AN OPTIMAL DRIVING BEHAVIOR TO MINIMIZE AUTOMOBILE EXHAUST EMISSIONS

by

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STATEMENT OF AUTHOR

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ABSTRACT

Is it possible to drive a vehicle in such a manner that the amount of smog-forming pollutants emitted from the exhaust is minimal? An affirmative answer to this question would have a potentially great impact on minimizing smog formation. In an attempt to answer this question, a dynamic programming model is formulated. An approach to the solution of this general model is proposed.

The special case of a one-stage process is solved. Examples of this one-stage process and the types of results which can be obtained from the analysis of this process are presented. Preliminary study shows that driving policies do exist which minimize the emission of smog-forming pollutants. These policies result in at least a one-order of magnitude decrease in emissions, and in many instances, two-orders of magnitude. In particular, there is an optimal policy which corresponds to cruising at the speed limit, if possible, and which, fortunately, results in minimal travel time.

Chapter 1

INTRODUCTION

During the past 25 years, an atmospheric phenomenon has evoked considerable attention from both the scientific and technical community as well as the general public because of its harmful effects on both man and his crops. Known by the general term "smog", this phenomenon can be observed in many geographical areas as a black dirty haze. Investigations of the phenomenon by public service organizations, the petroleum and automotive industries, independent research organizations and universities have contributed to the understanding of smog and its relationship to man and his environment.

This thesis proposes an approach to the alleviation of harmful smog effects by controlling the emission of smog-forming automotive exhaust pollutants. The emphasis thus far in automotive smog control has been placed on making the installation of mechanical smog-control devices mandatory. Taking another viewpoint, would it be possible to drive in such a way that harmful exhaust emissions are minimized? A dynamic programming formulation of the problem is proposed in an attempt to determine if such a driving policy exits, and if so, what difficulties arise in its determination. Specifically the general problem will be formulated and preliminary results for a one-block process obtained. An approach to the solution of the general problem is indicated, although a complete solution is beyond the scope of this study.

The thesis is divided into four major parts. Chapter 2 is devoted to a general survey of the smog phenomenon, its meteorological and chemical basis, the man-made contributions to the phenomenon - primarily the automotive aspects-and the attempts to alleviate, eliminate, and/or control the phenomenon. No familiarity with any aspect of the phenomenon has been assumed. This chapter does not dwell on the details or specifics involved in the smog phenomenon but attempts to describe the overall aspects of the phenomenon. However, there are sufficient citations throughout the chapter to aid anyone interested in further pursuing particular aspects of the phenomenon.

A mathematical model which describes the automotive exhaust emission phenomenon is presented in the third chapter. Utilizing Finite-State Machine Theory as a descriptive technique to identify the relevant parameters and variables, this chapter defines the scope of the phenomenon to be considered.

Chapter 4 is devoted to the mathematical model actually proposed. The dynamic programming model and all of its assumptions are discussed. A computer solution of the model is displayed and discussed.

Chapter 5 concludes the study with a discussion of the model's utility, its possible extensions and recommendations for further study.

Chapter 2

THE PHOTOCHEMICAL SMOG PROBLEM

2.1 The Smog Phenomenon

In Los Angeles, California and other metropolitan areas of the United States, there exists a unique combination of factors which results in a meteorological phenomenon known as photochemical smog (1). Historically, the pioneering investigations of this phenomenon were conducted relative to the Los Angeles conditions. Only recently have other areas discovered the similarity between their problem and that in Los Angeles. Consequently, even though the following discussion is equally valid in the other areas, for the most part it is involved with the Los Angeles phenomenon.

Originally, "smog" was applied to a combination of smoke and fog. However, in the Los Angeles area, smog is used to denote a complex mixture of smoke, fumes, gas and particulate matter including dust, metals, crystals and liquid droplets. It denotes an abnormal weather condition in which man-made contaminants in the air mingle with haze or fog to produce a dense, smoky atmosphere. Smog emits a general murkiness to the air, sharply reduces visibility, damages crops (2), and frequently results in irriation to the eyes, nose and throat.

Exactly what happens during smog formation remained a mystery until 1951, when Haagen-Smit (3) showed experimentally that mixtures of air and oxides of nitrogen after irradiation by natural sunlight produced effects similar to that of smog. In particular, he demonstrated that crop damage, eye irritation and rubber cracking were attributable to products in the atmosphere generated by a photochemical reaction involving hydrocarbons and oxides of nitrogen. The oxidizing action resulting from the interaction of oxides of nitrogen and ultraviolet light converts hydrocarbons, primarily the more reactive double bonded olefins, into peroxides, aldehydes and other acids. During this photochemical oxidation, ozone is formed which oxidizes other hydrocarbons to peroxides and aldehydes and also contributes to the cracking of rubber. These peroxides and aldehydes are irritating to both plants and animals. In addition, sulfur is oxidized. The sulfur and hydrocarbon derivatives form aerosols which further decrease visibility. See Figure 1.

Further work by Haagen-Smit and his associates as well as by others have confirmed his original findings (4, 5). Since then, the major efforts in this area have been directed toward identifying the hydrocarbon

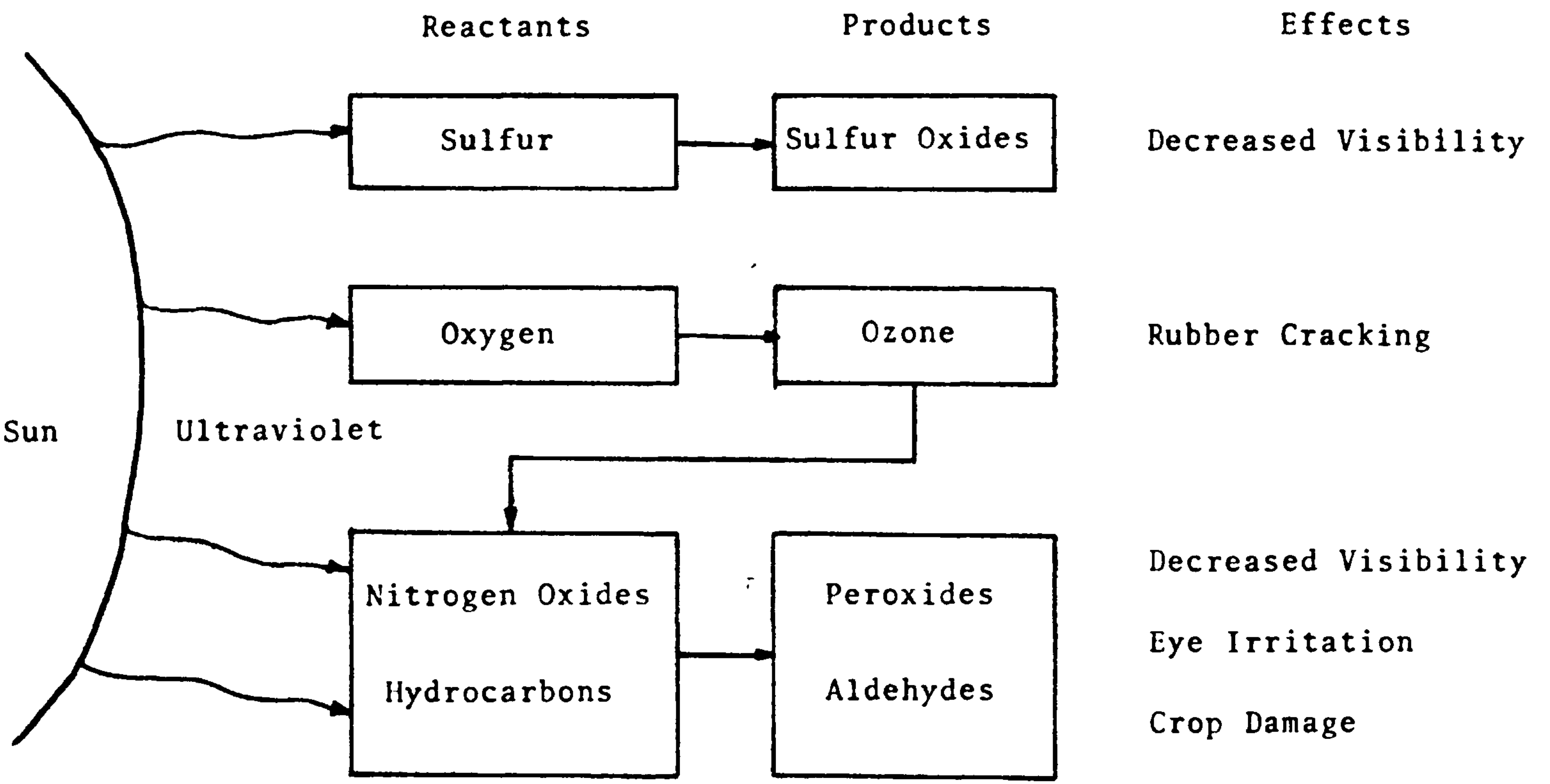


Figure 1

Photochemical Smog Reaction and Effects

derivatives, determining the identity of the reactants, determining the reaction constants and understanding the photochemical reaction (6, 7).

2.2 Meteorological Considerations

Although contaminants are always in the air, smog only occurs periodically. It is most frequent and intense during the summer and autumn when wind activity is at a minimum. It almost never occurs at night or in cloudy weather. Only when atmospheric conditions are favorable does smog occur. Three classes of air phenomena contribute to smog formation: an inversion layer, vertical air currents, and surface breezes (8).

Whereas normally air temperature decreases with increasing altitude, the opposite is true over the California coast and the Los Angeles area. This inversion layer acts as an invisible lid rising and falling between 1500 to 3000 feet in elevation during the daytime. Any smoke, dust, fumes or gases originating within the Los Angeles basin are carried via natural convection currents to the base of the inversion layer where they accumulate, building up a reservoir of contaminants that can produce smog. The inversion layer generally slopes upward in both easterly and westerly directions from the coast. It rests against the mountains on the east. Thus, it blocks any eastward or vertical flow of air from the basin which would eliminate the contaminants. Whenever the layer is low, smog is more likely to be noticed.

Vertical air currents are effected during the morning as sunlight warms the ground. The ground in turn radiates heat, warming the adjacent layers of air causing it to warm and rise. Cooler, denser air descending to replace the rising warm air carries with it the smog-producing particles from the base of the inversion layer down to street level where it is mixed throughout the air.

