

CANONIC ACTIVE RC NETWORKS

by

Richard Stanley Aikens

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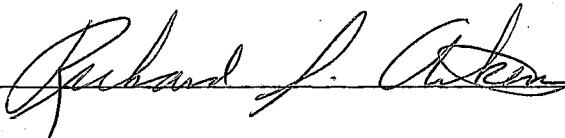
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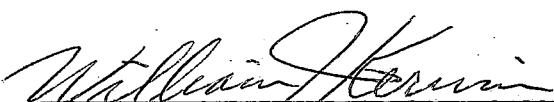
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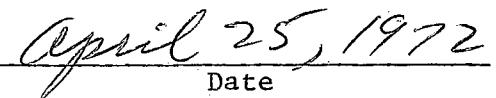
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## ABSTRACT

Over the past several years a variety of active RC networks have emerged, each optimized for one or several parameters usually at the expense of increased complexity involving additional active and passive elements.

The purpose of this thesis is to examine in detail a class of low pass networks which require the minimal number of passive elements (resistors and capacitors only) and that use only one active element, a positive gain amplifier. Thus, these networks are the simplest possible lumped element, active RC structures for any order. A recursive formula is derived in the analysis phase and used as an aid in realizing Thomson, Butterworth and Chebyshev responses for networks through the 6th order; a 7th order Thomson is also given. These solutions are tabulated for direct application to filter design problems.

Sensitivity to active and passive elements is discussed and optimal sets of element values are given. Theoretical derived results are verified on a laboratory model of a 6th order Butterworth filter.

## CHAPTER 1

### INTRODUCTION AND STATEMENT OF THE PROBLEM

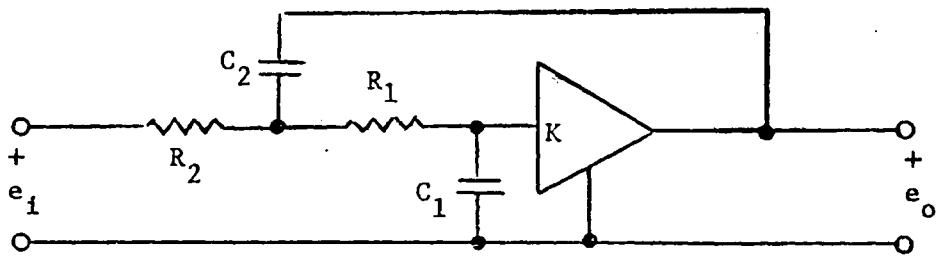
#### 1.1 Historical Background

Active RC networks became practical with the advent of reliable inexpensive solid state active elements in the mid-sixties. Early efforts yielded many variations of low pass, band pass, and high pass active RC networks, but the Sallen and Key [1955] positive gain active RC network shown in Fig. 1.1(a) was probably the simplest 2nd order, low pass, lumped element circuit developed during this period. Later work by Huelsman [1971] led to the 3rd order low pass network shown in Fig. 1.1(b). More recent work by Kerwin [1971] resulted in several MFM solutions to the 4th order form of this network type (see Fig. 1.2). This 4th order network still resembles, at least in a topological sense, the original 2nd order Sallen and Key structure of Fig. 1.1(a) and only one resistor-capacitor pair per pole of the transfer function is required. Because of this minimal RC form, the network is said to be canonic.

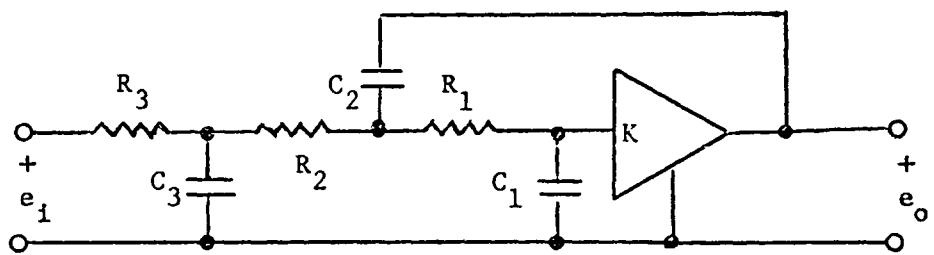
#### 1.2 Statement of the Problem

The fact that 2nd, 3rd and a few 4th order solutions for the canonic structure exist raises several questions.

1. How far can the configuration in Fig. 1.2 be extended beyond the 4th order for the common filter functions?



(a)



(b)

Fig. 1.1 Positive Gain Single Amplifier Second and Third Order Minimal RC Low Pass Networks

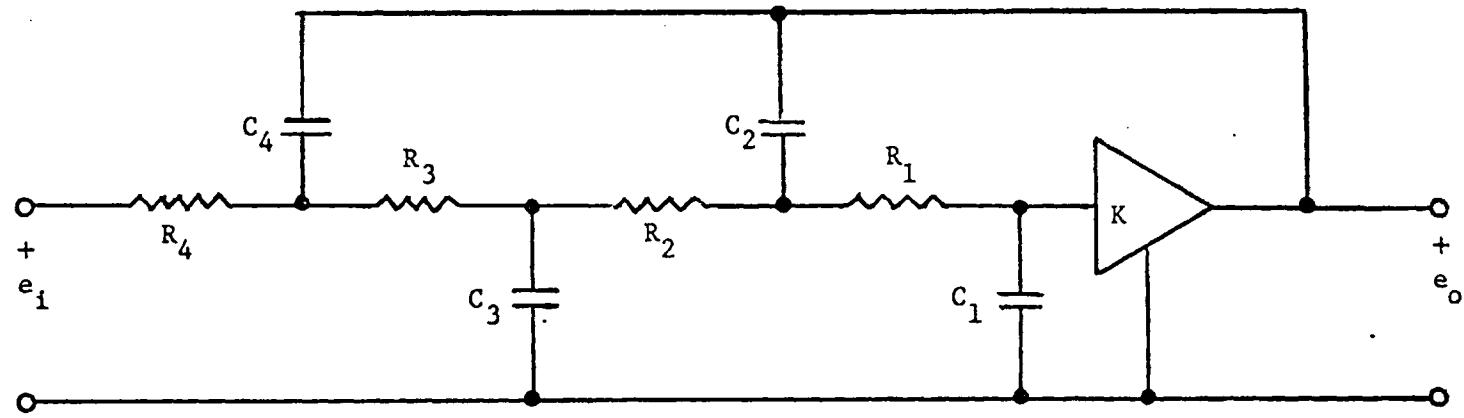


Fig. 1.2 A 4th Order Positive Gain Minimal RC Low Pass Network for Which MFM Solutions Were Found

12. If higher order realizations exist, what are their sensitivity characteristics? How shall we define sensitivity in cases where individual pole Q and frequency are relatively unimportant?
3. What limitations do the sensitivities to active and passive elements impose relative to the application of these networks to filtering problems?

These questions are investigated in the text which follows.

## CHAPTER 2

### THE ANALYSIS

#### 2.1 The nth Order Network

The extension of the 4th order network to order  $n$  is represented by Fig. 2.1. The arrangement of element subscripts, while unconventional, leads to the simplification of equations which arise later on. The analysis was made in the complex frequency domain where  $p$  is, as usual, the complex frequency variable.

This kind of analysis gives the functional relationship between network elements and the coefficients of the transfer function polynomials. The justification for a detailed analysis is that the method of synthesis discussed later requires a knowledge of this functional relationship.

#### 2.2 An Equivalent Circuit

Figure 2.2 is an equivalent circuit representation of the  $n$ th order network. The positive gain amplifier has been replaced by many voltage controlled voltage sources. Other voltage sources have been introduced, into the shunt capacitor legs to ground, for symmetry and convenience in the analysis. The form of the transfer function is

$$\frac{e_o}{e_i} = \frac{K}{d_n p^n + d_{n-1} p^{n-1} + \dots + d_1 p + d_0} \quad (2.1)$$

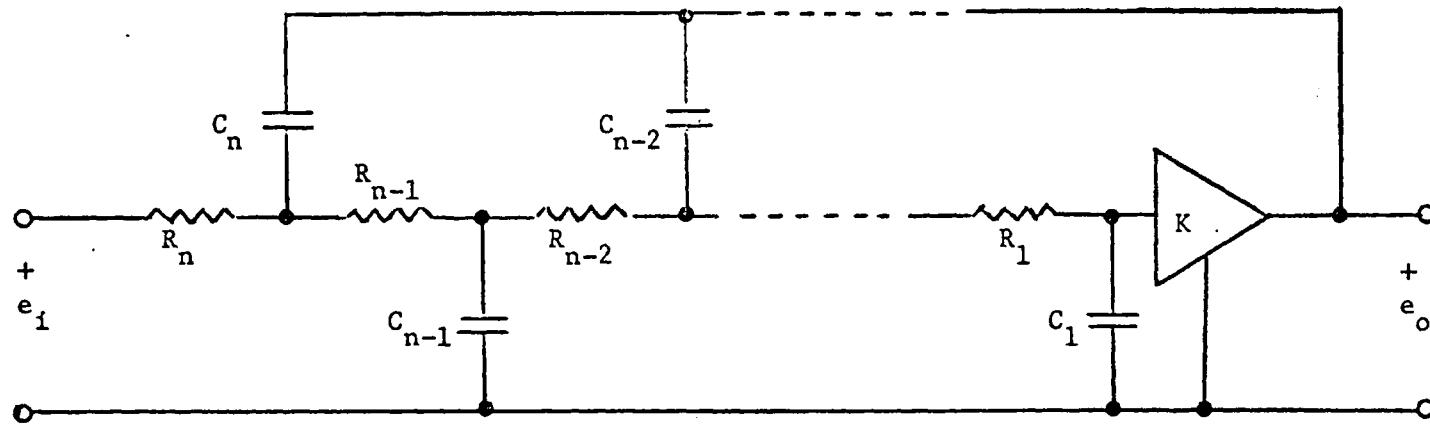


Fig. 2.1 The Minimal Active RC Low Pass Network of Order  $n$ ,  $K > 0$

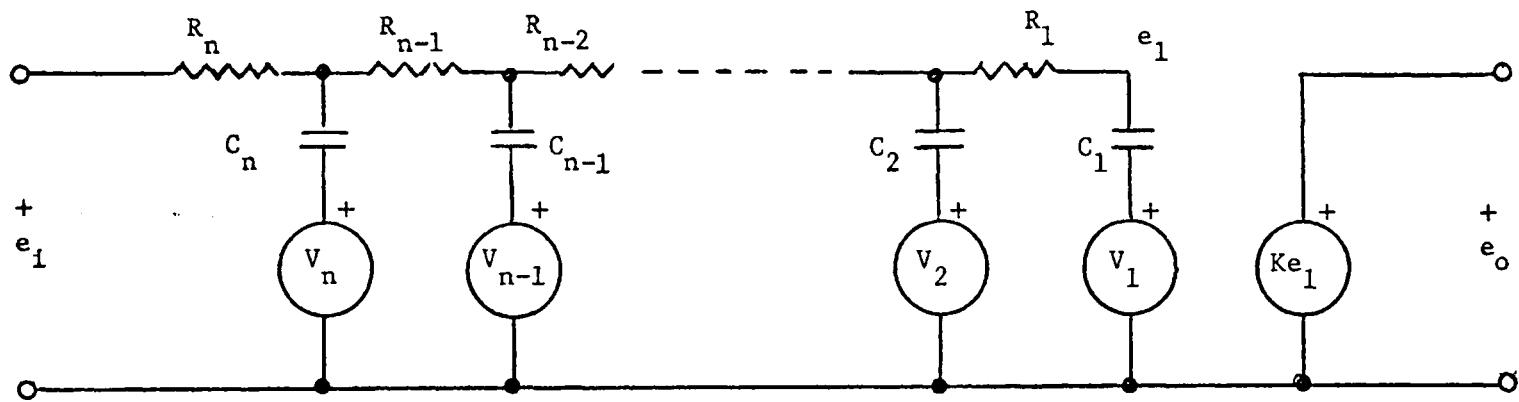


Fig. 2.2 . An Equivalent Circuit for the  $n$ th Order Network with  
Alternate Voltage Sources Added for Symmetry

where  $d_i$  are real positive coefficients and  $k$  is the amplifier gain.

