due to pattern position errors are inversely proportional to the local fringe spacing.

When encoding the CGH fringe pattern, a set of boundaries are given to limit the maximum position error of the segments, as shown in Fig. 2.2. The error boundary is defined by an error ratio $\varepsilon$, which represents the percentage of phase error to one fringe:

$$
\varepsilon = \frac{\sigma}{d}.
$$

The given boundary essentially determines the absolute peak-to-valley of the wavefront phase error. Given the error ratio, the maximum phase error is $\Delta \phi = -\varepsilon m \lambda$. In order to evaluate its spatial frequencies, we need to know the phase error as a function of the parameters of local fringe pattern.
Fig. 2.2. Segmenting the same fringe central line with different error boundaries.

$(\sigma_1=2\sigma_2, \varepsilon_1=2\varepsilon_2, L_1 = \sqrt{2}L_2)$. P-V phase error and segment length decreased as the boundary decreased.

As the simple geometry illustrated in Fig. 2.2, the error period $L$ depends on the local curvature of the fringe:

$$L = 4\sqrt{R \cdot \sigma} = 4\sqrt{\frac{\varepsilon d}{c}},$$

(2.3)

where $R$ is the local fringe radius, and $c = R^4$ is the local fringe curvature. The error frequency is the inverse of the error period:
This is the crucial parameter in characterizing the phase errors induced by encoding residuals.

There is a trade-off between the maximum encoding residuals and the number of polygons used, which is directly related to the size of the pattern data file. Tighter error boundary gives smaller phase error with higher spatial frequency, yet requires more polygon segments. For example, a 100mm size CGH pattern with error of 0.005\(\lambda\) usually will have a data size of about one Gigabyte.

In this study, a new method is introduced to estimate the encoding residual and its spatial frequencies during the CGH segmenting process. This method enables us to quantify encoding residual using a parametric model, and thus achieve required accuracy and improve the encoding algorithm.

2.2 ERROR FREQUENCY AS A FUNCTION OF GIVEN WAVEFRONT

Given any CGH pattern and error boundary, the encoding residual and its frequencies can be calculated by relating the phase function to the local parameters (e.g. fringe spacing, curvature) of the fringe pattern. The fringe type CGH is like a contour map. For a first order CGH, \(m=1\), each line adds \(\lambda\) to the wavefront phase. The local fringe spacing is simply related to the local slope of the given phase function and the wavelength \(\lambda\). The local fringe curvature, however, is
more complicated. Some concepts from differential geometry are introduced to relate between the fringe curvature and the phase function. [2.9].

Let \( f(x, y) \) denote a phase function created by the hologram. Then the gradient of this function in a 2D vector field is:

\[
\nabla f = (f_x, f_y),
\]

(2.5)

where \( f_x = \frac{\partial f(x, y)}{\partial x}, f_y = \frac{\partial f(x, y)}{\partial y} \).

Define the slope at a given location \( S(x, y) \) as the magnitude of \( \nabla f \):

\[
S(x, y) = |\nabla f| = \sqrt{f_x^2 + f_y^2}.
\]

(2.6)

The fringe local curvature can be expressed in Cartesian coordinates as:

\[
\kappa_x = -(f_y^2 f_{xx} - 2f_x f_y f_{xy} + f_x^2 f_{yy}) I S^3.
\]

(2.7)

Detail derivations are provided in appendix A. Using Eq.(2.7), the error frequency from Eq.(2.4) can be re-written as a function of local wavefront shape:

\[
Y(x, y) = \frac{1}{4} \sqrt{\frac{c}{\varepsilon d}} = \frac{1}{4} \sqrt{\frac{|\kappa_x|}{c \lambda / S}}
\]

\[
= \frac{1}{4S} \sqrt{f_y^2 f_{xx} - 2f_x f_y f_{xy} + f_x^2 f_{yy}} / \varepsilon \lambda.
\]

(2.8)

For example, the phase function of a Fresnel Zone Plate (FZP) can be approximated as:
where $F$ is the focal length of the FZP. Using Eq. (2.8), the error frequency for the FZP is then:

$$\gamma_{FZP} = \frac{1}{4\sqrt{\varepsilon \lambda F}}.$$  \hspace{1cm} (2.10)
of the detector. Errors with frequency beyond the system low-pass cutoff are filtered out, which may decrease the Root Mean Square (RMS) wavefront error. Including all the factors, a parametric model is built to estimate the encoding induced phase error and guide to select the error ratio $\varepsilon$.

2.3.1 Parametric model

To illustrate the parametric model, we used a FZP as a simple example. Fig. 2.3 shows a direct simulation of the encoding residual of a complete CGH. It is pattern for a FZP (300mm focal length, f/12). A boundary error of $0.2\lambda$ ($\varepsilon=20\%$), which is more than 40 times of our usual design rule, is used to exaggerate the encoding residual effect.
Fig. 2.3 Encoding induced phase error map of a FZP and its zoomed-in characters. (colorbar unit: nm)

Fig. 2.4 1D-PSD of a local area on a phase map when individual fringe is resolved. It has a shape close to a triangle (in dash red line).

A CGH with variable fringe spacing and curvature may be viewed as a collection of gratings with variable spatial frequency. As shown in Fig.2.3, initial phases of the sinusoidal errors along
the neighboring fringes are usually randomized to erase the fixed pattern error. In order to
analyze the properties of the encoding residuals, a localized phase error map is used in the
parametric model. Assuming the fringe’s radius of curvature is large compare to the width of the
area under analysis, we can approximate a constant fringe space and constant fringe curvature
within this region. Then the phase error along the fringe direction is approximated using sine
function with frequency equal the error frequency \( \gamma(x, y) \), defined in Eq.(2.8). The fringe spacing
is determined by the ratio of \( \lambda \), the wavelength to \( S(x,y) \), the local wavefront slope magnitude in
Eq.(2.6).

In order to characterize the frequency components of phase errors induced by encoding
residuals, the power spectral density (PSD) was used. In addition, 1-D PSD can be used to
represent the radial sum of the 2-D PSD as shown in Fig. 2.4.

The spatial frequency of the sharp raising ramp of the PSD profile is determined by the
error frequency \( \gamma(x, y) \), while the largest frequency component is determined by the fringe
spacing. In our parametric model, this 1D-PSD profile plotted in a standard linear scale is
simplified as a triangle as illustrated in Fig.2.5.

As we know, the RMS value of a phase map is the square root of the total area of its PSD [2.10]:

\[
\sigma^2 = \iint PSD(u,v) dudv.
\]  
(2.11)
Fig. 2.5. A PSD parametric model to calculate the actual RMS phase error using the ratio of the shaded area to the whole area of the triangle. ($\gamma$ is the error frequency from Eq.(2.8). $S/\lambda$ is the system cutoff frequency. $S/\lambda$ is the spatial frequency related to the fringe spacing, where $S$ is the local slope magnitude of the wavefront)

Ideally, the RMS phase error can be calculated from the area of the triangle. However, in real systems, individual fringe will not be resolved, meaning that the system cutoff is larger than the smallest spatial frequency in the model. Therefore the actual RMS phase error is proportional to the trapezoidal area between error frequency $\gamma(x,y)$ and the system cut-off frequency. From the area ratio illustrated in Fig.2.5, we can express the local RMS phase error as:

$$
\sigma_{RMS}(x,y) = \frac{\varepsilon \lambda}{\sqrt{2}} \sqrt{1 - \left[ \frac{S(x,y)}{\lambda} - f_{cutoff} \right]^2}\cdot
$$

(2.12)
where $\frac{\varepsilon \lambda}{\sqrt{2}}$ is the RMS value of a sinusoidal phase error before applying the low-pass filter from the real system, and it equals to the whole area of the PSD triangle. Eq.(2.12) is valid when $f_{\text{cutoff}} \geq f$. Otherwise, no phase error induced by encoding residuals will pass the system. This parametric model allows us to estimate the encoding error for a CGH even before actually digitizing the smooth fringe patterns.

2.3.2 Model application and analysis

Eq.(2.8) and Eq.(2.12) are the essential parts of the parametric model. Given a wavefront function $f(x, y)$, an error boundary $\varepsilon$ and a system cutoff frequency $f_{\text{cutoff}}$, the encoding induced RMS phase error can be estimated as a function of spatial coordinates of the CGH pattern. Two examples are employed -- a FZP (300mm focal length, f/12) and a general CGH (50 waves of spherical aberration with tilt carrier). MATLAB was used to generate the maps in Fig. 2.6 and Fig. 2.7. Wavefront phase function was digitized into a matrix. As shown in Fig.2.6a and Fig.2.7a, each fringe on the patterns represent 50 waves in a real CGH pattern. Meanwhile, the RMS phase error was calculated for each pixel. Fig.2.7b and Fig.2.8b delineate the encoding induced RMS phase error maps with error boundary ($\varepsilon=10\%$) and system cutoff frequency ($f_{\text{cutoff}} = 125\text{cyc/aperture}$).
Fig. 2.6. (a) Fringe map (FZP); (b) Encoding induced RMS phase error map (FZP)

Fig. 2.7. (a) Fringe map (Spherical aberration with tilt); (b) Encoding induced RMS phase error map (Spherical aberration with tilt)
In Fig 2.6b and 2.7b, the ‘hot spot’ on the phase error map is where the fringe curvature is small, or fringe spacing is large. To mitigate this locally high phase error, we can constraint the design to a tighter error boundary in that region. Moreover, tilt carrier will increase the slope and reduce RMS value. However, too much tilt will affect the accuracy of the CGH writing. Given the phase function and error ratio, we can calculate the peak frequency of the phase error and the RMS error. This helps to guide the design and encoding of the CGH.

2.4 EXPERIMENTAL VERIFICATION

In order to further verify the parametric model, a set of FZPs with different boundary errors were fabricated and then measured in an interferometric null test. Compared to other types of fabrication errors, the encoding residual for a generic CGH is small (ε < 0.5% λ) compared to other types of fabrication errors. By enlarging the boundary error, the encoding residual became dominant, which made experimental measurement possible.

2.4.1 Measurement Set-up

In order to measure the frequency components of the encoding residual, the interferometer should have a spatial resolution no less than two hundreds cycles per aperture. We used a
A Twyman-Green interferometer (PhaseCam, 4D technology ®, f/9 objective, 1000 X 1000 pixelated mask CCD) with a 7mm collimated output beam.

A set of FZPs (300mm focal length, f/12) were tested in transmission with different error boundaries. For each FZP under test, ring alignment patterns outside the clear aperture were used in reflection for positioning the substrate relative to the interferometer; the FZP itself collimated the beam. A coated flat mirror was placed behind the substrate to retro-reflect the light. In order to prevent undesired diffraction orders from entering the interferometer, a spatial filter was placed at the focus of the converging beam. This 2.1mm diameter stop also served as a 5.7 mm⁻¹ low-pass filter, which was included in the corresponding simulation results. The layout of the system is delineated in Fig. 2.8.

![Diagram of interferometric measurement for the phase error due to encoding the CGH pattern of a FZP](image)

The signature frequency components are above 30 cycles per aperture (1.3 cyc/mm for a 25mm aperture). This frequency range is susceptible to magnitude degradation resulted from wavefront
propagation [2.11]. Therefore, to constrain the effect of defocus to a negligible level, the return flat was placed within 20mm behind the FZP substrate. The detail analysis of the wavefront propagation effects is presented in Appendix B.

2.4.2 Results and analysis

Verification was performed by comparing the measurement results with: 1. the simulations of the encoding process; 2. the parametric model.

Table 2.1 lists the measured phase maps and 1D-PSD curves (boundary error: 0.5%, 5%, 10% and 20%) compared to the corresponding simulation results, respectively.

Since the target errors do not contain low frequencies, the first 37 terms of Zernike polynomials were subtracted from the phase error maps in Table 2.1. In the case with 0.5% boundary error, lowest frequency of the encoding residual was higher than the cut-off frequency of the low-pass filter. There was no frequency peak in the measured PSD as expected. This result demonstrated our ability to control the encoding residual down to a negligible level.

For the succeeding three cases (5%, 10%, and 20%), the frequency profiles matched the simulated results in the region where the frequency was above the value calculated from Eq. (2.8). The lower frequency regions in the measured PSDs had higher magnitude than the simulations. This may be due to the residual lower order phase error from the CGH substrate figure error or the surface errors on the flat return mirror. If more Zernike terms are subtracted from the measured map, the noise on the lower part of each PSD will be mitigated.
Table 2.1. Comparison of the measurement results and simulation results at different levels of encoding residual

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Phase error map (nm)</td>
</tr>
<tr>
<td></td>
<td>1D-PSD (nm&lt;sup&gt;2&lt;/sup&gt; mm&lt;sup&gt;-2&lt;/sup&gt;)</td>
</tr>
<tr>
<td>0.5% λ, Boundary error</td>
<td>RMS: 2.3nm</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>5% λ, Boundary error</td>
<td>RMS: 8.5nm</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>10% λ, Boundary error</td>
<td>RMS: 16.5nm</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>20% λ, Boundary error</td>
<td>RMS: 38.0nm</td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Graph" /></td>
</tr>
</tbody>
</table>
The profiles of these 1D-PSDs are the accumulative effect of different shapes of triangles discussed in the last section. Since the FZP has constant error frequency $\gamma$ globally, all the triangle shapes have the same starting frequency. Hence, a peak at the error frequency $\gamma$ is expected. In Table 2.2, the peak frequencies in each case were recorded and compared with the values from Eq. (2.8) and simulations. The differences of results are within 2%. In Table 2.3, the RMS phase errors of each case were compared with values from the corresponding simulations. The higher measured RMS values may also result in the lower order phase errors other than the encoding residuals.

Table 2.2. Comparison of peak frequencies among analytical, simulated and measured values

<table>
<thead>
<tr>
<th>Boundary error</th>
<th>Error frequency (1/mm) (first peak on the low end)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Eq. (2.10)</td>
</tr>
<tr>
<td>0.5 %</td>
<td>8.11</td>
</tr>
<tr>
<td>5 %</td>
<td>2.57</td>
</tr>
<tr>
<td>10%</td>
<td>1.81</td>
</tr>
<tr>
<td>20%</td>
<td>1.28</td>
</tr>
</tbody>
</table>
Table 2.3. Comparison of RMS phase errors between simulated and measured values

<table>
<thead>
<tr>
<th>Boundary error</th>
<th>Simulations</th>
<th>Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 %</td>
<td>0.0</td>
<td>2.1</td>
</tr>
<tr>
<td>5 %</td>
<td>8.1</td>
<td>8.5</td>
</tr>
<tr>
<td>10%</td>
<td>16.3</td>
<td>16.5</td>
</tr>
<tr>
<td>20%</td>
<td>34.6</td>
<td>38.0</td>
</tr>
</tbody>
</table>

After the simulation of the encoding process is verified, we applied the parametric model to this set of FZPs. We measured the local RMS phase errors (1 mm by 1 mm area) on certain locations of the FZPs and plotted the results in Fig.2.9. The differences of local RMS value between the parametric model and the measurements are within 10%. This agreement is sufficient to verify our parametric model, since the FZPs contain a large range of fringe spacing and curvature.
2.5 ANALYSIS OF A REAL-WORLD CGH

In previous subsections, we used exaggerated boundary errors and simple CGH patterns to verify our parametric model and analysis method. In this subsection, an example is presented with a CGH used in real test configuration. The first diffraction order of this CGH generates an aspheric wavefront to be used in a null test for an off-axis parabolic mirror. As illustrated in Fig. 2.10 (a), each fringe represents 300 fringes in the real CGH. The phase function of CGH pattern mainly consists of 360 waves of spherical aberration and over 5600 waves of tilt in the vertical direction at 632.8nm wavelength. The encoding error boundary is set at $\varepsilon = 0.25\%\lambda$. And the system cutoff

Fig. 2.9. Comparison results from measurements and parametric model for local RMS phase error.
frequency $f_{\text{cutoff}} = 500$ cyc/aperture. Using the parametric model we presented, the RMS induced phase error map is calculated and illustrated in Fig. 2.10(b). There is no induced phase error at both the lower left and lower right edge of the pattern. Even though the fringe spacing is large in these regions, the error frequency $\gamma$ is larger than the cutoff frequency $f_{\text{cutoff}}$. Therefore, the error frequency information is filtered out. The overall average value of phase error is 0.25nm RMS.

