INVESTIGATION OF THE FEASIBILITY OF DETERMINING THE OZONE DISTRIBUTION IN A RAYLEIGH ATMOSPHERE BY SOLUTION OF A FREDHOLM INTEGRAL EQUATION

by

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A Thesis Submitted to the Faculty of the

DEPARTMENT OF SYSTEMS ENGINEERING

In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE

In the Graduate College

THE UNIVERSITY OF ARIZONA

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ACKNOWLEDGMENTS

The author wishes to express his appreciation to the United States Army for providing the opportunity of a graduate study program.

A special note of thanks goes to Dr. Benjamin M. Herman and Mr. Douglas N. Yarger, both with the Institute of Atmospheric Physics, for providing guidance and necessary background information for this thesis.

The author would also like to express his sincere appreciation to Professor Samuel R. Browning under whose direction this thesis was completed. The guidance provided by Professor Browning and the interest he stimulated in the project combined to make this study both interesting and rewarding. In addition, he wishes to acknowledge the encouragement and assistance of the entire faculty and staff of the Department of Systems Engineering.

This thesis could not have been completed without the warm and gracious assistance of the author's wife, Connie and children, Becky, Jennifer, Jamie, and Galen, who, although often neglected, provided the necessary inspiration.

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ABSTRACT

The question of whether the vertical distribution of Ozone can be determined utilizing measurements taken from balloons or satellites is investigated. A numerical method for solving the equation of radiative transfer is verified for a plane parallel, horizontally homogeneous, conservative Rayleigh atmosphere (without polarization) to within $\pm.006$ units per units intensity for an incident flux of π units per unit normal area. A method is presented which provides a stable, but smoothed, solution to the problem of determining the vertical distribution of ozone. Results of several methods for solutions of the linear system produced by quadrature are presented graphically to show the usefulness of the methods.

CHAPTER 1

INTRODUCTION

Research in the atmospheric sciences during the past 15 years has uncovered a host of important and intriguing problems involving the structure and dynamics of the atmosphere. Research efforts directed toward treating these problems require knowledge of the fundamental physical processes affecting the atmosphere. Ozone is significantly involved, both directly and indirectly, in such processes. Thus, a broad program to increase knowledge of the physical properties and distribution of atmospheric ozone is of utmost importance if fundamental problems of the atmosphere are to be solved (1).

This thesis investigates a method for determining the vertical distribution of ozone with measurements taken from either balloons or satellites. The atmospheric model used is plane-stratified, with scattering of radiant energy according to Rayleigh's law, also referred to as molecular scatter. Radiative energy of the frequency considered is absorbed only by ozone. The model is further simplified by disregarding polarization. With the restrictions of the above model, a numerical approach to the determination of the vertical distribution of ozone is investigated to determine whether useful solution can be obtained.

The thesis is divided into three major parts. Chapter 2 is devoted to the description of a computer program for the numerical solution to the integral equation of radiative transfer as written by Chandrasekhar (2). The numerical scheme of Herman and Browning (5) has been simplified to disregard polarization. Thus, the scalar form of Rayleigh's phase function is used rather than the more complex vector form. Accuracy of the calculated emergent beams of radiation produced by the computer program (see Appendix A) is verfied by the comparison with published tables (3).

Chapter 3 shows that the integral equation of radiative transfer

$$\int_{\tau 1}^{\tau 2} I(\tau,(\mu,\phi))\omega(\tau)d\tau=g(\mu,\phi)+\epsilon(\mu,\phi) \tag{1.1}$$

corresponds to the Fredholm integral equation of the first kind

$$\int_a^b K(y,x)f(y)dy=g(x)+\epsilon(x). \tag{1.2}$$

The solution in Chapter 2 is concerned with finding intensities of radiation, $g(\mu,\phi)$, when the ozone distribution is known. Chapter 3 is concerned with the inverse problem where the solution is the ozone distribution, $\omega(\tau)$, and the intensities of radiation are now known. Numerical solutions to Eq. 1.2 have been found, by the inversion of the linear system produced by quadrature, using minimization techniques of Phillips (8) and Twomey (9). These minimization techniques are applied to Eq. 1.1 to determine whether useful solutions can be developed to determine the vertical distribution of ozone. Several solutions calculated by a computer program (Appendix B) are presented along with an example of convergence of the numerical scheme.

In the final chapter, conclusions are presented as to the validity and usefulness of the results obtained. Extensions of the research are also suggested which might further improve the accuracy of the results.

CHAPTER 2

SOLUTIONS TO THE EQUATION OF RADIATIVE TRANSFER

2.1. Introduction.

Solutions to the equation of radiative transfer through a non-absorbing Rayleigh atmosphere have been fully described in a paper by Herman and Browning (5). In this chapter the work presented by the above authors has been simplified to consider a mathematical model which produces emergent intensities at each of n-levels for the non-polarizing case. A numerical scheme for actual computations is provided along with the criterion for convergence of the scheme. As a check of the computational accuracy of the computer program, the calculated emergent intensities are compared with values published in an extensive set of tables by Coulsen, Dave and Sekera (3).

2.2. Model of the System.

A full discussion of the mathematical model for the case in which polarization is considered has been provided in a paper by Herman and Browning (5). For the case in which polarization is neglected, if one considers a plane-parallel, horizontally homogeneous atmosphere, illuminated at the top by plane-parallel radiation of intensity F_o; the formal solution to the transfer equation for the intensities of radiation, I, at optical depth τ, may be written as (6)

$$I(\tau,\mu,\phi)=\int_0^\tau J(\tau',\mu,\phi)\exp[-(\tau-\tau')/\mu]\omega(\tau')\frac{d\tau'}{\mu}$$

$$+\int_0^\tau P(\mu,\mu_o,\phi,\phi_o)F(\tau')\exp[-(\tau-\tau')/\mu]\omega(\tau')\frac{d\tau'}{\mu}. \quad (2.1)$$

Here $\mu=\cos\theta$ where θ is the angle between the local zenith and the direction of propagation of the scattered radiation, considered positive in the direction of increasing τ, ϕ is the azimuthal angle measured from some arbitrarly chosen x-axis, μ_o and ϕ_o are the polar and azimuthal angles of incident solar flux (see Fig. 1). The term $F(\tau')$ represents

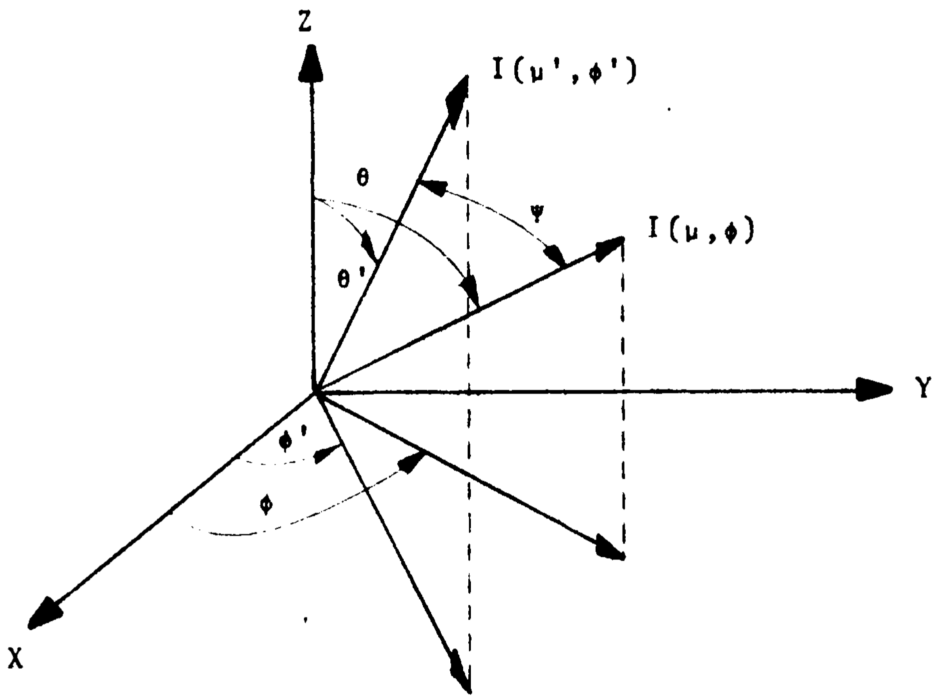


FIGURE 1

The scattering angle Ψ between the incident beam, $I(\mu',\phi')$ and the scattered beam, $I(\mu,\phi)$.

the contribution from the solar flux at level n, and is related to the incident radiation at the top of the atmosphere through the expression

$$F(\tau')=F_n(\mu_o,\phi_o)e^{-\tau'/\mu_o}. \qquad (2.2)$$

The term $\omega(\tau')$, called the albedo for single scattering is the ratio of the radiation scattered by molecules to input incident energy within τ'. For the purposes of this chapter, since no absorption is assumed, $\omega(\tau')=1.0$. The term $J(\tau,\mu,\phi)$, called the source function for the intensities of radiation, denotes the contribution, at any level τ, to the beam in the direction (μ,ϕ) arising from scattering into this direction from beams incident from all directions (μ',ϕ'). It is given by

$$J(\tau',\mu,\phi)=\int_0^{2\pi}\int_{-1}^{1}P(\mu,\mu',\phi,\phi')I(\tau',\mu',\phi')d\mu'd\phi'. \qquad (2.3)$$

The function P is the Rayleigh phase function, $P(\cos\Psi)=3/4$ $(1+\cos^2\Psi)$, given by Chandrasekhar (2), where Ψ, a function of μ,ϕ, is the angle between the incident and scattered beams. The term $I(\tau',\mu',\phi')$ represents the average intensity over τ' of incident beams.

Initially, only the inward directed intensities at the top and the bottom of the atmosphere are known. The outward directed intensities at the top and bottom are unknown and are the desired emergent intensities of radiation. Thus, the problem is a boundary value problem where a straightforward numerical integration is not possible (5).

2.3. Method of Calculation.

The numerical solution for the transmitted ($\mu>0$) intensities of radiation, I_n, in terms of quantities at the n-1 and the n-2 level is

$$\begin{aligned} I_n(\mu,\phi) = {} & I_{n-2}(\mu,\phi)e^{-2\Delta\tau/\mu} \\ & +(1-e^{-2\Delta\tau/\mu}) \sum_{\Delta\phi'} \sum_{\Delta\mu'} P(\cos\Psi) I_{n-1}(\mu',\phi')\Delta\mu'\Delta\phi' \qquad (2.4) \\ & +(1-e^{-2\Delta\tau/\mu})P(\cos\Psi_o)F_{n-1}(\mu_o,\phi_o), \quad n=1,\ldots,m. \end{aligned}$$

The first term on the right represents the part of the beam in the (μ,ϕ) direction that is not scattered in the interval n-2 to n. The second term represents the contributions in the (μ,ϕ) direction resulting from the scattering of all beams in the interval n-2 to n. The third term represents the contribution in the (μ,ϕ) direction from the scattering of the solar flux in the interval n-2 to n.

The numerical procedure is started by solving for the intensities of radiation at level n=1, in terms of each of the intensities at the top of the atmosphere (n=0, τ=0) except for F_o which is assumed equal to $F_{1/2}$. Thus, Eq. 2.4 written in forward difference form for the first computing equation, becomes

$$I_1(\mu,\phi)=(1-e^{-\Delta\tau/\mu})\sum_{\Delta\phi'}\sum_{\Delta\mu'}P(\cos\Psi)I_o(\mu',\phi')\Delta\mu'\Delta\phi'$$

$$+(1-e^{-\Delta\tau/\mu})P(\cos\Psi_o)F_{1/2}(\mu_o,\phi_o), \qquad (2.5)$$

where the first term on the right of Eq. 2.4 is set equal to zero by virtue of the boundary condition (solar flux F_o is the only input energy at the top). Thus, on the first downward pass only the third term evaluated at $n=1/2$ contributes to the calculation of the transmitted intensities of radiation at $n=1$. Subsequent intensitites of radiation, I_n, with $\mu>0$, are found progressing toward the bottom using a central difference form. Once all of the transmitted intensities have been computed, an analogous set of equations may be derived utilizing the boundary condition at the bottom of the atmosphere, $I_m(-\mu,\phi)=0$ for reflected intensities ($\mu<0$). Using the equations so obtained, reflected intensities of radiation are computed starting with the first level at the bottom and progressing upward until the emergent intensities at the top of the atmosphere are computed.

If the Gauss-Seidel iterative technique (see for example, Hildebrand (7)) is used, it is possible to converge to within some arbitrary criterion. For the purposes of this problem, where no absorption is assumed, the criterion selected was the law of conservation of energy where input energy is equal to output energy (normalized incident solar flux is equal to normalized output solar flux plus the normalized sum of emergent intensities at both the top and bottom of the atmosphere).

It was found that the first iteration resulted in values too small for the emergent flux. With successive passes, the flux increased to a value slightly less than .00006 of the normalized incident flux.

2.4. Results.

Calculations of emergent intensities for non-absorbing Rayleigh atmospheres have been published in an extensive set of tables by Coulson, Dave and Sekera (3) utilizing the solution by Chandrasekhar (2) for this problem. Comparing these values with values computed from the mathematical model of this chapter, it is possible to determine the accuracy of the computer program for the non-polarized case. For the comparison of results, increments of $\Delta\tau = .01$, $\Delta\phi = 30°$, and twenty increments of μ were selected, for a ten layer atmosphere, to provide a direct comparison for at least six values of μ in both $\mu > 0$ and $\mu < 0$ directions. Some of the results using these incremental values for an incident unpolarized solar flux of π units per unit normal area and for $\mu_o = 0.92$, and $\phi_o = 0°$ are shown in Figs. 2, 3. In these figures, the solid curves are drawn from the data of Coulson, Dave and Sekera (3), while the encircled points are the values

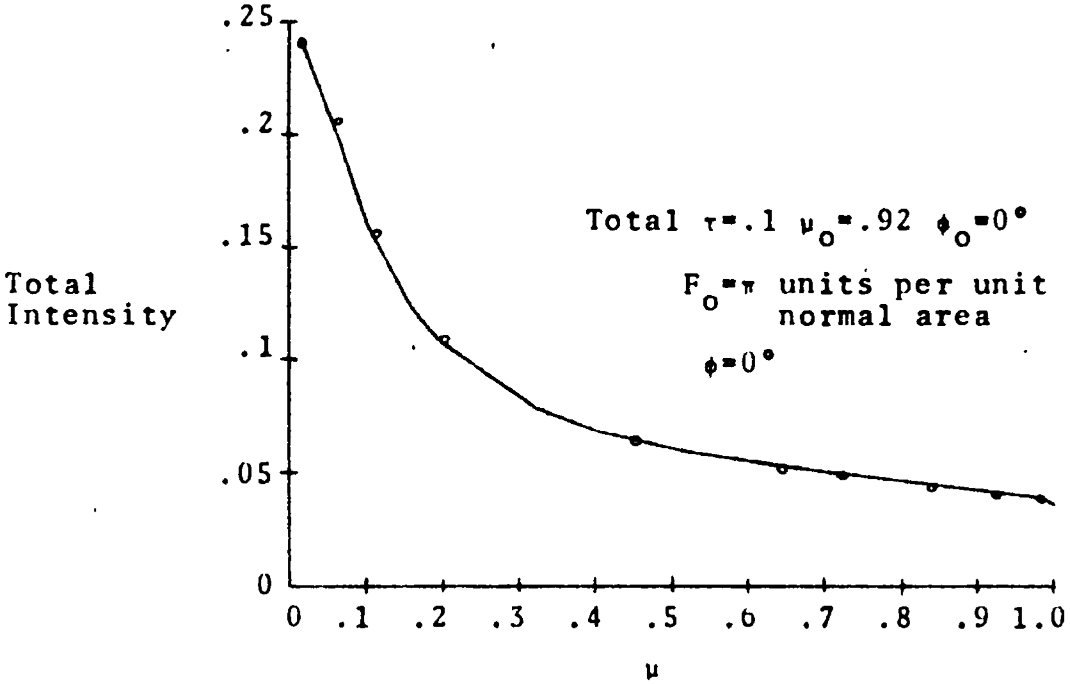


FIGURE 2

Downward Radiation Emergent from Bottom of the Atmosphere.