The term  $d_0$  will always be normalized to unity so the DC gain is  $k$  and the network response may be described completely by the denominator polynomial,  $D_n(p)$ . Let  $e_0$  take on the value  $k$  volts, then by Eq. (2.1),

$$e_i = D_n(p) \quad (2.2)$$

Now it is only necessary to find  $e_i$  to completely describe the network.

Figure 2.3 shows the section of the network at the input terminals where  $e_i$  is the input voltage. Other voltages and currents have also been assigned subscripts in keeping with the convention adopted earlier.

Summing currents at the  $e_n$  node, and solving for the input voltage  $e_i$

$$e_i = (e_n - e_{n-1}) \frac{R_n}{R_{n-1}} + e_n (1 + R_n C_n p) - R_n C_n V_n p \quad (2.3)$$

Equation (2.2) does not depend on  $n$ , so  $e_i$  could be the voltage  $e_n$  for the  $n-1$  network, or  $e_{n-1}$  for the  $n-2$  network, and so forth.

Consider severing the network at the point A indicated in Fig. 2.3 and eliminating the section to the left. Now  $e_n$  is an input voltage to a network of order  $n-1$ , and by Eq. (2.2), must equal the denominator polynomial  $D_{n-1}(p)$ . Similarly, cutting the network at B yields the result that

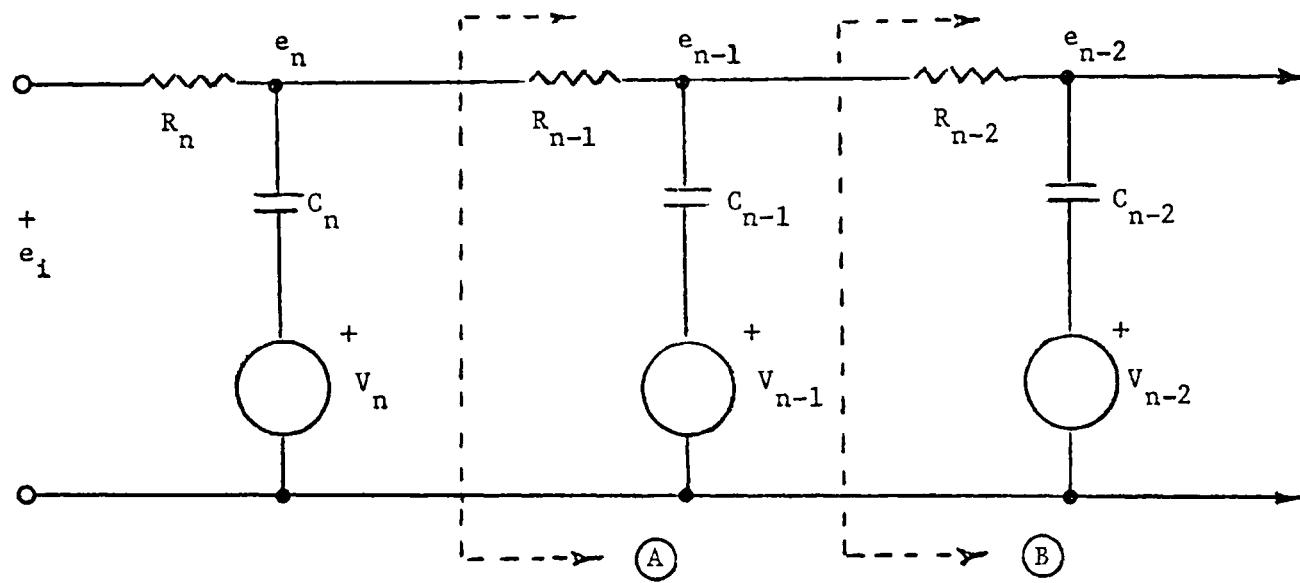


Fig. 2.3 The Input Section of the  $n$ th Order Equivalent Circuit

$$e_{n-2} = D_{n-3}(p) \quad (2.4)$$

and, in general

$$e_{n-k} = D_{n-1-k}(p) \quad K < n \quad (2.5)$$

Now Eq. (2.3) can be written in terms of  $D_n(p)$ ,  $D_{n-1}(p)$ , and  $D_{n-2}(p)$

$$\begin{aligned} D_n(p) &= [D_{n-1}(p) - D_{n-2}(p)] \frac{R_n}{R_{n-1}} \\ &\quad + D_{n-1}(p)(1 + R_n C_n p) - R_n C_n V_n p \end{aligned} \quad (2.6)$$

The result is that  $D_n(p)$  may be written as a function of the two next lower order polynomials, and the network exhibits a recursive nature which can be used now to great advantage. Defining the terms of the polynomials, let

$$D_{n-2}(p) = a_{n-2} p^{n-2} + a_{n-3} p^{n-3} + \dots + a_1 p + a_0 \quad (2.7a)$$

$$D_{n-1}(p) = b_{n-1} p^{n-1} + b_{n-2} p^{n-2} + \dots + b_1 p + b_0 \quad (2.7b)$$

and

$$D_n(p) = d_n p^n + d_{n-1} p^{n-1} + \dots + d_1 p + d_0 \quad . \quad (2.7c)$$

Substituting Equations 2.7(a), (b), and (c) into Eq. (2.6), collecting terms and equating coefficients of like powers in  $p$  gives the recursive relation between coefficients..

$$d_k = (b_k - a_k) \frac{R_n}{R_{n-1}} + b_k + b_{k-1} R_n C_n - \delta_k R_n C_n V_n \quad (2.8)$$

where

$$\begin{aligned}\delta_k &= 1, \text{ when } k = 1 \\ \delta_k &= 0, \text{ when } k \neq 1\end{aligned}$$

All of the terms in Eq. (2.8) have been identified with the exception of  $V_n$ .

Looking at Fig. 2.2, it can be seen that alternate voltage sources are zero and  $V_n$  can be written

$$V_n = \left[ \frac{(-1)^n + 1}{2} \right] \cdot K \quad (2.9)$$

Using the recursive relations and the following equations from the zeroth and first order networks, any  $D_n(p)$  may be found for any  $n$ .

$$D_0(p) = a_0 = 1 \quad (2.10)$$

$$D_1(p) = b_1 p + b_0 = R_1 C_1 p + 1 \quad (2.11)$$

An illustrative example will best demonstrate how this works.

Suppose it is required to find  $D_2(p)$ , the second order denominator polynomial. Applying Equations (2.8) and (2.9)

$$d_2 = (0 - 0) \frac{R_2}{R_1} + 0 + R_1 C_1 R_2 C_2 = R_1 C_1 R_2 C_2 \quad (2.12a)$$

$$d_1 = (R_1 C_1 - 0) \frac{R_2}{R_1} + R_1 C_1 + R_2 C_2 - V_n R_2 C_2 \quad (2.12b)$$

Now using Eq. (2.9) for  $V_n$  and rearranging terms, results in

$$d_1 = R_1 C_1 + R_2 C_1 + (1 - k) R_2 C_2 . \quad (2.12c)$$

Clearly,  $D_3(p)$  could now be found by applying the formula again.

For the trivial example shown here the computational simplification is small, but for networks above order 3, the recursive formula turns a nearly impossible task into a simple iterative procedure. The equations now, in this form, are quite amenable to computer evaluation. It should be realized that this recursive relationship is purely algebraic in nature and has no connection with specific filter functions. It provides a method for evaluating the coefficients of the nth order transfer function given an arbitrary set of element values. This feature does prove useful in the realization of specific filter functions.

For the reader who is unfamiliar with the complexity of terms which arise in conventional analysis, a coefficient from the 4th order

transfer function for this network would take up two lines of this page and there are four such coefficients, the fifth being unity.

## CHAPTER 3

### SYNTHESIS

#### 3.1 The Method

The synthesis problem presented here does not require finding a network configuration to realize a desired filter function, as in classical network synthesis. The network configuration is already prescribed and functional relationships, from the analysis phase, which give the transfer function coefficients in terms of the network elements have been developed previously. These coefficients must now be equated to the positive real numerical coefficients from the desired filter function. The resulting set of nonlinear simultaneous equations then need only be solved for the network element values and the network synthesis is complete.

#### 3.2 Use of Optimization

The equations which result from equating these coefficients are nonlinear in the network elements and there are  $n$  equations for an  $n$ th order network in  $2n + 1$  element variable unknowns so for an  $n$  of 3, the task is already formidable. Because the equations are nonlinear and because of the lack of any closed form method of solution, optimization techniques were brought to bear on the problem. Optimization, in the sense used here is outlined by the following.

Let  $X_i$  be real or complex variables which enter into some functional relationship

$$f(X_i) = G_i \quad (3.1)$$

Let  $R_i$  be some requirements which are to be met by  $G_i$ .

An error function is defined,  $E_f$ , which expresses the error between the values of  $G_i$  and the required values,  $R_i$ .

Optimization is achieved by operating on the variables,  $X_i$ , in any way which reduces the value of the error function until it reaches a specified minimum. Work by Huelsman [1968] has lead to the computer implementation of several optimization strategies, three of which were successfully used on the problem at hand. The component parts of the problem are readily identified with the various optimization functions and variables. The variables  $X_i$  are the network elements: resistors, capacitors, and gain. The functions  $G_i$  were found in the analysis and are the coefficients of  $D(p)$ . The requirements  $R_i$  are the coefficients of the transfer function which one is trying to synthesize. The error function  $E_f$  was an integral part of the software package,

$$E_f = \sqrt{\sum_{i=1}^n (G_i - R_i)^2} \quad (3.2)$$

All the necessary variables and functional relationships are now in a form which is amenable to the application of optimization.

### 3.3 Solutions for Common Filter Functions

Optimization techniques were applied to the problem with success up through the 7th order. Thomson (MFD), Butterworth (MFM), and Chebyshev (Equal Ripple) solutions were found for a variety of element combinations and orders.

The strategy used when only a vague notion of a solution existed, was a random direction and step size search routine. Usually the initial value of the error function could be reduced quickly by several orders of magnitude to a value less than 10. A pattern search routine was then used which systematically further reduced the error function by several orders of magnitude. Finally, when an error minimum was definitely nearby, the well-known Newton Raphson minimization method reduced the error to any desired level by about an order of magnitude per iteration. Once a specific solution had been found, other solutions for the same order could then be obtained by incremental optimization. Incremental optimization is achieved by incrementing one (or more) variables toward some desired final value (or values) and re-optimizing the solution each time. By this means it is then possible to step through solution space from one useful solution to the next.