![Fringe map with fringe interval = 300 waves](image)

![Fracturing induced RMS phase error (Unit: nm)](image)

Fig. 2.10. (a) Fringe map (CGH for testing an off-axis parabolic mirror); (b) Encoding induced RMS phase error map (average induced phase error is 0.25nm RMS)

2.6 CONCLUSIONS

Before the method we proposed in this chapter, the existing approach to address this problem was to perform a full simulation for the actual encoding process and create the induced phase error map. It is similar to the simulation results in Table.2.1. The disadvantage of this approach is time consuming. One may need to perform this trial and error approach until the errors are within
the budget. In comparison, the parametric model we presented will provide a systematic method to quickly estimate the encoding induced phase error and to analyse the CGH pattern design before we proceed to the encoding process.

In this section, we presented a method for estimating the phase error and its frequency profile when encoding the CGH. Firstly, a parametric model was built based on characteristics of encoding residual in the frequency domain. Secondly, the encoding induced RMS phase error was written as a function of the designed wavefront, error boundary and system spatial resolution. By varying the boundary error ratio and image resolution, the phase errors and its error frequencies can be tuned as required. Finally, verification tests with FZPs were performed. We demonstrated that the measurement results agreed with the simulation. Therefore, we proved our ability to quantify and control this type of error using parametric model with required accuracy.
3. THE DIFFRACTIVE OPTICS CALIBRATOR

This section provides an overview of the work and conclusions of the papers found in Appendices C- D.

Appendix C: Design, construction and functions of Diffractive Optics Calibrator (DOC).

Appendix D: Detail error analysis and the operation of DOC to measure the etching variations for CGH.
3.1 INTRODUCTION

The CGH substrate figure error is the dominant fabrication error, especially for large CGH (e.g. a 6” by 6” fused silica substrate with 0.25” thickness). Since the substrate irregularity affects all the diffraction orders equally, it can be calibrated using the measurement from the zero-order diffraction. On the other hand, the non-uniformities in etching parameters introduce wavefront errors for the zero-order and non-zero orders differently. After subtracting the zero-order measurement from the (non-zero order) surface measurement, the CGH substrate error is cancelled but the wavefront still contains some residual errors due to variations in duty-cycle or etching depth [3.1].

Local value of the duty-cycle and etching depth can be measured using white light interferometer, atomic force microscope or scanning electron microscope, which provide the surface relief of the object under test. However, these methods require expensive equipment and long inspection procedures. While they may be acceptable for small samples of diffractive optics, these methods are not suitable for inspecting large CGH substrates (usually 6 to 9 inch in diameter).

The alternative method is to use coherent illumination and analysis of the diffraction pattern in the Fourier plane [3.2]. In optical testing, the binary CGH often uses the tilt carrier to separate the desired diffraction order from other orders. The common range of the grating periods is from 5 μm to 30 μm, which is well within the limit of scalar diffraction theory [3.3]. Therefore, one can write analytical expression for the diffraction efficiency as a function of both the duty-cycle and the etching depth [3.4]. We employ this relationship to develop a new device: Diffractive Optics Calibrator (DOC).
From a collimated laser beam, the intensities of multiple diffraction orders were captured by a wide field of view camera lens. This measurement technique is illustrated in Fig. 3.1. The duty-cycle and etching depth are determined simultaneously by fitting the measured intensities to a parametric model. This device is able to scan through the whole area of the diffractive optics and generate a wavefront phase error map due to variations in duty-cycle and etching depth. The measurements are validated with a vertical scanning white-light interferometer and a Fizeau interferometer.

![System layout for the Diffractive Optics Calibrator (DOC).](image)

The system is capable of measuring the percentage of duty cycle variation that causes 1 nm P-V phase errors. This method is advantageous since it can be carried out rapidly with accurate and repeatable results, does not damage the sample, and uses low-cost equipment.

In this chapter, we will present the design, construction and functions of the DOC. Also we focus on presenting the detail error analysis and the operation of DOC to measure the etching variations for CGH. In subsection 3.2, the Fourier math for constructing the parametric model is
described. The derived sensitivity functions are discussed in detail to select the proper parameters for the CGH design. In subsection 3.3, the performance requirements are set based on a parametric model and Monte Carlo simulations. In subsection 3.4, the system hardware and software are described in details. In subsection 3.5, we quantify the performance of the system using a set of calibration gratings. In subsection 3.6, an example of using DOC to measure the phase error induced by etching variation is presented.

3.2 ANALYSIS OF ETCHING PARAMETERS FOR CGH

3.2.1 Parametric model

The performance of a CGH is directly related to the diffraction characteristics of a linear grating. In our previous works, a binary, linear grating model was used to study the wavefront sensitivity on fabrication uniformities.[3.4] However, in microfabrication of the binary CGHs, etches may undercut the masking layer and form cavities with sloping sidewalls. In this subsection, we introduced this new parameter to describe the sidewall slope in the model, which enable a more accurate prediction of the diffraction efficiency.
Fig. 3.2. Binary, linear grating profile: $t$ is the etching depth; $S$ is the grating period; $b/S$ is the duty-cycle; $c/S$ is the sidewall slope ratio and $A_0$ and $A_1$ are the amplitude of the output wavefront from the un-etched and etched areas of the grating, respectively. $\phi$ is the phase step between the two areas.

The scalar diffraction approximations can be applied when the wavelength of the incident light is much smaller in comparison to the grating period $S$. In this case, the output wavefront immediately past the grating, either reflected or transmitted, can be expressed as a simple product of the incident wavefront function and the grating profile function. In another word, the grating function modulates the incident wavefront directly. For a normal incident plane wavefront, the output wavefront function can be written as:

$$u(x) = A_0 + (A_1 e^{i\phi} - A_0) \cdot \text{rect} \left( \frac{x}{b} \right) \ast \frac{1}{c} \text{rect} \left( \frac{x}{c} \right) \ast \frac{1}{S} \text{comb} \left( \frac{x}{S} \right)$$

(3.1)

where the grating period is $S$ and the etching depth is $t$. The duty-cycle of the grating is defined as $D = b/S$, where $b$ is width of the un-etched area. The previous square profile is replaced by the trapezoidal profile. The sidewall slope ratio is defined as $P = c/S$, where $c$ is the width of the
sidewall. $A_0$ and $A_1$ correspond to the amplitudes of the output wavefront from the un-etched and etched areas of the grating, respectively. The phase depth $\phi$ represents the phase difference between these two areas, which equals $2\pi(n-1)t/\lambda$ for a grating used in transmission. $n$ is the refractive index of the grating substrate. Fig. 3.2 depicts a cross-sectional view of a binary grating with sloping sidewalls.

The far-field diffraction wavefront is related to the original wavefront via a simply Fourier transform relationship based on the Fraunhofer diffraction theory. Detail derivation is similar to Chang and Burge’s previous work [5]. The difference from the previous model is an additional convolution term $\frac{1}{c}\text{rect}\left(\frac{x}{c}\right)$ in Eq. (3.1), which will turn into a product term after the Fourier transform.

A summary of equations for the parametric model is presented in Table.3.1. The wavefront phase function $\tan\Psi$ can be defined using the ratio of the imaginary part to the real part of the complex far-field wavefront function. While, the diffraction efficiency $\eta$ is defined as the ratio of the intensity of the diffracted wavefront to the total intensity of the incident wavefront. As functions of duty-cycle and phase depth, both the zero order and non-zero order diffraction efficiency expressions were utilized in fitting the measured intensities. In Fig. 3.3, the diffraction efficiency of order 0 to 3 is plotted versus duty-cycle with 0.35 $\lambda$ phase depth.
Table 3.1. Summary of equations for parametric model analysis

<table>
<thead>
<tr>
<th></th>
<th>Zero order (m=0)</th>
<th>Non-zero order (m=±1,±2,…)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Diffraction wavefront</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \eta ) diffraction efficiency</td>
<td>( A_0^2(1-D)^2 + A_1^2D^2 + 2A_0A_1D(1-D)\cos \phi )</td>
<td>( (A_0^2 + A_1^2 - 2A_0A_1\cos \phi)D^2\text{sinc}^2(mD)\text{sinc}^2(mP) )</td>
</tr>
<tr>
<td>( \tan \Psi ) (( \Psi ) wavefront phase)</td>
<td>( \frac{A_0D\sin \phi}{A_0(1-D) + A_0D\cos \phi} )</td>
<td>( \frac{A_1\sin \phi}{-A_0 + A_1\cos \phi} )</td>
</tr>
<tr>
<td><strong>Sensitivity functions</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{\partial \eta}{\partial D} )</td>
<td>( -2A_0^2(1-D) + 2A_1^2D + 2A_0A_1(1-2D)\cos \phi )</td>
<td>( 2(A_0^2 + A_1^2 - 2A_0A_1\cos \phi)Dsinc(2mD)\text{sinc}^2(mP) )</td>
</tr>
<tr>
<td>( \frac{\partial \eta}{\partial \phi} )</td>
<td>( -2A_0A_1(1-D)\sin \phi )</td>
<td>( 2A_0A_1\sin \phi D^2\text{sinc}^2(mD)\text{sinc}^2(mP) )</td>
</tr>
<tr>
<td>( \frac{\partial \eta}{\partial P} )</td>
<td>0</td>
<td>( (A_0^2 + A_1^2 - 2A_0A_1\cos \phi)D^2\text{sinc}^2(mD) - \frac{2}{P^2}[\text{sinc}(2mP) - \text{sinc}^2(mP)] )</td>
</tr>
<tr>
<td>( \frac{\partial \Psi}{\partial D} )</td>
<td>( \frac{A_0A_1\sin \phi}{A_0^2D^2 + A_1^2(1-D)^2 + 2A_0A_1D(1-D)\cos \phi} )</td>
<td>0</td>
</tr>
<tr>
<td>( \frac{\partial \Psi}{\partial \phi} )</td>
<td>( \frac{A_1D^2 + A_0A_1D(1-D)\cos \phi}{A_0^2D^2 + A_1^2(1-D)^2 + 2A_0A_1D(1-D)\cos \phi} )</td>
<td>( \frac{A_1^2 - A_0A_1\cos \phi}{A_0^2 + A_1^2 - 2A_0A_1\cos \phi} )</td>
</tr>
<tr>
<td>( \frac{\partial \Psi}{\partial P} )</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Compared to the original model, the addition of sidewall slope ratio parameter does not affect the wavefront expression, only provides an extra term in the non-zero diffraction efficiency. In the case of anisotropic dry etching, the sidewall slopes are usually larger than 80 degree [3.5]. For example, a binary grating with period of 10 μm and etching depth of 500nm, the sidewall slope ratio $P$ is less than 1%. Since sidewall slope ratio $P$ is relatively small, the term $\text{sinc}^2(mP)$ gives a slow varying envelop, which will only affect the high orders. The intensity of the 7th diffraction order will have about 1% change, which is often below the random noise level of the detector and can be ignored.

Fig. 3.3. Diffraction efficiency as a function of duty-cycle for order 0 to 3 with 0.35 $\lambda$ phase depth.
3.2.2 Sensitivity functions

The wavefront phase sensitivity functions $\frac{\partial \Psi}{\partial D}$, $\frac{\partial \Psi}{\partial \phi}$ and $\frac{\partial \Psi}{\partial P}$ are defined as the deviations of the diffracted wavefront phase values due to variations in grating duty-cycle, phase depth and sidewall slope ratio, respectively. As shown in Table 3.1, errors in duty-cycle will not introduce wavefront phase error for non-zero diffraction orders. In a special case when duty-cycle is 50% ($D = 0.5$), the wavefront phase sensitivities to etching depth are the same for all the diffraction orders.

These functions are not only essential to setup the overall system requirements for DOC, but also to convert the measured etching variations to induced phase errors, or vice versa. In CGH substrate calibration, the phase error of zero order and that of non-zero order are compared. Therefore, we are interested in the sensitivity difference between zero and non-zero order. The variations of duty-cycle and that of phase depth that cause 1nm PV phase error between zero and non-zero diffraction orders are plotted in Fig.3.4 and Fig.3.5, respectively, with a range of commonly used values of duty-cycle and phase depth [3.6].

Derived from table 3.1, the wavefront phase sensitivity functions, $\frac{\partial \Psi}{\partial D}$ and $\frac{\partial \Psi}{\partial \phi}$, are used to estimate the wavefront error due to variation in duty-cycle and phase step:

$$\Delta W_D = \frac{1}{2\pi} \frac{\partial \Psi}{\partial D} \Delta D,$$

(3.2)
\[ \Delta W_\phi = \frac{\partial \Psi}{\partial \phi} \Delta \phi , \] (3.3)

where \( \Delta W_D \) and \( \Delta W_\phi \) are the wavefront errors in waves caused by small deviations in duty-cycle \( \Delta D \) and in phase step \( \Delta \phi \) (in waves), respectively. Eqs. (3.2)-(3.3) provide a connection between the fabrication errors and system performance. They can be used to estimate the amount of wavefront phase errors that result from the fabrication non-uniformities. Moreover, the information, when used in reverse, provides a set of error budgets for application using CGHs.

As depicted in Fig. 3.4, the duty-cycle variation induced phase sensitivity changes little as a function of the nominal duty-cycle. Meanwhile, in Fig.3.5, the phase depth variation induced phase sensitivity increases as the nominal duty-cycle approaches 50%. For example, there is a CGH with 49% nominal duty-cycle and 0.5\( \mu \)m etching depth (0.35\( \lambda \) phase depth at 632.8nm). In order to keep the error budget of the phase error induced by the duty-cycle variation within 1 nm P-V, duty-cycle variation must be controlled to within 0.25% P-V, which is a demanding requirement for CGHs that have a wide range of fringe spacing.

For the same scenario, in order to keep the induced phase error within 1nm P-V, phase depth must be controlled to within 0.03\( \lambda \) P-V (equivalent to about 40nm P-V of etching depth variation), which is easily achieved for current fabrication capability. From this example, we can make a preemptive conclusion that duty-cycle variation will be the dominant contributor to the induced phase error.
Fig. 3.4. Duty-cycle variations that cause 1nm PV wavefront phase error between zero and non-zero diffraction orders.

Fig. 3.5. Phase depths variation that cause 1nm PV wavefront phase error between zero and non-zero diffraction orders.
The diffraction efficiency sensitivity functions $\frac{\partial \eta}{\partial D}$, $\frac{\partial \eta}{\partial \phi}$ and $\frac{\partial \eta}{\partial P}$ are defined as the deviations of the diffraction efficiency values due to variations in grating duty-cycle, phase depth and sidewall slope ratio, respectively. These functions are used to analyze the influence of the intensity measurement noises on the fitting results of the etching parameters.

In Fig.3.6, the diffraction efficiency variations caused by 1% duty-cycle deviation are plotted versus the nominal duty-cycle for diffraction order 0 to 3. The sensitivity converges to zero at 50% duty-cycle. It implies that when the nominal duty-cycle value is close to 50%, deviations in duty-cycle have little effect on the intensity distribution among different orders. Large uncertainty will present when using the intensity information to get the duty-cycle value near 50%.

As for the diffraction efficiency variations caused by phase depth deviation plotted in Fig.3.7, the sensitivity remains at the same level. In the present of noise, with nominal duty-cycle value close to 50%, fitting result of phase depth (or etching depth) will be more accurate than that of duty-cycle.
Fig. 3.6. Diffraction efficiency variations due to 1% duty-cycle deviation. (Nominal phase depth is 0.35\(\lambda\). Diffraction order \(m=0,1,2,3\) are included)

Fig. 3.7. Diffraction efficiency variations due to 0.01\(\lambda\) phase depth deviation. (Nominal phase depth is 0.35\(\lambda\). Diffraction order \(m=0,1,2,3\) are included)
3.2.3 *Etching parameter design choice*

There are reasons for choosing duty-cycle at 50% in our previous work [3.6]: (1) It has the maximum diffraction efficiency at the first diffraction order, which will be used in the interferometric testing; (2) The wavefront sensitivities to etching depth are the same for the zero and the first order at 50% duty-cycle, which will not generate additional residual phase error when the CGH substrate is calibrated with zero order measurement.

![Monte Carlo simulation](image)

Fig. 3.8. Monte Carlo simulation of the duty-cycle fitting uncertainty (phase depth is 0.35λ and fitting includes ±7 orders)

As we discussed in section 3.2.2, however, it is difficult for DOC to make accurate measurement near 50% duty-cycle. To illustrate this effect, a Monte Carlo simulation is performed with 0.5%
random measurement noises. The fitting uncertainty shown in Fig.3.8 is represented by the $1-\sigma$ standard deviation of fitting results. The uncertainties increase dramatically when the nominal duty-cycle approaches 50%. Moreover, the mean value at 50% is offset by 0.3%. This is because diffraction efficiency is symmetry about duty-cycle at 50%. As illustrated in Fig. 3.3, for instance, the intensity distributions among all diffraction orders are identical for 51% and 49%, which the DOC measurement cannot distinguish. Therefore, the values above 50% will be degenerated into the values below 50% after the fitting. In this case, the mean value will be shifted from 50%. Another consequence is the sign of the induced phase error relative to 50% duty-cycle is unknown, while only the absolute magnitude of the induced wavefront phase error can be determined.