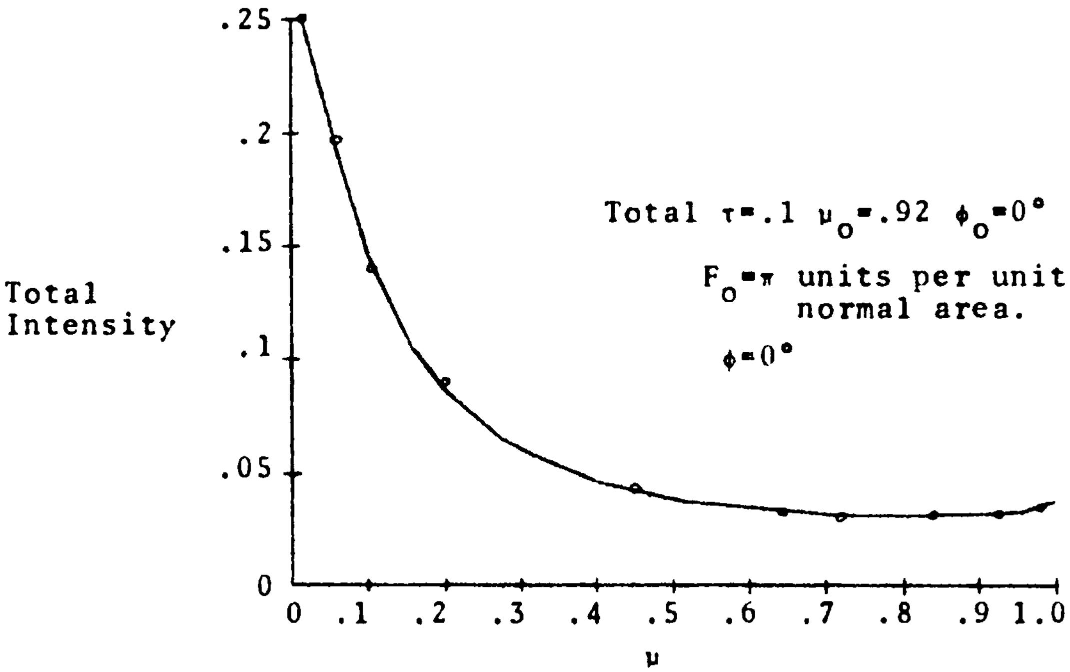


FIGURE 3

Upward Radiation Emergent from Top of the Atmosphere.

computed in the present work. Examination of these figures reveal that the maximum-deviation of the encircled values, from the solid curve is about 0.006 unit or approximately 2% error for the emergent intensities. Results for other azimuthal angles (not shown) indicate similar magnitudes of the deviations, in all cases within the maximum deviation computed above. Thus, it is now possible to determine, within the resultant accuracy of the computer program, intensities of radiation at each level of the atmosphere. These values are required for the quadrature formulation of Chapter 3.

CHAPTER 3

THE NUMERICAL SOLUTION TO INTEGRAL EQUATIONS OF THE FIRST KIND

3.1. Introduction.

The numerical solution of Fredholm integral equations of the first kind by the inversion of the linear system produced by quadrature is discussed in a paper by Twomey (9). In this chapter, the integral equation (2.1) is expressed as a quadrature, for the non-conservative case ($0 \leq \omega(\tau') \leq 1$). Twomey's numerical scheme for the solution of integral equations of the first kind is applied to the resultant quadrature form of Eq. 3.1 in an attempt to determine useful solutions to the present problem. Two theoretical distributions of albedos for single scatter are presented as true solutions, in order to determine the feasibility of the scheme as a numerical method for computing the solution vectors. The results include first, second, third, and fourth difference techniques which are applied over a range of γ for each of the two test distributions and a test of convergence for the iterative method of numerical computation.

3.2. Mathematical Model.

The numerical scheme of Herman and Browning (5) was used to solve the integral Eq. 2.1. The integral equation is of the form

$$\int_a^b K(y,x)f(y)\,dy = g(x)+\varepsilon(x), \qquad (3.1)$$

where $\varepsilon(x)$ always exists since $g(x)$ is a measured quantity. If the equation is written as a quadrature, it becomes

$$A\bar{f}=\bar{g}+\bar{\varepsilon}, \qquad (3.2)$$

where the matrix A consists of quadrature coefficients for the tabular points $y_1, y_2, \ldots, y_m$ and $x_1, x_2, \ldots, x_n$, and $\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n$ are the corresponding errors in measurement (9).

If Eq. 2.1 is to become a quadrature, it is first necessary to describe the mathematical system for a single beam of emergent radiation, any arbitrary $(-\mu,\phi)$ direction, at the top of the atmosphere, as a function of the contribution at each of n-1 lower layers. In order to describe the system, the following assumptions will be made. In Chapter 2, the numerical scheme computed intensities of radiation at the top, bottom, and at each of n-1 intermediate levels of the atmosphere. For the purposes of this problem midpoint values (those intensities between adjacent levels for the same (μ,ϕ) direction) are the required values, since the elements of the solution vector $\bar{\omega}$ are assumed to be the average value over each level. These midpoint values will be the mean values between adjacent levels (linear interpolation). Thus, a single beam of emergent radiation, for any arbitrary $(-\mu,\phi)$ direction, at the top of a ten layer atmosphere becomes

$$I_1(-\mu,\phi)=I_2(-\mu,\phi)e^{-2\Delta\tau/|\mu|}+\omega_1 a_1, \tag{3.3}$$

and subsequent lower layer intensities become

$$I_2(-\mu,\phi)=I_3(-\mu,\phi)e^{-2\Delta\tau/|\mu|}+\omega_2 a_2$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$I_9(-\mu,\phi)=I_{10}(-\mu,\phi)e^{-2\Delta\tau/|\mu|}+\omega_9 a_9$$

$$I_{10}(-\mu,\phi)=\omega_{10}a_{10},$$

where the first term on the right for $I_{10}(-\mu,\phi)$ is set equal to zero by virtue of the lower boundary condition. Thus, Eq. 3.3 written in terms of the contribution at each lower layer is

$$I_1(-\mu,\phi)=\omega_1 a'_1+\omega_2 a'_2+\ldots+\omega_{10}a'_{10}, \tag{3.4}$$

where ω_n represents the albedos for single scatter at each midpoint of the n layers, and a'_n represents sum of the midpoint contributions of the scattering of beams incident from all directions (μ',ϕ'), and the scattering of the solar flux, where prime denotes attenuation by $\exp[-(n-1)\Delta\tau/|\mu|]$. In order to describe the quadrature formulation, for the case where ten layers are assumed and ten equations are required, ten beams of emergent radiation are selected in different $(-\mu,\phi)$ directions as the measured emergent intensities $I_1(-\mu,\phi)$.

3.3. Computational Scheme.

A paper by Phillips (8) demonstrates well, the results when the exact solution of integral equations in the form of Eq. 3.2 is attempted with $\bar{\epsilon}$ set equal to zero. The result is almost always poor in the sense that the solution oscillates or displays some other feature which conflicts with *a priori* knowledge. In the present case using values computed in the last section when the direct inversion of Eq. 3.2 was attempted the matrix A was discovered to be singular (computationally singular) with rank seven. Therefore Twomey's method of finding numerical solutions to integral equations of the first kind was applied in order to determine the feasibility of this method to produce numerical solutions to the present problem.

Twomey (9) describes his method as follows; find an acceptable solution, in the sense of minimizing the second difference expression $\sum_{i=1}^{n}(f_i-2f_{i+1}+f_{i+2})^2$ where $f_k=0$, for all $k<1$, $k>n$, with the auxiliary conditions $A\bar{f}=\bar{g}+\bar{\epsilon}$ and $\sum_{i=1}^{n}\epsilon_i^2=\text{constant}$. The quantity to be minimized was then

$$\sum_{i=1}^{n}(f_i-2f_{i+1}+f_{i+2})^2+\gamma^{-1}\sum_{i=1}^{n}e_i^2, \qquad (3.5)$$

where γ is a weighting factor (γ defined ≥ 0) and the sum of squares of the errors is the computed error as γ is varied. Thus, the value for γ is selected by varying γ and computing the corresponding sum of squares of the errors until a value of γ has been found which makes the computed sum of squares of the errors of the same magnitude as the constant value for error specified in the auxiliary conditions (8).

If expression (3.5) is differentiated with respect to f_i there results

$$(f_{i-2}-4f_{i-1}+6f_i-4f_{i+1}+f_{i+2})+\gamma^{-1}\sum_{j=1}^{n} e_j a_{ij}=0, \qquad (3.6)$$

where a_{ij} is an element of A. This equation written in matrix form becomes

$$A^T\bar{\varepsilon}+\gamma H\bar{f}=0, \qquad (3.7)$$

where the matrix H for the second difference scheme is represented by

$$\begin{vmatrix}
1 & -2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-2 & 5 & -4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & -4 & 6 & -4 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -4 & 6 & -4 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -4 & 6 & -4 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & -4 & 5 & -2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & -2 & 1
\end{vmatrix}$$

The elimination of $\bar{\varepsilon}$ between Eq. 3.2 and 3.7 gives the solution in the form

$$\bar{f}=(A^TA+\gamma H)^{-1}A^T\bar{g}, \tag{3.8}$$

which does not require the matrix A to be square. The advantage of the non-square matrix will be shown in Chapter 4 where over-constrained systems will be discussed.

For the general solution of the present problem, the solution vector $\bar{f}$ represents $\bar{\omega}$ which is the vector of albedos for single scatter. The vector $\bar{g}$ represents the ten selected emergent intensities $I_1(-\mu,\phi)$. The matrix A is composed of the coefficients a'_n discussed in section 3.2. The error vector $\bar{\epsilon}$ for the auxiliary conditions is assumed to be the error in the measurement of the vector $\bar{g}$ at the top of the atmosphere. Once the constant value for the auxiliary sum of square of errors ($\sum_{i=1}^{n}\epsilon_i^2$=constant) has been specified, the value for γ can be selected by varying γ and computing the corresponding sum of squares of errors until a value of γ has been found that makes the computed sum of squares of errors of the same magnitude as the constant. Rewriting Eq. 3.8 yields

$$\bar{\omega}=(A^TA+\gamma H)^{-1}A^T\bar{g}. \tag{3.9}$$

The purpose of the investigation of Twomey's method is to determine whether his numerical solution of integral equations of the first kind can be applied to the mathematical model of the present problem. Thus, it is necessary to determine a specific γ and difference scheme, for the following formulation of H:

a) first difference form for H

$$\begin{vmatrix}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{vmatrix}$$

b) second difference form for H (see above).

c) third difference form for H

$$\begin{vmatrix}
1 & -3 & 3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-3 & 10 & -12 & 6 & -1 & 0 & 0 & 0 & 0 & 0 \\
3 & -12 & 19 & -15 & 6 & -1 & 0 & 0 & 0 & 0 \\
-1 & 6 & -15 & 20 & -15 & 6 & -1 & 0 & 0 & 0 \\
0 & -1 & 6 & -15 & 20 & -15 & 6 & -1 & 0 & 0 \\
0 & 0 & -1 & 6 & -15 & 20 & -15 & 6 & -1 & 0 \\
0 & 0 & 0 & -1 & 6 & -15 & 20 & -15 & 6 & -1 \\
0 & 0 & 0 & 0 & -1 & 6 & -15 & 19 & -12 & 3 \\
0 & 0 & 0 & 0 & 0 & -1 & 6 & -12 & 10 & -3 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 3 & -3 & 1
\end{vmatrix}$$

d) fourth difference form for H

$$\begin{vmatrix}
1 & -4 & 6 & -4 & 1 & 0 & 0 & 0 & 0 & 0 \\
-4 & 17 & -28 & 22 & -8 & 1 & 0 & 0 & 0 & 0 \\
6 & -28 & 53 & -52 & 28 & -8 & 1 & 0 & 0 & 0 \\
-4 & 22 & -52 & 69 & -56 & 28 & -8 & 1 & 0 & 0 \\
1 & -8 & 28 & -56 & 69 & -56 & 28 & -8 & 1 & 0 \\
0 & 1 & -8 & 28 & -56 & 69 & -56 & 28 & -8 & 1 \\
0 & 0 & 1 & -8 & 28 & -56 & 69 & -52 & 22 & -4 \\
0 & 0 & 0 & 1 & -8 & 28 & -52 & 53 & -28 & 6 \\
0 & 0 & 0 & 0 & 1 & -8 & 22 & -28 & 17 & 4 \\
0 & 0 & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1
\end{vmatrix}$$

which can be used to determine the solution vector $\bar{\omega}$ for any distribution of the vector elements.

3.4. Test Problems.

This investigation considers two general types of distributions. The first class of distributions, where the values of the albedos for single scatter are defined as ω'_i, yield a single maximum peak of partial pressure of ozone over the range of atmospheric pressure (see solid curve Fig. 4). The second class of distributions are similarly defined; however, double prime denotes the second case where the albedos ω''_i yield two peak values of partial pressure over the range of atmospheric pressure (see solid curve Fig. 5).

