AN AIRCRAFT POWER-CONTROL OPTIMALIZING COMPUTER

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This thesis has been approved on the date shown below:

Gordon M. Russell

8 May 1958

Date

TABLE OF CONTENTS

TABLE OF ILLUSTRATIONS

Table of Illustrations (cont'd)

Chapter 1

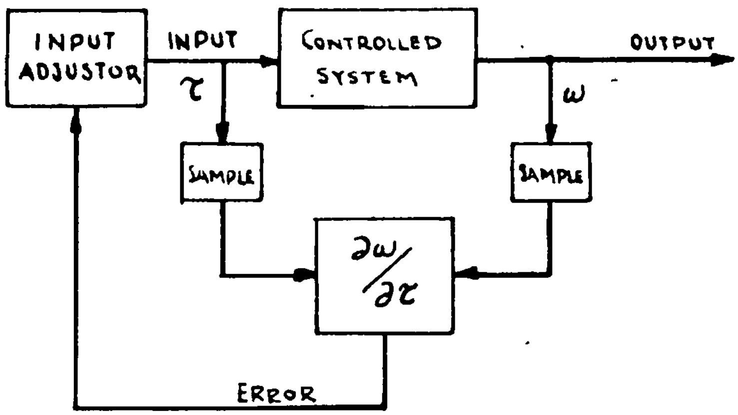
(1.1) Statement of the Problem.

In almost all controlled engineering systems or processes, there is an optimum state in which the system should operate.

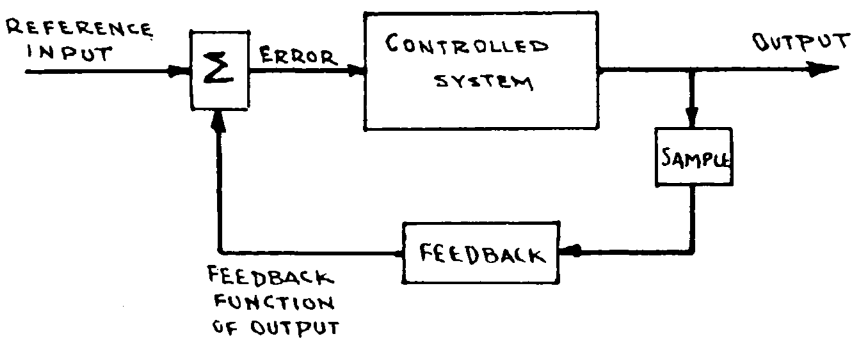
Optimalizing control systems are feedback systems in which the output is produced from an error signal bearing a relation to an optimum performance state of the controlled system. The error is used to modify its input so that the output approaches an optimum state and the error is minimized. The basic optimalizing principle is illustrated in Fig. 1.11A, in which $\partial\omega/\partial\tau$ is minimized corresponding to the optimum output ω for a given τ input.

In Fig. 1.11B, a simple servomechanism system is shown. Here the error is the difference between a reference input and some feedback function of the output. The error is minimized not by modifying the input but by actuating the control system to produce an output permitting the error's minimization.

The optimum state cannot be predicted exactly in advance, nor is this necessary. Unpredictable influences, such as enviromental changes, may affect the performance of the controlled system. Accomodation to such influences is a special trait of optimalizing systems.



A - BASIC OPTIMALIZING SYSTEM



B - BASIC SERVO SYSTEM

FIG 1.11

Therefore, if the purpose is to operate always near the optimum state, in spite of the drift of the system's characteristics, then the control device must be so designed as to (1) determine the optimum state for the existing conditions, (2) search out this state, and (3) confine the operation of the controlled system close to this state.

The necessity of optimalizing control and the problems associated with it are illustrated by choosing an airplane as an engineering system.

Today as the increase in range requirements are accentuated by large military and civilian aircraft, the design of a control system which maintains operation at optimum efficiency at all times during flight is of primary importance. For example, consider an airplane flying through icing weather conditions. The formation of ice on the wings, fuselage, etc., alters the aerodynamic properties of the aircraft considerably; moreover, it alters aircraft performance in a largely unpredictable way, depending upon other factors such as altitude. These changes of the aerodynamics of the aircraft degrade its performance. Thus, knowledge of the combination of engine throttle, engine rpm, and airplane trim settings that will result in the most efficient operation under these conditions is of primary importance. It is necessary to operate the aircraft at its maximum efficiency in order to conserve the fuel load. Under such adverse and changing conditions, previous knowledge of the airplane performance is rendered useless. The flight personnel, upon whom the responsibility of operating the aircraft at high efficiencies rests, have only their judgment and experience in interpreting instrument readings as a basis for adjustment of the controls.

The above considerations point out the desirability, if not the necessity, for an automatic system that will hold the aircraft at its optimum condition, as far as fuel administration is concerned, at all times during its flight.

This thesis undertakes the development of an automatic optimalizing controller which will cause the aircraft to operate at an airspeed resulting in the maximum range parameter (defined in Appendix I). Operating an aircraft with optimalized range parameter is known as Maximum Range Operation, and it is one of several common ways to cruise. Maximum range cruising requirements are assumed throughout this thesis.

(1.2) Previous Work

A systematic study of the possibilities of automatically holding the performance of a controllable process to an optimum was commenced in 1951 by C. S. Draper, Y. T. Li, and H. Lanning who first conceived the idea. Y. T. Li, in an article[1] that followed, classified the optimalizing controllers

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[1] Y. T. Li, "Optimalizing System for Process Control", Instruments, Vol. 25, January, 1952, pp. 72-77; 190-193.

into two distinct types. The "peak-holding" type and the "sensitivity" type.

All optimalizing controllers of the first type operate by producing an input correction signal based on the difference between the actual output and an optimum output indicated by the computer.

Optimalizing controllers of the second type compute the rate-of-change of the output with respect to some input variable, that is, the sensitivity. This is the arrangement illustrated in its simplest form in Fig. 1.11A.

The first experiment in optimalizing control was made on an internal combustion engine[1]. A controller of the peak holding type was built and used to adjust the air volume and ignition timing so that the power output of the engine always remained optimum with respect to fuel consumed as the load was varied. For the aircraft example considered in this thesis, the optimalizing controller is developed on the basis of this experimental work.

Figure 1.21 shows the functional block diagram of the peak-holding controller as described by Y. T. Li[1]. Reference will be made to this figure and Fig. 1.22 showing the performance characteristics to explain briefly its operation.

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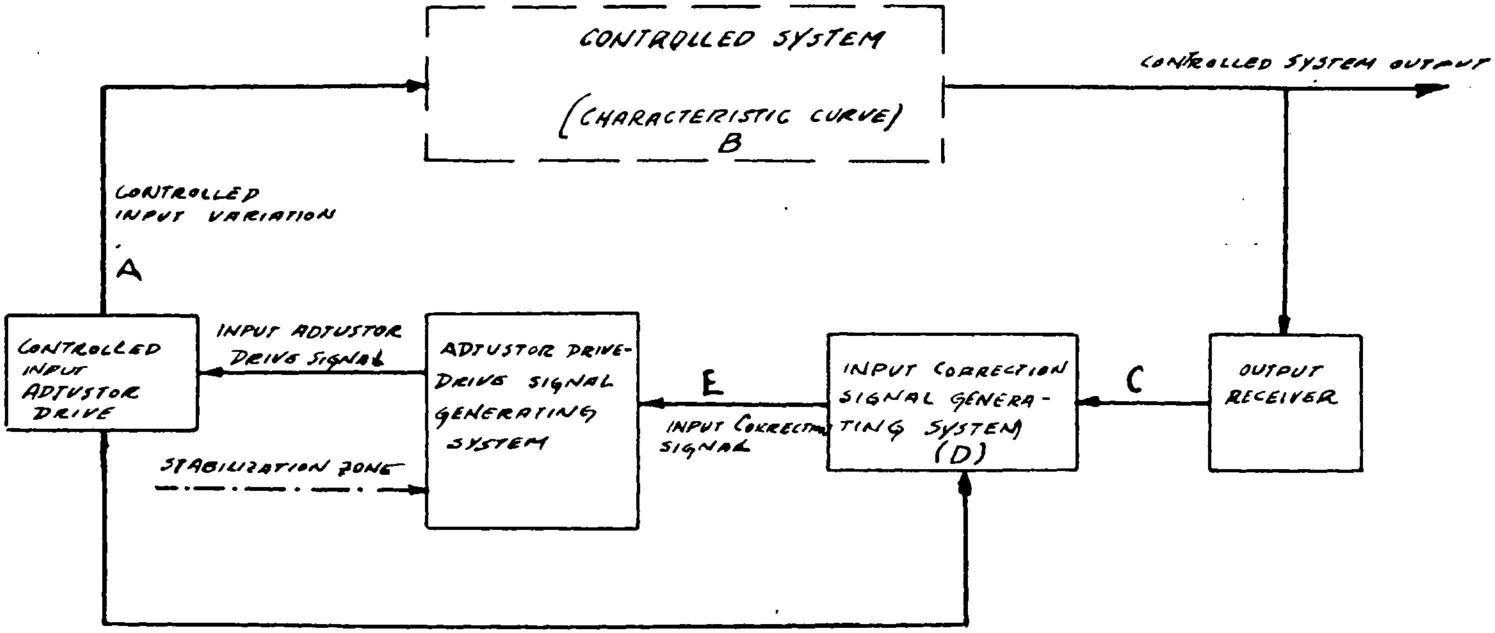
[1] Ibid. page 4.

