OPEN-CHANNEL CONTRACTIONS
FOR SUBCRITICAL FLOW

by

Habib M. Yaziji

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This thesis has been approved on the date shown below:

Thomas Carmody
Thomas Carmody
Associate Professor of
Civil Engineering

September 29, 1967
Date

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LIST OF SYMBOLS

Loss coefficient	C_L
Depth of water	D
Gravitational acceleration	g
Energy loss per unit weight	h_L
Average velocity	V
Average velocity in the upstream portion of the channel	V_1
Average velocity in the downstream portion of the channel	V_2
Constant velocity along the free-streamline transition	V_0
Bottom elevation from a certain datum	Z

Subscript 1 refers to the approach flow

Subscript 2 refers to the downstream flow

ABSTRACT

A study of subcritical flow in a rectangular horizontal open channel was performed to determine efficiencies of various shapes of vertical-wall contractions for two width ratios, the criteria being minimum length, maximum degree of uniformity of downstream velocity distribution, and minimum loss of energy. Results are given in the forms of velocity distributions for typical shapes and energy-loss coefficients for all shapes studied. Results indicate that the abrupt transition causes a significantly greater energy loss than do those either partially or extensively streamlined, and they indicate also that under certain conditions the quarter-ellipse transition (1:4 ratio) causes the least energy loss of all shapes studied, including two based on free-streamline analysis of discharge from a slot orifice.

CHAPTER I

INTRODUCTION

Since man-made open channels are used extensively as a means of transporting water, and since local conditions often require that channel cross sections be altered from point to point, it behooves the hydraulic engineer to know what must be sacrificed in the way of space, fluid energy, and man's money in order to accomplish the various transitions at desired efficiencies. An ideal transition, of course, produces a normal velocity distribution in the downstream section at no loss of energy, occupies a minimum of space, and consists of the simplest geometric form. An actual transition is subject to the effects of boundary-layer growth, streamline separation, and, associated especially with the latter, the phenomena of turbulence production, diffusion, transportation, and decay, with the resulting local conditions of comparatively high shear and large local rate of energy loss. Contraction transitions can be designed and operated to yield extremely efficient results, both because a contracting stream tends to suppress boundary-layer growth and because a minimum of streamlining considerably reduces separation and its effects.

One might expect a form rather closely related to the shape of the free streamline for irrotational flow from a two-dimensional slot to produce a satisfactory downstream velocity distribution at minimum loss of energy, which is indeed the case if the parameters controlling the shape of the free-streamline curve are selected judiciously. In order to adapt these principles to the present study, which includes only symmetric contraction transitions conveying open-channel flow from one horizontal rectangular conduit to another without change in bottom elevation, let us consider Fig. 1, representing the discharge from an infinite slot at the end of a conduit of finite upstream width. The walls ab and a'b', shown here making equal angles Θ with the walls of the channel, are essential if there is to be any contraction. The maximum length of ab (and its twin a'b') is that yielding bb' equal to the downstream conduit width cc', for which case the curve bc is of zero length and the velocity along that "curve" is infinite. As the length ab is reduced, bb' increases, the length of the curve bc increases, and the constant-magnitude velocity along the curve is diminished. At section cc' the curve blends into the wall of the downstream rectangular conduit, and here, in general, the velocity at and near the wall is greater than the velocity in the downstream channel. Thus the velocity beyond c must decrease, and in a real flow the pressure would increase. This adverse pressure gradient could cause separation, loss of energy, and a flow not well described by the theory. The minimum length of ab is that yielding a velocity along bc equal to that of the uniform stream in the contracted channel, for which case the length of bc is infinite. All lengths ab less than this minimum yield free-streamline flows that do not contract sufficiently to satisfy the downstream geometric requirements, and thus no further consideration need be given them here.

In applying this theory to real open-channel flow from a vertical slot of finite rather than infinite depth, one must consider viscous

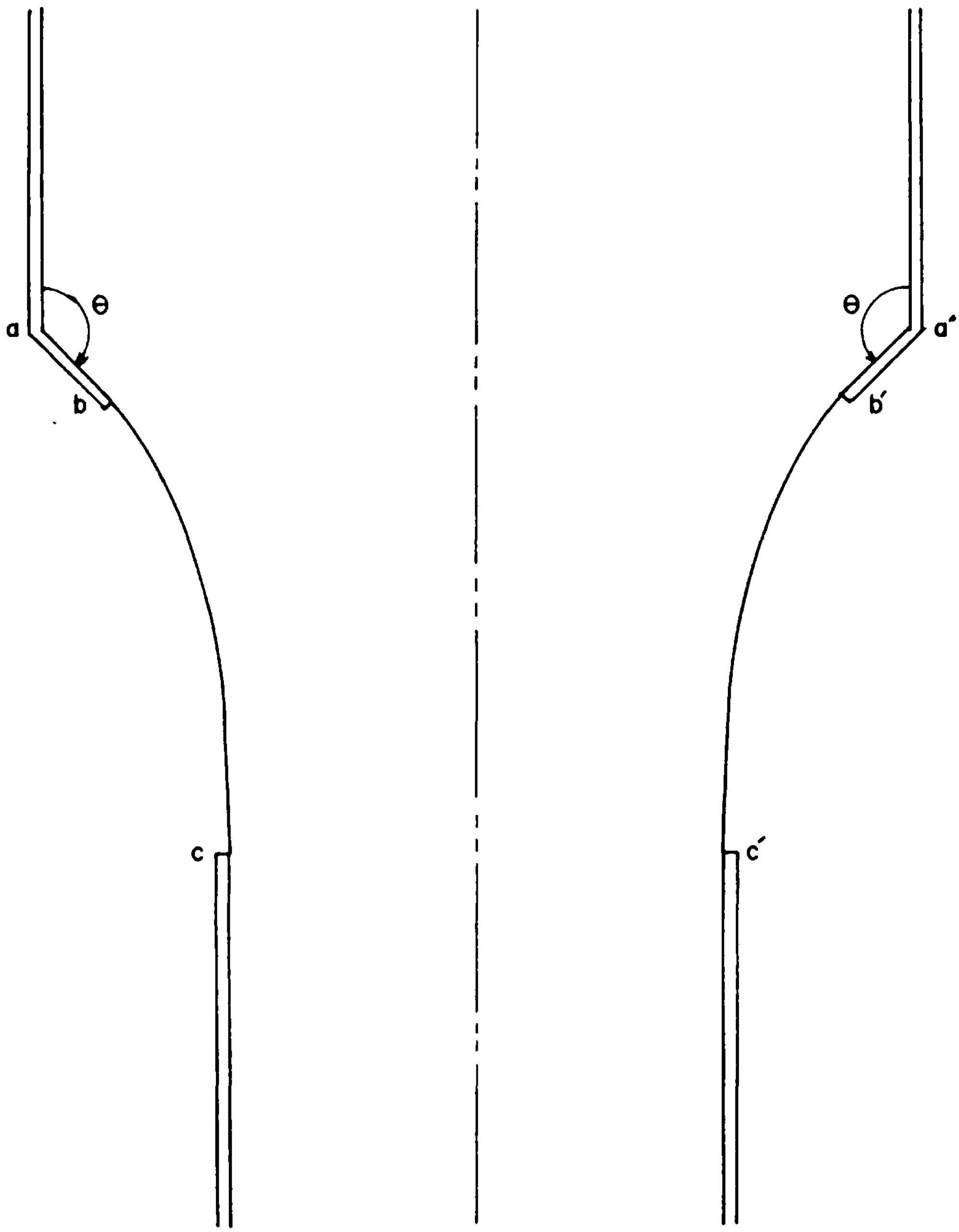

Figure 1. Discharge from a Slot

effects at the solid boundaries and gravity and energy effects at the free surface. If boundary conditions could be adjusted so that flow through a contraction were everywhere irrotational, a transition constructed along the free streamline generated by the minimum possible length ab as discussed above would convey fluid experiencing dissipation of energy by laminar shear only, especially at the boundary. In the downstream channel, however, there would be an adverse pressure gradient along the wall tending to induce separation. Unfortunately, the transition would be infinitely long. Since no such boundaries will ever exist, one need only adjust this hypothetical shape somewhat to account for the secondary effects, and the real transition produced might be expected to operate at minimum energy loss.

This procedure satisfies one of the three criteria mentioned above by which a transition is judged ideal, but it certainly does not satisfy those of minimum length and simplest geometric form. Since no one transition can possibly satisfy all three criteria, the present study was undertaken to determine the particular characteristics of various transitions of arbitrarily selected form. The contractions studied were of the following shapes, shown in Fig. 2 for the 1/2 width ratio.*

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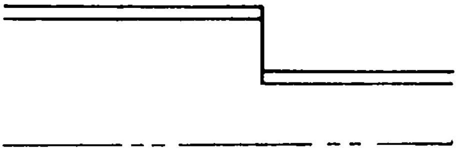
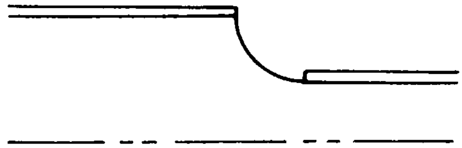
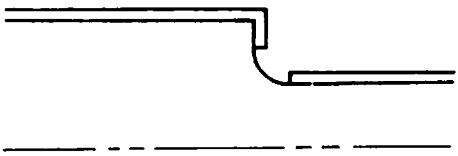
*From here on whenever "width ratio" is used it refers to the ratio of the width of the channel downstream from the contraction to the width of the channel upstream from the contraction.