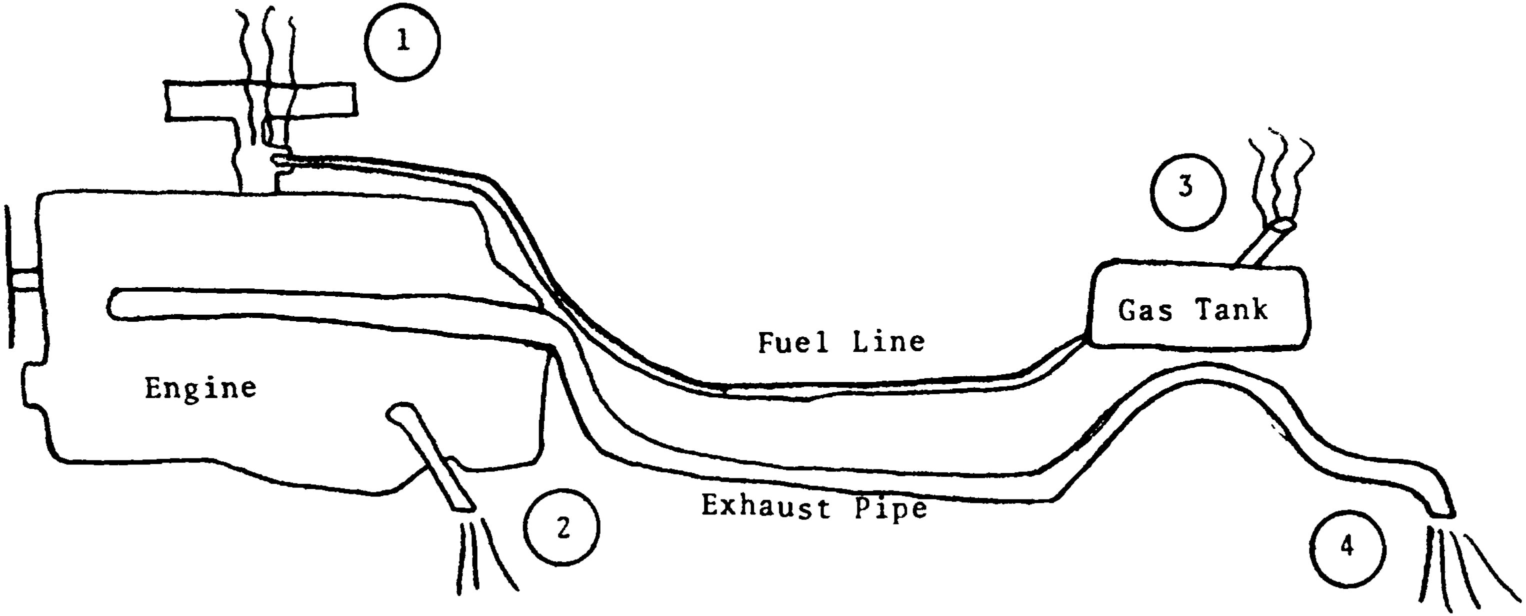
Surface breezes, which normally blow toward the ocean during the evening and night, blow inland during the day, moving the smog clouds to Los Angeles and the eastern part of Los Angeles County. Thus, smog is most intense in those communities near the mountains on the east where the smog clouds are blown and backed up by the inversion layer resting on the mountains.

It can be seen that the smog problem in Los Angeles could be alleviated by eliminating the inversion layer which not only traps pollutants but also the resulting smog. Eliminating the inversion layer would allow the natural vertically rising air currents to dissipate smog-forming contaminants into the upper atmosphere. However, this is beyond the scope of presently feasible solutions and also of this study.

2.3 Automotive Considerations

As industrial sources of hydrocarbons, dust, fumes and other pollutants were brought under control, it became more and more apparent that smog was not primarily the results of soot, black smoke and the like, but was, as originally proposed, a consequence of the invisible hydrocarbons in the atmosphere originating from automobiles. Four possible sources of hydrocarbons from automobiles are known: evaporation from the gasoline tank, gasoline evaporation from the carburetor vents, crankcase vent gases and exhaust emissions (9). See Figure 2.

Gasoline is a volatile liquid whose vapor pressure is highly dependent on the temperature. In order that the fuel be properly metered into the engine at all engine conditions (temperature), the vapor pressure is controlled by venting off the excess vapor. Venting is accomplished externally via a hole drilled into the top of the float bowl or internally by channeling the vapor into the engine air inlet. The latter method minimizes the fuel losses directly into the atmosphere but causes hot start difficulties and stalling or engine roughness during idle due to an excessively rich air-fuel mixture. External venting passes hydrocarbons directly into the atmosphere and causes a noticeable odor. Thus, whereas, neither method is entirely satisfactory, each contributes its share of atmospheric hydrocarbons (10).



1 Carburetor Evaporation

2 Road Draft Tube Emissions

3 Gas Tank Evaporation

4 Exhaust Emissions

Figure 2

Automotive Sources of Air Pollutants

Crankcase vent gases are a mixture of blowby gases and ventilation air. During normal engine operation, ventilation air entering through the breather is mixed with gases blown by the piston rings into the crankcase. The entire mixture leaves the engine via the road draft tube. Since a part of the crankcase gases comes from the cylinders its compositon is similar to that of exhaust emissions. It has also been shown that crankcase and exhaust emissions are of the same order of magnitude and consequently warrant equal consideration with respect to the smog problem (11).

Exhaust hydrocarbon emissions are a consequence of the "wall quenching" and exhaust gas dilution phenomena (12, 13). The wall quenching phenomenon occurs because the combustion front and flame fail to propagate throughout the cylinder up to the cylinder wall. Therefore, part of the air-fuel mixture admitted to the cylinder is unburned and, hence, available for direct expulsion into the atmosphere unaltered.

The exhaust gas dilution phenomenon is the result of dilution of the air-fuel mixture in the cylinder by exhaust gases to such an extent that combustion of the fuel is prevented. This occurs as follows. During the exhaust stroke of a four cycle engine not all of the exhaust gas is purged from the cylinder. Consequently, as the exhaust valve closes and the intake valve opens, some of the residual gases are drawn into the intake manifold due to the vacuum in the latter. During the intake stroke these residual gases are then sucked back into the cylinder, thereby, diluting the new air-fuel mixture. Depending upon the pressure differential between the intake manifold and the cylinder and the amount of valve overlap, the exhaust gas dilution phenomenon is more or less pronounced. The pressure differential is a function of the particular driving mode, deceleration, cruise, etc. The valve overlap is the amount of time both the intake and exhaust ports are open simultaneously.

Of these four automotive sources of harmful hydrocarbons, exhaust emissions and crankcase fumes constitute the major sources (14). Consequently, the largest portion of the effort in this area has been expended investigating the effect of engine design parameters and operating variables on these two sources. The operating variables include air-fuel ratio, spark timing, valve overlap, engine speed and combustion chamber deposits (15).

2.4 Possible Solutions

The understanding of this unique combination of events has shown that the effects of smog can be eliminated either by altering the environment in such a way that the ultraviolet energy is not available to drive the photochemical reaction or by eliminating and/or minimizing the reactants. At this time it is not desirable, practical or feasible to inhibit natural sunlight to effect the first approach.

Eliminating or minimizing the reactants can be accomplished either by dispersing the reactants after their formation or by inhibiting their formation. Due to the meteorological phenomena involved, dispersion could be attained two ways. One, move the inversion layer so that natural convection currents in the area would disperse the reactants into the upper atmosphere. Two, dig a channel through the mountain barrier to the east. This would permit surface currents to disperse the harmful pollutants into the desert where convection currents would disperse these pollutants into the upper atmosphere. However, neither of these approaches are presently feasible.

Since the primary source of harmful pollutants in the Los Angeles area is the automobile, inhibition of reactant formation requires control of each individual automobile. The automotive industry and air pollution control districts have focused their attention on controlling harmful hydrocarbon principally from the engine and exhaust system. Numerous devices and techniques have been evaluated which directly affect intake manifold conditions, either the amount of vacuum and/or the air-fuel ratio under all conditions. The effects of engine age (16), measured as the number of miles driven, and engine condition (17) have been studied. As a result of the effort expended to determine the impact of fuel composition and possible changes on exhaust emission of harmful olefinic hydrocarbons, the olefin content of automotive fuels has been limited. Linville, et.al. (18) presents a comprehensive discussion of the various methods of controlling vehicular emissions.

Presently, the most promising techniques for minimizing harmful hydrocarbon emissions are the afterburner type exhaust control devices, direct flame and catalytic, which complete, in the exhaust system, the oxidation process initiated in the cylinders. Of these two, the catalytic converter has been shown to be the most desirable. In this type the exhaust gases are passed around and through a bed of catalytic material which promotes the oxidation process (18).

As mechanically effective as the above devices and techniques may be, their effectiveness is highly dependent upon the mechanical condition to the engine and the device. Hence, periodic engine tune-ups and regular device maintenance are required. Economically, this maintenance of both engine and device may be unfeasible, for those cars which are the least mechanically fit are the highest emitters of harmful pollutants and belong to those individuals least able to maintain their cars properly (17).

Another approach to minimizing smog-forming exhaust pollutants requires that the driver regulate his speed at all times in such a manner that these emissions are minimized. In other words, it might be possible to minimize harmful emissions from a given vehicle regardless of its mechanical condition simply by controlling its velocity in a manner which would produce emissions at a minimal level of which that engine is capable. Stated more explititely: for a given engine (car), a given driver, and a given driver behavior during various driving modes, accelerating, cruising, decelerating and idling, utilize that combination of driving modes which will minimize harmful pollutant emissions under given driving conditions.

Assuming, of course, that the driver is desirous of contributing to the alleviation of smog, this approach would necessitate educating all drivers about the smog phenomenon and about the action required on his part. However, before this can be done, it must be determined whether or not such a combination of events exists. If so, what is it and how does it differ, if at all, from present driving behavior. This study, then, is an investigation of the first two questions.

Chapter 3

THE SCOPE OF THE PHENOMENON TO BE MODELED

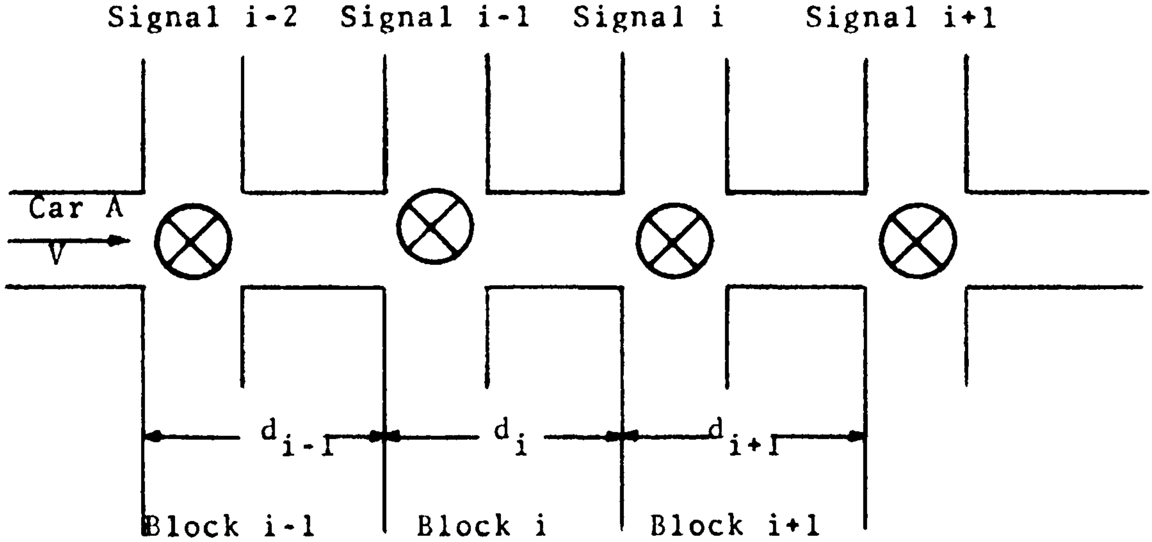
3.1 Assumptions and Notations

It is obvious that the determination of driving events which will minimize emissions involves an infinitely denumerable number of combinations of engines, engine characteristics, driver-driving characteristics, driver temperament, road and traffic conditions, environmental and meteorological conditions, legal restrictions, etc. However, in order to reduce the problem to manageable yet meaningful proportions, the following situation will be investigated.