The characteristic of the diffraction efficiency limits DOC’s ability to perform accurate measurement when duty-cycle is about 50%. In order to utilize the DOC measurement to do calibration analysis, we proposed to choose the nominal duty-cycle as 48%. The reasons are quad fold: (1) As shown in Fig.3.8, at 48%, duty-cycle fitting uncertainty is less than half of the uncertainty at 50%. (2) Diffraction efficiency at first order only decreases 0.4% compared to duty-cycle at 50%. (3) Assuming the etching variation of duty-cycle is less than 2%, there is no ambiguity of the duty-cycle value. (4) Although phase depth variation induced phase errors is different between zero order and non-zero order, the error magnitude is negligible compare to the duty-cycle induce phase error. It shows in Fig.3.9 that the etching depth induce phase error at 48% duty-cycle is about 8 times smaller than duty-cycle induce phase error, assuming the CGH has 0.5% duty-cycle variation and 5 nm etching depth variation.
Fig. 3.9. Induced phase error vs. nominal duty-cycle value. (0.5% duty-cycle variation and 5 nm etching depth variation)

3.3 PERFORMANCE REQUIREMENTS

To measure the variations in duty-cycle and etching depth, a grid of sampled points should be measured across the effective area of the CGH. The local values of duty-cycle and etching depth are determined by nonlinear least-square fitting the measured intensities of different diffraction orders to the parametric model described in the previous subsection. The measurement uncertainty can be estimated via a Monte Carlo simulation, based on which the performance requirements are determined.
3.3.1 Monte Carlo Analysis

In order to estimate the accuracy of the method with the presence of noises, a Monte Carlo simulation was performed to find the relationship between the number of diffraction orders measured and the measurement uncertainty of the duty-cycle or phase step. For a phase type CGH used in transmission, we assumed, the equations in table 3.1, $A_r$ and $A_1$ are both unities, the nominal duty-cycle is 49% and the etching depth is 0.5 μm (phase step of $0.35\lambda$ at 632.8nm). A ±1% of uniform-distributed noise is added to the intensity of each diffraction order in the simulation. Then a non-linear least square fit is performed on the simulated data to obtain the duty-cycle and phase step value.

The results depicted in Fig. 3.10 and Fig.3.11 are the simulated measurement uncertainty (1σ) of duty-cycle and etching depth, respectively, with the corresponding wavefront phase error of the 1st diffraction order from a 1000-trial Monte Carlo simulation. The duty-cycle uncertainty decreases when more orders of diffraction are included in the fitting process. This means that higher diffraction orders contribute to the accuracy of duty-cycle. On the other hand, the uncertainty of etching depth stays at the same level when higher orders are included. [3.1]
Fig. 3.10. Monte Carlo simulation on uncertainty of duty-cycle and corresponding RMS phase error of the 1st diffraction order. (Given ± 1% uniform-distributed errors in intensity measurements)

Fig. 3.11. Monte Carlo simulation on uncertainty of etching depth and corresponding RMS phase error of the 1st diffraction order. (Given ± 1% uniform-distributed errors in intensity measurements)
3.3.2 System requirements

In order to achieve measuring freeform aspheric surfaces with 1 nm RMS accuracy, the knowledge of phase error induced by duty-cycle variations must be within a fraction of 1 nm. The simulation result in Fig. 3.10 indicated that including more diffraction orders in the fitting decreases the measurement uncertainty. Thus, a wide-angle camera lens is preferred. Providing the intensity measurement error within 1%, the uncertainty of phase error is less than 0.25nm RMS, if up to ±7 orders are measured.

In addition, the entrance pupil of the lens system must coincide with the plane of the diffraction pattern. Otherwise, the vignetting effect will reduce the intensities for large angle incident and generate a systematic error for the fitting.[6]

The dynamic range of the detector must be large enough to cover the intensity difference between diffraction orders. For example, the intensity ratio between the 1st order and the 15th order is 225:1 for a grating with 50% duty-cycle and 0.5 μm etching depth (0.35λ phase step at 632.8 nm). Assuming the weakest signal is spread among 4 pixels, the signal-to-noise ratio (SNR) of the detector must be larger than 1000:1.

The requirements of important functions are list in Table 3.2.
Table 3.2. System requirements

<table>
<thead>
<tr>
<th>Features</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Optical subsystem</strong></td>
<td></td>
</tr>
<tr>
<td>FOV</td>
<td>Include up to ±7 diffraction orders for grating period: 5μm to 30μm</td>
</tr>
<tr>
<td>Entrance pupil</td>
<td>in front of the mechanical structure of the imaging lens</td>
</tr>
<tr>
<td>Image aberrations</td>
<td>Spot diameter &lt; ¼ spot spacing between adjacent orders</td>
</tr>
<tr>
<td>Detector Dynamic range</td>
<td>&gt;1000:1</td>
</tr>
<tr>
<td>Laser collimation</td>
<td>Prevent intensity cross-talk between adjacent orders</td>
</tr>
<tr>
<td><strong>Mechanical subsystem</strong></td>
<td></td>
</tr>
<tr>
<td>Scanning mechanism</td>
<td>Motorized 2-axis linear stages</td>
</tr>
<tr>
<td>Travel range</td>
<td>&gt;9 inches for each axis</td>
</tr>
<tr>
<td>Alignment mechanism</td>
<td>① Lockable adjustments between source and imaging system</td>
</tr>
<tr>
<td></td>
<td>② Plane of the CGH surface must be parallel to the motion direction</td>
</tr>
<tr>
<td><strong>Software</strong></td>
<td></td>
</tr>
<tr>
<td>Image acquisition</td>
<td>Shutter speed and gain control</td>
</tr>
<tr>
<td>Motion control</td>
<td>Programmable scanning routine</td>
</tr>
<tr>
<td>Image process and data output</td>
<td>Output local duty-cycle and etching depth value as a function of position</td>
</tr>
</tbody>
</table>
3.4 SYSTEM HARDWARE AND SOFTWARE

Fig. 3.12 illustrates the layout of the diffraction optics calibrator (DOC). It consists of three subsystems: optical system, mechanical system and software. In the optical subsystem, a collimated laser beam propagates through the CGH substrate and diffracts into multiple diffraction orders. The intensities of these orders are captured by a wide-angle camera and processed via LabVIEW to obtain the local duty-cycle and phase step. A two-axis motorized linear stage is implemented in the system, which is capable of scanning through a CGH with an aperture up to 9 inch. In order to measure duty-cycle variation across the CGH aperture, LabVIEW is used to synchronize the scanning motion of the stage with the image acquisition of the camera. The system will be used primarily for CGH quality control. Therefore, in this section, detail specifications of each component in the system are discussed in consideration of the requirements in Table 3.2.

![Fig. 3.12. Diffractive Optics Calibrator’s schematic layout](image-url)
3.4.1 Optical system

Based on diffraction grating equation, the half field of view (HFOV) of the imaging system can be defined as:

\[ HFOV = \theta_{\text{max}} = \sin^{-1}\left(\frac{m\lambda}{d}\right), \]

where \( m \) is the highest diffraction order measured, \( \lambda \) is the laser wavelength and \( d \) is the grating period. The maximum FOV requirement is determined by both the grating period and wavelength of test beam. As a result from the requirements in table 1, we implemented a commercial available camera lens with a focal length of 2.6mm F/2.5 and FFOV (full field of view) is 83.2° by 67.5°. The entrance pupil is located at 1mm in front of the lens.

For the same reason of including more orders, a short wavelength can be beneficial. However, too short a wavelength is not responsive to common CCD detectors. Moreover, angular separation between adjacent orders may not be enough for grating with large periods when using a short wavelength laser. In consideration of these trade-offs, we chose a compact collimated laser diode module from Thorlabs® at 532nm, 4mW and divergences <1.8mrad.

An iris aperture stop was placed to control the size of the laser beam footprint on the CGH pattern. The footprint should be small enough that the illuminated area can be approximated into linear grating. Otherwise the far field diffraction pattern will contain information about higher order effects. For instance, if the illuminated area is too large and contain curved pattern, the shape of high diffraction order spots may either be elongated by astigmatism or blurred by power term. Depending on local curvature of the CGH fringe pattern,
a threshold for the illuminated area can be set. We controlled the illuminated area on the pattern to 1mm in diameter. As for spatial resolution, 1 mm on a 6” CGH pattern is equivalent to 150 cycles per aperture. Higher frequency features were averaged within the illuminated area.

As we make measurements across the CGH pattern, the orientation of the local linear grating may change. Thus, a 2-D detector is required to cover diffraction pattern for all directions. In our system, we implemented a Pointgrey® FLEA camera with 1/3” 1024×768 pixels CCD and 12bit A-D convertor. The fill factor of the CCD sensor is close to 100% due to the micro-lenslet on top of each pixel. The measured SNR of this camera is about 3000:1, which is sufficient for our system requirements.

In summary, the optical system consists of a collimated laser, an aperture stop, a wide-field camera lens and a CCD array. They are selected to meet the performance requirements of our system.

3.4.2 Mechanical system

As depicted in Fig. 3.12, the light source and the camera were placed at different sides of the CGH substrate. The CGH substrate was mounted on a 2-axis linear stage in between to achieve the scanning ability. The stages are motorized using Schneider Electronic MDrive® 14 stepper motors and controlled by LabVIEW. The hardware in used was illustrated in Fig. 3.13. As mentioned in the lens configuration, there was only a 1mm clearance between the lens casing and the CGH substrate. Therefore the plane of the CGH surface must be parallel to the motion direction of the 2-axis linear stage before performing any measurement. This was done by adjusting the tip/tilt micrometers on the CGH kinematic mount.
3.4.3 Software

The DOC software was developed in LabVIEW to 1 communicate with both the camera and the stepper motors; 2 perform image processing and data fitting; and 3 output values of local grating period, duty-cycle and etching depth as a function of positions on the CGH pattern.

Via the graphical user interface (GUI) shown in Fig. 3.14, we can control DOC to individually measure a specific location on the CGH or program a routine to sample and scan through an area of interest. The positioning accuracy of the 2-axis stage is within 0.5 mm for each direction. Camera attributions, such as shutter speed, gain and frame rate, can be tuned before any measurements. In order to utilize the full dynamic range of the detector, the order with the highest intensity should be adjusted to just below saturation. A function to indicate any saturated pixel in real-time was built in the software.
In a standard measurement procedure, illustrated in Fig. 3.15, multiple frames of images were acquired and averaged for each location to reduce the random noise. The location of each diffraction order is found by centroiding in a defined area, and the intensity is calculated by averaging the intensities in the same area. Then non-linear least square fitting was performed to obtain duty-cycle and etching depth simultaneously. Additionally, the local grating period was calculated from the spacing between ±1 orders on the detector. Lastly, a log file was generated to
record the measured data for each location. One can either program in a routine to step and repeat through a region or can move the stage to an individual local of interest and perform measurements.

Fig. 3.15. Block diagram of the DOC measurement procedures.
To reduce measurement time, we implemented a new procedure for data processing with both LabVIEW and MATLAB. LabVIEW is used to do image acquisition. All the raw image data is stored with the measurement location information. Image data is then imported into MATLAB for image processing and non-linear fitting. For example, to test a 4” by 4” CGH pattern with 3mm by 3mm sampling grid, more than a thousand measurements should be taken. It will cost about 1 hour to finish. The procedure steps were illustrated in a flow chart in Fig.3.16.

![Fig. 3.16. Block diagram of the DOC measurement post-processing in MATLAB.](image-url)
3.5 SYSTEM PERFORMANCE

In order to measure freeform aspheric surfaces with 1 nm RMS accuracy, the knowledge of phase error induced by etching variations must be within a fraction of 1 nm. We will show in this section that DOC’s system performance meets this requirement.

3.5.1 Repeatability and accuracy of DOC

In order to test the repeatability and accuracy of DOC, three sets of linear gratings with varying duty-cycles was fabricated. The duty-cycle ranged from 41% to 53%. The detail specifications of the design are list in Table. 3.3.

Table 3.3. the specifications of a set of grating with different duty-cycle

<table>
<thead>
<tr>
<th>Duty-cycle range</th>
<th>41% to 53% (1% change/grating)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Etching depth</td>
<td>0.5 µm</td>
</tr>
<tr>
<td>Grating period</td>
<td>15.0 µm</td>
</tr>
<tr>
<td>Grating size</td>
<td>3mm × 20mm</td>
</tr>
<tr>
<td>Overall pattern size</td>
<td>39mm × 20mm</td>
</tr>
<tr>
<td>Number of sets</td>
<td>3</td>
</tr>
</tbody>
</table>
Fig. 3.17. The layout of a set of gratings with varying duty-cycle from 41% to 53%.

Repeatability tests were performed by measuring the same grating location multiple times. By changing the exposure time of the camera, we examined the repeatability of the measurements as the function of the light level reaching the CCD pixels. The results of the gratings with 46% and 49% duty-cycle are illustrated in Fig. 3.18 and Fig. 3.19. As the light level decreases from saturation, the relative low intensity diffraction orders will begin to drop below the CCD noise level. Therefore the fitting error may increase, which will lead to a larger uncertainty of measured value of duty-cycle and etching depth. Generally speaking, at light level above 60%, the 1σ uncertainty of the measured duty-cycle values is 0.05% and that of the measured etching depth values is 0.4 nm.
Fig. 3.18. Repeatability of measured duty-cycle as a function of light level

Fig. 3.19. Repeatability of measured etching depth as a function of light level
Since the wavefront phase error is induced by the variation of either duty-cycle or etching depth, system repeatability is important for CGH quality control. On the other hand, the system accuracy is crucial for calibrating the induced phase error. We compared the results between the Veeco white light interferometer and DOC in Fig. 3.20 and Fig. 3.21. As we predicted in subsection 3.2.3, a large discrepancy occurs at duty-cycle around 50%. For the rest of the duty-cycle values, the difference is around 0.25% between the two methods, which is the required accuracy to detect about 1 nm PV phase error due to duty-cycle variation. For the case of measuring etching depth, the general trends were matched for the measured values by both methods. The largest discrepancy between the two methods is 3 nm, which is equivalent to 1nm PV induced phase error at 40% duty-cycle and 0.15nm PV induce phase error at 49% duty-cycle.

Fig. 3.20. Duty-cycle measured values comparison between Veeco and DOC
Fig. 3.21. Etching depth measured values comparison between Veeco and DOC

3.5.2 Verification with interferometric measurement

We verified DOC’s performance on duty-cycle and etching depth measurements with a white light interferometer. Once the local values on duty-cycle and etching depth were obtained, the induced the wavefront phase can be calculated using the sensitivity functions in table. 3.1. A direct wavefront phase measurement was performed using a Fizeau type phase shift interferometer. The same set of gratings was measured with a transmission flat and a return flat, which is depicted in Fig.3.22.
Fig. 3.22. Interferometric test of a pattern with 3 sets of gratings with duty-cycle variations from 41% to 53%. The induced phase error was measured at 0 order (solid line setup). ±1st orders were used to calibrate the grating substrate error (dotted lines setups).

As we know, the CGH substrate error affects all diffraction orders equally, while the duty-cycle variation only affects the zero order wavefront. For example, in this specific case for linear gratings, the zero order contains both the substrate error and the duty-cycle variation induced phase errors, while the first diffraction orders contain only the substrate error. The wavefronts of both first orders and the zero order were measured. After the subtraction of the two wavefront maps, the duty-cycle induced error is the net result, which is shown in Fig. 3.23(a).
The same pattern was measured in DOC. As we mentioned in Sec. 3.2.3, DOC cannot tell if the duty-cycle value is below or above 50%. We constrained the fitting results to be always less than 50%. The measured values of duty-cycle were later adjusted to above 50% according to the nominal design. Then the measured values of duty-cycle and etching depth are input to the wavefront sensitivity functions to generate the induced phase error map, as shown in Fig. 3.23 (a). Although the resolution is not as good as the interferometric measurement, the induced phase error is well matched between these two methods.

