ANALYSIS OF EPOXY BONDED COMPOSITE BEAMS

by

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CHAPTER 1

INTRODUCTION

1.1. Introduction

In recent years considerable research has been conducted on beams comprised of two or more materials. The majority of this research has been concerned with obtaining knowledge of the behavior of composite beams for use in highway and building construction. The development of some early specifications for composite steel and concrete beams in the United States was based on familiar elastic concepts. It was tacitly assumed that the concrete slab and steel beam would act together as a unit up to ultimate load if the members were properly designed by the elastic method. Elastic design of the cross section was proven to be satisfactory except certain difficulties arose with respect to the design of shear connectors.

Semi-empirical formulas which predicted the behavior of mechanical shear connectors were developed for the 1957 revision of the AASHO specifications, but these formulas do not generally apply to an epoxy bonded composite beam which provides a continuous connection. It is because of this different type of bond and other uncertainties which arise from this type of connection that a more exact mathematical solution is desirable. The three material beam whose cross section consists of a common structural shape such as a T-beam will be referred to simply as the composite three material beam in this thesis.

1.2. The Problem Defined

The object of the research presented in this thesis is to develop mathematical expressions which predict the behavior of a simply supported three material beam whose cross section consists of two materials having rectangular sections with the third having a rolled structural shape and which is subjected to an arbitrary loading. Using this three material beam as a model, the results predicted by the mathematical expressions are to be compared with experimental results.

1.3. Method of Obtaining Mathematical Solution

The solution to a plane stress problem, using the principles of the theory of elasticity, for the composite three material simply supported beam consisted of using a stress function for each material. The problem concerned with all rectangular sections was solved by Endebrock (1964). See Figure 1. The development of a solution particular to the composite three material beam problem where one of the sections is a structural shape used certain concepts of transformed sections to replace the structural rolled section by an "equivalent" rectangular beam.

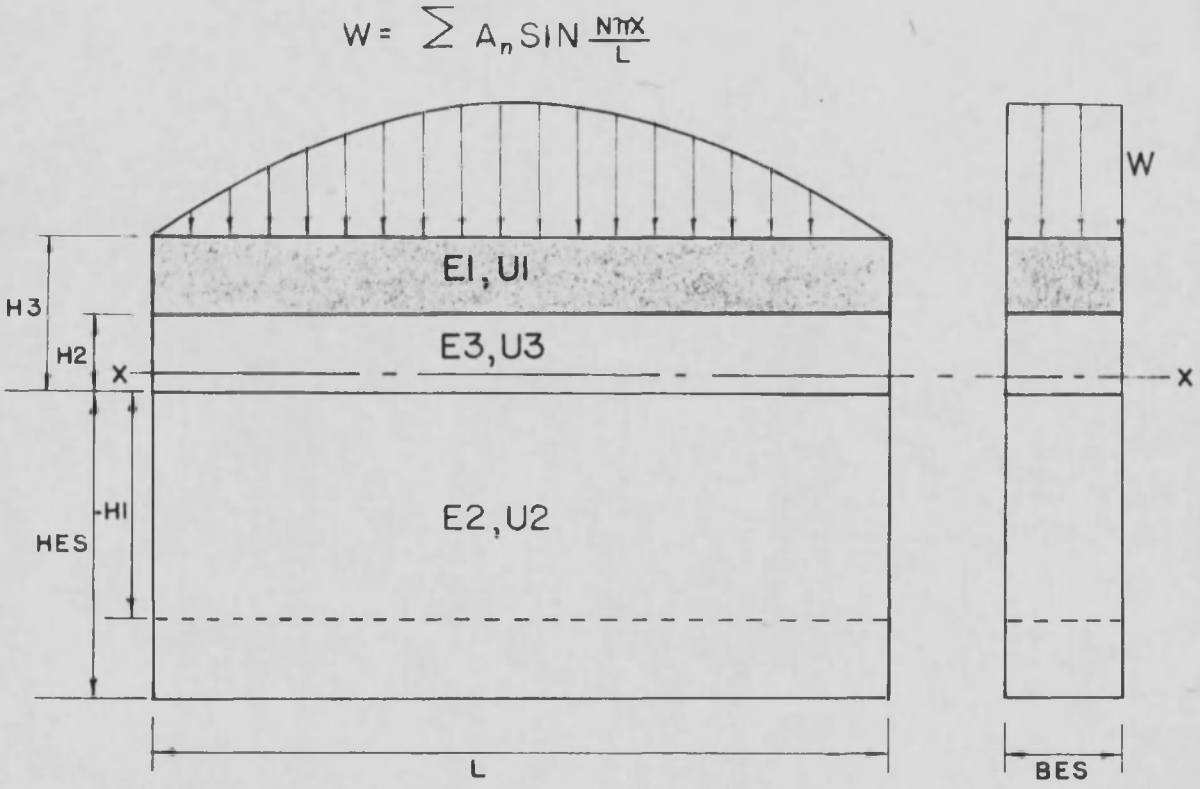


FIGURE 1

MATHEMATICAL MODEL FOR COMPOSITE THREE MATERIAL BEAM.

CHAPTER 2

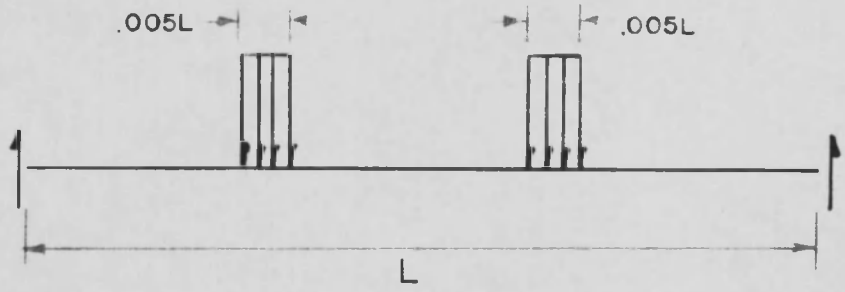
DEVELOPMENT OF MATHEMATICAL SOLUTION

2.1. Introduction

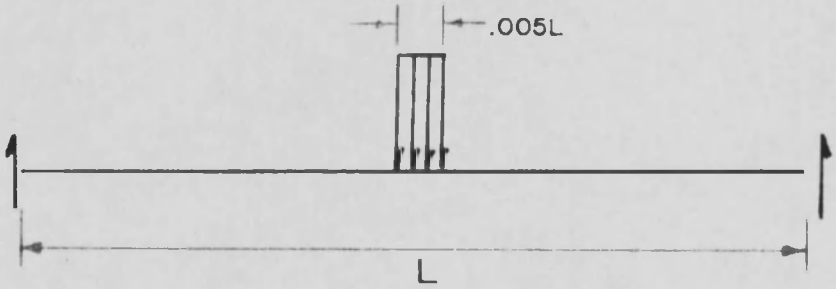
The development of the mathematical expressions used to predict the behavior of the composite three material simply supported beam employs the principles of the theory of elasticity and certain concepts of transformed sections.

The plane stress problem for rectangular sections has been solved using the principles of elasticity by Endebrock (1964). The solution consisted of generating a stress function which satisfied the biharmonic partial differential equation. The stresses, strains and displacements are determined from the stress function. The stress function was selected such that the stresses and displacements were satisfied on the boundaries of the region under consideration. In this problem there were actually three stress functions, one for each material layer. These stress functions differed only in that their constants of integration and the constants associated with the material properties were different.

A particular case of interest in the study involved concentrated loads; therefore, it was convenient to express the loads as a Fourier series expansion. Concentrated loads were approximated by using a uniformly distributed load over a length of five-thousandths of the span (0.005L), as shown in Figure 2.



THIRD POINT LOADING



MID POINT LOADING

FIGURE 2 APPROXIMATION OF CONCENTRATED LOADS BY A UNIFORMLY DISTRIBUTED LOAD.

2.2. Development

The stress function was assumed to be

$$\phi = f(y) \sin \frac{n\pi x}{L} \tag{1}$$

where f(y) is a function of y alone, L is the length of the beam and n is an integer. Substitution of ϕ into the biharmonic equation yields an ordinary linear differential equation in f(y) with constant coefficients. This equation has the solution (Timoshenko, 1951),

$$f(y) = C_1 \cosh \alpha y + C_2 \sinh \alpha y + C_3 y \cosh \alpha y + C_4 y \sinh \alpha y$$

where C_1, C_2, C_3 and C_4 are constants of integration and α is equal to $\frac{n\pi}{L}$.

The stress function then becomes

$$\phi = (C_1 \cosh \alpha y + C_2 \sinh \alpha y + C_3 y \cosh \alpha y + C_4 y \sinh \alpha y) \sin \alpha x \tag{2}$$

The stresses obtained from this stress function are

$$\sigma_x(x,y) = \frac{\partial^2 \phi}{\partial y^2} = \sin \alpha x \left[C_1 \alpha^2 \cosh \alpha y + C_2 \alpha^2 \sinh \alpha y + C_3 \alpha (2 \sinh \alpha y + \alpha y \cosh \alpha y) \right.$$

$$\left. + C_4 \alpha (2 \cosh \alpha y + \alpha y \sinh \alpha y) \right] \tag{3a}$$

$$\sigma_y(x,y) = \frac{\partial^2 \phi}{\partial x^2} = -\alpha^2 \sin \alpha x \left[C_1 \cosh \alpha y + C_2 \sinh \alpha y + C_3 y \cosh \alpha y \right.$$

$$\left. + C_4 y \sinh \alpha y \right] \tag{3b}$$

$$\tau_{xy}(x,y) = \frac{-\partial^2 \phi}{\partial x \partial y} = -\alpha \cos \alpha x \left[C_1 \alpha \sinh \alpha y + C_2 \alpha \cosh \alpha y + C_3 (\cosh \alpha y + \alpha y \sinh \alpha y) \right.$$

$$\left. + C_4 (\sinh \alpha y + \alpha y \cosh \alpha y) \right] \tag{3c}$$

As noted previously, the above derivation applies only to rectangular sections and therefore a modification using transformed

section concepts was necessary for the determination of the desired mathematical expressions for the case where one of the sections is a rolled structural shape.

Using certain concepts of transformed sections, the widths of the two top rectangular sections were changed by the ratio of their moduli of elasticity to the modulus of elasticity of the rolled structural member. The rolled structural shape was then transformed into an "equivalent" rectangle whose section properties were equal to those of the structural rolled shape. The "equivalent" rectangular beam representing the composite three material beam then used the width of this equivalent rectangle as its width. Having established an equivalent width for the entire section, the moduli of elasticity of the two top rectangular sections were changed by the ratio of their widths to this equivalent width, thus constituting the mathematical model as shown in Figure 3.

As mentioned above, an equivalent rectangle was established for the rolled section and thus two section properties had to be satisfied; namely the maintaining of the centroidal axis and the moment of inertia about its centroidal axis, as shown in Figure 3.

Using properties of the original section, the centroidal axis and moment of inertia of the composite beam are calculated as follows

$$\bar{y} = \frac{\frac{B_{s1}}{2}\left[(H_3)^2 - (H_2)^2\right] + \frac{B_{s3}}{2}(H_2)^2 - \frac{A_{sc}}{2}(H_1)}{B_{s1}(H_3 - H_2) + B_{s3}(H_2) + A_{sc}} \tag{4}$$

$$I_{(x-x)ss} = I_{ss} + A_{sc}\left(\frac{H_1}{2} + \bar{y}\right) \tag{5}$$

where $\bar{y}$ locates the centroidal axis of the section, I_{ss} is the moment of

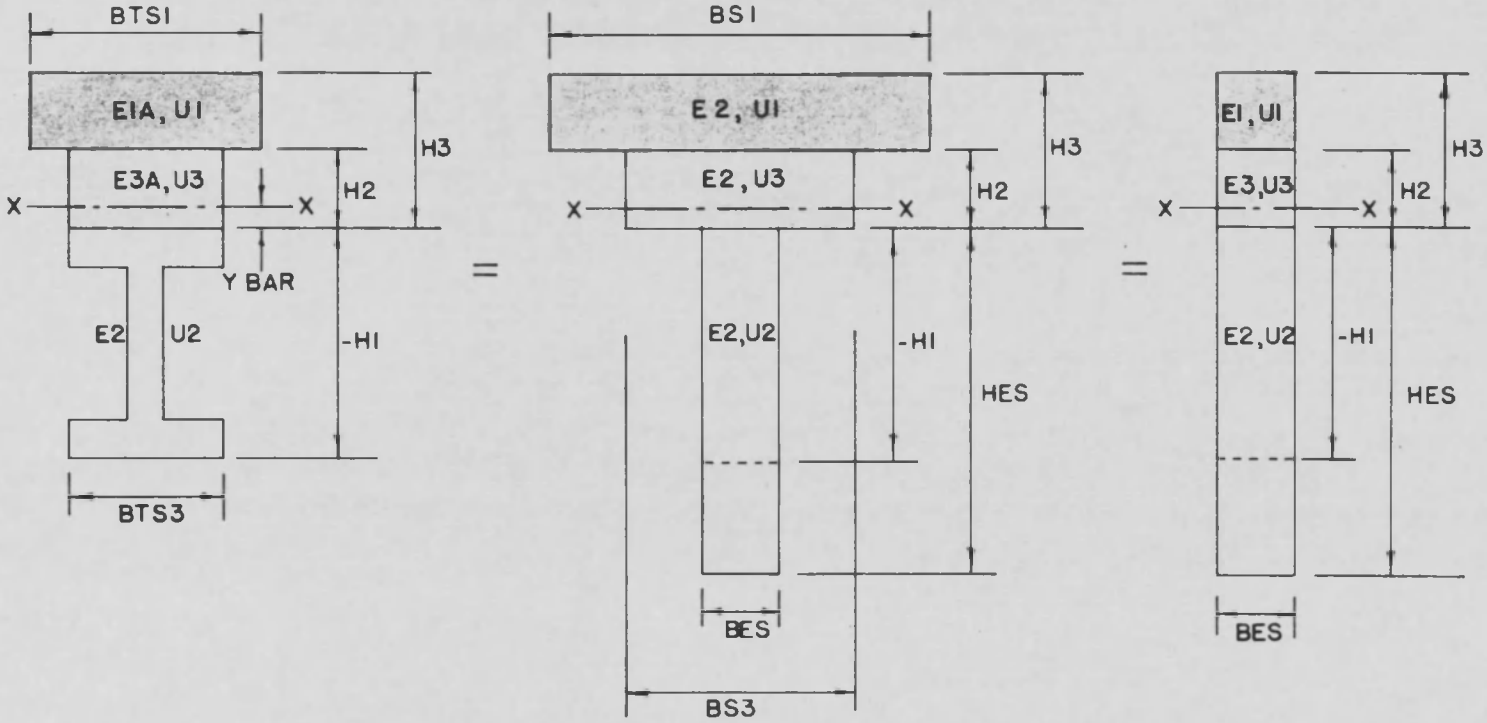


FIGURE 3
DEVELOPMENT OF MATHEMATICAL MODEL

inertia of the structural shape about the centroidal axis and the other terms are dimensions as shown in Figure 2.