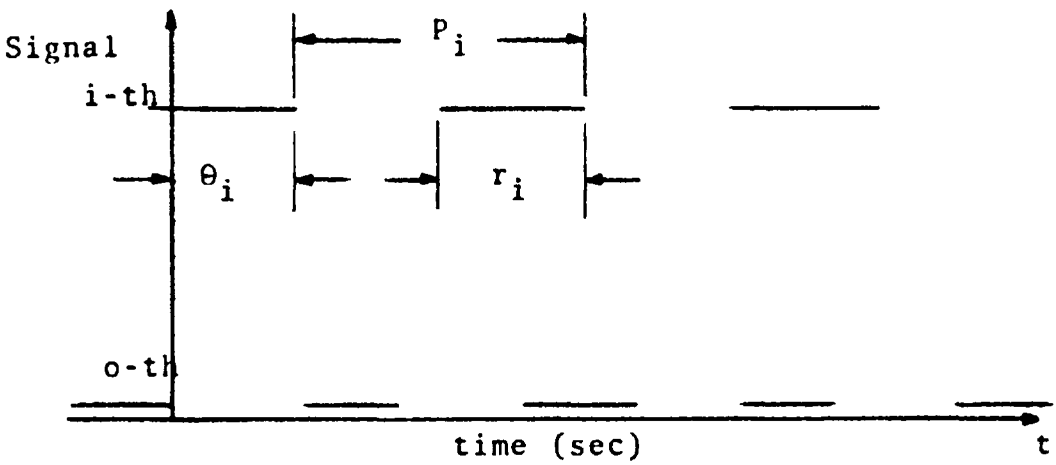
Car A is moving alone down Main Street, a straight, level, one-way, single lane, asphalt street with the usual 8-10 feet shoulders on each side. The street has no obstacles or obstructions on it, in it, projecting over it, onto it or even near it with two exceptions. Speed and traffic regulating devices such as three-phase overhanging traffic signals and stop and speed limit signs are placed near the street. At each traffic signal another street intersects Main Street at right angles. Traffic moving along the cross streets will either cross in front of Car A or, if it moves onto Main Street, does not interact in any way with Car A.

Car A must pass through (n+1) signals along Main Street with a distance d_i (miles) between the (i-1)st and i-th signal. A block is defined by the portion of Main Street between any two consecutive signals. This is measured from a point in the intersection farthest upstream to the corresponding point in the succeeding intersection. Consequently, there are n blocks in the system. The i-th signal has a cycle length or period of p_i (seconds). That is, the time lapse between the i-th signal initially turning green and its turning green again is p_i. It has a red phase of r_i (seconds). Its period is offset by a time θ_i (seconds) with respect to the initiating green phase of the zeroth signal. The speed limit in the i-th block is V_i (miles per hour). These relationships are illustrated in Figure 3.

Since present legislation requires crankcase exhaust control devices on all automobiles, the only remaining significant source of harmful automotive pollutants is the exhaust gas (19). Consequently, only harmful exhaust emissions are of interest. Of these emissions only hydrocarbons and carbon monoxide have been extensively monitored. Carbon monoxide has been monitored not so much because of



(a) Spacial relationship of signals, blocks and distances.



(b) Temporal relationship of period, red phase and phase offset for the i-th signal.

Figure 3

Temporal and Spacial Relationships of

Traffic Signals Along Main Street

its role in the smog phenomenon but because of its potential impairment of bodily functions in people with impaired respiratory and circulatory capacity (20). Therefore, only exhaust hydrocarbons are considered within the scope of this study.

Since hundreds of olefinic hydrocarbons have been identified as the smog-causing pollutants a major accomplishment has been the development of instrumentation capable of detecting these harmful exhaust hydrocarbons. However, as of now, not even the two principal detection techniques, flame ionization analysis (FIA) and the more popular, portable non-dispersive infrared detector (NDIR) measure the same olefins because of the difference in their principle of operation (21, 22). Consequently, gathering, interpreting and comparing data is difficult. The measurements taken by Way and Fagley (23) and Neerman and Millar (24) have been utilized in the solution and testing of the models derived below.

Four driving modes have been defined, idling, cruise, acceleration, deceleration, each with its corresponding emission rate, R_w, R_c, R_a, R_d respectively (25). In order that the emission rates reflect the amount of work done by the engine, the emission rates are expressed in pounds of pollutant per second. Whereas the emission rates for idling, acceleration and deceleration are more or less constant, R_c is a linear function of the velocity, i.e., $R_c = mV + b$ where m and b are constants and V is the vehicle's velocity.

It is assumed that a driver accelerates or decelerates at a steady uniform rate (a) or (-d) mph per second. This assumption appears reasonable since the majority of human beings tend to develop set ways or patterns in the manner in which they execute many of their daily activities. Even during the process of learning a task, although the technique employed after the task is learned differs from that at the start or at many intermediate stages, habits are formed during each of the stages and adhered to until a new or better way to do things is found. It is this constancy or uniformity of action at different stages in a person's development while performing a given task, which will be exploited during the formulation of any mathematical model herein.

Since constant or uniform acceleration and deceleration rates are involved, the equations of motion for uniform acceleration are used. The acceleration and deceleration rates are bounded by the mechanical condition of the car and human tolerance to velocity changes.

An investigation into the phenomenon has shown that it is characterized by two types of variables, those related to the physical situation and those related to the driver. In the former class are contained the position of the car, its velocity and acceleration, the traffic signal conditions, speed limits and the vehicle's emission. The latter class encompasses the driver's desired velocity and acceleration rates. The first class represents the environment as the driver finds it while the second class contains those variables which he can possibly manipulate.

Even if a driver aspires to operate the car at his desired velocity, existing speed limits may restrict his actual speed to the slower of two speeds, his desired speed or the speed limit; that is, the driver's actual speed never exceeds the speed limit. The car's actual rate of speed change is actually dependent upon the traffic situation facing the car and the desired acceleration and deceleration rates.

In order to be able to work with the situation it must be understood and described. As an aid to the understanding of the physical phenomenon a Finite-State Machine model was developed to describe more precisely the events and transitions involved (26).

3.2 Finite-State Machine Model

The Finite-State Machine description, M, of this phenomenon is defined by the set composed of a finite input alphabet X, a finite output alphabet Z, a finite state set S, a next state function f_s and an output function f_z all defined below. Mathematically, $M=\{X,S,Z,f_s,f_z\}$.

The input alphabet is defined as the Cartesian product of the seven discrete sets defined below. That is, $X = X^{(1)} \otimes X^{(2)} \otimes X^{(3)} \otimes X^{(4)} \otimes X^{(5)} \otimes X^{(6)} \otimes X^{(7)}$ where

$X^{(1)} = \{x_1 | x_1$ is the distance in miles from the next intersection to the vehicle, $x_1 \varepsilon \{0.00, 0.01, \ldots, 1.99, 2.00\}\}$,

$X^{(2)} = \{x_2 | x_2$ is the condition, Red (R), Amber (A), or Green (G), of the traffic signal at the next intersection, $x_2 \varepsilon \{R, A, G\}\}$,

$X^{(3)} = \{x_3 | x_3$ is the governing speed limit in miles per hour (mph) to the next intersection, $x_3 \varepsilon \{15, 16, \ldots, 44, 45\}\}$,

$X^{(4)} = \{x_4 | x_4$ is the driver's desired speed in miles per hour (mph), $x_4 \varepsilon \{0, 1, \ldots, 59, 60\}\}$,

$X^{(5)} = \{x_5 | x_5$ is the driver's desired acceleration in miles per hour per second (mph/sec), $x_5 \varepsilon \{0.0, 0.1, \ldots, 3.9, 4.0\}\}$,

$X^{(6)} = \{x_6 | x_6$ is the time in seconds spent in a particular state, $x_6 \varepsilon \{0, 1, \ldots, 359, 360\}\}$, and

$X^{(7)} = \{x_7 | x_7$ is the driver's desired deceleration rate in miles per hour per second (mph/sec), $x_7 \varepsilon \{-10.0, -9.9, \ldots, -0.1, 0.0\}\}$.

The domain of definition for each set has been determined, from traffic ordinances, mechanical and performance limitations of the car (27) or arbitrarily.

The state set is defined as the Cartesian product of three sets defined as follows.

$S = S^{(1)} \otimes S^{(2)} \otimes S^{(3)}$ where

$S^{(1)} = \{s_1 | s_1$ is the vehicle's actual speed, $s_1 \varepsilon \{0,1,\ldots,59,60\}\}$,

$S^{(2)} = \{s_2 | s_2$ is the vehicle's actual acceleration, $s_2 \varepsilon \{-10.0,-9.9,\ldots,3.9,4.0\}\}$, and

$S^{(3)} = \{s_3 | s_3$ is the number of the block the vehicle is in, $s_3 \varepsilon \{0,1,\ldots,n$, the number of intersections in the system$\}\}$.

The output alphabet is defined as the Cartesian product of three sets defined as follows.

$Z = Z^{(1)} \otimes Z^{(2)} \otimes Z^{(3)}$ where

$Z^{(1)} = \{z_1 | z_1$ is the distance in miles from the next intersection to the vehicle, $z_1 \varepsilon \{0.00,0.01,\ldots,1.99,2.00\}\}$,

$Z^{(2)} = \{z_2 | z_2$ is the number of the block the vehicle is in, $z_2 = s_3 \varepsilon \{0,1,\ldots,n\}\}$, and

$Z^{(3)} = \{z_3 | z_3$ is the total amount in pounds of vehicle emissions, $z_3 \varepsilon \{0.0000,0.0001,\ldots,0.9999,1.0000\}\}$.