Suppose that the input at time t = 0 is at a point a, in Fig. 1.22A, and the input adjustor drive is changing the input at a constant rate. The input operating through the characteristic curve of the controlled system (B) causes the output to have the shape shown in (C).

If an output sensing system is so designed as to follow the output, exactly, when it is increasing, but holds to the maximum value after the maximum is passed, then there will be a difference between the reading of this output sensing device, the "indicated optimum output signal" and the output itself, "output signal" shown in Fig. 1.22C. This difference is called the "indicated output deviation signal" and is shown in Fig. 1.22E. When the deviation signal reaches a pre-set value for a certain stabilization zone limit (at time 2), a signal is applied to the controlled input adjustor drive, Fig. 1.21, to reverse the direction of the controlled input signal. The input decreases after time 2, while the output increases until a maximum is again reached at time 3, etc.

Unfortunately, the simpler sensitivity-type system (Fig. 1.11A) is completely impractical for application to aircraft. The very slow effect on airspeed of engine power changes results in an extremely low sensitivity. Thus the available signal is easily masked by noise. Such systems of high inertia require an optimalizing computer employing forced hunting around the optimum state.



NOTE:

CAPITAL LETTERS REFER TO CORRESPONDING SIGNAL ILLUSTRATIONS OF FIG 1.22.

FIG. 1.21

PEAK-HOLDING OPTIMALIZING COMPUTER BLOCK DIAGRAM AS DESCRIBED BY Y.T. LI. [SEE REF. 1]

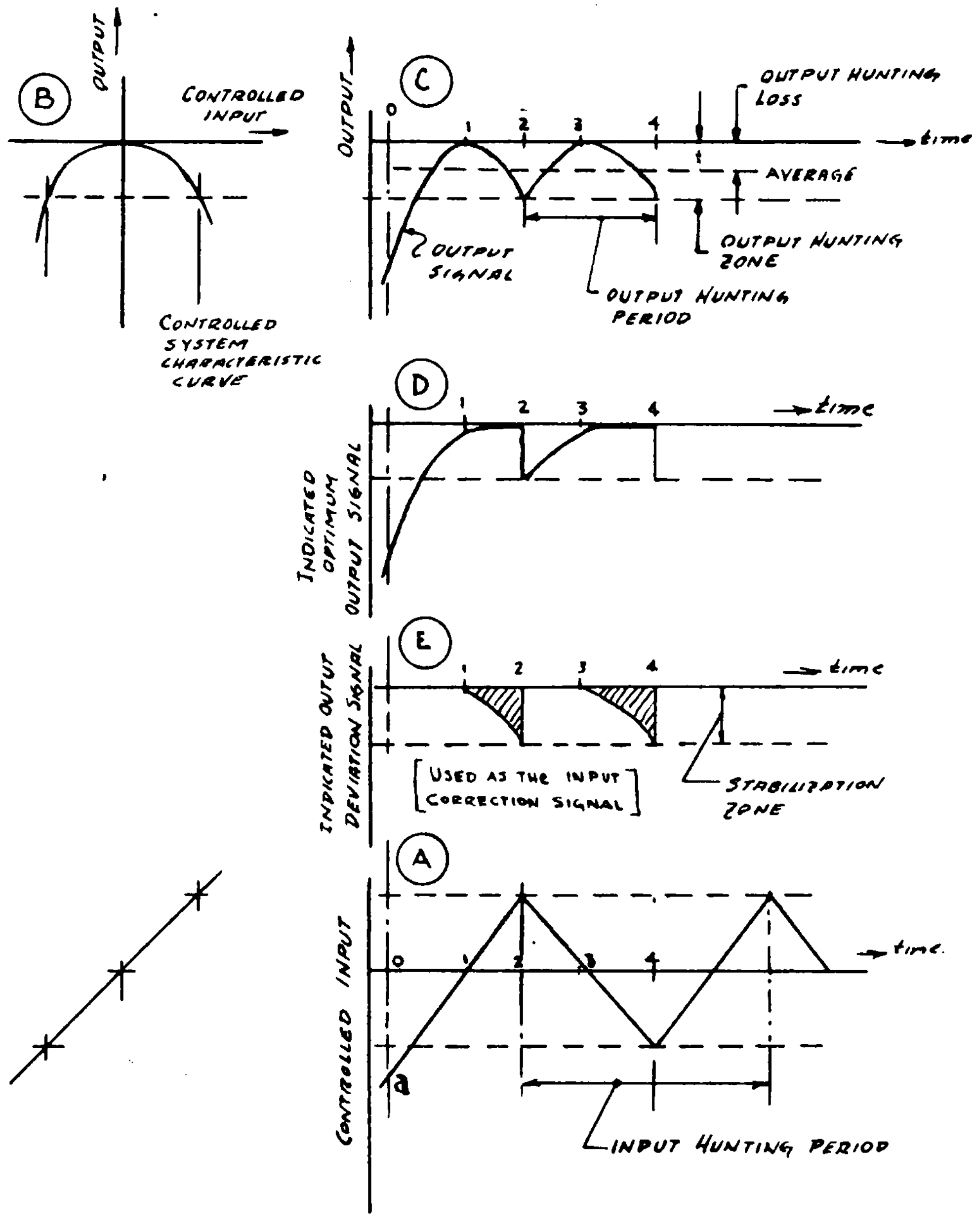


FIG. 1.12 - PEAK-HOLDING COMPUTER CHARACTERISTICS

J. R. Shull[2] described a peak-holding computer for use in aircraft. The optimalizing system was designed to control the engine rpm to hold the range parameter at an optimum value for given flight conditions.

The above system, although it points generally towards the solution, does not offer a complete answer to the problem for the following reasons: (1) The system described by Shull does not control the airspeed which is essential for optimum range operation, and (2) there are several operational variables which affect the range parameter and which were not considered in the above system.

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[2]J. R. Shull, "An Automatic Cruise Control Computer for Long Range Aircraft", IRE Trans. Prof. Group on Electronic Computers, December 1952, pp. 47.

Chapter 2

(2.1) Design Requirements.

The particular powerplant-airframe characteristic, upon which the Optimalizing Control System is based, is the dependence of the range parameter upon the airspeed. This relationship is shown in Fig. 7 of Appendix I. For optimum range performance the velocity which results in maximum range parameter, for given flight conditions, should be maintained.

In the system to be developed the Optimalizing Computer must receive an input proportional to the range parameter. The airplane airspeed is defined as the controlled system output, and the fuel flow as the controlled system input.

The Optimalizing System is also to control the propeller efficiency, through control of propeller rpm. As it is concluded from the discussion in section (2.3) such control is necessary for the successful optimalization of the range parameter. The conventional propeller-pitch regulator is omitted and replaced by this control.

The general form of the system to be developed is shown in the block diagram of Fig. 2.11. The following sections will develop this system in detail. The peak-holding optimalizing computer will be adapted from Li's design for this application.

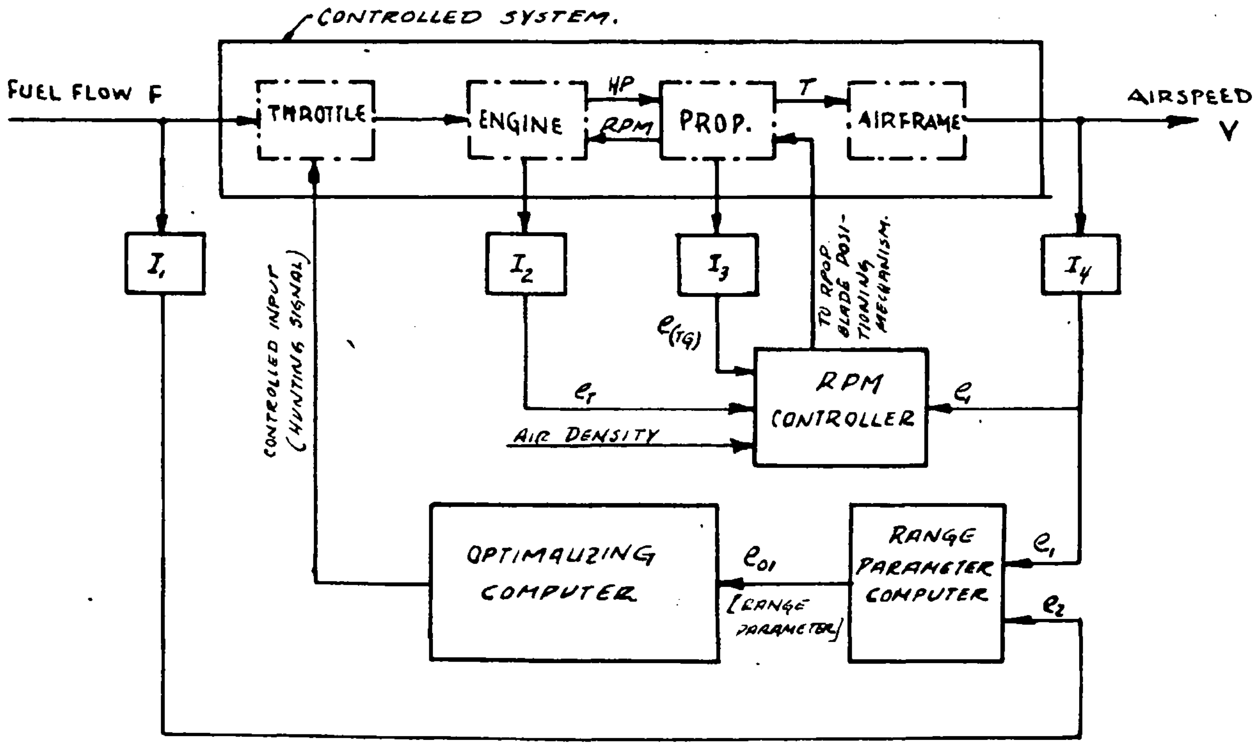

FIG 2.11

PROPOSED OPTIMALIZING SYSTEM

(2.2) The Range Parameter Computer.

