ANALOG COMPUTER SIMULATION OF THE DYNAMIC BEHAVIOR OF A FOURDRINIER PAPER MACHINE

by

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A Thesis Submitted to the Faculty of the DEPARTMENT OF CHEMICAL ENGINEERING In Partial Fulfillment of the Requirements For the Degree of MASTER OF SCIENCE In the Graduate College THE UNIVERSITY OF ARIZONA

1965
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ACKNOWLEDGMENTS

The author is indebted to the entire faculty of the Department of Chemical Engineering for their encouragement and assistance not only in preparing this thesis, but in other academic work as well. Special appreciation is extended to Dr. Edward J. Freeh who, as director of this research, gave invaluable assistance in all phases of the project. The critical comments and helpful suggestions of Dr. Richard M. Edwards were also appreciated. Financial assistance in the form of a National Science Foundation Cooperative Graduate Fellowship made this year of study possible, and is greatly appreciated.
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ABSTRACT

Mathematical models of the wet and dry ends of a fourdrinier paper machine were developed for implementation on a PACE 231R analog computer. A lumped-parameter approach was used in modeling the wire and the dryer train.

Feedback control of the wet end was simulated. The controlled variables were headbox consistency, wire speed, and basis weight and moisture at the couch. A penalty function was used to represent the relative cost attributable to the deviation of each of the controlled variables from their set point. The effect of sinusoidal variation in feed stock consistency and in fiber retention on the wire was studied. The controller settings were adjusted in each case to obtain a small value for the penalty function. The settings obtained were different for each disturbance, demonstrating that optimal control settings are dependent upon the nature of the disturbance to the system. The response of the dryer, without feedback control, to a variation of basis weight and moisture of the wet sheet and to variation of steam temperature is presented.

The models presented satisfactorily represent the dynamic variations in basis weight and moisture in a fourdrinier paper machine and should be suitable for future work involving computer control.
INTRODUCTION

History

There are two basic continuous papermaking systems in use today. Although they have been modified and improved considerably, they are basically the same as they were when invented in the early 1800's. The two different machines for implementing these systems are known as the cylinder machine and the fourdrinier machine. Both these machines have advantages and disadvantages and consequently the machine to be used will depend on the desired properties of the paper as well as the quality of paper to be produced.

The cylinder machine was invented in 1807 by Charles Kinsey. It is composed of a large porous cylinder, which is partially submerged in a tank containing fibers suspended in water. The cylinder is rotated and a layer of fibers, called the web, is formed on the cylinder surface as water flows through the porous walls of the cylinder. The web is removed from the cylinder and dried. This method is used for the production of rough, heavy paper and multi-layered board (Stephenson, 1953).

The fourdrinier paper machine was invented by L. Robert in 1799. Two brothers, Henry and Sealy Fourdrinier, purchased the patent rights in 1804 and promoted the use and
further development of the process, which now bears their names (Libbey, 1962).

**The Fourdrinier Machine: General**

On the fourdrinier machine the web is formed on a moving wire belt. The water-fiber slurry is discharged onto a belt formed of finely woven wire. Over 95% of the water in the slurry goes through the wire, along with up to 40% of the fiber. The web is removed from the wire and dried to produce the finished paper. Modern fourdrinier machines are used to produce all types of paper and light board. Paper up to 300 inches wide may be made at a rate of 2500 feet per minute or more on a high-speed fourdrinier.

The fourdrinier machine may be considered as composed of three major sections: the wet end, where web formation takes place; the press section for removal of water by mechanical means; and the dry end, which consists of final drying.

**The Fourdrinier Wet End**

Figure 1 shows a schematic drawing of a fourdrinier wet end. The machine chest serves as a storage tank for the fiber slurry and any special binders or additives. From the machine chest the slurry is pumped to the consistency regulator, where clear water is added as needed to maintain the consistency of the slurry. The consistency is usually defined
FIGURE 1.--Schematic Diagram of a Typical Four-drinier Wet End.
as the weight percentage of fiber. The consistency of the slurry leaving the consistency regulator is 1% to 5%. From the consistency regulator the slurry passes through the jordan, which is a device to subject the slurry to shear stresses, to break up any agglomerates of fiber and to obtain the desired fiber properties.

From the jordan the slurry, called the stock, passes through the stock valve to the fan pump where it is diluted to a consistency of 0.1% to 1.0% with water which is recycled from the wire pit. The diluted stock is screened to remove any large particles which might cause defects in the paper or damage to the wire.

The screened stock goes to a flow distributor which evenly spreads the stock the width of the headbox, which in large machines may be 200 inches or more. Proper flow distribution to the headbox is necessary to produce a sheet of paper which is uniform across the width of the sheet. Many different types of flow distributors have been developed and are being used today.

The purpose of the headbox is to apply sufficient pressure to the stock so that it contacts the wire at the proper speed. This pressure, or head, is maintained either by applying air pressure to a closed headbox, or by maintaining a large head of feed in an open headbox. Most modern high speed machines use a closed headbox and regulate the head pneumatically. A water shower in the headbox reduces foam and helps break up agglomerates of fiber.
The stock leaves the headbox through the slice, which is an adjustable slit along the bottom of the headbox. The slice opening is adjustable all along the width of the headbox to compensate for uneven flow into the headbox. The spouting velocity of the stock at the slice is dependent upon the slice opening and the headbox pressure. The desired spouting velocity is usually slightly less than the wire speed.

Web formation on the wire is a very complicated phenomenon and presently its control is more of an art than a science. The spouting velocity of the stock is made slightly less than the wire speed in order to cause a slight orientation of fibers in the machine direction. On some machines an alternating transverse motion, or "shaking" of the wire is used to promote uniform web formation.

Water removal on the wire is caused by gravitational forces to a slight degree, but mainly by mechanical means, especially on high-speed machines. Cylindrical rolls, called table rolls, support the wire for about half its length. As these rolls rotate they cause a suction on the wire which increases water drainage. Vacuum boxes, also called flat-boxes, are placed under the wire after the table rolls to further assist in water removal.

The wire "ends" at the couch roll. Here the web is removed from the wire and the uneven edges of the web are trimmed off. On some machines equipped with a vacuum couch
additional water is also removed at the couch. A water shower removes any fiber which adheres to the wire after the couch. The web is passed through the presses where additional water is mechanically squeezed out.

The wire pit receives most of the water and fiber which drain through the wire. This water-fiber mixture is known as white water and is used to dilute the feed stock at the fan pump. The excess white water flows to the couch pit. The couch pit receives water and fiber from the wire and the couch as well as the overflow from the wire pit. Material in the couch pit goes to the saveall where a final fiber recovery operation is performed before the water is discarded.

Table 1 below (Libbey, 1962, p. 193) shows typical values for the consistency at various points in the fourdrinier wet end.

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<td>End of table-roll section of wire</td>
<td>1.5-2.5</td>
</tr>
<tr>
<td>End of suction-box section of wire</td>
<td>10-18</td>
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<tr>
<td>After couch roll</td>
<td>18-25</td>
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The Fourdrinier Dry End

When the web leaves the presses it still contains about 70% water. Further removal of water by mechanical means would be uneconomical and might damage the web. The remainder of the water, therefore, must be removed by drying.

Figure 2 shows a schematic drawing of a typical fourdrinier dry end. The web is passed over steam heated cylindrical drums about 3 to 4 feet in diameter. The web is pressed tightly against the drum by a thick cloth mat called a felt. The pressure exerted by the felt increases the heat transfer coefficient between the web and the highly-polished drum surface.

The actual drying process has been considered by Nissan (1956) to take place in four stages at each drum. The first stage is the contact of the web with the drum before the felt contacts the web. During this stage very little drying takes place, but the web temperature rises. The second stage is the period when the web is covered by the felt. During this stage some water is absorbed into the felt, but very little drying takes place. The web temperature rises considerably during this stage. During the third stage the web is still in contact with the drum but no longer in contact with the felt. In this stage drying takes place at a fairly high rate. The fourth and final stage consists of the period when the web has been removed from the drum.
FIGURE 2.—Schematic Diagram of a Typical Four-Drinier Dry End.
surface but has not yet contacted the next drum. In this stage the drying rate is highest, since both sides of the heated web are exposed to circulating air. The number of drums which the web contacts depends on the size of the machine, but a large fourdrinier may have eighty or more drums.