The averaged profile values of the phase maps in Fig. 3.23 (a) are plotted in Fig. 3.23(b). The uncertainty of the DOC measurement is estimated by the Monte Carlo simulation result shown in Fig. 3.8. Other than the deviation around 50% duty-cycle as we expected, the rest of the regions have an agreement within 5nm with the interferometric results. This is the uncertainty level of the return flat surface quality.
Fig. 3.23. Zero order measurement comparison between interferometer and DOC.
3.6 APPLICATION USING DOC

In this section, we will utilize DOC to measure the etching parameters of a CGH. This CGH was also used in the analysis for encoding induced phase error in subsection 2.5.3. The CGH under test is a circle pattern about 90mm in diameter. The first diffraction order generates an aspheric wavefront to be used in a null test for an off-axis parabolic mirror. As illustrated in Fig.3.24 (a), each fringe represents 300 fringes in the real CGH, which is equivalent to 300 waves at 632.8nm wavelength. The shape of the pattern is mainly stemmed from spherical aberration and tilt. DOC was used to measure the CGH with a resolution of 3 mm in each direction. The scanning route is depicted in Fig.3.24 (b).

![Fig. 3.24. (a) CGH fringe pattern representation (fringe interval = 300 waves). (b) DOC scanning route (3mm resolution in both directions)](image-url)
After the scanning measurements in LabVIEW, data is imported into MATLAB for post processing, including extracting intensity information from the raw image data, non-linear least square fitting and generating induce phase error maps. Detailed information of each data point, including x and y coordinates, duty-cycle, etching depth, fringe spacing and fitting residuals, is logged in a spreadsheet for future analysis.

The statistics of the measured result is listed in Table 3.4. Compared to the nominal design values (duty-cycle is 50% and etching depth is 500nm), the measured average values are 48% for duty-cycle and 511.8nm for etching depth. This offset is mainly resulted from the etch bias, which is the undercutting distance compared to the design of the masking layer. The amount of etch bias is usually uniform across the CGH pattern. Therefore, small features will gain more duty-cycle offset in percentage.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Average</th>
<th>PV</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duty-cycle</td>
<td>48.00%</td>
<td>3.60%</td>
<td>0.40%</td>
</tr>
<tr>
<td>Etching depth</td>
<td>511.8 nm</td>
<td>19.8 nm</td>
<td>3.1 nm</td>
</tr>
<tr>
<td>Fringe spacing</td>
<td>8.2 μm</td>
<td>15.8 μm</td>
<td>2.6 μm</td>
</tr>
</tbody>
</table>

Each parameter can also be plotted as a function of locations in a 2D map as shown in Fig. 3.25, which is a more intuitive way to analyze the measured result. The variations of both duty-cycle
and etching depth manifest as the shape of coma aberration in the up-down direction, which is correlated to the fringe spacing variations illustrated in Fig. 3.24(a) and Fig. 3.25 (c).

Since fringe spacing is proportional to the local wavefront slope, the coma-shape map is stemmed from a coma-shape wavefront slope distribution, which comes from a wavefront contains spherical aberration. Assuming the etch bias is constant across the pattern, smaller fringe spacing will result in a larger duty-cycle offset, which results in the coma-shape variation demonstrated in Fig. 3.25 (a). To further verify this correlation, etch bias is calculated as the product of the fringe spacing and the duty-cycle offset in percentage. The etch bias map and the corresponding histogram are plotted in Fig. 3.26 (a) and (b), respectively. The majority of the values are concentrated around 0.15μm, which is a valuable parameter to evaluate the etching performance. In the knowledge of this fabrication error, we can optimize our CGH design to compensate the offset.

Fig. 3.25. 2D maps for etching parameters (a) Duty-cycle map (values in %); (b) Etching depth map (values in nm); (c) Fringe spacing map (values in μm)
Fig. 3.26. The etch bias is calculated as the product of the fringe spacing and the duty-cycle offset in percentage. (a) Etch bias map; (b) etch bias histogram.

Last but not least, the etching variation induced phase error maps are generated and shown in Fig. 3.27. Wavefront sensitivity functions are applied to the measured duty-cycle and etching depth. In Fig. 3.27(a) and (b), the induced phase errors are compared to the average value of 48% (duty-cycle) and 511.8nm (etching depth), respectively. As we mentioned before this type of induced errors are the difference between zero order and non-zero order. Therefore, when the CGH substrate error is calibrated using zero order measurement, the phase error due to etching variation can be taken into consideration to yield a more accurate measurement.
Fig. 3.27. Etching variation induced phase error maps (a) Duty-cycle variation induce phase error map; (b) etching depth variation induced phase error map; (c) fitting uncertainty of duty-cycle variation induce phase error estimated Monte Carlo simulation; (d) fitting uncertainty of etching depth variation induce phase error estimated Monte Carlo simulation.

3.7 CONCLUSION

This chapter described the diffractive optics calibrator (DOC), a new device that we developed to measure the variation of duty-cycle and etching depth of a binary diffractive optics used in transmission. A parametric model and the derived sensitivity functions were discussed. The
construction of the system hardware and software to meet a certain system requirement was described in details. The system performance is quantified using a set of calibrated gratings. An example of using DOC to measure the etch bias and the induced phase error was presented. The system is capable of measuring etching variations that cause 1 nm PV phase errors in the wavefront created by the CGH. The DOC can be used to evaluate the fabrication performance and provide feedback to etching procedure and future design compensations. It can also assess the uniformity of the diffractive optics for calibration or quality control.
4 CONCLUDING REMARKS

In this dissertation, the wavefront analysis and calibration for CGHs in optical testing were presented. Firstly, we introduced a comprehensive error analysis including test configurations, design considerations and fabrication errors. Second, the dominant sources of error were identified, two of which were discussed in details: (1) encoding induced phase error and (2) etching variation induced phase error.

Through the course of topic one, a parametric model was presented to estimate the RMS phase error introduced by encoding, which was verified by simulations and measurements of a set of special designed CGHs. The local RMS phase error can be written as a function of designed phase function of the CGH pattern, error boundary in the encoding algorithm and the test system spatial resolution. Instead of using either conservative estimation with average parameters or performing time-consuming full resolution simulation, the parametric model we demonstrated provides a systematic method to estimate the induced RMS phase error. Applying to real-world, the method enables a more robust budgeting for the encoding error of any CGH pattern. As part of future study, one can optimize the encoding error boundary in local regions with high RMS phase error to further mitigate the overall induced phase error.

In topic two, we discussed the diffractive optics calibrator (DOC), a new device for calibrating the etching variation of CGHs. The intensity distribution of far field diffraction pattern was captured and fitted to a parametric model to obtain local etching parameters such as duty-cycle, etching depth and grating period. DOC is capable of measuring 6 to 9 inch CGH with 0.5 mm resolution via the built-in 2-axis scanning stage. Detailed error analysis and system performance
validation was demonstrated with a set of special designed gratings. The information of etching variation can be used in quality control or calibration via wavefront sensitivity functions. Furthermore, it also serves as a feedback to the fabrication performance and future design parameter selection. To further improve the performance of the DOC in the future, (1) one can implement a CCD camera with large dynamic range (e.g. 14-bit ADC) to reduce the fitting uncertainty; (2) for measuring multi-level phase etched CGH, one only needs to provide a new parametric model for least square fitting and wavefront sensitivity calculation. Meanwhile, the hardware configuration and image acquisition software are both universal.

As shown in bold characters in table 4.1, this dissertation provides a rigorous analysis for the encoding error; the corresponding value in the “after” column is based on the example in subsection 2.5.3. Additionally, the DOC enables calibration of etching variation instead of budgeting. The once dominant error source is now a relative small contributor; the corresponding value in the “after” column is based on the fitting uncertainty illustrated in Fig. 3.27.

In summary, this research work provides (1) a new method to analyze the encoding error; and (2) a new device, DOC, for calibrating etching variations. These analysis and calibration improve the comprehensive error analysis for utilizing CGH in optical testing. They are the key components to achieve the goal of measuring freeform aspheric surfaces with 1 nm RMS accuracy.
Table 4.1. CGH errors and the corresponding Methodologies for budgeting or calibration before and after applying the methods discussed in this dissertation.

<table>
<thead>
<tr>
<th>Error source</th>
<th>Current methodology for budgeting or calibration</th>
<th>Wavefront error (nm RMS)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>before</td>
</tr>
<tr>
<td><strong>Design</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fitting residual</td>
<td>Independent ray-tracing</td>
<td>0.0</td>
</tr>
<tr>
<td>Encoding error</td>
<td><strong>Estimate RMS wavefront error with parametric model</strong></td>
<td>0.5</td>
</tr>
<tr>
<td>Fabrication</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pattern distortion</td>
<td>Measure positions of embedded Nikon marks</td>
<td>0.7</td>
</tr>
<tr>
<td>Etching variations</td>
<td><strong>Use diffractive optics calibrator (DOC) to generate a induce phase error map for calibration</strong></td>
<td>2.2</td>
</tr>
<tr>
<td>Substrate error</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Figure error</td>
<td>Calibrate with the transmission wavefront error</td>
<td>1.0</td>
</tr>
<tr>
<td>Laser wavelength</td>
<td></td>
<td>0.2</td>
</tr>
<tr>
<td>Refractive index</td>
<td>Budget the impact of the measurement uncertainty on wavefront accuracy with design model</td>
<td>0.0</td>
</tr>
<tr>
<td>thickness</td>
<td></td>
<td>0.0</td>
</tr>
<tr>
<td>wedge</td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>deformation</td>
<td></td>
<td>0.1</td>
</tr>
<tr>
<td>Alignment error</td>
<td>Use CGH alignment pattern to adjust tip/tilt and axial position</td>
<td>0.4</td>
</tr>
<tr>
<td>Root Sum Square</td>
<td></td>
<td>2.6</td>
</tr>
</tbody>
</table>
APPENDIX A - Analysis of Wavefront Error Introduced by Encoding the Computer Generated Holograms

Wenrui Cai, Ping Zhou, Chunyu Zhao, and James H. Burge

Published in *Applied Optics*, Dec 2013

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Analysis of wavefront errors introduced by encoding computer-generated holograms

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Received 4 September 2013; revised 25 October 2013; accepted 25 October 2013; posted 28 October 2013 (Doc. ID 196836); published 25 November 2013

The fabrication of computer-generated holograms (CGH) by e-beam or laser-writing machine specifically requires using polygon segments to approximate the continuously smooth fringe pattern of an ideal CGH. Wavefront phase errors introduced in this process depend on the size of the polygon segments and the shape of the fringes. In this paper, we propose a method for estimating the wavefront error and its spatial frequency, allowing optimization of the polygon sizes for required measurement accuracy. This method is validated with computer simulation and direct measurements from an interferometer. © 2013 Optical Society of America

OCIS codes: (050.1380) Binary optics; (120.2880) Holographic interferometry; (220.0000) Microstructure fabrication.

http://dx.doi.org/10.1364/AO.52.008324

1. Introduction

Interferometry with computer-generated holograms (CGH) has evolved into standard technology for optical testing and metrology. By controlling the phase of the diffracted light, CGHs are capable of generating reference wavefronts of any desired shape, which allow the use of interferometers for measuring complex aspheric surfaces [1]. The shape of a CGH pattern on a substrate defines the profile of the wavefront. The accuracy of plotting the CGH determines the accuracy of the wavefront. Therefore, fabrication errors, such as CGH pattern distortion errors and irregularities of CGH substrate, must be either calibrated or budgeted [2].

The most common way to generate a CGH pattern for optical testing involves four steps. First, a binary fringe pattern for the desired aspheric wavefront phase function is designed [3,4]. Second, the smooth pattern is digitized to a geometric pattern layout with adjustable tolerances. Third, the CGH encoding algorithm translates the pattern geometry into a standard exchange data format (e.g., GDSII). Last but not least, this intermediate data file is later converted to a machine-specific format (e.g., MEBES, etc.) [5].

The CGH fabrication process utilizes technology that was originally developed for the semiconductor industry. These e-beam or optical writing machines are optimized for writing x and y features in integrated circuit production. Therefore, the continuous smooth fringes required by the CGH must be encoded into a set of polygon segments that the machines can accommodate [6,7]. The schematic process of segmentation is demonstrated in Fig. 1. Smaller polygons yield higher performance yet require more data. Therefore, for a large CGH pattern (>100 mm in diameter) with small local periods (about 10 μm), the trade-off between the size of the data file and the tolerance on the wavefront error must be considered practically [8,9].

Any deviation from the ideal fringe causes a wavefront phase error in the application of the CGH. We defined this type of phase error as encoding residual, a particular type of error that relates to the design and fabrication of CGH. In this study, a new method is introduced to estimate the encoding residual and...
its spatial frequencies during the CGH segmenting process. This method enables us to quantify encoding residual using a parametric model and thus achieve the required accuracy and improve the encoding algorithm.

In Section 2, concepts and parameters in the segmenting process are defined. A method to analytically relate the desired phase function and the fringe pattern on the CGH is introduced in Section 3. Based on that, a parametric model is constructed to estimate the encoding-induced phase error in Section 4. In Section 5, special CGHs are designed and fabricated to verify the model. Matched results are illustrated between simulations and interferometric measurements.

2. Encoding Residual in the Polygon Approximation

This section defines the important parameters for encoding the fringe-type CGH. The wavefront phase generated by a CGH is a function of the position of the fringes [9,10]. Therefore, the deviation from the actual central line of the fringe is used to evaluate the accuracy of the polygon segment approximation of a smooth fringe. The amount of wavefront phase errors produced by the CGH pattern position error can be expressed as

$$\Delta \phi(x, y) = -m \sigma(x, y)$$  \hspace{1cm} (1)

where $\sigma$ is the projection of the fringe position error in the direction perpendicular to the pattern, $d$ is the local period of local fringe, $m$ is the diffraction order, and $\lambda$ is the wavelength. The resulting wavefront phase errors due to pattern position errors are inversely proportional to the local fringe spacing.

When encoding the CGH fringe pattern, a set of boundaries is given to limit the maximum position error of the segments, as shown in Fig. 2. The error boundary is defined by an error ratio $\epsilon$, which represents the percentage of phase error to one fringe:

$$\epsilon = \frac{\sigma}{d}$$  \hspace{1cm} (2)

The given boundary essentially determines the absolute peak-to-valley of the wavefront phase error. Given the error ratio, the maximum phase error is $\Delta \phi = -\epsilon d m$. In order to evaluate its spatial frequencies, we need to know the phase error as a function of the parameters of the local fringe pattern.

According to the simple geometry illustrated in Fig. 2, the error period $L$ depends on the local curvature of the fringe:

$$L = 4\sqrt{R \cdot \sigma} = 4\sqrt{\epsilon d / \lambda}$$  \hspace{1cm} (3)

where $R$ is the local fringe radius, and $c = R^{-1}$ is the local fringe curvature. The error frequency is the inverse of the error period:

$$\gamma = \frac{1}{L} = \frac{1}{4\sqrt{\epsilon / \lambda d}}$$  \hspace{1cm} (4)
This is the crucial parameter in characterizing the phase errors induced by encoding residuals.

There is a tradeoff between the maximum encoding residuals and the number of polygons used, which is directly related to the size of the pattern data file. A tighter error boundary gives smaller phase errors with higher spatial frequency, yet requires more polygon segments. For example, a 100 mm size CGH pattern with an error of 0.005λ usually will have a data size of about one gigabyte.

3. Error Frequency as Function of Given Wavefront

Given any CGH pattern and error boundary, the encoding residual and its frequencies can be calculated by relating the phase function to the local parameters (e.g., fringe spacing, curvature) of the fringe pattern. The fringe type CGH is like a contour map. For a first-order CGH, m = 1, each line adds λ to the wavefront phase. The local fringe spacing is simply related to the local slope of the given phase function and the wavelength λ. The local fringe curvature, however, is more complicated. Some concepts from differential geometry are introduced before presenting the relationship between the fringe curvature and the phase function [11].

Let \( f(x, y) \) denote a phase function created by the hologram. Then the gradient of this function in a 2D vector field is

\[
\nabla f = \langle f_x, f_y \rangle,
\]

where \( f_x = \frac{\partial f(x, y)}{\partial x}, f_y = \frac{\partial f(x, y)}{\partial y} \).

Define the slope at a given location \( S(x, y) \) as the magnitude of \( \nabla f \):

\[
S(x, y) = |\nabla f| = \sqrt{f_x^2 + f_y^2}.
\]

Define the aspect at each point as \( \theta(x, y) \) as the angle of \( \nabla f \) relative to the x axis:

\[
\theta(x, y) = \arctan \left( \frac{f_x}{f_y} \right).
\]

The tangential vector is defined as the direction perpendicular to the gradient:

\[
\nabla g = \langle f_y, -f_x \rangle,
\]

so \( \nabla g \cdot \nabla f = 0 \).