Legend

Symbol	Meaning
A	$(s_1+s_2x_6, s_2^*, s_3+1)$
B	$(s_1+s_2x_6, s_2^*, s_3)$
C	$(0,0,s_3)$ if $s_1+s_2x_6 \leq 0$ $(s_1+s_2x_6, x_5, s_3+1)$ otherwise
D	$(s_1, x_5, s_3)^{\dagger}$ if $x_1 < -s_1^2/(7200x_7)$ (s_1, x_7, s_3) " $=$ " $(s_1, 0, s_3)$ " $>$ "
E	$(s_1+s_2x_6, x_5, s_3)$ if $x_1 < -(s_1+s_2x_6)^2/(7200x_7)$ $(s_1+s_2x_6, x_7, s_3)$ " $=$ " $(s_1+s_2x_6, s_2^*, s_3)$ " $>$ "
s_2^*	$s_2^* = \begin{cases} x_5 & \text{if } s_1+s_2x_6 < \min(x_3,x_4) \\ 0 & \text{" " } = \text{ "} \\ x_7 & \text{" " } > \text{ "} \end{cases}$

Table 1

Next State Function

$\xi^{(i)}$ \ $\sigma^{(i)}$	$(0,0,s_3)$	$(0,s_2,s_3)$	$(s_1,0,s_3)$	(s_1,s_2,s_3) $s_2>0$	(s_1,s_2,s_3) $s_2<0$
$(0,R,x_3,x_4,x_5,x_6,x_7)$	$(0,0,s_3)$	A†	A†	A†	C
$(0,A,x_3,x_4,x_5,x_6,x_7)$	$(0,0,s_3)$	A	A	A	C
$(0,G,x_3,x_4,x_5,x_6,x_7)$	$(0,x_5,s_3+1)$	A	A	A	A
$(x_1,G,x_3,x_4,x_5,x_6,x_7)$	$(0,0,s_3)$	B	B	B	B
$(x_1,\genfrac{}{}{0pt}{}{R}{A}, x_3,x_4,x_5,x_6,x_7)$	$(0,x_5,s_3)$	B	D	E	E

† This will result in an illegal situation-running a red light.

Table 2

Output Function

$\sigma^{(i)}$ / $\zeta^{(i)}$	$(0,0,s_3)$	$(0,s_2,s_3)$	$(s_1,0,s_3)$	(s_1,s_2,s_3) $s_2>0$	(s_1,s_2,s_3) $s_2<0$
$(x_1,x_2,x_3,x_4,x_5,x_6,x_7)$	(x_1,s_3,ζ_3^1)	(x_1,s_3,ζ_3^2)	(x_1,s_3,ζ_3^3)	(x_1,s_3,ζ_3^4)	(x_1,s_3,ζ_3^5)

$$\zeta_3^1 = (0.528\text{x}10^{-4})x_6$$

$$\zeta_3^2 = (2.222\text{x}10^{-4})x_6$$

$$\zeta_3^3 = [(8.33s_1+250)\text{x}10^{-7}]x_6$$

$$\zeta_3^4 = (1.943\text{x}10^{-4})x_6$$

$$\zeta_3^5 = (0.8\text{x}10^{-4})x_6$$

The characterizing next state function, f_s, and output function, f_z, are defined in Table 1 and 2 respectively (23, 24). Since these functions can be written in closed form, only the generic terms are shown. Each term is indicative of a class of states, input, output or next state.

Notice that as far as state transitions and output are concerned, it makes no difference whether a car faces a yellow light or a red light. The same response and reaction is evoked. Consequently, these two conditions, red and yellow signal phase, will be collapsed into one phase, red, in the ensuing development.

Through the use of the f_s and f_z tables it is possible to simulate the various events that occur as Car A moves along Main Street. Given that Car A is in any state and the physical conditions, the next state and present output can be determined by reading the entry in the appropriate table at the intersection of the row and column corresponding to the given state and input conditions.

For example, suppose Car A is known to be traveling in the third block, whose speed limit is 35 mph, at a speed of 25 mph, accelerating at 2 mph/sec and sees a red light at the signal one mile ahead of him. The driver wants to drive at 40 mph. His normal acceleration and deceleration rates are 2 mph/sec and -1.5 mph/sec respectively. The car-driver has been in this state for 4 seconds. His next state is determined by reading the element at the intersection of state (25,2,3) and input (1.00,R,35,40,2.0,4,-1.5) in the f_s table. After a few simple calculations this state is found to be (33,2.0,3). The car's output at that time, i.e., during the time (4 sec) spent in that state, is similarly determined to be (33,0.0008,3).

Although this Finite-State Machine model is quite useful as a descriptive technique it does not readily lend itself to determining optimal driving speeds under different conditions. For this reason another mathematical model was developed which utilized the same definitions and notational conventions developed above but which appeared to be more amenable to optimization.

Chapter 4

OPTIMIZATION

4.1 Optimizing Model

In the Finite-State Machine model developed above, emphasis was placed on describing the vehicle's location and then determing the corresponding emission. However, by shifting attention to the primary objective, that is, to find a velocity policy which minimizes hydrocarbon emissions, a model more amenable to optimization can be developed. Specifically, what should a driver do when approaching a traffic signal? Should he maintain or decrease his speed in order to cruise through this signal? Or should he drive in a manner which will cause him to be stopped by a red light at this signal?

Toward this end a little consideration about the velocity-distance relationships yields the following conclusions about the velocity-acceleration combinations at all points along Main Street. In a zone immediately upstream of each intersection, the car can either be cruising or decelerating. At and downstream from each intersection there exists a zone within which the car can cruise, accelerate or decelerate. At all other points between these zones the vehicle cruises at a constant uniform speed.

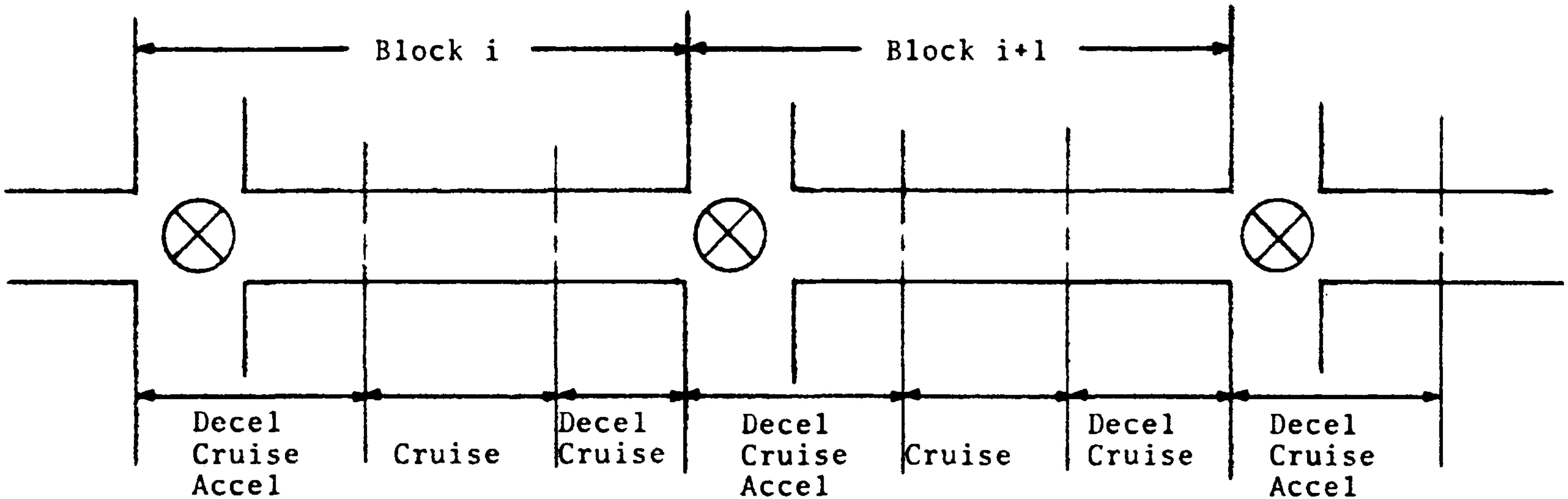


Figure 4

Speed-Acceleration-Intersection Relationships

These relationships are illustrated in Figure 4.

In order to determine exactly in which acceleration state the car is in at all points along Main Street, it must be determined whether or not the car is impeded at each intersection. First define the following new terms.

T_k = the total time spent in the k-th block.

$\Sigma_k T_k$, $k=1,\ldots,i$ = the total time lapse from the instant the car enters the first block until it emerges from the i-th block.

Char(X) = the integer portion of the argument X.

$j = \mathrm{Char}[(\Sigma_k T_k - \theta_i)/p_i]$, $k=1,\ldots,i$,

$T_o = jp_i + \theta_i$, and

$T_1 = (j+1)p_i - r_i + \theta_i$ for all i.

$[T_o, T_1]$ = the closed interval of time in the j-th cycle during which the i-th traffic signal is green.

To determine whether or not the car arrives at the i-th traffic signal when it is green, compare $\Sigma_k T_k$, $k=1,\ldots,i$, with the time interval $[T_o, T_1]$. If $\Sigma_k T_k \epsilon [T_o, T_1]$, $k=1,\ldots,i$, then the car moves through the i-th intersection at a speed of V_i. If $\Sigma_k T_k \not\epsilon [T_o, T_1]$, $k=1,\ldots,i$, the car is stopped and must wait for the next green phase before resuming its movement. It must wait for $(j+1)p_i - \Sigma_k T_k + \theta_i$ seconds, $k=1,\ldots,i$. In the latter case, T_i must be revised to $T_i = (j+1)p_i - \Sigma_k T_k + \theta_i$, $k=1,\ldots,i-1$.