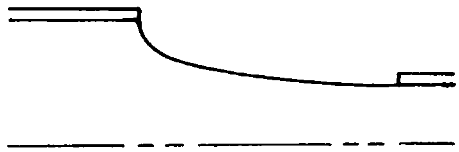
Abrupt



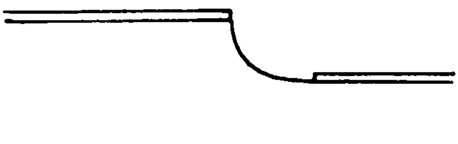
Quarter circle



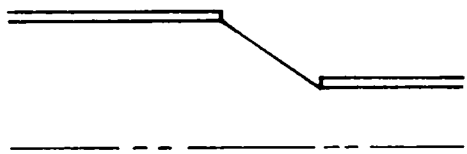
Quarter circle and tangent



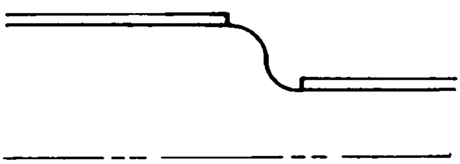
Quarter ellipse; axes ratio = 1/4



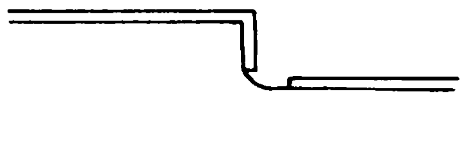
Quarter ellipse; axes ratio = 1/1.5



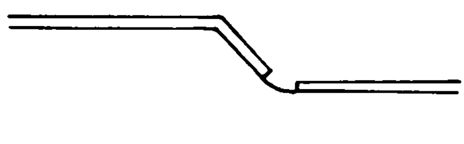
Straight line



Smooth S shape
(two equal circular arcs)



Free streamline shape, with
90° tangent and $\sigma = 0.4$



Free streamline shape, with
45° tangent and $\sigma = 0.4$

Figure 2. Transitions Selected for Study

Abrupt.

Quarter circle of radius equal to half the reduction in width.

Quarter circle of radius equal to half that of the above with a tangent perpendicular to the side wall of the upstream channel.

Quarter ellipse having its major axis parallel to the longitudinal axis of the channel and with axes ratio 1 to 4.

Quarter ellipse similar to the above except for the axes ratio which is 1 to 1-1/2.

Straight line with 1 to 2 inclination with the side wall.

Smooth S shape made of two equal quarter circles.

Free streamline with its upstream part connected to a straight line making 90 degrees with the channel side wall.

Free streamline with its upstream part connected to a straight line making 45 degrees with the channel side wall. (See appendix.)

Through inspection of Figure 2, one can appraise the variation in transition length and simplicity of form. The results of the study indicate rates of energy loss as functions of these shapes. Here it should be pointed out that for the 1:2 contraction, one quarter of a 1:4 ellipse is assumed to be that shape which approximates a desirable free-streamline contraction corrected qualitatively for boundary-layer effect. Figure 3 shows the quarter-ellipse and free-streamline ($\Theta = 90°$, $\sigma = 0.02$) curves superposed. One notices that the quarter ellipse contracts a bit more rapidly than the free-streamline curve, and thus it is taken as the "corrected" form, since the boundary layer developed in the free-streamline transition will tend to be compensated for in the elliptical transition. Of the shapes tested at this contraction ratio, this quarter ellipse exhibited the least energy loss.

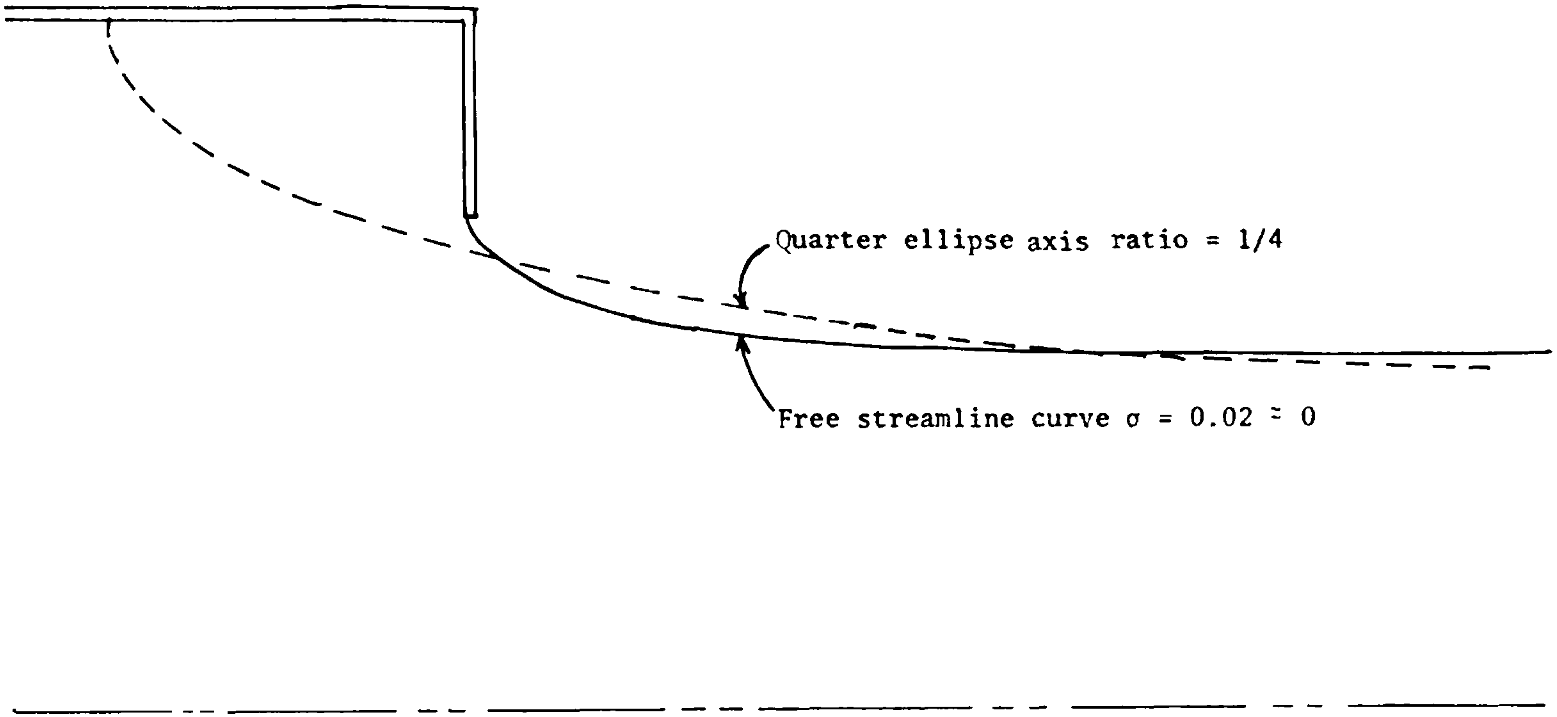


Figure 3. 1:4-Quarter-Ellipse and Free-streamline Curves Compared for 1:2 Contraction

Open-channel energy loss, considered as the difference between the total energies per unit weight upstream and downstream with respect to the contraction, may be expressed as follows:

$$h_L = (D_1 + Z_1 + \frac{V_1^2}{2g}) - (D_2 + Z_2 + \frac{V_2^2}{2g}) \qquad (1)$$

In this form, the velocity distribution at the two sections is considered to be uniform, or, the ratio of the average kinetic energy to the kinetic energy corresponding to the average velocity is taken as unity.

Loss may be expressed in terms of the difference between the downstream and upstream kinetic energies in the form

$$h_L = C_L \left(\frac{V_2^2}{2g} - \frac{V_1^2}{2g}\right) \qquad (2)$$

C_L is a nondimensional number called the loss coefficient, a function of boundary geometry, Froude Number, and Reynolds Number. Since the influence of boundary geometry is so much greater than the influence of the dynamic parameters, values of C_L are often listed as functions of boundary geometry only (2,3,4,5). A more convenient form of expressing the energy loss is in terms of the downstream kinetic energy only, as

$$h_L = C_L \frac{V_2^2}{2g} \qquad (3)$$

A part of the energy at any section is the kinetic energy, the magnitude of which depends on the velocity distribution. The velocity is considered to be uniform in the downstream channel and is determined by the continuity principle. This assumption is not expected to be true at the downstream end of a contraction where the velocity is higher near the boundary. In the case of the free-streamline shape contraction the theoretical boundary velocity is $1.18V_2$ for $\sigma = 0.4$. Using this value for the velocity at the boundary, assuming a linear velocity distribution, and satisfying continuity, one finds the kinetic energy equal to 1.026 of that of the uniform velocity distribution. In the case of this experiment, 0.026 of the kinetic energy is equal to 0.15 of the determined energy loss. The actual velocity distribution is expected to be a smooth curve which reduces the difference between the kinetic energies of the actual distribution and the uniform one.

In the case of the quarter circle contraction, the flow is considered irrotational, the circle theorem is applied for two circles -- a correct application if the two circles are very far apart -- and the velocity distribution curve is shifted parallel to itself to satisfy continuity. With the above approximations, the velocity along the boundary and along the centerline are $1.20V_2$ and $0.90V_2$ respectively and the kinetic energy is 1.028 of that of the uniform velocity. Due to the low value of the loss across the contraction, the excess kinetic energy (0.028 of the kinetic energy of a uniform flow) amounts to 50 per cent of the measured average energy loss.