In Fig. 3, the dashed lines represent the contours of an arbitrary phase function: \( f(x, y) = N\lambda \). The direction of the gradient at \( f(0, 0) \) is shown, as well as the aspect and the tangential vector. The local fringe curvature is defined as the contour curvature, which measures the curvature of the contour line in the x and y plane that goes through a certain location. It is the rate of \( \theta \) changes as we move in the direction of \( \nabla g \). We can define the contour curvature \( \kappa_c \) using directional derivative as [11]

\[
\kappa_c(x, y) = -\nabla \theta(x, y) \cdot (\nabla g(x, y)/S(x, y)),
\]

where \( \nabla \theta(x, y) = (\partial \theta/\partial x, \partial \theta/\partial y) \). Using Eq. (8) and \( (\text{d}/\text{d}x)[\arctan(x)] = 1/(1 + x^2) \), the contour curvature can be expressed in Cartesian coordinates as

\[
\kappa_c = -(f_x^2 f_{xx} + 2f_x f_y f_{xy} + f_y^2 f_{yy})/S^3.
\]

Contour curvature \( \kappa_c \) is positive for valleys and negative for ridges. But for our application, the sign of \( \kappa_c \) is not used. Using Eq. (10), the error frequency from Eq. (4) can be rewritten as a function of the local wavefront shape:

\[
\gamma(x, y) = \frac{1}{4} \sqrt{\frac{c}{\pi d}} = \frac{1}{4} \sqrt{\frac{\kappa_c}{S}} \bigg\| f_x f_{xx} - 2f_x f_y f_{xy} + f_y^2 f_{yy} \bigg\| / \sqrt{S}.
\]

For example, the phase function of a Fresnel zone plate (FZP) can be approximated as

\[
f_{\text{FZP}}(x, y) = \frac{x^2 + y^2}{2F}.
\]

where \( F \) is the focal length of the FZP. Using Eq. (11), the error frequency for the FZP is then

\[
\gamma_{\text{FZP}} = \frac{1}{4\sqrt{\pi de}F}.
\]

For this special case, the error frequency is a constant for a given error boundary and the focal length of the FZP. For a constant error ratio \( e \), the length of polygon segment is independent of the location on the CGH pattern. A high peak at \( \gamma_{\text{FZP}} \) is expected in the power spectral density (PSD) of the phase error over the CGH pattern. For a general CGH pattern, the error frequency is a function of the location \( (x, y) \), which corresponds to a continuum in the PSD of the phase error.
4. Parametric Model Simulations

Equation (11) directly relates error frequency to the wavefront function. It provides an insight of the characteristic of the encoding residual along the fringe direction. However, in a real system, the phase error and its frequency profile are also influenced by two factors: (1) error frequencies perpendicular to the fringe directions, which depend on local fringe spacing and the relative initial phase of the sinusoidal errors along the neighboring fringes, which are often randomized to reduce the fixed pattern error; (2) cut-off frequency of the system. A testing system usually serves as a low-pass filter with the cut-off frequency determined either by an aperture or the resolution of the detector. Errors with frequency beyond the system low-pass cutoff are filtered out, which may decrease the root mean square (RMS) wavefront error. Including all the factors, a parametric model is built to estimate the encoding induced phase error and guide to select the error ratio ε.

A. Parametric Model

To illustrate the parametric model, we used a FZP as a simple example. Figure 4 shows direct simulation of the encoding residual of a complete CGH. It is pattern for a FZP (300 mm focal length, f/12). A boundary error of 0.24ε (ε = 20%), which is more than 40 times of our usual design rule, is used to exaggerate the encoding residual effect. A CGH with variable fringe spacing and curvature may be viewed as a collection of gratings with variable spatial frequency. As shown in Fig. 4, initial phases of the sinusoidal errors along the neighboring fringes are usually randomized to erase the fixed pattern error. In order to analyze the properties of the encoding residuals, a localized phase error map is used in the parametric model. Assuming the fringe’s radius of curvature is large compared to the width of the area under analysis, we can approximate a constant fringe space and constant fringe curvature within this region. Then the phase error along the fringe direction is approximated using a sine function with a frequency equal to the error frequency Y(x, y), as defined in Eq. (11). The fringe spacing is determined by the ratio of λ, the wavelength to S(x, y), the local wavefront slope magnitude in Eq. (6).

In order to characterize the frequency components of phase errors induced by encoding residuals, PSD was used. It is a metric to analyze the mid to high-spatial-frequency components of the surface height errors of an optical surface. The amplitude of a 2D PSD is proportional to the modulus square of the Fourier transform of the phase error map:

$$\text{PSD}(u, v) = \frac{1}{A} |F[h(x, y)]|^2,$$  \hspace{1cm} (14)

where h(x, y) is the phase error map, and A is the map area.

In addition, 1D PSD can be used to represent the radial sum of the 2D PSD as shown in Fig. 5:

$$\text{PSD}_1(\rho) = \int_0^{2\pi} \text{PSD}(\rho, \theta) d\theta,$$  \hspace{1cm} (15)

where $\rho = \sqrt{u^2 + v^2}$, $\theta = \tan^{-1}(v/u)$ are in polar coordinate.

The spatial frequency of the sharp raising ramp of the PSD profile is determined by the error frequency Y(x, y), while the largest frequency component is determined by the fringe spacing. In our parametric model, this 1D-PSD profile plotted in a standard linear scale is simplified as a triangle, as illustrated in Fig. 6.

As we know, the RMS value of a phase map is the square root of the total area of its PSD [12]:

$$\sigma^2 = \int \text{PSD}(u, v) du dv.$$  \hspace{1cm} (16)

Ideally, the RMS phase error can be calculated from the area of the triangle. However, in real systems, individual fringe will not be resolved, meaning that the system cutoff is larger than the smallest spatial frequency in the model. Therefore the actual

![Fig. 4. Encoding-induced phase error map of a FZP and its zoomed-in characters. (color bar unit: nanometer)](image)

![Fig. 5. 1D PSD of a local area on a phase map when individual fringe is resolved. It has a shape close to a triangle.)](image)
RMS phase error is proportional to the trapezoidal area between error frequency \( Y(x,y) \) and the system cut-off frequency. From the area ratio illustrated in Fig. 6, we can express the local RMS phase error as
\[
\sigma_{\text{RMS}}(x,y) = \frac{\epsilon}{\sqrt{2}} \sqrt{1 - \left( \frac{S(x,y)S(y,x)}{S(x,y)S(y,x) - f_{\text{cutoff}}} \right)^2},
\]  
where \( \epsilon / \sqrt{2} \) is the RMS value of a sinusoidal phase error before applying the low-pass filter from the real system, and it equals to the whole area of the PSD triangle. Equation (17) is valid when \( f_{\text{cutoff}} \geq y \).

Otherwise, no phase error induced by encoding residuals will pass the system. The RMS phase error of the whole pattern \( \sigma_{\text{RMS}} \) can be expressed as the average of the local errors:
\[
\sigma_{\text{RMS}} = \frac{1}{MN} \sum_{x=1}^{M} \sum_{y=1}^{N} \sigma_{\text{RMS}}(x,y).
\]

This parametric model allows us to estimate the encoding error for a CGH, even before actually digitizing the smooth fringe patterns.

B. Model Application and Analysis

Equations (11) and (17) are essential parts of the parametric model. Given a wavefront function \( f(x,y) \), an error boundary \( \epsilon \) and a system cut-off frequency \( f_{\text{cutoff}} \), the encoding induced RMS phase error can be estimated as a function of the spatial coordinates of the CGH pattern.

Two examples are employed: a FZP (300 mm focal length, f/12) and a general CGH (50 waves of spherical aberration with tilt carrier). MATLAB was used to generate the maps in Figs. 7 and 8. A wavefront phase function was digitized into a matrix. As shown in Figs. 7(a) and 8(a), each fringe on the patterns represents 50 waves in a real CGH pattern. Meanwhile, the RMS phase error was calculated for each pixel. Figures 7(b) and 8(b) delineate the encoding induced RMS phase error maps with error boundary (\( \epsilon = 10\% \)) and system cut-off frequency (\( f_{\text{cutoff}} = 125 \text{cyc/aperture} \)).

In Figs. 7(b) and 8(b), the “hot spot” on the phase error map is where the fringe curvature is small or the fringe spacing is large. To mitigate this locally high phase error, we can constrain the design to a tighter error boundary in that region. Moreover, a tilt carrier will increase the slope and reduce the RMS value. However, too much tilt will affect the accuracy of the CGH writing.

Given the phase function and error ratio, we can calculate the peak frequency of the phase error and the RMS error. This helps to guide the design and encoding of the CGH.

5. Experimental Verification

In order to further verify the parametric model, a set of FZPs with different boundary errors were fabricated and then measured in an interferometric null test. Compared to other types of fabrication errors, the encoding residual for a generic CGH is small (\( \epsilon < 0.5\% \)) compared to other types of fabrication errors. By enlarging the boundary errors, the encoding residual becomes dominant, which made experimental measurement possible.

A. Measurement Setup

In order to measure the frequency components of the encoding residual, the interferometer should have a spatial resolution of no less than 200 cycles per aperture. We used a Twyman-Green interferometer (PhaseCam(T), 4D technology, f/9 objective, 1000 × 1000 pixelated mask CCD) with a 7 mm collimated output beam.

A set of FZPs (300 mm focal length, f/12) was tested in transmission with different error boundaries. For each FZP under test, ring alignment

Fig. 7. (a) Fringe map (FZP). (b) Encoding-induced RMS phase error map (FZP).

Fig. 8. (a) Fringe map (spherical aberration with tilt). (b) Encoding-induced RMS phase error map (spherical aberration with tilt).
patterns outside the clear aperture were used in reflection for positioning the substrate relative to the interferometer; the FZP itself collimated the beam. A coated flat mirror was placed behind the substrate to retroreflect the light. In order to prevent undesired diffraction orders from entering the interferometer, a spatial filter was placed at the focus of the converging beam. This 2.1 mm diameter stop also served as a 5.7 mm⁻¹ low-pass filter, which was

Table 1. Comparison of the Measurement Results and Simulation Results at Different Levels of Encoding Residual

<table>
<thead>
<tr>
<th>Measurements</th>
<th>Simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase error map (nm)</td>
<td>Phase error map (nm)</td>
</tr>
<tr>
<td>1D PSD (nm² / mm²)</td>
<td>1D PSD (nm² / mm²)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>0.5% ± Boundary error</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS 2.3 nm</td>
</tr>
<tr>
<td>1D PSD</td>
</tr>
<tr>
<td>RMS 0.8 nm</td>
</tr>
<tr>
<td>1D PSD</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5% ± Boundary error</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS 0.5 nm</td>
</tr>
<tr>
<td>1D PSD</td>
</tr>
<tr>
<td>RMS 0.1 nm</td>
</tr>
<tr>
<td>1D PSD</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>10% ± Boundary error</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS 1.6 nm</td>
</tr>
<tr>
<td>1D PSD</td>
</tr>
<tr>
<td>RMS 1.6 nm</td>
</tr>
<tr>
<td>1D PSD</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>20% ± Boundary error</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMS 3.8 nm</td>
</tr>
<tr>
<td>1D PSD</td>
</tr>
<tr>
<td>RMS 3.4 nm</td>
</tr>
<tr>
<td>1D PSD</td>
</tr>
</tbody>
</table>
included in the corresponding simulation results. The layout of the system is delineated in Fig. 9.

The signature frequency components are above 30 cycles per aperture (1.3 cyc/mm for a 25 mm aperture). This frequency range is susceptible to magnitude degradation resulting from wavefront propagation [13]. Therefore, to constrain the effect of defocus to a negligible level, the return flat was placed within 20 mm behind the FZP substrate.

8. Results and Analysis

Verification was performed by comparing the measurement results with (1) the simulations of the encoding process and (2) the parametric model.

Table 1 lists the measured phase maps and 1D-PSD curves (boundary error: 0.5%, 5%, 10%, and 20%) compared to the corresponding simulation results, respectively.

Since the target errors do not contain low frequencies, the first 37 terms of Zernike polynomials were subtracted from the phase error maps in Table 1. In the case with 0.5% boundary error, the lowest frequency of the encoding residual was higher than the cut-off frequency of the low-pass filter. There was no frequency peak in the measured PSD as expected. This result demonstrated our ability to control the encoding residual down to a negligible level.

For the succeeding three cases (5%, 10%, and 20%), the frequency profiles matched the simulated results in the region where the frequency was above the value calculated from Eq. (11). The lower-frequency regions in the measured PSDs had higher magnitude than the simulations. This may be due to the residual lower-order phase error from the CGH substrate figure error or the surface errors on the flat return mirror. If more Zernike terms are subtracted from the measured map, the noise on the lower part of each PSD will be mitigated.

The profiles of these 1D PSDs are the accumulative effect of different shapes of triangles discussed in the last section. Since the FZP has constant error frequency globally, all the triangle shapes have the same starting frequency. Hence, a peak at the error frequency \( \gamma \) is expected. In Table 2, the peak frequencies in each case were recorded and compared with the values from Eq. (11) and simulations. The differences of results are within 2%. In Table 3, the RMS phase errors of each case were compared with values from the corresponding simulations. The higher measured RMS values also may result in the lower-order phase errors other than the encoding residuals.

After the simulation of the encoding process is verified, we applied the parametric model to this set of FZPs. We measured the local RMS phase errors (1 mm by 1 mm area) on certain locations of the FZPs and plotted the results in Fig. 10. The differences of local RMS value between the parametric model and the measurements are within 10%. This agreement is sufficient to verify our parametric model, since the FZPs contain a large range of fringe spacing and curvature.

6. Discussion

Before the method we proposed in this paper, the existing approach to address this problem was to perform a full simulation for the actual encoding process and create the induced phase error map. It is similar to the simulation results in Table 1. The disadvantage of this approach is time consuming. One may need to perform this trial and error approach until the errors are within the budget. In comparison, the parametric model we presented will provide a systematic method to quickly estimate the encoding-induced phase error and to analyze the CGH pattern design before we proceed to the encoding process.

Table 2. Comparison of Peak Frequencies among Analytical, Simulated, and Measured Values

<table>
<thead>
<tr>
<th>Boundary Error (%)</th>
<th>Eq. (11)</th>
<th>Simulations</th>
<th>Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>8.11</td>
<td>8.16</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>2.57</td>
<td>2.56</td>
<td>2.55</td>
</tr>
<tr>
<td>10</td>
<td>1.81</td>
<td>1.80</td>
<td>1.79</td>
</tr>
<tr>
<td>20</td>
<td>1.28</td>
<td>1.28</td>
<td>1.28</td>
</tr>
</tbody>
</table>

Table 3. Comparison of RMS Phase Errors between Simulated and Measured Values

<table>
<thead>
<tr>
<th>Boundary Error (%)</th>
<th>Simulations</th>
<th>Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.9</td>
<td>2.1</td>
</tr>
<tr>
<td>5</td>
<td>8.1</td>
<td>8.5</td>
</tr>
<tr>
<td>10</td>
<td>16.3</td>
<td>16.5</td>
</tr>
<tr>
<td>20</td>
<td>34.6</td>
<td>38.0</td>
</tr>
</tbody>
</table>
7. Conclusion

In this paper, we presented a method for estimating
the phase error and its frequency profile when encod-
ing the CGH. First, a parametric model was built
based on characteristics of encoding residual in the
frequency domain. Second, the encoding-induced
RMS phase error was written as a function of the de-
signed wavefront, error boundary, and system spatial
resolution. By varying the boundary error ratio and
image resolution, the phase errors and its error fre-
quencies can be tuned as required. Finally, veri-
fication tests with PZPs were performed. We dem-
onstrated that the measurement results agreed with
the simulation. Therefore, we proved our ability to
quantify and control this type of error using a para-
metric model with the required accuracy.

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APPENDIX B - Wavefront Propagation Effects: Measurements of the CGH with Large Encoding Error

When performing the interferometric measurements of the CGH with large encoding error (20% boundary error), we found that some frequency components have low power spectral density (PSD) compared to the simulation results, which presents an oscillating profile in 1D-PSD, as shown in Fig. B.1(a). The smaller RMS phase error for the measurement also results from the missing error frequencies.