If, for the two test distributions selected, a value for γ and a difference scheme which gives an acceptable sum of squares of errors can be determined, then Twomey's

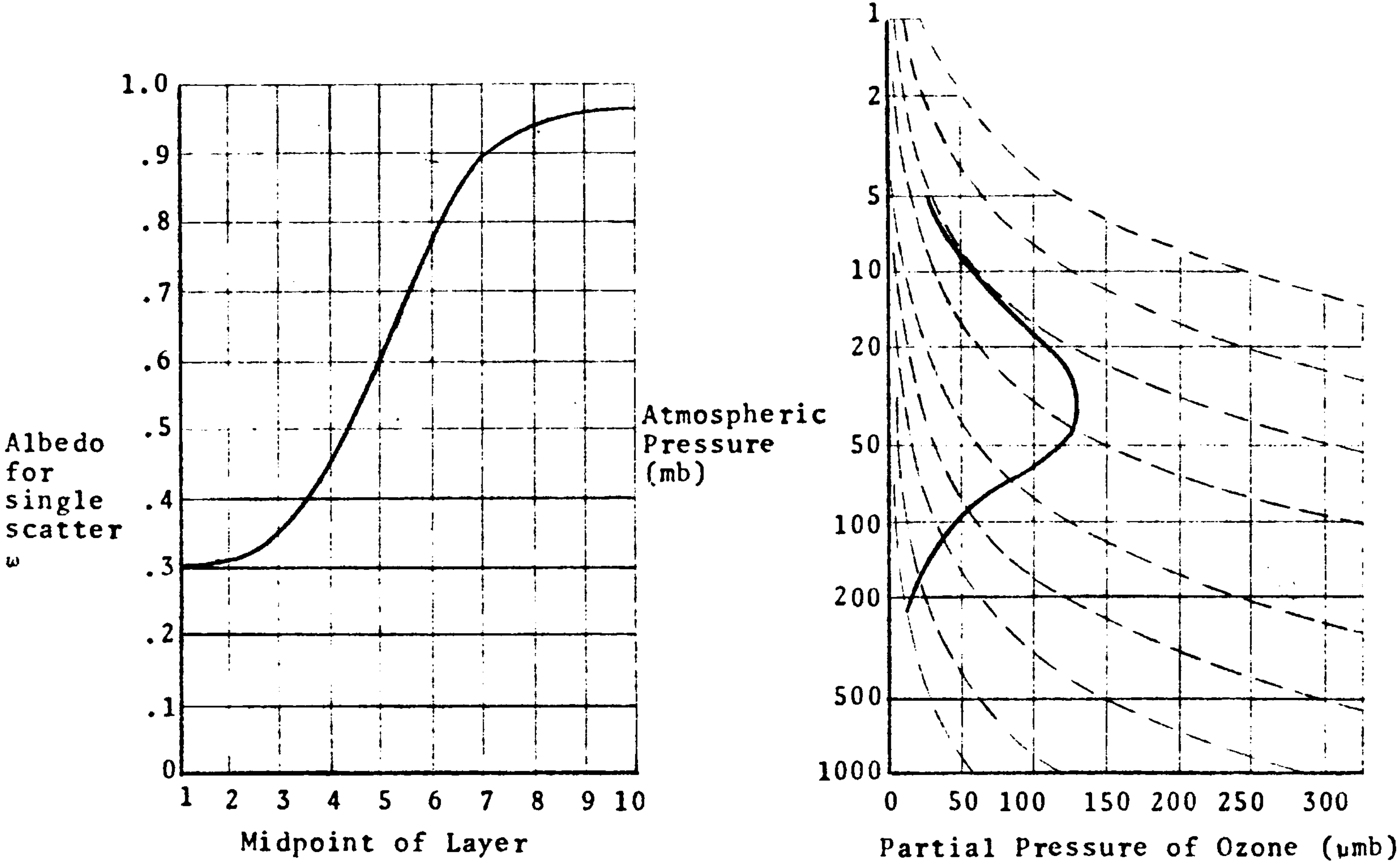


Figure 4a
ω vs. Midpoint of Layer

Figure 4b
Ozonagram of Vertical Ozone Distribution

FIGURE 4
Test Distribution I

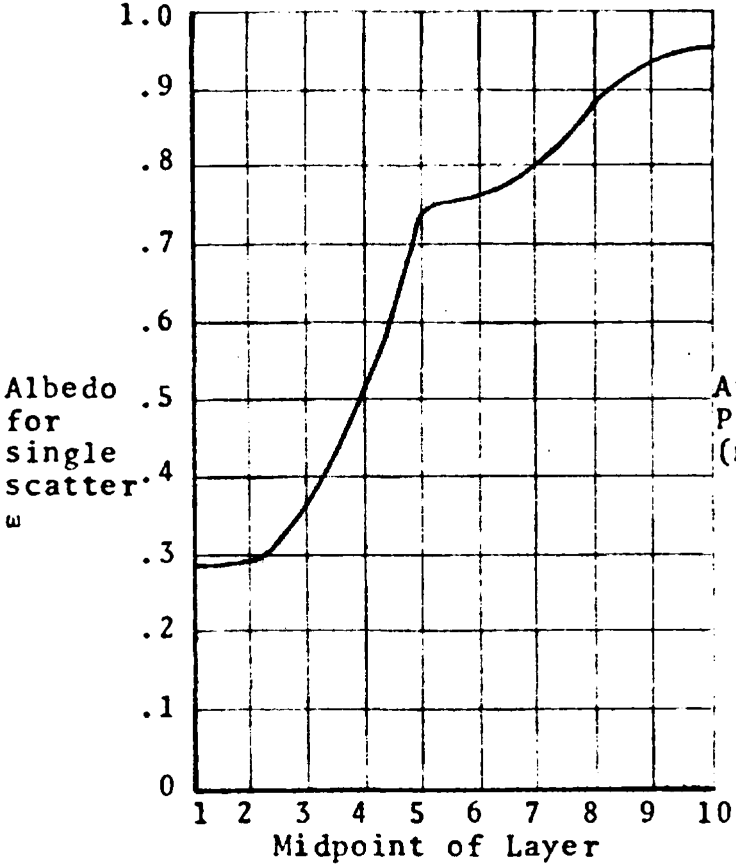


Figure 5a

ω vs. Midpoint of Layer

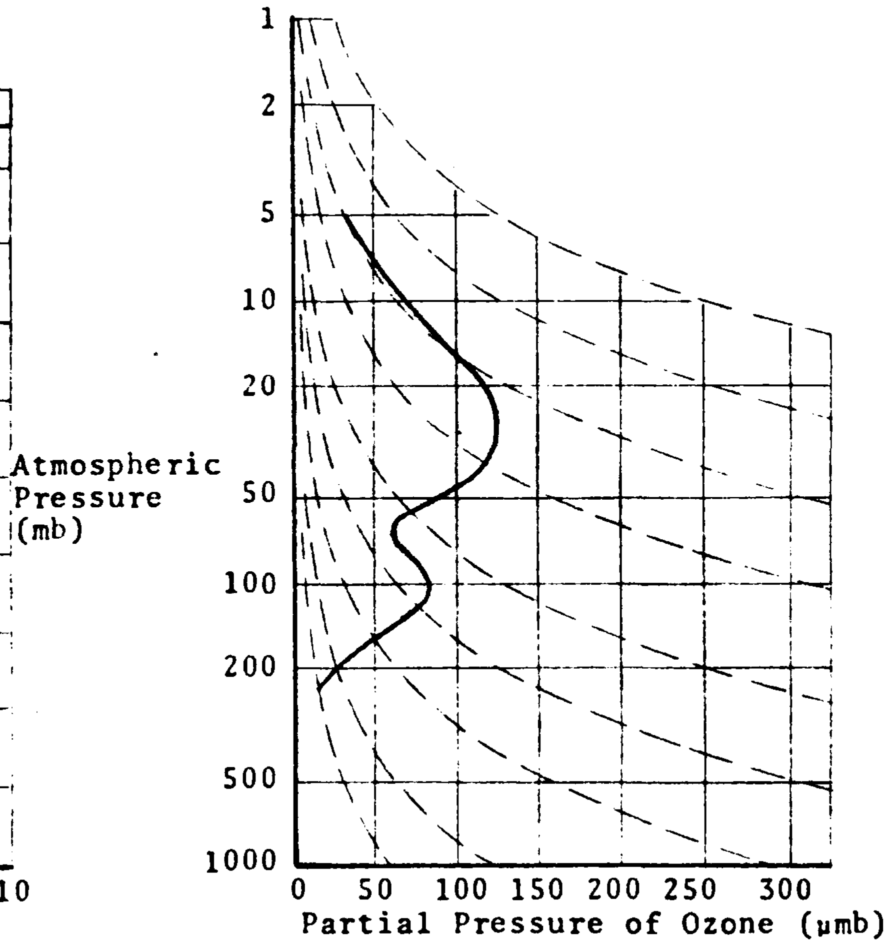


Figure 5b

Ozonagram of Vertical Distribution of Ozone

FIGURE 5

Test Distribution II

method will be considered a feasible numerical scheme for determining the distribution of the elements of the solution vector $\bar{\omega}$ for the general case.

3.4.1. Test Situation.

The numerical procedure is started, utilizing the first distribution $\bar{\omega}'$ and the computer program of Chapter 2, to calculate emergent intensities of radiation. From the set of emergent intensities of radiation at the top, ten beams in different $(-\mu,\phi)$ directions are selected as the correspondingly measured values for $\bar{g}'$.

A trial vector $\hat{\bar{\omega}}'$ is assumed and intensities of radiation at each of the ten levels of the atmosphere are computed. These intensities are averaged between adjacent levels, in corresponding $(-\mu,\phi)$ directions, to yield mid-point intensities required in the computation of the matrix A (see section 3.2). Thus, the necessary terms required for the solution of Eq. 3.2 are now available, but where $\bar{\omega}'$ is now the unknown solution vector. The equation is solved for $\bar{\omega}$, new intensities are computed and the process is repeated until no significant change in $\bar{\omega}$ occur.

In order to determine the best range of γ and difference technique required when using Twomey's method, it was decided to use the correct solution vector $\bar{\omega}$ as the first guess vector $\hat{\bar{\omega}}$. The matrix A was then computed as before. The constant for the auxiliary condition,

$\sum_{i=1}^{n} \varepsilon_i^2$=constant, is assumed equal to zero since the calculated values for $\bar{g}$ contain no errors in measurement but only those due to numerical computation. Thus, with the calculated values for $\bar{g}$, it is now possible to vary γ over the same range for each of the four difference schemes. The advantage that is gained by using this set of calculated values for $\bar{g}$ is that of selecting the value for γ that will yield the smallest computed sum of squares of the errors for each of the four difference schemes.

The computational scheme is applied to both test distributions to yield a corresponding best γ and matrix H. If the same γ and matrix H yield acceptable solutions, in the sense of minimizing expression (3.5), for both test cases without excessive smoothing of the solution vector (allowing the first term of expression (3.5) to become dominant) then Twomey's numerical scheme will be considered. a feasible scheme for solutions of the general case.

3.4.2. Test Results.

The results show that the minimum computed sum of squares of errors occurred in the γ_3=1x10^{-6} to γ_5=1x10^{-4} range with the magnitude of errors approximately equal to 1x10^{-7} for the first, second and third difference schemes for the matrix H (see Fig. 6).

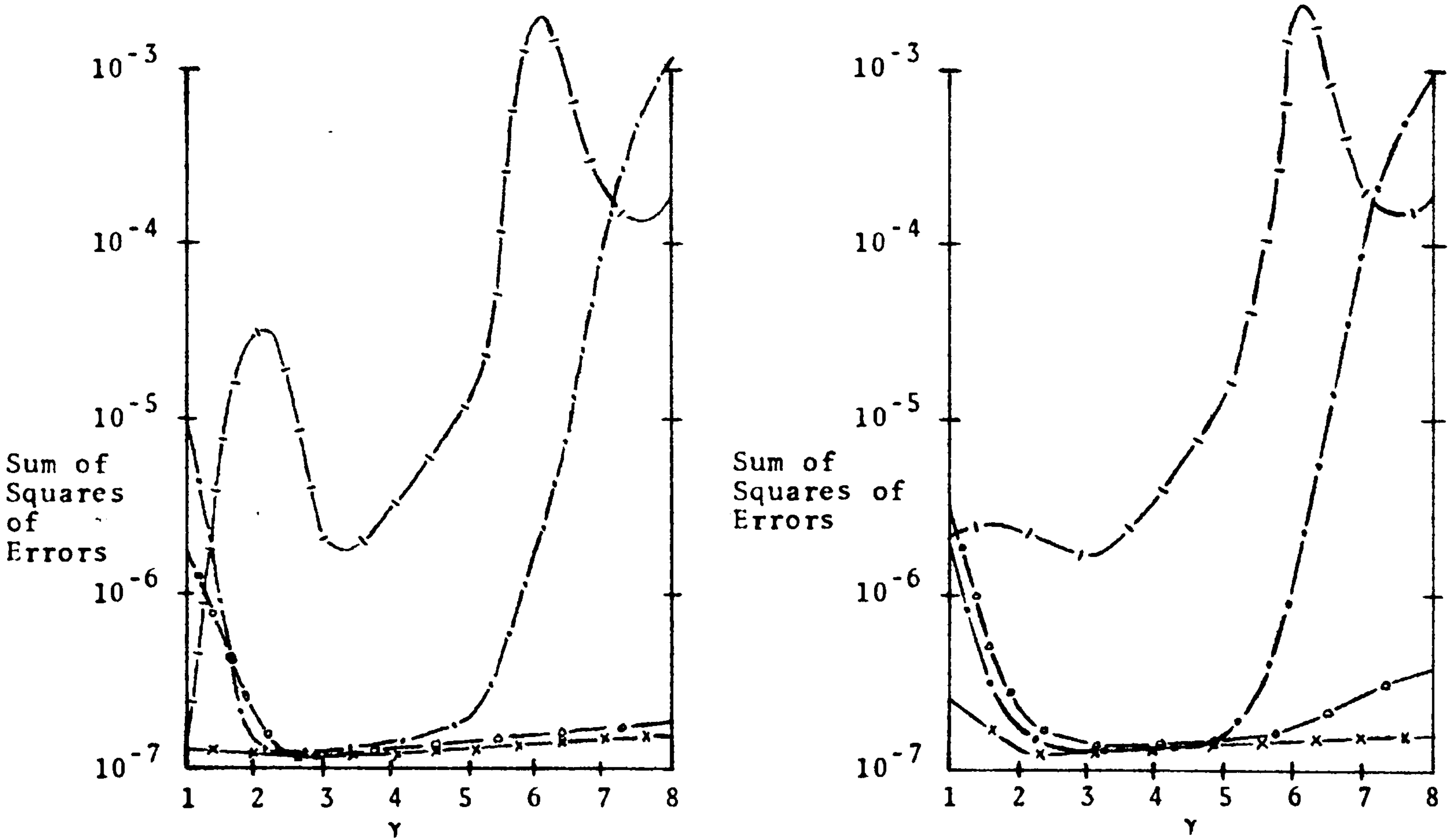


Figure 6a
Test Distribution I

Figure 6b
Test Distribution II

Legend: 1st Diff(-·-), 2d Diff(-∘-), 3rd Diff(-×-), 4th Diff(-ı̇-).