Since there were  $2n + 1$  variables, it was possible to apply constraints which limited results to more useful solutions. Two constraints were used: (1) making all resistors equal valued and solving for the capacitor values; (2) making all capacitors equal valued and solving for the resistor values. For the two solution sets,

the gain was incremented over a range for which solutions existed. Equal resistor solutions were found for gains near unity up to ten, while equal capacitor solutions existed only for gains between 2 and 3.

Equal capacitor networks have a decided cost advantage over the equal resistor type. The actual value to which all the capacitors must be matched, need only be specified to a few percent. The match of capacitors within a given circuit, however, is very important and must be made to better than 0.1% for some of the high order networks. Considering the matching problems and the cost of selecting capacitors compared with that of resistors, the only networks which are economically feasible to produce in any quantity are the equal capacitor types. Final frequency trimming for all the networks discussed here can be done by trimming resistor values.

### 3.4 The Data

The most useful data obtained for Thomson, Butterworth, and Chebyshev functions are tabulated in Appendix A. The transfer functions were all normalized to have leading and last coefficients unity for maximum computational efficiency. Table 3.1 gives the 3 db frequency for the Thomson, Butterworth, and Chebyshev functions, when normalized in this fashion. The Chebyshev cut-off frequency is defined as the frequency where the magnitude last contacts bottom of the ripple band before attenuation in the skirt. It was found that element value spread was the greatest near points where solutions ceased to exist. If these points are avoided when selecting an element set, then element spread is not a problem.

Table 3.1. Cut-off Frequencies in Radian per Second of Thomson, Butterworth, and Chebyshev Filters

Order	Filter Type		
	Thomson	Butterworth	Chebyshev
			0.5 db Ripple    1.0 db Ripple
2	0.787	1.0	0.810    0.938
3	0.712	1.0	1.120    1.270
4	0.661	1.0	1.274    1.380
5	0.617	1.0	1.410    1.521
6	0.579	1.0	
7	0.555		

All the solutions which were found for low pass networks are applicable to high pass networks. If the positions of the resistors and capacitors are interchanged, then a low pass network becomes high pass. The equal resistor low pass solutions then are equal capacitor high pass solutions and, thus, are also quite useful. The following chapter on sensitivity deals only with the low pass solutions.

## CHAPTER 4

### SENSITIVITY

#### 4.1 Definitions

Unlike passive networks in which poles are confined to the left half plane, active RC networks can have poles anywhere. For this reason, any stable solution to an active RC network, must be examined for sensitivity to changes in element values. Sensitivity of a characteristic,  $c$ , to some change in element  $X$  is traditionally defined as:

$$S_x^c = \frac{\partial c}{\partial x} \cdot \frac{x}{c} \quad (4.1)$$

Since approximately 2000 solutions covering a variety of orders, gains and filter functions were found, some sensitivity criteria had to be developed which yielded useful information in a simple, efficient manner. Pole position sensitivity and Q sensitivity definitions are inappropriate to this problem. Therefore, we define the following sensitivity criteria for use in evaluating network realizations.

For the Butterworth functions, the sensitivity to the change in an element value  $X$  is defined to be the maximum magnitude sensitivity, as given by Eq. (4.1), within the passband (DC to the 3 db frequency). For the normalized Butterworth functions the 3 db frequency is 1 radian per second.

$$S_x^{\text{mfm}} = \text{maximum} \left[ \frac{\partial |T(j\omega)|}{\partial x} \cdot \frac{x}{|T(j\omega)|} \right]_{\omega=0}^{\omega=1} \quad (4.2)$$

For the Thomson functions, the sensitivity to the change in an element X is defined to be the maximum group time delay sensitivity as given by Eq. (4.1) within the passband (DC to the 3 db frequency).

$$S_x^{\text{mfd}} = \text{maximum} \left[ \frac{\partial \left[ \frac{d\phi}{d\omega} \right]}{\partial x} \cdot \frac{x}{\left( \frac{d\phi}{d\omega} \right)} \right]_{\omega=0}^{\omega= \omega_{3\text{db}}} \quad (4.3)$$

where  $\phi$  is phase shift.

The sensitivity for the Chebyshev functions is defined in the same way as for the Butterworth functions except the passband is defined in the Chebyshev sense instead of by the 3 db frequency.

These criteria result in similar weighting for any order and are directly associated with the criteria for which the specific filter functions were derived. The Thomson functions are maximally flat in group time delay at zero frequency. The Butterworth functions are maximally flat in magnitude at zero frequency and the Chebyshev functions exhibit the closest approximation to a square magnitude response within a specified error.

#### 4.2 Discussion of Sensitivities

Sensitivities as defined in Equations (4.1), (4.2), and (4.3) were calculated for many of the solutions. The gain sensitivities are depicted graphically in Figures 4.1, 4.2, 4.3 and 4.4. The sensitivity data for all elements are tabulated in Appendix B. Since the sensitivity definition was arbitrary, the implication of these sensitivities in an absolute sense is somewhat vague, only the maxima are known.

Several conclusions, however, can be drawn immediately from the sensitivity data.

1. For the equal resistor case there is a gain sensitivity minimum near a gain of two and for the equal capacitor case, near a gain of .2.2 for all orders.
2. The equal resistor sensitivities are lower than the equal capacitor sensitivities for a given filter function in a given order.
3. The sensitivities increase with increasing pole Q for a given order, i.e., the Chebyshev functions exhibit a higher sensitivity than the Butterworth functions and the Butterworth functions have higher sensitivity than the Thomson functions.  
This is true for both active and passive element sensitivities.
4. The maximum gain sensitivity with respect to frequency occurred close to the cut-off frequency of the filters for all orders and filter functions. This is an especially important result to consider when implementing the active element.