The range parameter, as defined in Appendix I, is the ratio of the airplane airspeed to the fuel flow rate. The simple servo divider system[3], shown in Fig. 2.21, being capable of forming the ratio between two varying input quantities, is suitable for the computation of the range parameter.

The two instruments, (or transducers), measure and convert their respective inputs into electrical quantities. The instrument measuring the fuel flow can be a synchro transmitter-receiver system which, besides giving a visual indication of the fuel flow to the pilot, converts the fuel flow to an electrical d-c voltage by means of a potentiometer coupled to the shaft of the receiver. The potentiometer P_1 must be selected to have low torque and high resolution.

A second potentiometer of the same qualities can be used to convert the angular displacement of the airspeed indicator into an electrical signal e_1.

Assuming linear relations between the angle Θ_1 and the airspeed, the sensitivity of the measuring instrument can be defined as: $K_V = \frac{\Theta_1}{V}$ $\left(\frac{\text{deg-hr}}{\text{mile}}\right)$. To make the design general, assume that the fuel flow ranges, for example, from

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[3] W. W. Soroka, "Analog Methods in Computation and Simulation", McGraw-Hill Book Co, Inc., 1954, p. 74.

F_1 lbs/hr to $F_2 \frac{lbs.}{hr}$, and that this range is covered by the synchro angular displacement, Θ_2, ranging from 0 degrees to 360^o degrees. Then the equation for the fuel flow as a function of Θ_2 is: $F = (F_2 - F_1) + K_2\,\Theta_2$ (1)

where $K_F = \frac{\Delta F}{\Delta \Theta_2}\left(\frac{lbs}{hr\text{-}Deg.}\right)$ constant.

Solving for Θ_2:

$$\Theta_2 = \frac{F-(F_2-F_1)}{K_F} \quad \text{or} \quad \Theta_2 = \frac{F}{K_F} - \frac{F_2-F_1}{K_F} \tag{2}$$

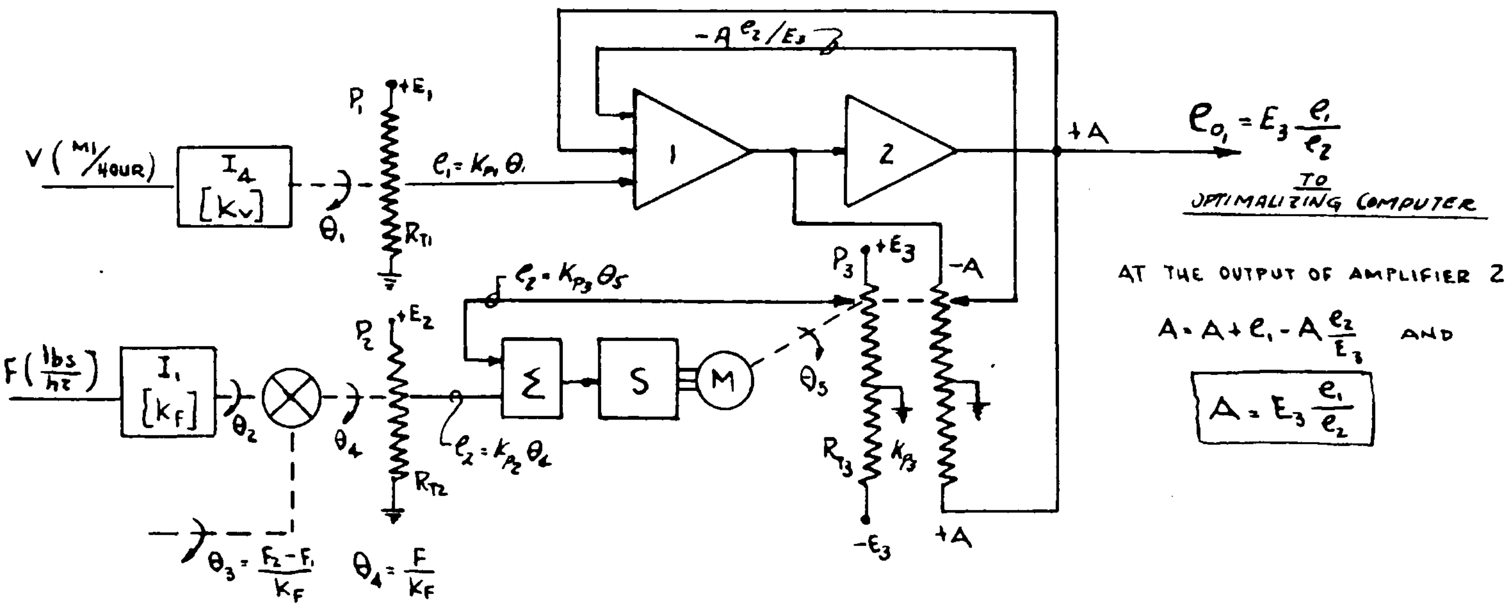
The above equation results in a non-linear relation between the signal output, and the range parameter. Equation (2) suggests a way to compensate for this non-linearity by adding to the angle Θ_2 another angle Θ_3 equal to the constant $\frac{F_2-F_1}{K_F}$. Then the resulting angle Θ_4 is equal to $\frac{F}{K_F}$.

The output of the range computer is:

$$e_{o_1} = \left(\frac{E_3 K_{p_1}}{K_{p_2}}\right) \times \left(K_V\ K_F\right) \times \frac{V}{F} \text{ volts} \tag{3}$$

The dynamical effects of the input instruments can be neglected because of the slow time variation of the inputs. The operation of the servo divider will not be described. The important relations existing between the components are stated in Fig. 2.21.

The servo amplifier S is conventional, a d-c input producing a reversible phase a-c output to actuate the a-c servo motor. Because of the low inertia load connected to this motor, its time constant is very small, resulting in



I_4 – AIRSPEED INDICATOR

I_1 – SYNCHRO SYSTEM

R_T = TOTAL POT. RESISTANCE

$K_V \left[\frac{\text{deg.-hr}}{\text{mile}}\right]$ – VELOCITY INSTRUMENT SENSITIVITY

$K_F - \left[\frac{\text{lbs}}{\text{hr-deg.}}\right]$ – FUEL FLOW INSTRUMENT SENSITIVITY

$K_{P_1} = \frac{E_1 m_1}{R_{T_1}} \left[\frac{\text{Volts}}{\text{deg.}}\right]$

$m_1 \left[\frac{\text{ohms}}{\text{deg.}}\right]$

FIG 2.21 RANGE PARAMETER COMPUTER

fast response. The power output requirements are very low. An output of 2 to 4 watts is ample to drive the ganged potentiometers which require a torque of approximately 5 to 6 oz-in.

From the above, the gain of the servo amplifier can be specified and its detailed design completed.

Good accuracy of the servo devider depends on identical characteristics of the two potentiometers P_3 and P_4 and the high gain of the operational amplifiers 1 and 2.

(2.3) Control of Rpm.

One important variable, affecting the optimum range operation, is the propeller efficiency, which is shown to vary with airspeed and rpm in Appendix I.

To obtain optimum propeller efficiencies, the propeller rpm must be corrected to the proper value after changes of altitude, airspeed, or attitude.

The simple regulator used in constant-speed propellers is ordinarily adequate since the efficiency of the propeller is nearly constant over the range of power settings of interest. This is illustrated in Fig. 5 of Appendix I. For the present control problem, however, the conditions for maximum range may very well call for brake mean effective pressures above the recommended maximums should the airplane operate with a constant speed propeller. For this reason the continous automatic control of the propeller efficiency is necessary.

The control scheme proposed is derived from the distinct treatment of the propeller coefficients as presented in Appendix III. The method of this treatment is credited to E. Kessler[4].

Maximum propeller efficiency is achieved, for a particular power coefficient C_p, when:

$$J = hC_p^{-k} \tag{4}$$

The control system for the rpm consists of an error device and two computing units, one for J and the other for hC_p^{-k}, in order to satisfy Equation (4)

<u>Computation of J</u>

The advance ratio J, is defined in Appendix III as:

$$J = \frac{88V}{Nd} = K_{(J)}\frac{V}{N} \tag{5}$$

where V is the airspeed in miles per hour

N is the propeller rpm

d is the propeller diameter in feet.