The web temperature is usually about 100° F. when it leaves the press. The web temperature (averaged over the four drying stages) rises rapidly the first few drums, then remains nearly constant during the constant drying rate period. Constant rate drying continues until the critical moisture content is reached. Nissan (1956, p. 209) gives the critical moisture content as 0.4 pounds water per pound fiber, but this varies with different types and grades of paper. Once the critical moisture content is reached, the web temperature rises and approaches the surface temperature of the drums. The exit water to fiber ratio usually varies between 0.05 and 0.08. Figure 3 shows a typical plot of web moisture and web temperature as a function of drum number.

The contact heat transfer coefficient between the web and drum surface decreases as web moisture content decreases until the web moisture content reaches the critical moisture content. Further drying does not cause any appreciable decrease in the coefficient (Cirrito, 1964, p. 118).

When the web leaves the last drying drum it passes through calender rolls where final surface finishing is
FIGURE 3.—Typical Web Moisture and Temperature in a Fourdrinier Dry End.
performed. The finished paper is wound on large rolls many tons in weight. The machine rolls are re-wound and the paper cut to customer specifications.

**Purpose and Objectives**

There has recently been a great increase in interest in computer control of continuous papermaking operations. Except for one investigation (Beecher, 1963) all the work to this author's knowledge has dealt with steady-state operations only. Since the major control problems existing are involved with dynamic conditions, a definite need exists for information regarding the development and optimization of dynamic control systems in the paper industry.

In most conventional control systems basis weight at the machine reel is determined by scanning across the web with a beta gauge to obtain an average basis weight. When a deviation of basis weight from the desired value occurs the stock valve is moved in the direction and magnitude indicated by the deviation. After a period of time amounting to a minute or more the beta gauge again scans the web to obtain a new value for basis weight. If a deviation from the desired value still exists the above procedure is repeated.

Continuous on-line measurements of sheet moisture is not used to a great extent in the paper industry today. However, the recent introduction of radio-frequency based moisture gauges is making such measurement practical. The
output of these gauges is used to control the steam pressure in the dryer or auxiliary dryers to provide control of moisture at the reel. No control on either moisture or basis weight at the couch is used presently.

The task of a computer control system is not only to do a better job of basis weight and moisture control, but also to optimize quality factors such as tear strength, opacity, and curl. If a computer control system is to account for the influence of process dynamics, it is necessary to develop at least a simplified transfer function for the process. The development of the transfer function can be on a purely experimental basis or it can be developed from a consideration of the principles involved in the sequence of steps making up the overall process.

The purpose of the work presented here is to investigate via simulation the dynamic behavior of a fourdrinier paper machine and to make a preliminary investigation of a relatively simple, but unconventional, wet end control system. In addition, this work will serve as a foundation for the investigation of more sophisticated control schemes employing analog, digital or hybrid computers. In particular it is planned to use the models developed herein in the simulation of direct digital control of a fourdrinier paper machine on a hybrid computer facility currently being developed.
THE WET END MODEL

General

An attempt to model simultaneously all parts of the fourdrinier wet end in detail would be a nearly impossible task at this time. In fact, a detailed analysis of the wire alone would be beyond the scope of this paper, since a practical analytic approach to the deposition of fibers of varying length on the wire has not yet been developed. Several papers (Beecher, 1963; Brewster, 1964; McMahon, 1964) have dealt briefly with the problem of modeling various papermaking apparatus mathematically, but to this author's knowledge a model suitable for analog computer control of the fourdrinier wet end has not been published.

For the purpose of this investigation, the model must supply the following information: stock feed rate, headbox consistency, fan pump flow, and basis weight and moisture at the couch. Figure 4 shows the simplified schematic diagram of the fourdrinier wet end which was used for modeling. The various streams are represented by arrows in Figure 4 and are numbered for identification. These numbers will be used as subscripts to identify fiber and water flow rates in the appropriate streams. We will use the following symbols to represent flow rates:
FIGURE 4.—Schematic Diagram of the Wet End Model.
\[ W_i = \text{Mass flow rate of stream } i \]
\[
\text{(water and fiber), lb/sec}
\]

\[ C_i = \text{Consistency of stream } i, \]
\[
\text{lb fiber/lb slurry}
\]

The starting point for the model was chosen to be the stock flow to the fan pump, labeled stream 1 in Figure 4. Any previous point, such as the input to the jordan, could have been chosen, but this would have introduced additional computations without any real increase in the value of the model for control purposes.

Two "classes" of fiber were assumed to exist in the system; "normal" fibers which are fibers of average or larger size, and "fine" fibers which are much shorter. These two classes behave quite differently on the wire, since a much larger fraction of the fine fiber passes through the wire into the white water. While this division of fibers into two classes is somewhat arbitrary, the existence of a rather high level of fine fibers, or fines, in the white water is recognized in the paper industry. Thus the classification of fiber as either fine or normal is used to enable the model to resemble an operating fourdrinier more closely. "Fine" fiber consistencies will be denoted by a primed "C." For example, the consistency of stream 1 is \(C_1\) pounds "normal" fiber per pound stock and \(C'_1\) pounds fines per pound stock.
The Fan Pump

The fan pump is assumed to cause instantaneous and complete mixing of streams 1 and 9, resulting in stream 2. Plug flow (no mixing) is assumed in all lines. The stock flow $W_1$ is controlled by the stock valve in line 1. The consistencies $C_1$ and $C'_1$ are determined by the type of pulp, degree of refining, and control in the consistency regulator, all factors which are "outside" the proposed model.

The Headbox

Stream 2 is delayed 7 seconds, represented by the circle in Figure 4. This delay corresponds to the transport delay from the fan pump to the slice. The transport delay between the inlet to the headbox and the slice is relatively small and is included in the transport delay mentioned above. Plug flow through the headbox was assumed, although considering the small volume of a pneumatically regulated headbox, it is really immaterial whether plug flow or perfect mixing occurs.

The slice flow rate, $W_4$, is controlled by opening or closing the slice. It was assumed that the headbox was maintained at a constant head and that increasing $W_4$ would result in an instantaneous corresponding increase in fan pump flow $W_2$. That is,

$$W_4 = W_2 \quad (1)$$
Equation (1) assumes that the flow rate of the headbox showers is negligible compared with the total flow rates involved. This is certainly a justifiable assumption for most if not all operations.

Since no mixing occurs in any lines or in the headbox we can write the following equations for the amount of each class of fiber leaving the slice.

\[
\dot{w}_4 c_4 = \left\{ \dot{w}_1 c_1 \right\}_\tau \tag{2}
\]

\[
\dot{w}_4 c'_4 = \left\{ \dot{w}_1 c'_1 + \dot{w}_9 c'_9 \right\}_\tau \tag{3}
\]

Where a function in brackets \( \{ \}_\tau \) indicates the function is delayed \( \tau \) seconds.

A material balance around the fan pump gives:

\[
\dot{w}_9 = \dot{w}_2 - \dot{w}_1 \tag{4}
\]

and, using Equation (1) gives

\[
\dot{w}_9 = \dot{w}_4 - \dot{w}_1 \tag{5}
\]

We define the total headbox consistency, \( C_{4C} \), by the following equation.

\[
C_{4C} = \frac{\dot{w}_1 c_1 + \dot{w}_4 c'_4}{\dot{w}_4} \tag{6}
\]
The Wire

A lumped-parameter approach was used in modeling the wire. The wire was modeled by assuming all the fiber and water flowing into the wire pit and couch pit pass through the wire at the point where the stock first contacts the wire. The remaining fiber and water then experience a transport delay which corresponds to the time required for the wire to travel between the slice and the couch. This delay is on the order of three to five seconds while the fan pump-to-headbox delay is three or more times this figure. Because the wire delay is much smaller than the fan pump delay the wire delay was neglected in the formulation of the model. This delay does not affect the dynamics of the wet end as considered here at any rate, and thus can be safely omitted.

