MATHEMATICAL DESCRIPTION OF BALL MILL PERFORMANCE AS A FUNCTION OF FEED RATE

by

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ABSTRACT

An attempt has been made in this investigation to describe mathematically the grinding characteristics of an open-circuit ball mill. A mathematical model has been developed by using a series of well-stirred tanks to describe the mixing action of the mill. Another mathematical model based on a multiple-gradient approach which appears applicable at high feed rates is also presented. Derivations of the models and comparisons of experimental and computed results are presented. General equations are presented for each model.

INTRODUCTION

The purpose of this investigation was to develop a mathematical model of an open-circuit comminution system. The importance of this model is to gain a better understanding of the grinding action and flow characteristics of a ball mill for the purpose of analysis and application. This should allow one to extend the model to process dynamics of complex comminution systems for the purpose of control and optimization.

Within the last few years several investigators have made attempts at mathematical descriptions of a ball mill system. Their approaches have been relatively semi-theoretical and utilized a matrix algebra description of the system. Meloy and Bergstrom (1964) employed matrices describing the feed distribution, the probability of a particle being broken, and the size distribution of a broken particle to describe the output size distribution of a ball mill. Probability vectors for both particle size and feed rate were empirically defined in order to predict the output size distribution of a hypothetical circuit. Lynch (1965), using a similar approach, also tried to describe the comminution system.

Kelsall and Reid (1965) used breakage and residence time functions and proposed that the rate of grinding is a function of feed rate in another attempt to describe the grinding system. All of these investigators used a ball mill feed having a fixed size distribution or composition.

The mathematical description of a comminution system presented in this thesis differs from the attempts made by other investigators. The model is based upon defining a rate of grinding as a function of particle size and allowing for the mixing action within the mill. A commercial ore was selected for the grinding test and the effect of feed rate upon the grinding action within the mill was studied.

EXPERIMENTAL WORK

A. Materials

The ore used in the grinding tests was obtained on September 14, 1965 from the Mission Operation of the American Smelting and Refining Company. The mine is located 20 miles south of Tucson, Arizona. The ore was removed from the conveyor belt feeding the rod mills at the Mission Mill and had the size distribution shown in Table 1.

Table 1 - Size Distribution of Ore Obtained From the Mission Operation

Sieve Size	Percent Retained
-1", +3/4"	0.0
-3/4", +3/8"	64.2
-3/8", +3 mesh	25.5
-3 mesh, +4 mesh	7.2
-4 mesh, +6 mesh	1.6
-6 mesh	1.5
	100.0

This silicious limestone ore, called argillite by the Mission geologist, was chosen because of its grindability. The work index of this ore is 19-20 kw-hr per ton, while the normal work index of the ore processed by the Mission Operation is nine kw-hr per ton. Because of the high work index of the ore, a ball mill feed could be obtained containing a small amount of very fine material. The specific gravity of the ore, determined by conventional methods, is 2.77 ± 0.09 at 95 percent confidence limits.

The as-received ore was too coarse to feed to the pilot-scale ball mill being used, and had to be further reduced in size. Approximately three tons of the ore was cycled through a pilot-scale roll crusher and vibrating screen until it was 100 percent minus 1/4 inch. The crushed ore was thoroughly mixed by several cone and quartering steps and was finally divided into two cones. One quarter of each cone was used to fill a 55 gallon drum, approximately 600 pounds of ore. Every tenth shovel of material being added to a drum was composited to form a sample of the ore in that drum. Each of the samples representing the ore in the different drums was reduced by splitters to approximately 1000 grams. These samples were then screened in order to determine the size distribution of the feed material. The ball mill feed distribution is presented in Table 2.

Table 2 - Size Distribution of Ball Mill Feed

Size Fraction Tyler Mesh	X_i	Percent Retained	95% Confidence Limits, %
-3, +6	X_1	45.6	9.5
-6, +10	X_2	27.4	5.2
-10, +20	X_3	10.2	2.3
-20, +35	X_4	5.7	1.9
-35, +65	X_5	3.6	1.6
-65, +150	X_6	2.8	1.3
-100, +270	X_7	2.0	0.9
-270	X_8	2.7	1.1

The 95 percent confidence limits of the ball mill feed shown in Table 2 were based on the variance due to the different drums of ore, the variance due to sampling, and the variance due to screening. The size distributions for the different drums are shown in Appendix A. The screen analyses for determining the sampling and screening variances are presented in Appendix B.

B. Apparatus

The experimental grinding data were developed in a 16 in. diameter, 16 in. long Denver Ball Mill having a grate discharge. The ball mill, driven by a 1.5 hp motor, operated at a fixed speed of 53.8 rpm. This speed was 78.5 percent of the critical speed.

The normal operating ball load was set at 230 pounds, and was comprised of 2.5, 2.0, 1.5, 1.0, and 0.75 in. diameter cast iron balls. The maximum size ball was determined by the equation (Bond, 1957)

$$B = \left[\frac{F}{K}\right]^{1/2} \left[\frac{SW_i}{C_s D}\right]^{1/3} \qquad [1]$$

where, B = maximum ball diameter, ft.

F = size in microns that 80 percent of the new feed passes

K = 350, a constant

S = specific gravity of the material being ground

W_i = work index at the feed size F, kw-hr/ton

C_s = percentage of the mill's critical speed

D = mill inside diameter, ft.

This calculation indicated that a 2.5 inch ball would be necessary to break the largest particle of ore, 0.25 inch.

Using the 2.5 inch ball as the maximum size, the ball distribution was adjusted to match the feed distribution of the ore. The ball distribution data are shown in Table 3.

Table 3 - Ball Data

Ball Diameter*, inches	95% Confidence Limits, in.	No. of Balls	Total Wt of Balls, lb	Total Surface Area of Balls, sq in.
0.756	0.039	171	10.5	612.0
1.028	0.020	121	18.5	922.8
1.496	0.071	95	45.1	696.0
1.993	0.039	73	83.7	403.8
2.547	0.020	30	72.2	313.2
			230.0	2947.8

* Ball Measurements are presented in Appendix C

The ore was added to the mill by a 6 inch wide, 18 inch long Denver Belt Ore Feeder which conveyed the ore from a feed hopper. The capacity of the feed hopper was about 100 pounds of ore. The feed rate was controlled by an adjustable gate on the feed hopper. Additional control on the feed rate could be achieved by a variable speed sheave on the belt feeder.

The water added to the mill was drawn from a constant head tank, controlled by a 1/2 inch needle valve, and measured by a 1/2 inch Fischer and Porter Co. Flowrater.

Pulp density measurements were made in a Mine and Smelter Supply Co. Direct Reading Pulp Density Scale. The amperage drawn by the mill was measured with an Amprobe.

C. <u>Experimental Procedure</u>

The following experimental procedure was employed in all of the grinding tests.

1. A weighed amount of ore was added to the ore hopper. This ore was weighed to the nearest 1/8 of a pound on a platform balance.

2. The feed gate on the hopper was adjusted until the desired feed rate was obtained. The feed rate was measured as follows:

   a) The belt feeder was run for a measured length of time.

   b) The ore was collected at the belt discharge.

   c) The ore was weighed to determine the pounds per minute being fed to the mill.

3. The water flow was then adjusted to obtain the desired percent solids.
4. The mill, belt feeder, and water flow were started at the same time. The starting time was recorded.
5. The discharge rate of solids from the mill was measured at regular intervals. This was accomplished as follows:
   a) The discharge was collected in a one-liter pulp density cylinder for a measured time interval.
   b) Water was added to finish filling the pulp density cylinder.
   c) The pulp density was measured on the pulp density scale.
   d) The pounds per minute of ore being discharged was then calculated.
6. Steady state conditions were achieved when the operating time had exceeded the mean residence time and when the discharge rate of solids, for repeated measurements, was approximately equal to the feed rate.
7. The amperage being drawn by the mill was measured by the amprobe.

8. Three samples were taken of the discharge of the mill in order to determine the size distribution of the ground ore. Each sample was taken for a time interval so that about one pound of ground ore was obtained. The time of obtaining each sample was recorded.
9. A pulp density sample was collected in the pulp density cylinder. Approximately one-third of the pulp density sample was collected after obtaining each of the three discharge samples described in step 8.
10. The samples collected in step 8 were then weighed to the nearest gram on a triple beam balance. The weight of the sample pans had been determined so that the wet weight of the sample could be obtained.
11. The mill, belt feeder, and water flow were then stopped. The total time elapsed during the test was recorded.
12. The ore remaining in the hopper was removed and weighed on the platform balance. The actual feed rate was then calculated on the basis of the total consumption of ore.

13. The samples obtained were dried over a hot plate and the dry weights of the samples were obtained.

14. The actual percent solids of the samples were calculated.

15. Two of the samples were wet screened through a 270 mesh screen in order to remove the fines and facilitate dry screening. The third sample was saved as a check and was not analyzed unless a disagreement existed between the first two samples.

16. After wet screening, the samples were dry screened through a set of 14 Tyler testing sieves. Each sub-set was allowed to run 20 minutes on the Rotap. A 20 minute screening time had been determined to be the minimum time that would give an accurate screen analysis. The first sub-set consisted of the 4, 6, 8, 10 and 14 mesh screens, the second sub-set consisted of the 20, 28, 35, 48, and 65 mesh screens, and the third sub-set consisted of the 100, 150, 200, and 270 mesh screens. The material held on each screen was weighed on a triple beam balance made by Ohaus

Scale Corp. This balance has a least-count of 0.1 gram.

17. A digital computer program was used in order to determine the specific surface of the sample using a sphericity of one for the ground product. The program is presented in Appendix D.

An example of the experimental data recorded in performing a test is presented in Appendix E.

DEVELOPMENT OF MODEL

A. Discussion of Variables

A mathematical model of an open-circuit grinding system can be developed as functions of several variables. The most important of these are feed rate, percent solids, ball load, ball area, grindability, and mill speed. Investigation of all of these variables is beyond the scope of this thesis. However, if a general model could be developed describing the grinding action and flow characteristics within the mill as a function of feed rate, the model probably could be modified to include the other variables by changing the constants in the general equations. This change in constants would transform them into functions of the variables considered.

In order to determine the effect that feed rate would have on the grinding action and resulting product size distribution, grinding tests were run varying the feed rate and holding the other variables constant. The feed rate was run at approximately 0.5, 1.0, 1.5, 2.0, and 3.0 pounds per minute. The other grinding variables were held constant at the following values:

percent solids	65%
ball load	230 lb
ball area	2948 sq in.
work index (grindability)	20 kw-hr/ton
mill speed	53.8 rpm (78.6% critical)
feed distribution	described in Table 2, page 5.

The results of the experimental tests are shown in Appendix F. Three tests had to be discarded because of poor control and are not presented herein.

B. Axial-Gradient Model

1. Derivation of Model

One approach that might be used to develop a mathematical model of a comminution system is the multiple-gradient approach (Himmelblau and Bischoff). This type of model can readily be developed by making a material balance about an element in the mill. In order to simplify the development, the radial gradient due to diffusion will be neglected. Since the net axial diffusion is much larger than the radial diffusion, this assumption is valid. A further simplification is made in assuming that the bulk flow velocity is uniform over the cross section of the mill that is occupied by the ore.