The computation of J is very similar to the computation of the range parameter. To compute J use of the servo divider is again made. The signal voltage e_1, used as an input to the operational amplifier of Fig. 2.21, can also be used as an input signal to the amplifier of the system shown in Fig. 2.31.

The rotational velocity (N) of the propeller is

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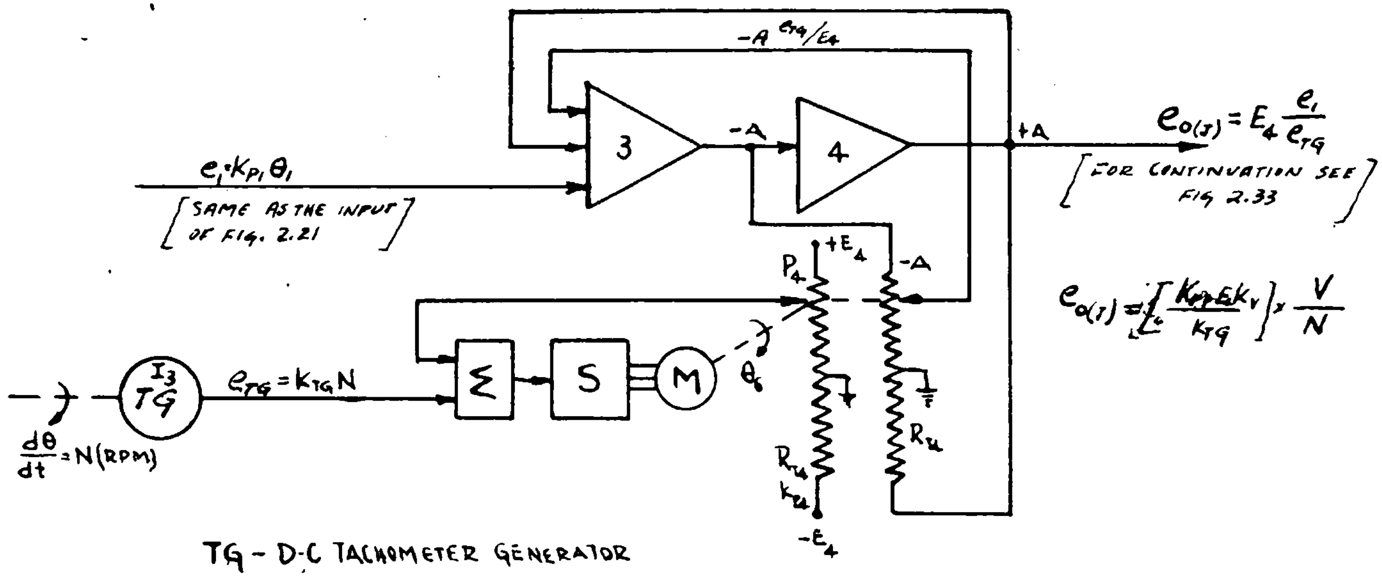
[4]E. Kessler, "Propeller Performance Analysis", Aero Digest, Vol. 52, June 1946, pp. 80.



TG - D-C TACHOMETER GENERATOR

$K_{P4} = \dfrac{E_4 m_4}{R_{T4}} \left[\dfrac{\text{VOLTS}}{\text{DEG.}}\right]$

$K_{TG}\left[\dfrac{\text{VOLTS}}{\text{RPM}}\right]$ - TACHOMETER SENSITIVITY.

$m_4 \left[\dfrac{\text{OHMS}}{\text{DEGREE}}\right]$

FIG 2.31 ADVANCE-RATIO COMPUTER

measured with a d-c tachometer generator whose output voltage is proportional to the rpm.

The output voltage $e_{o(j)}$ is:

$$e_{o(j)} = \left(\frac{K_{p1}\ K_V\ E_4}{K_{TG}}\right) \frac{V}{N} = K'_{(j)} \frac{V}{N} \qquad (6)$$

The d-c voltage E_4 must be selected after the range of possible values of N has been established and consequently the range of $e_{(TG)}$ is known. The value of E_4 must be at least as great as the maximum value that $e_{(TG)}$ assumes within this range. The same considerations should apply to the specification and design of the rest of the equipment as those of Fig. 2.21.

Computation of hC_p^{-K}

The propeller power coefficient is given by the expression:

$$C_p = 118 \times 10^6 \frac{HP}{\rho N^3 d^5} = K_{cp} \frac{HP}{\rho N^3 d^5}$$

and

$$hC_p^{-k} = h\left(\frac{K_{cp}\, HP}{\rho N^3 d^5}\right)^{-k}$$

The brake horsepower (HP) is measured with a torque-meter installed in the propeller planetary reduction gears. The expression is:

$$HP = T_p \times N_c \times K_+ \qquad (9)$$

where T_p is torque meter oil pressure

K_+ is a constant of proportionality

and N_c is the engine speed related to the propeller speed by the reduction gear ratio K_g-; therefore $N_c = K_g N$.

Equation (9) can then be written as:

$$HP = K_t K_g N T_p \tag{10}$$

Substituting equation (10) in to equation (8) yields:

$$h C_p^{-k} = \left[\frac{h^{1/k} d^5}{K_p K_t K_g} \right]^k \cdot \left[\frac{\rho N^2}{T_p} \right]^k$$

or for a particular airplane:

$$h C_p^{-k} = K_2 \left[\frac{\rho N^2}{T_p} \right]^k \tag{11}$$

Equation (11) will now be used to derive the system for the computation of hC_p^{-k}. This system is shown in Fig. 2.32. The voltage output e_{of}, of this figure will be compared with the voltage output $e_{o(j)}$ so that an error voltage will be produced whenever they are not equal. Care must be taken now, to make sure that the same scale factor is applied for the voltage representing J, and for the voltage representing hC_p^{-k}. This is necessary in order to insure that zero voltage error occurs when Equation (4) is satisfied.

The device producing the error signal is a differential servo amplifier driving a reversible d-c motor, as shown in Fig. 2.33. The angular displacement of the motor is proportional to the difference of $e_{o(j)}$ and e_{of}. This output controls the propeller blade angle and consequently the engine rpm as well as the propeller angle of attack. The feedback loop is completed through the controlled system.