A part of the kinetic energy due to the nonuniformity of the flow may be recovered as pressure head downstream where the surface elevation is measured to determine the total head. Another part is lost through internal shear during the adjustment of the velocity distribution, and this part is included in the loss due to the contraction. Since the energy dissipated due to the changing velocity distribution directly downstream from the contraction can not be determined from the measurements taken, the writer indicates that such loss is reported as a part of the loss due to the contraction and does not carry this subject any further.

The major goal of this project was to study different kinds of contractions in rectangular channels having two width ratios in order to determine their efficiencies. The scope was limited to the certain shapes of contractions which are mentioned above. In all cases the side walls were vertical. Also the flow was in the subcritical range, and the flow rate was limited to values such that reasonably accurate measurements could be taken. In all cases measurements were taken under steady-flow conditions.

In addition, velocity distribution in the channel downstream and away from the contraction was measured. This was done to find if the effect of the nonuniformity of the flow was negligible, as assumed. Also, the mean elevation of the water-surface profile in the transition area was measured for a few cases.

CHAPTER II

LITERATURE REVIEW

E. W. Lane[2], in 1915, conducted some experiments related to the subject at hand. He expressed the efficiency of certain contracting transitions in open channel in terms of d'Aubuisson* coefficient, C_{dA}, which is defined by the following equation:

$$V_2 = C_{dA}\sqrt{2gH_2}$$

where $$H_2 = \frac{V_2^{\,2}}{2g} + h_L$$

From the above two equations:

$$h_L = \frac{1 - C^2_{dA}}{C^2_{dA}} \; \frac{V_2^{\,2}}{2g}$$

According to the definition of loss coefficient, C_L, used in this thesis, the following relationship holds:

$$C_L = \frac{1 - C^2_{dA}}{C^2_{dA}}$$

For the width ratio of 1/6.75, Lane's values of loss coefficients for a quarter-circle contraction varied from a negative 0.2 to positive

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*D'Aubuisson de Voisins, a French scientist, who published in hydraulics, hydrology, and other related subjects, died in Toulouse, 1841.

0.2, with an average of +0.13. For an abrupt contraction, the transition coefficients varied from 0.27 to 0.51, with an average of 0.38.

Hinds[5] gave some relevant results from tests which consisted of measuring transition losses of existing structures. Some contractions were straight lines making different angles with the centerline, and the rest were smooth curves of different shapes. In several cases the losses appeared to be negative. Considering the negative losses as zero, the average loss coefficient was 0.04. Thirty per cent of the coefficients were greater than 0.1 and only ten per cent were greater than 0.14. None of the contractions were abrupt.

Scobey[3], in determining loss coefficients, included the effect of velocity head in the upstream channel. He gave the coefficients for different shapes of contractions without giving the width ratio. The following values of loss coefficients as defined in Eq. 2 were given:

Abrupt	0.20
Straight line making 30° to 40° with centerline	0.15
Quarter circle	0.10
Warped contractions with reversed parabolas	0.10

According to Scobey, in the last two cases the entrance loss, other than friction, comes within the zone of error.

Besides giving results of older experiments which were similar to those conducted in this project, the writer desires to indicate the poor accuracy which can be expected. Energy cannot be created at a transition, and so the only conceivable major reason for the negative values in Lane's results are errors in measurements, and as he mentioned, "A slight error in the observation of the water level would result in considerable error in the coefficient."

CHAPTER III

EXPERIMENT

The apparatus used in the present study consisted of two open storage tanks, a rectangular wooden test channel, two pumps, valves, metering devices, and necessary pipes. Water flowed by gravity from the higher tank, through the channel, into the lower one. It was pumped from the lower tank through two separate lines to the upper tank for recirculation.

The channel was 10 feet long, 3 feet wide, and about 18 inches deep. The downstream portion was converted into a narrower one where boards were used to form movable side walls within the original ones, resting on the same bottom. The centerline of the downstream channel was along the centerline of the original one. Experiments were conducted for the downstream channel's widths of 18 inches and 4 inches. Transition sections were constructed of sheet metal.

An adjustable overflow gate was constructed across the downstream end to control the water depth in the channel. To measure the bottom and water-surface elevations, five fixed point gages were set along the centerline of the channel. The gages were connected to steel angles lying across the channel and screwed into its side walls at the locations shown in Figure 4.

One of the recirculation lines contained a conventional elbow meter. In the other, two piezometric taps located 27 pipe diameters

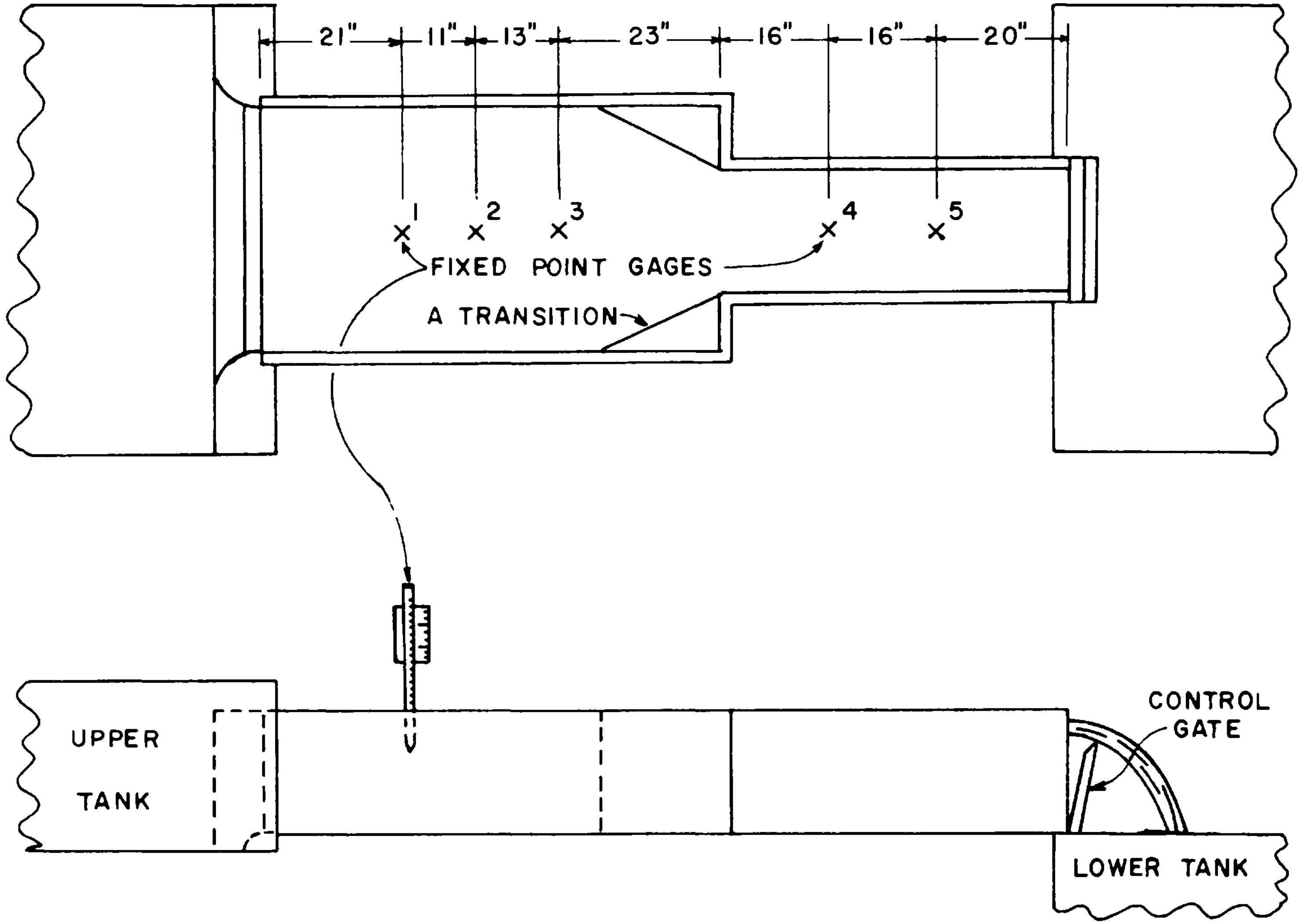


Figure 4. Plan and Side Views of the Experimental Set-up

apart, with an elbow between, were used to measure a head loss that was proportional to the flow rate. A sloping water-mercury manometer was attached to the elbow meter, and a vertical air-water manometer was attached to the other assembly. Flow over a sharp-crested weir in the original channel was used as the standard for calibrating these devices.

The procedure was to open the valves of recirculation lines to get a certain discharge rate. By lowering or raising the adjustable gate, different water depths in the channel were obtained. For each depth, referred to as a run, the water-surface elevation was measured using the fixed gages. To find the energy loss, the total energy at each gage location was calculated. The average energy of the first three was considered as the energy upstream from the contraction, and the average of the last two was taken as the energy downstream from the contraction. These averages were used because the water surface was not sufficiently smooth to rely upon one reading at one point for each reach. The loss coefficient for each run was calculated, and the average of these is reported as the loss coefficient for a certain contraction.

A point gage connected to a steel angle which could be moved in the upstream and downstream direction as well as transversely between the two side walls was used to measure water-surface elevation in the transition area. A conventional Pitot tube was used to measure the velocity distribution in the downstream channel.