FIGURE 6

Sum of Squares of Errors vs. γ

The constant for the test distribution in the first case was assumed equal to zero. The corresponding computed sum of squares of errors for the smallest non-oscillating solution vector, for second differences was 1.23432×10^{-7}. Following the same procedure for the second test distribution, the constant was again assumed equal to zero. Similarly the corresponding smallest computed sum of squares of errors was 1.15806×10^{-7}. Thus, the value for $\gamma = 1 \times 10^{-6}$ is selected, since in neither case is there a computed sum of squares of errors sufficiently small to be of the same magnitude as the above assumed constants.

In order to determine which difference scheme would provide a feasible range of solutions each element of the computed solution vectors, for $\gamma = 1 \times 10^{-6}$, was surveyed to determine whether elements of the vectors were within the *a priori* range $0 \leq \omega(\tau') \leq 1$. Table 1 shows that the solution vectors for the first and second difference schemes for H satisfy the above constraint. Thus, both first and second difference schemes are determined feasible schemes for the matrix H. In Figs. 7 and 8 the solid curve is the desired true solution, the curve which passes through the encircled values represents the computed solution using first differences, and the curve which passes through the crossed values represents the computed solution using second differences. In order to reduce the number of

Table 1a

Test Distribution I

	ω'_i	1st Diff.	2d Diff.	3rd Diff.	4th Diff.
1	.30382	.30806	.30848	.33457	-1.15530
2	.30523	.29115	.29720	.26554	3.08720
3	.35325	.35861	.34855	.31375	2.04380
4	.45228	.49640	.47849	.46041	-0.09266
5	.59935	.65732	.65249	.66172	-2.80780
6	.77466	.80009	.81552	.85844	-1.28350
7	.88380	.89687	.92415	.99599	1.68980
8	.93174	.94070	.96092	1.03140	5.07470
9	.95560	.94374	.94043	.94702	4.75200
10	.96708	.93635	.89595	.73715	-3.17430
Sum of squares of errors		$1.53604\text{x}10^{-7}$	$1.23432\text{x}10^{-7}$	$1.11563\text{x}10^{-7}$	$2.30931\text{x}10^{-6}$

Table 1b

Test Distribution II

	ω''_i	1st Diff.	2d Diff.	3rd Diff.	4th Diff.
1	.28587	.28265	.28369	.30588	-1.18880
2	.29314	.30028	.30641	.28038	3.11340
3	.36661	.39163	.38055	.34952	2.07880
4	.50576	.53516	.51630	.49356	-0.90893
5	.74859	.68886	.68401	.68905	-2.80080
6	.75717	.81719	.83311	.87020	-1.78420
7	.79946	.89400	.92362	.98944	1.69690
8	.88363	.91779	.94168	1.01010	5.08220
9	.93140	.90592	.90388	.91262	4.74180
10	.95551	.88998	.84294	.69418	-3.23600
Sum of squares of errors		$1.33058\text{x}10^{-7}$	$1.15806\text{x}10^{-7}$	$1.01027\text{x}10^{-7}$	$2.22665\text{x}10^{-6}$

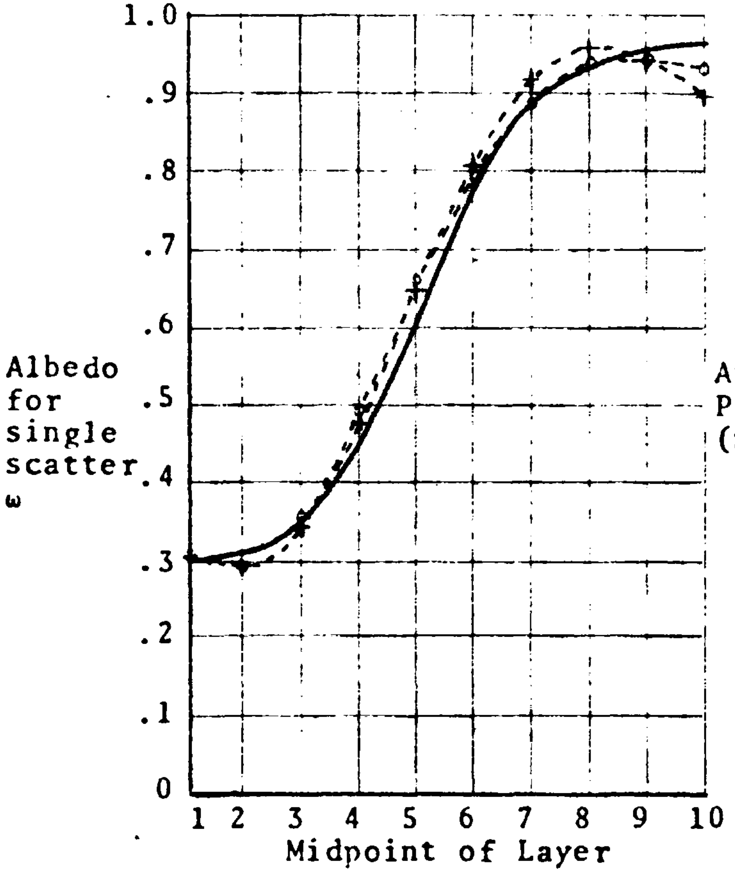


Figure 7a
ω vs. Midpoint of Layer

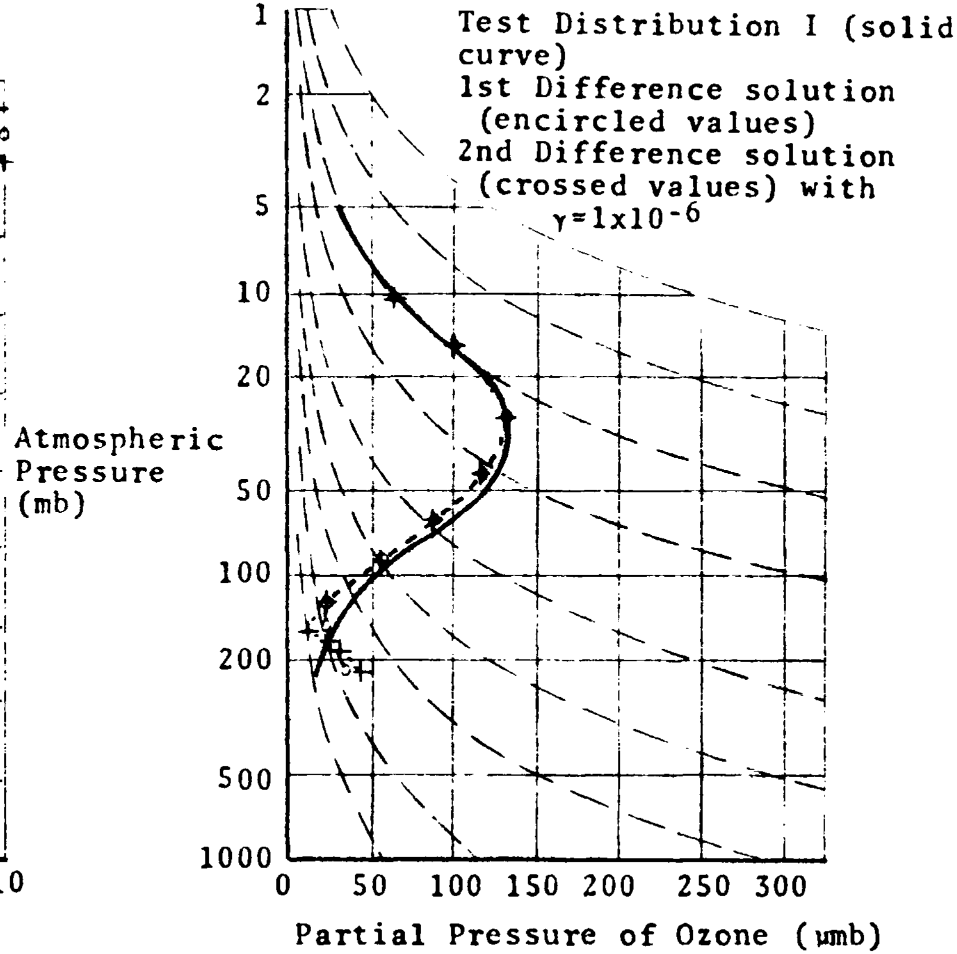


Figure 7b
Ozonagram of Vertical Distribution of Ozone

FIGURE 7

Numerical Solutions to Test Distribution I

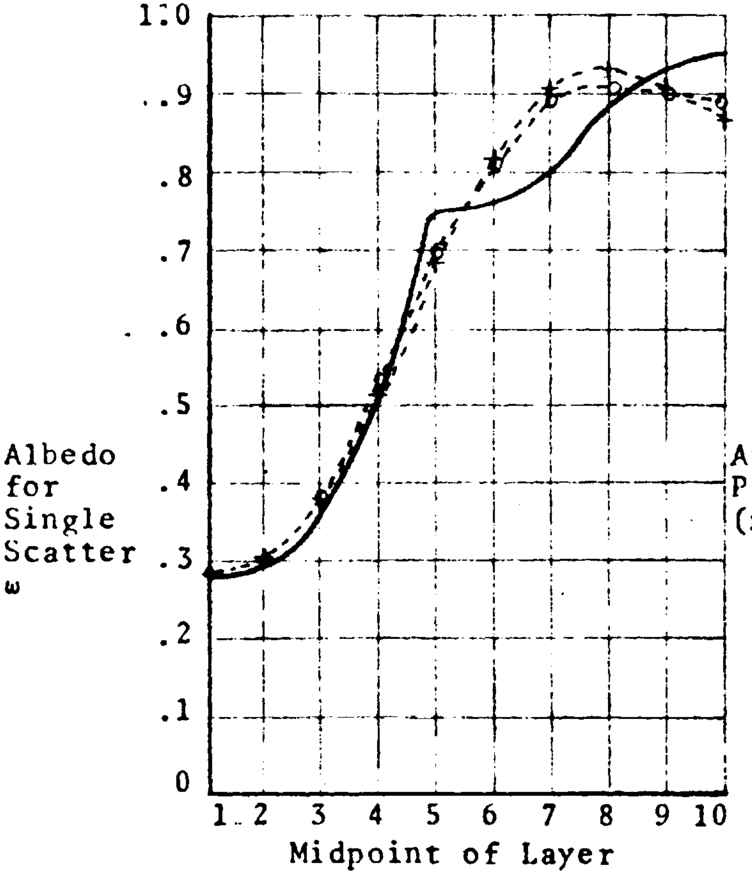


Figure 8a
ω vs. Midpoint of Layer

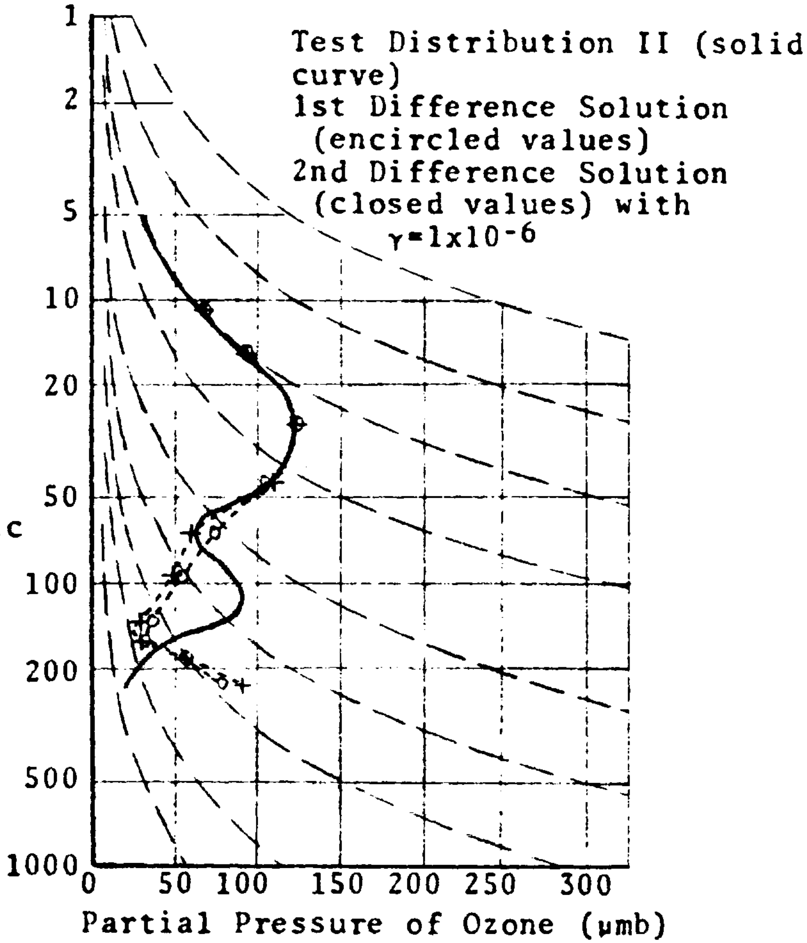


Figure 8b
Ozonagram of Vertical Distribution of Ozone

FIGURE 8

Numerical Solutions to Test Distribution II possible combinations which can be tested, only the second difference scheme for the matrix H is used throughout the rest of the chapter.

3.4.3. Test for Convergence.

The results of the previous section revealed that a best γ and matrix H could be selected which would yield acceptable results, in the sense of minimizing expression (3.5), for the two test distributions. These results were extended to include the case where an arbitrary first guess or trial solution is used. In order to determine convergence in this case one of several criteria could be selected. Since the albedos for single scatter were known for the two test cases, it was felt that the best criterion for convergence would be when there was no significant improvement in the elements of the computed solution vector. The results of the iterative scheme using an arbitrary first guess vector, shown as the curve which passes through the crosses of Fig. 9, produced after four iterations the curve which passes through the encircled values of the same figure. With the results for convergence for other arbitrary first guess vectors (not shown) along with convergence of the above scheme, it is reasonable to assume convergence for the general case.