The fraction of "normal" fibers leaving the slice which passes through the wire was defined as $q_p$. This fraction is a function of several variables, the most important of which are degree of refining of the stock and stock height on the wire. Stock height on the wire is defined as the weight of fibers leaving the slice per square foot of wire. That is,

$$ S = \frac{W_4 C_{HG}}{Lv} $$

(7)
where

\[ S = \text{stock height on the wire} \]
\[ L = \text{wire width} \]
\[ v = \text{wire speed} \]

The actual functional relationship between \( Q_F \) and the aforementioned quantities was not known, but a relation of the following form was assumed:

\[ Q_{tp} = \frac{a_1}{S} + a_2R \]  

(8)

where \( a_1, a_2 = \text{constants} \)

\( R \) = Refining coefficient, which is a function of degree of refining, type of pulp, etc.

\( R \) was assumed constant for any given set of input conditions.

Equation (8) should probably have a third term, \( \frac{a_3}{C_{4C}} \), to be strictly correct, but since the headbox consistency was a controlled variable and thus was not allowed to change drastically, it was assumed that this term would be essentially constant and thus could be included in the last term of equation (8).

The fraction of fines leaving the slice which pass through the wire was defined as \( Q'_F \). This fraction was assumed constant for any given set of input conditions. To be strictly correct, \( Q'_F \) should exhibit a functional relationship to the same variables as \( Q_F \). The fraction \( Q'_F \), though, should be more sensitive to stock height on the wire than \( Q_F \), but for simplicity \( Q'_F \) was assumed constant.
Once fibers passed through the wire they were assumed to be fines. That is,

\begin{equation}
C_6, C_7, C_8, C_9, C_{10} = 0
\end{equation}

This is a simplifying assumption, but closely approximates the practical situation. All fiber remaining on the wire was treated as normal fiber. That is, \( C'_5 = 0 \).

From the definitions of \( \alpha_F \) and \( \alpha'_F \) we can write the following equation:

\begin{equation}
W_6 C'_6 + W_7 C'_7 = \alpha_F W_4 C_4 + \alpha'_F W_4 C'_4
\end{equation}

The fraction of water leaving the slice which is retained by the stock on the wire was defined as \( \alpha_W \). That is,

\begin{equation}
W_5 (1-C_5) = \alpha_W W_4 (1-C_4 C)
\end{equation}

and, if we assume \( C'_6 \) and \( C'_7 \) are very small compared to 1,

\begin{equation}
W_6 + W_7 = (1- \alpha_W) W_4 (1-C_4 C)
\end{equation}

The above equations neglect the wire shower flow, which is very small compared with the other flow rates involved. It was assumed that \( \alpha_W \) could be controlled by adjusting the flat box vacuum; thus no functional relationship between \( \alpha_W \) and other variables was necessary.
The Wire Pit

The wire pit was simulated by two well-stirred tanks, the first emptying into the second and the second emptying into the couch pit and the fan pump. A constant mass of white water was assumed in each of the two tanks. A fiber balance around each of the tanks gives the following differential equations for white water consistency.

$$M_1 \frac{dC_8'}{dt} = W_6C_6' - W_8C_8'$$ (13)

$$M_{II} \frac{dC_9'}{dt} = W_8C_8' - (W_9 + W_{10})C_9'$$ (14)

Where $M_1$, $M_{II}$ = Mass of white water in tanks 1 and 2, respectively. But, since the total mass in each of the tanks is constant,

$$W_8 = W_6$$ (15a)

$$W_9 + W_{10} = W_8$$ (15b)

Thus,

$$M_1 \frac{dC_8'}{dt} = W_6C_6' - W_6C_8'$$ (16a)

$$M_{II} \frac{dC_9'}{dt} = W_6C_8' - W_6C_9'$$ (16b)
The ratio of water and fiber flowing through the wire which goes directly into the couch pit to that which goes into the wire pit was defined as $\beta$. That is,

$$\beta = \frac{\dot{w}_7}{\dot{w}_6} = \frac{\dot{w}_7 c'_7}{\dot{w}_6 c'_6}$$  \hspace{1cm} (17)$$

Equation (17) assumes that $c'_7 = c'_6$ which is not necessarily correct, but it is believed that the error introduced is negligible. Bear in mind that $\beta$ may be regarded as a parameter that is adjusted to make the lumped parameter model agree with experimental material balances in an actual machine. Combining equation (10) and (17) gives the following equation for the fiber flowing into the first stage of the wire pit.

$$\dot{w}_6 c'_6 = \frac{\alpha'_f \dot{w}_4 c'_4 + \alpha'_f \dot{w}_4 c'_4}{1 + \beta}$$

Combining equations (12) and (17) gives the following equation for the white water flow into the wire pit.

$$\dot{w}_6 = \frac{(1 - \alpha_w) \dot{w}_4 (1 - c'_4 c)}{1 + \beta}$$  \hspace{1cm} (18)$$

The Couch and Presses

A fiber balance around the wire gives

$$\dot{w}_5 c'_5 = \dot{w}_4 c'_4 (1 + \beta) \dot{w}_6 c'_6$$  \hspace{1cm} (19)$$
Now combining equations (11) and (19) we obtain an expression for the total flow of water and fiber to the couch.

$$W_5 = \alpha W_4(1-C_4c) + W_4C_4c - (1 + \beta)W_6C_6 \quad \text{(20)}$$

The deckle is removed at the couch and the trimmed web (stream 11 in Figure 4) goes to the presses. The basis weight at the couch is defined as the weight of 1,000 square feet of wet paper. That is,

$$B = \frac{1000 \, W_5}{L \nu} \quad \text{(21)}$$

where

- \( B \) = basis weight
- \( L \) = trim width
- \( \nu \) = wire speed

The term moisture is defined as the ratio of pounds water to pounds fiber in the web. Thus, at the couch,

$$M = \frac{W_5(1-C_5)}{W_5C_5} \quad \text{(22)}$$

where \( M \) = sheet moisture \quad \text{(23)}

The fraction of the sheet at the couch which goes to the press section was defined as \( \alpha_C \), and is given by

$$\alpha_C = \frac{W_{11}C_{11}}{W_5C_5} = \frac{W_{11}(1-C_{11})}{W_5(1-C_5)} \quad \text{(24)}$$
The fraction of water in the web removed in the press section was defined as $\alpha_p$; essentially no fiber is removed in the presses. Thus

$$W_{12}(1-C_{12}) = (1- \alpha_p)W_{11}(1-C_{11})$$  \hspace{1cm} (25)

and,

$$W_{12}C_{12} = W_{11}C_{11}$$  \hspace{1cm} (26)

The transport delay between the couch and the end of the press section must be applied to $W_{12}$ and $C_{12}$, determined as above, to provide inputs to the dryer section.

**Computer Implementation**

An analog computer program was prepared for a PACE 231R analog computer using the previously derived equations to give basis weight and moisture at the couch, headbox consistency, fan pump flow and wire pit consistency for any set of input conditions. Since the basis weight and moisture after the presses is a constant fraction of those values at the couch, the model did not include the press in order to reduce the computer hardware needed.

The fan pump delay $\tau$ was simulated by a second-order $\text{Pade}$ circuit, which appears in Figure 21, Appendix A. The computer circuit diagram for the wet end is shown in Figure 22, Appendix A.
Feedback control circuits were simulated for the headbox consistency, basis weight at the couch, moisture at the couch, and wire speed. The controlled and controlling variables and modes of control are related in Table 2 below.