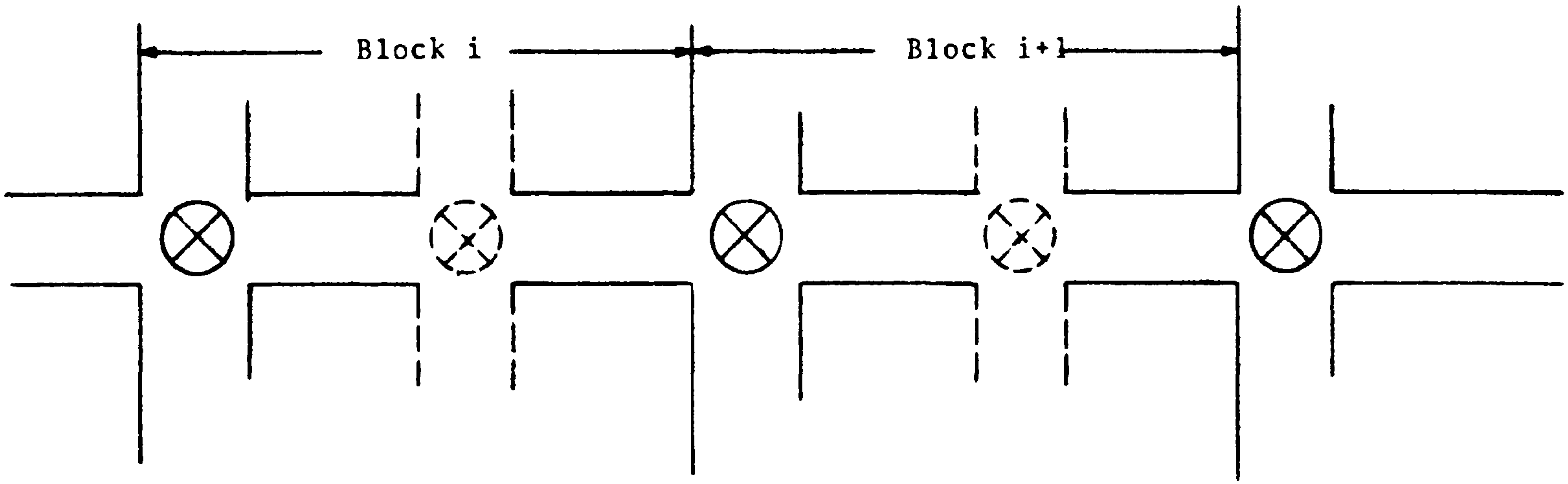


Figure 5

Relationship Between Real and Virtual Intersections

At this point, it is quite reasonable to assume that in the cruise zone of each block the car has reached its maximum speed in the block and develop an appropriate model. This maximum speed is equal to its cruise velocity, $V=\min(x_3,x_4)$. However, in keeping with the objective of this investigation, it is desirable not to specify x_3 in advance, but to determine an optimal x_3.

As a result, it becomes conceptually convenient to insert virtual intersections midway between the real intersections, vix, in the middle of the cruise zones. The relationship between the real and virtual intersections is illustrated in Figure 5. Therefore, if there were n blocks in the original system, in the model, there are 2n blocks, n+1 real intersections, and n virtual intersections. Now, if V_i indicates the velocity at each intersection in the model, $V_{2q}, q=0,1,\ldots,n$, represents the velocity at each real intersection and V_{2q+1} represents the velocity at the virtual intersections. This latter velocity is the desired velocity, x_3, for that block. Now if the model is developed and solved for the V_i, $i=1,\ldots,2n+1$, not only will the driver policy for each real intersection be determined, but also the desired speed in the real block.

With this approach, the following relationships hold. Consider the 2q-th block, that is, the portion of Main Street between the virtual intersection

2q-1 and real intersection 2q. Three conditions are possible: (i) The car cruises through the block and on through the intersection at $V_{2q}=V_{2q-1}$; (ii) The car cruises at V_{2q-1}, decelerates to V_{2q}, and cruises on through the 2q-th intersection at V_{2q} without stopping; (iii) The car cruises at V_{2q-1}, decelerates to $V_{2q}=0$, and waits for the light to turn green. Utilizing the equations of motion for uniform acceleration the space-time-emission relationships for each of these cases are tabulated below.

<u>Case (i) and Case (ii)</u>

i	decel,d	cruise,c
T_i	$(V_{2q-1}-V_{2q})/d$	$3600S_c/V_{2q-1}$
S_i	$(V_{2q-1}^2-V_{2q}^2)/(7200d)$	$S_c=d_{2q}-S_d$
E_i	$R_d t_d$	$R_c t_c$

$T_{2q}=t_d-t_c$, $E_{2q}=E_d+E_c$, and $\Sigma_k T_k$, $k=1,\ldots,2q$, must be contained in $[T_o,T_1]$ where $j=\mathrm{Char}[(\Sigma_k T_k-\Theta_{2q}]$, $k=1,\ldots,2q$, $T_o=jp_{2q}+\Theta_{2q}$, and $T_1=(j+1)p_{2q}-r_{2q}+\Theta_{2q}$.

Case (iii)

i	decel, d	cruise, c	idle, w
t_i	V_{2q-1}/d	$3600S_c/V_{2q-1}$	*
S_i	$V_{2q-1}/(7200d)$	$S_c=d_{2q}-S_d$	0
E_i	$R_d t_d$	$R_c t_c$	$R_w t_w$

* $t_w=(j+1)p_{2q}+\theta_{2q}-\Sigma_k T_k$ where $j=\text{Char}\ [(\Sigma_k T_k-\theta_{2q})/p_{2q}]$, $T_{2q}=t_d t_c$, and $k=1,\ldots,2q$. $E_{2q}=E_d+E_c+E_w$.

T_{2q} must be reset to $T_{2q}=t_d+t_c+t_w=(j+1)p_{2q}-\Sigma_k T_k+\theta_{2q}$, $k=1,\ldots,2q-1$.

Now, consider block 2q+1, that is, the portion of Main Street between intersections 2q, real, and 2q+1, virtual. Three possible conditions exist: (i) The car enters the intersection at velocity V_{2q}, accelerates to V_{2q+1}, and cruises at V_{2q+1}; (ii) The car enters the intersection at V_{2q}, decelerates to V_{2q+1}, and cruises at V_{2q+1}; (iii) The car enters the intersection at V_{2q} and cruises on through at $V_{2q+1}=V_{2q}$. Following the same procedure as above the following relationships apply in this block.

	(i) Decelerate	(ii) Cruise*	(iii) Accelerate
Δ_V	$V_{2q}-V_{2q+1}$	0	$V_{2q+1}-V_{2q}$
$t_{\Delta V}$	$\Delta V/d$	$t_c=3600S_c/V_{2q+1}$	$\Delta V/a$
$S_{\Delta V}$	$(V_{2q}^2-V_{2q+1}^2)/(7200d)$	$S_c=d_{2q+1}-S_{\Delta V}$	$(V_{2q+1}^2-V_{2q}^2)/(7200a)$
$E_{\Delta V}$	$R_d t_{\Delta V}$	$R_c t_{\Delta V}$	$R_a t_{\Delta V}$

* Notice that the cruise conditions are dependent upon the acceleration conditions and the length of the block.

The total pollutants emitted and the time lapse in the 2q-th block can be seen to be $E_{2q}=E_{\Delta V}+E_c$ and $T_{2q}=t_{\Delta V}+t_c$ respectively.

Now, since any model of a system of traffic signals involves only these two types of blocks, optimizing driver policy becomes a matter of linking these model blocks into the appropriate sequence and solving the resulting system of relationships. The latter may be accomplished by applying the optimality principle of dynamic programming to the model and optimizing the velocity at each intersection in the model.

The optimality principle of dynamic programming can be stated as follows. "An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision" (28).

From a descriptive and conceptual point of view, it is more convenient to model each real block as two separate blocks. However, from a computational point of view it is much more convenient to consider each actual block as an entity and combine the relationships which would have been associated with each half. Actually a combination of both approaches is used.

The velocity-time-emission relationships are obtained by considering all possible combinations of velocity-acceleration states at the intersections defining the actual block. The actual form of these relationships reflects the conceptual division of the block into halves. Since there are three possible velocity-acceleration states at each intersection, this yields nine possible cases, P_i, $i=1,...,9$, for each real block, only one of which is minimal. For that case, the maximum velocity $V_{max}=x_3$, in that block and the velocity, V_f, at the end of the block must also be found.

In order to formulate a dynamic programming model of the phenomenon, three conditions must be satisfied. First, there must exist a characteristic or feature associated with the phenomenon which can be used to signify the occurrence of a decision or judgment. This characteristic must be quantifiable and distinct in the sense that it can readily be determined whether or not a decision has been made. In this case, a "block" as defined above satisfies this criterion since (i) at the beginning of each block the driver must decide what his course of action for the coming block should be, and (ii) the location of the blocks and the instant the car enters the block is obvious. Thus, each stage of the decision process is associated with a block.

Second, an output must be definable. In this case, the car-street system output is the exhaust emissions from the car. However, the results obtained from the dynamic programming model will consist of the overall driver policy for that particular car-street system. This includes not only the exhaust emissions but also the maximum speed, V_{max}, and the terminal speed, V_f, for each block.

Third, there must exist some construct corresponding to the state of the system which contains all the information necessary to determine the optimal policy or output. Since knowledge of the car's velocity, V, as it enters the block, and the condition of the next signal, S, at the time of the vehicle's entry, is sufficient to determine the optimal case for that block, the state is a 2-tuple (V,S). In addition, associated with the i-th case, P_i, there is a set of constraints, R_i, which is a function of V and S, and upon which V_{max} and V_f depend. That is, $R_i=f(V,S)$, $V_{max}=f_1(V,S)$, and $V_f=f_2(V,S)$.

The functional equation which describes this phenomenon can now be written by combining all of these various ideas. The minimal total emissions from the car with N blocks left to traverse, knowing both the speed, V, of the car and the condition of the next signal, S, at the instant the car enters the first of these N blocks, is

$$f_N(V,S)=\min_{1\le i\le 9}\left\{\min_{V_{max},V_f\in R_i}\left[E_i(V,S,V_{max},V_f)+f_{N-1}(V,S')\right]\right\}$$

where i corresponds to the case number, E_i is the emission function for the i-th case, and S' corresponds to the next signal's condition at the instant the car moves into the (N-1)st remaining block. Let $f_o(V,S)=0$. The exact form of the E_i's and the R_i's can best be demonstrated by the following example.

4.2 Example

Car A is moving down a street with three traffic signals. This defines two blocks, the smallest non-trivial system, and a 2-stage decision process. Assume that the signal spacing (d), cycle length (p), red period (r), signal phasing (Θ), acceleration rates (a and d), emission rates ($R_a, R_d, R_w, R_c = mv+b$), and initial velocity (V_o) are known. The even subscripts denote the real signals, the odd subscripts the virtual signals.

First, consider the second block. For this one-stage process the functional equation reduces to

$$f_1(V_2,S) = \min_{1 \leq i \leq 9} \{ \min_{V_3,V_4 \varepsilon R_i} [E_i(V_2,S,V_3,V_4)]\}.$$

The car's velocity, V_2, entering the block and the condition of the second signal are assumed known. It is obvious that the condition of the next signal at the time the car arrives there is dependent upon (1) the total time spent in the previous blocks of the system $\Sigma_k T_k = T_1$, and (2) the velocity trajectory in this next block which in turn is dependent upon V_2.

Computationally, this condition is determined as follows. (1) Assume a particualr velocity trajectory for the next block. (2) Compute the total time lapse in the system including this trajectory, $\Sigma_k T_k$, $k=1,2$. (3) From this total subtract the phase Θ_4. (4) Divide the difference by the cycle length p_4. (5) The cycle number j is the integer portion of this quotient. Thus, $j = \mathrm{Char}[(T_1+T_2-\Theta_4)/p_4]$. (6) Multiply j by p_4. (7) By comparing T_1+T_2 with jp_4 the condition of the signal, for that particular velocity trajectory, at the time of arrival is determined. This technique is used in all nine cases which are discussed sequentially below. In addition, in all cases, $0 \le V_3 \le V_L$, $V_4 \le V_L$, and $d_3 = d_4$, one half the length of the second block. The superscripts on E or T corresponds to the case number. For example, E^i is the emission for the i-th case.