In developing an axial-diffusion model, the following nomenclature will be used:

A_X = cross sectional area, sq ft

Z = axial direction, ft

V_Z = velocity in Z direction, ft/min

X_i = mass fraction of size i, lb/lb

D_L = axial-diffusion coefficient, sq ft/min

j_i = mass flux of i-th component by diffusion, $lb/(min)(ft)^2$

R_i = reaction rate of i-th component, (lb/lb)/min

ρ_i = density of i-th component, lb/cu ft

k = comminution rate constant, min^{-1}

t = time, min

A material balance about the element shown in Figure 1 results in

$$\text{Transport In} - \text{Transport Out} + \text{Net Generation} = \text{Net Accumulation} . \qquad [2]$$

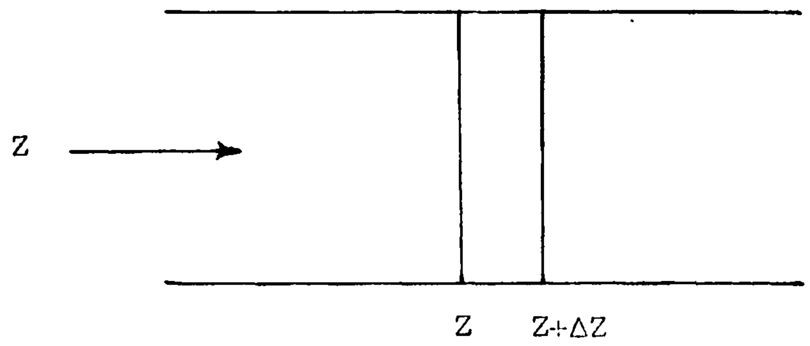


Figure 1 - Element Along the Length of the Mill

Contribution	In	Out
Transport by Convection	$(\Delta t A_X V_Z \rho_i X_i)_Z$	$(\Delta t A_X V_Z \rho_i X_i)_{Z+\Delta Z}$
Transport by Diffusion	$(\Delta t A_X j_{iZ})_Z$	$(\Delta t A_X j_{iZ})_{Z+\Delta Z}$
Accumulation	$(\Delta Z A_X \rho_i X_i)_t$	$(\Delta Z A_X \rho_i X_i)_{t+\Delta t}$
Reaction	$R_i(A_X \Delta Z \Delta t)$	

Combining the terms and dividing by $A_X \Delta Z \Delta t$ the equation becomes

$$\frac{(v_Z \rho_1 X_1) - (v_Z \rho_1 X_1)_{Z+\Delta Z}}{\Delta Z} + \frac{(j_{1Z})_Z - (j_{1Z})_{Z+\Delta Z}}{\Delta Z}$$

$$+ \rho_1 R_1 = \frac{(\rho_1 X_1)_t - (\rho_1 X_1)_{t+\Delta t}}{\Delta t} \quad . \qquad [3]$$

Taking the limit as ΔZ and Δt approach zero, the equation becomes

$$\frac{\partial (v_Z \rho_1 X_1)}{\partial Z} + \frac{\partial j_{1Z}}{\partial Z} + \rho_1 R_1 = \frac{\partial (\rho_1 X_1)}{\partial t} \quad . \qquad [4]$$

Using Fick's first law for the mass flux due to diffusion,

$$\text{i.e.,} \qquad j_{1Z} = \rho_1 D_L \frac{\partial X_1}{\partial Z} \quad , \qquad [5]$$

and dividing through by ρ_1(ρ = constant), the general equation becomes

$$V_Z \frac{dX_1}{dZ} - D_L \frac{d^2 X_1}{dZ^2} + R_1 = \frac{dX_1}{dt} \quad . \qquad [6]$$

Since this thesis presentation is concerned only with the steady state development of a mathematical description of a comminution system, the accumulation term becomes zero, and the equation reduces to

$$V_Z \frac{dX_1}{dZ} = D_L \frac{d^2 X_1}{dZ^2} + R_1 \quad . \qquad [7]$$

Several investigators (Roberts, 1950; Bowdish, 1960; Huttig, 1953) have shown that the rate of comminution is proportional to the amount of material present. Previous work (Freeh, Horst, and Kellner, 1966) has also demonstrated that a successful model of a batch grinding system can be achieved by utilizing a first-order comminution rate and coupled differential equations. Using the first-order comminution rate for a model of a continuous grinding system, and assuming that all of the material that is ground in one size fraction becomes material of the next smaller size fraction, the rate of grinding becomes

$$R_i = \text{depletion rate of size } i - \text{generation rate of size } i$$

$$= -k_i X_i + k_{i-1} X_{i-1} \quad , \qquad [9]$$

where k_i's are the comminution rate constants.

Substituting the reaction rate into the multiple-gradient equation, the general equation describing the rate of change of the i-th size fraction becomes

$$V_Z \frac{dX_i}{dZ} = -k_i X_i + k_{i-1} X_{i-1} + D_L \frac{d^2 X_i}{dZ^2} \quad . \qquad [10]$$

Feed and product size distributions were experimentally obtained in 16 size fractions ($\sqrt{2}$ Tyler series from 3 mesh to 270 mesh). In order to simplify the mathematical development, the size fractions to be described have been grouped into eight fractions (two instead of $\sqrt{2}$ Tyler series; i.e., -3, +6). Therefore, the model describing the comminution system requires eight coupled differential equations that must be solved simultaneously. These equations are compactly represented in the array below where the table entries denote the coefficients that precede the column headings (variables).

For example when i = 4,

$$V_Z \frac{dX_4}{dZ} = -k_4X_4 + k_3X_3 + D_L \frac{d^2X_4}{dZ^2} \quad . \quad . \qquad [11]$$

It is important to recognize here that the comminution rate constants (k_i's) are independent of feed rate but dependent upon the size of the material being ground. However, the comminution rate constants are also a function of the grinding variables that are being held constant; i.e., percent solids, ball load, ball area, grindability, and mill speed. It should, therefore, be clear that modification of this model to include other grinding variables would require the replacement of the comminution rate constants with a function of the grinding

i	$\frac{dX_i}{dZ}$	$\frac{d^2X_i}{dZ^2}$	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8
1	V_Z	D_L	$-k_1$							
2	V_Z	D_L	k_1	$-k_2$						
3	V_Z	D_L		k_2	$-k_3$					
4	V_Z	D_L			k_3	$-k_4$				
5	V_Z	D_L				k_4	$-k_5$			
6	V_Z	D_L					k_5	$-k_6$		
7	V_Z	D_L						k_6	$-k_7$	
8	V_Z	D_L							k_7	$-k_8$

variables being investigated. This would result in an equation of the following form:

$$V_Z \frac{dX_i}{dZ} = -\bar{k}_i X_i + \bar{k}_{i-1} X_{i-1} + D_L \frac{d^2 X_i}{dZ^2} \qquad [12]$$

where $\bar{k}_i$ = f (percent solids, ball load, ball area, grindability, mill speed . . .).

2. Analog Simulation of Grinding

Since differential equations have been used to describe the system, the solution of the equations lends itself to the analog computer. Each comminution rate constant can be represented by an individual potentiometer in the analog circuit and, therefore, can easily be adjusted in order to minimize the sum of the square of the deviation between experimental and computer results. Once the minimum sum has been obtained, the comminution rate constant can be determined from the potentiometer settings.

However, analog implementation is difficult since the general equation is unstable and not all of the initial conditions are known.

a. Stability of Equations

The stability of the general axial-diffusion equation can be determined by examining the roots of the characteristic equation of the transfer function. Taking the Laplace transform of the general equation, the following equation is obtained:

$$V_Z X_i s = -k_i X_i + k_{i-1} X_{i-1} + D_L X_i s^2 \quad . \qquad [13]$$

Solving for the transfer function results in

$$\frac{X_i}{k_{i-1} X_{i-1}} = \frac{1}{D_L s^2 - V_Z s - k_i} \quad . \qquad [14]$$

The characteristic equation of this transfer function is

$$D_L s^2 - V_Z s - k_i = 0 \quad , \qquad [15]$$

and the roots of this equation are

$$r_1 = \frac{V_Z - \sqrt{V_Z{}^2 + 4D_L k_i}}{2D_L} \quad \text{and}$$

$$r_2 = \frac{V_Z + \sqrt{V_Z{}^2 + 4D_L k_i}}{2D_L}$$

It can easily be seen in examining the roots that one of the roots, r_2, is positive and the other, r_1, is negative. If the positive root is transformed back into the time domain, a term of the form e^{at} is obtained. This causes X_i to increase with time and therefore, the general equation is unstable. For a system to be stable all of the roots of the characteristic equation must have negative real parts. The instability of the general equation caused difficulty in analog implementation.

b. Initial Conditions

Another difficulty in attempting to solve the equations on the analog computer is that the analog solution requires initial conditions on X_i and $\frac{dX_i}{dZ}$.

The initial condition on $\frac{dX_i}{dZ}$ can be determined by operating the computer in repetitive operation where the solution is completed every 20 milliseconds. This allows one to observe the shape of the first and second derivative on the oscilloscope and adjust the initial conditions on the first derivative until the two curves have the correct shapes.

Because of the diffusion term in the general equation, the amount of X_i immediately after entering the mill is different than the amount before entering the mill. The initial conditions on X_i as it enters the mill can be determined by (Himmelblau and Bischoff)

$$X_i(0^+) = X_i(0^-) + \frac{D_L}{V_Z} \frac{dX_i(0^+)}{dZ} \quad . \qquad [16]$$

Using the analog circuit shown in Figure 2, the potentiometers representing $X_i(0^+)$ and $\frac{dX_i(0+)}{dZ}$ could be adjusted until

$$X_i(0^+) - X_i(0^-) - \frac{D_L}{V_Z} \frac{dX_i(0^+)}{dZ} = 0. \qquad [17]$$

The adjustments on $X_i(0^+)$ must be made in order to maintain the correct shapes of the first and second derivative as discussed earlier.

c. Implementation

The first analog implementation of the equations used to describe the system was developed by solving the equations in terms of the highest derivatives. The analog representation of the i-th size using this approach is shown in Figure 3. As can be seen in

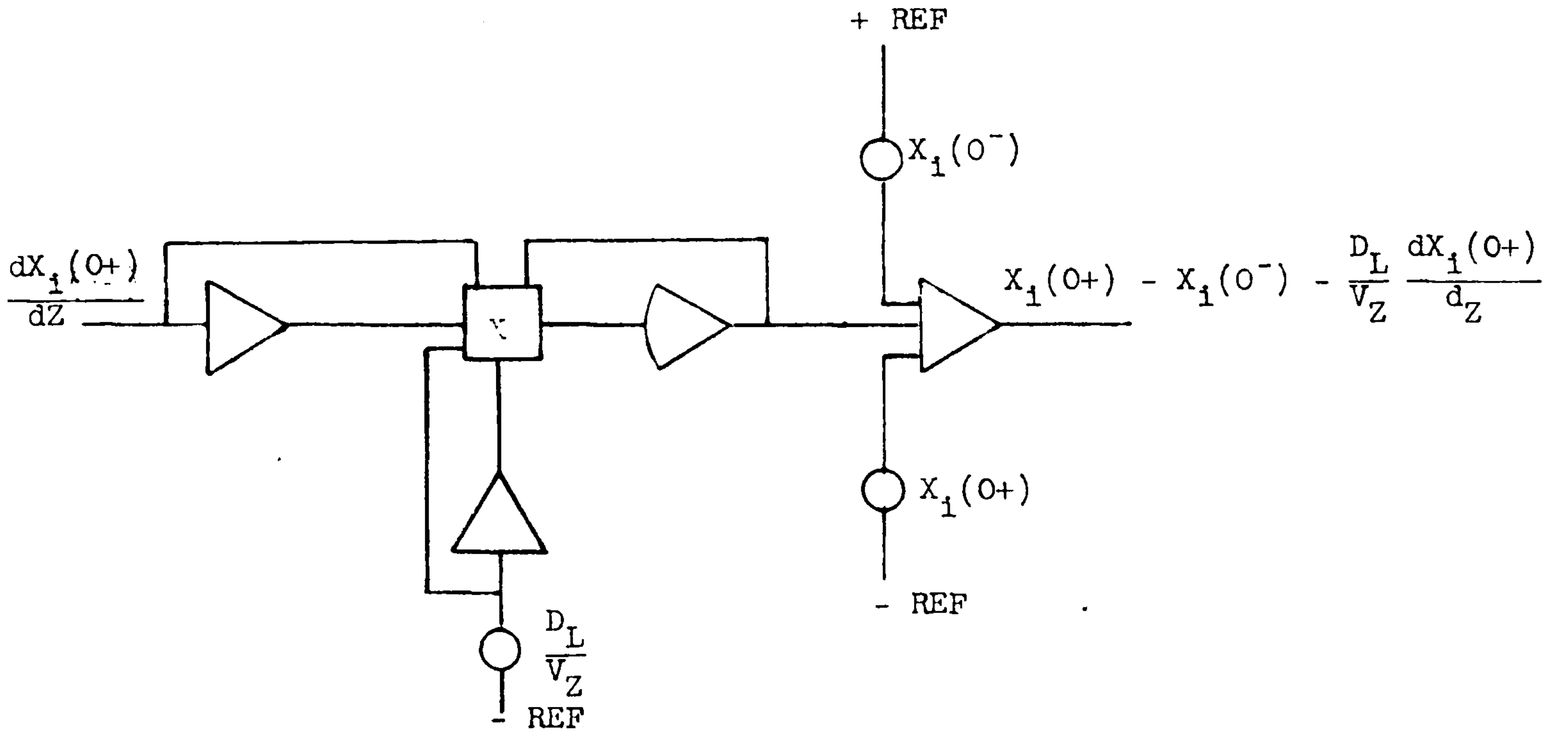


Figure 2 - Analog Circuit to Determine Initial Conditions

the figure, this implementation results in positive feedback, and consequently, control could not be achieved.