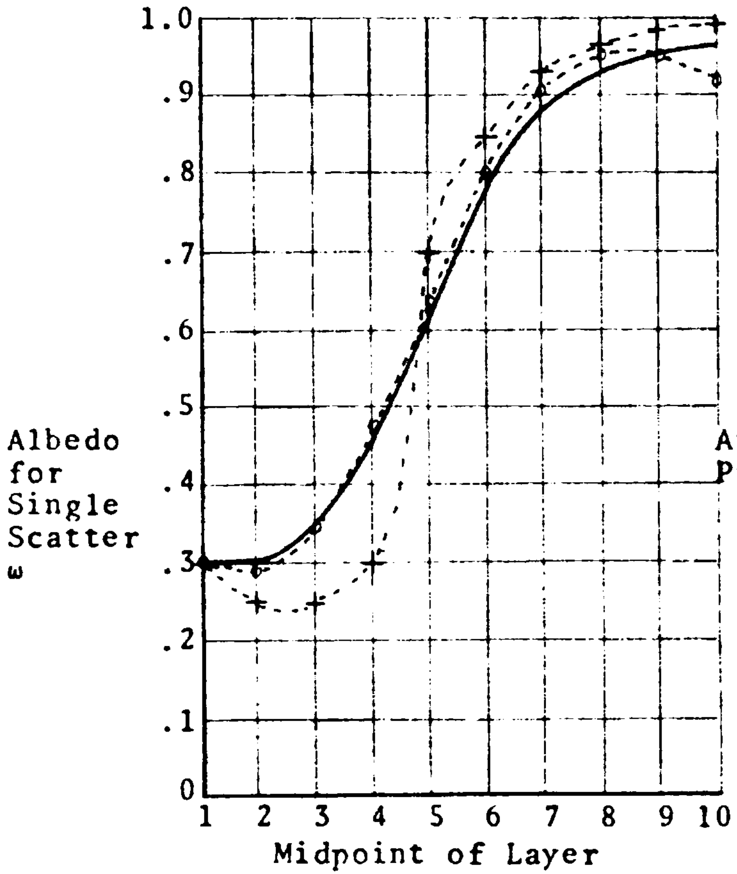


Figure 9a
ω vs. Midpoint of Layer

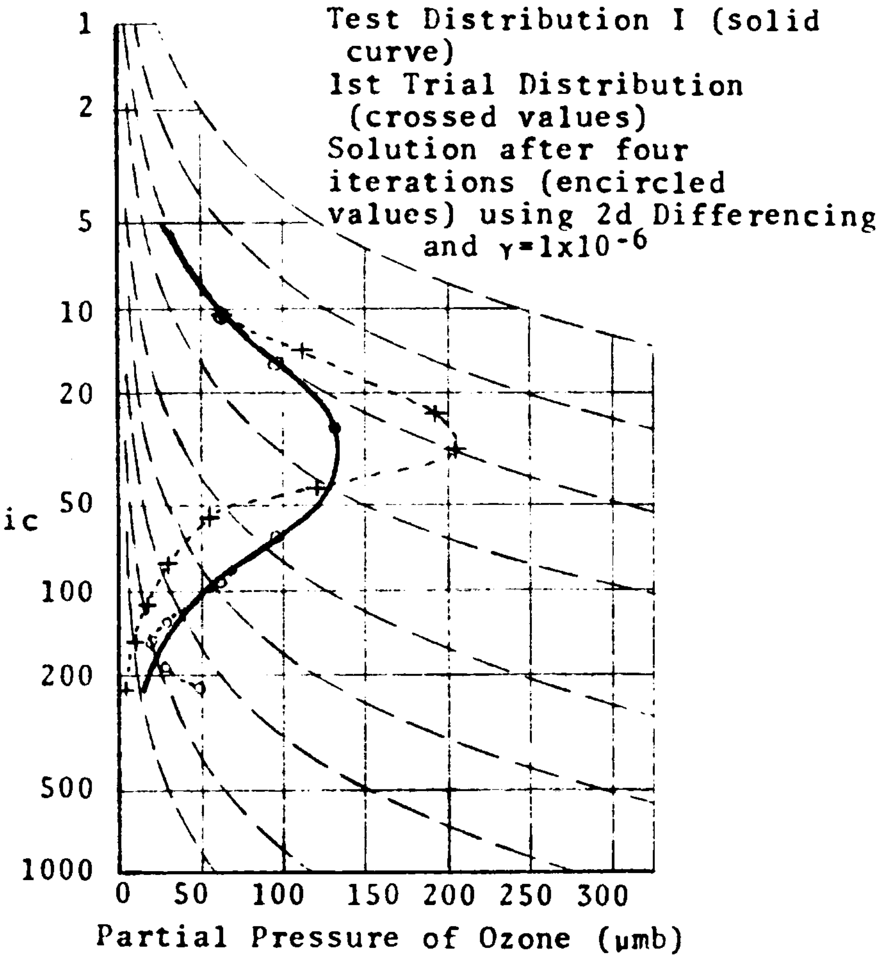


Figure 9b
Ozonagram of Vertical Distribution of Ozone

FIGURE 9

Test For Convergence

3.5. Results.

The results of this chapter indicate that Twomey's numerical method for solution of integral equations of the first kind is an appropriate method for determining solutions to the integral equation which describes the present problem. The most encouraging results presented in this chapter are probably those derived from the convergence of the numerical scheme. Other results showed that the resultant solution vector for the test distribution in the first situation appeared to be a very good approximation of the true solution. The results for the second test distribution gives a good resultant sum of squares of errors; however, the curve does not closely follow the true solution for the lower layers of the atmosphere. In fact neither solution closely represents the true solution well for the last two layers. Since no real improvement is made by iteration when using the true solution as the first trial solution it is apparent that more constraints should be included for better results. A range of possible constraints is briefly discussed in the following chapter. Some results of preliminary work are also provided along with recommendations for possible extension.

CHAPTER 4

EXTENSIONS, RECOMMENDATIONS, CONCLUSIONS

4.1. Introduction.

The philosophy underlying the development of the mathematical model of Chapter 3 was one of combining the work of Herman and Browning (5) with that of Phillips (8) and Twomey (9) to determine the feasibility of these combined methods in determining the distribution of ozone within the atmosphere. The results of Chapter 3 indicate that the overall computational scheme has merit in the sense that the iterative scheme was convergent and the resultant solution did provide an approximate solution without oscillations. The fact that the computed solution for the second distribution did not yield the lower peak values does not indicate that the computational method was not feasible for this distribution. It does, however, indicate that further information was required for better resolution of solution vectors. In this chapter several possible modifications which include other constraints are presented as extensions to the work previously presented.

4.2. Extensions.

From the multitude of possible extensions that exist for the present problem only two general areas are considered.

These are as follows: 1) modification of the numerical scheme of computation, and 2) introduction of *a priori* conditions to the system of equations prior to the inversion of the system.

Within the general area of modification of the numerical scheme for computation the following are considered of prime importance:

a) The expansion of the system of equations in Chapter 3 to consider more equations and more unknowns for the same optical depth could yield solutions with finer resolution to approximate the actual solution. For example, the number of layers could be doubled over the same total optical depth. The resultant system would consist of twenty equations with twenty unknowns. Thus, the use of finer increments would yield more values to determine a solution. No attempt was made to expand the present computational scheme of ten equations and ten unknowns, since operational computer time and the increase in storage capacity on the present computer system was not available.

b) The adaption of higher precision for the numerical computations of the inverse matrix would have allowed the use of a wider range for the value of γ.

The resultant advantage would have been such that fewer computationally singular matrices would have been calculated when the ill-conditioned matrix was inverted. Thus, the use of higher precision in the inversion of the matrix could and probably would have given better computed solutions since smaller values for γ could have been used.

c) The computation of the intensities of radiation in Chapter 2 was designed to yield intensities at the top, bottom, and each of nine intermediate levels of the atmosphere. If the numerical scheme were modified to yield intensities at the midpoint of the layers then no error in interpolation would have resulted. The consistency of computations for solutions between the numerical scheme for intensities and the numerical scheme for solutions to the inverse problem may lead to finer resolution of the solution vectors.

d) The value for γ chosen in Chapter 3 was a scalar quantity. Twomey (9) indicates that γ need not be a scalar but could and should be, for many problems, a vector quantity which would add weights to the matrix H. An example of where such weighted constraints could be applied is shown in Figs. 7 and 8. In each figure, it can be seen that there are small oscillations about the true solution for last few, lower layer, elements of the computed solution. If the corresponding last few rows of the matrix H are weighted, the resultant solution vector will be smoothed such that these small oscillations could be eliminated.

Several modifications resulting from the introduction of *a priori* conditions to the system of equations before solution of the inverse problem are considered, but are not limited to the following areas:

a) The system of equations could be increased to include the additional information that the $\sum_{i=1}^{n} \omega_i = \text{constant}$, since the sum of the values for albedos of single scatter is constant and can be determined experimentally for any distribution (4). The resultant increase in the number of equations is allowed, since there was no requirement in section 3.3 that the matrix A be square.

b) Several changes to the formulation of the matrix H are possible since much is known about the distribution of the albedos of single scatter at both the top and bottom of the atmosphere. As an example, one might assume $\omega_o = \omega_1$, rather than assuming $\omega_o = 0$ as in Chapter 3. The resultant formulation of the matrix H for second differences would be:

$$\begin{vmatrix} 2 & -3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & 6 & -4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 & 6 & -4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -4 & 5 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & -2 & 1 \end{vmatrix}$$

where the first two rows of the matrix H now contain larger coefficients. A similar result can be obtained by allowing $\omega_{10}=\omega_{11}$, where the weighted coefficients now appear in the last two rows. Finally, both $\omega_o=\omega_1$ and $\omega_{10}=\omega_{11}$ could be used to weight the top and bottom of the matrix H. The possible result in the computed solutions using these forms for the matrix H could be finer resolution plus the smoothing effect at either the top or bottom, or both, of the solution vector.

c) One other possible modification to the formulation of the matrix H, could be the assumption $\omega_{10}=1$. With this assumption, the quadrature form for the inversion will consist of a system of ten equations with nine unknowns. This result is accomplished by subtracting the now known quantities from the right side of Eq. 3.2 prior to inversion of the system.

4.3. Recommendations.

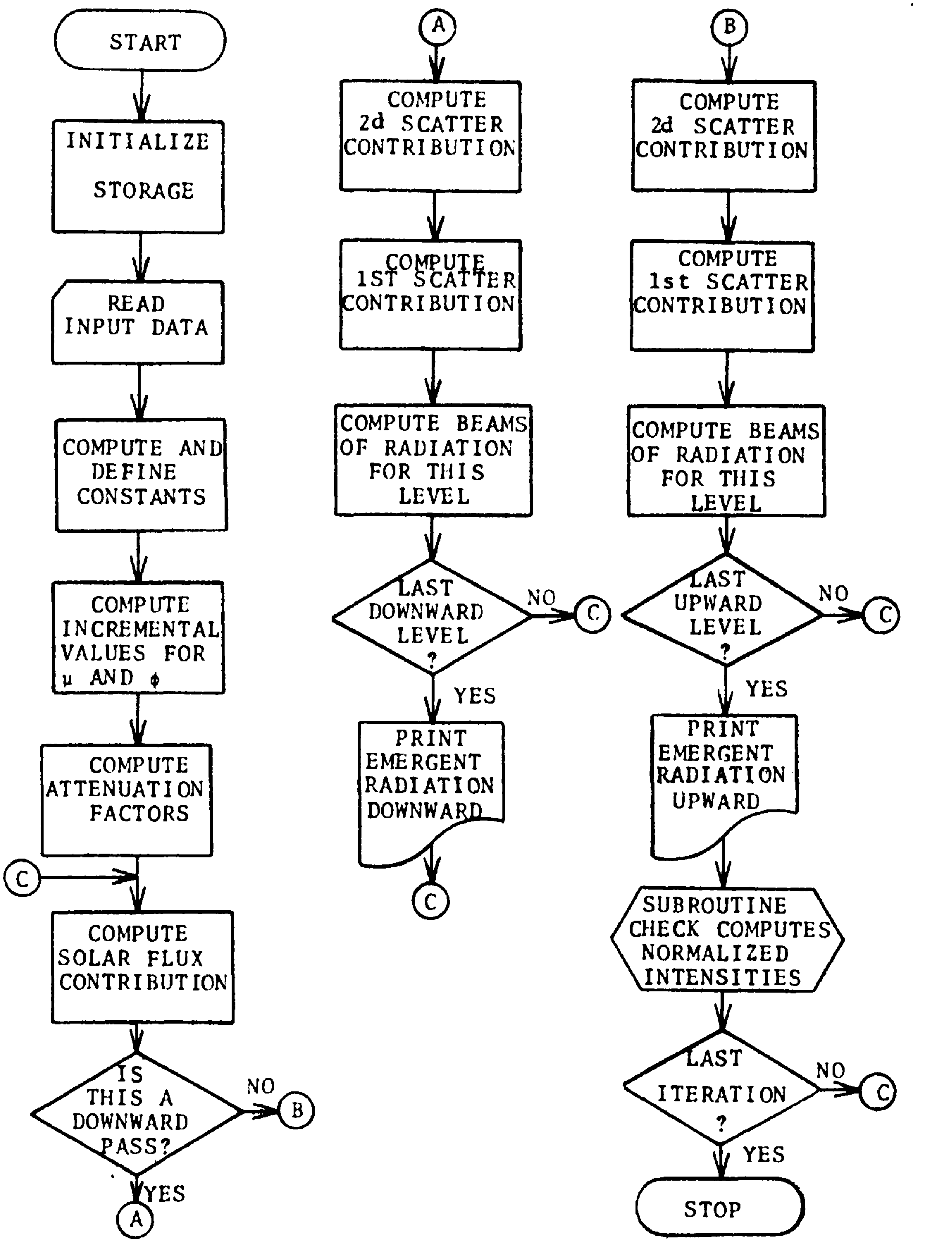
Of the many modifications listed in the previous section, there are several that show promise of producing potentially superior solution vectors. All of the modifications listed under the numerical scheme of computation are highly recommended where computer systems are available for implementation. Of the many modifications possible, listed under the introduction of *a priori* conditions, each modification listed should be evaluated separately, then combined to determine whether any improvement in the resolution of solutions can be achieved.

4.4. Conclusions.

It has been shown that the numerical methods described can provide an approximate solution to the problem of determining vertical ozone distributions in the atmosphere from measurements of emergent radiation at the top of the atmosphere. There is strong evidence that better approximations can be obtained by use of additional *a priori* information and by utilizing greater precision in the computer programs. Whether the solutions are, or will be, of sufficient accuracy to meet the requirements of the atmospheric physicists will have to be determined after further investigation and application to actual problems.

