A COMPARATIVE STUDY OF LINEAR PROGRAMMED INSTRUCTION
AND CONVENTIONAL INSTRUCTION IN
FIFTH GRADE MATHEMATICS

by
William Curtis Bourland, Jr.

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1972
STATEMENT BY AUTHOR

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SIGNED:

APPROVAL BY THESIS DIRECTOR

This thesis has been approved on the date shown below:

Glen I. Nicholson
Professor of Educational Psychology

August 31, 1971
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ABSTRACT

In the elementary school systems of the United States, a problem exists in the limited methods employed in teaching mathematics as a descriptive form of communication with logical patterns of expression.

The purpose of this study was to compare the teaching of mathematical functions and their associated graphs by two methods: Conventional Instruction versus Linear Programmed Instruction.

The subjects for the experiment were fifty-eight fifth-grade students who were divided into two groups: a control group taught by conventional instruction, and an experimental group taught by linear programmed instruction. Pre- and post-tests were administered to the groups in order to evaluate student achievement as a result of the study. The data was subjected to a one-way classification analysis of variance, and reliability coefficients were computed for the pre- and post-tests.

The main result of the study shows a significant difference in concept achievement in favor of the experimental group. An incidental objective of the study included observations of the two groups. It was noted that the experimental group encountered less frustration in learning from the experimental materials than the control group which was taught by conventional methods.

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CHAPTER I

INTRODUCTION

In recent years, beginning with the impetus of the first satellite launch in 1957 and its subsequent effects on the mathematics and science programs throughout the United States, numerous experimental programs have been undertaken in an attempt to develop more effective instruction in these disciplines at the elementary grade level. These programs have been directed toward the development of effective audio-visual instructional devices, the testing of various techniques of instruction within a given subject area, the development of programmed text material as well as many other programs aimed at generating the most effective learning of conceptual material.

In an effort to contribute to the effective teaching of mathematics, this researcher developed teaching materials comprised of linearly programmed lessons for the learning of functional operations and their graphs in the elementary school. This study is concerned with testing the effectiveness of those materials.

Justification and Rationale

Importance of teaching functions: It is widely agreed among mathematicians and mathematics teachers that functions are a very
powerful and descriptive method for analyzing patterns and relationships in mathematics. Evidence of the concept of functions as a unifying factor in mathematics was presented by Johnson and Cohen (1970, p. 306) when they stated some of the processes involved in working with functions:

"In working with functions, three typical situations arise. These can be described as substitution in a formula, equation solving, and curve fitting." It is this curve fitting property of functions that provides scientists with means of estimating values outside of their limited collected measurements. This notion of appreciating mathematical structure and patterns and their importance in scientific concepts is further emphasized by Dixon (1970, p. 828) in his urging an "approximations approach" to mathematics for applications in science.

Another important quality of mathematical functions and their graphs lies in its compatibility with the current notion of "Modern Mathematics." The main idea inherent in "Modern Mathematics" is that of presenting mathematics instruction in the more encompassing construct of developing concepts of quantitative analyses, and relations between processes and magnitudes. Presentation of these concepts is made beginning at the elementary grade school level. Therefore, the direction of "Modern Mathematics" is toward conceptual development rather than the primary goal of developing proficiency in computational skills typical of traditional elementary mathematics.
Implementing "Modern Mathematics:"

There is a common agreement among educators and the professional literature that many children are not learning mathematics adequately in the elementary school. The frustrations encountered by classroom teachers increase as they try to solve this problem by: achievement grouping within the classroom; departmentalization between classrooms; attempting individualized mathematics programs through the use of borrowed materials from other grade levels; as well as other methods. From these various approaches it can be seen that many of the difficulties encountered by the student in learning "Modern Mathematics" concepts may lie in the design and organization of instruction and instructional materials.

In order to approach the task of design of instruction, Jerome Bruner (1966, p. 40) suggests the formulation of a "theory of instruction."

A theory of instruction, in short, is concerned with how what one wishes to teach can best be learned with improving rather than describing learning. Bruner (p. 41) goes on to state that "...the merit of a (instructional) structure depends upon its power for simplifying information, for generating new propositions, and for increasing the manipulability of a body of knowledge."

The importance of design of instruction with respect to "Modern Mathematics" is further emphasized by Piaget's description of "Modern Mathematics" as developing concepts of "...wholes and structural
isomorphisms rather than traditional compartmentalization" (1970, p. 46).

At the same time, Piaget admits that this poses a serious problem in the development of understanding "Modern Mathematics" concepts since "The most general structures of modern mathematics are at the same time the most abstract as well" (p. 47). Consequently, the pedagogic problems of preparing acquaintances with and understandable experiences in "Modern Mathematics" poses quite a formidable task.