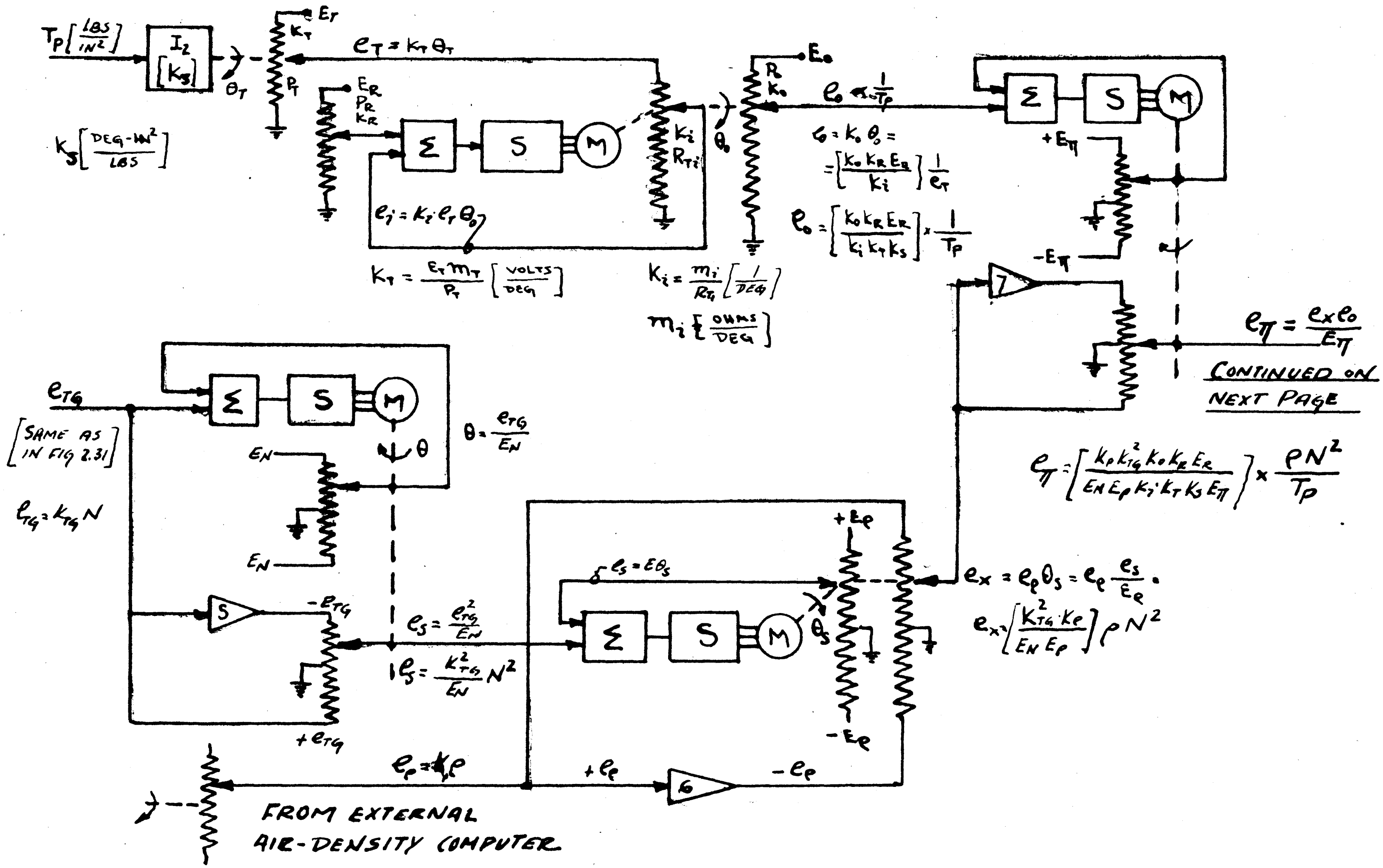
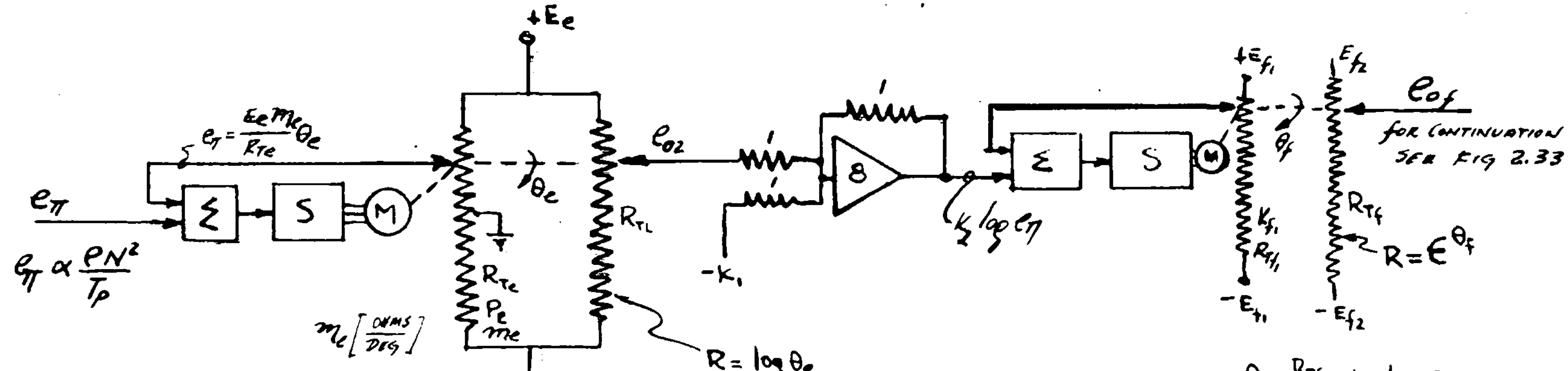


FIG 2.32 hC_p^{-k} COMPUTER



$$\theta_e = \frac{R_{Te}}{E_e m_e} e_\pi$$

$$\therefore\ e_{02} = \frac{E_e}{R_{TL}} \log\theta_e = \frac{E_e}{R_{TL}} \log \frac{R_{Te}}{E_e m_e} \times e_\pi =$$

$$= \frac{E_e}{R_{TL}}\left[\log \frac{R_{Te}}{E_e m_e} + \log e_\pi\right] =$$

$$= K_1 + K_2 \log e_\pi$$

$$\theta_f = \frac{R_{Tf_1}}{E_{f_1} K_{f_1}} k_2 \log e_\pi =$$

$$= K_{eq} \log e_\eta = \log e_\pi^{K_{eq}}$$

$$\therefore\ e_{0f} = \frac{E_{f2}}{R_{Tf}} \epsilon^{\theta_f} =$$

$$= \frac{E_{f2}}{R_{Tf}} \epsilon^{\log e_\pi^{K_{eq}}} =$$

$$= \frac{E_{f2}}{R_{Tf}} K_2 \left[\frac{\rho N^2}{T_p}\right]^{K_{eq}}$$

FIG 2.32 (CONTINUED FROM PREVIOUS PAGE)

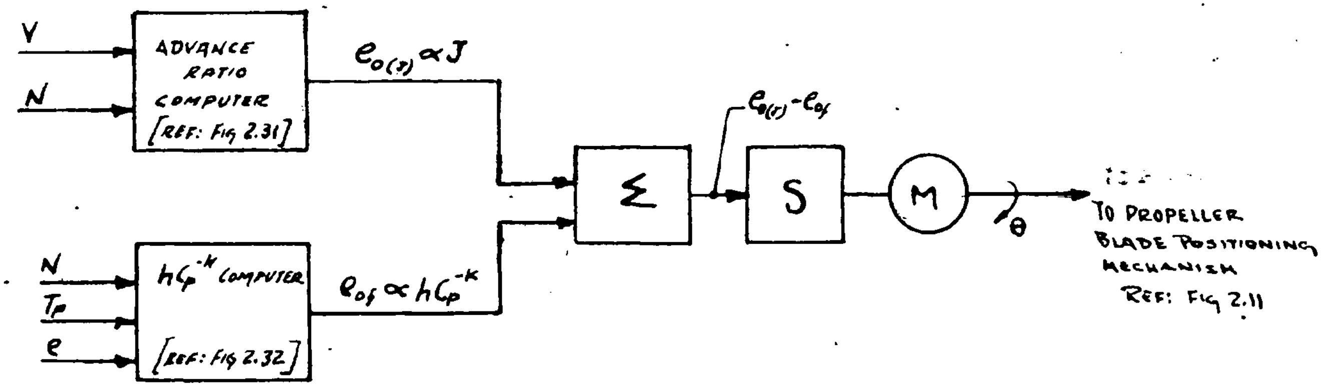


FIG 2.33 THE RPM CONTROLLER

(2.4) The Optimalizing Computer

The operation of the Optimalizing Computer was described in Section 2.1 with the aid of the functional block diagram Figure 1.21. Based on this, a specific design can be completed for any particular aircraft as discussed below.

Of great value to the designer is a quantity called the "hunting loss" of the controlled system. It is defined as the difference between the optimum output signal and the average output signal as shown in Figure 1.22C. For the design of efficient systems the hunting loss should be as small as possible. To accomplish this, one may reduce the output hunting zone or the amplitude of the input variation. The reduction of either parameter, however, reduces the stabilization zone of the input correction signal. The limit of such reduction is set by the level of noise (e.g. fluctuation of airspeed) present in the output of the controlled system. If the noise signal were to exceed the stabilization zone limit, then a spurious input-rate reversal would occur due to the action of the computer.

A prerequisite to the design of this computer is therefore the study of noise present in the airspeed within the normal cruising range. The average level and the fundamental frequency of noise must be determined by flight test measurement. The stabilization zone is then set to a wider value than the zone which is barely sufficient to prevent an input reversal due to interference alone.

Having specified the stabilization zone, which specifies the output hunting zone, the determination of the hunting loss is possible. Because the output is a series of approximately parabolic arcs, the hunting loss is almost exactly one-third of the hunting zone.

The static performance curve (as in Fig. 1-22B) of the particular engine in question, giving the range parameter vs airspeed, is used to find the corresponding input hunting zone.