APPENDIX A

COMPUTER PROGRAM FOR SOLUTIONS TO THE EQUATION OF RADIATIVE TRANSFER


START
INITIALIZE STORAGE
READ INPUT DATA
COMPUTE AND DEFINE CONSTANTS
COMPUTE INCREMENTAL VALUES FOR μ AND φ
COMPUTE ATTENUATION FACTORS
C
COMPUTE SOLAR FLUX CONTRIBUTION
IS THIS A DOWNWARD PASS?
NO
B
YES
A
A
COMPUTE 2d SCATTER CONTRIBUTION
COMPUTE 1ST SCATTER CONTRIBUTION
COMPUTE BEAMS OF RADIATION FOR THIS LEVEL
LAST DOWNWARD LEVEL ?
NO
C
YES
PRINT EMERGENT RADIATION DOWNWARD
C
B
COMPUTE 2d SCATTER CONTRIBUTION
COMPUTE 1st SCATTER CONTRIBUTION
COMPUTE BEAMS OF RADIATION FOR THIS LEVEL
LAST UPWARD LEVEL ?
NO
C
YES
PRINT EMERGENT RADIATION UPWARD
SUBROUTINE CHECK COMPUTES NORMALIZED INTENSITIES
LAST ITERATION ?
NO
C
YES
STOP

```
*     COMPILE FORTRAN,EXECUTE FORTRAN,DUMP

      SUBROUTINE CHECK
C    ***COMPUTES NORMALIZED INTENSITIES***
      DIMENSION AMU(20),DMU(20),SCATI(120,20)
      COMMON FO,AMU,DFI,SCATI,DMU,DFI,DTAU,NLEVS,FMUO,IP,
     1ICST
      TEN=NLEVS
      FOOUT=FO*EXPFF(-TEN   *DTAU/ABSF(FMUO))*FMUO
      PRINT 1,FOOUT
    1 FORMAT (//5X,38HFO OUT AT BOTTOM OF ATMOSPHERE (N)
     1,1PE16.7/)
      DNI=0.0
      DO 2 J=11,20
      DO 2 I=IP,ICST
    2 DNI=DNI+SCATI(I,J)*AMU (J)*DFI*DMU(J)
      PRINT 3,DNI
    3 FORMAT (5X,38HSUM OF DOWNWARD INTENSITIES NORMALIZED
     1,1PE16.7/)
      UPI=0.0
      DO 4 J=1,10
      DO 4 I=1,12
    4 UPI=UPI+SCATI(I,J)*AMU (J)*DFI*DMU(J)
      PRINT 5,UPI
    5 FORMAT (5X,38HSUM OF UPWARD INTENSITIES NORMALIZED
     1,1PE16.7/)
      SUMI=ABSF(FOOUT+DNI)+UPI
      PRINT 9,SUMI
    9 FORMAT (5X,38HSUM OF ALL INTENSITIES NORMALIZED
     1,1PE16.7/)
      RETURN
      END

C    ***MAIN PROGRAM***
C    ***INITIALIZE STORAGE***
      DIMENSION  XMU(21),OMEG(22),SFI(12),AMU(20),ANU(20),
     1EFO(23),ESC(2,20),ESCM1(2,20),SCATI(120,20),DMU(20)
      COMMON FO,AMU,DFI,SCATI,DMU,DFI,DTAU,NLEVS,FMUO,IP,
     1ICST
C    ***READ INPUT DATA***
      READ 1,NITR,NMU,NLEVS,NFI,PHI,DTAU,FO,FMUO,FIO
      READ 2,(XMU(I),I=1,NMU)
      LEVP1=NLEVS+1
      READ 2,(OMEG(I),I=1,LEVP1)
    1 FORMAT(4I5,5F10.0)
    2 FORMAT(8F10.0)
C    ***COMPUTE AND DEFINE CONSTANTS***
      ITR=1
      CST=3./(16.*3.1415927)
      DFI=PHI/57.295779
      NFIM1=NFI-1
      NSFI2=NFIM1*2
```

```
C    ***COMPUTE INCREMENTAL VALUES FOR MU AND PHI***
        DO 10 I=1,NSFI2
        XI=I-1
     10 SFI(I)=XI*DFI
        NMUM1=NMU-1
        DO 12 I=1,NMUM1
        AMU(I)=.5*(XMU(I)+XMU(I+1))
        ANU(I)=SQRTF(1.-AMU(I)**2)
     12 DMU(I)=XMU(I)-XMU(I+1)
C    ***COMPUTE ATTENUATION FACTORS***
        EFO(1)=EXPEF(-DTAU/(2.*ABSF(FMUO)))
        DO 14 K=2,NLEVS
        XK=K-1
     14 EFO(K)=EXPEF(-XK*DTAU/ABSF(FMUO))
        NLEV2=NLEVS*2
        LEVP2=NLEVS+2
        EFO(LEVP1)=EXPEF(-((2.*XK)+1.)*DTAU/(2.*ABSF(FMUO)))
        DO 15 L=LEVP1,NLEV2
        IL=NLEV2-L+1
        EFO(L+1)=EFO(IL)
     15 OMEG(L+1)=OMEG(IL)
        DO 16 I=1,2
        FX=I
        DO 16 J=1,NMUM1
        ESC(I,J)=EXPEF(-FX*DTAU/ABSF(AMU(J)))
     16 ESCM1(I,J)=1.-ESC(I,J)
        LEV=1
        ICST=NLEVS*NSFI2
     19 LEVM1=LEV-1
        LEVM2=LEV-2
        IF(LEV-NLEVS)20,20,21
     20 II=1
        III=NFI
        JJ=11
        JJJ=NMUM1
        GO TO 22
     21 II=1
        III=NFI
        JJ=1
        JJJ=10
     22 DO 29 I=II,III
        DO 29 J=JJ,JJJ
        STHET=0.0
        THET =0.0
C    ***COMPUTE SOLAR FLUX CONTRIBUTION***
        THETO=(ANU(J)*SQRTF(1.-FMUO**2)*COSF(SFI(I)-FIO)+
       1AMU(J)*FMUO)**2
        FTHET=(1.+THETO)*FO*EFO(LEV)
C    ***IS THIS THE LAST DOWNWARD PASS***
        IF(JJ-1)24,24,23
     23 IF(ITR-1)51,50,51
     50 SENSE LIGHT 2
```

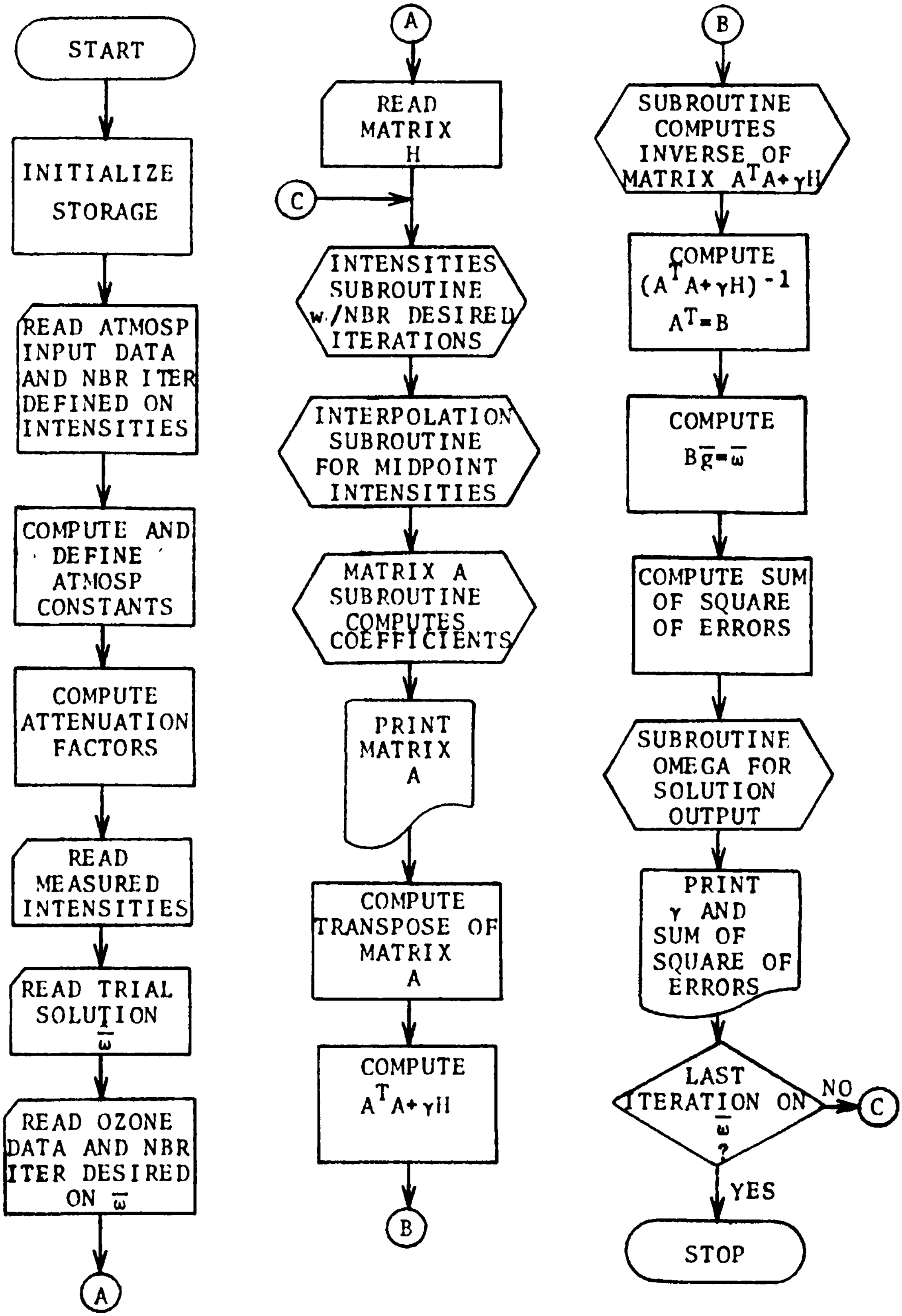
```
   51 IF(LEV-2)25,27,55
   25 ITEMP=1
      NLEV=I
C     ***COMPUTE SECOND SCATTER CONTRIBUTION***
C     ***COMPUTE FIRST SCATTER CONTRIBUTION***
C     ***COMPUTE BEAMS OF RADIATION FOR THIS LEVEL***
      IF(SENSE LIGHT 2)36,37
   36 PTHET=(STHET+FTHET)*ESCM1(ITEMP,J)*CST
      SCATI(NLEV,J)=PTHET*OMEG(LEV)
      GO TO 74
   37 SENSE LIGHT 1 .
      GO TO 55
   27 ITEMP=2
      SENSE LIGHT 3
   55 DO 28 K=1,NSFI2
      DFIS=  SFI(K)-SFI(I)
      IF(SENSE LIGHT 1)60,63
   63 ILEV=LEVM2*NSFI2+K
      LL=11
      LLL=20
      DO 26 L=LL,LLL
      THET=(ANU(J)*ANU(L)*COSF(DFIS)+AMU(J)*AMU(L))**2
   26 STHET=STHET+(1.+THET)*DMU(L)*DFI*SCATI(ILEV,L)
      IF(SENSE LIGHT 2)61,62
   61 SENSE LIGHT 2
      GO TO 28
   60 ILEV=K
      SENSE LIGHT 1
      GO TO 65
   62 ILEV=LEVM1*NSFI2+K
   65 LL=1
      LLL=10
      DO 30 L=LL,LLL
      THET=(ANU(J)*ANU(L)*COSF(DFIS)+AMU(J)*AMU(L))**2
   30 STHET=STHET+(1.+THET)*DMU(L)*DFI*SCATI(ILEV,L)
   28 CONTINUE
      PTHET=(STHET+FTHET)*ESCM1(ITEMP,J)*CST
      IF(SENSE LIGHT 1)70,72
   70 NLEV=I
   32 SCATI(NLEV,J)=PTHET*OMEG(LEV)
      GO TO 74
   72 NLEV=LEVM1*NSFI2+I
      IF(SENSE LIGHT 3)32,33
   33 LLEV=NLEV-NSFI2*2
      SCATI(NLEV,J)=SCATI(LLEV,J)*ESC(ITEMP,J)+PTHET*
     1OMEG(LEV)
      IF(SENSE LIGHT 2)74,74
C     ***COMPUTE SECOND SCATTER CONTRIBUTION***
C     ***COMPUTE FIRST SCATTER CONTRIBUTION***
C     ***COMPUTE BEAMS OF RADIATION FOR THIS LEVEL***
   24 IF(LEV-LEVP2)80,82,89
   80 ITEMP=1
```

```
      SENSE LIGHT 3
      GO TO 84
   82 ITEMP=2
      SENSE LIGHT 1
      GO TO 84
   89 IF(LEV-NLEV2)84,83,83
   83 SENSE LIGHT 4
   84 DO 48 K=1,NSFI2
      DFIS=SFI(K)-SFI(I)
      IF(SENSE LIGHT 3)85,86
   85 ILEV=ICST+K-NSFI2
      SENSE LIGHT 3
      GO TO 81
   86 ILEV=ICST-(LEVM1-NLEVS)*NSFI2+K
      LL=1
      LLL=10
      DO 49 L=LL,LLL
      THET=(ANU(J)*ANU(L)*COSF(DFIS)+AMU(J)*AMU(L))**2
   49 STHET=STHET+(1.+THET)*DMU(L)*DFI*SCATI(ILEV,L)
      ILEV=ICST-(LEV-NLEVS)*NSFI2+K
   81 LL=11
      LLL=NMUM1
      DO 31 L=LL,LLL
      THET=(ANU(J)*ANU(L)*COSF(DFIS)+AMU(J)*AMU(L))**2
   31 STHET=STHET+(1.+THET)*DMU(L)*DFI*SCATI(ILEV,L)
   48 CONTINUE
      PTHET=(STHET+FTHET)*ESCM1(ITEMP,J)*CST
      IF(SENSE LIGHT 3)90,92
   90 NLEV=ICST-NSFI2+I
   91 SCATI(NLEV,J)=PTHET*OMEG(LEV)
      GO TO 74
   92 NLEV=ICST-(LEV-NLEVS)*NSFI2+I
      IF(SENSE LIGHT 1)91,96
   96 LLEV=NLEV+NSFI2*2
      SCATI(NLEV,J)=SCATI(LLEV,J)*ESC(ITEMP,J)+PTHET*
     1OMEG(LEV)
   74 GO TO(29,75,76,77,78,79,29),I
   75 IR=NLEV+10
      GO TO 69
   76 IR=NLEV+8
      GO TO 69
   77 IR=NLEV+6
      GO TO 69
   78 IR=NLEV+4
      GO TO 69
   79 IR=NLEV+2
   69 SCATI(IR,J)=SCATI(NLEV,J)
   29 CONTINUE
C    ***LAST DOWNWARD LEVEL***
      IF(LEV-NLEVS)93,34,93
C    ***PRINT EMERGENT INTENSITIES DOWNWARD***
   34 PRINT 7,ITR
```

```
    7 FORMAT (5X,9HITERATION,I3,10X,20HINTENSITIES DOWNWARD
     1,/)
      PRINT 5
    5 FORMAT (1X,9HPHI =    0,14X,2H30,14X,2H60,14X,2H90,13X
     23H120,13X,
     13H150,13X,3H180,10X,3HAMU)
      IP=ICST-NSFI2+1
      IPP=ICST-NSFI2+NFI
      DO 35 J=11,NMUMI
   35 PRINT 6,(SCATI(I,J),I= IP,IPP),AMU(J)
    6 FORMAT (1P7E16.7,0PF8.3)
      GO TO 101
C     ***LAST UPWARD LEVEL***
   93 IF(SENSE LIGHT 4)94,101
C     ***PRINT EMERGENT INTENSITIES UPWARD***
   94 PRINT 8
    8 FORMAT (//28X,18HINTENSITIES UPWARD,/ )
      PRINT 5
      DO 95 J=1,10
   95 PRINT 6,(SCATI(I,J),I=1,NFI),AMU(J)
C     ***SUBROUTINE CHECK COMPUTES NORMALIZED INTENSITIES***
      CALL CHECK
C     ***LAST ITERATION***
      IF(ITR-NITR)102,103,103
  102 ITR=ITR+1
      LEV=1
      PRINT 100
  100 FORMAT (1H1)
      GO TO 19
  101 LEV=LEV+1
      GO TO 19
  103 STOP
      END
```