Many currently employed "Modern Mathematics" programs in the elementary schools may be detrimental to the achievement of some students from the outset. This results from a lack of foundation concepts or "readiness" with which to develop an understanding of the more sophisticated "Modern Mathematics" concepts encountered in the upper elementary grades. This lack of preliminary understanding and success experiences may tend to be compounded throughout the remainder of the child's elementary school life, thereby contributing to the development of a failure syndrome within the child. In the typical classroom, the "one" method of presenting mathematics by means of the current state-adopted text may be too difficult for many of the children and too boring for the few, more capable students. Consequently, satisfying feelings of success in mathematics may be limited to a very few children in the classroom and may be coincidental to the lesson presentation.
**Programmed instructional techniques:** In attempting to remedy some of the above deficiencies of single-text conventional instruction, many developments have been made in an attempt to develop more effective instructional materials and techniques. Included in these developments is the evolution of the programmed text and supplementary programmed materials.

Historically, the idea and use of programmed instruction is not the recent educational revolution it may appear to be. One of the earliest programmers was Socrates, who developed a program using the inductive method for teaching geometry. This was recorded by Plato in the dialogue "Meno." "It was Socrates' habit to guide his followers to knowledge by conducting them conversationally along a path from fact to fact and insight to insight" (Lysaught and Williams, 1963, p. 3).

In drawing an analogy between branched programmed instruction and Plato's Meno dialogue, L. M. Stolurow stated in his letter to the editor of the *Harvard Educational Review*:

A teacher is not only instructing a student, but he is also learning about that student and using what he learns in making decisions about what to do next in the course of his teaching. Similarly, the student is not only learning, but he is providing information to the teacher which, in turn, guides the teacher in the ongoing interaction. ...this dual process ... is explicitly attempted in programmed instruction (1963, pp. 384-5).

Programmed instruction, therefore, involves a gradual progression in the sequence of instruction. Throughout the course of instruction the student
is guided by small, discrete steps which are in the direction of required
learning goals. Each step in this gradual progression is called the frame
of instruction with its own built-in reinforcement agent which is intended
to motivate the learner to proceed to the next frame and so on. Taber,
Glaser, and Schaefer (1965, p. 87) refer to an instructional frame as
"...those stimuli necessary to elicit student performance which is a neces-
sary step toward mastery as defined by the analysis of terminal behavior."

There are two major criticisms of the use of programmed instruc-
tion. The first implies that it subjects the learner to skills learned in
the absence of their relevant application. The second, proposed by May
(1967, p. 12), suggests that "...excessive programming allowed the
student to succeed without extensive or intensive thought." In answer
to the first criticism, Lumsdaine and Glaser (1960, p. 570) argue that:
"Programmed learning sequences can be set up to provide a carefully
organized progression of instances by which the student is led to develop
discriminations and generalizations that enable him to learn and use com-
plicated concepts."

The second criticism is that the frame size is so small that any
aspect of challenge and the motivational value of its reinforcement are
destroyed. This implies that careful consideration should be given to
the level of ability of the student before prescribing a course of pro-
grammed instruction. Susan R. Meyer (Galanter, ed., 1959, p. 83) has
suggested that three factors be considered relative to the student:
"(1) what relevant behavior must be assumed to be available, (2) what behavior constitutes competence in arithmetic, and (3) what is the best order of steps from starting point to final competence?" In further accommodating individual differences relative to prescribing courses of programmed instruction, the U. S. Civil Service Commission Bureau of Training (1971, p. 7) proposed the introduction of "multi-track" linear programming: "Dependent on these results (pre-test achievement scores), the student is directed to one of several linear programs (Form A, B, C or D) each of which presents the same material but does so in varying step (frame) sizes."

In implementing programmed materials in mathematics instruction, Levin (1968) conducted a study involving three groups of fifth grade students in which he compared linear and branched programmed instruction of fractions with a control group taught in the conventional manner. He concluded that there was no significant difference between the two methods of programmed instruction but that both were superior to the performance of the control group. Bassler (1968) in his comparative study of programmed versus conventional instruction in fourth, sixth and eighth grade mathematics concluded that when mathematics instruction is provided by linear programmed instruction, a relatively high level of guidance should be provided. In answer to a survey conducted by Raymond (1964), ninth
and tenth grade teachers reported that the course of programmed instruction was academically sound but that it was lacking in the scope offered in conventional instruction. The teachers did, however, advocate its use in tutoring and in working with students with high absentee rates.

The principal advantage of using mathematics as a subject for programmed instruction lies in its "high degree of logical organization" (Heimer, 1961, p. 333). This quality of mathematics when applied to programmed instruction leads to another quality of "a high response probability" (Skinner, 1968, p. 180).