The accuracy in construction and reading was as follows: the deviation from the accurate trace drawn on the bottom did not exceed one sixteenth of an inch for the simple transitions such as the straight line and one eighth of an inch, or less than 3 per cent of the least radius of curvature, for the more complicated transitions such as the S shape. The point gages had divisions of one hundredth of a foot, and the thousandths were read from a vernier. The major source of error in elevation readings is believed to be caused by the fluctuation of the water surface, which was noticed to be maximum in the high ranges of velocity.

CHAPTER IV

RESULTS

The ratio of energy loss per unit weight across the transition to downstream mean-velocity head varied from 0.04 to 0.74. Excluding the abrupt transistions, the variation was between 0.04 and 0.2 for the width ratio of 1/2 and between 0.3 and 0.45 for the width ratio of 1/9. These results, as shown in Figures 5 and 6, were the average of the loss coefficients of the several runs, which varied in number from 7 to 22, for each contraction. Reporting results of single runs was believed to be of little value because of their large variation due to the small magnitude of energy loss compared to water surface fluctuation (see literature review).

The energy loss has a definite tendency to increase with increasing Froude number while the loss coefficients seem to be unchanged within the range of the measured velocities. The values of the coefficients resulting from varying the Froude number are shown in Figure 7 for two typical contractions. Because of the seemingly random appearance of the results, no attempt was made to establish a functional relationship between loss coefficient and Froude Number based on these data.