### TABLE 2.---Controlled and Controlling Variables for the Fourdrinier Wet End.

<table>
<thead>
<tr>
<th>Controlled Variable</th>
<th>Controlling Variable</th>
<th>Type of Control</th>
</tr>
</thead>
<tbody>
<tr>
<td>Headbox consistency</td>
<td>Slice opening</td>
<td>Proportional plus reset</td>
</tr>
<tr>
<td></td>
<td>(fan pump flow)</td>
<td></td>
</tr>
<tr>
<td>Basis weight at the couch</td>
<td>Stock feed rate</td>
<td>Proportional plus reset</td>
</tr>
<tr>
<td></td>
<td>Wire speed</td>
<td>Proportional only</td>
</tr>
<tr>
<td>Moisture at the couch</td>
<td>Flat box vacuum</td>
<td>Proportional plus reset</td>
</tr>
<tr>
<td></td>
<td>($\alpha_w$)</td>
<td></td>
</tr>
<tr>
<td>Wire speed</td>
<td>Wire speed</td>
<td>Reset only</td>
</tr>
</tbody>
</table>

The following equations relate the correction $\Delta$ to the controlling variable applied in response to a deviation $\epsilon$ of the controlled variable from the set point.

1. \[ \Delta w_4 = K_{C1} \epsilon_{c_{4C}} + K_{C1}^{RR1} \int \epsilon_{c_{4C}} dt \] (27)

2. \[ \Delta w_1 = -K_{C2} \epsilon_B - K_{C2}^{RR2} \int \epsilon_B dt \] (28)

3. \[ (\Delta v)_1 = K_{C3} \epsilon_B \] (29)
\[ \Delta \alpha_W = -K_{C4} \varepsilon_M + K_{C4}^R \int \varepsilon_M dt \]  
(30)

\[ (\Delta v)_2 = -RR \int \varepsilon_v dt \]  
(31)

where

\[ K_C = \text{Proportional gain} \]

\[ RR = \text{Reset rate} \]

\[ \varepsilon = \text{Set point} - \text{measured value} \]

\[ \Delta = \text{Change in value effected by the control system} \]

A penalty function was constructed to estimate the loss attributable to the deviation of controlled variables from their set points. A function of the following form was assumed.

\[ p = d_1 \varepsilon_M^2 + d_2 \varepsilon_B^2 + d_3 \varepsilon_{C_h C}^2 + d_4 \varepsilon_v^2 \]  
(32)

where

\[ \varepsilon = \frac{\varepsilon}{\text{Set Point}} \times 100\% \]

Controller gains and reset rates were varied to attempt to minimize the value of \( P \) where,

\[ P = \int_{t_1}^{t_2} p \, dt \]  
(33)
The values used for the constants in equation (32) are tabulated in Table 3 below. These values are based on the assumption that the following deviations from the set points are equivalent from a cost standpoint: basis weight, 3 lbs/1000 ft$^2$; sheet moisture, 0.25; headbox consistency, 0.1%; wire speed, 0.75 ft/sec. The set points for the above variables were as follows: basis weight, 225 lbs/1000 ft$^2$; sheet moisture, 4.00; headbox consistency, 0.5%; wire speed, 25 ft/sec.

**TABLE 3.--Penalty Function Parameters for the Wet End Model.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_1$</td>
<td>1.00</td>
</tr>
<tr>
<td>$d_2$</td>
<td>5.78</td>
</tr>
<tr>
<td>$d_3$</td>
<td>0.40</td>
</tr>
<tr>
<td>$d_4$</td>
<td>2.56</td>
</tr>
</tbody>
</table>

The integral P has arbitrary units and thus it is only significant to compare relative values of P for different controller gains. The integration in equation (33) was performed over one cycle of a periodic disturbance to the system.
**Nomenclature**

\( a_1, a_2 \) = constants in equation (8)

\( B \) = basis weight at the couch, lbs/1000 ft²

\( C \) = consistency, lbs fiber/lb slurry

\( d_1, d_2, d_3, d_4 \) = penalty function parameters

\( E \) = percent deviation from set point

\( K_c \) = proportional gain

\( M \) = moisture at the couch, lbs water/lb fiber 
   (not subscripted)

\( M \) = mass of white water, lb (subscripted)

\( p \) = penalty function

\( P \) = integrated penalty function

\( R \) = refining coefficient in equation (8), 
   dimensionless

\( RR \) = reset rate, repeats/minute

\( S \) = stock height on the wire, lbs/ft²

\( v \) = wire speed, ft/sec.

\( W \) = flow rate, lb/sec

\( \alpha_c \) = fraction of sheet at the couch which goes to 
   the press section, dimensionless

\( \alpha_F \) = fraction of normal fiber passing through the 
   wire, dimensionless

\( \alpha'F \) = fraction of fines passing through the wire, 
   dimensionless

\( \alpha_p \) = fraction of water removed at the press, 
   dimensionless

\( \alpha_W \) = fraction of water leaving the slice which 
   remains in the stock on the wire, dimensionless
\[ \beta = \text{fraction of water and fiber passing through the wire which goes directly into the couch pit, dimensionless} \]

\[ \Delta = \text{change in controlling variable effected by a deviation in the controlled variable} \]

\[ \epsilon = \text{deviation from set point (measured value - setpoint)} \]

Subscripts:

1, 2, 3, 4 = stream number in Figure 4

I, II = wire pit 1 and 2, respectively

B = basis weight

\( C_4C \) = headbox consistency

M = moisture

s = set point

v = wire speed
THE DRY END MODEL

General

A brief examination of the fourdrinier dry end was made. As in the wet end model, a lumped-parameter approach to modeling was used. A specific dryer, described by Cirrito (1962), was used as a basis for the model to obtain the necessary empirical data. The dryer train consisted of 82 five-foot diameter cast iron drums with a wall thickness of one and one quarter inches. Two lumped-parameter sections were used, the first consisting of the first 60 drums and the second consisting of the remaining 22 drums.

Figure 5 shows a schematic diagram of the dryer for modeling purposes. The circles represent transport delays of \( T \) seconds. The wet sheet from the presses is delayed \( T_1 \) seconds, representing the transport delay from the press to the center of the dryer Section I. The flow rates into Section I are \( W_{d1} \) pounds water per second and \( F_1 \) pounds solids/second. In Section I an amount of water \( E_1 \) pounds per second is removed by evaporation. The output of Section I is delayed \( T_2 \) seconds corresponding to the transport delay from the center of Section I to the center of Section II. In Section II \( E_2 \) pounds per second of water
FIGURE 5.--Schematic Diagram of the Dry End Model.
is removed from the sheet by evaporation. The output of Section II is delayed $\tau_3$ seconds, corresponding to the transport delay from the center of Section II to the reel.

This model obviously cannot be used to obtain a detailed moisture profile in the machine direction since only two lumped-parameter sections are modeled. If the information desired is simply output moisture, however, this simplified model can be expected to give reasonably accurate results. As should become obvious with the development of the model, this two-section approach may be easily extended to as many sections as desired to give a more accurate machine-direction moisture profile. It should be remembered, however, that computer hardware required for the solution of the model is almost directly proportional to the number of sections considered.

As has been previously mentioned, the drying process may be considered to take place in four different stages at each drum. Obviously a model which consists of only two lumped-parameter sections for the entire dryer train cannot and need not consider these four stages individually. The drying rate obtained from the model will give only an average drying rate for each section.
Heat Transfer Considerations

To obtain the average drying rate we consider the drying process to be represented by the schematic drawing in Figure 6. Since the heat supplied to the sheet for evaporation must be transferred through the drum-paper contact area, we will consider only this area in our model. The drum wall is planar in Figure 6, which is a very good assumption because of the large diameter to wall thickness ratio that exists.