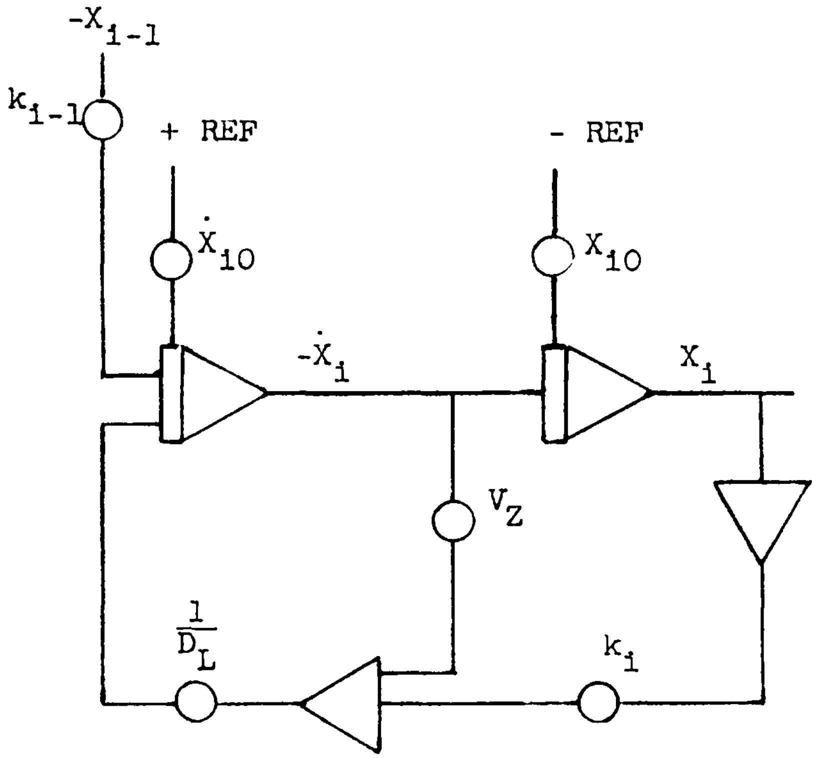


Figure 3 - Second Derivative Implementation of Axial Gradient Model

The second analog circuit, shown in Figure 4 for the i-th size fraction, consisted of differentiating the first derivative in order to obtain the second derivative.

However, this system could not be controlled because of the high gain amplifier in the differentiating loop.

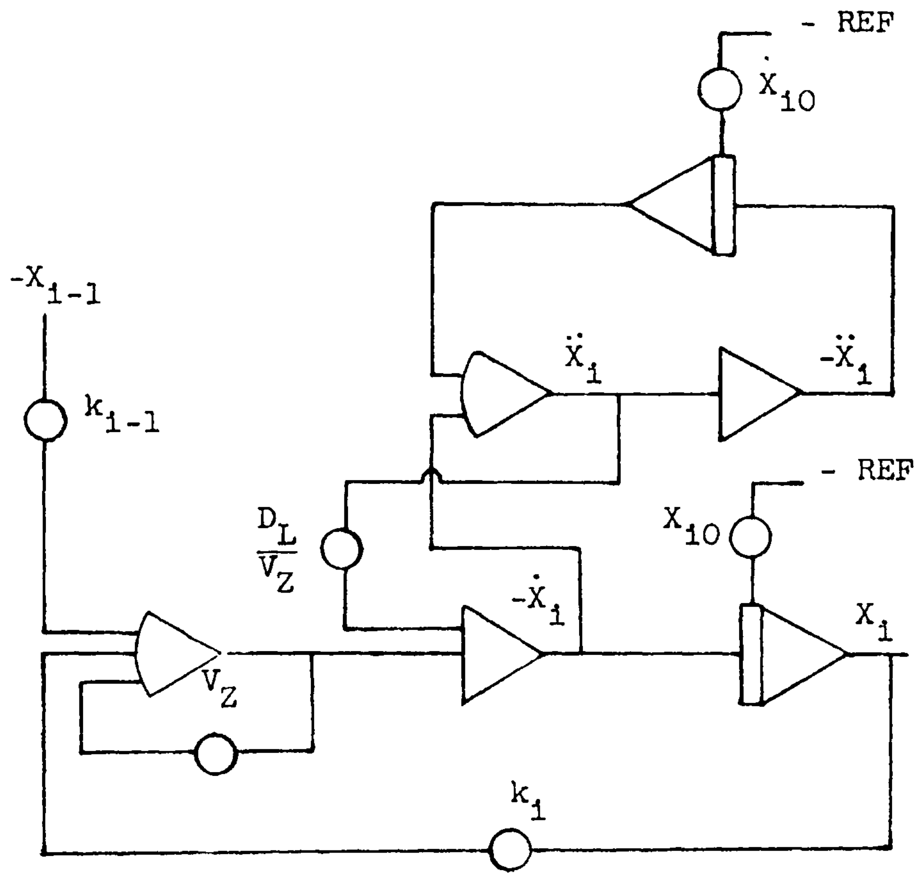


Figure 4 - Differentiating Circuit Implementation of Axial Gradient Model

The third analog circuit consisted of approximating the second derivative with a differentiation loop.

This circuit is shown for the i-th size fraction in Figure 5. This system could be made stable over a small range of D_L. A complete analog development including scaling factors, scaled equations, scaled circuit diagram, and static check is presented in Appendix G.

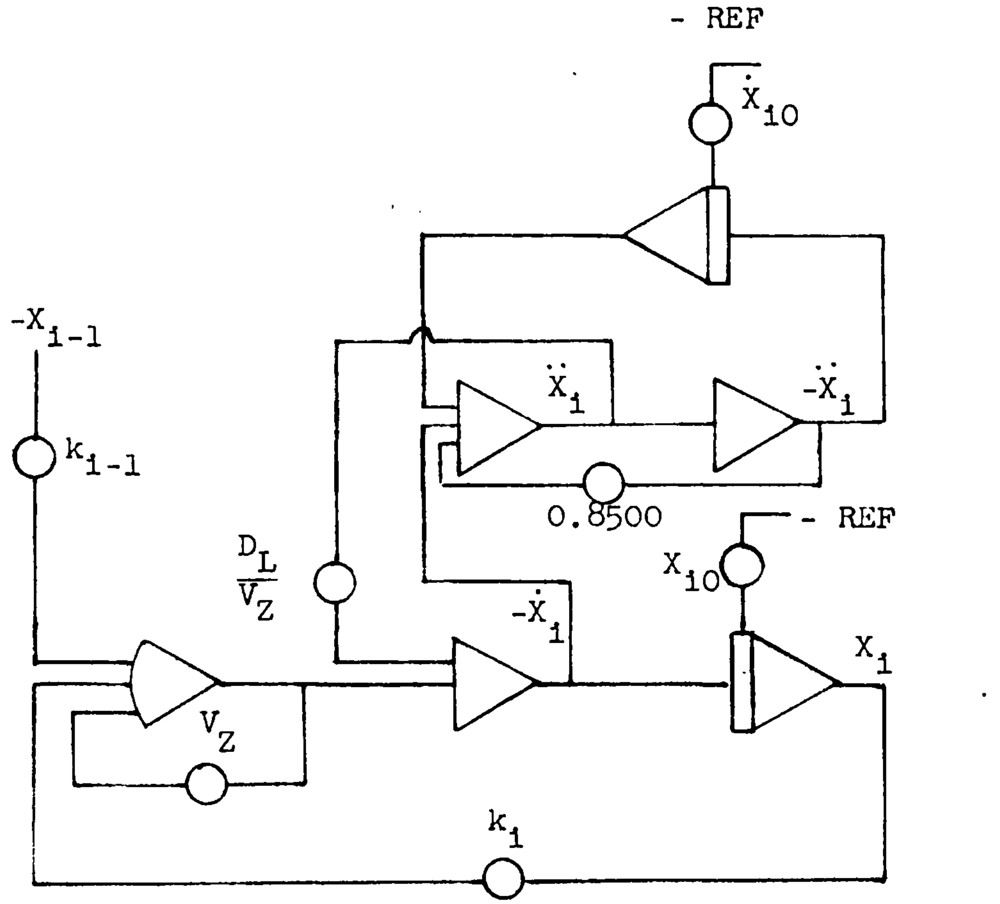


Figure 5 - Circuit for Approximation of Second Derivative

3. Results of Axial-Gradient Model

The implementation shown in Figure 5 yielded the constants shown in the following array.

For example when $i = 4$,

$$V_Z \frac{dX_4}{dZ} = 0.439X_3 - 0.418X_4 + 0.00625 \frac{d^2X_4}{dZ^2} \quad . \quad [18]$$

The flow in a ball mill is somewhere between plug flow and a well-stirred tank. In a plug flow model, D_L (the axial-diffusion coefficient) would be zero while in a well-stirred tank model, D_L would be infinite. In adjusting the comminution rate constants in order to fit the experimental data for the five feed rates, it became evident that a large enough D_L could not be obtained to match the lower feed rates. The feed rates had been varied over such a wide range that the higher feed rates approached plug flow while the lower feed rates approached the mixing of a well-stirred tank. The axial-diffusion model applies only to relatively small deviations from plug flow and fails as flow conditions approach those of a well-stirred tank. Because of the instability problems discussed earlier, an axial-diffusion coefficient greater than 0.016 sq ft/min could not be obtained on the analog computer. A comparison of computer and experimental

i	$\frac{dX_i}{dZ}$	X_1	X_2	X_3	X_4	X_5	X_6	X_7	X_8	$\frac{d^2X_i}{dZ^2}$
1	V_Z	-0.119								0.00625
2	V_Z	0.119	-0.229							0.00625
3	V_Z		0.229	-0.439						0.00625
4	V_Z			0.439	-0.418					0.00625
5	V_Z				0.418	-0.312				0.00625
6	V_Z					0.312	-0.250			0.00625
7	V_Z						0.250	-0.252		0.00625
8	V_Z							0.252	-0.000	0.00625

results with the axial-diffusion coefficient held constant at 0.00625 sq ft/min is shown in Table 4. The experimental data are shown with their 95% confidence limits.

In order to simplify the equations, the assumption was made that all of the material ground in a size fraction is fed forward directly to the next smaller size fraction. A feed-forward-two system was tried for the axial-gradient model, but did not significantly decrease the deviation between the experimental and computer results. It has also been demonstrated (Freeh, Horst, and Kellner, 1966) that a feed-forward-two model adequately described a batch grinding system. Therefore, a feed forward of a higher order would not significantly improve the model.