Case 1 The car enters the second block at a speed V_2, decelerates to and cruises at V_3, and decelerates to V_4, which is maintained through the third intersection.

From the relationships shown in the previous section, the following relationships hold.

$$E_2^1 = (E_d + E_c) + (E_c + E_d),$$
$$= R_d(V_2 - V_3)/d + (mV_3 + b)[7200d(d_3 + d_4) + V_4^2 - V_3^2]/(2dV_3).$$
$$T_2^1 = (t_d + t_c) + (t_c + t_d),$$
$$= (V_2 - V_4)/d + [7200d(d_3 + d_4) + V_4^2 - V_2^2]/(2dV_3).$$

In addition, $[T_1 + T_2^1] \varepsilon [t_o, t_1]$ where $t_o = jp_4 + \theta_4$, $t_1 = (j+1)p_4 + \theta_4 - r_4$, and $j = \mathrm{Char}[(T_1 + T_2^1 - \theta_4)/p_4]$.

The necessary conditions for E_2^1 to be minimized are: (1) $\partial E_2^1/\partial V_3 = 0$, which implies that $V_3 = \infty$, or that $V_4 = \sqrt{V_2^2 - 7200d(d_3 + d_4)}$, and (2) $\partial E_2^1/\partial V_4 = 0$, which implies that $V_3 = bV_4/(R_d - mV_4)$. Since local minima are also of interest, the case of $V_3 = V_L$ and $V_4 = R_d V_3/(mV_3 + b)$ must also be considered provided $V_L \le V_2$.

The sufficient conditions for any V_3 and V_4 resulting from the necessary conditions to yield a minimum requires that

$(\partial^2E_2^1/\partial V_3^2)(\partial^2E_2^1/\partial V_4^2)-(\partial^2E_2^1/(\partial V_3\partial V_4))^2>0$ and $\partial^2E_2^1/\partial V_3^2>0$.

$\partial^2E_2^1/\partial V_3^2=b[7200d(d_3+d_4)+V_4^2-V_2^2]/(dV_3^2)$, $\partial^2E_2^1/\partial V_3\partial V_4=0$,

and $\partial^2E_2^1/\partial V_4^2=(mV_3+b)/(dV_3)$.

Case 2 The car enters the second block at a speed V_2, decelerates to V_3, cruises at V_3, then stops at the third signal and waits for the next green phase.

Let $t_d=V_2/d$, $t_c=[7200(d_3+d_4)-V_2^2]/(2dV_3)$, $t_0=jp_4+\Theta_4$, $t_1=(j+1)p_4+\Theta_4-r_4$, and $j=\mathrm{Char}[(T_1+t-\Theta_4)/p_4)]$. Then $E_2^2=(E_d+E_c)+(E_c+E_d+E_w)$,

$=R_dt_d+(mV_3+b)t_c+R_w(t_1+r_4-T_1-t_d-t_c)$, and

$T_2^2=(j+1)p_4+\Theta_4-T_1$. In addition, $(T_1+t_d+t_c)>t_1$.

The necessary conditions for a minimal, $dE_2^2/dV_3=0$, implies that $V_3=\infty$, $R_w=b$, or $V_2=\sqrt{7200d(d_3+d_4)}$. Again the case $V_3=V_L$ must also be considered provided $V_L\leq V_2$. These roots must also satisfy the sufficient condition, $d^2E_2^2/dV_3^2>0$. This yields $V_3=\infty$, or $V_2>7200d(d_3+d_4)$. Since an explicit expression for V_3 is not obtained, V_3 must be calculated from the relationships $t=t_d+t_c$ and $j=\mathrm{Char}[T_1+t-\Theta_4)/p_4]$ by iterating on j. By hypothesis, $V_4=0$.

Case 3 The car enters the second block at a speed V_2, decelerates to V_3 and cruises through the third signal at $V_4=V_3$.

Let $t_d=(V_2-V_3)/d$, $t_c=[7200d(d_3+d_4)+V_3^2-V_2^2]/(2dV_3)$, $t_o=jp_4+\theta_4$, and $t_1=(j+1)p_4+\theta_4-r_4$.

Then $E_2^3=(E_d+E_c)+E_c=R_dt_d+(mV_3+b)t_c$, and $T_2^3=t_c+t_d$. In addition, $(T_1+T_2^3)\varepsilon[t_o,t_1]$.

The necessary condition for an extremum, $dE_2^3/dV_3=0$, yields the following expression from which V_3 can be determined.

$V_3^3+C_2V_3^2+C_4=0$ where $C_2=(b-2R_d)/(2m)$ and

$C_4=b[V_2^2-7200d(d_3+d_4)]/(2m)$. In order that any V_3 thus determined yield a minimal emission, it must also satisfy the sufficient condition $d^2E_2^3/dV_2^3>0$ which requires the $V_3^3>2C_4$.

Case 4 The car enters at a speed V_2, cruises at $V_3=V_2$, decelerates to V_4 and maintains that speed through the third intersection.

Let $t_c=[7200d(d_3+d_4)+V_4^2-V_2^2]/(2dV_2)$ and

$t_d=(V_2-V_4)/d$.

Then $E_2^4=E_c+(E_c+E_d)=(mV_2+b)t_c+R_dt_d$, and $T_2^4=t_c+t_d$. In addition $(T_1+T_2^4)\varepsilon[t_o,t_1]$ where $t_o=jp_4+\theta_4$, $t_1=(j+1)p_4+\theta_4-r_4$, and $j=\mathrm{Char}[(T_1+T_2^4-\theta_4)/p_4]$.

To determine V_4, $dE_2^4/dV_4=0$ yields $V_4=R_dV_2/(mV_2+b)$. Since $d^2E_2^4/dV_4^2=(mV_2+b)/(dV_2)$ is always positive, this V_4 will always yield a minimal E_2^4 provided that it satisfies the signal conditions.

<u>Case 5</u> The car enters thae second block at a speed V_2, cruises at $V_3=V_2$, decelerates to a stop, $V_4=0$, and waits for the next green phase of the third signal.

In this case, $t_c=[7200d(d_3+d_4)-V_2^2]/(2dV_2)$,

$t_d=V_2/d$, $t=t_c+t_d$, $j=\mathrm{Char}[(T_1+t-\theta_4)/p_4]$, $t_o=jp_4+\theta_4$, and $t_1=(j+1)p_4+\theta_4-r_4$.

Then $E_2^5=E_c+(E_c+E_d+E_w)=(mV_2+b)t_c+R_dt_d+R_w(t_o+p_4-t)$ and $T_2^5=(j+1)p_4-\theta_4-T_1$. In addition, $(T_1+t)>t_1$.

<u>Case 6</u> The car enters at a speed V_2 and cruises through the block and third intersection at $V_4=V_3=V_2$.

In this case, $E_2^6=E_c+E_c=(mV_2-b)3600(d_3+d_d)/V_2$ and $T_2^6=3600(d_3+d_4)V_2$. Since the car cruises through the third intersection, the light must be green. Therefore, $(T_1+T_2^6)\varepsilon[t_o,t_1]$ where $t_o=jp_4+\theta_4$, $t_1=(j+1)p_4+\theta_4-r_4$, and $j=\mathrm{Char}[(T_1+T_2^6-\theta_4)/p_4]$.

<u>Case 7</u> The car enters the second intersection at a speed V_2, accelerates to V_3, cruises at V_3, decelerates to and maintains a speed V_4 through the intersection.

Let $t_a=(V_3-V_2)/a$, $t_d=(V_3-V_4)/d$, and $t_c=[7200ad(d_3+d_4)+dV_2^2+aV_4^2-(a+d)V_3^2]/(2adV_3)$. Then $E_2^7=(E_a+E_c)+(E_c+E_d)=R_at_a+R_dt_d+(mV_3+b)t_c=E_2^7(V_3,V_4)$.

The necessary conditions for an extremum $\partial E_2^7/\partial V_3=0$ and $\partial E_2^7/\partial V_4=0$ yield (1) $V_3^3+C_2V_3^2+C_4=0$ and (2) $V_4=R_dV_3/(mV_3+b)$, respectively, where $C_2=[b(a+d)-2(dR_a+aR_d)]/[2m(a+d)]$ and $C_4=b[7200ad(d_3+d_4)+dV_2^2+aV_4^2]/[2m(a+d)]$. Combining equations (1) and (2) yields (3) $V_3^5+K_2V_3^4+K_3V_3^3+K_4V_3^2+K_5V_3+K_6=0$ where $A=C_2$, $B=b[7200ad(d_3+d_4)+dV_2^2]/[2m(a+d)]$, $C=abR_d/[2m(a+d)]$, $K_2=2b/m+A$, $K_3=(b^2+2bmA)/m^2$, $K_4=(Ab^2+Bm^2+C)/m^2$, $K_5=2bB/m$, and $K_6=b^2B/m^2$. By solving (3) for V_3, V_4 may be obtained by substitution into (2).

For any V_3 and V_4 thus determined, the sufficient conditions for a minimum must also be satisfied. That is, $\partial^2E_2^7/\partial V_3^2>0$ and $(\partial^2E_2^7/\partial V_3^2)(\partial^2E_2^7/\partial V_4^2)-(\partial^2E_2^7/(\partial V_3\partial V_4))^2>0$, where $\partial^2E_2^7/\partial V_3^2=-m(a+d)/(ad)+b[7200ad(d_3+d_4)+dV_2^2+aV_4^2]/(adV_3^3)$, $\partial^2E_2^7/\partial V_4^2=(mV_3+b)/(dV_3)$, and $\partial^2E_2^7/(\partial V_3\partial V_4)=-bV_4/(dV_3^2)$.

In addition, since the car proceeds through the third intersection $(T_1+T_2^7)\varepsilon[t_o,t_1]$ where $t_o=jp_4+\Theta_4$, $t_1=(j+1)p_4+\Theta_4-r_4$, $j=\mathrm{Char}[(T_1+T_2^7-\Theta_4)/p_4]$, and $T_2^7=t_a+t_c+t_d$.

Case 8 The car enters at a speed V_2, accelerates to and cruises at V_3, then decelerates to a stop and waits for the next green phase.

Here let $t_c=[7200ad(d_3+d_4)+dV_2^2-(a+d)V_3^2]/(2adV_3)$, $t_a=(V_3-V_2)/a$, $t_d=V_3/d$, $t=t_a+t_c+t_d$, $t_1=(j+1)p_4+\Theta_4-r_4$, and $j=\mathrm{Char}[(T_1+t-\Theta_4)/p_4]$.

Then $E_2^8=(E_a+E_c)+(E_c+E_d+E_w)$,

$$=R_a t_a+(mV_3+b)t_c+R_d t_d+R_w[(j+1)p_4+\Theta_4-T_1-t].$$

The necessary condition for an extremum $\partial E_2^8/\partial V_3=0$ yields the following relationship from which V_3 may be determined.