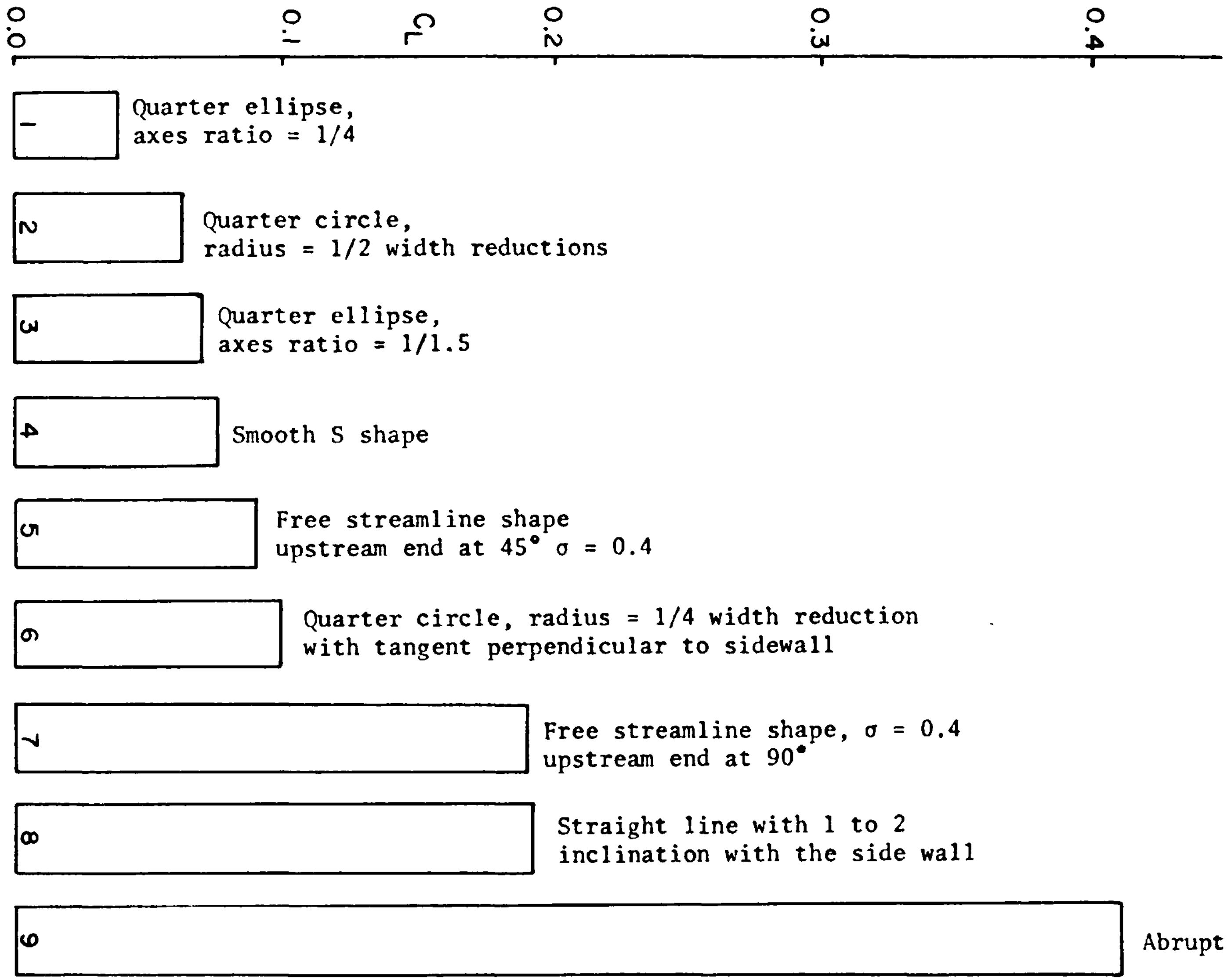


Figure 5. Loss Coefficients for Width Ratio of 1/2

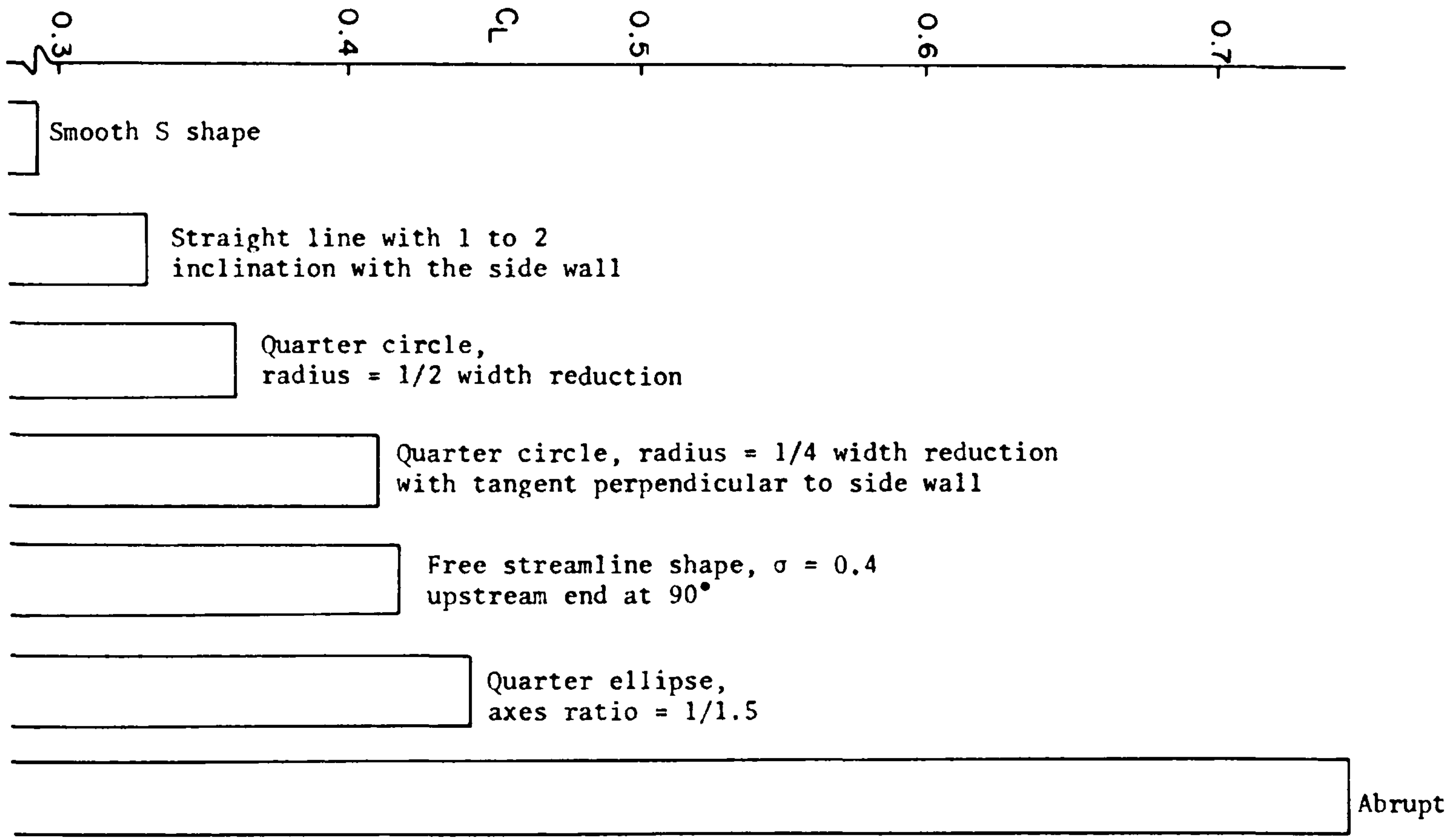


Figure 6. Loss Coefficients for Width Ratio of 1/9

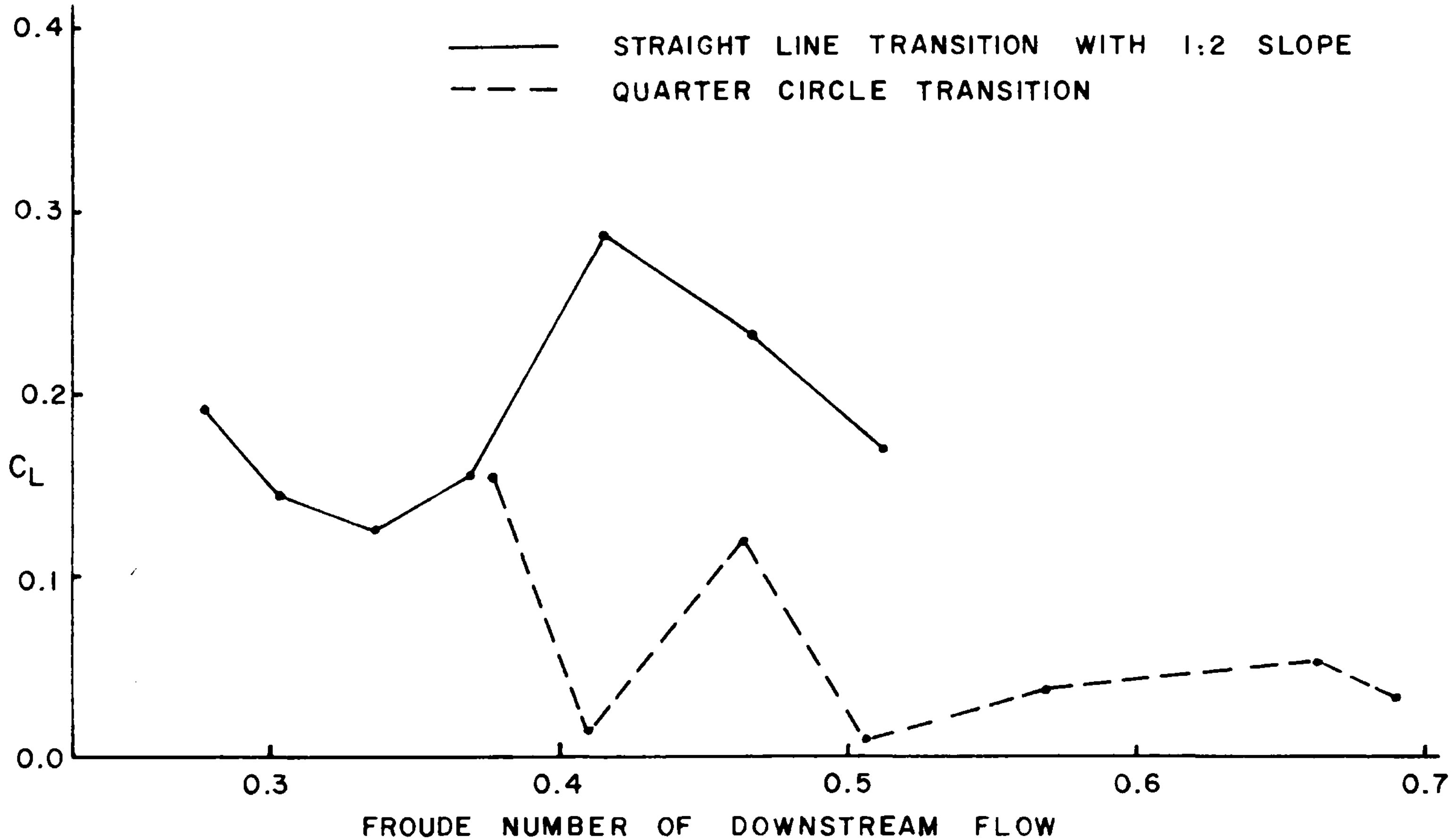


Figure 7. C_L vs. F_2 for Two Typical Contractions at Width Ratio of 1/2

Discussion of Results

The rate of loss of energy varies at a contraction in proportion to the turbulence generated, and so the transition which causes the maximum amount of turbulence is expected to have the highest loss coefficient. When streamlines are forced to change their direction, turbulence may occur. When the change is gradual, as along smooth curves, and the velocity is low, very little turbulence is generated. On the other hand, when the change in the flow direction is sudden, as around sharp corners, and at high velocity, the turbulence is high. It can be seen that the amount of turbulence and, consequently, the loss of energy depend on many factors. For the same shape of contraction the head loss increases with increasing values of Froude number due to the increase in laminar shear stresses and the shear stresses caused by increased turbulence. However, the results indicate that the loss coefficient does not change systematically with the Froude number; such differences as were measured were probably misleading and a result of the fixed measuring stations. Thus it is the shape of the contraction rather than the dynamic parameters that play the major role. The ratio of the length of a contraction to its width may give an idea about its efficiency when compared with similar curves as two quarter-ellipse contractions with different axes ratios. On the other hand a contraction with 90 percent of its width being normal to the flow and the rest is a straight line forming small angle with the centerline may be very long and still very inefficient.

The results for the width ratio of ½ shown in Figure 5, where the different transitions are numbered according to increasing values of mean loss coefficients, seem to agree with the above argument. By examining Figure 8, one can see that the contractions numbered 1 to 4 are

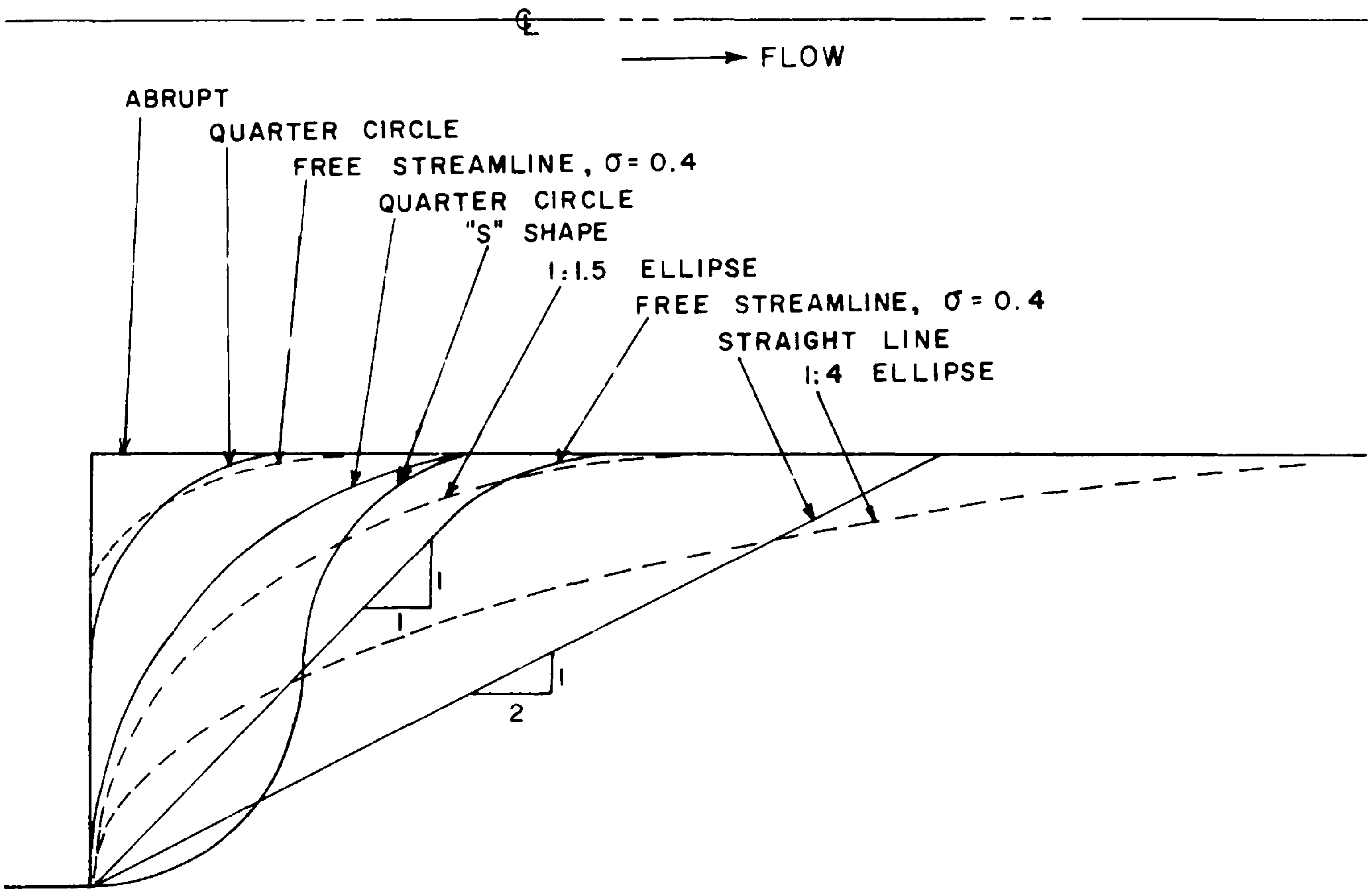


Figure 8. Relative Shapes of Different Contractions Tested

comparatively smoothly curved with gradual change of direction. Contractions 5 and 6 have more of a sudden change at the upstream end. Contraction 7 has a sudden change in direction at the upstream end and a large portion which is perpendicular to the approach flow. Contraction 8 has sudden change of direction at both the upstream and downstream ends. Contraction 9 is abrupt with a sharp corner, and the whole reduction in width is perpendicular to the approach flow.