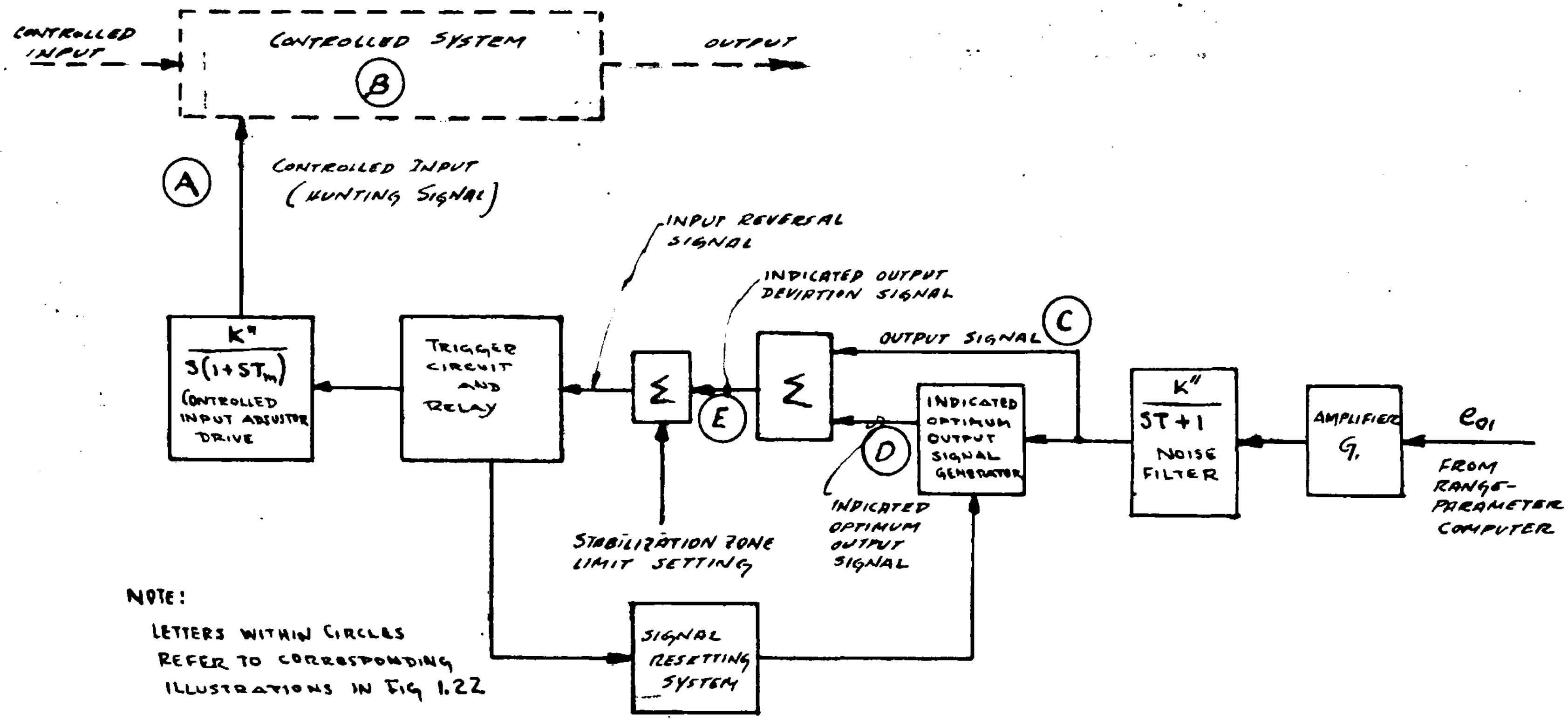
Should it be necessary to reduce the noise in the measured controlled-system output in order to permit a satisfactory hunting zone, the output may be filtered before applying it to the optimalizing computer. The delay introduced by this filter will usually be small compared to the total time delay, as discussed below, and will not, therefore, have a serious effect on the control characteristics.

The output hunting period must be made large with respect to the total time delay of the optimalizing system. The total time delay is made up of the input time delays, such as fuel-flow control valve and its driving motor, the engine's time delay arising from the inertia of the moving parts, the aero-dynamic response time of the airframe, and the time delay of the noise filter. All but the last of these would be determined from flight-test measurements of the particular aircraft.

Finally the rate of the controlled input (hunting) variation may be computed as:

$$\text{input variation rate} = \frac{\text{input hunting zone}}{\text{output hunting period}}.$$

The general form of the resulting optimalizing computer is illustrated in the block diagram of Figure 2.41.



NOTE:
LETTERS WITHIN CIRCLES REFER TO CORRESPONDING ILLUSTRATIONS IN FIG 1.22

FIG. 2.41 PROPOSED OPTIMALIZING COMPUTER

Chapter 3

(3.1) Summary of Results and Conclusions.

In Chapter 2, methods were developed for computing the range parameter and for controlling engine rpm. Design parameters were established to permit design of equipment suited to a particular aircraft. The last section of that chapter adapted the general design of the Optimalizing Computer as given by Y. T. Li, to apply to this aircraft power-control system. The final result is the synthesis of the system postulated in Section 2.1, and as shown in Figure 2.11.

The system will compute the optimum state of the range parameter for any given flight conditions, will search for this state, and finally will hold the operation close to this state.

The Optimalizing Computer together with the range parameter computer, insure optimum power plant efficiencies as represented by the range parameter _vs_ airspeed curve. The rpm controller compliments the action of the Optimalizing Computer by insuring optimum aerodynamic efficiency of the propeller.

The proposed control system could be studied profitably by simulation on an analog computer of the non-linear characteristics of the power-plant airframe combination, and the several response characteristics of the system.

APPENDIX I - Maximum Range Performance Requirements and Characteristics

It is evident that an economically optimum performance is only possible by optimalizing the overall efficiency of the airplane. In an engineering system like this, one is confronted with two separate efficiencies.

The first is the mechanical efficiency of the power plant, and the second is the aerodynamic efficiency of the airframe.

Efficiency however, is a rather complex and sensitive variable to control. The number of factors affecting it, and the way they affect it, is rather complicated for any engineering system.

In this Appendix therefore, the quantities affecting the most efficient performance, most critically, are discussed.

The first quantity in the list of the most critical controllable variables, is the total drag of the airplane. The drag (D) is one of the prime factors that appear in the equation for the power required to maintain the aircraft at level cruising speeds as given in Appendix II. It is desirable then, in principle, to keep the drag at a minimum. Although the drag is an aircraft design constant, it can be controlled, however, by controlling the angle of attack at which the airplane is flown.

By use of wind tunnel models, it is possible to determine the drag induced at various angles of attack.

Figure 1 shows the relationship between the lift to drag "ratio" and the angle of attack. Of course, the best angle of attack to fly an airplane is that which results in a maximum L/D ratio or, in other words, at that angle which results in the required lift produced with minimum drag.

However, to maintain the aircraft at level flight with this favorable angle of attack, a particular airspeed is required for every gross weight of the airplane, and every altitude. Figure 2 shows the relationship between the lift to drag ratio and airspeed for a particular gross weight and altitude.

The next critical variable is again an aerodynamic quantity. It is the propeller efficiency which is defined as:

$$\eta = \frac{C_T}{C_P} J$$

A certain comparison can be made between the L/D ratio, as discussed previously, and the propeller efficiency. Any propeller blade element is an airfoil in motion through the air, and as such, it is acted upon by a lift and a drag force. Consequently, there is an angle of attack which results in a maximum L/D ratio as shown in Figure 1.

In the case of a wing, the lift force is in the upward direction because the area of decreased pressure is above the wing. In the case of a propeller the area of decreased pressure is in front of the propeller and therefore the lift force, called now the thrust (T), is acting in the forward direction.