The heat flux from the steam to the drum wall may be approximated in terms of steam temperature $T_S$ and midpoint wall temperature $T_m$ by the following equation:

$$q_{in} = U(T_S - T_m) \quad (34)$$

where $U$ is an overall heat transfer coefficient defined in terms of the inside film coefficient $h_i$, wall thickness $s$, and thermal conductivity $k$ as follows:

$$\frac{1}{U} = \frac{1}{h_i} + \frac{s}{2k} \quad (35)$$

Similarly, the heat flux out of the wall may be approximated by the equation:

$$q_{out} = \frac{2k}{s} (T_m - T_0) \quad (36)$$

where $T_0$ is the outside drum surface temperature. The
FIGURE 6.—Schematic Drawing of Dryer Drum Wall.
accumulation of heat within the wall is:

\[ Q_{acc} = M A C_p \frac{dT_m}{dt} \]  \hspace{1cm} (37)

where

\[ M = \text{mass of drum wall per unit heat transfer area} \]

\[ A = \text{heat transfer area} \]

\[ C_p = \text{heat capacity of drum wall} \]

Multiplying equations (34) and (36) by the heat transfer area \( A \) to obtain heat flows and combining them with equation (37) gives an approximate equation for the mid-point wall temperature, \( T_m \):

\[ M A C_p \frac{dT_m}{dt} = U A (T_S - T_m) - \frac{2k}{s} A(T_m - T_0) \] \hspace{1cm} (38)

But we can also calculate the heat flow out of the wall by the following equation:

\[ Q_{out} = h_0 A(T_0 - T_p) \] \hspace{1cm} (39)

where:

\[ h_0 = \text{contact film coefficient between the web and the drum surface} \]

\[ T_p = \text{psuedo paper temperature} \]

Combining equations (36) and (39) gives an implicit expression for the outside drum surface temperature, \( T_0 \):

\[ T_0 = T_m - \frac{sh_0}{2k} (T_0 - T_p) \] \hspace{1cm} (40)
Now, replacing the output term in equation (38) with equation (39) and dividing by the coefficient of the derivative, gives

\[
\frac{dT_m}{dt} = \frac{U}{MC_p} (T_S - T_m) - \frac{h_0}{MC_p} (T_0 - T_p)
\]  
(41)

Now, if \( T_p \) and \( h_0 \) are known, equations (40) and (41) may be used to calculate \( T_0 \). Then, equation (39) may be used to obtain the heat transferred to the paper. We will assume that some fraction \( R \) of the heat which flows out of the drum is effective in evaporating water from the sheet. The remaining heat represents heat losses by conduction and radiation from the drums and sheet, and sensible heat to the sheet.

**Correlation with Published Data**

Thus we have three parameters, \( T_p \), \( h_0 \), and \( R \) which are necessary to determine the drying rate \( E \). The contact coefficient \( h_0 \) was assumed to be a function only of the sheet moisture. Although felt pressure can affect \( h_0 \), for a given machine this variation may be assumed negligible. Cirrito (1964) calculated values for \( h_0 \) at several different moisture levels. These values are plotted in Figure 7, and were used for the model presented here.

Cirrito (1962) considered the dryer train in five sections. He calculated the average heat flux, \( q_{out} \), for each of the sections and measured the drum surface temperature
FIGURE 7.—Contact Heat Transfer Coefficient $h_0$. 
$T_0$ at each drum. With these values, and the values for $h_0$, equation (39) was used to calculate the pseudo paper temperature, $T_p$, at various sheet moisture contents.

The paper temperature $T_p$ is not actually uniform throughout the sheet as we have indicated in Figure 6. Nissan and Hansen (1960) showed experimentally that the actual sheet temperature varies through the thickness of the sheet as much as $20^\circ F$ at a moisture level of 0.60. The pseudo paper temperature $T_p$, then, corresponds most closely to the paper temperature at the point of contact with the drum. The lack of a definite physical meaning for $T_p$ does not detract from the ability of the model to relate dryer performance to drum steam pressure and input variations in basis weight and moisture.

To obtain the heat flux for a dry sheet, the drum surface temperature was noted at the end of the dryer where the sheet moisture was very low (0.08). Since the steam temperature was known, the heat flux was calculated from the equation:

$$q_{dry} = U' (T_S - T_0)$$

where

$$U' = \frac{1}{h_1} + \frac{s}{k}$$

The value $q_{dry}$ was used in equation (39) to give the sheet temperature at zero moisture.
An equation of the following form was used to define $T_p$:

$$T_p = b_0 + \gamma(T_0 - b_1)$$  \hspace{1cm} (44)

where

$b_0$, $b_1$ = constants

$\gamma$ = function of sheet moisture

Since $T_0$ is a function of steam temperature, equation (44) indicates that the sheet temperature is a function only of moisture and steam temperature. Actually, paper temperature is almost certainly a function of basis weight as well as moisture and steam temperature. Because of a lack of data with varying basis weight, however, the nature of the relation between basis weight and moisture could not be determined. For future work, equation (44) could be easily modified to include basis weight if further data become available.

It was assumed that at zero moisture in the sheet $\gamma$ would be 0.90 and at a moisture level greater than 1.00 (pounds water per pound dry fiber) $\gamma$ would be 0.10. No theoretical basis is claimed for equation (44) or the values for $\gamma$ defined above. The form and values, however, are reasonable and give results which are in good agreement with the limited experimental evidence at hand.
The constants $b_0$ and $b_1$ were calculated from equation (44) using the calculated paper temperature and defined values for $\gamma$ at a moisture level of 1.1 and for the dry sheet.

The values calculated were:

\[
\begin{align*}
  b_0 &= 201 \\
  b_1 &= 234
\end{align*}
\]

Calculated values for $T_p$ were used to calculate $\gamma$ at other moisture levels. That is, $\gamma$ may be viewed as an arbitrary function employed to correlate $T_p$ with moisture such that calculated values of heat flux agree with experimentally observed heat transfer rates.

The fraction $R$ of the heat transferred which is effective in evaporation was calculated from the data of Cirrito (1962), and the following defining equation for $R$ for each of five sections of the dryer.

\[
R \ q_{\text{out}} = q_{\text{eff}}
\]  

These values appear in Table 10, Appendix B, and range from 0.23 at the first few drums to 0.96 in the fourth section. The low value for the first drums is explained by the sensible heat required to raise the temperature of the wet sheet to the point where significant evaporation occurred.

We now may write an expression for the drying rate, $E$, as follows:
The heat transfer area, $A$, is

$$A = n \rho \pi D L$$  \hspace{1cm} (47)$$

where $n =$ number of drums

$\rho =$ fraction of total drum surface contacting sheet.

Values for $R$ for each of the model sections were calculated as follows: The input and output moisture level and flow rates were used to calculate the water removal in each section. Assuming a constant value for $\Delta H$, $Q_{eff}$ was calculated for each section. Values for $T_0$ and $h_0$ evaluated at the average moisture content in each section were used to calculate the total heat flow $Q_{out}$. The ratio $Q_{eff}/Q_{out}$, or $R$, was then obtained. For Section I, $R$ was 0.91. For Section II, however, $R$ was slightly over 1.0, which is impossible. This indicated an error in $T_p$, hence $\gamma$, since $h_0$ is essentially constant in the second section moisture range. A value of 0.90 was assumed for $R$ and the proper $T_p$ and $\gamma$ were calculated. The above value for $\gamma$ along with the previous values calculated are listed in Table 11, Appendix B.
Since R must equal zero when the sheet moisture is zero, Figure 8 was prepared showing R as a function of sheet moisture. Strictly speaking, R should probably be a function of basis weight also, but for the reasons explained earlier in discussing $\gamma$, R was assumed a function of sheet moisture alone.

**Computer Implementation**

A PACE 231R analog computer was used to implement the model. A fourth-order Padé approximation was used to simulate the transport delay $\tau_2$ between the dryer sections. The delays $\tau_1$ and $\tau_3$ were produced by the application of a bias to the output equipment, although if control were to be added to this model, other methods for producing an actual lag, such as the Padé circuit, would have to be used. Since variations in basis weight were assumed to have negligible effect on the drying rate, no sheet speed term appears in the model. Thus the velocity may be assumed constant, and the time delays remain constant. If the effect of basis weight, hence wire speed, is added to the model, the transport delays would, of course, have to be dependent upon sheet velocity.

Variable diode function generators were used to generate $\gamma$, $h_0$, and R as functions of average moisture for Section II. Since the average moisture in Section I
FIGURE 8.--Heat Effectiveness, R, as a Function of Sheet Moisture.
remained greater than 1.0 at all times, \( \gamma \) was 0.1 and \( R \) was assumed to be 0.91 regardless of moisture level.