Having established the centroidal axis and moment of inertia, the dimensions of the equivalent rectangle are found as follows

$$\bar{y} = \frac{\frac{B_{s1}}{2}\left[(H_3)^2 - (H_2)^2\right] + \frac{B_{s3}}{2}(H_2)^2 - \frac{(B_{es})(H_{es})^2}{2}}{B_{s1}(H_3 - H_2) + B_{s3}(H_2) + (B_{es})(H_{es})} \tag{6}$$

$$I_{(x-x)ss} = \frac{1}{12}(B_{es})(H_{es})^3 + (B_{es})(H_{es})\left(\frac{H_{es}}{2} + \bar{y}\right)^2 \tag{7}$$

where H_{es} and B_{es} are the width and height of the equivalent rectangle respectively.

Using expressions (6) and (7), the two unknowns, B_{es} and H_{es}, are found by solving the two resulting simultaneous equations, thus satisfying the two necessary conditions.

The composite three material beam problem has now been reduced to the problem solved by Endebrock (1964). In forming the composite beam, using all rectangular sections, the three rectangular sections were joined or connected. In the mathematical model this was accomplished by equating the normal and shearing stresses and displacements at the interfaces of the adjoining material layers.

Also, in forming a mathematical model, the problem involving all rectangular sections of equal width does not alter the distortion effects. However, in forming a mathematical model for a composite three material beam, the transformation used replaces the structural shape by an "equivalent" rectangular beam which resulted in only true consideration of shear distortions present in the central and top

layers. The shear distortions in the structural shape are negligible since the modulus of shear rigidity of the central layer for the problem considered here is always less than that of the other layers by at least a factor of ten. Since in the transformation, the shape of the central layer remained the same; namely, rectangular, the mathematical model does yield the essentially exact shear distortion effects present in the composite three material beam.

A consideration of the shear distortion present in the bottom layer may be derived using the following relationships between shearing strain and shear stress:

$$\gamma_r = \frac{\tau_{ave}}{G_r} = \frac{V}{(1.5)(B_{es})(H_{es})(G_s)} \tag{7a}$$

$$\gamma_s = \frac{\tau_{ave}}{G_s} = \frac{V}{(t_w)(H_1)(G_s)} \tag{7b}$$

where G_r and G_s are the shear moduli of the equivalent rectangle and the structural section respectively, τ_{ave} is the average shear stress and t_w is the web thickness of the structural shape.

Equating these shearing strains, γ_r and γ_s, and noting

$$G = \frac{E}{2(1+\mu)} \tag{7c}$$

a relationship is obtained which gives a Poisson's ratio to be used for the equivalent rectangle in the transformation; namely,

$$U_r = \frac{(1.5)(B_{es})(H_{es})(1+U_s)}{(t_w)(H_1)} - 1 \tag{7d}$$

where U_r and U_s are Poisson's ratio for the equivalent rectangle and the structural shape respectively.

Note that in forming the mathematical model, the horizontal displacement u and the vertical displacement v entered into the boundary conditions and therefore had to be determined. The relationships between strains, displacements, and stresses are

$$\epsilon_x = \frac{\partial u}{\partial x} = \frac{1}{E}(\sigma_x - \mu\sigma_y) \tag{8a}$$

$$\epsilon_y = \frac{\partial v}{\partial y} = \frac{1}{E}(\sigma_y - \mu\sigma_x) \tag{8b}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} = \frac{2(1+\mu)}{E}\tau_{xy} \tag{8c}$$

where E is the modulus of elasticity and μ is Poisson's ratio.

The stress equations (3) were substituted into the strain equations (8). Equations (8a) and (8b) were then integrated to obtain the displacements u and v. Integration of equations (8a) and (8b) gave two arbitrary functions which were evaluated by substituting the displacement expressions u and v into equation (8c). As a result the following expressions for u and v were obtained:

$$u = -\frac{\cos\alpha x}{E}\Big\{C_1\,\alpha(1+\mu)\cosh\alpha y + C_2\,\alpha(1+\mu)\sinh\alpha y + C_3\left[2\sinh\alpha y + \alpha y(1+\mu)\cosh\alpha y\right] + C_4\left[2\cosh\alpha y + \alpha y(1+\mu)\sinh\alpha y\right]\Big\} + Ay + B \tag{8d}$$

$$v = \frac{\sin\alpha x}{E}\Big\{C_1\alpha(1+\mu)\sinh\alpha y+C_2\alpha(1+\mu)\cosh\alpha y+$$

$$+C_3\left[\alpha y(1+\mu)\sinh\alpha y+(-1+\mu)\cosh\alpha y\right]+$$

$$+C_4\left[\alpha y(1+\mu)\cosh\alpha y+(-1+\mu)\sinh\alpha y\right]\Big\}-Ax+F, \qquad (8e)$$

where A, B, and F are constants of integration. These constants are associated with rigid body motions; hence, for this problem they may be taken to be zero. The displacements expressions then reduce to

$$u = -\frac{\cos\alpha x}{E}\Big\{C_1\alpha(1+\mu)\cosh\alpha y+C_2\alpha(1+\mu)\sinh\alpha y+$$

$$+C_3\left[2\sinh\alpha y+\alpha y(1+\mu)\cosh\alpha y\right]+C_4\left[2\cosh\alpha y+\right.$$

$$\left.+\alpha y(1+\mu)\sinh\alpha y\right]\Big\} \qquad (8d)$$

$$v = \frac{\sin\alpha x}{E}\Big\{C_1\alpha(1+\mu)\sinh\alpha y+C_2\alpha(1+\mu)\cosh\alpha y+$$

$$+C_3\left[\alpha y(-1+\mu)\sinh\alpha y+(-1+\mu)\cosh\alpha y\right]+$$

$$+C_4\left[\alpha y(1+\mu)\cosh\alpha y+(-1+\mu)\sinh\alpha y\right]\Big\} \qquad (8e)$$

The remaining portion of the problem was to evaluate the constants of integration from the boundary conditions. There is a stress function for each material and, from equation (1), four constants of integration for each stress function; therefore there are twelve independent constants of integration for which there must be twelve independent boundary conditions in order that the solution may be obtained. The stress functions were denoted as ϕ_1, ϕ_2, and ϕ_3

where the subscripts correspond to the material layers with the same number as shown in Figure 1. The constants of integration were denoted as C_1^1, C_2^1, etc., corresponding to ϕ_1; C_1^2, C_2^2, etc., corresponding to ϕ_2, etc.

The boundary conditions are listed below. The superscripts denote the corresponding material layers as indicated in Figure 1.

$$\tau^1_{xy}(x,-h)=0 \tag{9a}$$

$$\sigma^1_y(x,-h)=0 \tag{9b}$$

$$\tau^1_{xy}(x,0)= \tau^2_{xy}(x,0) \tag{9c}$$

$$\sigma^1_y(x,0)= \sigma^2_y(x,0) \tag{9d}$$

$$u^1(x,0)=u^2(x,0) \tag{9e}$$

$$v^1(x,0)=v^2(x,0) \tag{9f}$$

$$\tau^2_{xy}(x,t)=\tau^3_{xy}(x,t) \tag{9g}$$

$$\sigma^2_y(x,t)= \sigma^3_y(x,t) \tag{9h}$$

$$u^2(x,t)=u^3(x,t) \tag{9i}$$

$$v^2(x,t)=v^3(x,t) \tag{9j}$$

$$\tau^3_{xy}(x,c)=0 \tag{9k}$$

$$\sigma^3_y(x,c)=\sum a_n \sin\frac{n\pi x}{L} \tag{9l}$$

$$\int_A \sigma_x(0,y)dy=0 \tag{10a}$$

$$\int_A \sigma_x(L,y)dy=0 \tag{10b}$$

$$\int_A \sigma_x(0,y)ydy=0 \tag{10c}$$

$$\int_A \sigma_x(L,y)ydy=0 \tag{10d}$$

$$\int_A \tau_{xy}(0,y)dy=\text{Reaction} \tag{10e}$$

$$\int_A \tau_{xy}(L,y)dy=\text{Reaction} \tag{10f}$$

where the letter A associated with the integral sign denotes the cross-sectional area of the beam.

Boundary conditions (9) were used to evaluate the twelve constants of integration whereas boundary conditions (10) were identically satisfied by the choice of the stress function. The procedure used to evaluate these constants was covered by Endebrock (1964) and therefore will not be covered here.

CHAPTER 3

ANALYSIS OF COMPOSITE THREE MATERIAL BEAMS

3.1. Introduction

Composite three material beams were analyzed using the method developed in Chapter 2. The results of these studies are discussed and compared to the experimental results using elementary relationships of composite beam action. In the examples used emphasis was placed upon the material properties of the central layer. The material properties of the top and bottom layers were selected so that the centroidal axis of the entire section was located in or near the central layer thereby subjecting the central layer to the maximum shearing stress in the beam.

The dimensions of the three composite three-material beams used in the investigations presented in this chapter were kept the same; however, their central layers were of different materials, one being of rubber and the other two of epoxy compounds. A beam of 108 inches in length and four inches in total depth, where the top and central layers were both one-half inch and the bottom layer consisted of a 3I2.59 rolled aluminum shape, was used for the investigation. The widths of the top and central layers were the same as the width of the aluminum rolled shape; namely, two inches, as shown in Figure 3. The moduli of elasticity used for the top and bottom layers were 30×10^6 psi and 10×10^6 psi, respectively, with corresponding

Poisson's ratios of 0.33 and 0.30. The three moduli of elasticity and Poisson's ratios of the central layers were 1270 psi (Rubber), 80,000 psi and 675,000 psi, and 0.45 (Rubber), 0.40, and 0.40 respectively.

Two types of loading were investigated; namely, mid-point and third point loadings, as shown in Figures 22 and 23. For the third point loading the concentrated loads were applied at 36 inches and 72 inches along the axis of the beam. The strains and displacements were measured at the quarter and mid-points for each type of loading. It is noted that the concentrated loads were approximated by using a uniformly distributed load over approximately 1/200 of the span length. All beams analyzed here were symmetrically loaded; however, the modified solution is not limited to only symmetrically loaded beams.

3.2. Effective Composite Action

Strains and displacments were measured for the three composite three material beams under the two different type loadings. For each beam and each type of loading the effective composite action was investigated. Using elementary relationships the theoretical vertical displacements at the quarter and mid-points were calculated. Using these values as criteria, the displacements obtained from the modified and experimental solutions were compared and the effective composite action of the beam was determined. The relative values of strain on the top and bottom of the aluminum beam, when compared, were also used as a criteria for establishing the effective composite action.

The illustration of these methods used in establishing the composite action is shown in Table 1 and Figures 4 through 9.

TABLE 1

EFFECTIVE COMPOSITE ACTION FOR THREE THREE-MATERIAL BEAMS

CRITERIA		RUBBER: (CENTRAL LAYER) (E = 1270 psi)				EPOXY CENTRAL LAYER (E=80,000 psi)				EPOXY: (CENTRAL LAYER) (E = 675,000 psi)			
		Third Point Loading		Mid Point Loading		Third Point Loading		Mid Point Loading		Third Point Loading		Mid Point Loading	
		Experimental (%)	Theoretical (%)	Experimental (%)	Theoretical (%)	Experimental (%)	Theoretical (%)	Experimental (%)	Theoretical (%)	Experimental (%)	Theoretical (%)	Experimental (%)	Theoretical (%)
DISPLACEMENT	1/4L	16.6	13.1	18.9	13.0	86.5	93.4	83.4	90.7	93.4	96.5	94.2	97.5
	1/2L	17.7	12.3	18.3	12.1	86.6	93.0	84.6	91.0	93.5	97.0	96.3	97.4
STRAIN	1/4L	18.8	11.3	18.5	12.0	91.8	92.3	89.6	92.9	94.5	97.5	95.6	97.5
	1/2L	18.5	11.5	18.8	13.3	91.5	93.5	89.3	92.1	94.5	97.0	95.2	97.2