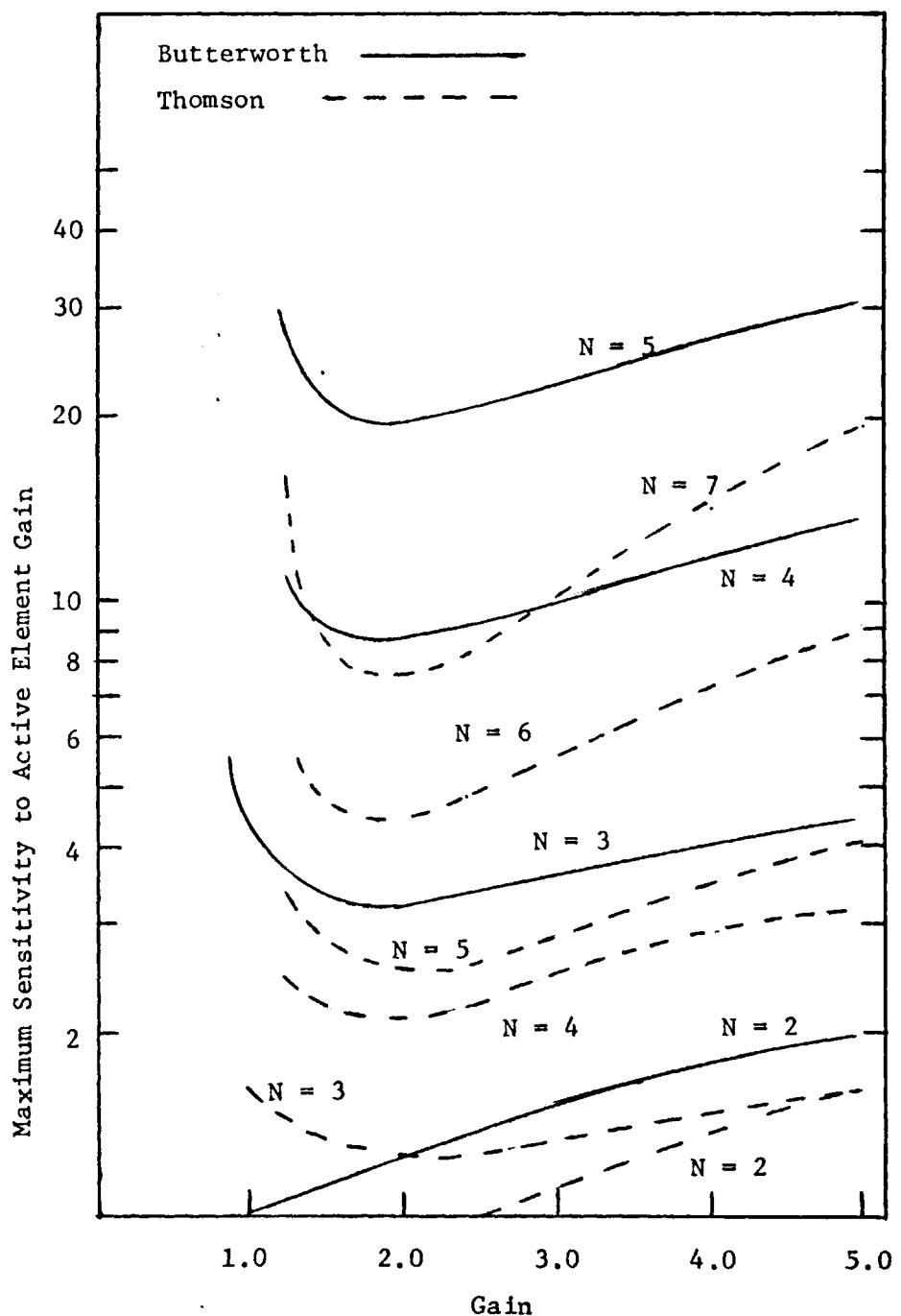


Fig. 4.1 Gain Sensitivities for Equal Resistor Butterworth and Thomson Filters

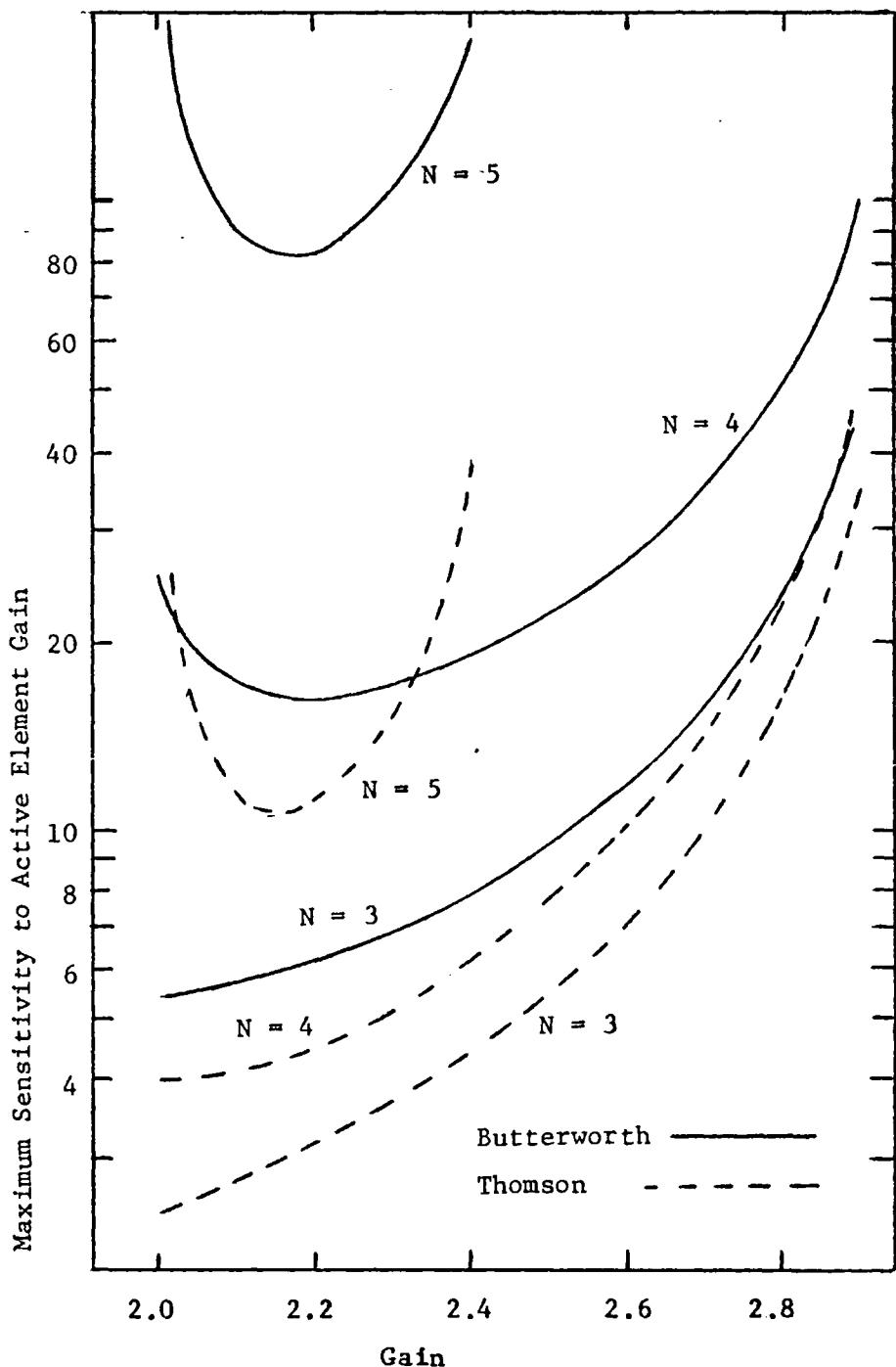


Fig. 4.2 Gain Sensitivities for Equal Capacitor Butterworth and Thomson Filters

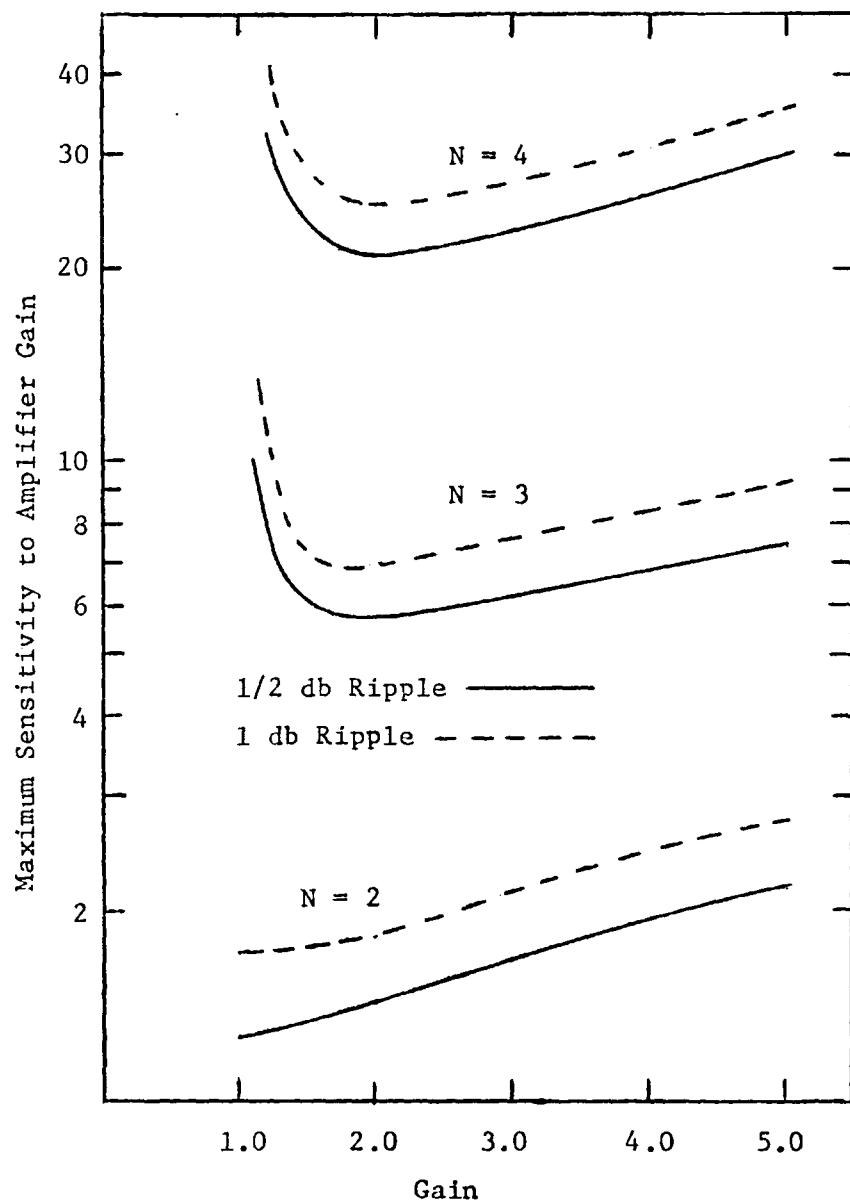


Fig. 4.3 Gain Sensitivities for Equal Resistor Chebyshev Filters

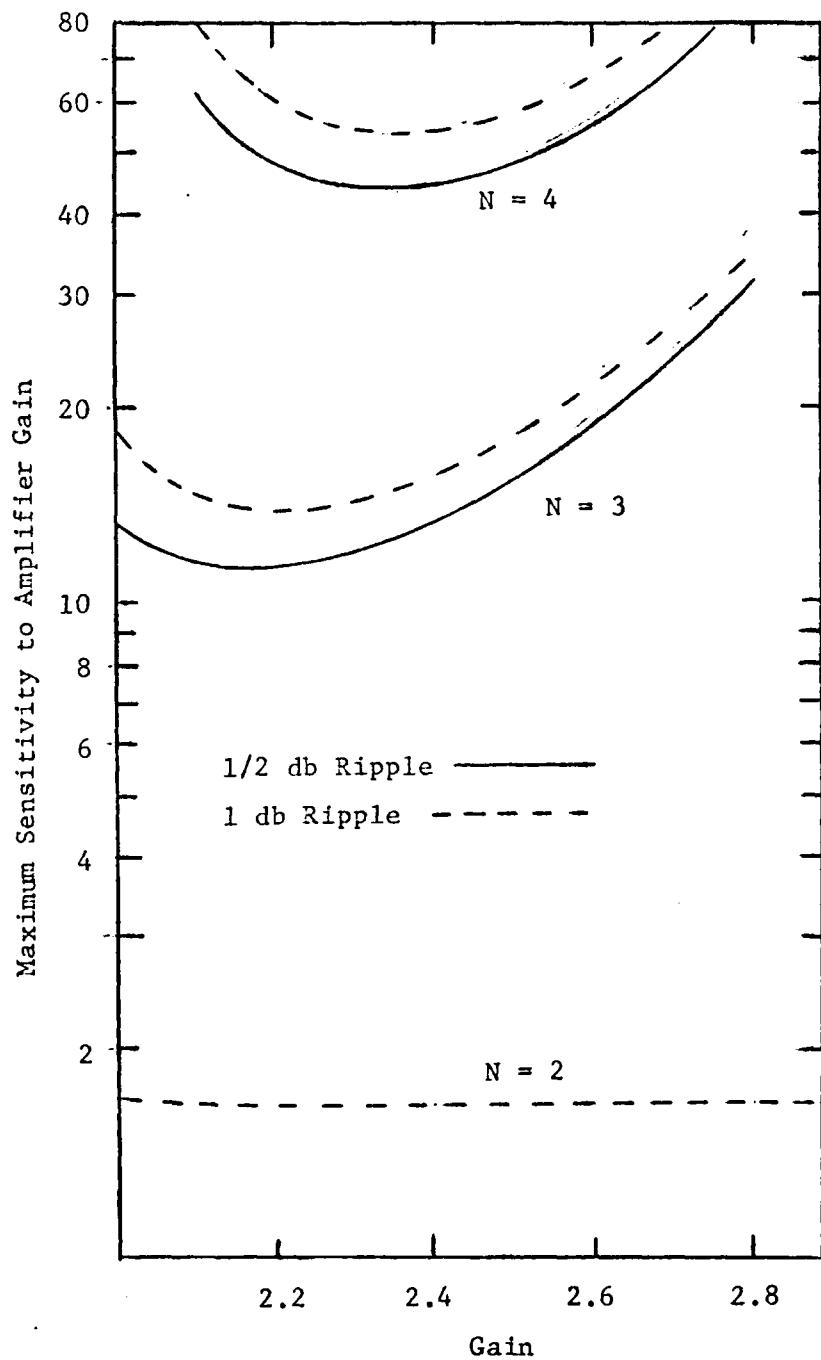


Fig. 4.4 Gain Sensitivities for Equal Capacitor Chebyshev Filters

### 4.3 Optimal Element Sets

The sensitivity data was significant enough to indicate a choice of optimal element sets. For the equal resistor case, a gain of 2.0 is optimum independent of order.

An optimum choice of gain for the equal capacitor case is between values of 2.2 and 2.3. The sensitivities to active element gain were invariably higher than any of the passive element sensitivities for all orders and filter functions. Passive element sensitivity varied in magnitude over a wide range depending on the gain. The equal capacitor networks exhibited large passive element sensitivities near gains of 2 and 3 where solutions went out of bounds. Figure 4.5 shows a set of passive and active element sensitivities for the 4th order Butterworth equal resistor network, as a function of gain.

The high order networks demonstrate high sensitivity but, if one is willing to employ more than one active element high order, canonic synthesis is still possible in the passive element sense, but with decreased sensitivity. A 4th order filter can be implemented by cascading two 2nd order sections, each having the proper response. A 5th order filter can be made by cascading 2nd and 3rd order sections. Finally, a 6th order filter can be made by cascading three 2nd order sections.