The lumped-parameter approach to the model presents a problem of evaluation of the quantities \( \gamma \), \( R \), and \( h_0 \), since the average values of these variables in each section must be used. Since in general the average value of a function is not the same as the function evaluated at the average value of the independent variable, we are not strictly correct if we evaluate \( \gamma \), \( R \) and \( h_0 \) at the average moisture content for each section. The values which appear in Figures 7, 8, and Table 11, Appendix B, however, were calculated from data which were averaged over a range of moisture level, and thus represent average values to a certain degree. In addition, the curves for \( \gamma \) and \( h_0 \) are not exceedingly nonlinear and thus minimize the averaging error. With the above considerations in mind, it was decided that sufficient accuracy could be obtained by evaluating \( \gamma \), \( h_0 \) and \( R \) at the average moisture content for the section.

A definite improvement would result with the addition of more lumped parameter sections to the model, since the moisture variation would be smaller within each section and the averaging error would accordingly be less.

Figure 23, Appendix A, shows the computer program used for a fourth-order Pade circuit used to simulate the transport delay \( T_2 \). Two such circuits are necessary
between each pair of dryer sections. Figure 24, Appendix A, shows the computer program used for Section II of the model. The program for Section I is similar except that the quantities $\gamma$ and $R$ are constant and multiplication by these quantities is performed with hand-set potentiometers.
Nomenclature

\( A = \) heat transfer area, \( \text{ft}^2 \)

\( b_0, b_1 = \) constants in equation (44), \( ^\circ\text{F} \)

\( C_p = \) drum heat capacity, Btu/lb-\( ^\circ\text{F} \)

\( D = \) drum diameter, ft

\( E = \) evaporation rate, lb/sec

\( F = \) fiber flow rate, lb/sec

\( h_i = \) inside drum surface condensing film heat transfer coefficient, Btu/sec-ft\(^2\)-\( ^\circ\text{F} \)

\( h_0 = \) contact heat transfer coefficient between drum surface and sheet, Btu/sec-ft\(^2\)-\( ^\circ\text{F} \)

\( \Delta H = \) latent heat of vaporization of water, Btu/lb

\( k = \) thermal conductivity of drum, Btu/sec-ft\(^2\)-\( ^\circ\text{F}/\text{in} \)

\( L = \) drum width, ft

\( M = \) drum wall mass, lb/ft\(^2\)

\( n = \) number of drums

\( q = \) heat flux, Btu/sec-ft\(^2\)

\( Q = \) heat flow rate, Btu/sec

\( R = \) heat effectiveness, dimensionless

\( s = \) drum wall thickness, in

\( T_m = \) mid-point drum wall temperature, \( ^\circ\text{F} \)

\( T_0 = \) outside drum surface temperature, \( ^\circ\text{F} \)

\( T_p = \) pseudo paper temperature, \( ^\circ\text{F} \)

\( T_S = \) steam temperature, \( ^\circ\text{F} \)

\( U, U' = \) overall heat transfer coefficients, Btu/sec-ft\(^2\)-\( ^\circ\text{F} \)
\( W_d \) = water flow rate, lb/sec

\( \gamma \) = paper temperature adjustment factor, dimensionless

\( \epsilon \) = fraction of total drum surface contacting sheet, dimensionless

\( \tau \) = transport delay, sec

Subscripts

1, 2 = dryer section I and II, respectively
Controller Schematic

A schematic diagram of a typical control loop as considered in this paper is shown in Figure 9. The controller transfer function for a proportional-plus-reset controller is shown. The controlled element transfer function is $K_v$. The operator $P(u)$ in Figure 9 is a rather complex function of the input vector since the process considered here (the fourdrinier machine) is non-linear. The feedback element transfer function is $K_f$. The loop gain of the system shown in Figure 9 cannot be calculated unless the process gain is known. In implementing the control loop, $K_v$ was combined with the controller transfer function.

Dynamic Response of the Wet End Model to Variation in Stock Consistency

The dynamic behavior of the wet end model to a sinusoidal variation in feed stock consistency was studied. A disturbance of twelve cycles per hour with a peak-to-peak magnitude of 0.5% was superimposed on a stock consistency of 4.0% normal fiber. A stock consistency of 1.0% fine fibers was allowed to remain constant. Table 4 summarizes the
FIGURE 9.--Schematic Diagram of a Typical Control Loop as Used in the Wet End Model.
values specified for the set points and constants used.

The process variables $W_1$, $W_4$, $\alpha_W$ and $v$ were adjusted so the controlled variables $C_{4C}$, $B$, $M$, and $v$ were at their respective set point values before the disturbance was applied to $C_1$. At a stock consistency of 4.0%, $\alpha_F$ was 0.116.

TABLE 4.--Set Points and Constants for Wet End Model With Stock Consistency Variations.

<table>
<thead>
<tr>
<th>Set Point or Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C'_1$</td>
<td>1.0%</td>
</tr>
<tr>
<td>$M_1$</td>
<td>25,000 lb</td>
</tr>
<tr>
<td>$M_{II}$</td>
<td>25,000 lb</td>
</tr>
<tr>
<td>$a_1$</td>
<td>0.063</td>
</tr>
<tr>
<td>$a_{2R}$</td>
<td>0.050</td>
</tr>
<tr>
<td>$\alpha'_F$</td>
<td>0.200</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.20</td>
</tr>
<tr>
<td>$B_S$</td>
<td>225 lb/1000 ft$^2$</td>
</tr>
<tr>
<td>$M_S$</td>
<td>4.0 lb water/lb fiber</td>
</tr>
<tr>
<td>$C_{4CS}$</td>
<td>0.5%</td>
</tr>
<tr>
<td>$v_S$</td>
<td>25 ft/sec</td>
</tr>
<tr>
<td>$\tau$</td>
<td>15 sec</td>
</tr>
</tbody>
</table>

The responses of the basis weight and moisture at the couch, headbox consistency, and wire speed with all controllers disconnected are shown in Figure 10. The value of the penalty function $p$ is also shown.
FIGURE 10.—Response of Wet End Model Without Control to Stock Consistency Variation.
Various combinations of controller gains were used in an attempt to minimize the value of \( P \), the integral of the penalty function, over one cycle of the disturbance. Table 5 below gives controller settings which resulted in a small value for \( P \).

The response of the wet end to the stock consistency variation mentioned above, with the controllers connected with gains as indicated in Table 5, is shown in Figure 11. The value of the penalty function \( p \) is also shown. Figure 12 shows the variation in the controlling variables stock flow rate, fan pump flow rate, flat box vacuum \( (\alpha_w) \), and wire speed. Note that variation in fan pump flow is virtually zero, indicating that little control of headbox consistency is necessary.

Figure 13 shows the response of the wet end to the stock consistency variation with only the basis weight controller connected. The gains on the basis weight controller are those indicated in Table 5. The integrated penalty function \( P \) was roughly twice the value obtained with all controls connected, but was still several times less than the value obtained with no control at all.

Note that the uncontrolled variables in Figure 16 are somewhat out of phase with the disturbance. This phase shift is due to the transport delay between the fan pump and headbox as well as the capacitance of the wire pit.
<table>
<thead>
<tr>
<th>Controlled Variable</th>
<th>Controlling Variable</th>
<th>$K_cK_v$</th>
<th>RR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Headbox consistency</td>
<td>Fan pump flow</td>
<td>$3 \times 10^2 \frac{\text{lb/sec}}{% \text{ consistency change}}$</td>
<td>0.54 repeats/min</td>
</tr>
<tr>
<td>Moisture at the couch</td>
<td>Flat box vacuum $(\alpha_w)$</td>
<td>$1.25 \times 10^2 \frac{\text{unit } \alpha_w}{\text{unit moisture}}$</td>
<td>5.37 repeats/min</td>
</tr>
<tr>
<td>Basis weight at the couch</td>
<td>Stock feed rate</td>
<td>$6.6 \frac{\text{lb/sec}}{\text{lb/1000 ft}^2}$</td>
<td>0.45 repeats/min</td>
</tr>
<tr>
<td>Basis weight at the couch</td>
<td>Wire speed</td>
<td>$5.5 \times 10^{-2} \frac{\text{ft/sec}}{\text{lb/1000 ft}^2}$</td>
<td>--</td>
</tr>
<tr>
<td>Wire Speed</td>
<td>Wire speed</td>
<td>--</td>
<td>0.60 repeats/min</td>
</tr>
</tbody>
</table>
FIGURE 11. -- Response of Wet End Model With Feedback Control to Stock Consistency Variation.
FIGURE 12.--Response of Wet End Controlling Variables to Stock Consistency Variation.
FIGURE 13.--Response of Wet End With Basis Weight Control Only to Stock Consistency Variation.
STOCK CONSISTENCY

HEADBOX CONSISTENCY

SHEET MOISTURE

BASIS WEIGHT

WIRE SPEED

PENALTY FUNCTION

0.5%

0.05%

0.4

3 lb/M ft^2

25 ft/sec.