As shown in Table 4, the difference between the experimental and computer results becomes larger as the feed rate decreases. The mean standard deviation for the highest feed rate is 1.59 percent. In attempting to fit the two highest feed rates the mean standard deviation becomes 2.68 percent, and in fitting all three feed rates, it becomes 6.60 percent. Additional work indicated that if the axial-diffusion coefficient could be increased in going to lower feed rates, much better

Table 4 - Comparison of Experimental and Computer Results Using An Axial-Gradient Model

Size	Feed Rate				
		lb/min	3.0	2.0	1.5
		ft/min	0.125	0.0870	0.0651
	Basis		Weight Percent Retained		
X_1	Experimental		13.8±6.8	7.4±2.4	6.4±0.9
	Computer		13.8	7.5	5.5
X_2	Experimental		12.2±1.3	7.8±0.8	6.0±0.7
	Computer		12.6	7.9	5.5
X_3	Experimental		7.6±0.5	5.2±0.3	4.1±0.2
	Computer		8.2	5.2	3.7
X_4	Experimental		8.8±0.9	7.3±0.4	5.9±0.1
	Computer		9.6	6.8	4.8
X_5	Experimental		11.4±1.3	11.8±0.9	10.8±0.3
	Computer		13.2	10.4	8.0
X_6	Experimental		12.7±1.4	16.2±1.6	16.5±0.6
	Computer		14.6	13.8	11.0
X_7	Experimental		9.3±1.1	12.5±1.1	13.7±0.7
	Computer		10.7	12.6	11.3
X_8	Experimental		24.2±2.1	31.7±1.8	36.6±0.5
	Computer		21.0	41.0	66.5

results could be achieved. However, as discussed earlier, stability could not be maintained on the analog computer at high values of D_L. Large axial-diffusion coefficients can be obtained, however, by using a series of well-stirred tanks to describe the flow characteristics of the grinding system.

C. Well-Stirred-Tank Model

Himmelblau and Bischoff have shown that the axial-diffusion coefficient can be related to the number of well-stirred tanks in series by

$$\frac{1}{j} = 2\,\frac{D_L}{V_Z L} - 2\left(\frac{D_L}{V_Z L}\right)^2 \quad 1 - \exp\left(-\frac{V_Z L}{D_L}\right) \qquad [19].$$

where, j = the number of well-stirred tanks in series

D_L = axial-diffusion coefficient, sq ft/min

V_Z = feed rate, ft/min

L = mill length, ft

As can be seen from the above relationship, a D_L of zero, plug flow, would correspond to an infinite number of tanks in series. A D_L of infinity, perfect mixing, would correspond to a single well-stirred tank. Therefore, by using the correct number of well-stirred tanks in series, the mixing action in the ball mill can be simulated.

1. Derivation of Model

In developing a model on the basis of well-stirred tanks in series, the following nomenclature will be used.

F = flow rate, lb/min

$X_{i,j}$ = mass fraction of size i that leaves tank j, lb/lb

ρ = density, lb/cu ft

V = volume of tank, cu ft

τ = tank retention time, min

k_i = comminution rate constant of i-th fraction, min^{-1}

$T_{i,j}$ = j-th tank in series for size fraction i.

The assumptions made in developing the axial-gradient model will also be made in developing the well-stirred-tank model; i.e., steady state is achieved, grinding is a first-order-rate phenomena, and all of the material that is ground in one size fraction becomes material of the next smaller size fraction. A further assumption is that all tanks have the same volume and residence times.

Using the above nomenclature and assumptions, a mass balance can be developed for the j-th tank and i-th size fraction as shown in Figure 6.

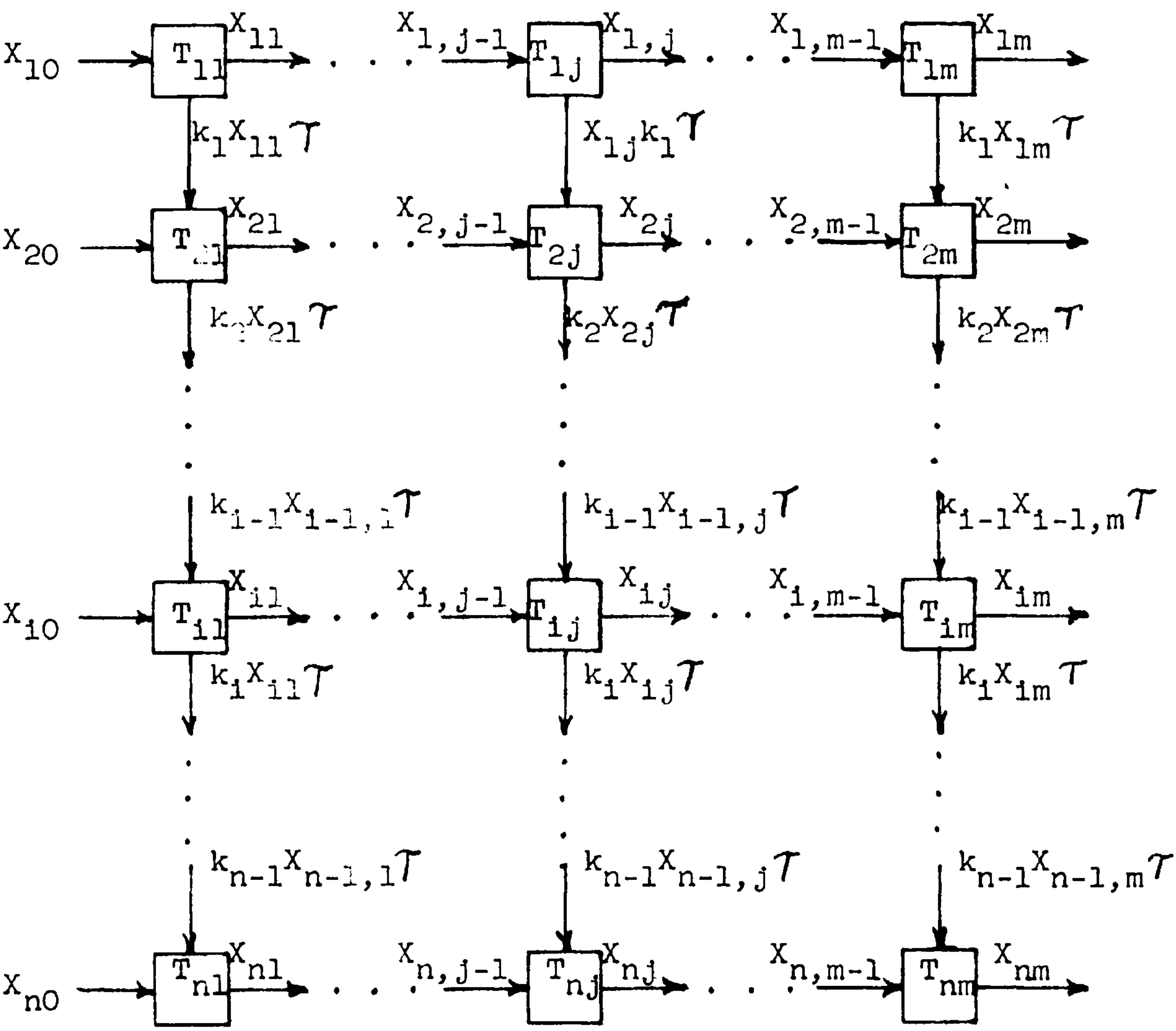


Figure 6 - Material Flow For Well-Stirred Tanks in Series

Transport In - Transport Out + Net Generation = 0 [20]

$$(FX_{i,j-1} + k_{i-1} X_{i-1,j} \rho V) - (FX_{i,j}) + (-k_i X_{i,j} \rho V) = 0 \qquad [21]$$

Dividing through by F and simplifying, the general equation describing the mass fraction of size i leaving the j-th tank becomes

$$X_{ij} = \frac{X_{i,j-1} + k_{i-1} X_{i-1,j} \tau}{1 + k_i \tau} \quad . \qquad [22]$$

Since the feed and product distributions are described by eight size fractions, eight coupled algebraic equations are necessary to describe the grinding system for one stirred tank. If two stirred tanks in series are used, 16 equations are required to describe the system, and if three tanks are used, 24 equations are required, etc.

Since algebraic equations have been used to describe the grinding systems, solution of the equations is suited for the digital computer. The difficulty in solving the equations is that only the initial feed to the first tank of each size, X_{iO}, and the final product from the last tank of each size, X_{im}, are known. The internal flow between tanks is not known and must be calculated. The equations that result when considering a two-tank model are shown below.

Output of 1st tank

$$X_{11} = \frac{X_{10}}{1+\tau k_1} \qquad [23]$$

$$X_{21} = \frac{X_{20}+\tau k_1 X_{11}}{1+\tau k_2} \qquad [25]$$

$$X_{31} = \frac{X_{30}+\tau K_2 X_{21}}{1+\tau k_3} \qquad [27]$$

$$X_{41} = \frac{X_{40}+\tau k_3 X_{31}}{1+\tau k_4} \qquad [29]$$

$$X_{51} = \frac{X_{50}+\tau k_4 X_{41}}{1+\tau k_5} \qquad [31]$$

$$X_{61} = \frac{X_{60}+\tau k_5 X_{51}}{1+\tau k_6} \qquad [33]$$

$$X_{71} = \frac{X_{70}+\tau k_6 X_{61}}{1+\tau k_7} \qquad [35]$$

$$X_{81} = \frac{X_{80}+\tau k_7 X_{71}}{1} \qquad [37]$$

Output of 2nd tank

$$X_{12} = \frac{X_{11}}{1+\tau k_1} \qquad [24]$$

$$X_{22} = \frac{X_{21}+\tau k_1 X_{12}}{1+\tau k_2} \qquad [26]$$

$$X_{32} = \frac{X_{31}+\tau k_2 X_{22}}{1+\tau k_3} \qquad [28]$$

$$X_{42} = \frac{X_{41}+\tau k_3 X_{32}}{1+\tau k_4} \qquad [30]$$

$$X_{52} = \frac{X_{51}+\tau k_4 X_{42}}{1+\tau k_5} \qquad [32]$$

$$X_{62} = \frac{X_{61}+\tau k_5 X_{52}}{1+\tau k_6} \qquad [34]$$

$$X_{72} = \frac{X_{71}+\tau k_6 X_{62}}{1+\tau k_7} \qquad [36]$$

$$X_{82} = \frac{X_{81}+\tau k_7 X_{72}}{1} \qquad [38]$$

The equations for the two-tank model can be solved in a sequential manner; i.e., solving for the comminution rate constants for the first size fraction, and using that comminution rate constant to solve for the constant for the second size fraction, etc. Substituting for the X_{i1} values in the equation describing the output of the second tank, and solving the equation for k_i results in the quadratic equation of

$$k_i^2(X_{i2}\tau^2) + k_i(2\tau X_{i2} - \tau^2 k_{i-1}X_{i-1,2}) +$$

$$(X_{i2} - X_{i0} + \tau k_{i-1}X_{i-1,1} + \tau k_{i-1}X_{i-1,2}) = 0. \quad [39]$$

It can easily be seen that solving the equations in this manner for three tanks results in a cubic equation; for four tanks it results in a quartic equation, etc. In going to a higher number of tanks, it would be necessary to use an iterative technique. However, the k_i's for a two-tank model can be easily solved since a negative grinding coefficient is physically impossible and the negative solution can be ignored.

2. Results of Well-Stirred-Tank Model

A digital computer program was written to solve for the comminution rate constant for the different size fractions and feed rates. This program is shown in Appendix H.

Since the comminution rate constants (presented in Appendix H) are not a function of feed rate, the constants were averaged for each size over the feed rates. The comminution rate constants determined by this method are shown in Table 5.

It is important to note here that it is easier for the smaller particles to be discharged from the ball mill than it is for the larger particles. A probability of particles being discharged from the mill was defined in order to account for this phenomena, and is shown in Table 5.