![Fig. B.1](image)

Fig. B.1 Comparison of encoding residuals and the corresponding 1D-PSDs between measurement and simulation (at 20% boundary error). The 1D-PSD of the interferometric measurement (a) shows lower magnitude at certain frequencies compared to the simulation (b).
The reason of this effect is: the amplitude and phase modulation of the periodic structure has a sinusoidal variation as a function of the propagation distance. When the CGH pattern is not perfectly in focus for the imaging system of the interferometer, the transfer function of different frequency component will have different values. Because of the nature of the double-pass testing set-up, the CGH surface and its image from the mirror cannot be focused at the same time. Hence, the oscillating profile in the measured 1D-PSD is due to wavefront propagation, which contains a set of different frequency components. Talbot effect can be used to analyze the transfer function for different frequencies, which is discussed in details by Zhou and Burge in [2.11].

As analyzed in chapter 2, the encoding induced phase error of a Fresnel zone plate (FZP) has a periodic character along the fringe direction, which has a special error frequency for a given error boundary. According to Talbot effect, an exact image of this phase structure will appear at certain distances by free-space propagation. When collimated coherent light illuminates a periodic object, the self-image planes are equispaced and well defined by Talbot distance:

\[ z_T = \frac{2p^2}{\lambda} = \frac{2}{\lambda \gamma^2} , \]  

where \( p \) is the period of the periodic object, \( \gamma = 1/p \) is the spatial frequency and \( \lambda \) is the wavelength of the illumination. When the periodic objects have weak phase distribution, the attenuation of the phase ripple as it propagates a distance \( z \) can be described by a cosine transfer function [B.1]:

\[ W' = W \cos\left(2\pi \frac{z}{z_T}\right) = W \cos\left(\pi z \lambda \gamma^2\right) , \]  

(B.2)
where \( W \) is the magnitude of the original phase ripple.

For spherical wavefront, the replication of the object is amplified and the distance between the self-image planes varies with propagation. The spherical illumination can be converted to a collimated beam by an ideal lens with focal length \( f \), which can be obtained from Gaussian reduction of the image system of the interferometer. The schematic conversion is depicted in Fig. B.2.

![Fig. B.2. Propagation of a spherical wavefront is converted to equivalent propagation in a collimated space by an ideal lens. (The propagation is from right to left, which is consistent with the convention as the light is collected by the interferometer in Fig. 2.9)](image)

When a spherical wavefront with radius of curvature \( R_1 \) and phase ripples of spatial frequency \( \gamma \), propagates to a position with radius of curvature \( R_2 \), the effective propagation distance in collimated space is:

\[
\Delta z_{\text{effective}} = z_2 - z_1 = f^2 \left( \frac{1}{R_2} - \frac{1}{R_1} \right),
\]

(B.3)
where \( z_1 \) and \( z_2 \) are the distances to conjugate planes of wavefront at \( R_1 \) and \( R_2 \), respectively. The effective spatial frequency of the phase ripple in the collimated space is:

\[
\gamma_{\text{effective}} = \frac{R_1}{f} \gamma.
\]  

(B.4)

As the wavefront propagates in collimated space, the equivalent phase attenuation can be expressed as:

\[
W_{\text{effective}} = W \cos \left[ \pi (\Delta z_{\text{effective}}) \gamma_{\text{effective}} \lambda^2 \right] = W \cos \left[ \pi \lambda \gamma^2 \frac{R_1 (R_1 - R_2)}{R_2} \right].
\]  

(B.5)

Fig. B.3. The absolute magnitude of the transfer function for phase ripple with different spatial frequencies. Two different propagation distances 20mm and 400mm are shown.

The transfer function can be calculated as a function of the spatial frequency using Eq.(B.5). In our measurement set-up, \( R_1 = 300\text{mm} \) is the focal length of the FZP. \( R_1 - R_2 \) is approximately twice the distance between the CGH and the mirror. As shown in Fig. B.3, the longer distance...
the wavefront propagates, the more spatial frequency information is attenuated. The multiple-valley profile of the transfer function is similar to the oscillating profile of the 1-D PSD shown in Fig. B.1(a).

In order to verify the measurement effect is due to the wavefront propagation, we conducted an experiment by taking a set of measurements with different distances between CGH and the return mirror. The measurement set-up is illustrated in Fig. B.4. While the measurement results are presented in Fig. B.5.

Fig. B.4. Interferometric measurement for the phase error due to encoding the CGH. (a) wavefront propagation effects are minimized when return mirror close to the CGH; (b) wavefront propagation effects become more prominent when return mirror moves away from the CGH.
Fig. B.5. phase error maps and 1-D PSDs as the return mirror moves away from the CGH step by step. CGH-mirror distance: (a) 10, 25, 50mm; (b)75, 100, 125mm; (c)150, 175, 200mm; (d)225, 250, 275mm.
We can see from measurement results in Fig. B.5, the loss of frequency information varies for wavefront propagation caused by different amount of defocus. For example, a 200 mm CGH-mirror distance will cause transfer function local minimums at 2 mm\(^{-1}\), 3.5 mm\(^{-1}\), and 4.4 mm\(^{-1}\), which will cause oscillating profile in the PSD. In Fig. B.6, the spatial frequencies of transfer function’s first minimum are plotted for different CGH-mirror distances. And the result is matched with the values calculated from Eq. (B.5).

Fig. B.6 The spatial frequencies of transfer function’s first minimum vs. the CGH-mirror distance.
For our specific application about analyzing the 1D-PSD profile of encoding residual, the error frequency range is susceptible to magnitude degradation resulted from wavefront propagation. As a result, we need to constrain the effect of defocus to a negligible level. For instance, in order to maintain the transfer function above 80% within the cut-off frequency, the defocus must be less than 20 mm.
APPENDIX C - The Diffractive Optics Calibrator: Design and Construction

Wenrui Cai, Ping Zhou, Chunyu Zhao, and James H. Burge

Published in Optical Engineering, SPIE, Dec. 2013

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Diffractive optics calibrator: design and construction

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Diffractive optics calibrator: design and construction

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Abstract. We present a new device, the diffractive optics calibrator, for measuring duty cycle and etching depth for computer-generated holograms (CGHs). The system scans the CGH with a collimated laser beam and collects the far-field diffraction pattern with a CCD array. The relative intensities of the various orders of diffraction are used to fit the phase shift from etching and the duty cycle of the binary pattern. The system is capable of measuring variations that cause 1-nm peak-to-valley (PV) phase errors in the waveform created by the CGH. The measurements will be used primarily for quality control of CGHs, but the data can also be used to provide a lookup table for corrections that allow calibration of the lithography errors. Such calibration is necessary to achieve our goal of measuring freeform aspheric surfaces with 1-nm RMS accuracy. © 2003 Society of Photo-Optical Instrumentation Engineers (SPIE)
[DOI: 10.1117/1.1624198]

Subject terms: computer-generated holograms; duty cycle; etching depth; instrumentation; quality control.

Paper 131960P received Sep. 5, 2003; revised manuscript received Nov. 4, 2003; accepted for publication Nov. 7, 2003; published online Dec. 4, 2003.

1 Introduction

Interferometry with computer-generated holograms (CGHs) has evolved to be a standard technology for aspheric optical testing and metrology. CGHs are capable of generating a wavefront of any desired shape.1 However, errors and uncertainties in the fabrication processes of diffractive optics produce wavefront errors. When applying the finished part in optical testing, accuracy of the measurements will be affected consequently. Therefore, CGH fabrication errors must be either calibrated or budgeted.

Irregularities of the CGH substrate are the dominant fabrication error, especially for large CGHs. Since the substrate irregularities influence all the diffraction orders equally, it can be calibrated using the measurement from the zero-order diffraction. The nonuniformities in duty cycle and etching depth are coupled together, and introduce wavefront errors for the zero order and nonzero orders differently. (Except for a special case of 50% duty cycle, all orders have the same sensitivity to the etching depth errors.) After subtracting the zero-order measurement from the (nonzero order) surface measurement, the CGH substrate error is canceled but the wavefront still contains some residual errors due to variations in duty cycle or etching depth.

The local value of duty cycle and etching depth can be measured using a white light interferometer, an atomic force microscope, or a scanning electron microscope, which provide the surface relief of the object under test. However, these methods require expensive equipment and long inspection procedures. While they may be acceptable for small samples of diffractive optics, these methods are not suitable for inspecting large CGH substrates (usually 6 to 9 in. in diameter).

The alternative method is to use coherent illumination and analysis of the diffraction pattern in the Fourier plane.2 In optical testing, the binary CGH often uses the tilt carrier to separate the desired diffraction order from other orders.

The common range of the grating periods is from 5 to 30 μm, which is well within the limit of scalar diffraction theory.4 Therefore, one can write an analytical expression for the diffraction efficiency as a function of both the duty cycle and the etching depth.5 We utilize this relationship in the device. From a collimated laser beam, the intensities of multiple diffraction orders were captured by a wide-field-of-view camera lens. This measurement technique is illustrated in Fig. 1. The duty cycle and etching depth are determined simultaneously by fitting the measured intensities to a parametric model. This device is able to scan through the whole area of the diffractive optics and generate a wavefront phase error map due to variations in duty cycle and etching depth. The measurements are validated with a vertical scanning white-light interferometer.

The system is capable of measuring the percentage of duty cycle variation that causes 1-nm PV phase errors. This method is advantageous since it can be carried out rapidly with accurate and repeatable results, does not damage the sample, and uses low-cost equipment.

In Sec. 2, the performance requirements are set based on a parametric model and Monte Carlo simulations. In Sec. 3, the system hardware and software are described in detail. In Sec. 4, diffraction optics calibrator’s (DOC’s) performance is evaluated by a set of specially designed gratings and is compared with the measurement from a white light interferometer.

This article presents the design, construction, and function of the DOC® 1®, this paper is based on SPIE Proceedings Paper (vol. 8839-6) with minor revisions, whereas the detailed analysis and operation of the system for calibrating CGHs will be presented in another article.6

2 Performance Requirements

To measure the variations in duty cycle and etching depth, a grid of sampled points should be measured across the effective area of the CGH. The local values of duty cycle and etching depth are determined by nonlinear least-square fitting the measured intensities of different diffraction orders to a
parametric model. Although the calibration method only requires the intensity information of diffraction orders, the diffracted wavefront phase is also presented in order to relate the fabrications error to the induced wavefront phase error and serve as a guideline to set up the system requirements. The measurement uncertainty can be estimated via a Monte Carlo simulation, based on which the performance requirements are determined.

$$\eta = \begin{cases} A_0^2(1 - D)^2 + A_1^2D^2 + 2A_0A_1D(1 - D)\cos \phi & m = 0 \\ (A_0^2 + A_1^2 - 2A_0A_1\cos \phi)D^2\sin^2(mD) & m = \pm 1, \pm 2, \ldots \end{cases}$$ (1)

The diffracted wavefront phase $\Psi$ in the far field can be defined as

$$\tan \Psi = \begin{cases} \frac{\Delta \phi}{A_0 + 2A_1\sin \phi \sin(mD)} & m = 0 \\ \frac{-\Delta \phi}{A_0(1 + 2A_1\cos \phi \sin(mD))} & m = \pm 1, \pm 2, \ldots \end{cases}$$ (2)

Derived from Eq. (2), the wavefront phase sensitivity functions, $\partial \Psi / \partial D$ and $\partial \Psi / \partial \Delta \phi$, are used to estimate the wavefront error due to the variation in the duty cycle and phase step,

$$\Delta W_D = \frac{1}{2\pi} \frac{\partial \Psi}{\partial D} \Delta D,$$ (3)

$$\Delta W_{\Delta \phi} = \frac{\partial \Psi}{\partial \Delta \phi},$$ (4)

where $\Delta W_D$ and $\Delta W_{\Delta \phi}$ are the wavefront errors in waves caused by small deviations in duty cycle $\Delta D$ and in phase step $\Delta \phi$ (in waves), respectively. Equations (3) and (4) provide a connection between the fabrication errors and system performance. They can be used to estimate the amount of wavefront phase errors that result from the fabrication non-uniformities. Moreover, the information, when used in reverse, provides a set of error budgets for application using CGHs. For example, there is a CGH with 49% nominal duty cycle and 0.5-μm etching depth (0.35 μm phase step at 632.8 nm). In order to keep the error budget of the phase error induced by the duty-cycle variation within 1-nm peak-to-valley (P-V), $\Delta D$ must be controlled to within 0.25% P-V. Therefore, a threshold can be set when measuring the duty-cycle variation to indicate whether the fabricated part meets the system requirement. For the same scenario, in order to keep the induced phase error within 1-nm P-V, $\Delta \phi$ must be controlled to within 0.039 μm P-V, which is equivalent to 40 nm P-V of etching depth variation.

### 2.1 Parametric Model

The parametric model based on scalar diffraction theory was first developed by Chang and Barge. It assumes that the wavelength of the incident light is much smaller than the grating period (in our case, grating period ranges from 5 to 30 μm with visible light.) Figure 2 depicts a cross-sectional view of a binary grating. It is defined by period $S$ and the etching depth $t$. The duty cycle of the grating is defined as $D = h/S$, where $h$ is width of the unetched area, $n$ is the refractive index of the grating, and $A_0$ and $A_1$ correspond to the amplitudes of the output wavefront from the unetched and etched areas of the grating, respectively. The phase step $\phi$ represents the phase difference between these two areas, which equals $2\pi(n - 1)/\lambda$ for a grating used in transmission.

Based on Fraunhofer diffraction theory, the diffraction efficiency $\eta$ in the far field can be derived as

In order to estimate the accuracy of the method with the presence of noises, a Monte Carlo simulation was performed to find the relationship between the number of diffraction orders measured and the measurement uncertainty of the duty cycle or phase step. For a phase type CGH used in transmission, we assumed, in Eq. (2), $A_0$ and $A_1$ are both unity, the nominal duty cycle is 49%, and the etching depth is 0.5 μm (phase step of 0.35 μm at 632.8 mm). A ±1% of uniform-distributed noise is added to the intensity of each diffraction order in the simulation. Then, a nonlinear least square fit is performed on the simulated data to obtain the duty cycle and phase step value.

The results depicted in Figs. 3 and 4 are the simulated measurement uncertainty (1σ) of the duty cycle and etching depth, respectively, with the corresponding wavefront phase error of the first diffraction order from a 1000-μm Monte Carlo simulation. The duty cycle uncertainty decreases when more orders of diffraction are included in the fitting process. This means that higher diffraction orders contribute to the accuracy of duty cycle. On the other hand, the uncertainty of etching depth stays at the same level when higher orders are included.

### 2.3 System Requirements

In order to achieve measuring freeform aspheric surfaces with 1-nm RMS accuracy, the knowledge of phase error...
induced by duty cycle variations must be within a fraction of 1 nm. The simulation result in Fig. 3 indicated that including more diffraction orders in the fitting decreases the measurement uncertainty. Thus, a wide-angle camera lens is preferred. Providing the intensity measurement error within 1%, the uncertainty of phase error is <0.25-μm RMS, if up to ±7 orders are measured.

In addition, the entrance pupil of the lens system must coincide with the plane of the diffraction pattern. Otherwise, the vignetting effect will reduce the intensities for large angle incident and generate a systematic error for the fitting.

The dynamic range of the detector must be large enough to cover the intensity difference between diffraction orders. For example, the intensity ratio between the 1st order and the 15th order is 225:1 for a grating with 50% duty cycle and 0.5-μm etching depth (0.35 μm) phase step at 632.8 nm. Assuming the weakest signal is spread among four pixels, the signal-to-noise ratio (SNR) of the detector must be larger than 1000:1.

The requirements of important functions are listed in Table 1.