20

0

TIME, SECONDS
The wire pit capacitance does not have as noticeable an effect with the disturbance applied here as it would if a disturbance with a much higher frequency were used. The use of a step function disturbance would best demonstrate this capacitance, but the Pade approximation used to represent the transport lag is not suitable for use with such rapidly changing functions.

It is important to note that the controller gains in Table 5 are not claimed to be the optimum settings. Because of a lack of multichannel recording equipment and other computer hardware, the search procedure used to obtain the minimum value for $P$ was a very tedious and time-consuming task. The settings presented in Table 5 gave the lowest value for $P$ of some 50 to 60 different combinations investigated. The controlled response, however, does show a great decrease in the penalty function and does not indicate excessive oscillation of the controlled or controlling variables.

**Dynamic Response of the Wet End Model to Variation in $\alpha_F$**

The dynamic response of the wet end model to a variation in $\alpha_F$ was also examined. A sinusoidal variation was applied to the refining coefficient term in equation (8) for $\alpha_F$. The term $a_{1R}$ in equation (8) was varied $\pm 0.10$ about a value of 0.15 at a rate of twelve cycles per hour.
With \( a_1^R = 0.15 \), \( \alpha_F \) was 0.208. The set points and constants used are summarized in Table 6 below. As in the previous case, the process variables \( W_1, W_4, \alpha_W \) and \( v \) were adjusted so the controlled variables \( C_{40}, B, M, \) and \( v \) were at their respective set point values before the disturbance was applied.

The responses of the basis weight and moisture at the couch, headbox consistency, and wire speed with all controllers disconnected are shown in Figure 14 along with the variation of \( \alpha_F \) and the value of the penalty function \( p \). The unsymmetric first peak on the basis weight and moisture curves is due to the effect of the capacitance of the two wire pits, which at time zero contained white water of uniform consistency.

**TABLE 6.**--Set Points and Constants for the Wet End Model with Variation in \( \alpha_F \)

<table>
<thead>
<tr>
<th>Set Point or Constant</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C'1 )</td>
<td>1.0%</td>
</tr>
<tr>
<td>( M_I )</td>
<td>25,000 lb</td>
</tr>
<tr>
<td>( M_{II} )</td>
<td>25,000 lb</td>
</tr>
<tr>
<td>( a_1 )</td>
<td>0.063</td>
</tr>
<tr>
<td>( \alpha_{F} )</td>
<td>0.300</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.20</td>
</tr>
<tr>
<td>( B_S )</td>
<td>225 lb/1000 ft(^2)</td>
</tr>
<tr>
<td>( M_S )</td>
<td>4.0 lb water/lb fiber</td>
</tr>
<tr>
<td>( C_{4CS} )</td>
<td>0.5%</td>
</tr>
<tr>
<td>( v_S )</td>
<td>25 ft/sec</td>
</tr>
<tr>
<td>( \tau )</td>
<td>15 sec</td>
</tr>
</tbody>
</table>
FIGURE 14.---Response of Wet End Model Without Feedback Control to Variation in $\alpha_F$. 
As with the previous case, various controller settings were used to attempt to minimize the value of P over one cycle of the disturbance. The controller settings which gave the minimum value for P of some 40 to 50 different combinations used are listed in Table 7. These settings appeared to result in minimum deviation from set points with very little oscillation.

Figure 15 shows the response of headbox consistency, basis weight and moisture at the couch and wire speed with all controls connected with settings as given in Table 7. The disturbance to $\alpha_P$ and the penalty function are also shown in Figure 15.

Figure 16 shows the response of the controlling variables stock feed rate, fan pump flow, flat box vacuum ($\alpha_W$), and wire speed to the control actions.

Note that a much higher reset rate was necessary for the headbox consistency controller than was required when the stock consistency was varied. This is certainly reasonable considering the fact that variation in $\alpha_P$ has a strong effect on the white water consistency. Since the white water flow is used to control headbox consistency, variation in the white water consistency makes headbox control more difficult.
TABLE 7
Controller Settings for Wet End
With Variation in $\alpha_F$

<table>
<thead>
<tr>
<th>Controlled Variable</th>
<th>Controlling Variable</th>
<th>$K_cK_v$</th>
<th>RR</th>
</tr>
</thead>
<tbody>
<tr>
<td>Headbox consistency</td>
<td>Fan pump flow</td>
<td>$3 \times 10^3 \frac{lb/sec}{% \text{ consistency change}}$</td>
<td>36 repeats/min</td>
</tr>
<tr>
<td>Moisture at the couch</td>
<td>Flat box vacuum $(\alpha_H)$</td>
<td>$9.37 \times 10^{-2} \frac{\text{unit } \alpha_H}{\text{unit moisture}}$</td>
<td>4.8 repeats/min</td>
</tr>
<tr>
<td>Basis weight at the couch</td>
<td>Stock feed rate</td>
<td>$8.25 \times 10^{-1} \frac{lb/sec}{lb/1000 \text{ ft}^2}$</td>
<td>1.6 repeats/min</td>
</tr>
<tr>
<td>Basis weight at the couch</td>
<td>Wire speed</td>
<td>$2.75 \times 10^{-2} \frac{\text{ft/sec}}{lb/1000 \text{ ft}^2}$</td>
<td>--</td>
</tr>
<tr>
<td>Wire speed</td>
<td>Wire speed</td>
<td>--</td>
<td>1.5 repeats/min</td>
</tr>
</tbody>
</table>
FIGURE 15.--Response of Wet End Model With Feedback Control to Variation in $\alpha_F$. 
FIGURE 16.--Response of Wet End Controlling Variables to Variation in $\alpha_p$. 
Dynamic Behavior of the Dry End Model

The data from Cirrito (1962) were used as input to the model. These data are summarized in Table 8 below.

TABLE 8.--Data Used for Dry End Model.

1. 60-in diameter dryers x 19 ft trim width
2. Steam temperature: $350^\circ F$ ($340^\circ F$ for first 5 drums)
3. Dry fiber flow rate: 16.9 lb/sec
4. Water flow rate: 35.8 lb/sec
5. Moisture in feed: 2.12 lb water/lb fiber
6. Paper-drum contact length: 10.17 ft/drum
7. Speed 1366 ft/min
8. Basis weight of product: 42 lbs/1000 ft$^2$
9. Type of product: W. F. Board

Table 9 gives the values for the other constants used for Sections I and II.

TABLE 9.--Constants Used for Dry End Model.

<table>
<thead>
<tr>
<th>Constant</th>
<th>Dimensions</th>
<th>Value for Section I</th>
<th>Value for Section II</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>Btu/sec*ft$^{20F}$/in</td>
<td>$8.89 \times 10^{-2}$</td>
<td>$8.89 \times 10^{-2}$</td>
</tr>
<tr>
<td>s</td>
<td>inches</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>A</td>
<td>ft$^2$</td>
<td>$1.16 \times 10^4$</td>
<td>$4.25 \times 10^4$</td>
</tr>
<tr>
<td>$\Delta H$</td>
<td>Btu/lb</td>
<td>990</td>
<td>990</td>
</tr>
<tr>
<td>M</td>
<td>lb/ft$^2$</td>
<td>45.5</td>
<td>45.5</td>
</tr>
<tr>
<td>$C_p$</td>
<td>Btu/lb$^\circ F$</td>
<td>0.119</td>
<td>0.119</td>
</tr>
</tbody>
</table>
With the sheet velocity given in Table 8 the time delays were determined to be as follows:
\[ \tau_1 = 20 \text{ sec} \]
\[ \tau_2 = 25 \text{ sec} \]
\[ \tau_3 = 10.3 \text{ sec} \]

When the model was implemented the output moisture from the first section agreed exactly with the data of Cirrito previously mentioned. The second section moisture was slightly greater than the desired figure, however. This was assumed due to a slight error in the generation of \( \gamma \) by means of the diode function generator, and a slight adjustment of the constant \( b_1 \) in equation (44) was made to bring the output moisture to the proper level. The adjusted value for \( b_1 \) was 230, while the original value was 234.