The other important factor, though not the only one, influencing the amount of energy loss is the transition length. Shorter transitions are expected, in general, to have higher coefficients. For the width ratio of 1/2 the ratios of the transition length to the width of the downstream channel in order of increasing coefficients are as follows:

Transition Number (According to Increasing Coefficients)	Length Ratio	Loss Coefficient
1	2.00	0.04
2	0.50	0.06
3	0.75	0.07
4	0.50	0.07
5	0.66	0.08
6	0.25	0.10
7	0.34	0.18
8	1.00	0.18
9	0.00	0.41

It is clear that the length criterion does not stand alone in judging a contraction efficiency and, as an example the 6th and 7th transitions are to be considered. The 7th transition has a length ratio of 0.34 and has a higher coefficient than the 6th, the length ratio of which is 0.25. By examining the shapes of the two transitions more closely, one can see that the longer transition has a straight line portion, perpendicular to the flow direction, the length ratio of which is 0.36, while the shorter transition has a similar portion with a length ratio 0.25. Another obvious failure of the length criterion is the position of the 8th transition. This transition has two sudden changes in direction, as mentioned before, and it seems logical to have more turbulence caused by this than by a smooth change over shorter distances.

For the width ratio of 1/9 great reduction in depth was noticed through the transition and undulations observed on the surface of the downstream channel. This leads to more energy loss due to the three-dimensional effect which is independent of the form of the contraction, and consequently high values of loss coefficients are expected. The undulations render accurate measurements of the water depth extremely difficult, which adds to the inaccuracy of the results. Because of all this, it seems more realistic to classify the contractions into abrupt and nonabrupt with distinct ranges of values. Subdividing the nonabrupt contractions into groups as was done with the width ratio of 1/2 shows that the contractions do not fall in the same sequence when grouped according to increasing coefficients. The disagreement, however, is minor except in the case of the straight-line and the quarter-ellipse contractions.

The above discussion refers to the general shapes of the contractions. It is felt that more justification is required for the free-streamline shape. Theoretically, there is no pressure and, consequently, no velocity change along the free-streamline part of the contraction. This condition is expected to exist if the flow were irrotational and two-dimensional. In the actual flow the free water surface was noticed to drop at the transition section in inverse proportion to the contraction ratio, and this makes the flow three-dimensional. Thus the limited depth of the water and the boundary layer which may exist along the bottom cause the flow to be three-dimensional.

The shape and position of the free-streamline curve depends - for a certain channel - on the σ value, defined as $(\frac{Vo}{V2})^2 - 1$, which may vary from infinity to zero. It was found that when σ goes to infinity the free-streamline curve coincides with an abrupt change in width. For $\sigma = 0$, the free-streamline curve is found to be similar to quarter ellipse with axes ratio of 2 to 1 with the upstream nose cut off, which makes the energy loss increase.

The free-streamline transition tested had a value for σ of 0.4 and, from Figure 8, it can be seen that the transition which approximates it most is that of the quarter circle shape with radius equal to 1/4 of the width reduction. The free-streamline transition has the higher coefficient although it is longer. However, the effect of the free-streamline contraction length is counteracted by the greater length of the straight line normal to the flow and the higher curvature of the upstream end.

CHAPTER V

CONCLUSION

When the amount of energy loss is not a critical factor in the design and the transition is composed of erosion-resistant material, the choice of a shape for a contraction - excluding the abrupt - may depend on the cost of construction. For most actual field situations and when the energy loss is in consideration, the contractions tested can be grouped as abrupt and nonabrupt.

The loss coefficients seem to depend on the width ratio as well as on the shape of the contraction. The abrupt contraction, which is the simplest, obviously, has the highest coefficient. The quarter circle contraction seems to be the best when energy loss, space, and geometric simplicity are considered simultaneously. When energy loss is critical, more study is needed to choose the contraction shape which is most suitable for the required width ratio.

APPENDIX

Some measurements and calculations which are not directly related to the determination of the loss coefficients will be included in this section. These subjects are divided into three parts and are presented in the following sequence:

A. Velocity distribution.

B. Water surface profile in the transition region.

C. Derivation of the free-streamline curve.

APPENDIX A

VELOCITY DISTRIBUTION

As mentioned before, the velocity distribution in the rectangular channels is assumed to be uniform. To check the above assumption, to get an idea about the actual velocity distribution, and to check the accuracy of the discharge measurements, the velocity across the downstream channel some distance from the transition was measured. The results are shown graphically on the following two pages for two typical cases.

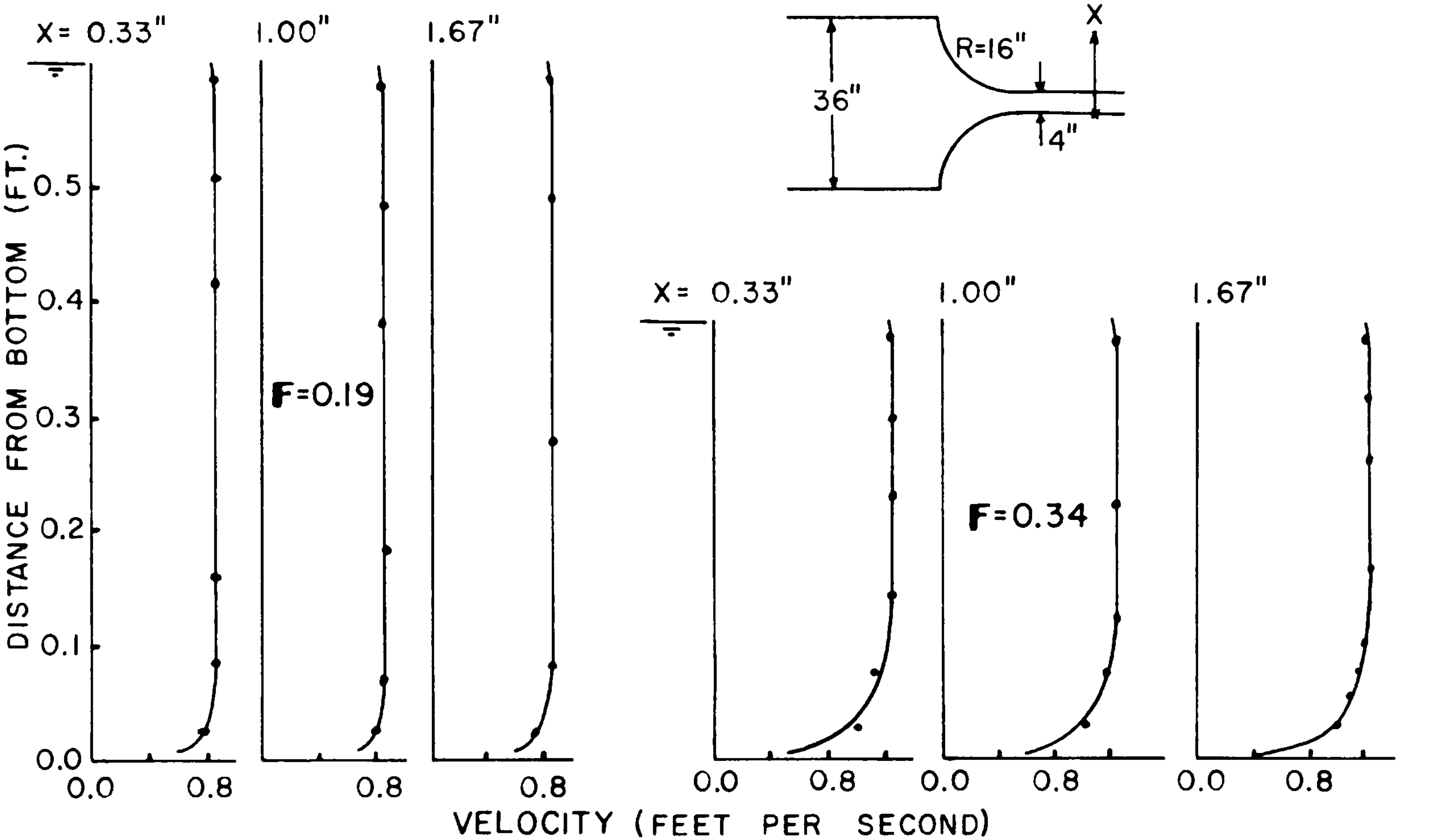


Figure 9. Velocity Distribution for 16" Quarter-circle Transition for Two Depths of Flow