$V_3^3+C_2V_3^2+C_4=0$ where

$$C_2=[(R_w+b)(a+d)-2(dR_a+aR_d)]/[2m(a+d)] \text{ and}$$

$$C_4=d(b-R_w)[V_2^2+7200a(d_3+d_4)]/[2m(a+d)].$$

Any V_3 thus determined must also satisfy the sufficient condition for a minimum, namely, $d^2E_2^8/dV_3^2>0$, which yields $V_3^3<2C_4$. In addition, the condition $(T_1+t)>t_1$ must be satisfied. For any V_3 satisfying these conditions, $T_2^8=(j+1)p_4-\Theta_4-T_1$. By hypothesis, $V_4=0$.

Case 9 The car enters the second block at a speed V_2, accelerates to V_3, and cruises on through the third intersection at a speed $V_4=V_3$.

In this case, let $t_c=[7200a(d_3+d_4)+V_2^2-V_3^2]/(2aV_3)$ and $t_a=(V_3-V_2)/a$. Then $E_2^9=(E_a+E_c)+E_c=R_a t_a+(mV_3+b)t_c$. The necessary conditions for an extremum, $dE_2^9/dV_3=0$, yields $V_3^3+C_2V_3^2+C_4=0$, where $C_4=b[7200a(d_3+d_4)+V_2^2]/(2m)$ and $C_2=(b-2R_a)/(2m)$, from which V_3 can be computed. Any V_3 must also satisfy the sufficient conditions $d^2E_2^9/dV_3^2>0$ which reduces to $V_3^3<2C_4$.

In addition, the following condition must also be satisfied. $(T_1+T_2^9)\varepsilon[t_o,t_1]$ where $T_2^9=t_a+t_c$, $t_o=jp_4+\theta_4$, $t_1=(j+1)p_4+\theta_4-r_4$, and $j=\mathrm{Char}[(T_1+T_2^9-\theta_4)/p_4]$.

Now that the E_i's and R_i's are known, the emissions for each case may be obtained by performing the operations specified. Then, the driver policy for that block or stage is obtained by choosing the case with the smallest emissions. Notice that there may be 3, 6, or 9 cases to be scanned for a minimal emission policy depending upon the magnitude of V_2 and V_L.

Due to the sheer number of calculations, the logic involved and the possibility of performing these same calculations for different drivers, different temperaments and different traffic conditions, a computer solution to this one-stage process is warranted. Such an approach was taken. The results of a computer solution are discussed in the following section. A more complete discussion of the actual program is found in Appendix A.

Next, consider the first block and the two-block optimization situation. The functional equation takes the form

$$f_2(V_o,S) = \min_{1\leq i\leq 9} \{ \min_{V_1,V_2\epsilon R_i} [E_i(V_o,S,V_1,V_2)+f_1(V_2,S')]\}$$

where

$$f_1(V_2,S') = \min_{1\leq i\leq 9} \{ \min_{V_3,V_4\epsilon R_i} [E_i(V_2,S',V_3,V_4)]\}.$$

It can be seen that in order to perform the inner-most minimization the sum of the two functions E_i and f_1 must be determined for each case. Due to the nature of the function f_1, it is difficult to write the sum in closed form. While it could be done for this "simple" case it was not within the scope of this study to do so.

4.3 Results of the Computer Solution of a One-Stage Process

As an example of the results from this dynamic programming model for the one-block system, consider the situation defined by the parameter and variables given in Table 3. The corresponding emission, velocities and time lapse for each case of this situation are displayed in Table 4. The "∞" symbol is an arbitrary code used to signify that the original entry was not included in R_i.

TABLE 3

Example: Parameters of the Situation

$R_a = 2.222 \times 10^{-4}$ #/sec	$d_3 = d_4 = 0.5$ miles
$R_d = 8.000 \times 10^{-5}$ #/sec	$p_4 = 60.0$ seconds
$R_w = 5.280 \times 10^{-5}$ #/sec	$\Theta_4 = 0.0$ seconds
$m = 8.330 \times 10^{-7}$ #/sec/mph	$r_4 = 30.0$ seconds
$b = 2.500 \times 10^{-5}$ 3/sec	$V_2 = 25$ mph
$a = -d = 2$ mph/sec	$V_L = 40$ mph

TABLE 4

Example: Computer Results for Various Driver Policies

Case No.	Maximum Speed (mph)	Final Speed (mph)	Time Lapse (secs)	Total Emissions (pounds)
1†	40.0	54.9	∞	∞
2	9.91	0.00	360	1.256×10^{-2}
3†	40.0	40.0	∞	∞
4†	∞	43.6	∞	∞
5	25.0	0.00	180	8.883×10^{-3}
6	25.0	25.0	144	6.599×10^{-3}
7†	40.0	54.9	∞	∞
8	40.0	0.00	120	8.558×10^{-3}
9†	∞	∞	∞	∞

†Non-allowable case: Possible velocity or time lapse not included in R_i.

Block Policy

6	25.0	25.0	144	6.599×10^{-3}

Notice that, for this particular example, the optimal block policy corresponds to Case 6. That is, the car enters the second block at 25 mph, cruises through the block and third intersection at 25 mph. The total time lapse, 144 seconds, corresponds to an emission of 6.599×10^{-3} pounds of smog-forming hydrocarbons.

To demonstrate the potential of this model, the following question was asked. "Given the same traffic situation, what should the entering velocity, V_2, be in order to minimize emissions?" To answer this question, a series of calculations for various values of V_2 was made. The results of these calculations are shown in Figure 6 and in Table 5.

It can be seen from Figure 6 that the initial velocity which will yield the minimal emission for the block is 40 mph. Referring to Table 5 to determine the corresponding case policy, this calls for the car to enter the block at 40 mph and maintain that speed through the intersection (Case 6). From Table 5 it can also be seen that this case yields almost the minimal time lapse, 90 seconds. At this stage any generalizations and comments about trends would be premature. However, further study is being carried out to determine the relationship between some of these variables and emission.

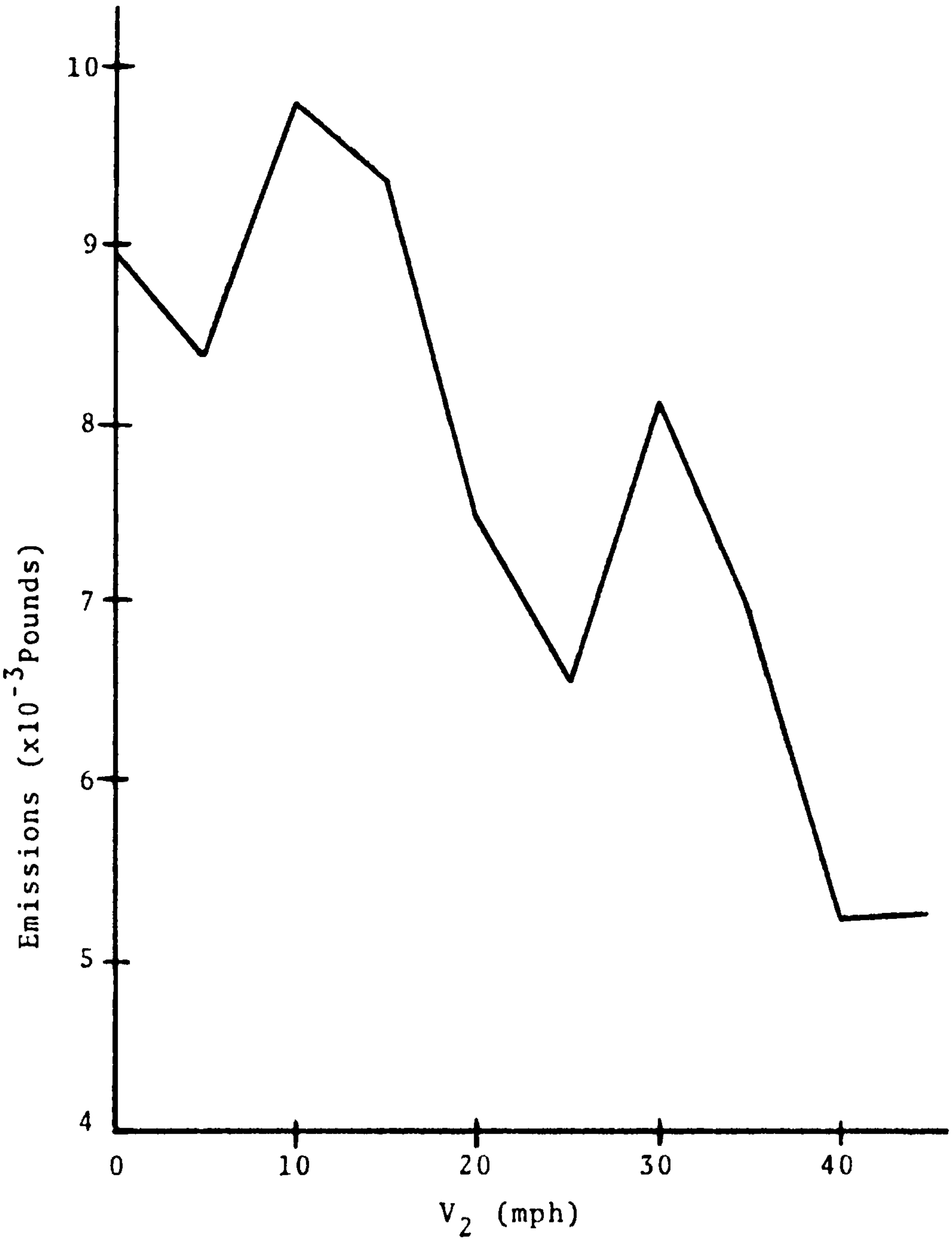


Figure 6

Emissions vs. Initial Speed for a One-Stage

Process with a 40 Mph Speed Limit

TABLE 5

Example: Driver Policy for Various Values of Intitial Velocity

Initial Speed (mph)	Policy No.	Maximum Speed (mph)	Final Speed (mph)	Time Lapse (secs)	Minimal Emission ($x10^{-3}$ lbs)
0	9	31.3	31.3	123	8.95
5	9	31.3	31.3	120	8.41
10	8	40.0	0	120	9.81
15	8	40.0	0	120	9.39
20	6	20.0	20.0	180	7.50
25	6	25.0	25.0	144	6.60
30	2	20.5	0	180	8.14
35	5	35.0	0	120	6.94
40	6	40.0	40.0	90	5.25
45	3	40.0	40.0	90	5.29
50	3	40.0	40.0	89	5.32

Although, for this example, only the initial speed was varied, it is obvious that a parametric study of emission varying V_L, a, d, d_3, d_4, p_4, Θ_4 and r_4 could be carried out in a similar manner to determine their effect on emission. While a complete and detailed investigation of this one-stage process would lead to some generalizations, they would only be valid for a 2 signal tandem The validity of the extension of these generalizations to an n-signal system would be questionable. However, some insight into the overall phenomenon would be gained.