### Table 1 System requirements.

<table>
<thead>
<tr>
<th>Features</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Optical subsystem</td>
<td></td>
</tr>
<tr>
<td>Field of view (FOV)</td>
<td>Include up to ±7 diffraction orders for grating period: 5 to 30 μm</td>
</tr>
<tr>
<td>Entrance pupil</td>
<td>In front of the mechanical structure of the imaging lens</td>
</tr>
<tr>
<td>Image aberrations</td>
<td>Spot diameter &lt;1/4 spot spacing between adjacent orders</td>
</tr>
<tr>
<td>Detector dynamic range</td>
<td>&gt;1000:1</td>
</tr>
<tr>
<td>Laser calibration</td>
<td>Prevent intensity cross-talk between adjacent orders</td>
</tr>
<tr>
<td>Mechanical subsystem</td>
<td></td>
</tr>
<tr>
<td>Scanning mechanism</td>
<td>Motorized two-axis linear stages</td>
</tr>
<tr>
<td>Travel range</td>
<td>&gt;6 in. for each axis</td>
</tr>
<tr>
<td>Alignment mechanism</td>
<td>(1) Lockable adjustments between source and imaging system</td>
</tr>
<tr>
<td></td>
<td>(2) Plane of the CGH surface must be parallel to the motion direction</td>
</tr>
<tr>
<td>Software</td>
<td></td>
</tr>
<tr>
<td>Image acquisition</td>
<td>Shutter speed and gain control</td>
</tr>
<tr>
<td>Motion control</td>
<td>Programmable scanning routine</td>
</tr>
<tr>
<td>Image process and data output</td>
<td>Output local duty cycle and etching depth value as a function of position</td>
</tr>
</tbody>
</table>

3 System Hardware and Software

Figure 5 illustrates the layout of the DOC. It consists of three subsystems: optical system, mechanical system, and software. In the optical subsystem, a collimated laser beam propagates through the CGH substrate and diffracts into multiple diffraction orders. The intensities of these orders are captured by a wide-angle camera and processed via LabVIEW to obtain the local duty cycle and phase step. A two-axis motorized linear stage is implemented in the system, which is capable of scanning through a CGH with an aperture up to 9 in. In order to measure the duty-cycle variation across the CGH aperture, LabVIEW is used to synchronize the scanning motion of the stage with the image acquisition of the camera. The system will be used primarily for CGH quality control. Therefore, in this section, detailed specifications of each component in the system are discussed in consideration of the requirements in Table 1.
HFOV = \theta_{\text{max}} = \sin^{-1}\left(\frac{\lambda^2}{d^2}\right), \tag{5}

where \(m\) is the highest diffraction order measured, \(\lambda\) is the laser wavelength, and \(d\) is the grating period. The maximum FOV requirement is determined by both the grating period and wavelength of test beam. As a result from the requirements in Table 1, we implemented a commercial available camera lens with a focal length of 2.6 mm F/2.5, and full field of view is 83.2 deg by 67.5 deg. The entrance pupil is located at 1 mm in front of the lens.

For the same reason of including more orders, a short wavelength can be beneficial. However, too short a wavelength is not responsive to common CCD detectors. Moreover, angular separation between adjacent orders may not be enough for gratings with large periods when using a short wavelength laser. In consideration of these trade-offs, we choose a compact collimated laser diode module from Thorlabs\textsuperscript{b} at 532 nm, 4 mW, and divergences \(<1.8\) mrad.

An iris aperture stop was placed to control the size of the laser beam footprint on the CGH pattern. The footprint should be small enough that the illuminated area can be approximated into linear grating. Otherwise the far-field diffusion pattern will contain information about higher-order effects. For instance, if the illuminated area is too large and contains a curved pattern, the shape of high-diffraction-order spots may either be elongated by accommodation or blurred by power term. Depending on local curvature of the CGH fringe pattern, a threshold for the illuminated area can be set. We controlled the illuminated area on the pattern to 1 mm in diameter. As for spatial resolution, 1 mm on a 6-in. CGH pattern is equivalent to 150 cycles/aperture. Higher-frequency features were averaged within the illuminated area.

As we make measurements across the CGH pattern, the orientation of the local linear grating may change. Thus, a two-dimensional (2-D) detector is required to cover diffraction for all directions. In our system, we implemented a Pointgrey\textsuperscript{c} FLEA camera with \(\frac{1}{2}\)-in. 1024 x 768 pixels CCD and 12-bit A-D converter. The fill factor of the CCD sensor is close to 100% due to the micro-lenset on top of each pixel. The measured SNR of this camera is about 3000:1, which is sufficient for our system requirements.

In summary, the optical system consists of a collimated laser, an aperture stop, a wide-field camera lens, and a CCD array. They are selected to meet the performance requirements of our system.

![Fig. 5 DOC's schematic layout.](image)

3.2 Mechanical System

As depicted in Fig. 5, the light source and the camera were placed at different sides of the CGH substrate. The CGH substrate was mounted on a two-axis linear stage in between to achieve the scanning ability. The stages are motorized using Schneider Electronic MDrive\textsuperscript{d} 14 stepper motors and controlled by LabVIEW. The hardware used was illustrated in Fig. 6. As mentioned in the lens configuration, there was only a 1 mm clearance between the lens casing and the CGH substrate. Therefore, the plane of the CGH surface must be parallel to the motion direction of the two-axis linear stage before performing any measurement. This was done by adjusting the tripod micrometers on the CGH kinematic mount.

3.3 Software

The DOC software was developed in LabVIEW to (1) communicate with both the camera and the stepper motors; (2) perform image processing and data fitting; and (3) output values of local grating period, duty cycle, and etching depth as a function of positions on the CGH pattern.

Via the graphical user interface shown in Fig. 7, we can control DOC to individually measure a specific location on the CGH or program a routine to sample and scan through an area of interest. The positioning accuracy of the two-axis stage is within 0.5 mm for each direction. Camera attributions, such as shutter speed, gain, and frame rate, can be tuned before any measurements. In order to utilize the full dynamic range of the detector, the order with the highest intensity should be adjusted to just below saturation. A
Fig. 7 DOC's graphical user interface in LabVIEW.

Fig. 8 Block diagram of the DOC measurement procedures.
function to indicate any saturated pixel in real time was built in the software.

In a standard measurement procedure, illustrated in Fig. 8, multiple frames of images were acquired and averaged for each location to reduce the random noise. The location of each diffraction order is found by centroiding in a defined area, and the intensity is calculated by averaging the intensities in the same area. Then nonlinear least square fitting was performed to obtain duty cycle and etching depth simultaneously. Additionally, the local grating period was calculated from the spacing between ±1 orders on the detector. Last, a log file was generated to record the measured data for each location. One can either program in a routine to step and repeat through a region or can move the stage to an individual local of interest and perform measurements.

Table 2. The specifications of a set of gratings with different duty cycle values.

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duty cycle range</td>
<td>40% to 52% (1% change/grating)</td>
</tr>
<tr>
<td>Etching depth</td>
<td>0.5 μm</td>
</tr>
<tr>
<td>Grating period</td>
<td>15.0 μm</td>
</tr>
<tr>
<td>Grating size</td>
<td>3 × 20 mm²</td>
</tr>
<tr>
<td>Overall pattern size</td>
<td>39 × 20 mm²</td>
</tr>
</tbody>
</table>

Fig. 9. The layout of a set of gratings with varying duty cycle from 40% to 52%.

Fig. 10. Repeatability of measured duty-cycle as a function of light level.

Fig. 11. Repeatability of measured etching depth as a function of light level.

4 Measurement Results and Analysis

In order to test the repeatability and accuracy of DOC, a set of 13 linear gratings with varying duty cycles was fabricated with specifications listed in Table 2 and illustrated in Fig. 9. Repeatability tests were performed by measuring the same grating location multiple times. By changing the exposure time of the camera, we examined the repeatability of the measurements as the function of the light level reaching the CCD pixels. The results of the gratings with 46% and 49% duty cycle are illustrated in Figs. 10 and 11. As the light level decreases from saturation, the relative low intensity diffraction orders will begin to drop below the CCD noise level. Therefore, the fitting error may increase, which will lead to larger uncertainty of measured value of duty cycle and etching depth. Generally speaking, at light level >60%, the 1σ uncertainty of the measured duty cycle values is 0.05%, and that of the measured etching depth values is 0.4 nm.

Since the wavefront phase error is induced by the variation of either duty cycle or etching depth, system repeatability is important for CGH quality control. On the other hand,
the system accuracy is crucial for calibrating the induced phase error. We compared the results between a white light interferometer and DOC. The final results are presented in Figs. 12 and 13.

The direct measurement result from the white light interferometer was a 2-D phase map of the grating with a field of view about 150 μm, which contains 10 periods of grating. After the data postprocessing in MATLAB®, the averaged values of both the duty cycle and the etching depth can be obtained. Three measurements within the same duty cycle region were performed and averaged. The measurement procedures and the data processing of the whole set of gratings will take several hours to finish (make measurements every 1 mm, totally 39 sample points). On the other hand, using DOC, the local duty cycle and etching depth were calculated simultaneously. The fitted results are already the average duty cycle and etching depth over an illuminated area of 1 mm in diameter. The results of the whole set of gratings were received within 15 min (39 sample points).

For the case of measuring duty cycle, the largest discrepancy between the two methods occurred at duty cycle around 50%. According to Eq. (2), at 50% duty cycle, the even non-zero orders have zero efficiencies, which are more susceptible to background noise level. This may cost a relatively large fitting error.4 The rest of the regions have a difference of around 0.25% between the two methods, which is the required accuracy to detect about 1-μm PV phase error due to duty cycle variation. For the case of measuring etching depth, the general trends were matched for the measured values by both methods. The largest discrepancy between the two methods is 3 nm.

5 Conclusions
This article describes the DOC, a new device we developed to measure the variation of duty cycle and etching depth of a binary diffractive optics used in transmission. The system is capable of measuring variations that cause 1-μm PV phase errors in the wavefront created by the CGH. The measurements can be used to assess uniformity of the diffractive optics for calibration or quality control.

Acknowledgments
The authors acknowledge help from Todd Horne, Caleb Pocock, and James Upton for the development of DOC’s mechanical system.

References

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APPENDIX D - The Diffractive Optics Calibrator: Measurement of Etching Variations for Binary Computer Generated Holograms

Wenrui Cai, Ping Zhou, Chunyu Zhao, and James H. Burge

Submitted to Applied Optics

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The Diffractive Optics Calibrator: Measurement of Etching Variations for Binary Computer Generated Holograms

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We present a new device, the Diffractive Optics Calibrator (DOC), for measuring etching variations, such as duty-cycle and etching depth, of computer generated holograms (CGHs). The intensity distribution of far field diffraction patterns is captured and fitted to a parametric model to obtain local etching parameter such as duty-cycle, etching depth, and fringe spacing. The sensitivity of such etching parameter is analyzed and the design choice is provided. For the wavefront created by the CGH, DOC is capable of measuring variations in these parameters that cause 1 nm peak-to-valley (P-V) phase errors. System performance is verified by measurements from a phase shift Poincaré interferometer. The device will be used primarily for quality control of the CGHs. As an example, DOC is utilized to measure etching parameters of a CGH. The results can be used to evaluate the fabrication performance and guide future design. DOC is also capable of generating an induced phase error map for calibration. Such calibration is essential for measuring freeform aspheric surfaces with 1 nm root mean square (RMS) accuracy.

OCIS codes: (050.1380) Binary optics; (090.2880) Holographic interferometry; (120.4000) Optical inspection

1. Introduction

In the field of optical testing and metrology, computer generated holograms (CGHs) are often used in interferometric systems to produce reference wavefronts. By controlling the wavefront phase of the diffracted light, the application of CGHs in optical interferometry allows complex non-spherical surfaces to be measured [1]. Fabrication errors in CGHs, however, result in phase errors in the diffracted wavefront, which directly affects the accuracy and validity of the interferometric measurements. Therefore, CGH fabrication errors must be either budgeted or calibrated [2].

Duty-cycle and etching depth can be measured using white light interferometer, atomic force microscope or scanning electron microscope, which provide the surface relief of the object under test. However, these methods require expensive equipment and long inspection procedures. While they may be acceptable for small samples of diffractive optics, these methods are not suitable for inspecting large CGH substrates (usually 8 to 9 inch in diameter).

An alternative method is to use coherent illumination and analyze the diffraction pattern in the Fourier plane [3]. In optical testing, the binary CGH often uses the tilt carrier to separate the desired diffraction order from other orders. The common range of the grating periods is from 5 μm to 30 μm, which is well within the limit of scalar diffraction theory [4]. Therefore, one can write analytical expressions for the diffraction efficiency as a function of both the duty-cycle and the etching depth [5]. This relationship is applied to a new device: Diffractive Optics Calibrator (DOC).

From a collimated laser beam, the intensities of multiple diffraction orders were captured by a wide field of view camera lens. This measurement technique is illustrated in Fig. 1. The duty-cycle and etching depth are determined simultaneously by fitting the measured intensities to a parametric model. This device is able to scan through the whole area of the diffractive optics and generate a wavefront phase error map due to variations in duty-cycle and etching depth. The measurements are validated with a vertical scanning white-light interferometer and a Poincaré interferometer.

Fig. 1. The system layout for the Diffractive Optics Calibrator (DOC).

The system is capable of measuring the percentage of duty cycle variation that causes 1 nm P-V phase errors. This method is advantageous since it can be carried out rapidly with accurate and repeatable results, does not damage the sample, and uses low-cost equipment.

In our previous works, the design, construction and functions of the DOC was demonstrated [6]. In this paper, we focus on presenting the detail error analysis and the operation of DOC to measure the etching variations for
CGH. In section 2, the Fourier math for constructing the parametric model is described. The derived sensitivity functions are discussed in detail to select the proper parameters for the CGH design. In section 3, the performance of the system is quantified by a set of calibration gratings. In section 4, an example of using DRC to measure the phase error induced by etching variation is presented.

2. Analysis for CGH calibration

2.1 Parametric model

The performance of a CGH is directly related to the diffraction characteristics of a linear grating. In our previous works, a binary, linear grating model was used to study the wavefront sensitivity on fabrication uniformities [2, 5]. However, in microfabrication of the binary CGHs, etches may undercut the masking layer and form cavities with sloping sidewalls. In this paper, a new parameter is introduced to describe the sidewall slope in the model, which enables a more accurate prediction of the diffraction efficiency. Fig. 2 depicts a cross-sectional view of a binary grating with sloping sidewalls.

![Binary, linear grating profile](image)

**Fig. 2.** Binary, linear grating profile; \( t \) is the etching depth; \( S \) is the grating period; \( b \) is the duty-cycle; \( c \) is the sidewall slope ratio and \( A_e \) and \( A_r \) are the amplitude of the output wavefront from the un-etched and etched areas of the grating, respectively. \( \phi \) is the phase step between the two areas.

The scalar diffraction approximations can be applied when the wavelength of the incident light is much smaller in comparison to the grating period \( S \). In this case, the output wavefront immediately past the grating, either reflected or transmitted, can be expressed as a simple product of the incident wavefront function and the grating profile function. In another words, the grating function modulates the incident wavefront directly. For a normal incident plane wavefront, the output wavefront function can be written as:

\[
u(x) = A_r + (A_e - A_r) \cdot \cos \left( \frac{x}{c} \right) + \frac{1}{S} \cdot \cos \left( \frac{x}{c} \right) \cdot \sin \left( \frac{x}{S} \right) \]

where the grating period is \( S \) and the etching depth is \( t \). The duty-cycle of the grating is defined as \( D = b/S \), where \( b \) is width of the un-etched area. The previous square profile is replaced by the trapezoidal profile. The sidewall slope ratio is defined as \( c = S \cdot d \), where \( c \) is the width of the etched or etched areas of the grating, respectively. The phase depth \( \phi \) represents the phase difference between these two areas, which equals \( 2\pi (n - 1) / \lambda \) for a grating used in transmission. \( n \) is the refractive index of the grating substrate.

The far-field diffraction wavefront is related to the original wavefront via a simply Fourier transform relationship based on the Fraunhofer diffraction theory. The derivatives are similar to Chang and Burge's previous work [5]. The difference from the previous model is an additional convolution term \( \frac{1}{c} \cdot \cos \left( \frac{x}{c} \right) \) in Eq. (1), which will turn into a product term after the Fourier transform.

A summary of equations for the parametric model is presented in Table 1. The wavefront phase function \( \tan \Psi \) can be defined using the ratio of the imaginary part to the real part of the complex far-field wavefront function. While, the diffraction efficiency \( \eta \) is defined as the ratio of the intensity of the diffracted wavefront to the total intensity of the incident wavefront. As functions of duty-cycle and phase depth, both the zero order and non-zero order diffraction efficiency expressions were utilized in fitting the measured intensities. In Fig. 3, the diffraction efficiency of order 0 to 3 is plotted versus duty-cycle with 0.35 \( \lambda \) phase depth.

<table>
<thead>
<tr>
<th>Table 1. Summary of equations for parametric model analysis</th>
</tr>
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<tbody>
<tr>
<td><strong>Zerorder (m=0)</strong></td>
</tr>
<tr>
<td>----------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Diffracted wavefront</strong></td>
</tr>
<tr>
<td>( \eta ) dffraction efficiency</td>
</tr>
<tr>
<td>( \tan \Psi ) ( \Phi ) wavefront phase</td>
</tr>
<tr>
<td>- ( A_r^2 + A_e^2 - 2A_rA_e \cdot \cos \phi \cdot D \cdot \sin^2 (mD) \cdot \sin^2 (m\phi) )</td>
</tr>
<tr>
<td>Sensitivity functions</td>
</tr>
<tr>
<td>( \partial \eta_{DD} )</td>
</tr>
<tr>
<td>( \partial \eta_{\phi} )</td>
</tr>
<tr>
<td>( \partial \eta_{DP} )</td>
</tr>
</tbody>
</table>
2.2 Sensitivity functions

The wavefront phase sensitivity functions \( \frac{\partial \phi}{\partial D} \) and \( \frac{\partial \phi}{\partial \delta} \) are defined as the deviations of the diffraction phase values due to variations in grating duty-cycle, phase depth and sidewall slope ratio, respectively. These functions are not only essential to setup the overall system requirements for DOC [8], but also to convert the measured etching variations to induced phase errors, or vice versa.