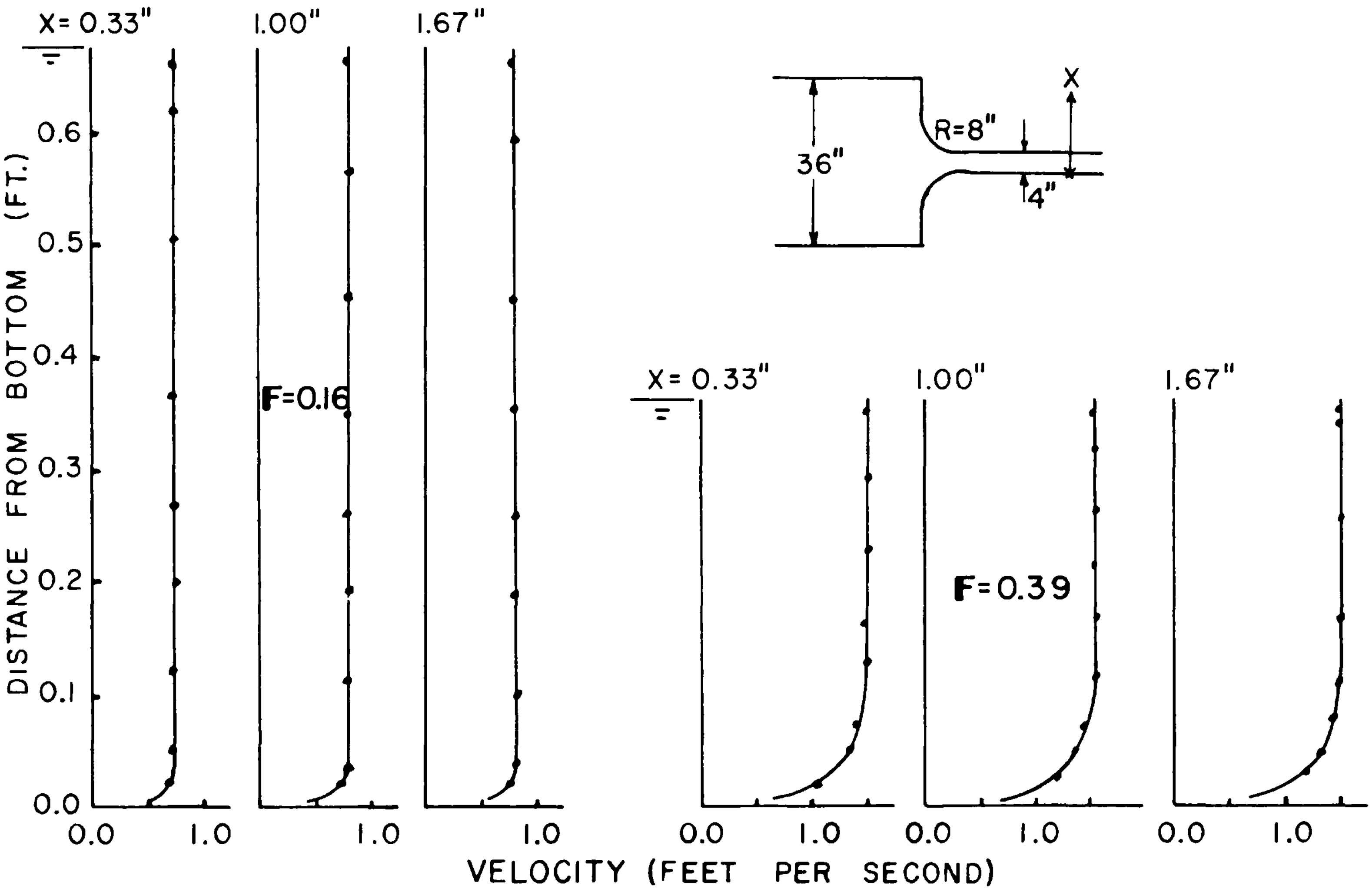


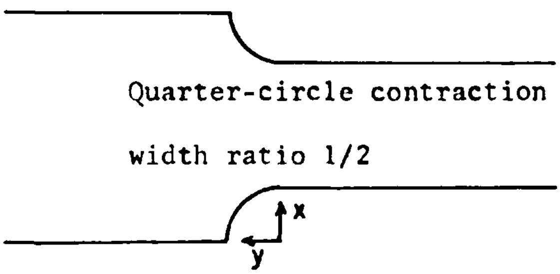
Figure 10. Velocity Distribution for 8" Quarter-circle Transition for Two Depths of Flow

APPENDIX B

WATER SURFACE PROFILE IN THE TRANSITION REGION

The water surface profile was found at the transition area for a few runs. The results of a typical case are given below in tabular form and are plotted on the following page.

x (inches)	Depth* expressed as a fraction of the average depth at upstream channel at:			
	y=o"	y=3"	y=6"	y=9"
35.5	---	---	---	1.000
30.0	---	---	---	1.012
28.5	---	---	0.990	---
27.0	0.969	0.971	---	---
24.0	0.979	0.985	0.995	1.005
18.0	0.985	0.995	0.996	1.000



Quarter-circle contraction

width ratio 1/2

*Elevation above the bottom, which is considered as horizontal in this region.

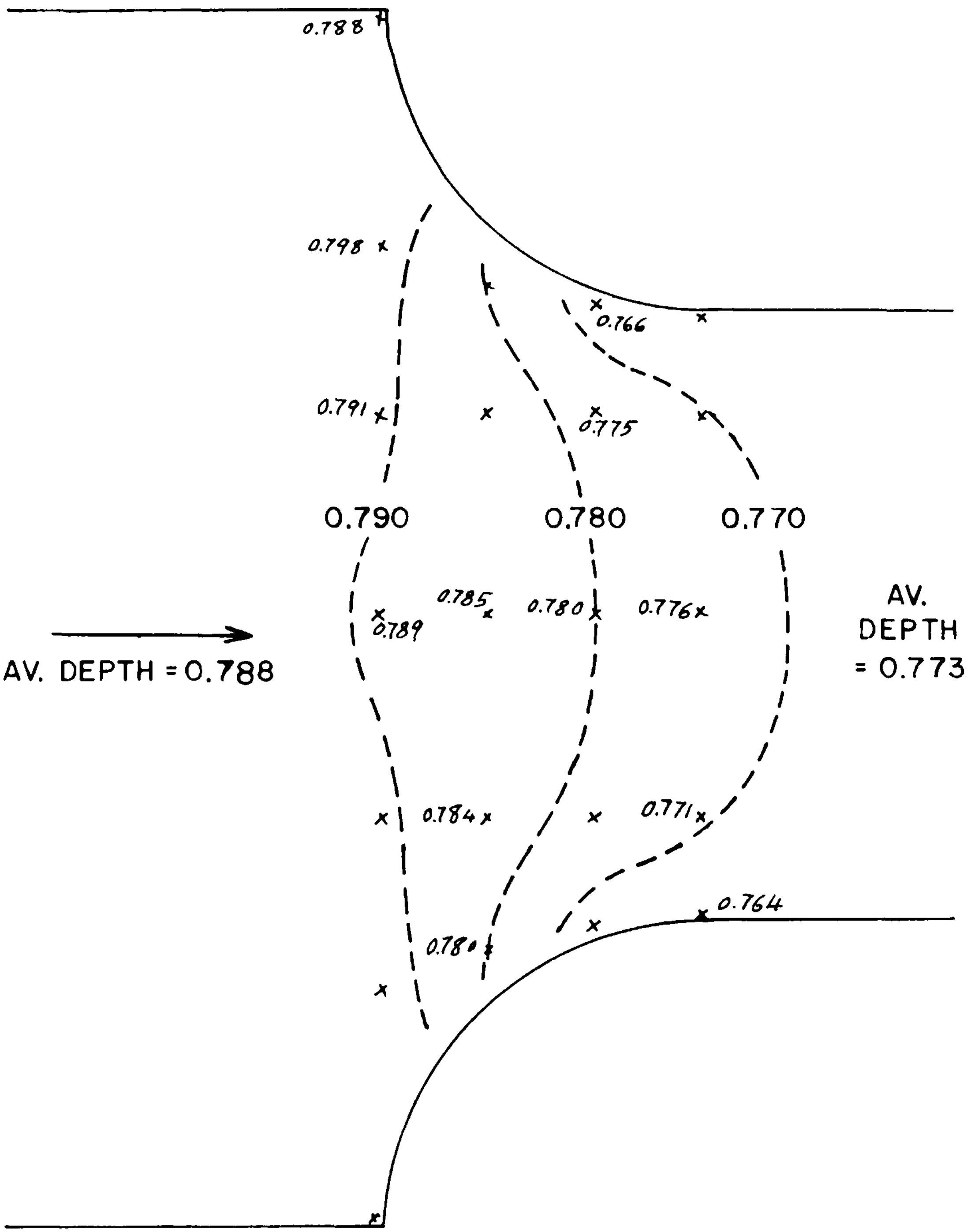


Figure 11. Water Surface Contours at Quarter-circle Transition (Depth in feet)

APPENDIX C

DERIVATION OF THE FREE-STREAMLINE CURVE

It was decided to include two types of contractions of the free-streamline shape. The first has a tangent making 90° with the channel sidewall and its equation is derived in Reference 1. The tangent of the second makes 45° with the channel sidewall and the derivation of its equation is given below following the same steps as in the above mentioned reference.

Figure 12 shows the actual shape of the channel in the Z plane and this mapped into the three other planes as explained below.

Ω Plane

Ω is defined as $\ln(-V_0 \frac{dz}{dw})$

so $$\Omega = \ln\left(-\frac{V_0}{\frac{dw}{dz}}\right) = \ln\left(\frac{V_0}{qe^{-i\Theta}}\right) = \ln\frac{V_0}{q} + i\Theta = -\ln\frac{q}{V_0} + i\Theta$$

where $\frac{dw}{dz} = -u + iv$

and u and v are the velocity components. Using these definitions, the axes of the Ω plane are $-\ln\frac{q}{V_0}$ and Θ, and the points A, B, C, D in the Z plane correspond to A, B, C, D in the Ω plane.