Table 5 - Comminution Rate Constants for Well-Stirred-Tank Model

Size Fraction		Probability of Particles being Discharged	Comminution Rate Constant
X_i	Tyler Mesh		
X_1	-3, +6	0.8	0.151
X_2	-6, +10	0.8	0.279
X_3	-10, +20	0.8	0.449
X_4	-20, +35	0.8	0.341
X_5	-35, +65	0.9	0.211
X_6	-65, +150	0.9	0.110
X_7	-150, +270	1.0	0.095
X_8	-270	1.0	0.000

The comminution rate constants presented in Table 5 were then used to calculate the produce size distribution for a two-tank model. The digital computer program used to perform these calculations is presented in Appendix I. A comparison of the computer results for the two-tank model and experimental results is presented in Table 6. The experimental data are shown with their 95 percent confidence limits.

No attempt has been made to optimize the comminution rate constants or the discharge probability vector. However, as should be evident from Table 6, it has been demonstrated that a model using two well-stirred tanks in series can be used to describe the steady state conditions of an open-circuit ball mill at the feed rates evaluated. Using a two-tank model corresponds to an axial-diffusion coefficient of approximately 0.042 sq ft/min. A diffusion coefficient of this magnitude cannot be used in the analog model because of the stability problems discussed earlier. However, it should be recognized that the axial-gradient model will be useful in modeling higher feed rates where D_L is somewhat smaller. It is also important to recognize that industrial feed rates in closed-circuit grinding systems are higher than the feed rates studied here. It

Table 6 - Comparison of Computer and Experimental Results for Well-Stirred-Tank Model

Size	Feed Rate lb/min	3.0	2.0	1.5	1.0	0.5
	Feed Rate ft/min	0.125	0.0870	0.0651	0.0418	0.0228
	Basis	Weight Percent Retained				
X_1	Experimental	13.8±6.8	7.4±2.4	6.4±0.9	2.4±1.8	0.5±0.7
	Computer	11.2	7.8	5.6	3.1	1.2
X_2	Experimental	12.2±1.3	7.8±0.8	6.0±0.7	2.2±1.0	0.8±0.3
	Computer	9.8	7.1	5.2	3.0	1.2
X_3	Experimental	7.6±0.5	5.2±0.3	4.1±0.2	1.7±0.2	0.9±0.1
	Computer	7.0	5.3	4.0	2.3	1.0
X_4	Experimental	8.8±0.9	7.3±0.4	5.9±0.1	3.1±0.5	1.7±0.1
	Computer	9.3	7.5	5.9	3.7	1.6
X_5	Experimental	11.4±1.3	11.8±0.9	10.8±0.3	7.5±1.5	4.6±0.4
	Computer	13.9	12.8	11.0	7.7	3.7
X_6	Experimental	12.7±1.4	16.2±1.6	16.5±0.6	16.4±1.5	11.0±0.5
	Computer	15.6	17.4	17.3	14.5	8.4
X_7	Experimental	9.3±1.1	12.5±1.1	13.7±0.7	20.7±4.5	16.6±1.0
	Computer	10.2	13.7	15.6	15.8	11.3
X_8	Experimental	24.2±2.1	31.7±1.8	36.6±0.5	45.9±3.9	64.0±1.2
	Computer	23.0	28.4	35.4	49.9	71.6

is, therefore, possible that in modeling the ball mill circuit over a range of feed rates, the well-stirred-tank model applies at the lower feed rates and the axial-gradient model applies at higher feed rates.

PROPOSED FUTURE WORK

Further development of a model for comminution systems would include:

1. Adding other operating variables into the well-stirred-tank model and axial-gradient model.
2. Optimizing comminution rate constants and particles probability of discharge in well-stirred-tank model by using a digital computer program for method of steepest ascent.
3. Developing a plug flow model for the continuous ball mill system.
4. Developing a model of a closed-circuit comminution system as a function of operating variables.

CONCLUSIONS

On the basis of this investigation the following conclusions are advanced:

1. A well-stirred-tank model using two tanks in series can be used to describe a pilot-scale, open-circuit ball mill at the feed rates studied.
2. An axial-gradient model appears to be applicable in describing the open-circuit ball mill at higher feed rates. This model would apply at commercial feed rates.
3. A first-order comminution rate and a feed forward of the ground material to the next smaller size fraction adequately describes the open-circuit grinding system.

APPENDIX A

FEED DISTRIBUTION DATA

Feed Distribution Data

Mesh Size	Weight Percent Retained					
	Drum No.1	Drum No.2	Drum No.3	Drum No.4	Drum No.5	Drum No.6
-3, +4	22.98	17.67	19.40	18.70	19.26	13.75
-4, +6	27.99	27.36	28.98	27.92	25.64	24.06
-6, +8	15.17	16.64	18.38	17.34	15.52	17.06
-8, +10	9.08	10.68	11.15	11.25	10.59	11.56
-10, +14	5.16	6.00	5.91	5.98	6.14	6.98
-14, +20	3.70	4.17	3.81	3.92	4.41	5.03
-20, +28	2.93	3.27	2.65	2.95	3.58	4.02
-28, +35	2.23	2.46	1.87	2.20	2.69	3.16
-35, +48	1.72	1.94	1.32	1.64	2.12	2.41
-48, +65	1.63	1.85	1.22	1.53	1.99	2.31
-65, +100	1.58	1.78	1.16	1.46	1.89	2.19
-100, +150	1.11	1.22	0.81	1.02	1.27	1.49
-150, ±200	1.17	1.27	0.80	1.02	1.23	1.48
-200, +270	0.95	0.79	0.54	0.65	0.84	0.93
-270	2.60	2.90	2.00	2.42	2.83	3.57
	100.00	100.00	100.00	100.00	100.00	100.00
Sample Wt. Grams	908.58	738.75	773.21	1067.92	906.40	911.68

APPENDIX B

DATA FOR FEED VARIANCE CONTRIBUTION

Data For Sample Variance Contribution

Mesh Size	Weight Percent Retained				
	Sample No.1	Sample No.2	Sample No.3	Sample No.4	Sample No.5
-3, +4	20.91	16.55	17.41	19.67	18.97
-4, +6	27.44	27.42	28.18	28.74	27.81
-6, +8	16.07	19.04	18.08	16.78	16.72
-8, +10	10.48	12.42	10.87	11.43	11.05
-10, +14	5.95	6.56	5.90	5.70	5.79
-14, +20	3.91	4.13	3.85	3.80	3.91
-20, +28	2.94	2.97	2.93	2.84	3.06
-28, +35	2.25	2.10	2.22	2.10	2.31
-35, +48	1.67	1.51	1.72	1.55	1.76
-48, +65	1.58	1.36	1.64	1.42	1.66
-65, +100	1.52	1.29	1.59	1.33	1.60
-100, +150	1.05	0.90	1.12	0.93	1.09
-150, +200	1.06	0.91	1.11	0.91	1.09
-200, +270	0.67	0.58	0.72	0.59	0.70
-270	2.50	2.26	2.66	2.21	2.48
	100.00	100.00	100.00	100.00	100.00
Sample Wt. Grams	1057.44	999.37	1100.89	972.77	1209.15

Data For Screening Variance Contribution

Mesh Size	Weight Percent Retained		
	Run No. 1	Run No. 2	Run No. 3
-3, +4	23.52	23.31	22.98
-4, +6	27.96	27.95	27.99
-6, +8	15.16	15.10	15.17
-8, +10	8.99	8.98	9.08
-10, +14	5.09	5.16	5.16
-14, +20	3.67	3.68	3.70
-20, +28	2.90	2.90	2.93
-28, +35	2.22	2.20	2.23
-35, +48	1.70	1.77	1.72
-48, +65	1.58	1.59	1.63
-65, +100	1.53	1.56	1.58
-100, +150	1.06	1.08	1.11
-150, +200	1.11	1.13	1.17
-200, +270	0.71	0.79	0.95
-270	2.80	2.80	2.60
	100.00	100.00	100.00
Sample Wt. Grams	908.16	910.71	909.58

APPENDIX C

BALL MEASUREMENTS

N	Ball Size, Inches				
	3/4" balls	1" balls	1-1/2" balls	2" balls	2-1/2" balls
1	0.745	1.023	1.492	1.997	2.531
2	0.744	1.046	1.482	2.014	2.554
3	0.742	1.036	1.550	1.961	2.551
4	0.740	1.025	1.511	1.966	2.550
5	0.746	1.034	1.578	1.982	2.553
6	0.753	1.030	1.470	1.990	2.548
7	0.770	1.044	1.500	1.964	2.526
8	0.742	1.004	1.540	2.079	2.550
9	0.792	1.022	1.507	1.971	2.566
10	0.783	1.014	1.498	2.057	2.540
11			1.487	2.005	
12			1.491	1.982	
13			1.488	1.980	
14			1.476	1.972	
15			1.499	2.024	
16			1.470	1.963	
17			1.490	1.988	
18			1.486	1.996	
19			1.505	1.971	
20			1.450	2.000	
21			1.448	2.030	
22			1.530	1.960	
23			1.515	2.006	
24			1.497	1.962	
25			1.550	2.004	
26			1.477		
27			1.602		
28			1.478		
29			1.469		
30			1.488		
31			1.518		
32			1.458		
33			1.432		
34			1.470		
35			1.470		
36			1.530		
37			1.453		
38			1.556		
39			1.460		
40			1.462		

APPENDIX D

SPECIFIC SURFACE PROGRAM

In order to calculate a specific surface for the feed and product distributions, a Fortran program was written for a 7072 digital computer. This program calculated the specific surface based on the weight of material held on each Tyler screen in the $\sqrt{2}$ series from 3 to 270 mesh. The mass fraction held on each screen and that screen's contribution to the total specific surface was calculated. A least-square line was determined using the mass fraction held on the 65, 100, 150, 200 and 270 mesh screens, and this line was extrapolated to obtain mass fractions and specific surface contributions for the 400, 565, 800, 1130, and 1600 meshes. Another least-square line based on the material held on the 100, 150, 200, and 270 mesh screens was also determined and extrapolated to obtain the material for the 400 to 1600 meshes. The two least-square lines were compared and the one having the smallest variance of experimental and calculated points was used in the determination of the specific surface. In all of the specific surface calculations, a sphericity of 1.0 was assumed.