APPENDIX B

COMPUTER PROGRAM FOR THE NUMERICAL SOLUTION TO INTEGRAL EQUATIONS OF THE FIRST KIND


START
INITIALIZE STORAGE
READ ATMOSP INPUT DATA AND NBR ITER DEFINED ON INTENSITIES
COMPUTE AND DEFINE ATMOSP CONSTANTS
COMPUTE ATTENUATION FACTORS
READ MEASURED INTENSITIES
READ TRIAL SOLUTION $\bar{\omega}$
READ OZONE DATA AND NBR ITER DESIRED ON $\bar{\omega}$
A
A
READ MATRIX H
C
INTENSITIES SUBROUTINE w/NBR DESIRED ITERATIONS
INTERPOLATION SUBROUTINE FOR MIDPOINT INTENSITIES
MATRIX A SUBROUTINE COMPUTES COEFFICIENTS
PRINT MATRIX A
COMPUTE TRANSPOSE OF MATRIX A
COMPUTE $A^TA+\gamma H$
B
B
SUBROUTINE COMPUTES INVERSE OF MATRIX $A^TA+\gamma H$
COMPUTE $(A^TA+\gamma H)^{-1}A^T=B$
COMPUTE $B\bar{g}=\bar{\omega}$
COMPUTE SUM OF SQUARE OF ERRORS
SUBROUTINE OMEGA FOR SOLUTION OUTPUT
PRINT γ AND SUM OF SQUARE OF ERRORS
LAST ITERATION ON $\bar{\omega}$?
NO
C
YES
STOP

```
*     COMPILE FORTRAN,EXECUTE FORTRAN,DUMP IF ERROR

      SUBROUTINE OMAGA(DTAU,ALPHA,BETA,ITER,OGA,LAST,OMEG)
C    ***SUBROUTINE PRODUCES SOLUTION OUTPUT***
      DIMENSION OMEG(22),OGA(22),PB(20),P3(20)
      PRINT 22
   22 FORMAT (36H DETERMINATION OF OZONE DISTRIBUTION//)
      PN=0.0
      OMGTL=0.0
      PRINT 23, DTAU,ALPHA,BETA,ITER
   230FORMAT (11H DELTA TAU=1F7.4,8H  ALPHA=1F6.3,7H  BETA=
     11F10.8,3X,11H ITERATION I3/)
      PRINT 40
   40 FORMAT( 38H FIRST PBAR AND P3BAR POINT DETERMINES )
      PRINT 60
   60 FORMAT(27H CONSTANT MIXING RATIO LINE )
      PRINT 61
   61 FORMAT(43H REMAINDER OF POINTS ARE PRESSURE MIDPOINTS)
      PRINT 24
   240FORMAT (22X,88H SINGLE SCATTERING ALBEDO      OZONE
     1PRESSURE      AVERAGE     OZONE PARTIAL)
      PRINT 25
   25 FORMAT(31X,10H (OMEGA 0)10X9HINCREMENT13X
     110H INCREMENT4X8HPRESSURE5X8HPRESSURE   )
      PRINT 26
   260FORMAT(24X,7H 1 DTAU,8X,7H 2 DTAU,3X11H (M-ATM-CM)
     115X4H(MB)9X4H(MB)7X10H(MICRO MB)
      NNN=LAST+1
      OMEG(NNN)=OGA(LAST)
      OMEG(1)=OGA(1)
      DO 45 J=2,LAST
   45 OMEG(J)=(OGA(J-1)+OGA(J))*.5
      DO 41 I=1,LAST
      DOMEG=DTAU*(1.-OGA(I))/ALPHA
      DIMEG=1000.*DOMEG
      DELP=OGA(I)*DTAU*0.980665/BETA
      IF(I-1) 42,42,43
   42 PBAR=DELP
      POZ=DIMEG/0.79
      PN=DELP
      PNB4=PN
      GO TO 99
   43 PBAR=(2.*PNB4+DELP)/2.
      PN=PN+DELP
      IF(PN) 7,7,8
    8 IF(PNB4) 7,7,9
    9 PPP=PN/PNB4
      PNB4=PN
      IF(I-1) 17,99,17
   99 P3BAR=POZ
      GO TO 7
   17 P3BAR=.55*DIMEG/  LOGF(PPP)
```

```
    7 OMGTL=OMGTL+D1MEG
      PRINT 52,I,OGA(I),OMEC(I),D1MEG,DELP,PBAR,P3BAR
   52 FORMAT(16X,I3,3X,1PE11.4,4XF11.4,2XF11.4,8X3F13.4)
      PR(I)=PBAR
      P3(I)=P3BAR
   41 CONTINUE
      PRINT 53, OMEG(NNN)
   53 FORMAT(37X1PE11.4)
      PRINT 60,OMGTL
   60 FORMAT (14H TOTAL OZONE =E11.3,9H M-ATM-CM)
      S=0.0
      DO 12 I=1,LAST
   12 S=S+OGA(I)
      PRINT 13,S
   13 FORMAT(17H SUM OF OMEGA'S =,F10.6)
      RETURN
      END

      SUBROUTINE INVER(B,NEQ,AINV)
C    ***SUBROUTINE INVERSE COMPUTES INVERSE MATRIX***
      DIMENSION A(10,10) ,B(10,10), AINV(10,10), ICHG(10)
      DO17I=1,NEQ
      DO15J=1,NEQ
      A(I,J)=B(I,J)
   15 AINV(I,J)=0.
      AINV(I,I)=1.
   17 ICHG(I)=1
      DET=1.
      NEQP1=NEQ
      DO30L=1,NEQ
      K=1
      DO24I=1,NEQ
      N1=ICHG(I)
      GO TO(18,24),N1
   18 GO TO (19,20),K
   19 C=ABSF(A(I,I))
      IR=I
      IC=I
   20 DO23J=1,NEQ
      N1=ICHG(J)
      GO TO(21,23),N1
   21 IF(C-ABSF(A(J,I)))22,23,23
   22 C=ABSF(A(J,I))
      IR=J
      IC=I
   23 CONTINUE
      K=2
   24 CONTINUE
      ICHG(IC)=2
      DO25I=1,NEQP1
      C=AINV(IR,I)
      AINV(IR,I)=AINV(IC,I)
```

```
      AINV(IC,I)=C
      C=A(IR,I)
      A(IR,I)=A(IC,I)
   25 A(IC,I)=C
      C=A(IC,IC)
      IF (IR-IC) 35,34,35
   35 DET =-DET
   34 DET =DET*C
      IF(DET)31,32,31
   32 CONTINUE
      JJJ=L-1
      PRINT 33, JJJ
   33 FORMAT(10X,25HMATRIX IS SINGULAR, RANK=   ,I3   )
      RETURN
   31 CONTINUE
      DO26I=1,NEQP1
      AINV(IC,I)=AINV(IC,I)/C
   26 A(IC,I)=A(IC,I)/C
      DO29I=1,NEQ
      IF(I-IC)27,29,27
   27 C=A(I,IC)
      DO28J=1,NEQP1
      AINV(I,J)=AINV(I,J)-C*AINV(IC,J)
   28 A(I,J)=A(I,J)-C*A(IC,J)
   29 CONTINUE
   30 CONTINUE
      RETURN
      END

      SUBROUTINE AVEI
C    ***INTERPOLATION SUBROUTINE FOR MIDPOINT INTENSITIES***
      DIMENSION  XMU(21),OMEG(22),SFI(12),AMU(20),ANU(20),
     1EFO(23),ESC(2,20),ESCM1(2,20),SCATI(120,20),NPHI(10),
     2EFO1(10),A(10,10),AT(10,10),H(10,10),GH(10,10),
     3AX(10,10),B(10,10),DMU(20),G(10,10),ATAGH(10,10)
      COMMON CST,DFI,DTAU,FIO,FMUO,FO,ICST,IP,LEVP1,LEVP2,
     1NLEVS,NMUM1,NSFI2,A,AMU,ANU,AT,ATAGH,AX,B,DMU,EFO,
     2G,GH,H,NPHI,OMEG,SCATI,SFI,XMU,NFI,NLEV2,ESC,ESCM1,
     3EFO1
      DO 1 K=1,9
      IK=NSFI2*(K-1)+1
      IKK=NSFI2*(K-1)+NSFI2
      DO 1 I=IK,IKK
      DO 1 J=1,10
    1 SCATI(I,J)=.5*(SCATI(I,J)+SCATI(I+12,J))
      IK=IKK+1
      IKK=IKK+NSFI2
      DO 2 I=IK,IKK
      DO 2 J=1,10
    2 SCATI(I,J)=.5*(SCATI(I,J))
      DO 3 K=1,9
      IK=ICST-(K*NSFI2)+1
```

```
      IKK=ICST-(K*NSFI2)+12
      DO 3 I=IK,IKK
      DO 3 J=11,20
    3 SCATI(I,J)=.5*(SCATI(I,J)+SCATI(I-12,J))
      DO 4 I=1,NSFI2
      DO 4 J=11,20
    4 SCATI(I,J)=.5*(SCATI(I,J))
      RETURN
      END

      SUBROUTINE MATRX
C    ***SUBROUTINE COMPUTES ELEMENTS FOR MATRIX A***
      DIMENSION  XMU(21),OMEG(22),SFI(12),AMU(20),ANU(20),
     1EFO(23),ESC(2,20),ESCM1(2,20),SCATI(120,20),NPHI(10),
     3AX(10,10),B(10,10),DMU(20),G(10,10),ATAGH(10,10),MU(10)
     4MU(10)
      COMMON CST,DFI,DTAU,FIO,FMUO,FO,ICST,IP,LEVP1,LEVP2,
     1NLEVS,NMUM1,NSFI2,A,AMU,ANU,AT,ATAGH,AX,B,DMU,EFO,
     2G,GH,H,NPHI,OMEG,SCATI,SFI,XMU,NFI,NLEV2,ESC,ESCM1,
     3EFO1,MU
      DO 1 J=1,NLEVS
      M=MU(J)
      I=NPHI(J)
      DO 1 LEV=LEVP1,NLEV2
      IF(LEV-LEVP1)3,2,3
    2 SENSE LIGHT 3
      GO TO 4
    3 IF(SENSE LIGHT 3)4,4
    4 LEVM1=LEV-1
      STHET=0.0
      THET =0.0
      THETO=(ANU(M)*SQRTF(1.-FMUO**2)*COSF(SFI(I)-FIO)+
     1AMU(M)*FMUO)**2
      LE=NLEV2-LEV+1
      FTHET=(1.+THETO)*FO*EFO1(LE)
      DO 48 K=1,NSFI2
      DFIS=SFI(K)-SFI(I)
      IF(SENSE LIGHT 3)85,86
   85 ILEV=ICST+K-NSFI2
      SENSE LIGHT 3
      GO TO 81
   86 ILEV=ICST-(LEVM1-NLEVS)*NSFI2+K
      LL=1
      LLL=10
      DO 49 L=LL,LLL
      THET=(ANU(M)*ANU(L)*COSF(DFIS)+AMU(M)*AMU(L))**2
   49 STHET=STHET+(1.+THET)*DMU(L)*DFI*SCATI(ILEV,L)
      ILEV=ICST-(LEV-NLEVS)*NSFI2+K
   81 LL=11
      LLL=NMUM1
      DO 31 L=LL,LLL
      THET=(ANU(M)*ANU(L)*COSF(DFIS)+AMU(M)*AMU(L))**2
```

```
31 STHET=STHET+(1.+THET)*DMU(L)*DFI*SCATI(ILEV,L)
48 CONTINUE
   NA=NLEV2-LEV+1
 1 A(J,NA)=(STHET+FTHET)*ESCM1(1,M)*CST*ESC(1,M)**(LE-1)
   RETURN
   END

   SUBROUTINE CHECK
C  ***SUBROUTINE COMPUTES NORMALIZED INTENSITIES***
   DIMENSION  XMU(21),OMEG(22),SFI(12),AMU(20),ANU(20),
  1EFO(23),ESC(2,20),ESCM1(2,20),SCATI(120,20),NPHI(10),
  2EFO1(10),A(10,10),AT(10,10),H(10,10),GH(10,10),
  3AX(10,10),B(10,10),DMU(20),G(10,10),ATAGH(10,10),
   COMMON CST,DFI,DTAU,FIO,FMUO,FO,ICST,IP,LEVP1,LEVP2,
  1NLEVS,NMUM1,NSFI2,A,AMU,ANU,AT,ATAGH,AX,B,DMU,EFO,
  2G,GH,H,NPHI,OMEG,SCATI,SFI,XMU,NFI,NLEV2,ESC,ESCM1,
  3EFO1
   TEN=NLEVS
   FOOUT=FO*EXPEF(-TEN  *DTAU/ABSF(FMUO))*FMUO
   PRINT 1,FOOUT
 1 FORMAT (//5X,38HFO OUT AT BOTTOM OF ATMOSPHERE (N)
  1,1PE16.7/)
   DNI=0.0
   DO 2 J=11,20
   DO 2 I=IP,ICST
 2 DNI=DNI+SCATI(I,J)*AMU (J)*DFI*DMU(J)
   PRINT 3,DNI
 3 FORMAT (5X,38HSUM OF DOWNWARD INTENSITIES NORMALIZED
  1,1PE16.7/)
   UPI=0.0
   DO 4 J=1,10
   DO 4 I=1,12
 4 UPI=UPI+SCATI(I,J)*AMU (J)*DFI*DMU(J)
   PRINT 5,UPI
 5 FORMAT (5X,38HSUM OF UPWARD INTENSITIES NORMALIZED
  1,1PE16.7/)
   SUMI=ABSF(FOOUT+DNI)+UPI
   PRINT 9,SUMI
 9 FORMAT (5X,38HSUM OF ALL INTENSITIES NORMALIZED
  1,1PE16.7/)
   RETURN
   END

   SUBROUTINE INT (NITR)
C  ***SUBROUTINE COMPUTES INTENSITIES REQD FOR MATRIX A***
   DIMENSION  XMU(21),OMEG(22),SFI(12),AMU(20),ANU(20),
  1EFO(23),ESC(2,20),ESCM1(2,20),SCATI(120,20),NPHI(10),