**Conventional mathematics texts:** Much of the previous discussion on functions, graphs, and "Modern Mathematics" has pointed to the fact that mathematics is a "whole" science, a science which provides the forms and patterns in terms of which regularities in nature can be comprehended (Bruner, 1966, p. 36). However, in analyzing the current mathematics texts commonly considered for state adoption, there appears very limited coverage with respect to unifying concepts and pattern development in elementary school mathematics. This researcher has taught in elementary public schools for four years and has had the opportunity of evaluating these texts. Most of the texts did present the concept of functions but usually in the form of fragmented exercises divorced from graphical representation. In two instances, very limited exercises in which the two concepts of functions and graphs were related, were presented in the last chapter of the books. Very seldom are teachers
able to "cover the book," including the last chapters, by the end of a school year.

Statement of the Problem

The primary elements of the problem contributing to this study include (1) the problem, (2) importance of the study, and (3) the objectives of the study.

The problem: The purpose of this study is to compare the achievement of two groups of children being taught similar concepts of fifth grade mathematical functions and associated graphs. A minor purpose of this study is to note incidents of attention and motivation displayed by the students, as well as differences in teacher behaviors relative to each group.

Importance of the study: Through the development of the experimental materials based on a linear program model, this researcher attempted to reduce those negative characteristics in teaching mathematics which include: providing material not understood by all students, failing to compensate for individual rates of learning, and providing insufficiently descriptive and self-explanatory mathematics material. An important aspect taken into consideration by this researcher in the development of the experimental materials is that of making the functions of numbers and graphs illustrate that mathematics can be a descriptive language with its own discernible logical patterns of expression.
Objective of the study: The primary objective of this study is to detect gains in computation achievement and/or concept achievement of the experimental group over the control group. The control group was taught by a conventional textbook presentation while the experimental group was taught using the experimental materials developed by this researcher.

A minor objective was to list incidental observations regarding attitudes and over-all discipline during the periods of the days that the programs were in operation.

Hypotheses

The following null hypotheses were considered in the study:

1. There are no differences in total achievement between the control group and the experimental group as measured by the researcher-prepared pre- and post-tests.

2. There are no differences in computation skills between the groups as measured by the researcher-prepared pre- and post-tests.

3. There are no differences in concept achievement between the groups as measured by the researcher-prepared pre- and post-tests.

Limitations of the Study

This study was limited to two groups of fifth grade children enrolled at Elvira Elementary School, Sunnyside School District, Tucson,
Arizona. A total of 58 children were involved in the study: 30 children constituted the control group while 28 children made up the experimental group.

**Definition of Terms**

Certain terms relative to mathematics instruction have several definitions. For the purpose of this study, these terms will be defined as follows:

**Conventional instruction:** Conventional instruction is defined as that series of lessons selected from *Elementary School Mathematics Book 5* published by Addison-Wesley (Eicholz et al., 1967), used to teach functions and their associated graphs according to the procedures recommended in the teacher's manual of the text.

**Functions:** "If there is assigned to each element of a set $x$ exactly one element of another set $y$, then such an assignment is called a function" (Read, 1963, p. 726).

**Experimental material:** Experimental material refers to that set of lessons developed by this researcher which is being compared with "conventional instruction."

**Organization of the Remainder of the Thesis**

An introduction to the study, justification and rationale, statement of the problem, hypotheses, limitations, and definitions of terms are
included in Chapter I. Chapter II contains information on: (1) procedures employed in the study; (2) instructional materials employed in the study; (3) pre- and post-tests used in collecting data; and (4) statistical methods employed in analyzing the data. Chapter III contains an analysis of the data with an accompanying discussion. Chapter IV includes a summary of the study and conclusions and recommendations based on the study.
CHAPTER II

PROCEDURES

Introduction

The purpose of this chapter is to describe the procedures employed in the study. The procedures include a description of (1) the design and development of the instructional materials being tested; (2) conventional instruction; (3) the construction of the sample groups; (4) the pre- and post-tests used to gather the data; and (5) statistical tests employed in analyzing the data.

Instructional Materials

Experimental material: The objectives in using the experimental material were to demonstrate that mathematics can be a descriptive language and to give functional operations with numbers a concrete utility in the child's cognitive processes. In using the experimental material, the student needs the data he produces in each individual functional operation in order to reproduce a pre-planned pattern. The material progresses from simple point-plotting exercises to complex functional operations. As the skill of the student increases, the picture he is able to decode becomes more interesting.

The control of error factor is contained entirely within the material, freeing the child to work independently of the teacher. An error
will result in an obvious deviation from the developing pattern. An answer key is available so that the student can locate and identify his mistake and note where it occurred, thus providing a motivational feedback loop.

The experimental material tested in this study is the result of approximately two years spent on its design, development and testing. Upon completion of the initial prototype, the material underwent three major revisions according the following procedures:

1. Once the initial twenty-four problem sets were completed and the appropriate graph blanks drawn, five sets of the material were distributed among five of the more capable sixth-grade students. These students worked under this researcher's supervision so that any problems, whether typographical or conceptual, could be immediately detected and noted. While the students were working on problem sets twenty-three and twenty-four it was observed that considerable difficulty was encountered in plotting a very large quantity of points on a small scale graph. In an attempt to remedy this difficulty, this researcher decided to simplify the patterns in twenty-three and twenty-four and also add one more problem set bringing the total to twenty-five. The reason for the addition of number twenty-five was to maintain approximately the same number of exercises in developing the concept of scale.