4.4 An Approach to the Solution of the Model

From the preceding discussion, it is apparent that: (i) for a system composed of many blocks, an analytic formulation in terms of a dynamic programming model with functional equations is feasible and (ii) a closed-form analytic solution for this model rapidly approaches unmanageable proportions as the number of blocks in the system increases beyond two.

At this point, in order to solve the general case, it would appear that to overcome this latter difficulty, a combination of search techniques would be desirable. That is, a digital computer should be programmed to follow some combination of search techniques to locate an optimal solution for the n-stage process within the overall framework of the analytic model.

This would probably involve an iterative procedure wherein a rough grid of acceptable values would correspond to the domain of the independent variables. For the first iteration, the optimal solution would be localized to some portion of the grid. On successive iterations, the domain would be confined to that part of the grid. With each iteration, both the resolution and magnification of that part of the grid is increased until the desired accuracy is obtained.

Another, but yet similar, approach to the solution would utilize a modification of the Finite-State Machine model presented in section 3.2 as the analytic model. That is, perform a computer search for the optimal solution within the framework of the Finite-State Machine model.

Chapter 5

EXTENSIONS, RECOMMENDATIONS, CONCLUSIONS

5.1 Extensions

The philosophy underlying the development of the model was to choose a point in time, consider one car alone, choose a particular driver and street, and assume that the driver and traffic conditions, signal spacing, setting, etc., over which the driver has no control, at that time remains constant for all time. Therefore, this was a static model. It is obvious that there is more than one type of driver, for a given driver many different mental states, many combinations of streets and traffic conditions, and many different vehicles in various mechanical condition. However, there are only a few variables over which a driver has immediate control at a given time, such as his desired speed and acceleration.

Applying this model to the dynamic case requires that the effect of environment and mental state upon these two driver controlled variables be determined. These two variables can be entered into the model at the beginning of the computations for any block and can be changed at similar points to reflect changes in environment, driver temperament, etc.

Similarly, the effects of the mechanical condition of the car on emission rates should be and, to a certain extent, have already been determined. However, the mechanical condition of the car is not likely to change as rapidly with time as do the environmental conditions or the driver's mental state. Therefore, this effect is of little consequence in those situations to which this model is likely to be applied.

If it is empirically determined that the cruise emission rates, in pounds per second per mph, are not a linear function of speed, in mph, but of some other form, new emission equations can be written. These relationships can be readily incorporated into the model without altering the basic logic and form. However, the type of calculations may actually present computational difficulties.

The lack of comparable data for other pertinent pollutants involved in the photochemical smog phenomenon limited this study to hydrocarbon emissions. When this data becomes available the model can be utilized with the obvious substitution of emission rates.

The extension of this model to incorporate more than one pollutant simultaneously, that is, minimization of total harmful pollutants, only requires that decisions be based on the sum of the individual emissions at any particular stage of the decision process. Therefore, to model reality, the contribution of each pollutant must be computed independently, summed for each case, and the decision dependent upon the magnitude of these sums. In the case where each pollutant is of unequal importance to the phenomenon, a weighting factor, whose magnitude is a function of the pollutant's relative importance, can be introduced into each term of this sum, and the model used as before.

Similarly, the model could be extended to incorporate tire wear and gasoline consumption into the objective function to be minimized. That is, determine the optimal driving policy which would minimize the sum of tire wear, gasoline consumption, and exhaust emissions.

5.2 Recommendations For Further Study

An investigation of the relationships between the variables and parameters of the one-block process should be conducted and the effects of changes in these variables on pollutant emissions, determined. These results and conclusions will then form the "boundary conditions" which must be satisfied by the solution of any general model.

At this point, a number of possible avenues of investigation exist. One, an algorithm for solving the n-block process should be sought and the emissions for the n-blocks determined. Again, a parametric study of the effects of the variables on emissions should be conducted.

Two, the effects on emission of the interaction of two, or more cars in single-file, follow-the-leader type movement be investigated. The techniques and results of car-following studies would contribute to this endeavor.

Three, other factors, such as tire and brake wear, and gasoline consumption, can be incorporated into the model. Their effect on the corresponding driving policy and exhaust emissions should be determined.

Four, further studies should determine the effects on emission levels of other traffic phenomena, such as merging traffic, passing, left turns, buses and trucks. Obviously, the effects of various combinations of traffic phenomena and number of types of pollutants considered should be the eventual objective.

In all cases, the resulting emissions should be extrapolated in an appropriate manner to determine the resulting change in smog hours per day, smog-free days, human discomfort, smog-influenced fatalities, crop damage, visibility, irritation level and rubber damage.

A later study should be conducted to determine the most effective method for educating drivers of the optimal policies generated by these studies and the receptivity of thepublic to such techniques. In addition, a cost-effectiveness study should be conducted to determine the relative merits of these policies.

5.3 Conclusions

There exists evidence that driver behavior does affect the emission of smog-forming pollutants and hence smog formation. Specifically, various combinations of acceleration, deceleration, and constant velocities can change pollutant emissions by as many as two orders of magnitude. The optimal driving policy consist, of accelerating at a uniform rate to some speed and maintaining that speed if possible. That cruising speed is dependent upon signal synchronization, signal spacing, speed limits and the initial speed. This also tends to yield the minimal travel time. The lower initial speeds tend to yield higher emissions levels while higher initial speeds, those at the speed limit, yield lower emission levels.

Appendix A

Computer Solution To The Single-Stage Process

Since the driver of the car has three options - decelerate, maintain speed, accelerate - upon entering a block, the nine cases were grouped into three classes for discussion puposes - Class I for an initial deceleration, Class II for no initial velocity change, and Class III for an initial acceleration. Depending upon the relationship of V_2 to V_L, one or more of the classes of cases must be considered in the determination of the driving policy for a given block.

The program was actually set up to handle each of the three classes individually and execute the computations for only the necessary class(es). For each case, regardless of class, the maximum velocity, terminal velocity, total emission, and total time spent in the block was determined. Upon completion of all the computations for the appropriate classes, the emission totals for each case were compared to determine the minimal. For that case, its policy became the policy for that block. A logic diagram of the overall process is shown in Figure 7. Except for the above discussion, the logic and calculations followed the discussion presented in section 4.2.

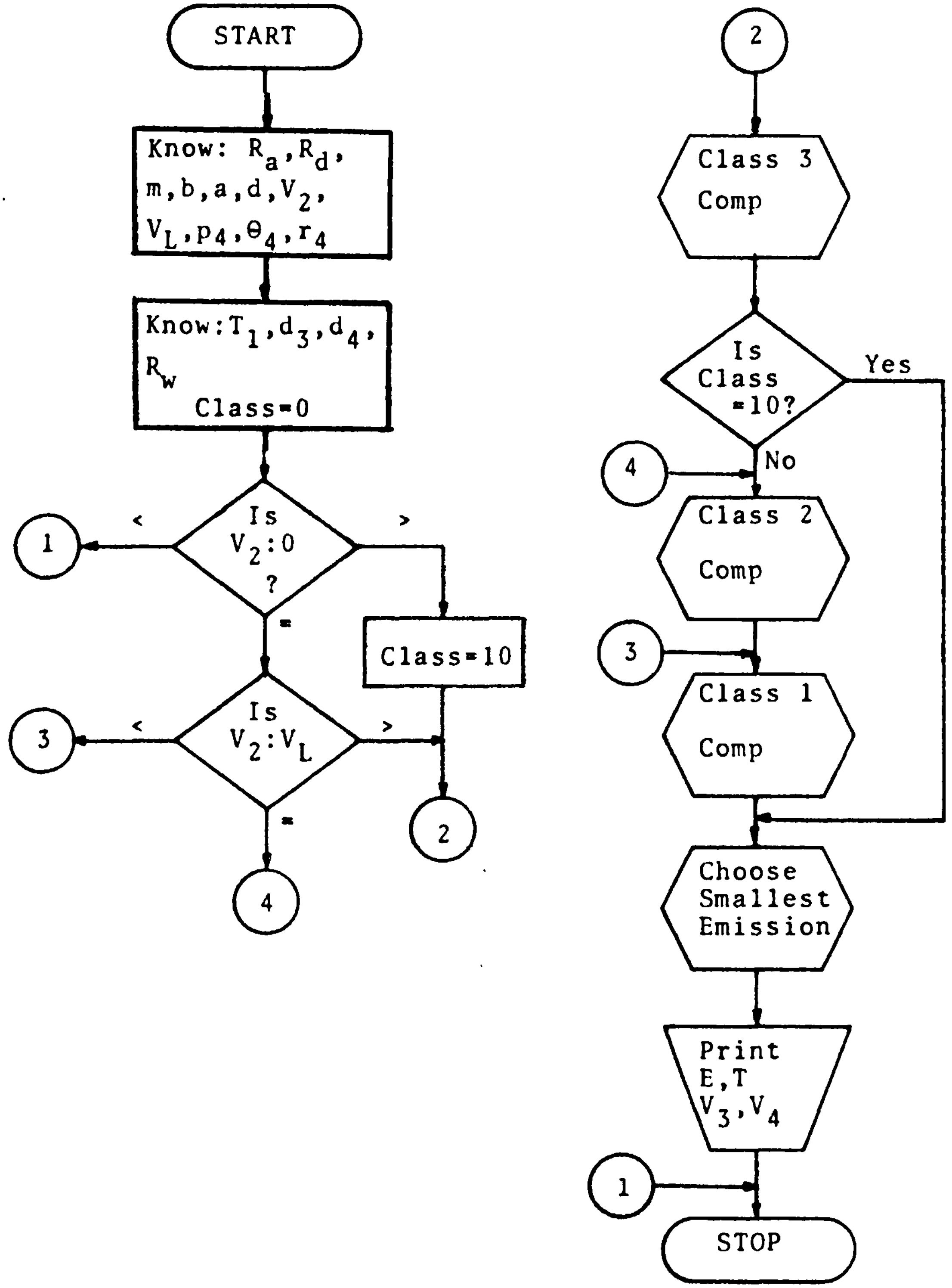


Figure 7

Logic Diagram for The Single Stage Decision Process

Only one computational difficulty was encountered in the numerical solution of this process - the determination of the zeros of the polynomial. The numerical technique which proved to be the most satisfactory was a modified Newton's Method. The effectiveness criteria used to evaluate potential program solutions were its ability (1) to handle any size polynomial, (2) to converge on a root with sufficient accuracy and (3) to evaluate complex roots if necessary. In this program, Newton's Method is used to extract a single complex root and to iterate to the desired accuracy. The degree of the polynomial is reduced by one with the use of synthetic division. This process is repeated until all roots have been extracted. This technique failed only for four roots symmetrically placed on the unit circle and gave results with only three significant figures for roots of multiplicity greater than two.

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