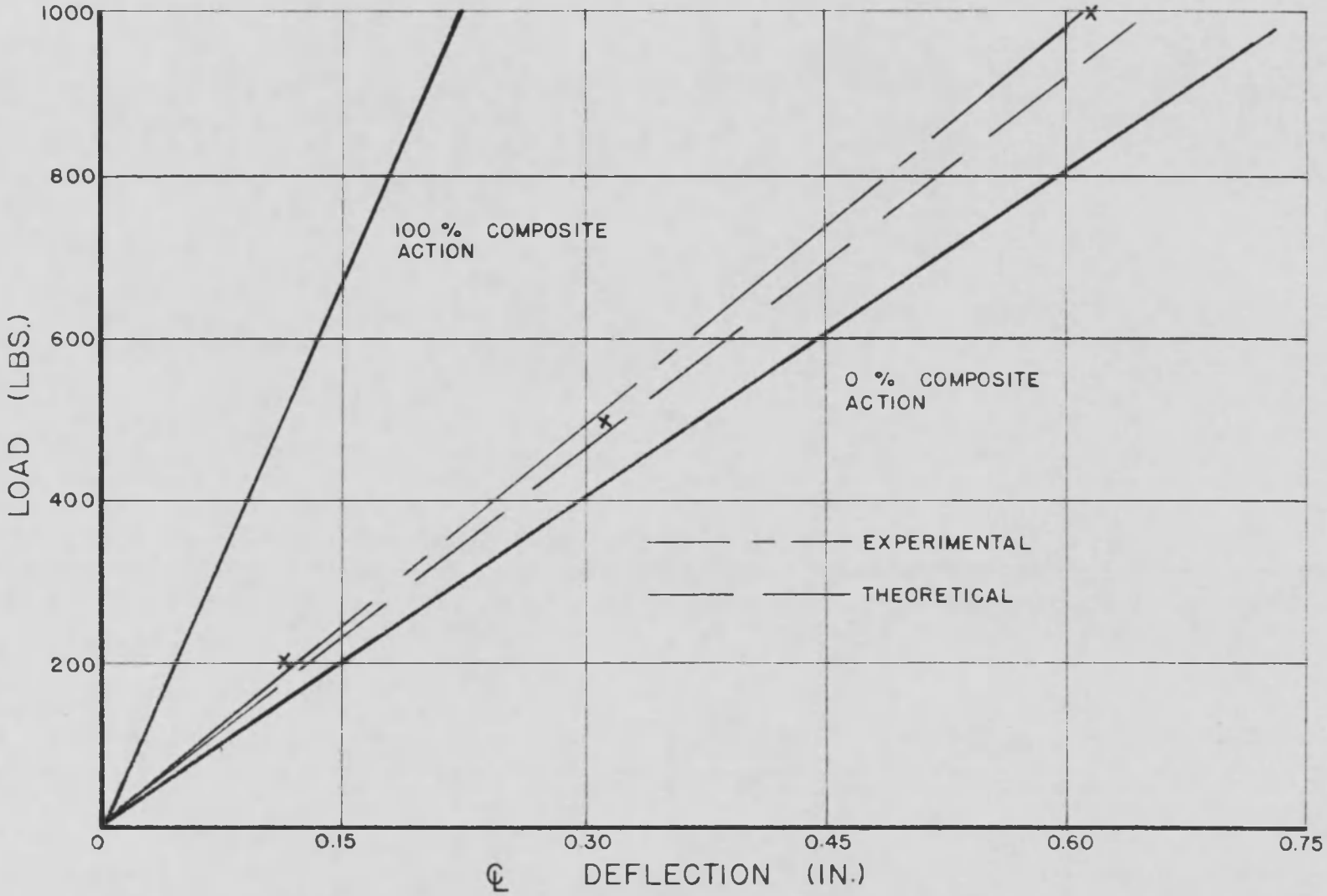


FIGURE 4 LOAD VS CENTERLINE DEFLECTION: RUBBER CENTRAL LAYER, THIRD POINT LOADING

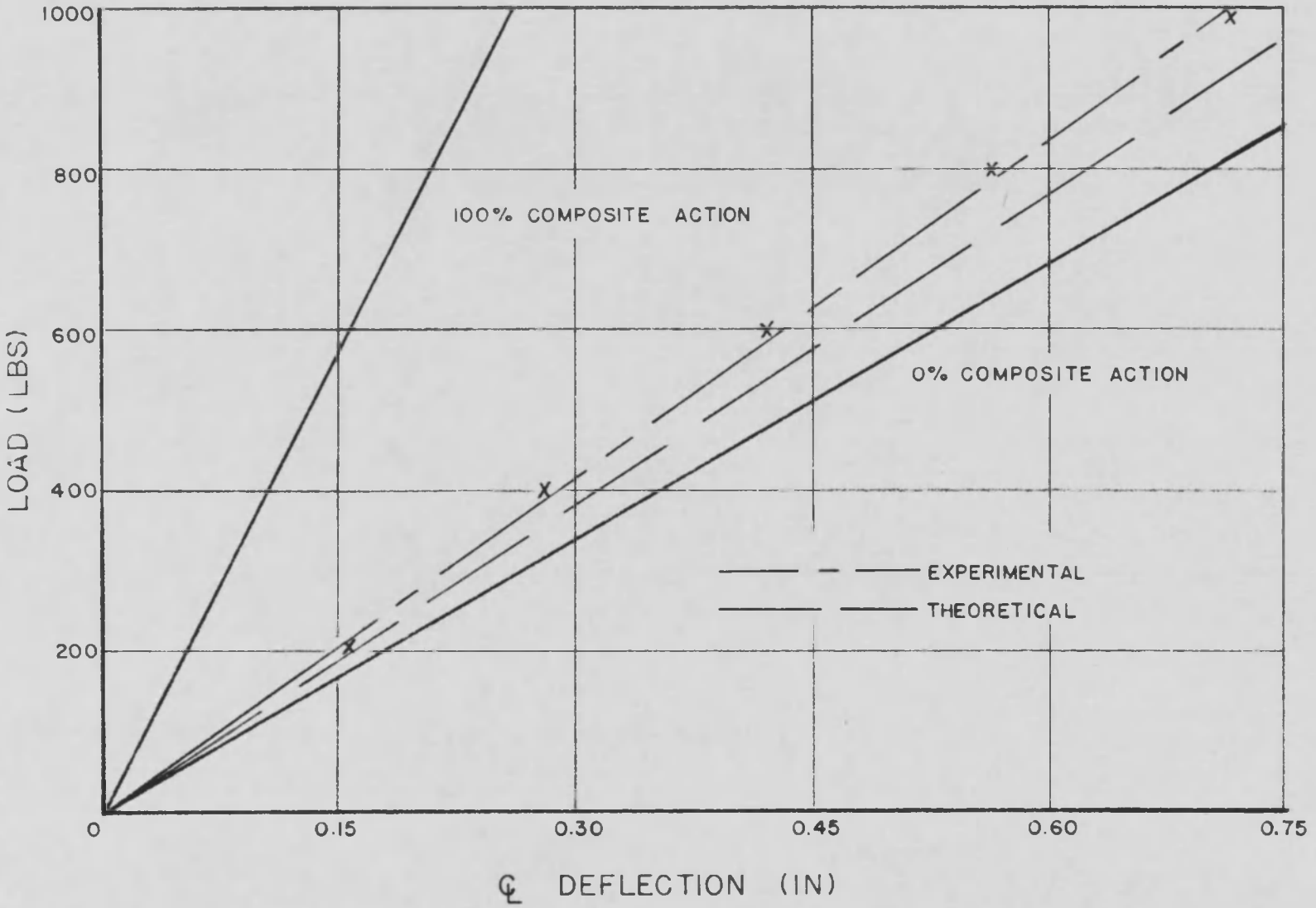


FIGURE 5 LOAD VS CENTERLINE DEFLECTION: RUBBER CENTRAL LAYER, MID POINT LOADING

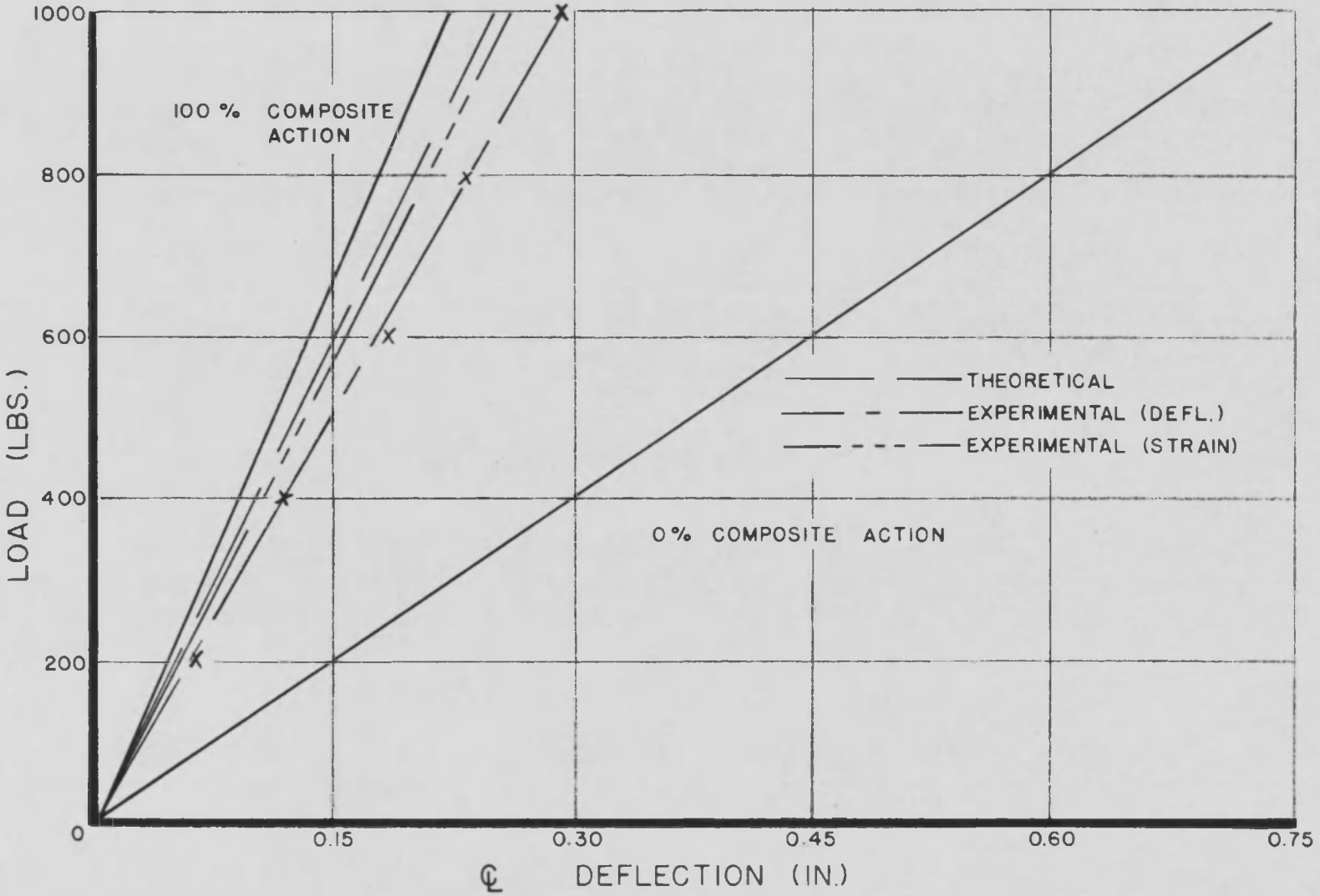


FIGURE 6 LOAD VS. CENTERLINE DEFLECTION: EPOXY CENTRAL LAYER (E = 80,000 PSI), THIRD POINT LOADING

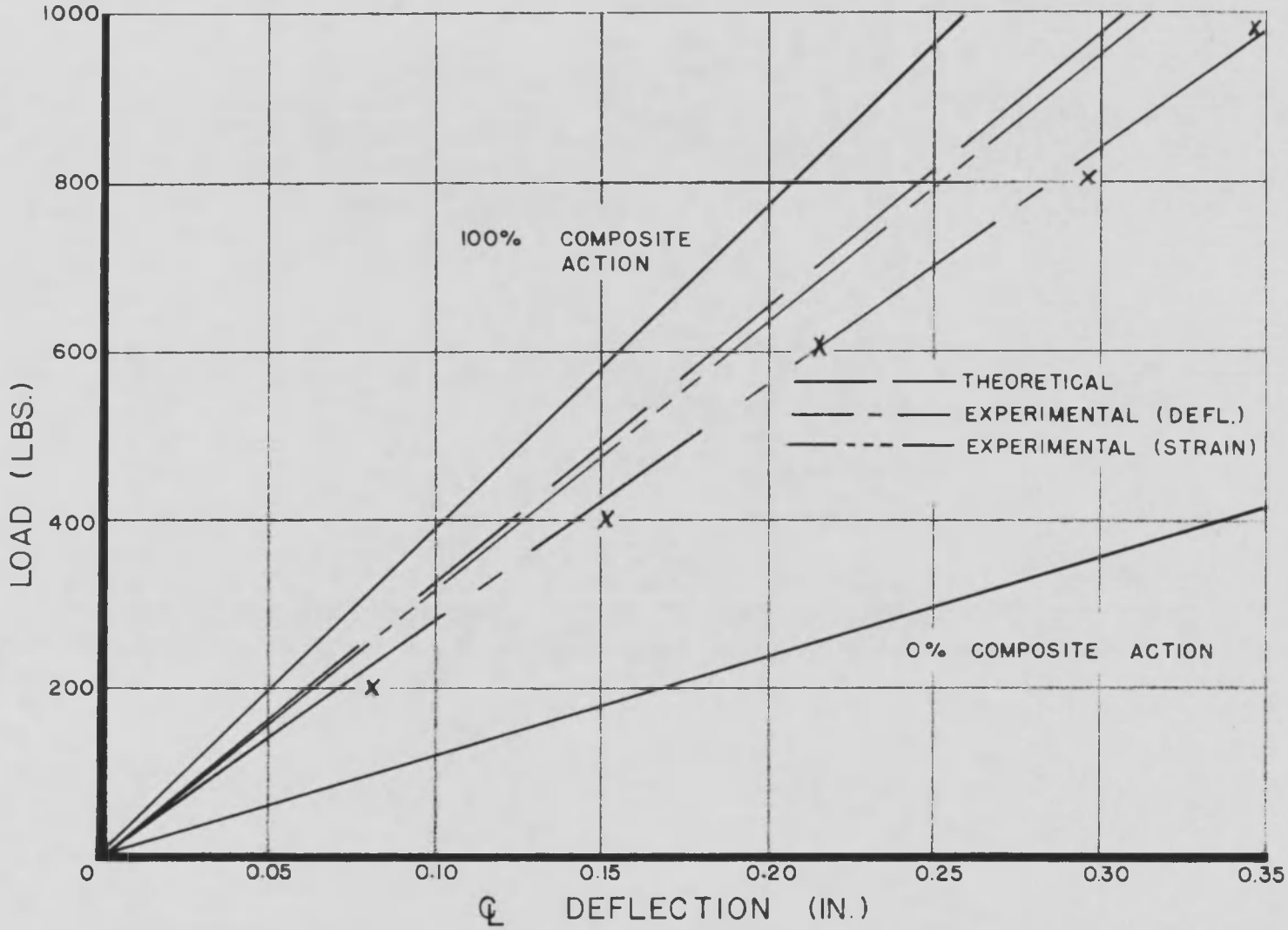


FIGURE 7 LOAD VS CENTERLINE DEFLECTION EPOXY CENTRAL LAYER (E=80,000 PSI), MID POINT LOADING

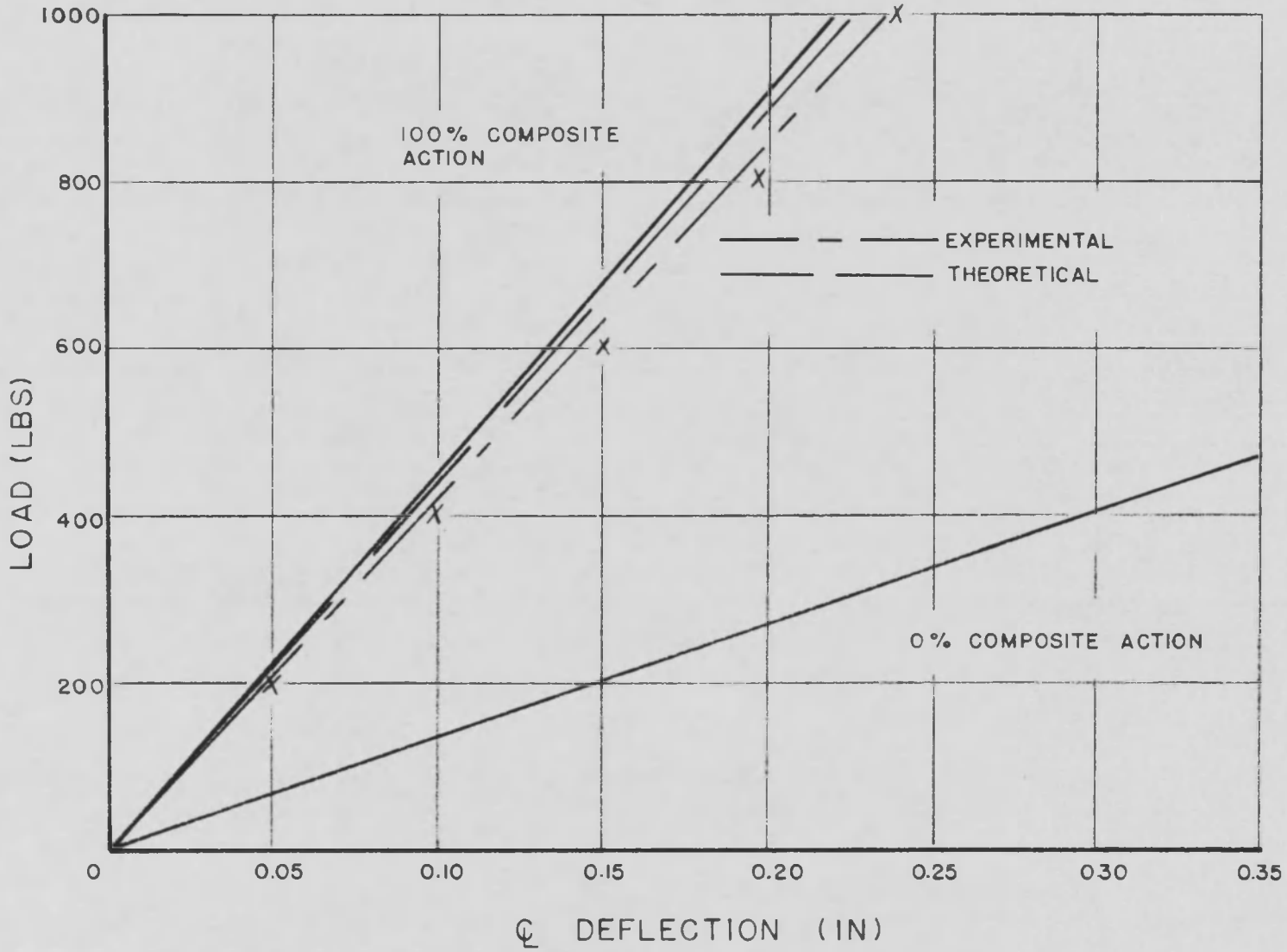


FIGURE 8 LOAD VS CENTERLINE DEFLECTION: EPOXY CENTRAL LAYER (E= 675,000 PSI) , THIRD POINT LOADING

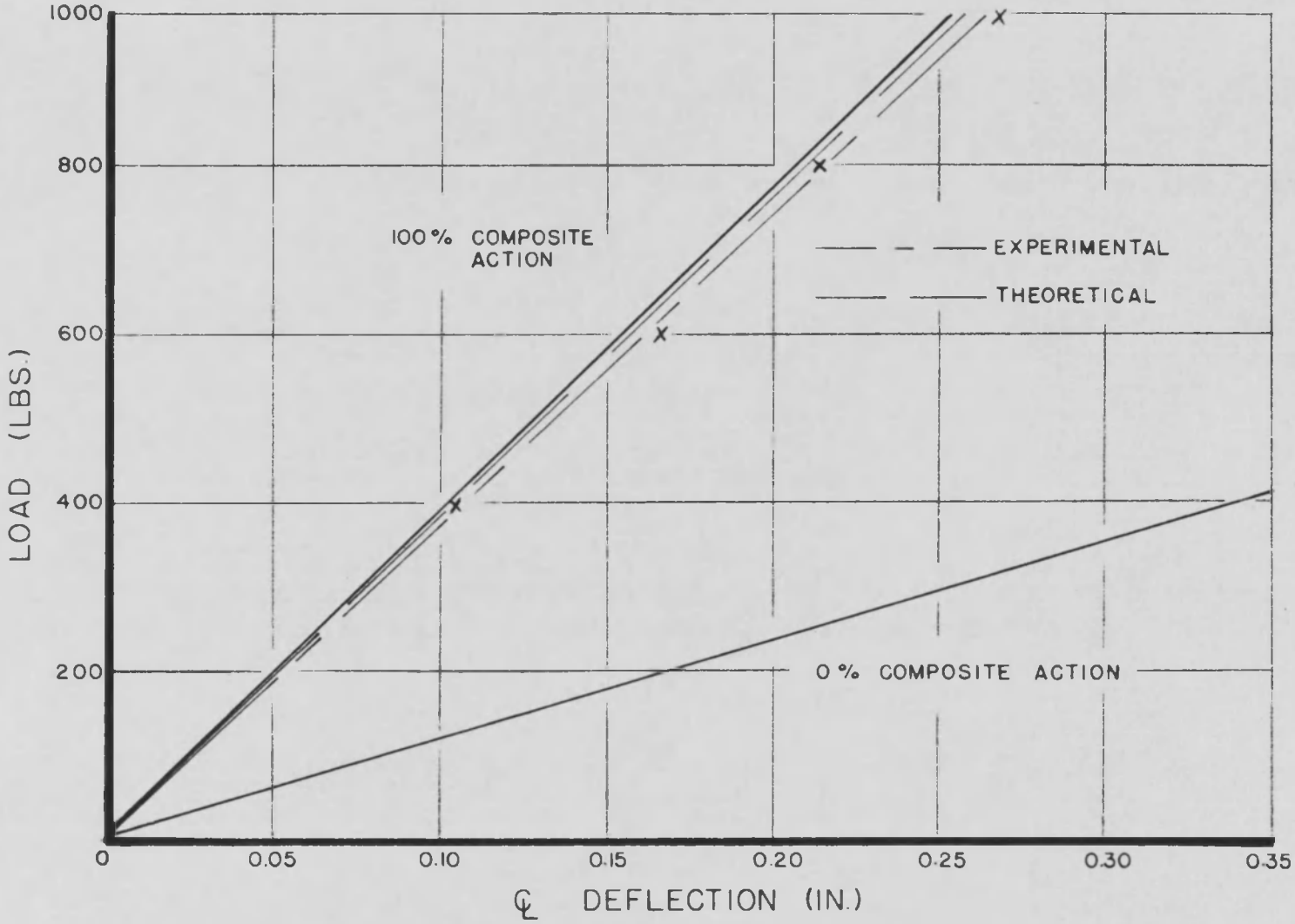


FIGURE 9 LOAD VS CENTERLINE DEFLECTION : EPOXY CENTRAL LAYER (E = 675,000), MID POINT LOADING.

CHAPTER 4

EXPERIMENTAL TEST RESULTS

4.1. Introduction

Three composite three material beams were constructed, instrumented and tested. Two of the beams consisted of aluminum, steel, and an epoxy resin while the other beam was of aluminum, steel and rubber. The results from the experimental tests were compared to the results predicted by the mathematical solution.

4.2. Description of a Model Beam

A composite three material beam was constructed of an aluminum rolled structural shape, steel and epoxy resin. The steel and aluminum constituted the top and bottom layers respectively whereas the epoxy resin constituted the central layer. A standard 3I2.59 rolled aluminum structural shape was used while the dimensions of the steel and epoxy were both 1/2 by 2 inches. The beam was 2 inches wide and 4 inches in total depth. The total length of the beam was 120 inches. During testing the beam was simply supported with a span length of 108 inches. The material constants (Modulus of Elasticity and Poisson's Ratio) for the steel and aluminum were taken from a handbook whereas the modulus of elasticity for the epoxy resin was obtained by testing tensile specimens, and Poisson's ratio was taken from experimental tests conducted by Endebrock (1964). The tensile specimens were molded at the