SPECIFIC SURFACE FORTRAN PROGRAM

```
KELLNER
COMPILE FORTRAN, EXECUTE FORTRAN
 DIMENSION SCR(20,2),WT(20),AM(20)
 SCR(1,1) = 3.0
 SCR(1,2) = 0.673
 SCR(2,1) = 4.0
 SCR(2,2) = 0.476
 SCR(3,1) = 6.0
 SCR(3,2) = 0.3327
 SCR(4,1) = 8.0
 SCR(4,2) = 0.2362
 SCR(5,1) = 10.0
 SCR(5,2) = 0.1651
 SCR(6,1) = 14.0
 SCR(6,2) = 0.1168
 SCR(7,1) = 20.0
 SCR(7,2) = 0.0833
 SCR(8,1) = 28.0
 SCR(8,2) = 0.0589
 SCR(9,1) = 35.0
 SCR(9,2) = 0.0417
```

```
SCR(10,1) = 48.0
SCR(10,2) = 0.0295
SCR(11,1) = 65.0
SCR(11,2) = 0.0208
SCR(12,1) = 100.0
SCR(12,2) = 0.0147
SCR(13,1) = 150.0
SCR(13,2) = 0.0104
SCR(14,1) = 200.0
SCR(14,2) = 0.0074
SCR(15,1) = 270.0
SCR(15,2) = 0.0053
SCR(16,1) = 400.0
SCR(16,2) = 0.0037
SCR(17,1) = 565.0
SCR(17,2) = 0.0026
SCR(18,1) = 800.0
SCR(18,2) = 0.00185
SCR(19,1) = 1130.0
SCR(19,2) = 0.0013
SCR(20,1) = 1600.0
```

```
      SCR(20,2) = 0.00093
    2 FORMAT (3F10.3,3I10)
    3 FORMAT (F20.8)
   10 FORMAT (10X,1H-,I4,2H,+,I4,5X,4F20.8/)
   11 FORMAT (10X, 1H-,I4,11X,4F20.8/)
   12 FORMAT (5X, 11HSUMMATION =,30X,3F20.8/)
   13 FORMAT (5X, 30HVOLUME-SURFACE MEAN DIAMETER =,F20.8/)
   16 FORMAT (37X, 13HTEST NUMBER =,I4,5X,4HN = ,I3/)
   17 FORMAT (12X,  9HMESH SIZE,12X,13HDIAMETER (CM),7X,14HWEIGHT (GRAMS
     1),7X,13HMASS FRACTION,7X,12HSURFACE AREA/)
   80 FORMAT (38X, 25HSURFACE AREA CALCULATIONS/)
   81 FORMAT (1H1,38X,25HSURFACE AREA CALCULATIONS/)
   82 FORMAT (1H1,10X, 3HY =, F10.5,1H+,F10.5,1HX,15X,21H5 POINT EXTRAPO
     1LATION/)
   83 FORMAT (25X, 67H65 MESH        100 MESH        150 MESH        200 ME
     1SH        270 MESH/)
   84 FORMAT (5X, 8HACTUAL =,4X,5F15.5/)
   85 FORMAT (10X,10HVARIANCE =,F10.5///)
   86 FORMAT (5X,12HCALCULATED =,5F15.5/)
   87 FORMAT (1H ,10X, 3HY =, F10.5,1H+,F10.5,1HX,15X,21H4 POINT EXTRAPO
     1LATION/)
```

```
   READ 2, RHO, SH, SMES, N, NTEST, N1
   PRINT 80
15 SUM = 0.0
   PRINT 16, NTEST, N
   PRINT 17
   DO 20 NN =1,N
   READ 3, DATA
   WT(NN) = DATA
20 SUM = SUM+WT(NN)
   L = 0
22 L = L+1
   IF (SMES-SCR(L,1)) 22, 23, 22
23 ARSU = 0.0
   L1 = L
   AMSU = 0.0
   NK = 0
   I = 0
   GO TO 25
24 L = L+1
25 DAV = (SCR(L,2)+SCR(L-1,2))/2.0
27 NK = NK+1
```

```
      MES = SCR(L-1,1)
      MES1 = SCR(L,1)
      AM(NK) = WT(NK)/SUM
      AREA = 6.0*SH*AM(NK)/(RHO*DAV)
      AMSU = AMSU+AM(NK)
      ARSU = ARSU+AREA
      IF (NK-(N-1)) 29,29,40
   29 PRINT 10, MES, MESI, DAV, WT(NK), AM(NK), AREA
      IF (NK-(N-1)) 24,30,40
   30 IF (N1) 31,31,32
   32 L11 = L
      NK1 = NK
      N11 = N
      XY2 = ARSU
      XY1 = AMSU
      DIMENSION WTPP(20)
      DIMENSION WFPP(5)
      SUMLS = 0.0
      J = I
      COEA2=0.0
      YC2=0.0
```

```
      COEB2=0.0
      COEFA =0.0
      YC=0.0
      COEFB = 0.0
      A1 = 1.0
      SUMP = 0.0
      N6 = N1-L1
      DO 33 N2=1,N6
   33 SUMP = SUMP+ AM(N2)
      N7 = N-1
      N8 = N6 +1
      DO 34 N3=N8,N7
      SUMP = SUMP + AM(N3)
      J = J + 1
      WTPP(J) = (1.0-SUMP)*100.0
      WFPP(J) = 1.0-SUMP
      Y1 = LOGF(WTPP(J))
      N9 = N3+L1-1
      X1 = LOGF(SCR(N9,1))
      YC = YC + Y1
      COEFB = COEFB + X1
```

```
   COEFA = COEFA + A1
   Y2 = Y1*X1
   A2 = A1*X1
   X2 = X1*X1
   YC2 = YC2 + Y2
   COEB2 = COEB2 + X2
34 COEA2 = COEA2 + A2
   A = (COEB2*YC-COEFB*YC2)/(COEFA*COEB2-COEFB*COEA2)
   B = (YC2-COEA2*A)/COEB2
35 XEXT = SCR(L+1,1)
   YLOG = A + B*LOGF(XEXT)
   PERP = 10.0**YLOG
   WT(N) = (WTPP(J)-PERP)*SUM/100.0
   J = J+1
   WTPP(J) = PERP
   N = N+1
   L = L+1
   IF (L-20) 35,36,36
36 N1 = N1-20
   L = 15
   WT(N) = PERP*SUM/100.0
```

```
      GO TO 24
   31 DAV = 0.5*SCR(L,2)
      L = L+1
      GO TO 27
   40 PRINT 11, MES, DAV, WT(NK), AM(NK), AREA
      PRINT 12, SUM, AMSU, ARSU
      VSMD = 6.0*SH/(ARSU*RHO)
      PRINT 13, VSMD
      IF (L-16) 1,1,38
   38 J=1
      I=I+1
      DIMENSION PERP1(5)
      IF (I-2) 42,43,43
   42 PRINT 82, A,B
      GO TO 44
   43 PRINT 87, A,B
   44 PRINT 83
      DO 37 M = 11,15
      YLOG = A + B*LOGF(SCR(M,1))
      PERP1(J) = 10.0**YLOG/100.0
   37 J = J+1
```

```
      DO 41 J=I,5
   41 SUMLS = SUMLS + ((PERP1(J)-WFPP(J))*100.0)**2.0
      EN = J-I+1
      SUMLS = SUMLS/EN
      PRINT 84, WFPP
      PRINT 86, PERP1
      PRINT 85, SUMLS
      IF (I-2) 39,1,1
   39 N1 = N1 +21
      AMSU = XY1
      ARSU = XY2
      N = N11
      NK = NK1
      L = L11
      GO TO 32
    1 READ 2, RHO, SH, SMES, N, NTEST, N1
      IF (N) 50,50,5
    5 PRINT 81
      GO TO 15
   50 STOP
      END
```

APPENDIX E

EXAMPLE OF TEST DATA COLLECTED

Date: 12/14/65 Test 7

Desired Feed rate = 2 lb/min

Measured feed rate in adjusting ore hopper gate = 912 gms/min
= 2.01 lb/min

Ore addition to hopper

Minutes elapsed during test	lbs added to hopper*	Notes
0	59-1/4	Test started at 1:35 P.M.
7	57-1/4	
31	55-1/4	Ore remaining in hopper at end of test = 62-1/2 lb.

* Ore weights include weight of bucket (3-1/4 lb).

Water addition to mill

Minutes elapsed during test	Percent of Calibration Scale	Grams water per minute
0	30%	490
17	30%	490
31	30%	490

Amperage Measurements

Wire	Amps	
	Run No. 1	Run No. 2
1	4.1	4.1
2	4.2	4.2
3	4.2	4.2

Discharge Rate of Solids (Measured on Pulp Density Scale)

Minutes elapsed during test	Sample Collection Time, Seconds	Grams Solids Discharged per minute
30	30	902
35	30	886
40	30	836
42	30	890

Samples Collected to Determine Product Size Distribution

Sample Number	Minutes elapsed during test	Sample Collection Time, Seconds	Wet Wt. of Sample, Grams	Dry Wt. of Sample, Grams
7-1	44-1/3	30	684	438
7-2	46	30	662	433
7-3	47	30	649	488

Test terminated after 52 minutes of elapsed time.

Pulp density measured on Pulp Density Scale = 65%

Test Control Checks:

Actual feed rate (based on total ore added to mill during elapsed time of test) = 1.98 lb/min

Measured feed rate (used to set ore hopper gate) = 2.01 lb/min

Measured discharge rate (based on data obtained with pulp density scale) = 1.96 lb/min

Calculated discharge rate (based on the three samples collected to determine the size distribution of the product) = 1.89 lb/min

Actual percent solids (based on percent solids of three samples collected to determine product size distribution) = 64.7 percent

Measured percent solids (measured on pulp density scale) = 65.0 percent

The product size distributions are in Appendix F.

APPENDIX F

PRODUCT SIZE DISTRIBUTION FOR DIFFERENT FEED RATES

Test 1

Date - 12/4/65
Feed Rate = 2.03 lb/min
Percent Solids = 64.7%
Mill Retention Time = 15.14 min

Mesh Size	Sample 1		Sample 2	
	Weight Retained, grams	Percent Retained	Weight Retained, grams	Percent Retained
-3, +4	9.90	2.19	7.18	1.64
-4, +6	22.00	4.87	18.75	4.28
-6, +8	17.49	3.87	18.38	4.19
-8, +10	16.58	3.67	14.53	3.31
-10, +14	11.24	2.49	11.22	2.56
-14, +20	11.97	2.65	11.40	2.60
-20, +28	14.78	3.27	14.46	3.30
-28, +35	18.60	4.12	18.02	4.11
-35, +48	22.63	5.01	22.43	5.11
-48, +65	31.88	7.06	31.82	7.26
-65, +100	41.77	9.25	41.49	9.46
-100, +150	33.23	7.35	33.22	7.57
-150, +200	35.07	7.76	34.32	7.83
-200, +270	21.58	4.78	21.93	5.00
-270	143.00	31.66	139.34	31.78
	451.72	100.00	438.49	100.00
Specific Surface sq cm/g	903		903	

Test 2

Date - 12/4/65
Feed Rate = 2.80 lb/min
Percent Solids = 63.9%
Mill Retention Time = 10.78 min

Mesh Size	Sample 1		Sample 2	
	Weight Retained, grams	Percent Retained	Weight Retained, grams	Percent Retained
-3, +4	9.38	2.36	17.03	4.20
-4, +6	27.84	7.01	35.21	8.69
-6, +8	28.42	7.16	25.48	6.29
-8, +10	22.66	5.71	21.08	5.20
-10, +14	15.83	3.99	15.13	3.74
-14, +20	15.70	3.95	14.60	3.60
-20, +28	17.61	4.43	17.18	4.24
-28, +35	19.47	4.90	19.39	4.79
-35, +48	20.99	5.28	21.58	5.33
-48, +65	26.25	6.61	26.90	6.64
-65, +100	30.40	7.65	30.39	7.50
-100, +150	22.85	5.75	23.23	5.74
-150, +200	23.32	5.87	24.41	6.03
-200, +270	14.28	3.60	15.70	3.88
-270	102.20	25.73	97.72	24.13
	397.20	100.00	405.03	100.00
Specific Surface sq cm/g	742		687	

Test 3

Date - 12/9/65
Feed Rate = 0.52 lb/min
Percent Solids = 64.3%
Mill Retention Time = 58.60 min

Mesh Size	Sample 1		Sample 2	
	Weight Retained, grams	Percent Retained	Weight Retained, grams	Percent Retained
-3, +4	0.00	0.00	0.00	0.00
-4, +6	0.49	0.11	1.03	0.23
-6, +8	1.47	0.34	1.38	0.31
-8, +10	1.26	0.29	0.71	0.39
-10, +14	1.54	0.35	1.84	0.41
-14, +20	2.02	0.46	2.14	0.48
-20, +28	2.90	0.66	2.99	0.68
-28, +35	4.27	0.98	4.42	1.00
-35, +48	6.72	1.53	7.45	1.68
-48, +65	12.63	2.88	13.92	3.15
-65, +100	22.70	5.18	22.67	5.13
-100, +150	25.11	5.73	24.91	5.63
-150, +200	39.75	9.08	39.58	8.95
-200, +270	35.30	8.06	33.02	7.47
-270	281.90	64.35	285.18	64.49
	438.06	100.00	442.24	100.00
Specific Surface sq cm/gm	2100		2121	

Test 4

Date - 12/9/65
Feed Rate = 0.51 lb/min
Percent Solids = 63.5%
Mill Retention Time = 58.57 min

Mesh Size	Sample 1		Sample 2	
	Weight Retained, grams	Percent Retained	Weight Retained, grams	Percent Retained
-3, +4	1.28	0.30	1.49	0.34
-4, +6	1.91	0.44	1.91	0.44
-6, +8	2.09	0.49	1.91	0.44
-8, +10	1.93	0.45	2.04	0.47
-10, +14	1.71	0.40	1.92	0.44
-14, +20	1.98	0.46	2.27	0.52
-20, +28	2.88	0.67	3.06	0.70
-28, +35	4.25	0.99	4.57	1.05
-35, +48	6.66	1.55	7.20	1.65
-48, +65	12.32	2.86	13.30	3.04
-65, +100	23.03	5.35	23.02	5.27
-100, +150	25.84	6.01	25.38	5.81
-150, +200	38.62	8.98	38.67	8.86
-200, +270	33.72	7.84	31.26	7.16
-270	271.90	63.21	278.58	63.81
	430.12	100.00	436.58	100.00
Specific Surface sq cm/g	2051		2094	