The Butterworth functions are particularly interesting with respect to application of the 2nd order Sallen and Key structure in cascade. The 2nd order form has solutions with simultaneously equal capacitors and equal resistors for any pole pair on a circle, about

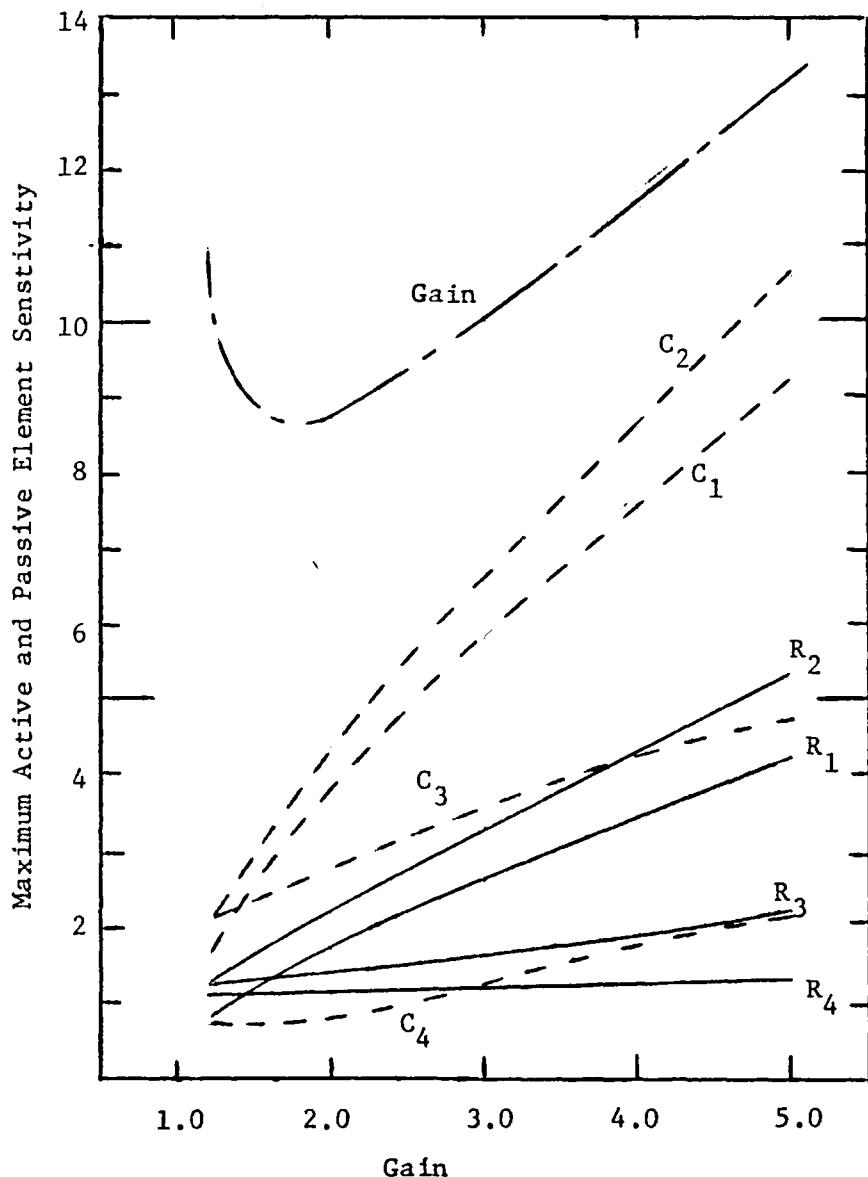


Fig. 4.5 Active and Passive Element Sensitivities vs. Gain  
for the 4th Order MFM Equal Resistor Networks

the origin. The Butterworth poles also lie on a circle about the origin so it is only necessary to choose the proper gain for each section of the filter to realize high, even order, Butterworth functions having both equal capacitors and equal resistors.

Figures 4.6 and 4.7 show 4th and 6th order Butterworth filters having equal capacitors and equal resistors. The required gains are not particularly convenient, but the sensitivity is quite reasonable, about like that for a 2nd order 1 db Chebyshev filter.

None of the multi-amplifier structures have been investigated in depth here as emphasis has been placed on the single amplifier canonic form.

#### 4.4 The Active Element

The active element, thus far, has been assumed to be an ideal positive gain amplifier with infinite input impedance and a gain characteristic which is independent of frequency. The first assumption, that of infinite input impedance, is sufficiently satisfied by the circuit in Fig. 4.8. The second assumption, that of a flat gain frequency characteristic, is not so easily met. Furthermore, the sensitivity data indicated that the gain sensitivity maxima always occurred near the filter cut-off frequencies. Therefore, the gain sensitivities found here must be applied at, or near, the cut-off frequency when selecting an active element configuration. The circuit in Fig. 4.8 has excellent DC characteristics, assuming a large open loop gain, but the gain error increases with frequency as the roll-off characteristics of

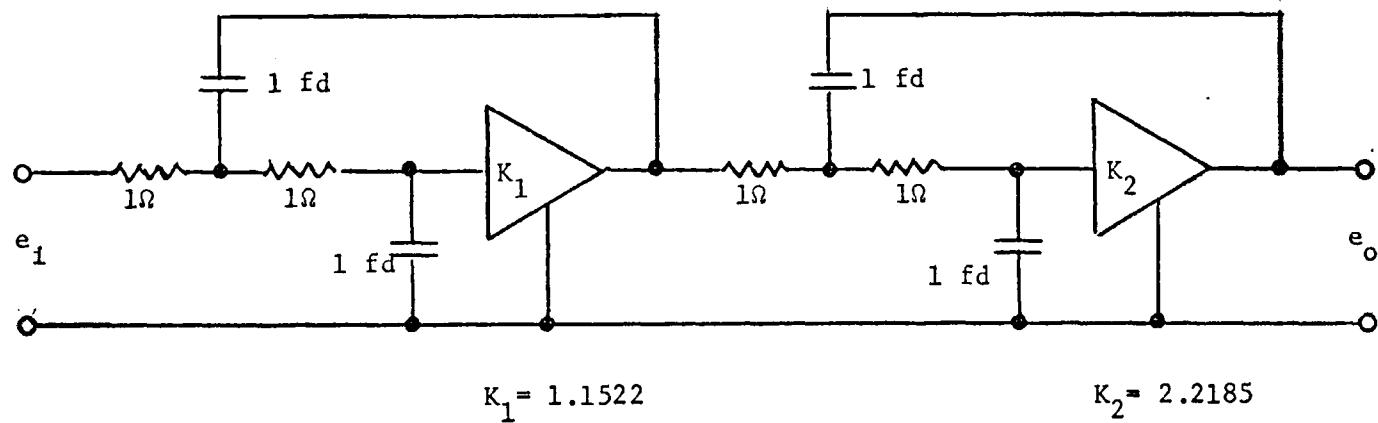
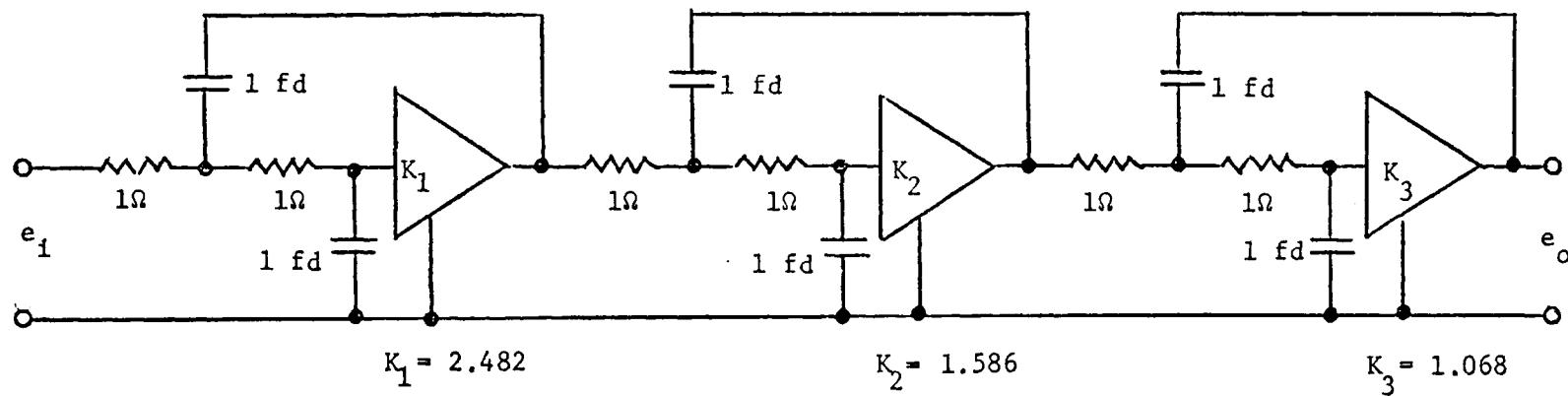
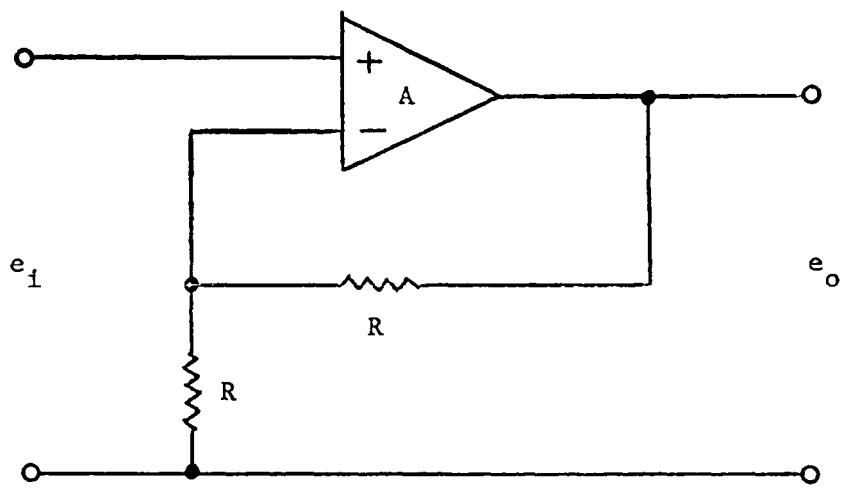


Fig. 4.6 A 4th Order Equal Resistor Equal Capacitor Butterworth Filter



**Fig. 4.7** A 6th Order Equal Resistor Equal Capacitor Butterworth Filter with Overall Gain of 4.20



$$\frac{e_o}{e_i} \approx 2.0 \quad \text{assuming } A \gg 1.0$$

Fig. 4.8 A Possible Realization of an Active Element with a Gain of 2.0

the operational amplifier begins to dominate the response. By the time the filter cut-off frequency is reached, this error could become large enough to cause serious degradation of the filter response. The point being made here is that the proper implementation of the active element is not as simple as it appears at first glance.

#### 4.5 Experimental Results

Two 6th order single amplifier Butterworth filters were constructed: one having equal valued capacitors with a gain of 2.2 and a second equal valued resistor type with a gain of 2.0. All passive elements were selected within 0.1% of the prescribed value and the cut-off frequency was 800 Hz. Frequency response measurements were made, and the maximum error was 2% peaking for the equal resistor case and 4% peaking for the equal capacitor case. The peaking indicated that the sensitivity was primarily due to the poles nearest the  $j\omega$  axis, certainly not a surprising result. Figure 4.9 shows the effect of a 1% gain change on the frequency response for the 6th order Butterworth equal resistor circuit. The effect of a 1% gain change on a 6th order equal resistor Thomson filter is shown for comparison. The Thomson function in Fig. 4.9 has been normalized to have a 3 db frequency of 1 radian per second.