time the beam was constructed and kept in the same environment as the beam.

4.3. Description of Test

The model beam was simply supported on knife edge supports 108 inches apart. The beam was positioned on the supports such that the steel constituted the top layer. Instrumentation consisted of electric strain gages and 0.001 dial deflection gages. The electric strain gages were placed on the top and bottom of both the steel and aluminum layers at the midpoint and quarter points of the span (27, 54 and 81 inches in this case). The dial deflection gages were mounted on the outer surface of the aluminum rolled structural shape and placed at the midpoint and quarter points of the span length. A single concentrated load at midspan and a symmetrically placed two point loading located at 36 and 72 inches was used. The load was measured by means of a mechanically driven Tinius Olsen testing machine. The mathematical solution applied only in the elastic range; hence, during testing the maximum load applied was kept below the yield load. Several views of the model beam and test setup are shown in Figures 22 through 24.

4.4. Discussion and Comparison of Results

The results predicted by the mathematical solution and those obtained by experimental tests are shown in Table 1 and Figures 4 through 9 and 10 through 21. Additional results which are predicted by the mathematical solution but which were not measured in the experimental tests are shown in the Appendix in the Example Output. The maximum

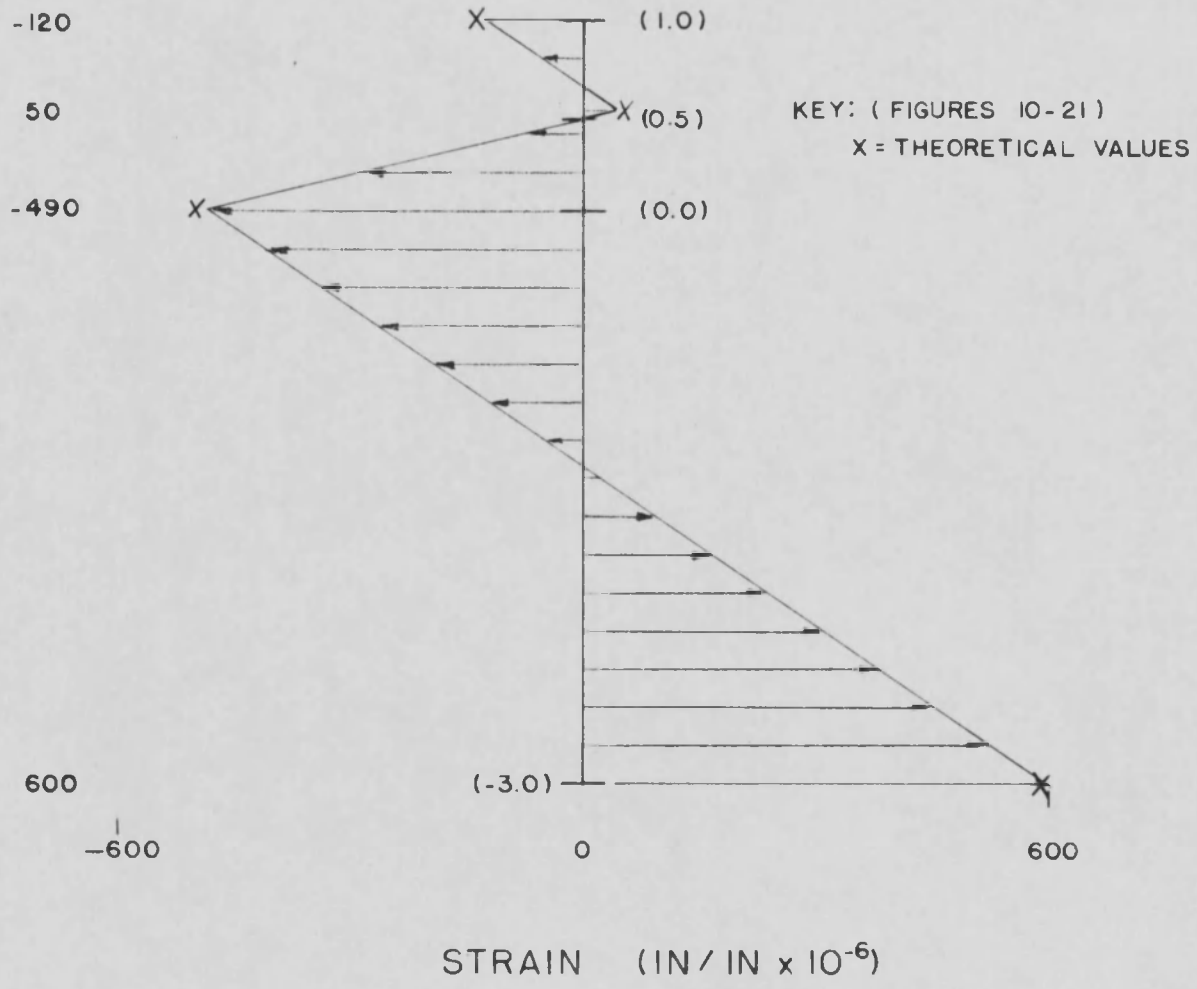


FIGURE 10 EXPERIMENTAL CROSS-SECTIONAL STRAINS FOR BEAM WITH RUBBER CENTRAL LAYER AT QUARTER POINT WITH THIRD POINT LOADING.

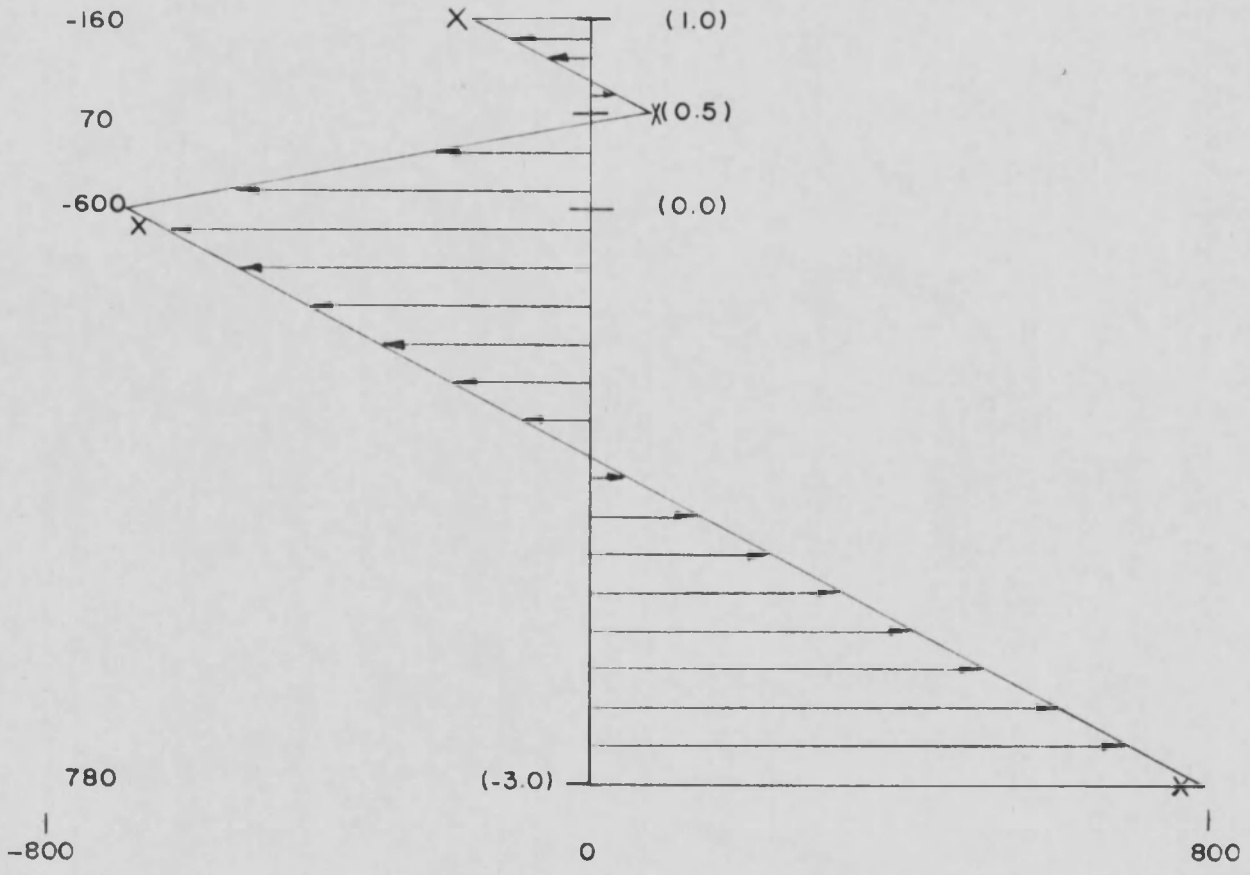


FIGURE 11 EXPERIMENTAL CROSS-SECTIONAL STRAINS FOR BEAM WITH RUBBER CENTRAL LAYER AT MID POINT UNDER THIRD POINT LOADING.