2. Once the material was revised according to the above mentioned errors and observations, it was again distributed to four different
six-grade students who worked through the twenty-five problem sets.

Again, many errors, mostly typographical, were encountered.

3. The material was again revised according to the previously detected errors. This third revision was the material used in the experimental program.

The following are samples of the types of exercises included in the experimental material:

A. Simple point plotting - single coordinate.

(1) \( N = 2, \ E = 3 \)
(2) \( N = 1, \ E = 4 \)
(3) \( N = 0, \ E = 2 \)

B. Drawing a figure - two coordinates.

(1) \( N = 1, \ E = (N \times 1) + 1 \)
(2) \( W = 3, \ N = (W \times 3) \div 3 \)
(3) \( N = 3, \ E = (N \times 4) \div 6 \)
(4) \( N = 1, \ W = (N + 7) - 5 \)

Connect the points in the following orders:
START 1, 3, 2, 4, 1 STOP

Figure 1. Samples of Exercises Included in Experimental Material

Conventional material: The control group was presented lessons from the text, Elementary School Mathematics, Book 5, published by
Addison-Wesley (Eicholz et al., 1967). Lessons were selected from the text which most closely matched the concepts being promoted in the experimental material. The notion of a function given in the teacher's edition of the text states that "...the function associates with each number of a given set, exactly one number from another set" (Eicholz et al., 1967, p. 42). This function concept coincides with the one used as the basis of the experimental material. Further similarities between the two methods of instruction used in this study are included in four of the five objectives listed for the chapter on graphing. These objectives are:

1. To introduce graphing sets of points on a number line
2. To introduce graphing pairs of numbers
3. To establish a relationship between product sets and graphing number pairs
4. To introduce the idea of graphing number sentences or functions (p. 344).

The experimental material complies with objective number 1 by teaching concepts of distance and direction on the number line. It complies with objective number 2 by requiring a pair of numbers for plotting a point on a graph. Objective number 3 is observed by the experimental material in its required arithmetic operations for deriving the necessary pair of numbers. The fourth objective is included in the format of the problems of the experimental materials.
It was noted by this researcher that the chapter covering graphing was the last chapter of the text and the lesson dealing specifically with the graphs of functions was designated as an optional lesson, to be done at the teacher's discretion.

A sample of the problems included in Chapter 13 of *Elementary School Mathematics, Book 5* (Eicholz et al., 1967).

5. Give the number pairs for the points A through F.

![Figure 2. Sample of Exercises Included in Conventional Instruction](p. 355)

**Organization of Sample Groups**

At the start of the 1970-71 school year this researcher was the assigned teacher of one of the fifth grade classes used in the study. In the beginning of that year, arrangements were made with Mr. William Condit, another fifth grade teacher in the same school, for this researcher to teach mathematics to both classes throughout the entire year. Since the actual comparative study began during the second semester of the school year, both groups began with a common base of having been taught mathematics for one full semester by this researcher using the
Elementary School Mathematics, Book 5 series published by Addison-Wesley (Eicholz et al., 1967). By working with Mr. Condit's class in mathematics during the semester before the study, this researcher hoped to limit the Hawthorne effect usually introduced by the novelty of the "new teacher." It is possible that the Hawthorne effect was still active within the experimental group since no student in that group had ever had the type of mathematics instruction as presented through the experimental materials.

Since this researcher had control over both of the groups involved in the study, it was therefore possible to introduce unnoticed biases which might have favored one group over the other. As one measure taken in an attempt to limit this bias, Mr. Condit's class was set up as the group in which the experimental materials were tested, and this researcher's own classroom was maintained as the control group which continued to be taught in the conventional manner. This procedure was implemented since traditionally teachers tend to favor their own classes over those of other teachers.

**Pre- and Post-Tests**

The pre- and post-tests used in this study were composed of the following parts:

Part I. Fifteen problems involving simple arithmetic computations;

Part II. Twelve problems involving the associative (grouping) principle in computation;
Part III. Eight problems involving function concepts;

Part IV. Six problems testing the ability to graph functions.

In the analysis of the data, parts I and II were combined to form the computation division of the test while parts III and IV were combined to form the concept division. The pre- and post-tests used in this study may be found in Appendices A and B, respectively.

In order to determine the reliability of the tests, coefficients were computed for the pre- and post-tests administered to the study groups. The technique employed in computing the reliability coefficients was the split-half method with an applied Spearman-Brown correction. In using this technique the pre- and post-test results for the groups were divided into odd-numbered correct responses and even-numbered correct responses after which Pearson Product-Moment correlation coefficients were computed. Since this coefficient is actually calculated for half of the test, the Spearman-Brown formula corrects for this by projecting the calculated coefficient to the entire test. The coefficients calculated for the performance of each group on the pre- and post-tests were then combined using a Fisher Z transformation. This procedure yielded a reliability coefficient of .916 for the pre-test and .835 for the post-test.