The dynamic behavior of the model was observed by perturbing the water and fiber flow rates to the first section and the steam temperature in the second section. Except for the perturbed quantities, all values listed in Tables 8 and 9 above were used in the model.

With the steam temperature held constant at 350°F, the water and fiber flow rates from the press were changed as shown in Figure 17. Note that while both water and fiber rates increase, the net effect is an increase in sheet moisture from 2.12 to 2.20 at the peak flow and a decrease to 2.07 at the minimum flow. These conditions might correspond to the production of a heavier, wetter sheet and a
FIGURE 17.--Water and Fiber Flow Rate Inputs to Dry End Model.
thinner, dryer sheet respectively at the couch. The response of both sections of the dryer to this disturbance is shown in Figure 18. Note that the response of Section I (delayed \( \mathcal{V}_1 \) + \( \mathcal{V}_2 \) from press flow rates) indicates that essentially the same amount of water is evaporated for moderate variations in moisture input. This response corresponds to practical experience for the first two-thirds of the dryer. The response of Section II (delayed \( \mathcal{V}_3 \) to show reel conditions) shows the drying rate to be more responsive to input moisture changes. The input water (output water from Section I, delayed \( \mathcal{V}_2 \)) increases 3.4 pounds per second, while the output water increases only 1.9 pounds per second. When the input water decreases 1.07 pounds per second the output water decreases only 0.18 pounds per second. This is in accordance with actual non-linear behavior for the last one-third of a drier train and is caused by the increasing difficulty of removing moisture during the falling rate period.

To test dynamic response to steam temperature changes, water and fiber rates from the press were held constant at 35.8 and 16.9 pounds per second. Steam temperature in the first section was kept at 350°F. Steam temperature in the second section was changed as shown in Figure 19. The steam temperature varied from a maximum of 385°F, to a minimum of 335°F. The water rate out of the second section showed the response indicated in Figure 20. The minimum water flow
FIGURE 18.--Water Output from Dryer Sections I and II Subjected to Input Disturbance shown in Figure 17.
FIGURE 19.--Steam Temperature Variation in Dryer Section II.

FIGURE 20.--Water Output from Dryer Section II for Steam Temperature Variation Shown in Figure 19.
rate was 0.72 pounds per second and the maximum water flow rate was 1.40 pounds per second. The output sheet moisture range was 0.038 to 0.083. The thermal lag demonstrated by the model is in accordance with the lag observed in actual dryers.

While no data were available to check the quantitative accuracy of the above results, the qualitative responses were certainly what would be expected from an operating dryer. If quantitative errors should be found to exist, these errors can almost certainly be attributed to inaccuracies resulting from the averaging of moisture for each section to obtain values for $R$, $\gamma$, and $h_0$. Some of these errors may be reduced to a negligible amount by modifying the functions $R$ and $\gamma$. As has previously been pointed out, agreement with an actual dryer can also be improved by adding more dryer sections to the model. The addition of two or more sections should practically eliminate the averaging errors, but would add considerably to the computer hardware necessary for solution.
CONCLUSIONS AND RECOMMENDATIONS

Conclusions

The model of the fourdrinier paper machine presented here gives a useful representation of the dynamics of an operating machine. The thermal lag in the dryer and the wire pit capacitance observed in the model correlate well with actual experience. For the purpose for which the model was desired, namely as a foundation for future study of computer control of the fourdrinier machine, the models developed here are entirely adequate.

The control simulation performed indicates that before the interacting controls on a fourdrinier machine can be "tuned" to give optimal performance the type, magnitude, and frequency of the disturbance imposed upon the system must be known. This indicates that some compromise must be made among the various optimal settings, or the controller settings must be adapted to changes in the system or environment.

The use of a penalty function was a great convenience in obtaining efficient controller settings, and appears to offer a good criterion for measuring the effectiveness of a control system of this type.
Recommendations

If further control optimization is carried out with the model as presented here, a multichannel recorder or multichannel oscilloscope should be used to facilitate an efficient search procedure to obtain the optimum controller settings.

An improvement in the wet end model would be effected by adding a filler consistency to the two fiber consistencies included in the model presented here. A dynamic material balance could then be made on the chemicals and fillers present as well as on the fiber and water. This would be useful in quality control considerations where the amount of chemicals and filler in the web are important. Since the retention of filler on the wire is much less than the retention of fiber, the change in white water dynamics brought about by the inclusion of filler consistency in the model would be significant and would make the model a much better representation of an actual fourdrinier machine.

To implement both the wet and dry end models simultaneously as they are presented here would require a large amount of computer hardware. A detailed examination of the behavior of the model should make possible the formulation of a simplified transfer function for the wet end which would require less equipment. With the inclusion of special feedback networks the computer hardware might be reduced to one-half or less of the current requirement.
APPENDIX A

COMPUTER DIAGRAMS
FIGURE 21.--Computer Schematic for Second-Order Padé Approximation.
FIGURE 22.--Computer Schematic for the Wet End Model.
FIGURE 23.--Computer Schematic for Fourth-Order Padé Approximation.
FIGURE 24.--Computer Schematic for Section II of the Dry End Model.
KEY:
QSM - QUARTER SQUARE MULTIPLIER
SM - SERVOMULTIPLIER
DFG - DIODE FUNCTION GENERATOR
APPENDIX B

DRYER DATA AND PARAMETER TABULATION
**TABLE 10**

Heat Effectiveness $R$

<table>
<thead>
<tr>
<th>Dryer Section</th>
<th>Drums Numbered</th>
<th>Moisture In</th>
<th>Moisture Out</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 - 5*</td>
<td>2.12</td>
<td>2.07</td>
<td>0.23</td>
</tr>
<tr>
<td>2</td>
<td>6 - 22</td>
<td>2.07</td>
<td>1.53</td>
<td>0.95</td>
</tr>
<tr>
<td>3</td>
<td>23 - 42</td>
<td>1.53</td>
<td>0.72</td>
<td>0.89</td>
</tr>
<tr>
<td>4</td>
<td>43 - 62</td>
<td>0.72</td>
<td>0.20</td>
<td>0.96</td>
</tr>
<tr>
<td>5</td>
<td>63 - 82</td>
<td>0.20</td>
<td>0.076</td>
<td>0.95</td>
</tr>
</tbody>
</table>

* Data from Cirrito (1962)
TABLE 11

Paper Temperature Correlation Factor

<table>
<thead>
<tr>
<th>Average Moisture</th>
<th>Sheet Temperature</th>
<th>Drum Surface Temperature</th>
<th>( \gamma^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.8</td>
<td>210</td>
<td>285</td>
<td>0.00</td>
</tr>
<tr>
<td>1.1</td>
<td>210</td>
<td>318</td>
<td>0.00</td>
</tr>
<tr>
<td>0.45</td>
<td>238</td>
<td>326</td>
<td>0.34</td>
</tr>
<tr>
<td>0.29</td>
<td>245</td>
<td>324</td>
<td>0.42</td>
</tr>
<tr>
<td>0.14</td>
<td>279</td>
<td>335</td>
<td>0.83</td>
</tr>
<tr>
<td>0.00</td>
<td>293</td>
<td>335</td>
<td>1.00</td>
</tr>
</tbody>
</table>

* From equation:

\[ T_p = 210 + \gamma (T_0 - 234) \]
LITERATURE CITED