Test 5

Date - 12/9/65
Feed Rate = 0.94 lb/min
Percent Solids = 62.9%
Mill Retention Time = 31.27 min

Mesh Size	Sample 1		Sample 2	
	Weight Retained, grams	Percent Retained	Weight Retained, grams	Percent Retained
-3, +4	7.03	1.54	4.58	1.02
-4, +6	9.12	2.00	7.10	1.58
-6, +8	6.43	1.41	6.61	1.47
-8, +10	5.54	1.22	5.76	1.28
-10, +14	3.55	0.78	4.12	0.92
-14, +20	4.04	0.89	3.99	0.89
-20, +28	5.22	1.14	5.39	1.20
-28, +35	7.63	1.67	7.88	1.76
-35, +48	11.08	2.43	11.12	2.48
-48, +65	19.86	4.36	19.69	4.39
-65, +100	34.30	7.52	34.36	7.66
-100, +150	37.09	8.14	36.38	8.11
-150, +200	47.00	10.31	48.18	10.74
-200, +270	46.22	10.14	52.92	11.79
-270	211.72	46.45	200.64	44.71
	455.83	100.00	448.72	100.00
Specific Surface sq cm/g	1292		1214	

Test 6

Date - 12/14/65
Feed Rate = 0.99 lb/min
Percent Solids = 66.6%
Mill Retention Time = 32.52 min

Mesh Size	Sample 1		Sample 2	
	Weight Retained, grams	Percent Retained	Weight Retained, grams	Percent Retained
-3, +4	1.22	0.30	4.78	1.06
-4, +6	4.08	1.01	5.52	1.22
-6, +8	3.91	0.97	4.18	0.92
-8, +10	3.22	0.80	3.89	0.86
-10, +14	3.01	0.74	3.50	0.77
-14, +20	3.68	0.91	4.47	0.99
-20, +28	5.28	1.30	6.10	1.35
-28, +35	8.18	2.02	8.92	1.97
-35, +48	12.49	3.09	13.52	2.98
-48, +65	21.12	5.22	22.98	5.07
-65, +100	34.38	8.50	37.41	8.25
-100, +150	35.61	8.80	38.43	8.48
-150, +200	44.95	11.11	46.98	10.36
-200, +270	45.42	11.22	32.79	7.23
-270	178.08	44.01	219.82	48.49
	404.63	100.00	453.29	100.00
Specific Surface sq cm/g	1175		1378	

Test 7

Date - 12/14/65
Feed Rate = 1.98 lb/min
Percent Solids = 64.7%
Mill Retention Time = 15.53 min

Mesh Size	Sample 1		Sample 2	
	Weight Retained, grams	Percent Retained	Weight Retained, grams	Percent Retained
-3, +4	10.09	2.30	13.28	3.06
-4, +6	24.11	5.50	25.03	5.78
-6, +8	20.32	4.64	18.52	4.27
-8, +10	16.61	3.79	15.60	3.60
-10, +14	11.87	2.71	10.92	2.52
-14, +20	12.01	2.74	11.68	2.70
-20, +28	13.97	3.19	13.65	3.15
-28, +35	17.34	3.96	16.92	3.90
-35, +48	21.00	4.79	20.47	4.72
-48, +65	29.72	6.78	28.71	6.63
-65, +100	38.71	8.84	36.30	8.38
-100, +150	31.74	7.25	29.10	6.72
-150, +200	33.62	7.67	31.19	7.20
-200, +270	22.80	5.21	19.58	4.52
-270	134.22	30.63	142.35	32.85
	438.13	100.00	433.30	100.00
Specific Surface sq cm/g	876		970	

Test 8

Date - 12/14/65
Feed Rate = 2.91 lb/min
Percent Solids = 64.7%
Mill Retention Time = 10.56 min

Mesh Size	Sample 1		Sample 2	
	Weight Retained, grams	Percent Retained	Weight Retained, grams	Percent Retained
-3, +4	29.58	6.05	31.64	6.83
-4, +6	52.47	10.74	44.02	9.50
-6, +8	35.78	7.32	29.89	6.45
-8, +10	25.80	5.28	24.85	5.36
-10, +14	18.68	3.82	17.33	3.74
-14, +20	18.11	3.71	17.46	3.77
-20, +28	19.40	3.97	18.87	4.07
-28, +35	21.48	4.39	20.68	4.46
-35, +48	23.33	4.77	22.59	4.88
-48, +65	28.85	5.90	28.19	6.09
-65, +100	33.37	6.83	32.49	7.01
-100, +150	24.85	5.08	24.41	5.27
-150, +200	26.11	5.34	25.64	5.53
-200, +270	15.82	3.24	16.68	3.60
-270	115.16	23.56	108.60	23.44
	488.79	100.00	463.34	100.00
Specific Surface sq cm/g	685		676	

Test 9

Date - 12/23/65
Feed Rate = 1.54 lb/min
Percent Solids = 65.6%
Mill Retention Time = 20.36 min

Mesh Size	Sample 1		Sample 2	
	Weight Retained, grams	Percent Retained	Weight Retained, grams	Percent Retained
-3, +4	9.49	2.09	9.40	2.04
-4, +6	17.49	3.85	20.57	4.46
-6, +8	13.38	2.95	13.21	2.86
-8, +10	12.81	2.82	12.99	2.82
-10, +14	8.90	1.96	9.89	2.14
-14, +20	9.20	2.03	9.62	2.08
-20, +28	11.40	2.51	11.53	2.50
-28, +35	15.08	3.32	15.40	3.34
-35, +48	20.02	4.41	20.17	4.37
-48, +65	29.74	6.55	29.85	6.47
-65, +100	41.21	9.08	51.32	8.96
-100, +150	35.22	7.76	34.85	7.56
-150, +200	38.90	8.57	38.50	8.35
-200, +270	25.57	5.64	23.37	5.07
-270	165.50	36.46	170.58	36.98
	453.91	100.00	461.25	100.00
Specific Surface sq cm/g	1000		1030	

Test 10

Date - 12/23/65
Feed Rate = 1.51 lb/min
Percent Solids = 65.3%
Mill Retention Time = 20.63 min

Mesh Size	Sample 1		Sample 2	
	Weight Retained, grams	Percent Retained	Weight Retained, grams	Percent Retained
-3, +4	11.91	2.57	8.31	1.91
-4, +6	20.75	4.48	18.52	4.26
-6, +8	15.73	3.40	15.58	3.58
-8, +10	12.72	2.75	12.73	2.92
-10, +14	9.31	2.01	9.00	2.07
-14, +20	9.68	2.09	9.20	2.11
-20, +28	11.67	2.52	11.08	2.55
-28, +35	15.52	3.35	14.51	3.33
-35, +48	20.03	4.33	19.22	4.42
-48, +65	29.18	6.31	27.81	6.39
-65, +100	40.40	8.73	38.32	8.81
-100, +150	34.33	7.42	32.61	7.49
-150, +200	37.61	8.13	36.02	8.28
-200, +270	24.83	5.37	23.38	5.37
-270	169.11	36.54	158.86	36.51
	462.78	100.00	435.15	100.00
Specific Surface sq cm/g	1020		1014	

APPENDIX G

ANALOG DEVELOPMENT OF AXIAL-GRADIENT MODEL

Equations

1) $$V_Z \frac{dX_1}{dZ} = -k_1X_1 + D_L \frac{d^2X_1}{dZ^2}$$

2) $$V_Z \frac{dX_2}{dZ} = -k_2X_2 + D_L \frac{d^2X_2}{dZ^2} + k_1X_1$$

3) $$V_Z \frac{dX_3}{dZ} = -k_3X_3 + D_L \frac{d^2X_3}{dZ^2} + k_2X_2$$

4) $$V_Z \frac{dX_4}{dZ} = -k_4X_4 + D_L \frac{d^2X_4}{dZ^2} + k_3X_3$$

5) $$V_Z \frac{dX_5}{dZ} = -k_5X_5 + D_L \frac{d^2X_5}{dZ^2} + k_4X_4$$

6) $$V_Z \frac{dX_6}{dZ} = -k_6X_6 + D_L \frac{d^2X_6}{dZ^2} + k_5X_5$$

7) $$V_Z \frac{dX_7}{dZ} = -k_7X_7 + D_L \frac{d^2X_7}{dZ^2} + k_6X_6$$

8) $$V_Z \frac{dX_8}{dZ} = + D_L \frac{d^2X_8}{dZ^2} + k_7X_7$$

Prob. Var.	Max. Value	Scale Factor	Comp. Var.
X_1	50	2	$(2X_1)$
$\dot{X}_1$	1000	0.1	$(.1\dot{X}_1)$
$\ddot{X}_1$	1000	0.1	$(.1\ddot{X}_1)$
X_2	25	4	$(4X_2)$
$\dot{X}_2$	500	0.2	$(.2\dot{X}_2)$
$\ddot{X}_2$	500	0.2	$(.2\ddot{X}_2)$
X_3	10	10	$(10X_3)$
$\dot{X}_3$	500	0.2	$(.2\dot{X}_3)$
$\ddot{X}_3$	500	0.2	$(.2\ddot{X}_3)$
X_4	10	10	$(10X_4)$
$\dot{X}_4$	500	0.2	$(.2\dot{X}_4)$
$\ddot{X}_4$	500	0.2	$(.2\ddot{X}_4)$
X_5	10	10	$(10X_5)$
$\dot{X}_5$	500	0.2	$(.2\dot{X}_5)$
$\ddot{X}_5$	500	0.2	$(.2\ddot{X}_5)$
X_6	20	5	$(5X_6)$
$\dot{X}_6$	500	0.2	$(.2\dot{X}_6)$
$\ddot{X}_6$	500	0.2	$(.2\ddot{X}_6)$
X_7	20	5	$(5X_7)$
$\dot{X}_7$	500	0.2	$(.2\dot{X}_7)$
$\ddot{X}_7$	500	0.2	$(.2\ddot{X}_7)$
X_8	100	1	(X_8)
$\dot{X}_8$	500	0.2	$(.2\dot{X}_8)$
$\ddot{X}_8$	500	0.2	$(.2\ddot{X}_8)$

Scaled Equations

$$1) \quad (.1\dot{X}_1) = -0.05 \frac{k_1}{V_Z}(2X_1) + \frac{D_L}{V_Z}(.1\ddot{X}_1)$$

$$2) \quad (.2\dot{X}_2) = -0.05 \frac{k_2}{V_Z}(4X_2) + \frac{D_L}{V_Z}(.2\ddot{X}_2) + 0.1 \frac{k_1}{V_Z}(2X_1)$$

$$3) \quad (.2\dot{X}_3) = -0.02 \frac{k_3}{V_Z}(10X_3) + \frac{D_L}{V_Z}(.2\ddot{X}_3) + 0.5 \frac{k_2}{V_Z}(4X_2)$$

$$4) \quad (.2\dot{X}_4) = -0.02 \frac{k_4}{V_Z}(10X_4) + \frac{D_L}{V_Z}(.2\ddot{X}_4) + 0.02 \frac{k_3}{V_Z}(10X_3)$$