Treatment of the Data

The collected data were submitted to a one-way classification analysis of variance. This statistical test was used in evaluating all of
the hypotheses at the .05 level of significance. The analysis of variance was chosen because it provides "a method for dividing the variation observed in experimental data into different parts, each part assignable to a known source, cause, or factor" (Ferguson, 1959, p. 281).

Tables were drawn displaying the results of the analyses of variance applied to the hypotheses to be tested.
CHAPTER III

ANALYSIS OF THE DATA AND DISCUSSION

The analysis of the data collected during this study is presented in three major categories which correspond to the hypotheses being tested. These categories include total achievement between groups, computation achievement between groups and concept achievement between groups. A one-way analysis of variance was applied to pre-test results as well as post-test results in order to illustrate the initial equality of the control and experimental groups and to indicate any statistical differences between the groups at the end of the program.

Pre-Test Achievement Between Groups

Total achievement between groups (pre-test): A minor hypothesis being tested at this point is that there is no difference in total mathematics achievement between the groups on pre-test results. Inspection of Table I shows an obtained F-ratio of .063. Since an F-ratio of 4.02 was required at the .05 level of significance and 7.12 at the .01 level, the relatively small calculated F-ratio of .063 supports the acceptance of the minor null hypothesis that there is no difference between the groups in total achievement, based on pre-test scores.
TABLE I
TOTAL ACHIEVEMENT BETWEEN GROUPS (PRE-TEST)
ANALYSIS OF VARIANCE

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Variance Estimate</th>
<th>F-ratio</th>
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</thead>
<tbody>
<tr>
<td>Between</td>
<td>3.62</td>
<td>1</td>
<td>3.62</td>
<td>.063</td>
</tr>
<tr>
<td>Within</td>
<td>3233.50</td>
<td>56</td>
<td>57.74</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>3237.12</td>
<td>57</td>
<td></td>
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</table>

Computation achievement between groups (pre-test): In analyzing the equality of the groups in more detail, separate one-way analyses of variance were calculated for each of the two major divisions of the pre-test: computation and concept attainment.

The minor hypothesis tested here states that there is no difference between the groups in computation ability at the beginning of the study, based on pre-test computation results. Table II provides a summary of the analysis of variance, and, again, inspection shows that the calculated F-ratio of .019 falls far short of the required 4.02 for the .05 level of significance. Therefore, the null hypothesis that there is no difference between the groups on pre-test computation achievement is retained.

Concept achievement between groups (pre-test): The third minor hypothesis to be tested was that there was no difference between the groups at the concept attainment level of the pre-test. A summary of the analysis of variance is included in Table III. The calculated F-ratio of
### TABLE II

**COMPUTATION ACHIEVEMENT BETWEEN GROUPS (PRE-TEST)**

**ANALYSIS OF VARIANCE**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Variance Estimate</th>
<th>F-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>.75</td>
<td>1</td>
<td>.75</td>
<td>.019</td>
</tr>
<tr>
<td>Within</td>
<td>2127.66</td>
<td>56</td>
<td>37.99</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>2128.41</td>
<td>57</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### TABLE III

**CONCEPT ACHIEVEMENT BETWEEN GROUPS (PRE-TEST)**

**ANALYSIS OF VARIANCE**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Variance Estimate</th>
<th>F-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>7.69</td>
<td>1</td>
<td>7.69</td>
<td>.924</td>
</tr>
<tr>
<td>Within</td>
<td>465.16</td>
<td>56</td>
<td>8.31</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>472.85</td>
<td>57</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

.924 does not meet the required 4.02 for significance at the .05 level and therefore the null hypothesis of no difference between groups with respect to pre-test concept achievement is retained.

**Total Achievement Between Groups (Post-Test)**

Hypothesis one stated that there would be no differences in total achievement between the control group and the experimental group as
measured by the researcher-prepared pre- and post-tests. In testing this hypothesis, the main interest was in gains in total mathematics achievement of the experimental group over the control group.

An inspection of Table IV shows that the calculated F-ratio of 14.93 is considerably larger than the required 4.02 at the .05 level or 7.12 required for significance at the .01 level. Therefore, null hypothesis one was rejected at the .01 level and a conclusion was drawn that the experimental group displayed a gain in total achievement over the control group, based on post-test scores.