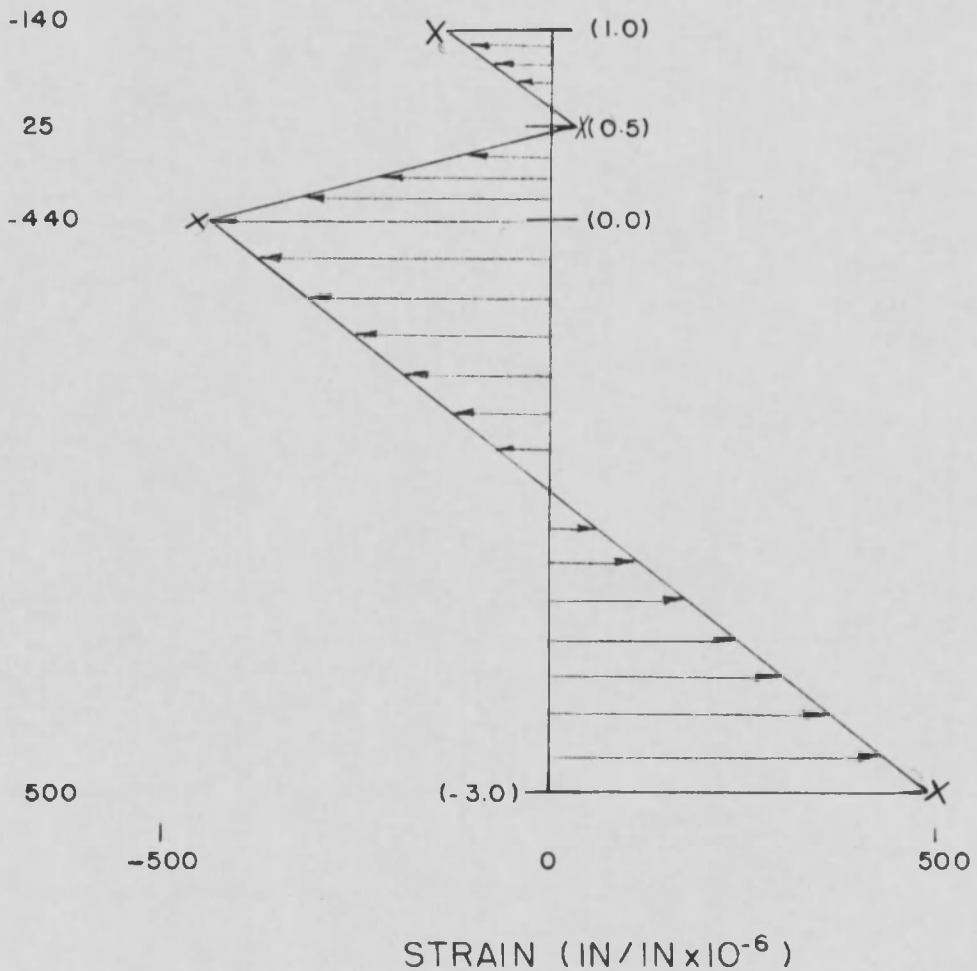


FIGURE 12 EXPERIMENTAL CROSS-SECTIONAL STRAINS FOR BEAM WITH RUBBER CENTRAL LAYER AT QUARTER POINT UNDER MID POINT LOADING.

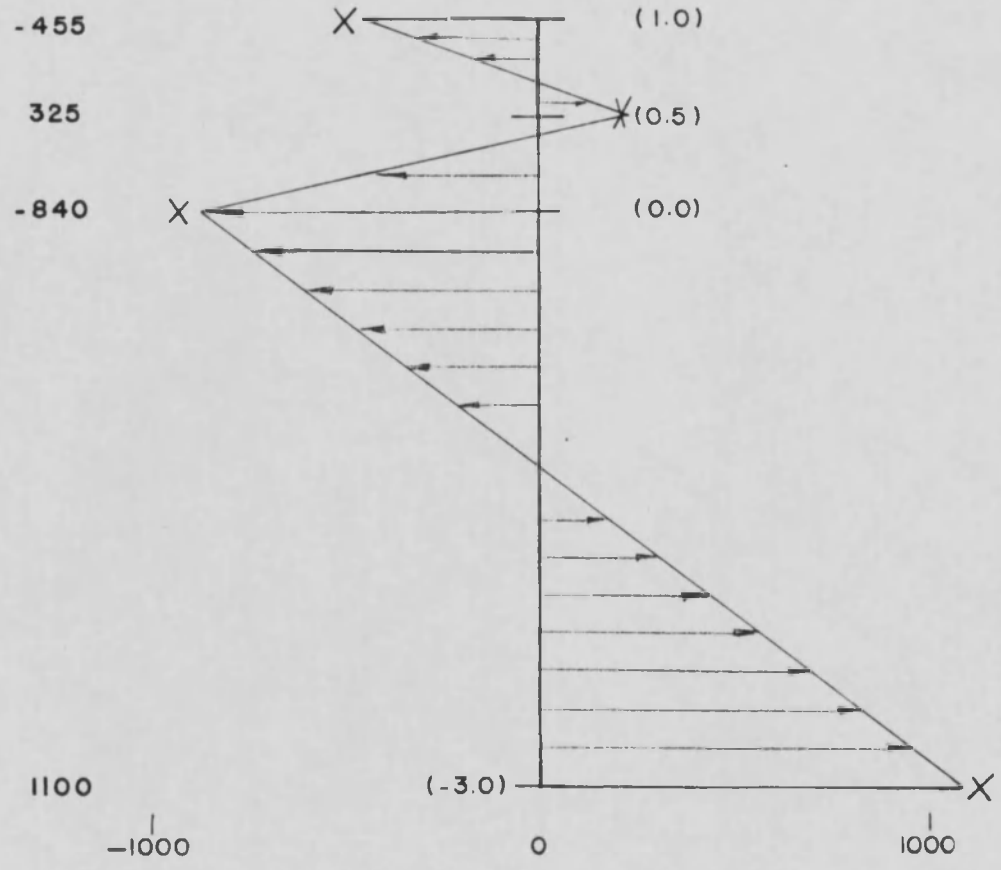


FIGURE 13 EXPERIMENTAL CROSS-SECTIONAL STRAINS FOR BEAM WITH RUBBER CENTRAL LAYER AT MID POINT UNDER MID POINT LOADING.

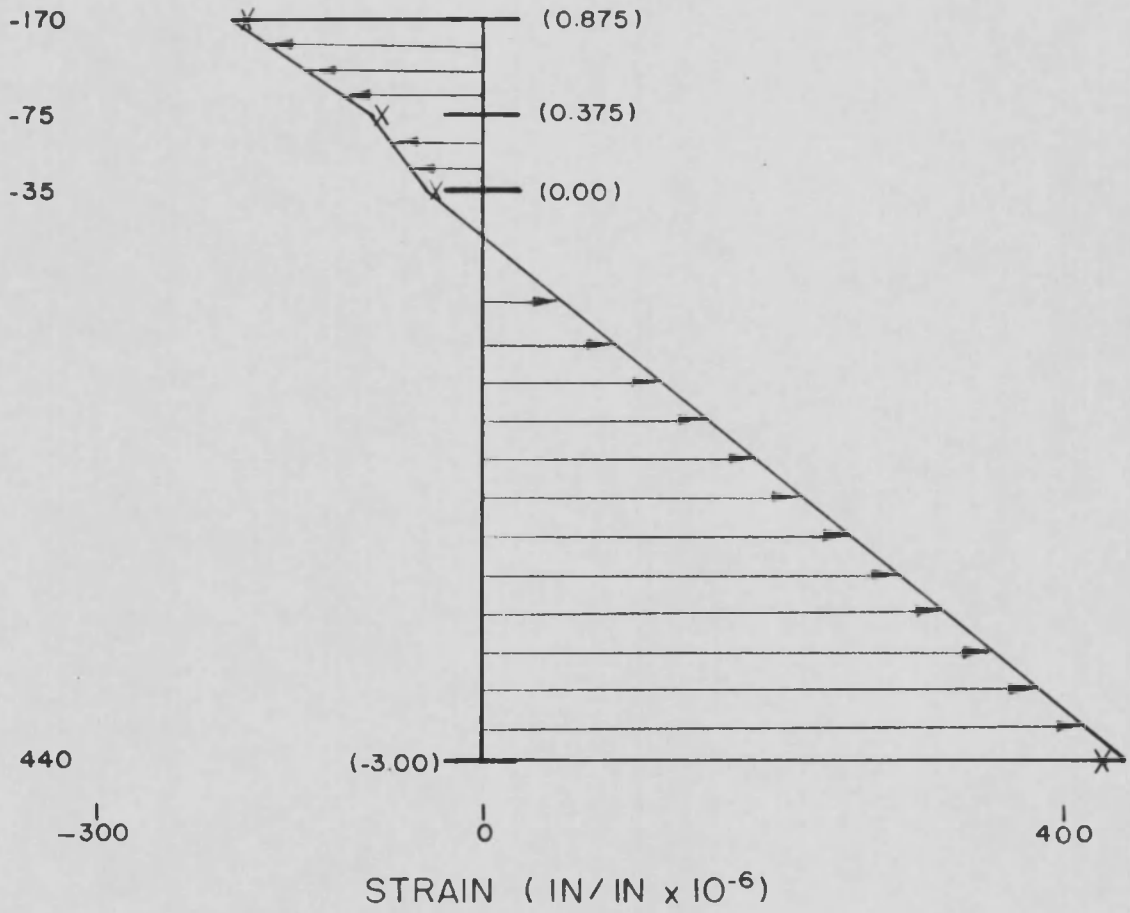


FIGURE 14 EXPERIMENTAL CROSS-SECTIONAL STRAINS EPOXY CENTRAL LAYER (E = 80,000 PSI) AT QUARTER POINT UNDER THIRD POINT LOADING.

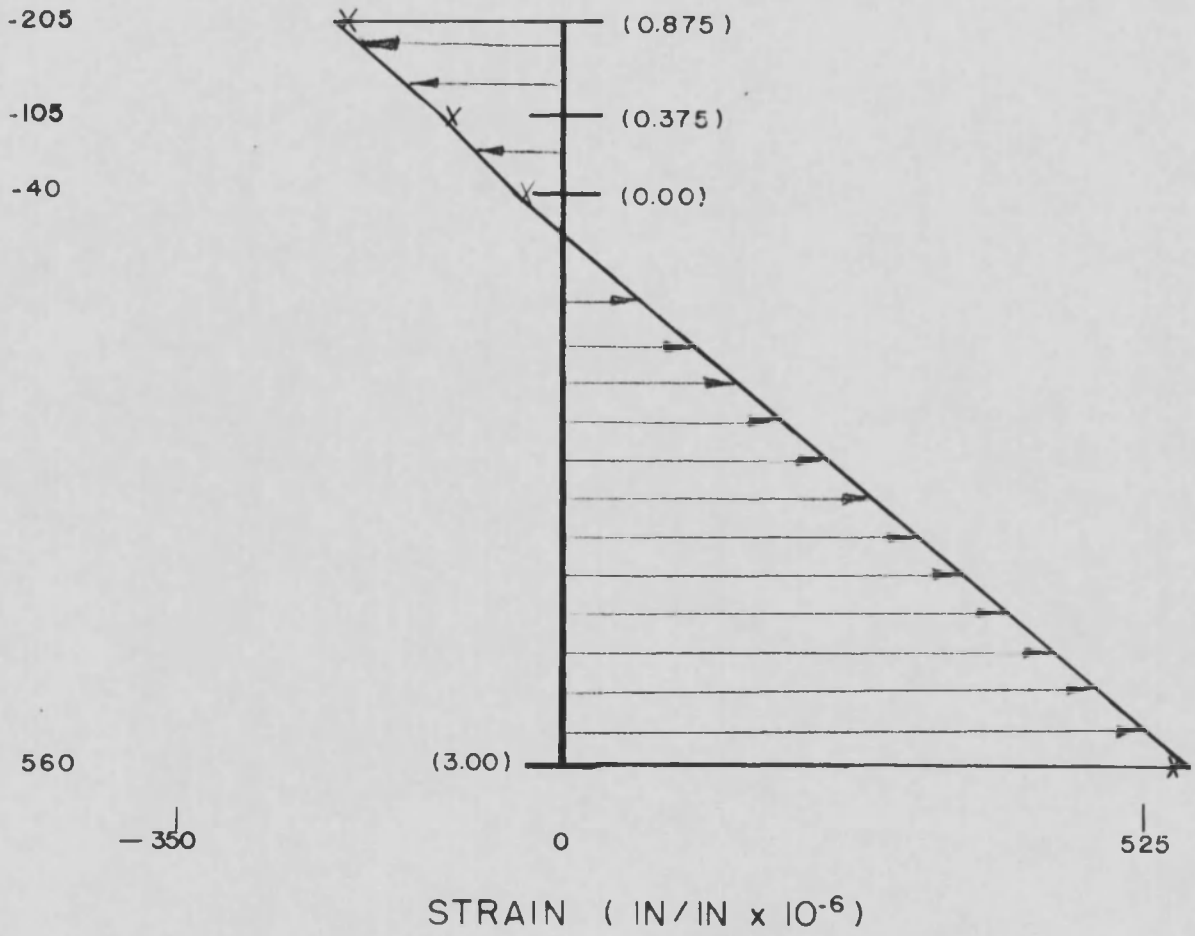


FIGURE 15 EXPERIMENTAL CROSS-SECTIONAL STRAINS WITH EPOXY CENTRAL LAYER (E = 80,000 PSI) AT MID POINT UNDER THIRD POINT LOADING.