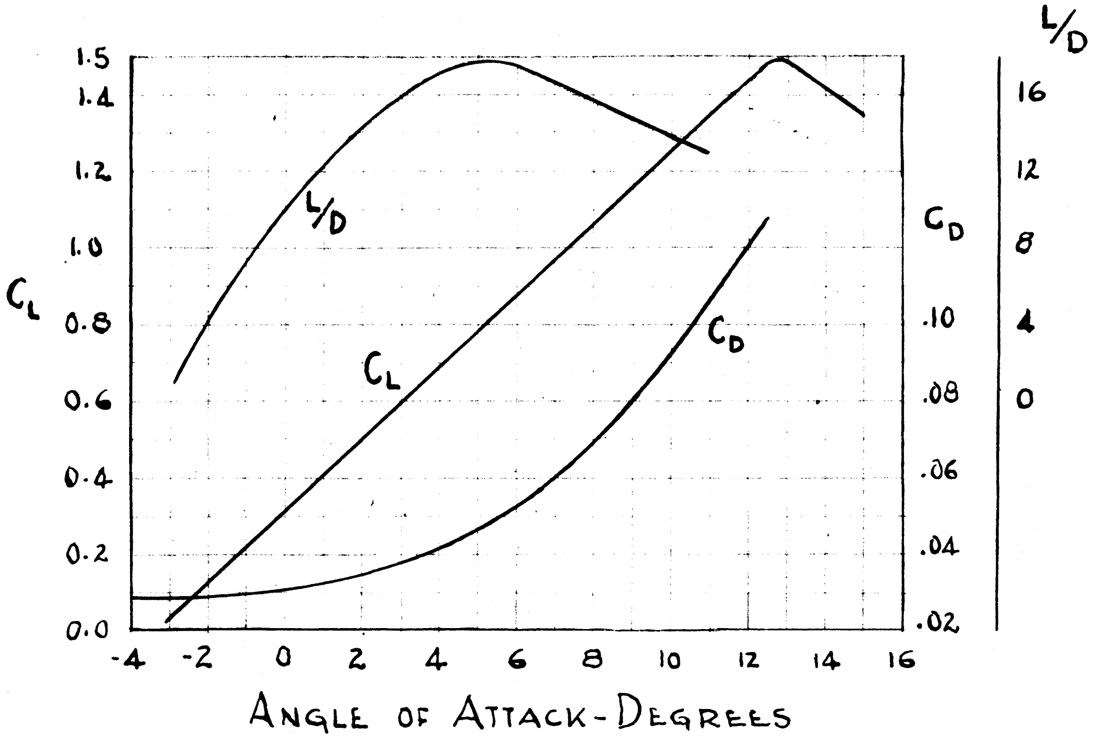


FIG.[5] 1

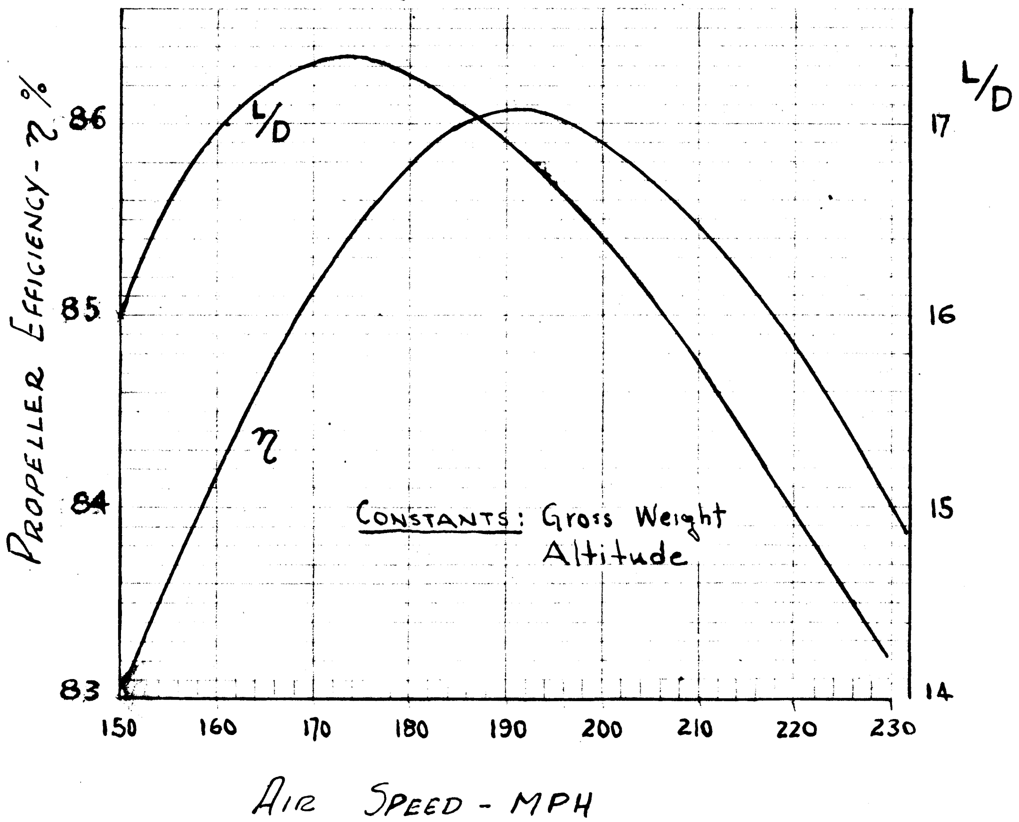


FIG.[5] 2

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5. R.D SPEAS, J.T FARAH, "CRUISE CONTROL FOR FLYING EFFICIENCY" AVIATION, JULY 1943 P.P- 193-197

Aerodynamically then, the thrust is the result of the propeller shape and the angle of attack of the propeller blade.

Referring to Figure 3, the blade angle β includes the angle of attack α, which for maximum efficiency, must be kept small.

The actual blade angle of the propeller, however, necessary to maintain small angles of attack varies with the forward velocity of the airplane (V), and the rotational speed of the propeller. (See Figure 3.)

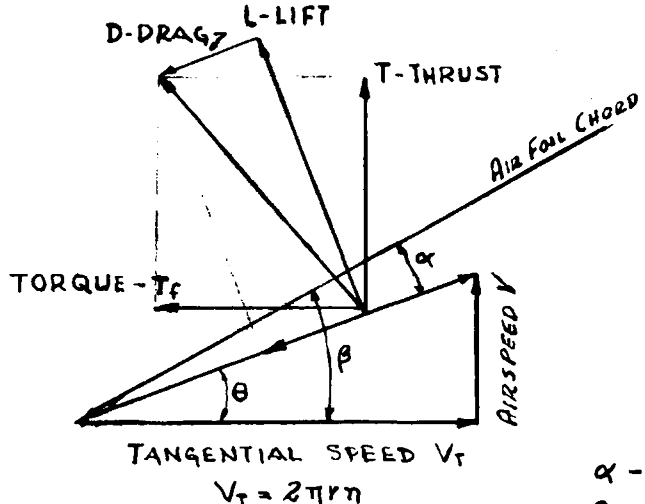
On variable pitch propellers, the blade angle is adjusted to provide the most efficient angle of attack at all engine speeds and airframe velocities.

Figure 2 shows the variation of propeller efficiency with airspeed. For a particular altitude and gross weight, the proper engine speed, for any desired efficiency is given in Figure 4.

The blade angle again, is an excellent means for adjusting the load on the propeller to control the engine rpm.

Similar curves to those shown in Figure 4 can be plotted for the propeller efficiency vs. engine rpm at different altitudes but for constant airspeed.

Figure 5 shows again, the variation of efficiency with engine rpm, but for different horsepower outputs.



α - ANGLE OF ATTACK

β - PITCH OR BLADE ANGLE

$\theta = \tan^{-1} \frac{V}{2\pi r n}$

FIG. 3

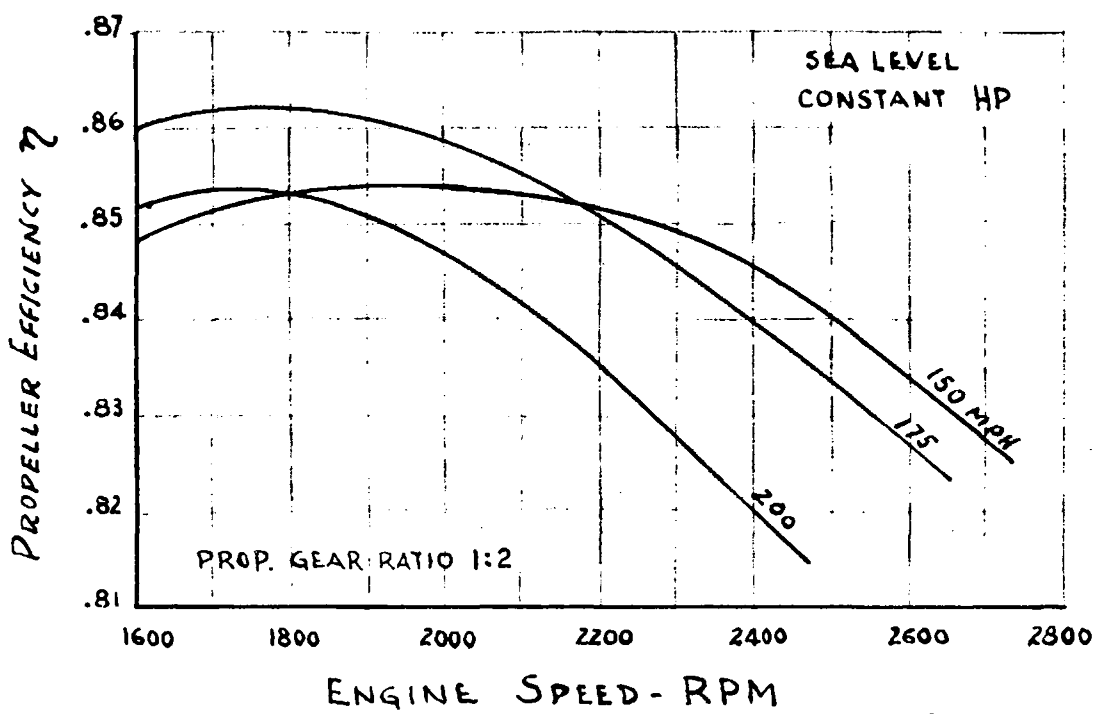


FIG.[5] 4

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[5] Ibid. Page

The important fact to notice is that the propeller efficiency is constant for all horsepower outputs if the proper engine speeds (circled) are used. In all these curves it is evident that the efficiency can be controlled within ±0.5% throughout a rather wide range of any of the influencing variables.

The specific fuel consumption, SFC, is the third variable whose proper control is necessary for an optimum airplane performance. The SFC is the ratio of the brake horsepower to the fuel consumed. For optimum efficiency then, the necessary brake horsepower must be realized with the minimum possible fuel consumption. Such a condition can exist with a particular engine rpm and airplane velocity. The variation of SFC with engine rpm and airspeed is shown in Figure 6.

The rpm thus, influences both the SFC and the propeller efficiency. It is evident that for maximum range operation, the propeller efficiency must be maximum and the SFC a minimum.

The criterion for the degree of efficient operation however, is a quantity called the range parameter. The range parameter (R) is defined as:

$$R = \frac{V}{F} \left(\frac{\text{miles}}{\text{lbs}} \right)$$

and of course, an airplane attains its best operating efficiency when it is making its best miles per pound of fuel, or when the range parameter is optimalized.