<table>
<thead>
<tr>
<th>TABLE IV</th>
<th>TOTAL ACHIEVEMENT BETWEEN GROUPS (POST-TEST) ANALYSIS OF VARIANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Source of Variation</td>
<td>Sum of Squares</td>
</tr>
<tr>
<td>Between</td>
<td>517.24</td>
</tr>
<tr>
<td>Within</td>
<td>1939.60</td>
</tr>
<tr>
<td>Total</td>
<td>2456.84</td>
</tr>
</tbody>
</table>

Computation Achievement Between Groups (Post-Test)

The second major hypothesis stated that there would be no difference in computation achievement between the groups as measured by the researcher-prepared post-test. Again, the main concern was to detect any gains of the experimental group over the control group. As can be seen
in Table V, the calculated F-ratio of 1.828 is not large enough to warrant rejection of the null hypothesis at the .05 level which requires an F-ratio of 4.02. It is therefore concluded that there are no differences between the groups in computation as measured by the researcher-prepared post-test.

**TABLE V**

**COMPUTATION ACHIEVEMENT BETWEEN GROUPS (POST-TEST)**

**ANALYSIS OF VARIANCE**

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Variance Estimates</th>
<th>F-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>31.66</td>
<td>1</td>
<td>31.66</td>
<td>1.828</td>
</tr>
<tr>
<td>Within</td>
<td>968.91</td>
<td>56</td>
<td>17.27</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>1000.57</td>
<td>57</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Concept Achievement Between Groups (Post-Test)**

The last major hypothesis to be tested stated that there would be no differences in concept achievement as measured by the researcher-prepared pre- and post-tests. Inspection of Table VI shows that a substantial gain, by the experimental over the control group, resulted since the calculated F-ratio of 613.68 is considerably larger than the 7.12 required for significance at the .01 level. Therefore, the third null hypothesis of no differences between the groups on post-test concept achievement is not accepted.
TABLE VI
CONCEPT ACHIEVEMENT BETWEEN GROUPS (POST-TEST)
ANALYSIS OF VARIANCE

<table>
<thead>
<tr>
<th>Source of Variation</th>
<th>Sum of Squares</th>
<th>Degrees of Freedom</th>
<th>Variance Estimates</th>
<th>F-ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Between</td>
<td>4651.76</td>
<td>1</td>
<td>4651.76</td>
<td>613.68</td>
</tr>
<tr>
<td>Within</td>
<td>423.93</td>
<td>56</td>
<td></td>
<td>7.58</td>
</tr>
<tr>
<td>Total</td>
<td>5075.69</td>
<td>57</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Discussion

General comments on results: A re-examination of the computed pre-test F-ratios shows them all to be very small relative to the required F-ratio at the .05 level of significance. The computation achievement pre-test F-ratio and the concept achievement pre-test F-ratio each shows no significant differences between the experimental and control groups in those achievement areas. Also, the combination of computation achievement and concept achievement to yield a total achievement F-ratio shows a very small difference which falls far short of being significant at the .05 level. Based on these results, plus the fact that the groups had been initially randomly assigned to their respective classes, this researcher concluded that there are no significant statistical differences in mathematics achievement between the two groups at the beginning of the study.

The post-test results, however, did vary considerably with the exception of computation achievement. There is no significant difference
in post-test computation achievement between the groups at the .05 level. At the same time, the calculated F-ratio for post-test concept achievement is an extremely large 613.8. Therefore, it was concluded that the large significant difference displayed by the total achievement F-ratio owes its strength primarily to the differences in concept achievement between the two groups.

General comments on group behavior: There were several differences not only in group behavior but also in teacher behavior during the study. In the control group, the lessons were presented by this researcher according to the procedures outlined in the teacher's manual of Elementary School Mathematics, Book 5 published by Addison-Wesley (Eicholz et al., 1967). For the most part the lesson presentations took the following pattern: this teacher introduced a concept, worked a few examples on the chalkboard, and made an assignment. The children worked on the assignment in class. Several times during the assignment phase of the lesson, many children would encounter a difficulty with a particular problem and consequently this researcher would have to interrupt the more capable students while the example was explained to the class. By the end of the mathematics period there were always a few students who were not able to finish the assignment and, unfortunately, a few who still did not understand the concept(s) promoted in the lesson. On the following day, the previous day's assignment was reviewed in an effort to strengthen the respective concept(s).
In order to be fair to the majority of the class, new material was then presented, an assignment made, and so on. At the end of the program, the post-test was administered to all of the students in the control group at the same time.

The procedure employed with the experimental group was quite different from that utilized with the control group. After the initial presentation of the experimental materials was made, the students received their booklets and began work on the program sets. Many questions came up while working on the initial problems regarding procedures of recording and checking answers, however, very few questions resulted regarding basic conceptual understandings. It seemed to this researcher that the main advantage in using the experimental materials was that the students in this group did not "get behind." Instead, work on the experimental materials was a continuous operation which could proceed from one day to the next, regardless of where the individual student was at the end of the period. It is noted here that the students worked with the experimental materials only during the mathematics period of the day and were not allowed to take them home. Practically all of the children thoroughly enjoyed working with the materials and decoding the pictures. They were eager to begin working with the materials as soon as the mathematics period began.