As shown in Table 1, errors in duty-cycle will not introduce wavefront phase error for non-zero diffraction orders. In a special case when duty-cycle is 50% (\( D = 0.5 \)), the wavefront phase sensitivities to etching depth are the same for all the diffraction orders.

Questions may arise here: why bother measuring the duty-cycle variations if there is no influence on the diffraction order that will be used in the optical testing (usually it is the first order)? The reason lies in the in COH substrate calibration, where the phase error of zero order and that of non-zero order are compared. Therefore, we are interested in the sensitivity difference between zero and non-zero order. The COH substrate figure error is the dominant fabrication error, especially for large COHs (e.g. a 6" by 6" fused silica substrate with 0.25" thickness). Since the substrate irregularity affects all the diffraction orders equally, it can be calibrated using the measurement from the zero-order diffraction. On the other hand, the non-uniformities in etching parameters introduce wavefront errors for the zero-order and non-zero orders differently. After subtracting the zero-order measurement from the (non-zero order) surface measurement, the COH substrate error is cancelled but the wavefront still contains some residual errors due to variations in duty-cycle or etching depth.

The variations of duty-cycle and that of phase depth that cause 1nm PV phase error between zero and non-zero diffraction orders are plotted in Fig. 3 and Fig. 4, respectively, with a range of commonly used values of duty-cycle and phase depth [8].

As depicted in Fig. 4, the duty-cycle variation induced phase sensitivity changes little as a function of the nominal duty-cycle. Meanwhile, in Fig. 5, the phase depth variation induced phase sensitivity increases as the nominal duty-cycle approaches 50%. For example, there is a COH with 48% nominal duty-cycle and 0.5 µm etching depth (0.35 λ phase depth at 632.8nm). In order to keep the error budget of the phase error induced by the duty-cycle variation within 1 nm PV, duty-cycle variation must be controlled to within 0.25% PV, which is a demanding requirement for COHs that have a wide range of fringe spacing.

For the same scenario, in order to keep the induced phase error within 1nm PV, phase depth must be controlled to within 0.03 λ PV (equivalent to about 40nm PV of etching depth variation), which is easily achieved for current fabrication capability. From this example, we can make a prediction that duty-cycle variation will be the dominant contributor to the induced phase error.
The diffraction efficiency sensitivity functions $\frac{\delta \eta}{\delta \delta}$, $\frac{\delta \eta}{\delta \phi}$, and $\frac{\delta \eta}{\delta \phi}$ are defined as the deviations of the diffraction efficiency due to variation in grating duty-cycle, phase depth and sidewall slope ratio, respectively. These functions are used to analyze the influence of the intensity measurement noises on the fitting results of the etching parameters.

In Fig. 6, the diffraction efficiency variations caused by 1% duty-cycle deviation are plotted versus the nominal duty-cycle for diffraction order 0 to 3. The sensitivity converges to zero at 50% duty-cycle. It implies that when the nominal duty-cycle value is close to 50%, deviations in duty-cycle have little effect on the intensity distribution among different orders. Large uncertainty occurs when using the intensity information to get the duty-cycle value near 50%.

As for the diffraction efficiency variations caused by phase depth deviation plotted in Fig. 7, the sensitivity remains at the same level. In the presence of noise, with nominal duty-cycle value close to 50%, fitting result of phase depth (or etching depth) will be more accurate than that of duty-cycle.

Fig. 6. Diffraction efficiency variations due to 1% duty-cycle deviation. (Nominal phase depth is 0.35λ. Diffraction order m=0,1,2,3 are included)

Fig. 7. Diffraction efficiency variations due to 0.013 phase depth deviation. (Nominal phase depth is 0.35λ. Diffraction order m=0,1,2,3 are included)

2.3 Etching parameter design choice
There are reasons for choosing duty-cycle at 50% in our previous work [8]: (1) It has the maximum diffraction efficiency at the first diffraction order, which will be used in the interferometric testing; (2) The wavefront sensitivities to etching depth are the same for the zero and the first order at 50% duty-cycle, which will not generate additional residual phase error when the CSH substrate is calibrated with zero order measurement.
However, as discussed in section 2.2, it is difficult for DOC to make accurate measurement near 50% duty-cycle. To illustrate this effect, a Monte Carlo simulation is performed with 0.5% random measurement noise. The fitting uncertainty shown in Fig. 8 is represented by the 1-σ standard deviation of fitting results. The uncertainties increase dramatically when the nominal duty-cycle approaches 50%. Moreover, the mean value at 50% is offset by 0.3%. This is because diffraction efficiency is symmetric about duty-cycle at 50%. As illustrated in Fig. 5, for instance, the intensity distributions among all diffraction orders are identical for 51% and 49%, which the DOC measurement cannot distinguish. Therefore, the values above 50% will be degenerated into the values below 50% after the fitting. In this case, the mean value will be shifted from 50%. Another consequence is the sign of the induced phase error relative to 50% duty-cycle is unknown, while only the absolute magnitude of the induced wavefront phase error can be determined.

The characteristic of the diffraction efficiency limits DOC’s ability to perform accurate measurement when duty-cycle is about 50%. In order to utilize the DOC measurement to do calibration analysis, we proposed to choose the nominal duty-cycle at 48%. The reasons are: (1) As shown in Fig. 8, at 48%, duty-cycle fitting uncertainty is less than half of the uncertainty at 50%. (2) Diffraction efficiency at first order only decreases 0.4% compared to duty-cycle at 50%. (3) Assuming the etching variation of duty-cycle is less than 2%, there is no ambiguity of the duty-cycle value. (4) Although phase depth variation induced phase error is different between zero order and non-zero order, the error magnitude is negligible compare to the duty-cycle induced phase error. It shows in Fig. 9 that the etching depth induce phase error at 48% duty-cycle is about 8 times smaller than duty-cycle induce phase error, assuming the CGH has 0.5% duty-cycle variation and 5 nm etching depth variation.

3. System performance

In order to measure freeform aspheric surfaces with 1 nm RMS accuracy, the knowledge of phase error induced by etching variations must be within a fraction of 1 nm. It is shown in this section that DOC’s system performance meets this requirement.

3.1 DOC hardware and software

Fig. 10 (a) and (b) illustrates the layout of the diffraction optics calibrator (DOC). It consists of three subsystems: optical system, mechanical system and software. In the optical subsystem, a collimated laser beam propagates through the CGH substrate and diffracts into multiple diffraction orders. The intensities of these orders are captured by a wide-angle camera and processed via LabVIEW™ to obtain the local duty-cycle and phase step. A two-axis motorized linear stage is implemented in the system, which is capable of scanning through a CGH with an aperture up to 9 inch. In order to measure duty-cycle variation across the CGH aperture, LabVIEW™ is used to synchronize the scanning motion of the stage with the image acquisition of the camera. Detailed functions of each subsystem are discussed in our previous work [6].

To reduce the measurement time, a new procedure is implemented for data process with both LabVIEW™ and MATLAB™. LabVIEW™ is used to do image acquisition. All the raw image data is stored with the measurement location information. Image data is then imported into MATLAB™ for image processing and non-linear fitting. For example, to test a 4” by 4” CGH pattern with 3mm by 3mm sampling grid, more than a thousand measurements should be taken. It takes about 1 hour to finish. The procedure steps were illustrated in a flow chart in Fig. 10(c).
3.2 Repeatability and accuracy of DOC

In order to test the repeatability and accuracy of DOC, three sets of linear gratings with varying duty-cycles were fabricated. The duty-cycle range is from 41% to 53%. The detail specifications of the design are list in Table 2.

<table>
<thead>
<tr>
<th>Duty cycle range</th>
<th>41% to 53% (1% change/duty-cycle)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Etching depth</td>
<td>0.5 μm</td>
</tr>
<tr>
<td>Grating period</td>
<td>15.0 μm</td>
</tr>
<tr>
<td>Grating size</td>
<td>3mm × 20mm</td>
</tr>
<tr>
<td>Overall pattern size</td>
<td>30mm × 20mm</td>
</tr>
<tr>
<td>Number of sets</td>
<td>3</td>
</tr>
</tbody>
</table>

Repeatability tests were performed by measuring the same grating location multiple times. The uncertainty of the measured duty-cycle values is 0.05% and that of the measured etching depth values is 0.4 nm, which is equivalent to 0.2 nm RMS phase error and 0.01 nm RMS phase error, respectively.

Since the wavefront phase error is induced by the variation of either duty-cycle or etching depth, system repeatability is important for CGH quality control. On the other hand, the system accuracy is crucial for calibrating the induced phase error. The results between the Veeco white light interferometer and DOC are compared in Fig. 11 and Fig. 12. The error bars are determined by the fitting uncertainty estimated by the Monte Carlo simulation result shown in Fig. 8. As predicted in section 2.3, a large discrepancy occurs at duty-cycle around 50%. For the rest of the duty-cycle values, the difference is around 0.25% between the two methods, which is the required accuracy to detect about 1 nm PV phase error due to duty-cycle variation. For the case of measuring etching depth, the general trends were matched for the measured values by both methods. The largest discrepancy between the two methods is 3 nm, which is equivalent to 1nm PV induced phase error at 40% duty-cycle and 0.15nm PV induce phase error at 49% duty-cycle.
3.3 Verification with interferometric measurement

In the previous subsection, DOC's performance on duty-cycle and etching depth measurements is verified with a white light interferometer. Once the local values on duty-cycle and etching depth were obtained, the induced wavefront phase can be calculated using the sensitivity functions in Table 1. A direct wavefront phase measurement was performed using a Fizeau type phase shift interferometer. The same set of gratings was measured with a transmission flat and a return flat, which is depicted in Fig. 13.

![Image](image_url)

**Fig. 13.** Interferometric test of a pattern with 3 sets of gratings with duty-cycle variations from 41% to 59%. The induced phase error was measured at 0 order (solid line setup). ±1st orders were used to calibrate the grating substrate error (dotted line setups).

OGH substrate error affects all diffraction orders equally, while the duty-cycle variation only affects the zero order wavefront. In this specific case for linear gratings, the zero order contains both the substrate error and the duty-cycle variation induced phase offset, while the first diffraction orders contains only the substrate error. The wavefronts of the first orders and the zero order were measured. The net result of subtraction of the two wavefront maps is the duty-cycle induced phase offset, which is shown in Fig. 14(a).

The same pattern was measured in DOC. As we mentioned in Sec. 2.3, DOC cannot tell if the duty-cycle value is below or above 50%. We constrained the fitting results to be always less than 50%. The measured values of duty-cycle were later adjusted to above 50% according to the nominal design. Then the measured values of duty-cycle and etching depth are input to the wavefront sensitivity functions to generate the induced phase error map, as shown in Fig. 14(a). Although the resolution is not as good as the interferometric measurement, the induced phase error is well matched between these two methods.

The averaged profile values of the phase maps in Fig. 14(a) are plotted in Fig. 14(b). The uncertainty of the DOC measurement is the combination of the phase error from duty-cycle variations and etching depth variations, which are estimated using the Monte Carlo simulation result, as depicted in Fig. 8. Other than the deviation around 50% duty-cycle mark as expected, the rest of the regions have an agreement within 5nm with the interferometric results. This is the uncertainty level of the return flat's surface quality.
4. Application using DOC

In this section, we will employ DOC to measure the etching parameters of a CGH. The information can be used in quality control, calibration. Furthermore, it also serves as a feedback to the fabrication performance and future design parameter selection.

The CGH under test is a circle pattern about 90mm in diameter. The first diffraction order generates an aspheric wavefront to be used in a null test for an off-axis parabolic mirror. As illustrated in Fig 15 (a), each fringe represents 300 fringes in the real CGH, which is equivalent to 300 waves at 632.8nm wavelength. The shape of the pattern is mainly stemmed from spherical aberration and tilt. DOC was used to measure the CGH with a resolution of 3 nm in each direction. The scanning route is depicted in Fig 15 (b).

After the scanning measurements acquired using LabVIEW™ data is imported into MATLAB™ for post processing, including extracting intensity information from the raw image data, non-linear least square fitting and generating induced phase error maps. Detailed information of each data point, including x and y coordinates, duty-cycle, etching depth, fringe spacing and fitting residuals, is logged in a spreadsheet for future analysis.

The statistics of the measured result is listed in table 3. Compared to the nominal design values (duty-cycle is 50% and etching depth is 500nm), the measured average duty-cycle is 48%, while the measured average etching depth is 511.8nm. This offset is mainly resulted from the etch bias, which is the undercutting distance compared to the design of the masking layer. The amount of etch bias is usually uniform across the CGH pattern. Therefore, small features will gain more duty-cycle offset in percentage.
Table 3. DOC measurement results of the etching parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Average</th>
<th>PV</th>
<th>RMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duty-cycle</td>
<td>48.00%</td>
<td>0.60%</td>
<td>0.40%</td>
</tr>
<tr>
<td>Etching depth</td>
<td>311.8 mm</td>
<td>19.8 mm</td>
<td>51 mm</td>
</tr>
<tr>
<td>Fringe spacing</td>
<td>3.5 µm</td>
<td>15.8 µm</td>
<td>26 µm</td>
</tr>
</tbody>
</table>

Each parameter can also be plotted as a function of location in a 2D map as shown in Fig. 16, which is a more intuitive way to analyze the measured result. The variations of both duty-cycle and etching depth present as a shape of coma aberration in the up-down direction, which is correlated to the fringe spacing variations illustrated in Fig. 18(a) and Fig. 18(c).

Since fringe spacing is proportional to the local wavefront slope, the coma-shape map is stemming from a coma-shape wavefront slope distribution, which comes from a wavefront contains spherical aberration. Assuming the etch bias is constant across the pattern, smaller fringe spacing will result in a larger duty-cycle offset, which results in the coma-shape variation demonstrated in Fig. 16. To further verify this correlation, etch bias is calculated as the product of the fringe spacing and the duty-cycle offset in percentage. The etch bias map and the corresponding histogram are plotted in Fig. 17(a) and Fig. 17(b), respectively. The majority of the values are concentrated around 0.15 µm, which is an essential parameter for evaluating the etching performance. With the knowledge of this fabrication error, the CGH design can be optimized to compensate the offset.

![Fig. 16. 3D maps for etching parameters (a) Duty-cycle map (values in %). (b) Etching depth map (values in nm). (c) Fringe spacing map (values in µm).](image)

Last but not least, the etching variation-induced phase error maps are generated and shown in Fig. 18. Wavefront-sensitive functions are applied to the measured duty-cycle and etching depth. In Fig. 18(a) and (b), the induced phase errors are compared to the average value of 48% (duty-cycle) and 011.8 nm (etching depth), respectively. As mentioned before, this type of induced error is the difference between zero-order and non-zero-order. Therefore, when the CGH substrate error is calibrated using zero-order measurement, the phase error induced by etching variations can be taken into consideration to yield a more accurate measurement.

![Fig. 17. The etch bias is calculated as the product of the fringe spacing and the duty-cycle offset in percentage. (a) Etch bias map. (b) Etch bias histogram.](image)
Fig. 10. Etching variation induced phase error maps. a) Duty cycle variation induced phase error map. b) Etching depth variation induced phase error map. c) Fitting uncertainty of duty cycle variation induced phase error and Monte Carlo simulation. d) Fitting uncertainty of etching depth variation induced phase error estimated Monte Carlo simulation.

5. Conclusion

This paper described the diffractive optics calibrator (DOC), a novel device that we developed to measure the variation of duty cycle and etching depth of a binary diffractive optics used in transmission. A parametric model and the derived sensitivity functions were discussed. The system performance was quantified using a set of calibrated gratings. An example of using DOC to measure the etch bias and the induced phase error was presented. With the exception at 50% duty cycle, the system is capable of measuring etching variations that cause 1 nm PV phase errors in the wavefront created by the CGH. The DOC can be used to evaluate the fabrication performance and provide feedback to etching procedure and future design compensations. It can also assess the uniformity of the diffractive optics for calibration or quality control.

Reference


REFERENCE