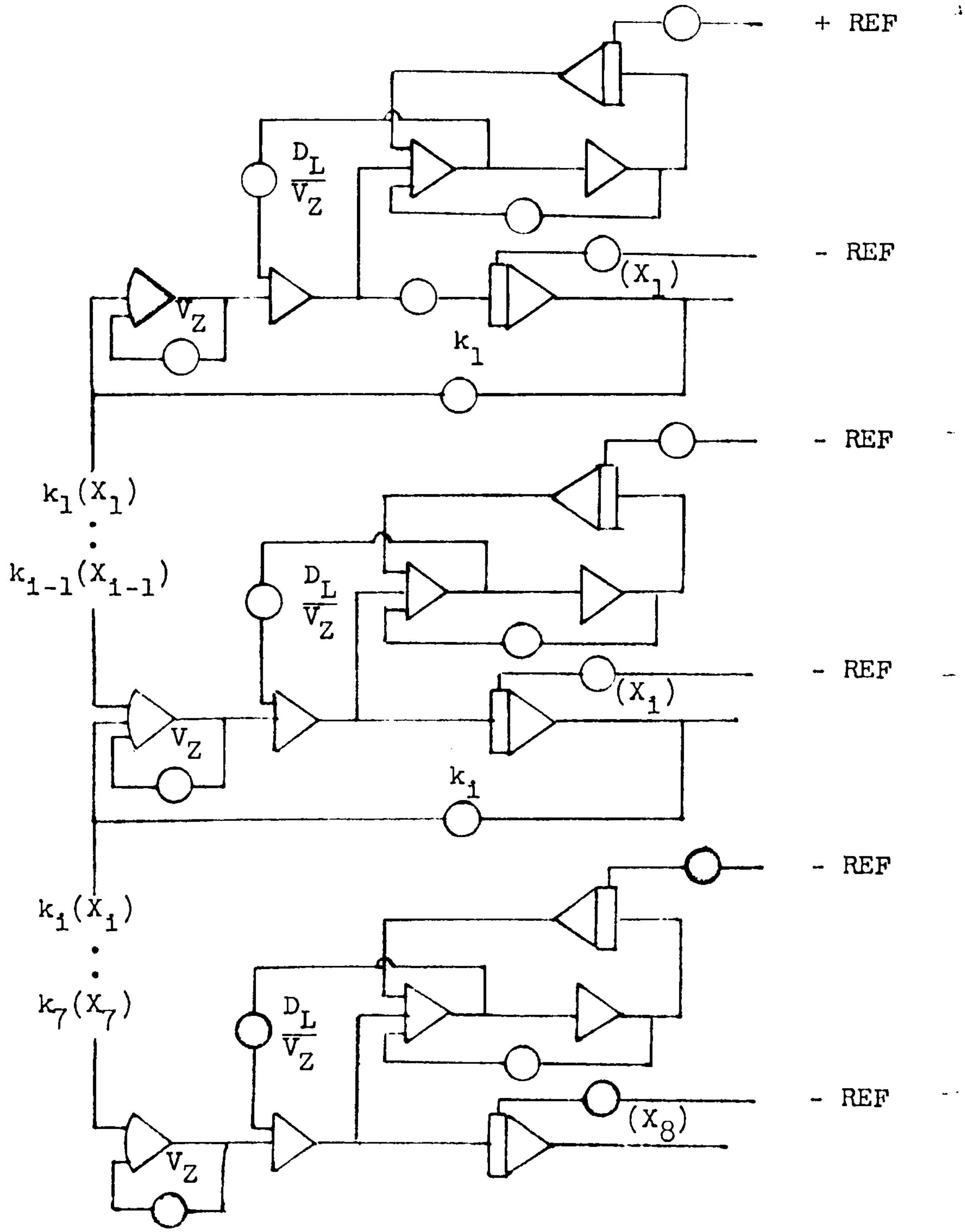
$$5) \quad (.2\dot{X}_5) = -0.02 \frac{k_5}{V_Z}(10X_5) + \frac{D_L}{V_Z}(.2\ddot{X}_5) + 0.02 \frac{k_4}{V_Z}(10X_4)$$

$$6) \quad (.2\dot{X}_6) = -0.04 \frac{k_6}{V_Z}(5X_6) + \frac{D_L}{V_Z}(.2\ddot{X}_6) + 0.02 \frac{k_5}{V_Z}(10X_5)$$

$$7) \quad (.2\dot{X}_7) = -0.04 \frac{k_7}{V_Z}(5X_7) + \frac{D_L}{V_Z}(.2\ddot{X}_7) + 0.04 \frac{k_6}{V_Z}(5X_6)$$

$$8) \quad (.2\dot{X}_8) = + \frac{D_L}{V_Z}(.2\ddot{X}_8) + 0.04 \frac{k_7}{V_Z}(5X_7)$$

Analog Circuit Diagram



APPENDIX H

DETERMINATION OF COMMINUTION RATE CONSTANTS FOR WELL-STIRRED-TANK MODEL

In order to determine the comminution rate constants for a model using two well-stirred tanks in series, a Fortran program was written for a 7072 digital computer. This program calculated the comminution rate constants as a function of particle size and the probability of the particles being discharged from the mill. The results of these calculations using the best probability vector obtained by trial and error is also presented in this Appendix.

COMMINUTION RATE CONSTANT FORTRAN PROGRAM

```
  KELLNER
  COMPILE FORTRAN, EXECUTE FORTRAN
   DIMENSION X(8), THEDA(10), CX(8,2), GK(8,10), GG(8,10), A(8)
   DIMENSION XX(8,10)
90 FORMAT (F10.5)
91 FORMAT (10F10.3)
92 FORMAT (2F10.5)
93 FORMAT (8F5.2)
   X(1) = 0.4566
   X(2) = 0.2739
   X(3) = 0.1020
   X(4) = 0.0566
   X(5) = 0.0361
   X(6) = 0.0282
   X(7) = 0.0192
   X(8) = 0.0274
   THEDA(1) = 58.57
   THEDA(2) = 58.60
   THEDA(3) = 31.27
   THEDA(4) = 32.52
   THEDA(5) = 20.36
```

```
      THEDA(6) = 20.63
      THEDA(7) = 15.14
      THEDA(8) = 15.53
      THEDA(9) = 10.78
      THEDA(10) = 10.56
      I = 0
   21 I=I+1
      DO 20 N=1,8
      READ 90, P
   20 XX(N,I)=P
      IF (I-10) 21,22,22
   22 READ 93, A
      IF (A(1)) 70,70,23
   23 I = 0
   40 I=I+1
      T=THEDA(I)/2.0
      CX(1,2)=XX(1,I)/A(1)
      GK(1,I) = ((X(1)/CX(1,2))**0.5-1.0)/T
      CX(1,1) = X(1)/(1.0+T*GK(1,I))
      M=0
      DO 50 N=2,8
```

```
      CX(N,2)=XX(N,I)/A(N)
      M=M+1
      B=2.0*T*CX(N,2)-T**2.0*GK(M,I)*CX(M,2)
      AC=-T**2.0*CX(N,2)*(X(N)+T*GK(M,I)*CX(M,1)+T*GK(M,I)*CX*(M,2)-CX(N,
     12))
      GK(N,I)=(-B+(B**2.0-4.0*AC)**0.5)/(2.0*CX(N,2)*T**2.0))
   50 CX(N,1) = (X(N)+T*GK(M,I)*CX(M,1))/(1.0+T*GK(N,I))
      IF (I-10) 40,60,60
   60 PRINT 91, ((GK(N,I),I=1,10),N=1,8)
      GO TO 22
   70 STOP
      END
```

Calculated Comminution Rate Constants

Size Fraction	Probability of Discharge	Feed Rate, lb/min									
		0.51	0.52	0.94	0.99	1.54	1.51	2.03	1.98	2.80	2.91
		Comminution Rate Constants									
X_1	0.8	0.203	0.466	0.157	0.216	0.140	0.131	0.181	0.141	0.151	0.092
X_2	0.8	0.322	0.346	0.310	0.392	0.259	0.233	0.272	0.241	0.219	0.195
X_3	0.8	0.443	0.415	0.587	0.540	0.437	0.426	0.473	0.432	0.390	0.350
X_4	0.8	0.313	0.302	0.428	0.355	0.346	0.336	0.358	0.350	0.319	0.303
X_5	0.9	0.173	0.167	0.238	0.194	0.212	0.209	0.233	0.232	0.231	0.220
X_6	0.9	0.083	0.086	0.104	0.093	0.113	0.113	0.126	0.129	0.134	0.123
X_7	1.0	0.062	0.061	0.066	0.070	0.113	0.112	0.128	0.126	0.117	0.098
X_8	1.0	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

APPENDIX I

CALCULATED PRODUCT DISTRIBUTION FOR WELL-STIRRED-TANK MODEL

A Fortran program was written for the 7072 digital computer in order to calculate the size distribution of the product at the different feed rates. The comminution rate constants presented in Appendix H were averaged for each particle size and then used to calculate the product size distribution. The probability vector presented in Appendix H was used in the calculations.

PRODUCT DISTRIBUTION FORTRAN PROGRAM

```
  KELLNER
  COMPILE FORTRAN, EXECUTE FORTRAN
   DIMENSION P(8), GK(8), XX(8,5), X(8), THETA(5)
90 FORMAT (8F5.3)
91 FORMAT (5F10.3)
   X(1) = 0.4566
   X(2) = 0.2739
   X(3) = 0.1020
   X(4) = 0.0566
   X(5) = 0.0361
   X(6) = 0.0282
   X(7) = 0.0192
   X(8) = 0.0274
   THETA(1) = 58.585
   THETA(2) = 31.895
   THETA(3) = 20.495
   THETA(4) = 15.335
   THETA(5) = 10.67
10 READ 90, GK
   IF(GK(1)) 70,70,11
11 I=0
```

```
   READ 90, P
20 I=I+1
   T=THETA(I)/2.0
   TT=X(1)/((1.0+T*GK(1))**2.0)
   XX(1,I)=P(1)*TT
   FFA=X(1)/(1.0+T*GK(1))
   DO 30  N=2,8
   TT=          ((X(N)+FFA*T*GK(N-1))/(1.0+T*GK(N))+TT*T*GK(N-1))/(1.
  10+T*GK(N))
   XX(N,I)=P(N)*TT
30 FFA=(X(N)+T*GK(N-1)*FFA)/(1.0+T*GK(N))
   IF(I-5) 20,21,21
21 PRINT 91, ((XX(N,I),I=1,5),N=1,8)
   GO TO 10
70 STOP
   END
```

LIST OF REFERENCES

Bond, F. C., Grinding Ball Size Selection, presented at the Annual Meeting of Society of Mining Engineers of AIME, October, 1957.

Bowdish, F. W., Theoretical and Experimental Studies of the Kinetics of Grinding in a Ball Mill, Transactions AIME, vol. 217, 1960, pp. 194-202.

Freeh, E. J., Horst, W. E., and Kellner, R. C., A Mathematical Model for Ball Mill Grinding, Part I, Batch Grinding, submitted to Transactions AIME, April 18, 1966.

Himmelblau, D. M., and Bischoff, K. B., Chemical Process Analysis, to be published by John Wiley and Sons.

Huttig, G. F., The Kinetics of Grinding Processes, Z. Electrochem., 1965, pp. 534-539.

Kelsall, D. F., and Reid, K. J., The Influence of the Environment Within a Continuous Wet Ball Mill on the Grinding Characteristics of the System, presented at the Fifty-Eighth Annual Meeting of American Institute of Chemical Engineers, Dec. 1965.

Lynch, A. J., A Mathematical Model of a Multi-Stage Comminution System, presented at the Fifty-Eighth Annual Meeting of American Institute of Chemical Engineers, December 1965.

Meloy, T. P., Dynamo, Computers in the Mineral Industries, Part I, Stanford University Publications, Geological Sciences, vol. 9, number 1, 1964, pp. 312-323.

Meloy, T. P., and Bergstrom, B. H., Matrix Simulation of Ball Mill Circuits Considering Impact and Attrition Grinding, presented at the International Mineral Dressing Congress, New York, September 1964.

Meloy, T. P., and Bergstrom, B. H., Computer Simulation of Comminution Circuits, Quarterly of the Colorado School of Mines, October 1964.

Roberts, E. J., The Probability Theory of Wet Ball Milling and its Application, Transaction AIME, vol. 187, 1950, pp. 1267-72.