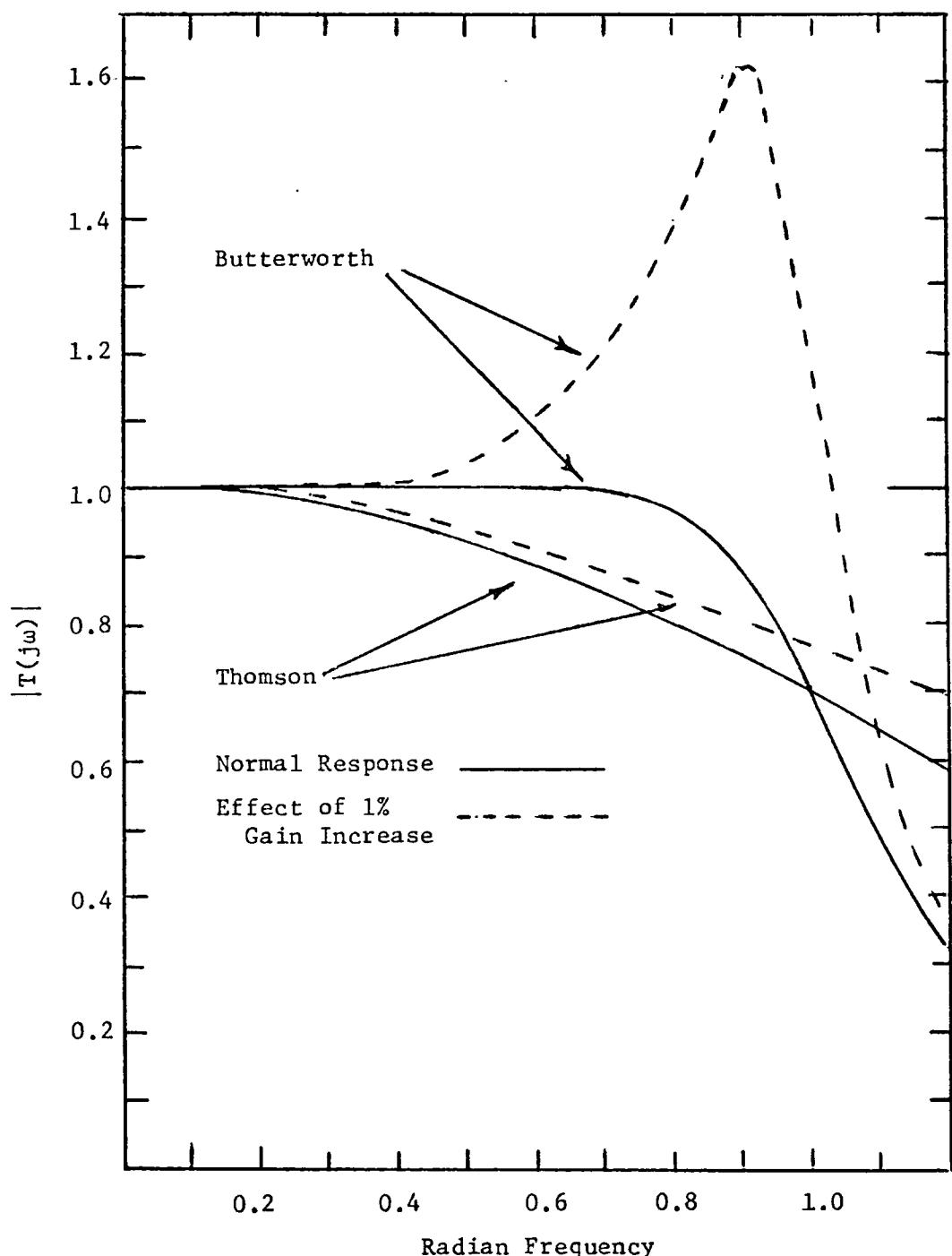


Fig. 4.9 Normalized Magnitude Response of 6th Order Equal Resistor Butterworth and Thomson Networks. The Gain Chosen was 2.0 Where the Gain Sensitivity is Minimum

## CHAPTER 5

### CONCLUSIONS

Many of the high order canonic networks studied here can be applied directly to filtering problems. The primary considerations, when selecting an element set for realizing a specific filter function, are active and passive element sensitivities. These studies indicate that the use of optimal element sets can minimize the sensitivity of network response to active element change. With regard to the implementation of a stable active element, the element gain is assumed to be constant over the bandpass of the filter and care must be taken to assure that this assumption is met. This aspect of the active element cannot be overemphasized.

Further work on the networks described here could be done. There are infinitely many solutions which were not studied. It is possible that applying constraints minimizing active element sensitivity, and solving for passive element values could result in more useful higher order realizations. This problem is non-trivial as it requires a two loop optimization procedure and a great deal of computer time.

In concluding, one has to be impressed with the effect of a single low gain active element upon the many poles which are normally confined to the real axis. An element with a gain of only 2 can move

these poles from their relatively uninteresting real axis positions to useful, prescribed locations in the complex plane. The author chooses to call real poles which are given this additional degree of freedom, through the application of some active element, active complex poles.

## APPENDIX A

### ELEMENT VALUES FOR THOMSON, BUTTERWORTH, AND CHEBYSHEV FILTERS

The following tables give element values as a function of gain for Thomson, Butterworth, and Chebyshev filters. The element subscripts are in keeping with the notation developed in the text. The filter functions are normalized so that leading and last coefficients are unity. All element values are in ohms for resistors and farads for capacitors. For the equal resistor networks, the resistor values are 1.0 ohm for the equal capacitor networks; the capacitors have a value of 1.0 farad.

Table A.1. Capacitor Values for Equal Resistor Thomson Networks

Order	Gain	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$
2	1	0.8660	1.1547				
	2	1.2622	0.7923				
	3	1.5227	0.6567				
	4	1.7321	0.5774				
	5	1.9120	0.5230				
3	1	0.3579	2.0069				
	2	0.7510	0.7644	1.7419			
	3	0.9207	0.5594	1.9417			
	4	1.0406	0.4586	2.0955			
	5	1.1365	0.3958	2.2231			
4	2	0.4852	0.8001	2.3730	1.0855		
	3	0.6224	0.5552	2.9894	0.9681		
	4	0.7159	0.4422	3.5090	0.9002		
	5	0.7887	0.3739	3.9735	0.8534		
	6	0.3412	0.7084	1.6441	1.0892	2.3102	
5	2	0.4392	0.4578	2.0164	0.8702	2.8344	
	3	0.4982	0.3487	2.3191	0.7632	3.2522	
	4	0.5402	0.2852	2.5849	0.6956	3.6091	
	5	0.2510	0.6482	1.3520	1.2622	2.9941	1.2030
6	2	0.3377	0.4117	1.7229	0.9663	3.9328	1.0985
	3	0.3896	0.3100	2.0197	0.8302	4.7484	1.0398
	4	0.4262	0.2513	2.2779	0.7465	5.4929	0.9999

Table A.2. Capacitor Values for Equal Resistor Thomson Networks

Order	Gain	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$	$C_7$
7	1.2	0.0626	1.5623	0.6870	2.8187	1.4748	1.8199	1.9670
	1.6	0.1438	0.7762	0.9392	1.5432	1.8278	1.4057	2.4057
	2.0	0.1915	0.5548	1.0976	1.1910	2.1212	1.2281	2.7642
	2.6	0.2363	0.3992	1.2767	0.9481	2.5072	1.0846	3.2195
	3.0	0.2570	0.3387	1.3759	0.8532	2.7416	1.0235	3.4873
7	4.0	0.2924	0.2475	1.5861	0.7065	3.2758	0.9236	4.0766
	5.0	0.3151	0.1958	1.7631	0.6189	3.7603	0.8604	4.5897
	6.0	0.3313	0.1623	1.9195	0.5587	4.2110	0.8154	5.0524

Table A.3. Resistor Values for Equal Capacitor Thomson Networks

Order	Gain	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>	R <sub>6</sub>
2	2	1.7321	0.5774				
	3	2.1889	0.4569				
	4	2.5243	0.3961				
	5	2.8025	0.3568				
3	2.0	0.4870	1.0376	1.9790			
	2.2	0.4613	0.8010	2.7061			
	2.4	0.4091	0.6347	3.8513			
	2.6	0.3347	0.4925	6.0671			
	2.8	0.2315	0.3443	12.5497			
4	2.0	0.2406	2.0999	2.9604	0.6685		
	2.2	0.3186	1.2190	4.2133	0.6111		
	2.4	0.3246	0.8750	6.1415	0.5732		
	2.6	0.2921	0.6367	9.8629	0.5452		
	2.8	0.2192	0.4197	20.7728	0.5231		
5	2.2	0.1161	1.3452	1.0929	0.9753	6.0084	
	2.4	0.1004	0.6753	1.0725	0.6856	20.0547	
6	2.2	0.0950	1.4848	1.0782	1.3084	8.1726	0.6151
	2.4	0.0933	0.7144	1.1456	0.8228	27.6322	0.5761

Table A.4. Capacitor Values for Equal Resistor Butterworth Networks

Order	Gain	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$
2	1	0.7071	1.4142				
	2	1.1441	0.8740				
	3	1.4142	0.7071				
	4	1.6283	0.6141				
	5	1.8113	0.5521				
3	1	0.2025	3.5468	1.3926			
	2	0.6761	0.8671	1.7058			
	3	0.8660	0.6172	1.8708			
	4	1.0000	0.5000	2.0000			
	5	1.1064	0.4285	2.1091			
4	2	0.4338	0.8519	2.3041	1.1746		
	3	0.5748	0.5752	2.9187	1.0361		
	4	0.6704	0.4526	3.4396	0.9581		
	5	0.7444	0.3799	3.9071	0.9050		
	6	0.3087	0.7416	1.6294	1.2165	2.2039	
5	2	0.4124	0.4715	2.0179	0.9485	2.6865	
	3	0.4752	0.3565	2.3309	0.8224	3.0798	
	4	0.5198	0.2902	2.6049	0.7443	3.4198	
	5	0.2262	0.6512	1.3294	1.3644	2.9118	1.2853
6	2	0.3128	0.4088	1.7088	1.0216	0.8487	1.1642
	3	0.3650	0.3060	2.0103	0.8696	4.6683	1.0977
	4	0.4019	0.2472	2.2718	0.7766	5.4193	1.0530

Table A.5. Resistor Values for Equal Capacitor Butterworth Networks

Order	Gain	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>	R <sub>6</sub>
2	2	1.4142	0.7071				
	3	1.9319	0.5176				
	4	2.2882	0.4370				
	5	2.5779	0.3879				
	2.0	0.4348	1.4694	1.5652			
3	2.2	0.4538	1.0086	2.1849			
	2.4	0.4196	0.7590	3.1401			
	2.6	0.3520	0.5709	4.9762			
	2.8	0.2483	0.3903	10.3195			
	2.0	0.1339	3.8931	2.4792	0.7736		
4	2.2	0.2696	1.4915	3.6435	0.6826		
	2.4	0.2963	0.9992	5.3667	0.6294		
	2.6	0.2779	0.7012	8.6661	0.5921		
	2.8	0.2154	0.4500	18.2994	0.5639		
	2.2	0.1072	1.5131	1.1422	1.0955	4.9266	
5	2.4	0.0986	0.7187	1.1343	0.7447	16.7005	
	2.2	0.0839	1.5988	1.0805	1.4769	7.0400	0.6635
6	2.4	0.0884	0.7375	1.1800	0.8835	24.0192	0.6123

Table A.6. Capacitor Values for Equal Resistor Chebyshev 0.5 db Ripple Networks

Order	Gain	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
2	1	0.5788	1.7275			
	2	1.0535	0.9492			
	3	1.3305	0.7516			
	4	1.5479	0.6460			
	5	1.7330	0.5770			
3	1	0.0681	8.5676	1.7136		
	2	0.5483	0.8891	2.0512		
	3	0.7320	0.6202	2.2026		
	4	0.8645	0.4990	2.3183		
	5	0.9712	0.4263	2.4150		
4	2	0.3281	0.8135	2.4043	1.5584	
	3	0.4588	0.5383	3.0750	1.3167	
	4	0.5495	0.4208	3.6386	1.1888	
	5	0.6204	0.3521	4.1428	1.1050	
	6					
5	2	0.2136	0.6461	1.5477	1.5666	2.9883
	3	0.3072	0.4118	1.9998	1.1638	3.3966
	4	0.3682	0.3133	2.3555	0.9843	3.7392
	5	0.4135	0.2564	2.6616	0.8764	4.0436
	6					