  2EFO1(10),A(10,10),AT(10,10),H(10,10),GH(10,10),
  3AX(10,10),B(10,10),DMU(20),G(10,10),ATAGH(10,10)
   COMMON CST,DFI,DTAU,FIO,FMUO,FO,ICST,IP,LEVP1,LEVP2,
  1NLEVS,NMUM1,NSFI2,A,AMU,ANU,AT,ATAGH,AX,B,DMU,EFO,
  2G,GH,H,NPHI,OMEG,SCATI,SFI,XMU,NFI,NLEV2,ESC,ESCM1,MU,
```

```
  3EF01
   ITR=1
   LEV=1
   DO 17 L=LEVP1,NLEV2
   IL=NLEV2-L+1
17 OMEG(L+1)=OMEG(IL)
19 LEVM1=LEV-1
   LEVM2=LEV-2
   IF(LEV-NLEVS)20,20,21
20 II=1
   III=NFI
   JJ=11
   JJJ=NMUM1
   GO TO 22
21 II=1
   III=NFI
   JJ=1
   JJJ=10
22 DO 29 I=II,III
   DO 29 J=JJ,JJJ
   STHET=0.0
   THET =0.0
   THETO=(ANU(J)*SQRTF(1.-FMUO**2)*COSF(SFI(I)-FIO)+
  1AMU(J)*FMUO)**2
   FTHET=(1.+THETO)*FO*EFO(LEV)
   IF(JJ-1)24,24,23
23 IF(ITR-1)51,50,51
50 SENSE LIGHT 2
51 IF(LEV-2)25,27,55
25 ITEMP=1
   NLEV=I
   IF(SENSE LIGHT 2)36,37
36 PTHET=(STHET+FTHET)*ESCM1(ITEMP,J)*CST
   SCATI(NLEV,J)=PTHET*OMEG(LEV)
   GO TO 74
37 SENSE LIGHT 1
   GO TO 55
27 ITEMP=2
   SENSE LIGHT 3
55 DO 28 K=1,NSFI2
   DFIS=  SFI(K)-SFI(I)
   IF(SENSE LIGHT 1)60,63
63 ILEV=LEVM2*NSFI2+K
   LL=11
   LLL=20
   DO 26 L=LL,LLL
   THET=(ANU(J)*ANU(L)*COSF(DFIS)+AMU(J)*AMU(L))**2
26 STHET=STHET+(1.+THET)*DMU(L)*DFI*SCATI(ILEV,L)
   IF(SENSE LIGHT 2)61,62
61 SENSE LIGHT 2
   GO TO 28
60 ILEV=K
```

```
      SENSE LIGHT 1
      GO TO 65
   62 ILEV=LEVM1*NSFI2+K
   65 LL=1
      LLL=10
      DO 30 L=LL,LLL
      THET=(ANU(J)*ANU(L)*COSF(DFIS)+AMU(J)*AMU(L))**2
   30 STHET=STHET+(1.+THET)*DMU(L)*DFI*SCATI(ILEV,L)
   28 CONTINUE
      PTHET=(STHET+FTHET)*ESCM1(ITEMP,J)*CST
      IF(SENSE LIGHT 1)70,72
   70 NLEV=I
   32 SCATI(NLEV,J)=PTHET*OMEG(LEV)
      GO TO 74
   72 NLEV=LEVM1*NSFI2+I
      IF(SENSE LIGHT 3)32,33
   33 LLEV=NLEV-NSFI2*2
      SCATI(NLEV,J)=SCATI(LLEV,J)*ESC(ITEMP,J)+PTHET*
     1OMEG(LEV)
      IF(SENSE LIGHT 2)74,74
   24 IF(LEV-LEVP2)80,82,89
   80 ITEMP=1
      SENSE LIGHT 3
      GO TO 84
   82 ITEMP=2
      SENSE LIGHT 1
      GO TO 84
   89 IF(LEV-NLEV2)84,83,83
   83 SENSE LIGHT 4
   84 DO 48 K=1,NSFI2
      DFIS=SFI(K)-SFI(I)
      IF(SENSE LIGHT 3)85,86
   85 ILEV=ICST+K-NSFI2
      SENSE LIGHT 3
      GO TO 81
   86 ILEV=ICST-(LEVM1-NLEVS)*NSFI2+K
      LL=1
      LLL=10
      DO 49 L=LL,LLL
      THET=(ANU(J)*ANU(L)*COSF(DFIS)+AMU(J)*AMU(L))**2
   49 STHET=STHET+(1.+THET)*DMU(L)*DFI*SCATI(ILEV,L)
      ILEV=ICST-(LEV-NLEVS)*NSFI2+K
   81 LL=11
      LLL=NMUM1
      DO 31 L=LL,LLL
      THET=(ANU(J)*ANU(L)*COSF(DFIS)+AMU(J)*AMU(L))**2
   31 STHET=STHET+(1.+THET)*DMU(L)*DFI*SCATI(ILEV,L)
   48 CONTINUE
      PTHET=(STHET+FTHET)*ESCM1(ITEMP,J)*CST
      IF(SENSE LIGHT 3)90,92
   90 NLEV=ICST-NSFI2+I
   91 SCATI(NLEV,J)=PTHET*OMEG(LEV)
```

```
      GO TO 74
   92 NLEV=ICST-(LEV-NLEVS)*NSFI2+1
      IF(SENSE LIGHT 1)91,96
   96 LLEV=NLEV+NSFI2*2
      SCATI(NLEV,J)=SCATI(LLEV,J)*ESC(ITEMP,J)+PTHET*
     1OMEG(LEV)
   74 GO TO(29,75,76,77,78,79,29),I
   75 IR=NLEV+10
      GO TO 69
   76 IR=NLEV+8
      GO TO 69
   77 IR=NLEV+6
      GO TO 69
   78 IR=NLEV+4
      GO TO 69
   79 IR=NLEV+2
   69 SCATI(IR,J)=SCATI(NLEV,J)
   29 CONTINUE
      IF(LEV-NLEVS)93,34,93
   34 PRINT 7,ITR
    7 FORMAT (5X,9HITERATION,I3,10X,20HINTENSITIES DOWNWARD
     1,/)
      PRINT 5
    5 FORMAT (1X,9HPHI =     0,14X,2H30,14X,2H60,14X,2H90,13X
     2,3H120,13X,
     13H150,13X,3H180,10X,3HAMU)
      IP=ICST-NSFI2+1
      IPP=ICST-NSFI2+NFI
      DO 35 J=11,NMUM1
   35 PRINT 6,(SCATI(I,J),I= IP,IPP),AMU(J)
    6 FORMAT (1P7E16.7,0PF8.3)
      GO TO 101
   93 IF(SENSE LIGHT 4)94,101
   94 PRINT 8
    8 FORMAT (//28X,18HINTENSITIES UPWARD,/ )
      PRINT 5
      DO 95 J=1,10
   95 PRINT 6,(SCATI(I,J),I=1,NFI),AMU(J)
      CALL CHECK
      IF(ITR-NITR)102,103,103
  102 ITR=ITR+1
      LEV=1
      PRINT 100
  100 FORMAT (1H1)
      GO TO 19
  101 LEV=LEV+1
      GO TO 19
  103 RETURN
      END

C     ***MAIN PROGRAM***
C     ***INITIALIZE STORAGE***
```

```
      DIMENSION  XMU(21),OMEG(22),SFI(12),AMU(20),ANU(20),
     1EFO(23),ESC(2,20),ESCM1(2,20),SCAT1(120,20),NPHI(10),
     2EFO1(10),A(10,10),AT(10,10),H(10,10),GH(10,10),
     3AX(10,10),B(10,10),GMA(10,10),ER(10),OMG(22),MU(10),
     4DMU(20),G(10,10),ATAGH(10,10)
      COMMON CST,DFI,DTAU,FIO,FMUO,FO,ICST,IP,LEVP1,LEVP2,
     1NLEVS,NMUM1,NSFI2,A,AMU,ANU,AT,ATAGH,AX,B,DMU,EFO,
     2G,GH,H,NPHI,OMEG,SCAT1,SFI,XMU,MU,NFI,NLEV2,ESC,ESCM1
     3,EFO1
C    ***READ ATMOS DATA-NBR OF ITER REQD FOR INTENSITIES***
      READ 1,NITR,NMU,NLEVS,NFI,PHI,DTAU,FO,FMUO,FIO
      READ 2,(XMU(I),I=1,NMU)
      LEVP1=NLEVS+1
    1 FORMAT(4I5,5F10.0)
    2 FORMAT(8F10.0)
C    ***COMPUTE AND DEFINE CONSTANTS***
      CST=3./(16.*3.1415927)
      DFI=PHI/57.295779
      NFIM1=NFI-1
      NSFI2=NFIM1*2
      DO 10 I=1,NSFI2
      XI=I-1
   10 SFI(I)=XI*DFI
      NMUM1=NMU-1
      DO 12 I=1,NMUM1
      AMU(I)=.5*(XMU(I)+XMU(I+1))
      ANU(I)=SQRTF(1.-AMU(I)**2)
   12 DMU(I)=XMU(I)-XMU(I+1)
C    ***COMPUTE ATTENUATION FACTORS***
      EFO(1)=EXPEF(-DTAU/(2.*ABSF(FMUO)))
      DO 14 K=2,NLEVS
      XK=K-1
   14 EFO(K)=EXPEF(-XK*DTAU/ABSF(FMUO))
      NLEV2=NLEVS*2
      LEVP2=NLEVS+2
      EFO(LEVP1)=EXPEF(-((2.*XK)+1.)*DTAU/(2.*ABSF(FMUO)))
      DO 15 L=LEVP1,NLEV2
      IL=NLEV2-L
   15 EFO(L+1)=EFO(IL+1)
      DO 16 I=1,2
      EX=I
      DO 16 J=1,NMUM1
      ESC(I,J)=EXPEF(-EX*DTAU/ABSF(AMU(J)))
   16 ESCM1(I,J)=1.-ESC(I,J)
      ICST=NLEVS*NSFI2
C    ***READ VALUES OF MEASURED INTENSITY***
      READ 3,(MU(I),I=1,NLEVS)
      READ 3,(NPHI(I),I=1,NLEVS)
    3 FORMAT(8I10)
      READ 2,(G(1,J),J=1,NLEVS)
      DO 104 I=1,NLEVS
      K=NPHI(I)
```

```
  104 G(K,I)=G(1,I)
      DO 105 I=1,NLEV2,2
      IE=I/2+1
  105 EFO1(IE)=EFO(1)**I
C     ***READ TRIAL SOLUTION VECTOR***
      READ 2,(OMEG(I),I=1,LEVP1)
      ITER=0
      L9=NLEVS
C     ***READ OZONE DATA-NBR ITER REQD ON SOLUTION VECTOR***
      READ 116,NITER,ALPHA,BETA,GAM
  116 FORMAT(I10,3F10.0)
C     ***READ MATRIX H***
      DO 108 J=1,L9
      READ 2,(H(J,I),I=1,L9)
      DO 108 I=1,L9
  108 GH(J,I)=GAM*H(J,I)
C     ***INTENSITIES SUBROUTINE W/NBR ITER REQD***
  122 CALL INT(NITR)
C     ***INTERPOLATION SUBROUTINE FOR MIDPT INTENSITIES***
      CALL AVEI
C     ***SUBROUTINE COMPUTES COEFFICIENTS FOR MATRIX A***
      CALL MATRX
C     ***PRINT MATRIX A***
      PRINT 4
    4 FORMAT(1H1,4X,8HA MATRIX/)
      DO 106 J=1,10
      PUNCH 103,(A(J,I),I=1,10)
  106 PRINT 107,(A(J,I),I=1,10)
  103 FORMAT (5E15.8)
  107 FORMAT(1X,1P5E16.7/17X,5E16.7/)
      OMEG1=0
      DO 20 I=1,NLEVS
      K=NPHI(I)
   20 GMA(K,I)=G(K,I)-A(I,10)*OMEG1
C     ***COMPUTE TRANSPOSE OF MATRIX A***
      DO 118 I=1,L9
      DO 118 J=1,NLEVS
  118 AT(I,J)=A(J,I)
  121 DO 109 K=1,L9
      DO 109 J=1,L9
      ATEMP=0
      DO 110 I=1,NLEVS
  110 ATEMP=ATEMP+AT(J,I)*A(I,K)
C     ***COMPUTE MATRIX TO BE INVERTED***
  109 ATAGH(J,K)=ATEMP+GH(J,K)
C     ***SUBROUTINE COMPUTES INVERSE OF MATRIX ATAGH***
      CALL INVER(ATAGH,L9,AX)
      DO 111 K=1,NLEVS
      DO 111 J=1,L9
      ATEMP=0
      DO 112 I=1,L9
  112 ATEMP=AX(J,I)*AT(I,K)+ATEMP
```

```
C     ***COMPUTE B=(ATA+(GAMMA)H)EXP(-1)AT***
  111 B(J,K)=ATEMP
      DO 113 J=1,L9
      ATEMP=0
      DO 114 I=1,NLEVS
      K=NPHI(I)
  114 ATEMP=B(J,I)*GMA(K,I)+ATEMP
C     ***COMPUTE SOLUTION VECTOR OMEGA***
  113 OMG ( J)=ATEMP
      SSER=0
      DO 124 I=1,L9
      K=NPHI(I)
      ER(I)=-GMA(K,I)
      DO 125 J=1,L9
  125 ER(I)=ER(I)+A(I,J)*OMG (J)
C     ***COMPUTE SUM OF SQUARES OF ERRORS***
  124 SSER=SSER+ER(I)**2
      ITER=ITER+1
C     ***SUBROUTINE FOR SOLUTION VECTOR OUTPUT***
      CALL OMAGA(DTAU,ALPHA,BETA,ITER,OMG ,NLEVS,OMEG)
C     ***PRINT GAMMA AND SUM OF SQUARES OF ERRORS***
      PRINT 5,GAM,SSER
    5 FORMAT (1X,7HGAMMA =,F10.8,
     13X,25HSUM OF SQUARES OF ERRORS = 1PE16.7//)
C     ***LAST ITERATION ON SOLUTION VECTOR***
      IF(ITER-NITER)122,120,120
  120 STOP
      END
```

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