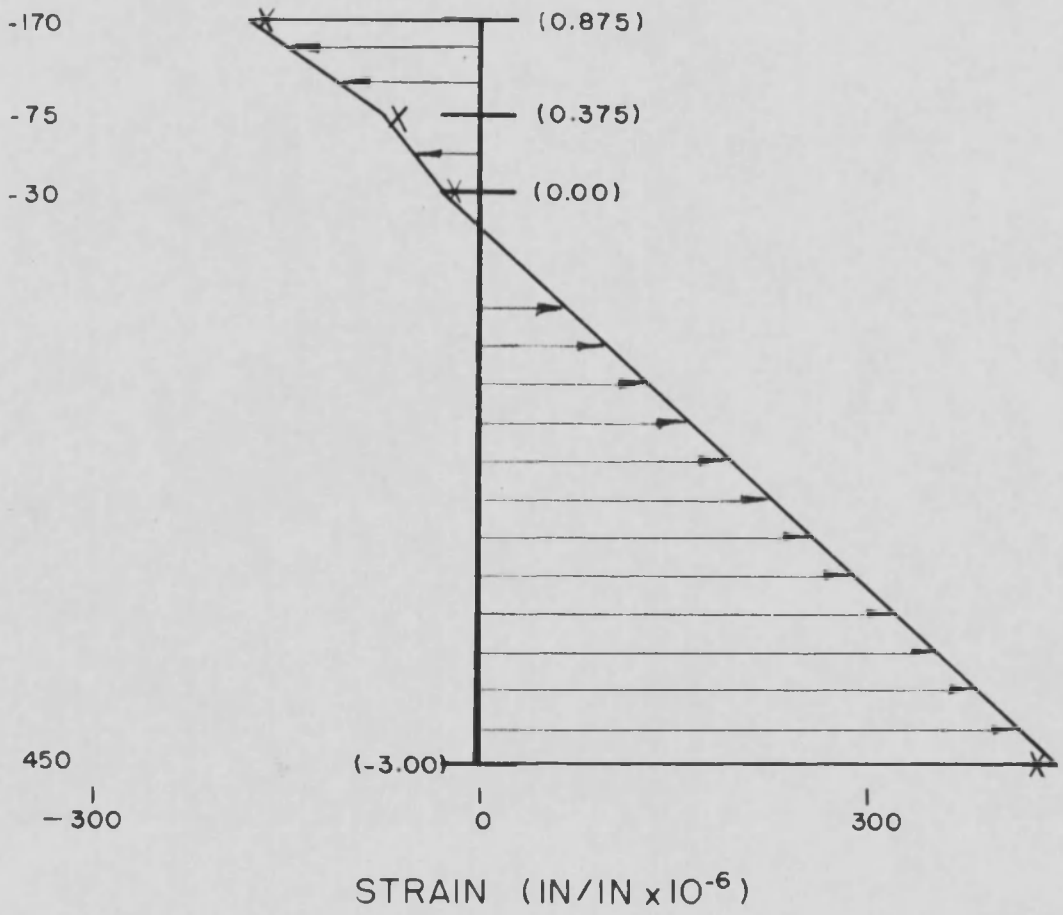


FIGURE 16 EXPERIMENTAL CROSS-SECTIONAL STRAINS WITH EPOXY CENTRAL LAYER (E = 80,000 PSI) AT QUARTER POINT UNDER MID POINT LOADING.

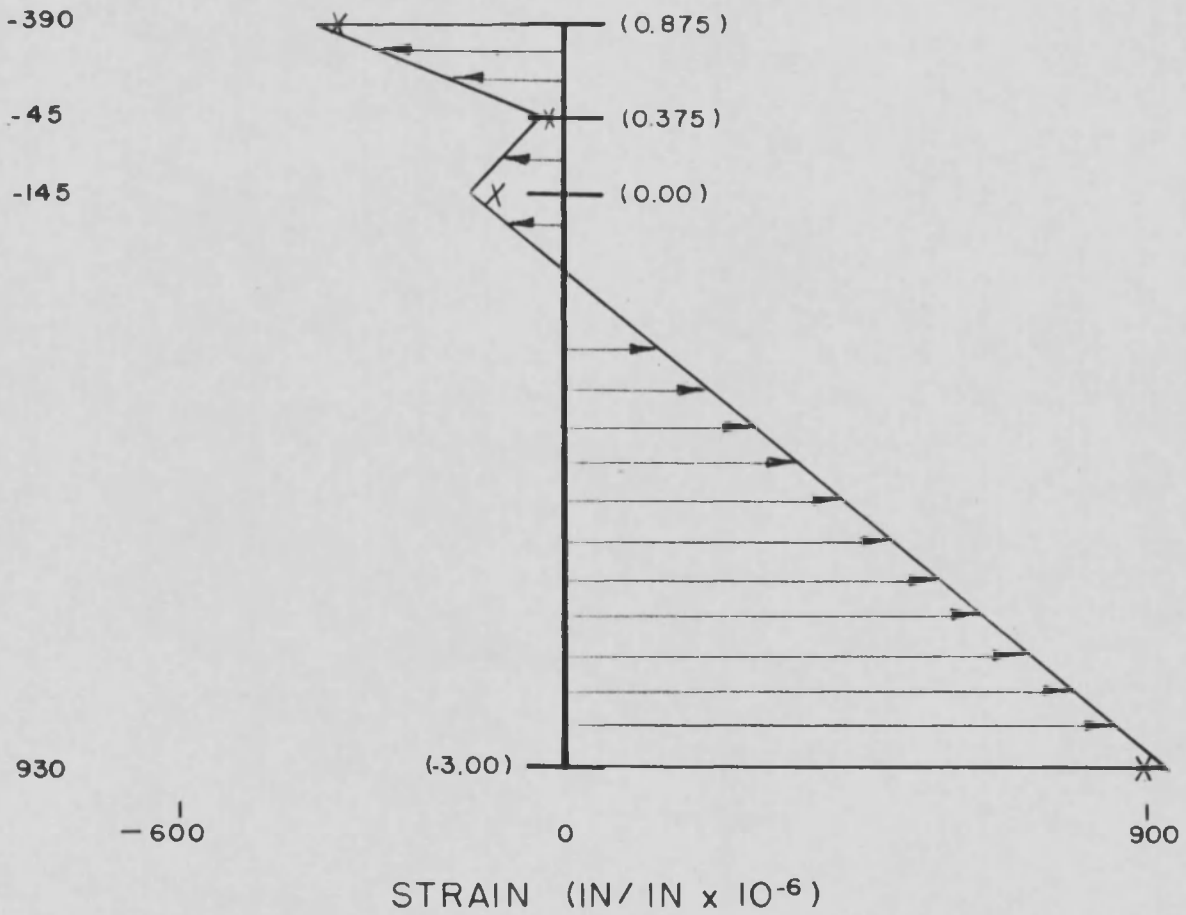


FIGURE 17 EXPERIMENTAL CROSS-SECTIONAL STRAINS WITH EPOXY CENTRAL LAYER (E=80,000 PSI) AT MID POINT UNDER MID POINT LOADING.

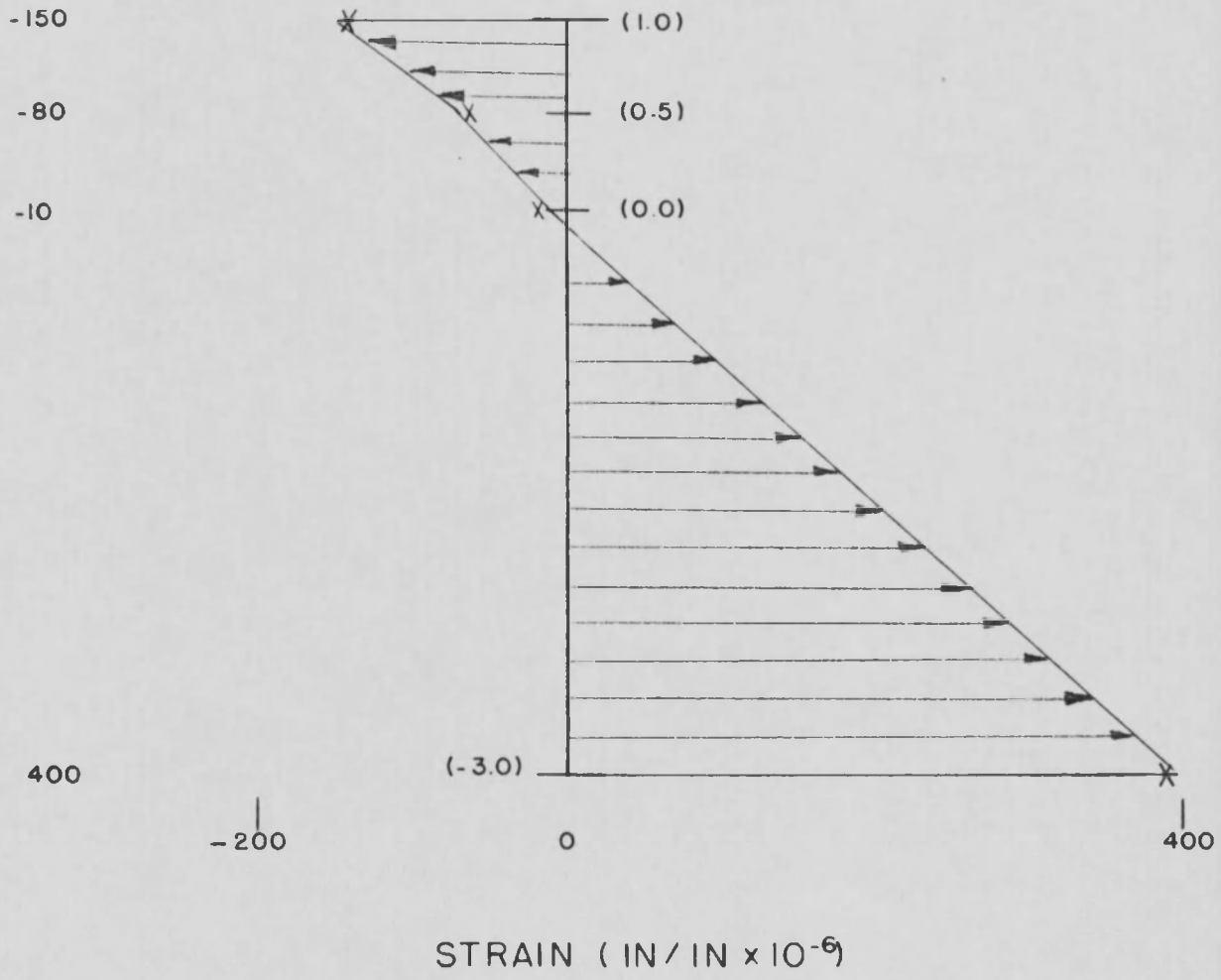


FIGURE 18 EXPERIMENTAL CROSS-SECTIONAL STRAINS FOR BEAM WITH EPOXY CENTRAL LAYER (E = 675,000 PSI) AT QUARTER POINT UNDER THIRD POINT LOADING.

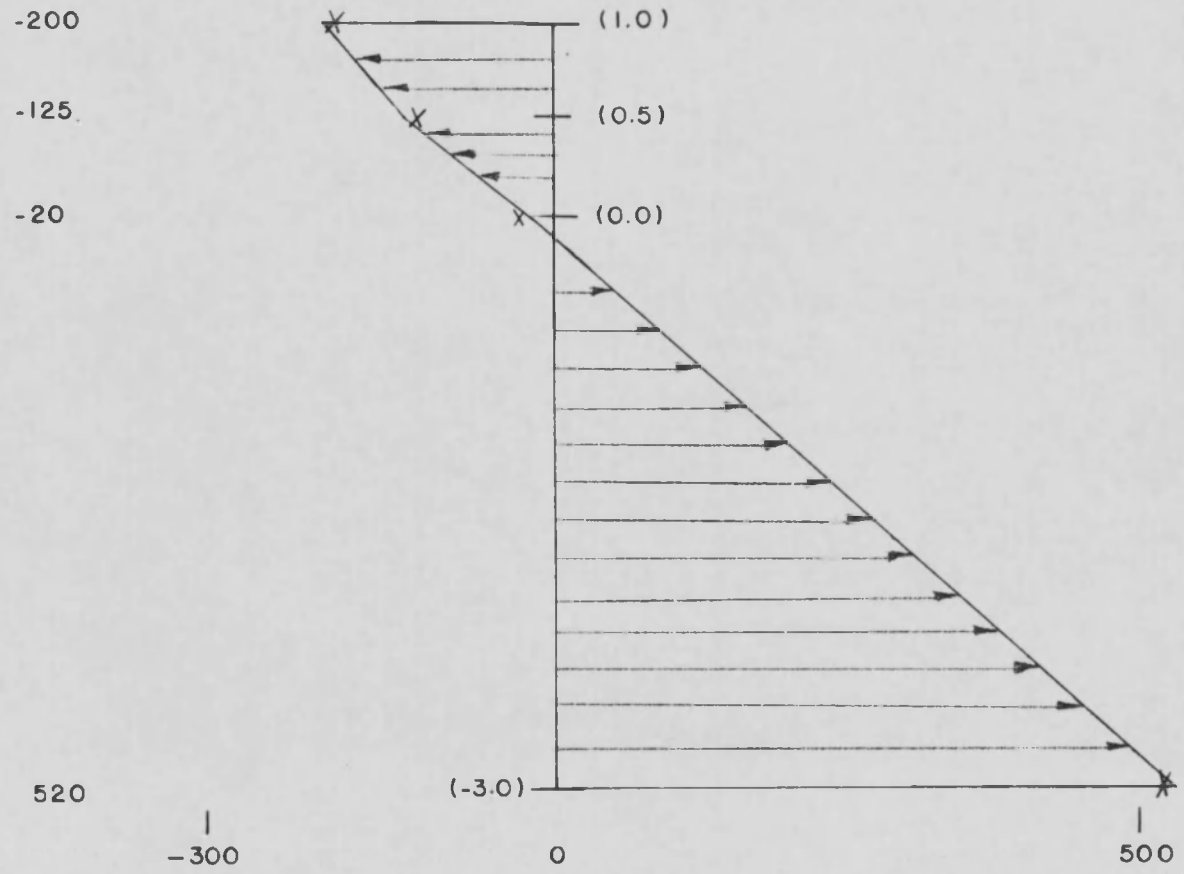


FIGURE 19 EXPERIMENTAL CROSS-SECTIONAL STRAINS FOR BEAM WITH EPOXY CENTRAL LAYER (E=675,000 PSI) AT MID POINT UNDER THIRD POINT LOADING.