W Plane

$W = \phi + i\psi$

Where ϕ is the velocity potential and ψ is the stream function. Taking

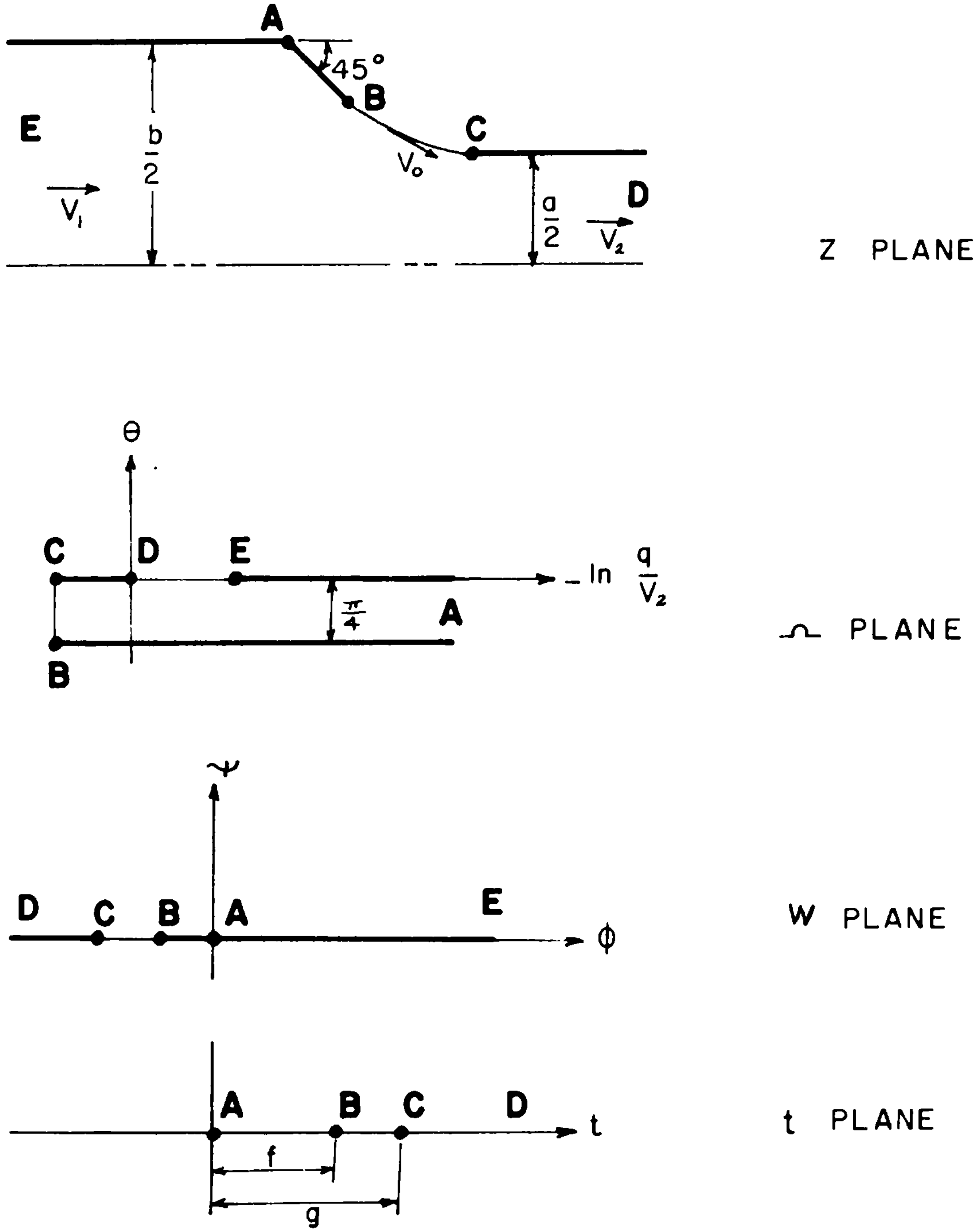


Figure 12. Conformal-mapping Planes

the boundary of the channel as the streamline $\psi_{\bullet} = 0$ along which the velocity potential ϕ changes from ∞ to $-\infty$ with zero value at A, the points A, B, C, D fall as shown in the W plane.

<u>t Plane</u>

To locate the points in the t plane, the Schwarz-Christoffel transformation is used. As in Reference 1, D is assigned at infinity and A at zero. Also f and g are related by the parametric equations

$$f = \frac{m}{m+1}, \quad g = \frac{m}{m-1}$$

Schwarz-Christoffel transformation is:

$$\frac{d\Omega}{dt} = (t-t_1)^{\frac{-B_1}{\pi}} (t-t_2)^{\frac{-B_2}{\pi}} (t-t_3)^{\frac{-B_3}{\pi}}$$

where t_1, t_2, t_3,.... are the locations of the vertices of an Ω plane polygon in the t plane and B_1, B_2, B_3,.... are the exterior angles of the polygon. For the case at hand, after integration and evaluation of the constants, the result is

$$\Omega = -\ln \frac{V_0}{V_2} + \frac{i}{4} \cos^{-1} \left[\frac{m(t-1)}{t}\right] \qquad (1)$$

From the values of t and Ω at point D, it is found that

$$m = \frac{1}{2} \left[\left(\frac{V_0}{V_2}\right)^4 + \left(\frac{V_2}{V_0}\right)^4 \right]$$

Mapping the w plane onto the t plane and taking the distance of point E as h, the following equation is obtained:

$$w = - \frac{V_2 a}{2\pi} \ln (t+h)$$

Evaluating Eq. 1 at point E, the following result is obtained

$$\ln \left(\frac{V_0}{V_1}\right) = \frac{i}{4} \cos^{-1} \left[\frac{m(h+1)}{h}\right]$$

and from this it can be found that

$$h = \frac{m}{n-m}$$

where
$$n = \frac{1}{2} \left[\left(\frac{V_1}{V_0}\right)^4 + \left(\frac{V_0}{V_1}\right)^4\right]$$

Using Eq. 1, and the equation

$$\Omega = -\ln \frac{V_0}{V_2} + i\Theta$$

The following result can be reached

$$t = \frac{m}{n-\cos 4\Theta} \cdot$$

By substituting for t and h in Eq. 2, w is found to be

$$w = -\frac{V_2 a}{2\pi} \ln \left[\frac{m}{n-\cos 4\Theta} + \frac{m}{n-m}\right] \quad (3)$$

Choosing the streamline $\psi_\circ = 0$ and using Eq. 3, the following equation is obtained

$$\phi = -\frac{V_2 a}{2\pi} \ln \left[\frac{m}{n-\cos 4\Theta} + \frac{m}{n-m}\right]$$

Knowing
$$\frac{\partial \phi}{\partial S} = -V_0$$

or
$$S = \frac{1}{V_0} \phi$$

then the distance from B to a point along the boundary where the angle is Θ will be

$$S = \frac{V_2 a}{2\pi} \ln \left[\frac{m}{n-\cos 4\Theta} + \frac{m}{n-m}\right]$$

Using $\qquad dx = ds \cos\Theta$

and $\qquad dy = ds \sin\Theta$

the final required equations are

$$\frac{dx}{d\Theta} = -\frac{2aV_2}{\pi V_0} \frac{(n-m)\sin 4\Theta \cos\Theta}{(n-\cos 4\Theta)(m-\cos 4\Theta)} \qquad (4)$$

and

$$\frac{dy}{d\Theta} = -\frac{2aV_2}{\pi V_0} \frac{(n-m)\sin 4\Theta \sin\Theta}{(n-\cos 4\Theta)(m-\cos 4\Theta)} \qquad (5)$$

To find the free-streamline curve, Eqs. 4 and 5 are solved by the numerical method using the equation

$$y(x+\Delta) = y(x) + \Delta y'(x) + \frac{\Delta^2}{2} y''(x) + \text{---------} \qquad (6)$$

and the coordinates of different points along the curve are found as outlined below.

Taking the pressure parameter as 0.4, the following ratios of the velocities, for the case of width ratio of 1/2, are found as

$$\frac{V_0}{V_2} = 1.18 \text{ and } \frac{V_1}{V_0} = 0.59$$

Using the above values it is found that

$$m = 1.226$$

$$n = 4.16$$

and $$a = 18''$$

The first approximation of Eq. 6 leads to

$$\chi_{n+1} = \chi_n + \Delta_\Theta \left(\frac{dx}{d\Theta}\right)_n \quad (7)$$

and

$$Y_{n+1} = Y_n + \Delta_\Theta \left(\frac{dy}{d\Theta}\right)_n \quad (8)$$

Θ varies between $-\frac{\pi}{8}$ and 0. Δ_Θ is taken as 5 degrees or $\frac{\pi}{36}$.

From Eqs. 7 and 8, the following values of χ and y corresponding to each value of Θ are found.

n	Θ	x	y
1	-45°	0	10.08
2	-40°	0	10.08
3	-35°	0.06	10.03
4	-30°	0.17	9.95
5	-25°	0.40	9.82
6	-20°	0.76	9.65
7	-15°	1.30	9.46
8	-10°	2.08	9.26
9	- 5°	3.08	9.08
10	0	3.99	9.00

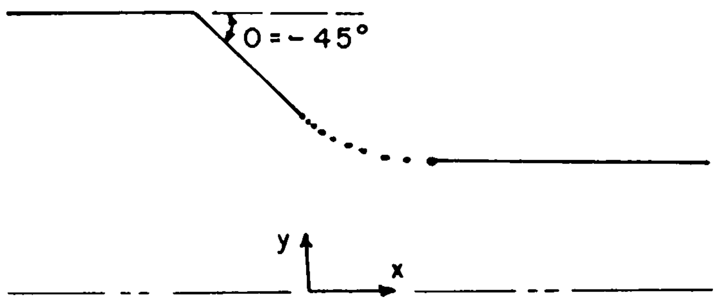


Figure 13. Plot of the Free-streamline Curve ($\sigma = 0.4$ and Tangent Angle = 45°)

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