Another interesting observation was made with respect to the learning and working rates of the children in the experimental group.
About a ten-day span resulted from the time the more capable students finished the lessons to the time that the slower learners completed their sets. In keeping with the spirit of the individualized program of the experimental group, each child was tested as he finished his problem set.
CHAPTER IV

SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

This study was designed to compare the performance of two fifth grade classes in learning about mathematical functions and their graphs under varying forms of instruction. The areas of interest regarding significant differences between the two classes were in computation achievement and concept achievement.

This chapter contains a general summary of the study, conclusions arrived at through the statistical analysis of the data, and recommendations based on these data.

Summary

This study was limited to two groups of fifth-grade children. The control group included 30 students who were taught by methods of conventional instruction using the Elementary School Mathematics, Book 5 series published by Addison-Wesley. (Eicholz et al., 1967). The experimental group included 28 students who were taught by means of experimental materials, developed by this researcher, based on a linear program model.

The primary objective of this study was to detect any gains in computation achievement and/or concept achievement in favor of the experimental group.
The three major hypotheses tested in the study stated that there would be no significant statistical differences between the groups with respect to: (1) total achievement; (2) computation achievement; and (3) concept achievement. A one-way classification analysis of variance was used to test these hypotheses at the .05 level of significance. The results of the data analysis are:

1. There are no significant differences between the groups regarding total achievement, computation achievement, and concept achievement at the beginning of the study based on pre-test scores.

2. There is a significant difference in favor of the experimental group in total achievement based on post-test scores.

3. There is no significant difference between the groups in computation achievement at the end of the study.

4. There is a significant difference in favor of the experimental group in concept achievement at the end of the study based on post-test scores.

Two principal advantages of the instruction given to the experimental group over the conventional instruction given to the control group were: (1) the accommodation of different rates of working and learning; and (2) the continuity of activity afforded by working with the experimental materials. The comparative studies cited indicate that the use of programmed instruction in teaching elementary school mathematics is as effective as or better than conventional methods of instruction.
Conclusions

The two main conclusions drawn from the study pertain to:

(1) the difference in concept achievement between the two groups; and
(2) the usefulness of the experimental method in teaching computations and concepts of functions and their associated graphs.

At the beginning of this thesis, it was mentioned that the main objective of "Modern Mathematics" is that of developing concepts of mathematical processes. Since there was a statistically significant difference in concept achievement in favor of the experimental group, this researcher concludes that instruction of concepts of mathematical functions and their graphs by means of the experimental material, does satisfy this objective of "Modern Mathematics."

It is felt that the study has considerable applicability, especially based on this researcher's observations of the behavior exhibited by each group. The experimental group vividly demonstrated that the students did have individual rates of learning and working and that these rates were to a large extent, accommodated by the experimental materials. It was therefore concluded that the children of the experimental group derived greater satisfaction in working with mathematics understandings of functions and their graphs than did the control group which was taught in the conventional manner.
Recommendations

Based on the findings in this study, the following recommendations are proposed:

1. It is recommended that the study be extended to include a larger number of subjects in order to obtain a more representative sample.

2. It is recommended that more than one teacher be used in order to control for experimental bias.

3. It is recommended that further studies be directed toward incorporating methods of programmed instruction to other areas of elementary school mathematics.

4. It is recommended that an attitude scale be designed which would measure student attitudes on programmed instruction versus conventional instruction in elementary school mathematics.
APPENDIX A

PRE-TEST
A

Name ___________________________ Date ________________

I. a) \(5 \times 6 = N\)  
\(N = \) ________  

b) \(8 + 7 = R\)  
\(R = \) ________  

c) \(15 - 9 = S\)  
\(S = \) ________  

d) \(48 \div 6 = A\)  
\(A = \) ________  

e) \(4 \times 9 = C\)  
\(C = \) ________  

f) \(8 + 9 = T\)  
\(T = \) ________  

\(32 \div 8 = B\)  
\(B = \) ________  

\(8 + 7 = R\)  
\(R = \) ________  

\(13 - 8 = B\)  
\(B = \) ________  

\(49 \div 7 = S\)  
\(S = \) ________  

g) \(9 \times 8 = M\)  
\(M = \) ________  

\(5 \times 8 = A\)  
\(A = \) ________  

\(10) 4 \times 9 = C\)  
\(C = \) ________  

\(7 \times 6 = M\)  
\(M = \) ________  

\(35 \div 5 = R\)  
\(R = \) ________  

II. a) \((4 + 8) + 6 = C\)  
\(C = \) ________  

g) \(33 \div (5 + 6) = E\)  
\(E = \) ________  

\(6 \times (4 + 3) = D\)  
\(D = \) ________  

\(15 - (6 + 7) = S\)  
\(S = \) ________  

e) \(3 \times (12 \div 4) = B\)  
\(B = \) ________  

\((3 + 6) - 5 = N\)  
\(N = \) ________  

\((63 \div 7) \times 2 = J\)  
\(J = \) ________  

\((6 \times 4) + (15 \div 3) = W\)  
\(W = \) ________  

d) \(6 \times 4 = M\)  
\(M = \) ________  

\(14 - 6) \times (4 + 2) = N\)  
\(N = \) ________  

\(14 - 6) \times (4 + 2) = N\)  
\(N = \) ________ 
f) \( (12 \times 4) \div (6 \times 2) = M \)

\[ M = \underline{_______} \]

l) \( (56 \div 7) + 14 = S \)

\[ S = \underline{_______} \]