Table A.7. Resistor Values for Equal Capacitor Chebyshev 0.5 db Ripple Networks

Order	Gain	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>
2	2	1.1578	0.8637			
	3	1.7344	0.5766			
	4	2.1070	0.4746			
	5	2.4051	0.4158			
	2.0	0.2398	2.4854	1.6781		
3	2.2	0.3388	1.2859	2.2954		
	2.4	0.3483	0.8938	3.2118		
	2.6	0.3131	0.6419	4.9748		
	2.8	0.2336	0.4232	10.1141		
	2.2	0.1670	1.7439	3.3868	1.0141	
4	2.4	0.2105	1.0431	5.0773	0.8970	
	2.6	0.2125	0.6921	8.2762	0.8214	
	2.8	0.1743	0.4257	17.5762	0.7668	
	2.2	0.0714	1.5687	1.1796	1.5637	4.8436
5	2.4	0.0791	0.7048	1.2301	0.9168	15.9009

Table A.8. Capacitor Values for Equal Resistor Chebyshev 1.0 db Ripple Networks

Order	Gain	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$
2	1	0.5284	1.8927			
	2	1.0190	0.9813			
	3	1.2985	0.7701			
	4	1.5171	0.6592			
	5	1.7029	0.5872			
3	1	0.0462	11.6895	1.8503		
	2	0.5176	0.8798	2.1958		
	3	0.6969	0.6118	2.3454		
	4	0.8269	0.4918	2.4591		
	5	0.9320	0.4201	2.5538		
4	2	0.3116	0.8028	2.3922	1.6711	
	3	0.4401	0.5299	3.0783	1.3930	
	4	0.5295	0.4139	3.6529	1.2492	
	5	0.5996	0.3463	4.1662	1.1561	
	6					
5	2	0.2017	0.6248	1.5107	1.6044	3.2735
	3	0.2925	0.3988	1.9672	1.1856	3.6751
	4	0.3524	0.3039	2.3274	1.0005	4.0113
	5	0.3970	0.2491	2.6377	0.8896	4.3100

Table A.9. Resistor Values for Equal Capacitor Chebyshev 1.0 db Ripple Networks

Order	Gain	R <sub>1</sub>	R <sub>2</sub>	R <sub>3</sub>	R <sub>4</sub>	R <sub>5</sub>
2	2	1.0567	0.9463			
	3	1.6594	0.6026			
	4	2.2038	0.4907			
	5	2.3392	0.4275			
	2.0	0.1936	2.8776	1.7954		
3	2.2	0.3065	1.3384	2.4377		
	2.4	0.3239	0.9126	3.3836		
	2.6	0.2963	0.6484	5.2042		
	2.8	0.2245	0.4236	10.5172		
	2.2	0.1528	1.7781	3.2575	1.1299	
4	2.4	0.1976	1.0437	4.9297	0.9835	
	2.6	0.2022	0.6863	8.0784	0.8921	
	2.8	0.1674	0.4194	17.2160	0.8274	
	2.2	0.0663	1.5408	1.1517	1.6517	5.1547
5	2.4	0.0756	0.6913	1.2236	0.9402	16.6267

## APPENDIX B

### ACTIVE AND PASSIVE ELEMENT SENSITIVITIES

The sensitivities given in these tables were calculated using the sensitivity definitions developed in the text.

Table B.1. Resistor Sensitivities for Equal Resistor Thomson Networks, Orders 2 - 5

Order	Gain	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$	$S_{R_5}$
2	1	0.50	0.50			
	2	0.73	0.45			
	3	0.88	0.53			
	4	1.00	0.62			
	5	1.10	0.71			
3	1	0.60	0.38	0.71		
	2	0.62	0.42	0.70		
	3	0.69	0.42	0.71		
	4	0.77	0.41	0.71		
	5	0.84	0.39	0.72		
4	2	0.93	0.36	0.64	0.44	
	3	1.19	0.61	0.78	0.57	
	4	1.46	0.92	0.91	0.71	
	5	1.72	1.21	1.02	0.86	
	6					
5	2	1.16	-0.64	0.62	0.57	0.63
	3	1.58	-1.12	0.76	0.86	0.69
	4	2.03	-1.64	0.93	1.14	0.69
	5	2.51	-2.20	1.11	1.41	0.71
	6					

Table B.2. Capacitor and Gain Sensitivities for Equal Resistor Thomson Networks, Orders 2 - 5

Order	Gain	$S_{C_1}$	$S_{C_2}$	$S_{C_3}$	$S_{C_4}$	$S_{C_5}$	$S_K$
2	1	1.00	0.62				-0.66
	2	1.46	0.99				-0.91
	3	1.76	1.24				+1.14
	4	1.99	1.43				-1.33
	5	2.21	1.60				-1.51
3	1	0.53	0.62	0.56			-1.63
	2	0.92	1.02	0.71			-1.24
	3	1.12	1.17	0.79			-1.36
	4	1.28	1.25	0.85			-1.49
	5	1.42	1.31	0.90			-1.61
4	2	1.16	0.96	-1.50	0.97		-2.18
	3	1.70	1.71	-1.99	1.33		-2.47
	4	2.21	1.62	-2.41	1.67		-2.78
	5	2.72	2.08	-2.81	2.00		-3.09
	6						
5	2	1.49	-1.13	-1.71	1.47	0.59	-2.55
	3	2.28	-1.89	-2.28	2.18	0.72	-2.86
	4	3.12	-2.70	-2.81	2.86	0.87	-3.39
	5	4.01	-3.58	-3.31	3.51	0.92	-4.03

Table B.3. Resistor Sensitivities for Equal Resistor Thomson Networks, Orders 6, 7

Order	Gain	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$	$S_{R_5}$	$S_{R_6}$	$S_{R_7}$
6	1.6	1.25	-1.10	1.08	0.91	0.52	0.37	
	2.0	1.58	-1.48	1.21	1.14	0.57	0.38	
	3.0	2.44	-2.57	1.68	1.71	-0.70	0.49	
	4.0	3.38	-3.81	2.21	2.31	-0.87	0.62	
	5.0	4.42	-5.17	2.79	2.94	-1.03	0.77	
7	1.6	1.29	-2.02	1.73	0.71	-5.90	0.39	0.57
	2.0	1.68	-2.54	1.95	0.66	-0.71	0.51	0.59
	3.0	2.69	-4.05	2.91	-1.29	-1.08	0.89	0.63
	4.0	3.81	-5.72	4.19	-2.31	-1.48	1.34	0.67
	5.0	5.04	-7.49	5.73	-3.60	-1.91	1.84	0.69

Table B.4. Capacitor and Gain Sensitivities for Equal Resistor Thomson Networks, Orders 6, 7

Order	Gain	$S_{C_1}$	$S_{C_2}$	$S_{C_3}$	$S_{C_4}$	$S_{C_5}$	$S_{C_6}$	$S_{C_7}$	$S_K$
6	1.6	1.81	-1.91	1.54	2.03	-1.20	0.70		-4.49
	2.0	2.38	-2.54	1.91	2.61	-1.46	0.84		-4.49
	3.0	3.95	-4.24	2.97	4.03	-2.06	1.18		-5.61
	4.0	5.71	-6.11	4.14	5.47	-2.60	1.51		-7.20
	5.0	7.66	-8.16	5.40	6.94	-3.11	1.83		-9.01
7	1.6	2.43	-3.50	2.59	2.37	-2.05	1.10	0.44	-7.82
	2.0	3.31	-4.51	3.18	2.68	-2.56	1.36	0.51	-7.64
	3.0	5.73	-7.78	5.24	3.00	-3.92	2.17	0.64	-10.10
	4.0	8.48	-11.70	7.88	-3.95	-5.36	3.07	0.75	-14.00
	5.0	11.50	-15.90	11.00	-6.01	-6.84	4.07	0.85	-18.80

Table B.5. Resistor Sensitivities for Equal Capacitor Thomson Networks

Order	Gain	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$	$S_{R_5}$
2	2	1.00	0.62			
	3	1.26	0.83			
	4	1.46	0.99			
	5	1.62	1.12			
	2.0	0.71	0.40	0.80		
3	2.2	0.87	0.36	0.88		
	2.4	1.08	0.25	0.94		
	2.6	1.38	0.49	0.98		
	2.8	1.99	1.06	1.02		
	2.0	0.69	0.48	0.92	0.56	
4	2.2	0.92	0.35	1.05	0.62	
	2.4	1.20	0.47	1.50	0.68	
	2.6	1.62	0.87	1.23	0.74	
	2.8	2.44	1.69	1.30	0.80	
	2.0	0.65	0.26	0.95	0.71	0.79
5	2.2	1.29	1.09	1.61	1.06	0.92
	2.4	2.68	3.44	4.09	2.20	1.62

Table B.6. Capacitor and Gain Sensitivities for Equal Capacitor Thomson Networks

Order Order	Gain	$S_{C_1}$	$S_{C_2}$	$S_{C_3}$	$S_{C_4}$	$S_{C_5}$	$S_K$
2	2	1.33	0.89				-0.67
	3	1.53	1.05				-0.79
	4	1.69	1.18				-0.91
	5	1.82	1.29				-1.03
3	2.0	1.42	1.69	0.80			-2.45
	2.2	1.61	2.21	1.10			-3.13
	2.4	1.98	3.13	1.56			-4.36
	2.6	2.80	5.06	2.73			-6.96
	2.8	5.32	11.10	6.17			-15.90
4	2.0	1.87	1.79	-1.13	0.73		-4.00
	2.2	1.99	2.57	-1.61	0.71		-4.57
	2.4	2.47	3.93	-2.49	0.77		-6.12
	2.6	3.54	6.71	-4.18	0.80		-10.00
	2.8	6.85	15.40	-9.00	0.83		-23.00

Table B.7. Resistor Sensitivities for Equal Resistor Butterworth Networks

Order	Gain	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$	$S_{R_5}$
2	1	-0.48	-0.48			
	2	-0.79	-0.17			
	3	-0.98	-0.21			
	4	-1.13	-0.32			
	5	-1.26	-0.43			
3	1	-0.40	0.20	-0.94		
	2	-0.95	0.55	-0.95		
	3	-1.21	0.80	-0.97		
	4	-1.43	1.01	-0.99		
	5	-1.61	1.21	-1.00		
4	1.2	-0.75	1.23	-1.20	-1.09	
	2	-1.70	2.21	-1.40	-1.13	
	3	-2.61	3.30	-1.68	-1.19	
	4	-3.44	4.32	-1.95	-1.25	
	5	-4.24	5.29	-2.20	-1.31	
5	1.2	-1.07	3.13	-2.73	-1.01	-1.01
	2	-2.39	4.75	-2.99	-0.85	-1.09
	3	-3.75	6.68	-3.84	-1.19	-1.17
	4	-5.04	8.53	-4.77	-1.87	-1.23
	5	-6.31	10.30	-5.74	-2.61	-1.28

Table B.8. Capacitor and Gain Sensitivities for Equal Resistor Butterworth Networks