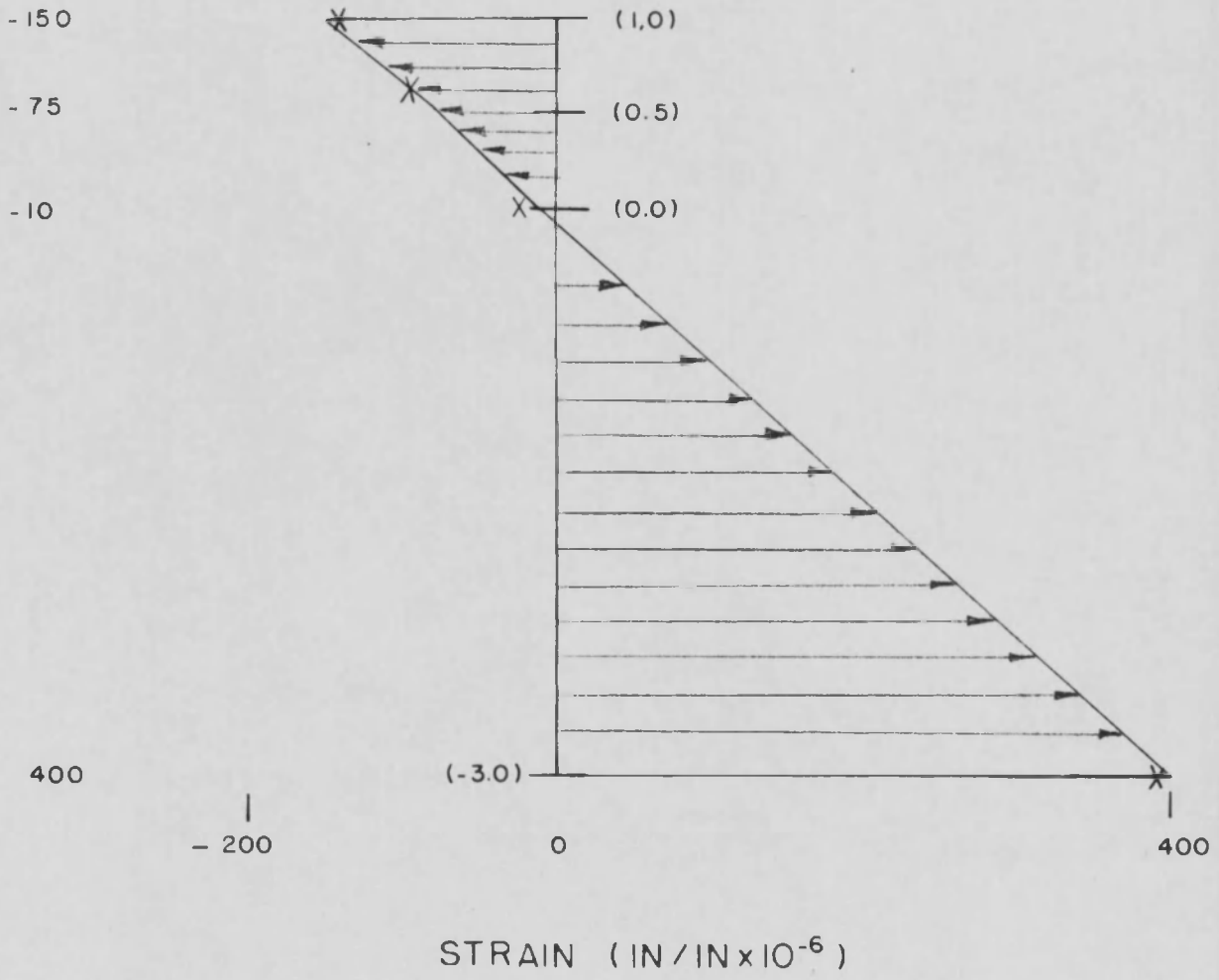


FIGURE 20 EXPERIMENTAL CROSS-SECTIONAL STRAINS FOR BEAM WITH EPOXY CENTRAL LAYER (E = 675,000 PSI) AT QUARTER POINT UNDER MID POINT LOADING.

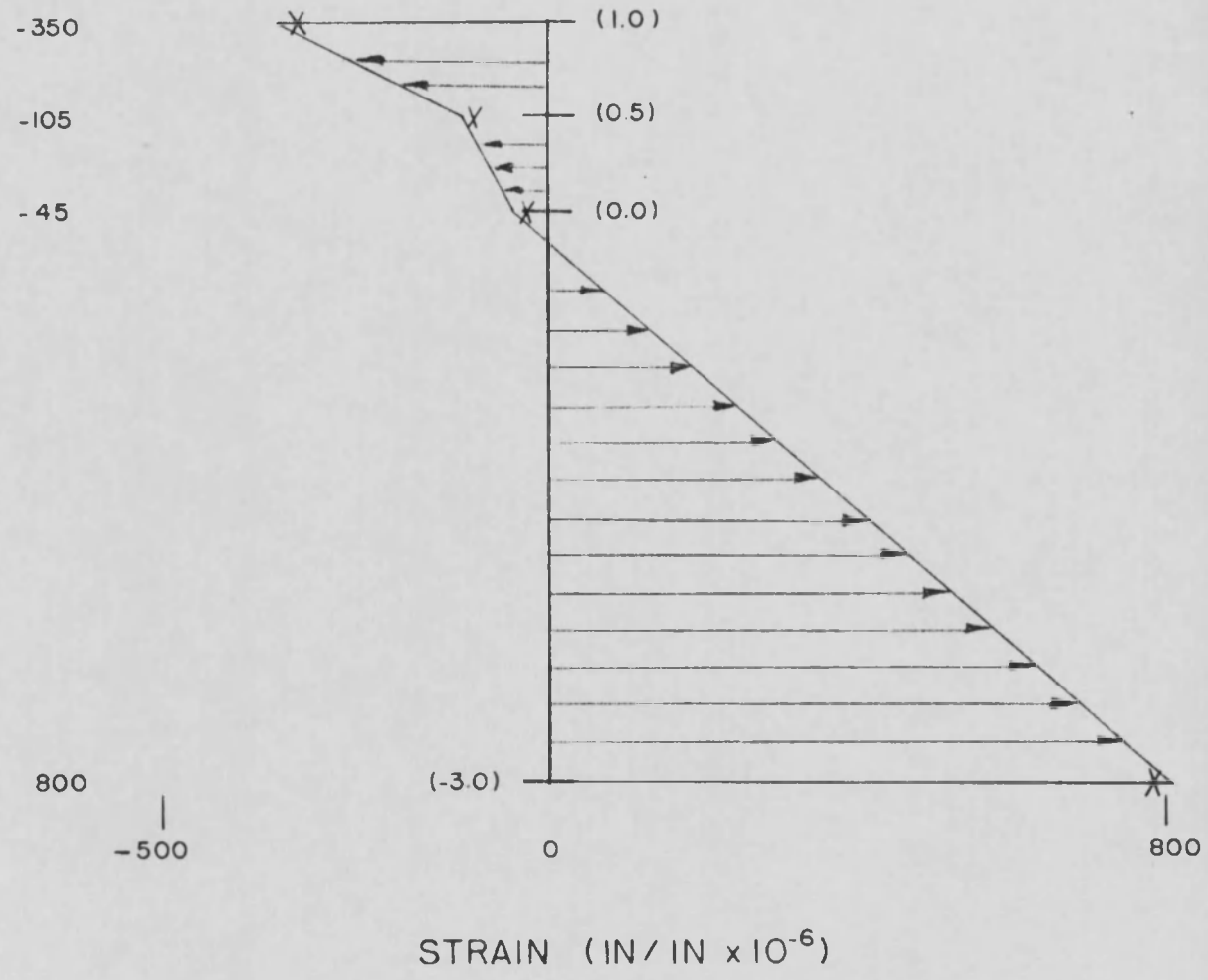


FIGURE 21 EXPERIMENTAL CROSS-SECTIONAL STRAINS FOR BEAM WITH EPOXY CENTRAL LAYER (E = 675,000 PSI) AT MID POINT UNDER MID POINT LOADING.

load applied in the experimental tests was 1000 pounds, and the theoretical results were obtained by multiplying the respective values of the computer output by the value of the reaction for the particular type loading.

The theoretical and experimental load versus deflection curves are shown in Figures 4 through 9 from which the respective composite action can also be determined by comparing these curves to the load deflection curves obtained by calculating a deflection using elementary methods for both full and zero composite action. From Table 1 and Figures 4 through 9 it was noted that the correlation between the theoretical and experimental results was very good. In fact, the differences between the theoretical and experimental maximum values were less than 4 percent for the two beams whose central layer consisted of a rubber material for one and an epoxy solution ($E = 675{,}000$ psi) for the other under both loading conditions. For the beam with a central layer of epoxy ($E = 80{,}000$ psi), the correlation between the theoretical and experimental results was also within 4 percent for the strain values; however, it was a minimum of 6 percent and a maximum of 9 percent for those values of the deflections. This, however, is not an excessive amount and could be attributed to the variant characteristics of the epoxy solutions used in this experimental test.

The experimental cross-sectional strains for each beam and for each type loading at the quarter and midpoints were also plotted as shown in Figures 10 through 21. From these figures the composite action for the various beams and types of loading can be observed.