A - 2

III. a) If \( E = 8 \), and \( N = E + 6 \)

Then \( N = \underline{_______} \)

b) If \( S = 4 \), and \( E = 8 - S \), 

\[ E = \underline{_______} \]

c) If \( N = 8 \), and \( W = (N \times 4) + 3 \)

\[ W = \underline{_______} \]

d) If \( S = 4 \), and \( W = S + 6 \)

\[ W = \underline{_______} \]

IV. Locate and label the following points on the graph:

(1) \( N = 1, E = (N \times 2) + 2 \)

(2) \( S = 3, W = (S + 4) \div 7 \)

(3) \( W = 2, N = (W \times 3) \div (W \times 3) \)

(4) \( S = 3, E = S \)

(5) \( E = 1, N = (E \times 8) - 5 \)

Connect the points in the following order:

START 4, 5, 2, 1, 3, 4 STOP
APPENDIX B

POST-TEST
Name ___________________________ Date ____________________________

I.  a) $4 \times 8 = N$  
    $N = $ _______  
 f) $8 + 9 = T$  
    $T = $ _______  
 k) $48 - 6 = B$  
    $B = $ _______  

 b) $9 + 7 = R$  
    $R = $ _______  
 g) $14 - 9 = B$  
    $B = $ _______  
 l) $63 - 7 = S$  
    $S = $ _______  

 c) $14 - 8 = S$  
    $S = $ _______  
 h) $9 \times 7 = M$  
    $M = $ _______  
 m) $5 \times 9 = A$  
    $A = $ _______  

 d) $42 - 7 = A$  
    $A = $ _______  
 i) $5 + 8 = D$  
    $D = $ _______  
 n) $64 - 8 = T$  
    $T = $ _______  

 e) $3 \times 8 = C$  
    $C = $ _______  
 j) $7 \times 8 = N$  
    $N = $ _______  
 o) $35 - 5 = R$  
    $R = $ _______  

II. a) $(5 + 7) + 4 = C$  
    $C = $ _______  
 g) $44 - (8 + 3) = E$  
    $E = $ _______  

 b) $7 \times (6 + 4) = D$  
    $D = $ _______  
 h) $44 - (8 + 4) = S$  
    $S = $ _______  

 c) $16 - (6 - 3) = F$  
    $F = $ _______  
 i) $(6 + 3) - 5 = N$  
    $N = $ _______  

 d) $(54 - 6) \times 3 = J$  
    $J = $ _______  
 j) $(7 \times 3) + (16 - 8) = W$  
    $W = $ _______  

 e) $4 \times (18 - 6) = B$  
    $B = $ _______  
 k) $(15 - 5) \times (3 + 4) = M$  
    $M = $ _______
III. a) If \( E = 7 \), and \( N = E + 4 \)

Then \( N = \) ________

b) If \( S = 6 \), and \( E = 9 - S \)

\( E = \) ________

c) If \( N = 9 \), and \( W = (N \times 4) + 3 \)

\( W = \) ________

d) \( S = 8 \), \( W = S + 7 \)

\( W = \) ________

e) \( N = 7 \), \( E = (N \times 4) - (N \times 2) \)

\( E = \) ________

f) \( W = 16 \), \( S = (W - 8) \times 7 \)

\( S = \) ________

g) \( W = 42 \), \( N = (W - 7) + (W - 6) \)

\( N = \) ________

h) \( E = 18 \), \( N = (E - 6) \times 8 \)

\( N = \) ________

IV. Locate and label the following points on the graph:

1) \( N = 1 \), \( E = (N \times 2) + 2 \)

2) \( S = 3 \), \( W = (S \times 2) - 6 \)

3) \( W = 2 \), \( N = (W \times 4) - (W \times 4) \)

4) \( S = 3 \), \( E = S \)

5) \( E = 1 \), \( N = (E \times 7) - 4 \)

Connect the points in the following order:

START 4, 5, 2, 1, 3, 4, STOP
LIST OF REFERENCES


Raymond, Roger A. "Teaching Algebra to Ninth and Tenth Grade Pupils with the Use of Programmed Materials and Teaching Machines," South Dakota: Sioux Falls Public Schools, October, 1964.