Order	Gain	$S_{C_1}$	$S_{C_2}$	$S_{C_3}$	$S_{C_4}$	$S_{C_5}$	$S_K$
2	1	-0.98	0.21				1.00
	2	-1.59	0.73				1.24
	3	-1.97	1.08				1.50
	4	-2.28	1.37				1.74
	5	-2.53	1.62				1.95
3	1	-0.75	0.35	-0.60			6.16
	2	-2.18	1.69	-0.85			3.24
	3	-2.88	2.48	-0.99			3.60
	4	-3.41	3.08	-1.09			4.01
	5	-3.87	3.60	-1.17			4.41
4	1,2	-1.66	2.04	-2.03	0.74		10.90
	2	-3.82	4.36	-2.80	0.83		8.68
	3	-5.78	6.61	-3.54	1.30		9.97
	4	-7.54	8.64	-4.17	1.74		11.60
	5	-9.21	10.60	-4.74	2.15		13.30
5	1,2	-2.37	5.42	-4.59	1.63	-0.57	31.00
	2	-6.11	9.41	-5.97	2.07	-0.82	19.30
	3	-9.47	13.90	-8.06	3.50	-1.05	22.00
	4	-12.60	18.20	-10.20	4.98	-1.25	26.20
	5	-15.50	22.30	-12.30	6.50	-1.41	30.80

Table B.9. Resistor Sensitivities for Equal Capacitor Butterworth Networks

Order	Gain	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$	$S_{R_5}$
2	1.4	-0.45	0.46			
	2	-0.98	0.21			
	3	-1.34	0.50			
	4	-1.59	0.73			
	5	-1.80	0.91			
3	2.0	-0.66	0.35	-0.96		
	2.2	-0.88	0.63	-1.14		
	2.4	-1.10	0.97	-1.28		
	2.6	-1.42	1.40	-1.39		
	2.8	-2.07	2.14	-1.47		
4	2.0	-0.51	0.75	-1.51	-0.82	
	2.2	-1.22	1.58	-1.77	-0.75	
	2.4	-1.98	2.39	-1.95	-0.71	
	2.6	-2.97	3.48	-2.09	-0.68	
	2.8	-4.87	5.48	-2.21	0.66	
5	2.1	-1.19	4.19	-4.71	-1.09	-1.40
	2.2	-1.66	6.09	-6.16	-1.80	-1.56
	2.4	-6.23	15.50	-15.10	-6.66	-1.82

Table B.10. Capacitor and Gain Sensitivities for Equal Capacitor Butterworth Networks

Order	Gain	$S_{C_1}$	$S_{C_2}$	$S_{C_3}$	$S_{C_4}$	$S_{C_5}$	$S_K$
2	1.4	-1.28	0.45				1.16
	2	-1.48	0.62				1.00
	3	-1.71	0.83				1.10
	4	-1.90	1.01				1.24
	5	-2.07	1.17				1.37
3	2.0	-3.33	2.76	-0.73			5.42
	2.2	-3.59	3.35	-1.15			6.08
	2.4	-4.26	4.56	-1.79			7.84
	2.6	-5.65	7.08	-3.02			11.60
	2.8	-9.54	14.70	-6.62			23.30
4	2.0	-12.50	12.70	-1.81	0.66		25.50
	2.2	-8.28	8.67	-2.55	0.62		16.30
	2.4	-9.13	10.50	-3.73	0.65		18.70
	2.6	-11.70	15.30	-6.06	0.68		26.00
	2.8	-19.40	30.70	-12.90	0.72		50.10
5	2.1	-37.50	44.50	-7.10	3.84	-1.37	88.20
	2.2	-34.80	43.10	-10.00	5.18	-2.11	82.70
	2.4	-63.40	95.01	-30.00	16.00	-8.11	184.00

Table B.11. Resistor Sensitivities for Equal Resistor Chebyshev (0.5 db Ripple) Networks

Order	Gain	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$
2	1	-0.19	-0.19		
	2	-0.55	0.21		
	3	-0.75	0.38		
	4	-0.91	0.53		
	5	-1.05	0.67		
3	1	0.54	0.60	-1.08	
	2	-1.08	1.40	-1.10	
	3	-1.55	1.87	-1.09	
	4	-1.92	2.25	-1.12	
	5	-2.24	2.58	-1.13	
4	1,2	-3.80	4.17	-2.30	-1.74
	2	-6.12	6.59	-2.49	-2.08
	3	-8.34	9.24	-2.88	-2.50
	4	-10.30	11.70	-3.26	-2.91

Table B.12. Capacitor and Gain Sensitivities for Equal Resistor Chebyshev  
(0.5 db Ripple) Networks

Order	Gain	$S_{C_1}$	$S_{C_2}$	$S_{C_3}$	$S_{C_4}$	$S_K$
2	1	-0.62	0.27			1.29
	2	-1.33	0.94			1.41
	3	-1.74	1.36			1.68
	4	-2.06	1.36			1.92
	5	-2.34	1.96			2.15
3	1	0.43	0.60	-0.83		25.40
	2	-2.08	3.04	-1.15		5.63
	3	-3.93	4.33	-1.31		6.11
	4	-4.80	5.34	-1.43		6.74
	5	-5.54	6.20	-1.53		7.36
4	1,2	-5.16	5.68	-3.40	-2.11	32.60
	2	-9.77	10.80	-4.14	-2.79	20.60
	3	-14.30	15.90	-5.00	-3.64	22.60
	4	-18.40	20.70	-5.76	-4.46	25.90

Table B.13. Resistor Sensitivities for Equal Capacitor Chebyshev (0.5 db Ripple) Networks

Order	Gain	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$
2	2	-0.62	0.27		
	3	-1.85	0.67		
	4	-1.33	0.94		
	5	-1.55	1.17		
3	2.0	0.40	0.69	-0.99	
	2.2	-0.85	1.29	-1.24	
	2.4	-1.27	1.88	-1.40	
	2.6	-1.83	2.57	-1.54	
	2.8	-2.92	3.77	-1.64	
4	2.1	-3.25	3.59	-2.38	-2.05
	2.2	-4.33	4.53	-2.45	-2.08
	2.4	-6.39	6.50	-2.57	-2.15
	2.6	-9.09	9.24	-2.68	-2.23

Table B.14. Capacitor and Gain Sensitivities for Equal Capacitor Chebyshev  
(0.5 db Ripple) Networks

Order	Gain	$S_{C_1}$	$S_{C_2}$	$S_{C_3}$	$S_{C_4}$	$S_K$
2	2	-1.26	0.88			1.29
	3	-1.48	1.10			1.29
	4	-1.68	1.30			1.41
	5	-1.86	1.48			1.55
	2.0	-6.74	6.72	0.81		13.10
3	2.2	-5.89	6.17	-1.34		11.30
	2.4	-6.67	7.58	-2.12		13.20
	2.6	-8.64	10.90	-3.65		18.20
	2.8	-14.30	21.00	-8.14		33.90
	2.1	-29.00	31.50	-3.13	-2.37	61.20
4	2.2	-23.00	25.10	-3.67	-2.32	47.00
	2.4	-21.90	24.90	-5.32	-2.29	44.40
	2.6	-25.70	32.10	-8.70	-2.30	55.30

Table B.15. Resistor Sensitivities for Equal Resistor Chebyshev (1.0 db Ripple) Networks

Order	Gain	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$
2	1	-0.35	-0.35		
	2	-0.80	0.29		
	3	-1.06	0.48		
	4	-1.26	0.64		
	5	-1.44	0.79		
3	1	0.65	0.76	-1.10	
	2	-1.90	1.75	-1.14	
	3	-2.54	2.72	-1.17	
	4	-3.05	2.81	-1.19	
	5	-3.50	3.25	-1.21	
4	1.2	-3.65	5.30	-2.62	-1.91
	2	-6.64	8.14	-2.75	-2.33
	3	-9.51	11.30	-3.14	-2.85
	4	-12.10	14.30	-3.52	-3.34

Table B.16. Capacitor and Gain Sensitivities for Equal Resistor Chebyshev  
(1.0 db Ripple) Networks

Order	Gain	$S_{C_1}$	$S_{C_2}$	$S_{C_3}$	$S_{C_4}$	$S_K$
2	1	-0.84	0.31			1.75
	2	-1.74	1.08			1.82
	3	-2.26	1.57			2.14
	4	-2.66	1.97			2.44
	5	-3.00	2.31			2.72
3	1	-0.71	0.70	-0.88		42.10
	2	-3.88	3.55	-1.22		6.84
	3	-5.33	5.05	-1.37		7.47
	4	-6.48	6.28	-1.49		8.29
	5	-7.46	7.32	-1.60		9.10
4	1.2	-5.48	7.03	-3.76	-2.42	41.50
	2	-11.50	13.00	-4.40	-3.29	24.80
	3	-17.10	19.10	-5.25	-4.35	26.90
	4	-22.00	24.80	-6.04	-5.36	30.70

Table B.17. Resistor Sensitivities for Equal Capacitor Chebyshev (1.0 db Ripple) Networks

Order	Gain	$S_{R_1}$	$S_{R_2}$	$S_{R_3}$	$S_{R_4}$
2	2	-0.84	0.31		
	3	-1.40	0.76		
	4	-1.74	1.08		
	5	-2.02	1.34		
3	2.0	-0.74	0.84	-1.01	
	2.2	-1.60	1.59	-1.27	
	2.4	-2.32	2.33	-1.46	
	2.6	-3.27	3.39	-1.61	
	2.8	-5.10	5.37	-1.74	
4	2.1	-2.92	4.48	-2.58	-2.36
	2.2	-4.26	5.62	-2.62	-2.40
	2.4	-5.81	7.99	-2.70	-2.50
	2.6	-10.10	11.30	-2.79	-2.60

Table B.18. Capacitor and Gain Sensitivities for Equal Capacitor Chebyshev  
(1.0 db Ripple) Networks

Order	Gain	$S_{C_1}$	$S_{C_2}$	$S_{C_3}$	$S_{C_4}$	$S_K$
2	2	-1.71	1.05			1.75
	3	-1.95	1.27			1.67
	4	-2.19	1.51			1.82
	5	-2.41	1.72			1.98
3	2.0	-9.70	9.13	-0.85		18.50
	2.2	-7.75	7.49	-1.40		14.00
	2.4	-8.55	8.92	-2.23		15.80
	2.6	-10.70	12.60	-3.87		21.20
	2.8	-16.80	23.80	-8.67		38.60
4	2.1	-37.70	41.60	-3.38	-2.86	82.00
	2.2	-28.80	31.80	-3.92	-2.77	59.60
	2.4	-26.50	30.20	-5.63	-2.71	23.80
	2.6	-30.50	37.80	-9.19	-2.72	65.30

#### LIST OF REFERENCES

- Huelsman, L. P. GOSPEL, Engineering Experiment Station, University of Arizona, September 1968.
- "An Equal-Valued Capacitor Active RC Network Realization of a Third Order Low-Pass Butterworth Characteristic," Elec. Ltrs., Vol. 7, No. 10, May 1971.
- Kerwin, W. J. "Active Network Synthesis with Voltage Amplifiers," Int'l. IEEE Conf. on Systems, Networks and Computers, Daxtepec, Mexico, January 1971.
- Sallen, R. P., and E. L. Key. "A Practical Method of Designing RC Active Filters," IRE Trans. on Circuit Theory, Vol. Ct-2, No. 1, March 1955.

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