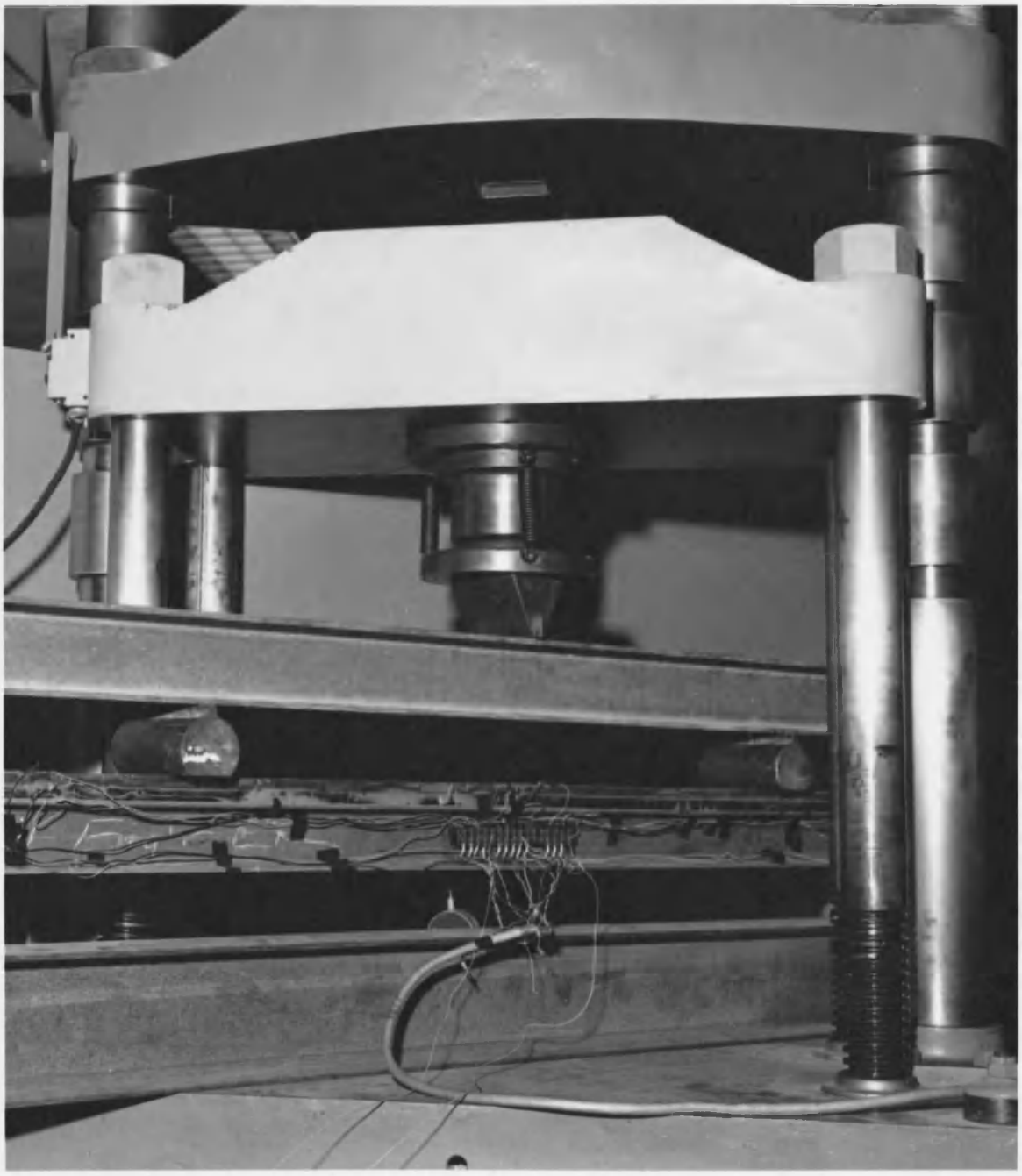


FIGURE 22 TEST APPARATUS

FIGURE 23 INSTRUMENTATION OF MODEL BEAM AND MEASURING EQUIPMENT

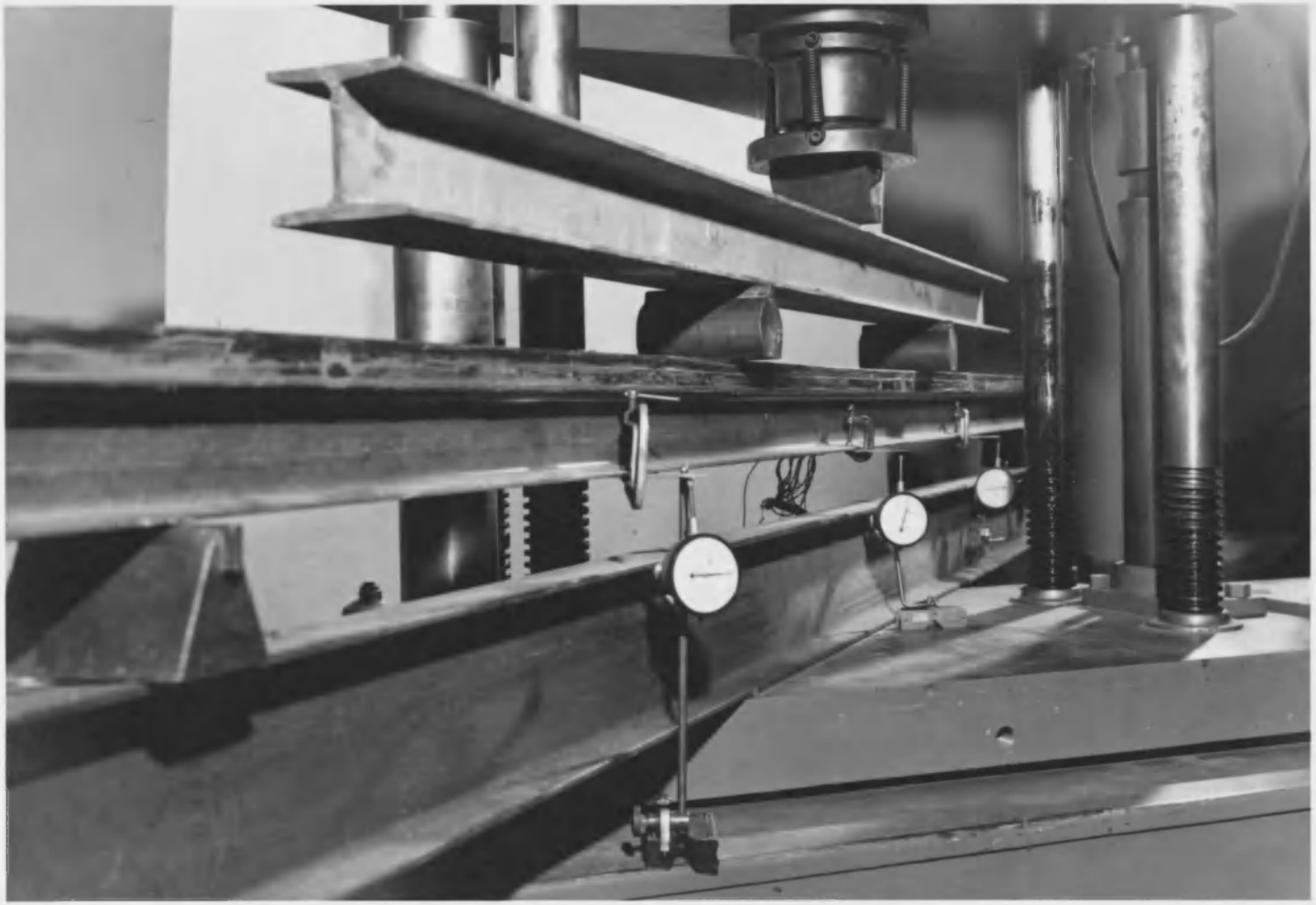


FIGURE 24 THIRD POINT LOADING WITH DEFLECTION GAGES

CHAPTER 5

SUMMARY AND CONCLUSIONS

5.1. Summary

A mathematical solution to the plane stress composite three material simply supported beam was developed for the case where one of the elements is a structural shape. The mathematical solution was obtained by modifying a solution for the case where all the elements were rectangular in cross section (Endebrock, 1964). Endebrock's solution consisted of generating a stress function for each material in the form of a product of two single independent variable functions. The stress functions were selected such that the stresses and displacements determined from the stress function satisfied the boundary conditions.

The composite three material beam was reduced to an "equivalent" rectangular beam using certain concepts of transformed sections. To form the beam, the three rectangular sections were joined or connected. In the mathematical model this was accomplished by equating the shearing stresses and displacements at the interfaces of the adjoining layers.

In transforming the composite three material beam into an "equivalent" rectangular beam only those shear distortions present in the central layer were considered to be important and further analysis showed that consideration of the shear distortion effects

present in the bottom layer had a negligible effect on the results predicted by the mathematical solution.

5.2. Conclusions

Three composite three material beams, in which the material properties and dimensions of the top and bottom layers remained the same while those of the central layer were changed, were constructed, tested and examined. In analyzing the material properties of the central layer of the three beams, it was noted that the modulus of elasticity had a variable effect on the composite action of the beam. A change of 10,000 psi within the range of 10,000 psi to 100,000 psi had a considerable effect while the same change within the range of 100,000 psi to 700,000 psi had little effect on the composite action.

The composite action for the three beams under third and mid-point loading was calculated and plotted. The theoretical and experimental results were shown to be in good correlation. All comparable quantities obtained from the mathematical solution and the experimental tests differed by at most 4 percent for the two beams whose central layer consisted of a rubber layer for one and an epoxy solution (E = 675,000 psi) the other, under both loading conditions. The correlation of the results for the beam with an epoxy central layer (E = 80,000 psi) under both types of loading was also within 4 percent for the strain values; however it was a minimum of 6 percent and a maximum of 9 percent for those values of the deflections. This was not considered an excessive amount and could be considered to be within the

acceptable limits of the test and specimen preparation; thus verifying the validity of the mathematical solution.

REFERENCES

1. Endebrock, Elton G., An Elasticity Solution of the Multi-Material Beam, 1964.

2. Timoshenko, S. and J. Goodin, Theory of Elasticity, McGraw-Hill Book Company, Inc., New York, 1951.

3. Timoshenko, S. and D. H. Young, Elements of Strength of Materials, D. Van Nostrand Company, Inc., New Jersey, 1962.

APPENDIX

FRAGMENT OF COMPUTER PROGRAM

```
***  GEIGER
*    COMPILE FORTRAN , EXECUTE FORTRAN


      DIMENSION LABEL (176)
      READ 68,LABEL
   68 FORMAT (16A5)
   52 READ 88,1,B1,B2,B3,E1A,E2,E3A,U1,U2,U3,H1,H2,H3,ESP,
     1SSI,BTS1,BTS3,ASC,UUS
C     UUS EQUALS TWEB HERE (SHEAR DISTORTION)
   88 FORMAT (8F10.0)
      READ 89, RS,TT,CC,TL,TL1,TL2,SL
   89 FORMAT (7A5)
      PRINT 168,LABEL
  168 FORMAT (16A5)
      BS1 = BTS1*E1A/E2
      BS3 = BTS3*E3A/E2
      AT13 = BS1*(H3 - H2) + BS3*H2
      TM1 = (H3 - H2)*BS1*((H3 - H2)/2. + H2)
      TM3 = H2**2*BS3/2.
      YBAR = (TM1 + TM3 - (ASC*H1/2.))/(ASC + AT13)
      SSIT = SSI + ASC*(H1/2. + YBAR)**2
      SSIT1 = SSIT*YBAR
      GA = TM1 +TM3 - YBAR*AT13
      HA = (SSIT/2. - (GA*YBAR))*3./GA
      HB = -(SSIT1 - (GA*YBAR)*YBAR)*3./GA
      HES = (HA + SQRTF(HA**2 - 4.*HB))/2.
      BES = GA/(YBAR*HES + HES**2/2.)
      E1 = BTS1*E1A/BES
      E3 = BTS3*E3A/BES
      U2 = ((1.5*BES*HES*(1.+U2))/(H1*UUS))-1.
C     U2 ACCOUNTS SHEAR DISTORTION BOTTOM LAYER
```

EXAMPLE COMPUTER OUTPUT

DESCRIPTION OF TERMS
Q = HORIZONTAL COORDINATE, Z = VERTICAL COORDINATE
BES = EQUIVALENT RECTANGULAR WIDTH
HES = EQUIVALENT RECTANGULAR HEIGHT
YBAR = CENTROIDAL AXIS OF SECTION
STRXS = HORIZONTAL STRESS, STRYS = NORMAL STRESS
STIX = STRAIN IN HORIZ. DIRECTION
STIY = STRAIN IN NORMAL DIRECTION
SHRS = SHEAR STRESS, TAXY = SHEAR STRAIN
UUS = DISPLACEMENT IN U DIRECTION
VVS = DISPLACEMENT IN V DIRECTION

Q	Z	STRXS TAXY	STRYS SHRS	STIX UUS	STIY VVS
0.00	-3.61	0.000E 00	0.000E 00	0.000E 00	0.000E 00
		-6.870E-13	-9.905E-07	-4.573E-05	0.000E 00
0.00	-3.00	0.000E 00	0.000E 00	0.000E 00	0.000E 00
		-1.355E-07	-1.953E-01	-3.777E-05	0.000E 00
0.00	0.00	0.000E 00	0.000E 00	0.000E 00	0.000E 00
		-4.301E-07	-6.201E-01	9.375E-07	0.000E 00
0.00	0.00	0.000E 00	0.000E 00	0.000E 00	0.000E 00
		-2.664E-06	-6.201E-01	9.375E-07	0.000E 00
0.00	0.25	0.000E 00	0.000E 00	0.000E 00	0.000E 00
		-2.658E-06	-6.187E-01	4.002E-06	0.000E 00
0.00	0.50	0.000E 00	0.000E 00	0.000E 00	0.000E 00
		-2.642E-06	-6.150E-01	7.066E-06	0.000E 00
0.00	0.50	0.000E 00	0.000E 00	0.000E 00	0.000E 00
		-5.331E-08	-6.151E-01	7.055E-06	0.000E 00
0.00	0.75	0.000E 00	0.000E 00	0.000E 00	0.000E 00
		-3.225E-08	-3.721E-01	1.025E-05	0.000E 00
0.00	1.00	0.000E 00	0.000E 00	0.000E 00	0.000E 00
		2.014E-11	2.324E-04	1.344E-05	0.000E 00
27.00	-3.61	9.476E 00	-1.052E-05	9.476E-07	-2.339E-06
		-1.649E-12	-2.378E-06	-3.294E-05	-3.219E-04
27.00	-3.00	7.842E 00	-1.095E-04	7.842E-07	-1.936E-06
		-1.347E-07	-1.942E-01	-2.718E-05	-3.232E-04
27.00	0.00	-2.179E-01	-6.366E-03	-2.022E-08	5.315E-08
		-4.274E-07	-6.161E-01	6.586E-07	-3.135E-04
27.00	0.00	-1.423E-02	-1.284E-03	-2.022E-08	7.581E-09
		-2.647E-06	-6.161E-01	6.586E-07	-3.135E-04
27.00	0.25	-6.071E-02	-1.500E-03	-8.894E-08	3.825E-08
		-2.637E-06	-6.137E-01	2.851E-06	-3.135E-04
27.00	0.50	-1.073E-01	-1.757E-03	-1.579E-07	6.896E-08
		-2.620E-06	-6.097E-01	5.043E-06	-3.134E-04

Q	Z	STRXS TAXY	STRYS SHRS	STIX UUS	STIY VVS
27.00	0.50	-4.632E 00	-1.757E-03	-1.544E-07	4.627E-08
		-5.285E-08	-6.098E-01	5.035E-06	-3.134E-04
27.00	0.75	-6.687E 00	-3.920E-03	-2.229E-07	6.674E-08
		-2.698E-08	-3.113E-01	7.353E-06	-3.134E-04
27.00	1.00	-8.742E 00	-5.965E-03	-2.914E-07	8.722E-08
		1.432E-11	1.653E-04	9.672E-06	-3.134E-04
54.00	-3.61	1.264E 01	-1.947E-05	1.264E-06	-3.119E-06
		-5.588E-21	-8.057E-15	2.602E-12	-4.533E-04
54.00	-3.00	1.046E 01	-2.811E-04	1.046E-06	-2.581E-06
		-1.513E-14	-2.181E-08	2.158E-12	-4.551E-04
54.00	0.00	-2.931E-01	-1.160E-02	-2.645E-08	7.119E-08
		-3.254E-14	-4.692E-08	-6.950E-14	-4.412E-04
54.00	0.00	-1.891E-02	-2.340E-03	-2.645E-08	9.138E-09
		-2.016E-13	-4.692E-08	-6.950E-14	-4.412E-04
54.00	0.25	-8.120E-02	-2.648E-03	-1.185E-07	5.021E-08
		-2.034E-13	-4.735E-08	-2.488E-13	-4.412E-04
54.00	0.50	-1.437E-01	-3.012E-03	-2.109E-07	9.135E-08
		-1.988E-13	-4.627E-08	-4.274E-13	-4.412E-04
54.00	0.50	-6.169E 00	-3.012E-03	-2.056E-07	6.159E-08
		-4.010E-15	-4.627E-08	-4.265E-13	-4.412E-04
54.00	0.75	-8.916E 00	-6.072E-03	-2.972E-07	8.896E-08
		-8.512E-15	-9.821E-08	-5.952E-13	-4.411E-04
54.00	1.00	-1.167E 01	-8.967E-03	-3.887E-07	1.164E-07
		-1.183E-18	-1.365E-11	-7.638E-13	-4.411E-04

THIRD PT. LOAD L= 108.0

TRANSFORMED ROLLED SECTION PROPERTIES

YBAR	BES	HES	H2
-0.079	0.504	3.608	0.50

H3	E1	E2	E3
1.000	1.487E 08	1.00E 07	3.35E 06

COMPOSITE ROLLED SECTION PROPERTIES

H1	H2	H3	E1A	E2
3.000	0.500	1.000	3.00E 07	1.00E 07

E3A	U1	U2	U3	ROLL SECTION
6.750E 05	0.30	0.33	0.45	3I2.59 Alum

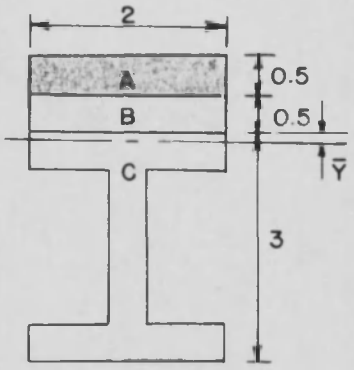
EXAMPLE ELEMENTARY CALCULATIONS

MODULUS OF ELASTICITY

Material A - 30×10^6 psi

Material B - 7×10^5 psi

Material C - 10×10^6 psi



MODULAR RATIOS

$$B_a = \frac{2E_a}{E_c} \qquad B_b = \frac{2E_b}{E_c}$$

3I2.59

$A_I = 2.21$

$I_I = 2.93$

DETERMINATION OF NEUTRAL AXIS

$$\bar{y} = \Sigma Ay \Big/ \Sigma A$$

$$\bar{y} = \frac{(1/2)(14/100)(1/4)+(1/2)(6)(3/4)-(2.21)(1.5)}{(1/2)(14/100)+(1/2)(6)+2.21}$$

$$\bar{y} = -0.0982$$

MOMENT OF INERTIA

$$I = I_o + Ad^2$$

$$I = \frac{(6)(1/2)^3}{12} + (6)(1/2)(1.0982)^2 + \frac{(14/100)(1/2)^3}{12}$$

$$+ (14/100)(1/2)(.3482)^2 + 2.93 + (2.21)(1.4)^2$$

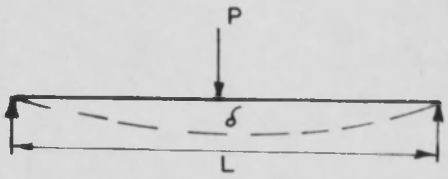
$$I = 9.56$$

EXAMPLE ELEMENTARY CALCULATIONS--Continued.

MAXIMUM DEFLECTIONS

Concentrated load at midspan

$$\delta = \frac{PL^3}{3E_c I}$$



Concentrated load symmetrically placed about midspan.

P = 500 LBS.

L = 108 IN.

$$\delta = \frac{(500)(108)^3}{(3)(10\times10^6)(9.56)} = .276$$

where δ is value of maximum deflection assuming 100% composite action.

PERCENT COMPOSITE ACTION

δ (0% composite action) = .875

δ (experimental or theoretical) = .367

$$\% \text{ composite action} = 100 - \frac{.367 - .276}{.875 - .276}$$

% composite action = 100 - 15.1 = 84.9%