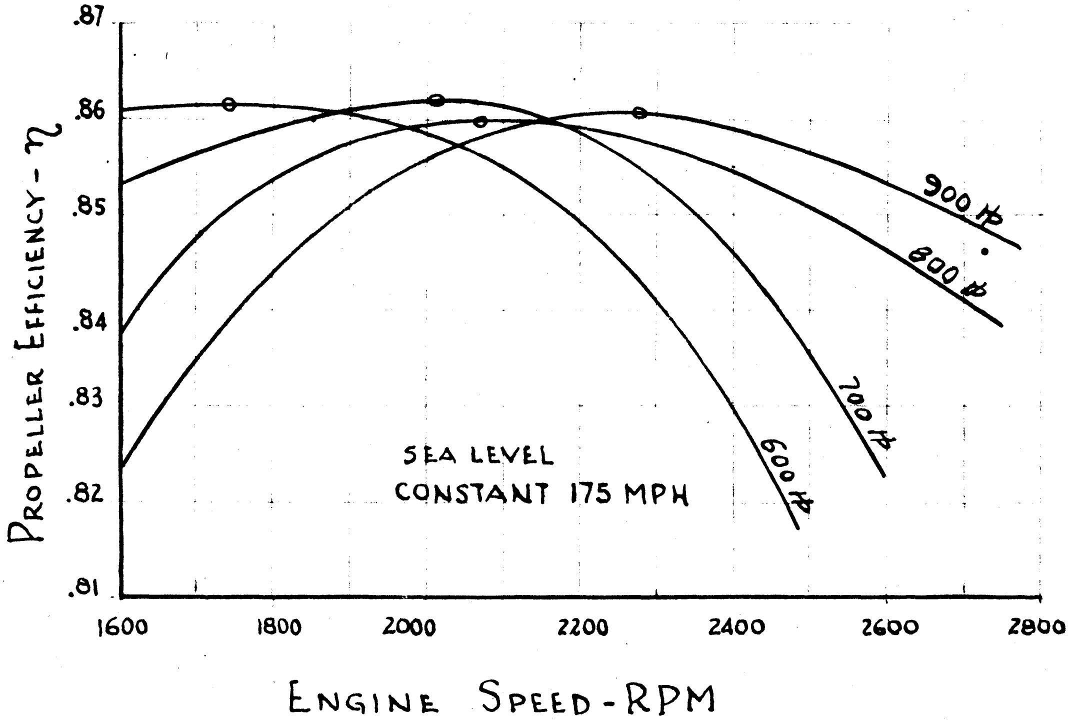

FIG.[5] 5

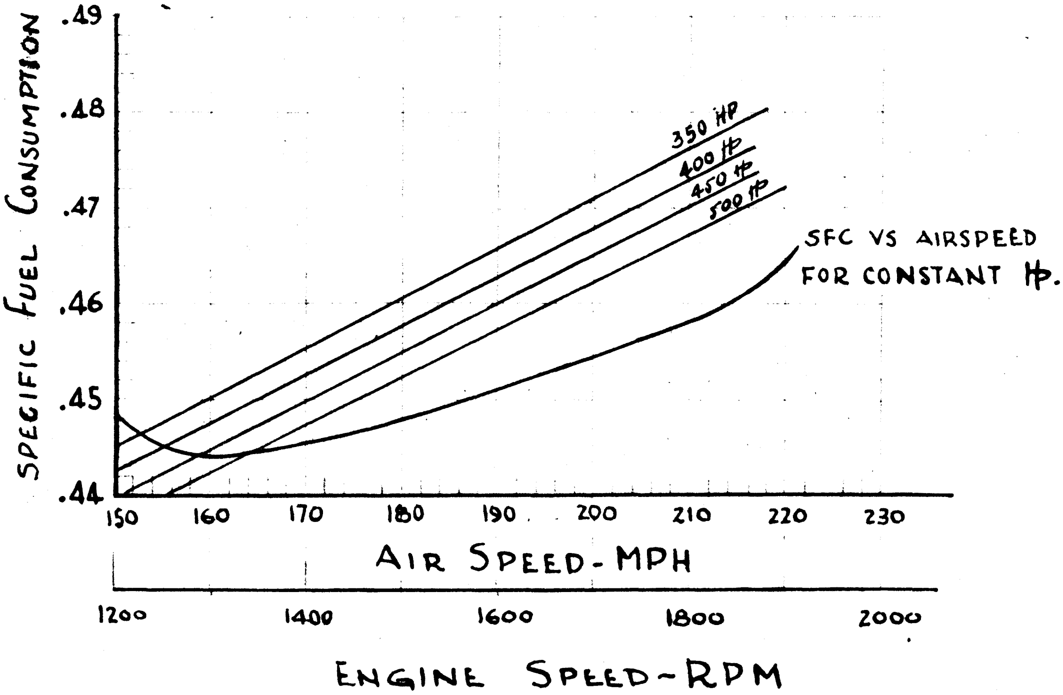

FIG.[5] 6

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5. Ibid. Page

The range parameter becomes maximum at a particular airplane velocity. At this airplane velocity, the effects of the previously discussed variables on the operational efficiency, are properly accounted for. In other words, this airplane velocity is the one at which the value of:

$$\frac{L/D \times \eta}{SFC} = \text{Maximum.}$$

This is illustrated in Figure 7, where the three variables propeller efficiency, specific fuel consumption, and lift to drag ratio have been plotted together with the range parameter vs. the airplane airspeed.

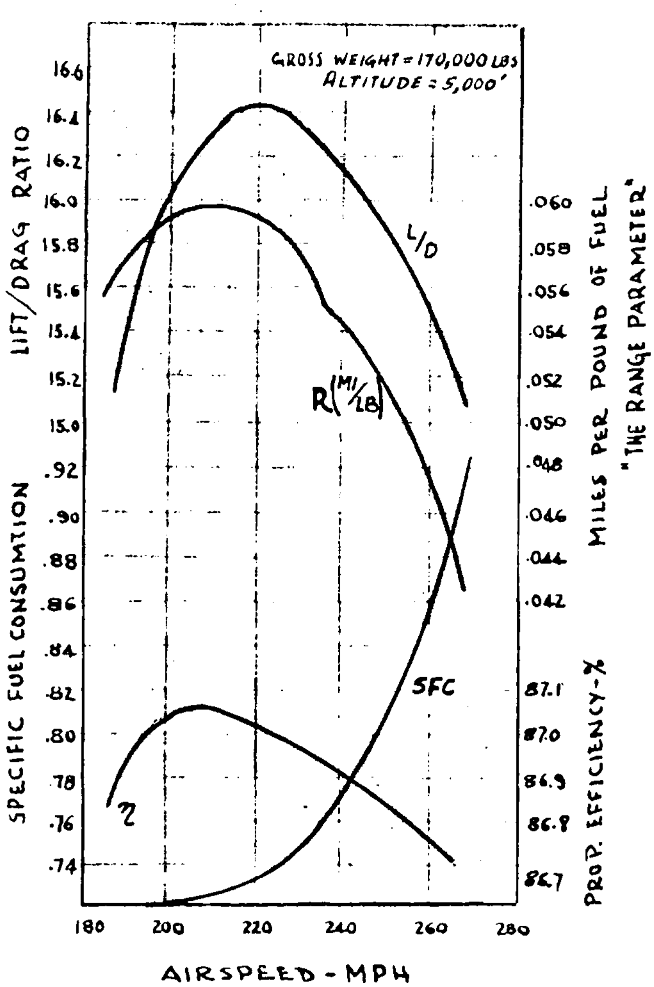


FIG. 7 [6]

6. U.S DEPARTMENT OF AIRFORCE, "AIRCRAFT PERFORMANCE ENGINEERING FOR RECIPROCATING ENGINES", AF MANUAL. 51-9, DEC. 1954, P. 236

APPENDIX II - Equations of Level Flight

Level flight is defined as a steady flight in a straight line, perpendicular to the direction of gravity, with all forces acting on the body, in equilibrium. The forces are:

(1) The gross weight (W) of the aircraft.

(2) The lift force (L) which is produced by the motion of the wings through the air, and which acts in a direction opposite the direction of gravity.

(3) The drag force (D) which is connected with the lift force and which acts in a direction opposite to that of the direction of flight.

(4) The thrust force (T) which is produced by the propeller and its existance is necessary for balancing the drag force.

(5) The torque moment (Q) which is invariably connected with the propelled thrust. It acts about the propeller axis and in a sense, opposite to that of the propeller rotation.

(6) The brake moment (M_{br}) is a necessary moment to balance the propelled torque moment. This force is supplied by the engine, and acts in the same sense as the propelled rotation.

In form of equations these forces are:

$$W = L; \quad D = T; \text{ and } Q = M_{br} \tag{1}$$

In order to deal with power equations instead of force equations, it is customary to multiply the second and third equations by the airspeed V and by the angular velocity $\omega = 2\pi N$, respectively. This yields $W = L$; $DV = TV$, and $\omega Q = \omega M_{br}$ (2)

The product DV is known as power required, while the product TV as power available. Then for level flight:

$$P_{req} = DV; \quad P_{av} = TV; \text{ and } P_{req} = P_{av} \qquad (3)$$

Equation (3) indicates that P_{req} and P_{av} are functions of airspeed. Plots of these quantities vs airspeed form basic charts for performance calculations.

APPENDIX III - Propeller Coefficients

The existence of the thrust force and the torque moment and their importance have already been introduced in Appendix II. The thrust force is given by:

$$T = \rho N^2 d^4 C_t \qquad (1)$$

The engine supplies the brake horsepower through a rotating shaft, and the propeller is absorbing and converting it into thrust horsepower, which is the product of the thrust force (T) and the airplane velocity (V). The propeller power (P), which equals the engine brake horsepower, is given by:

$$P = \omega Q = \rho N^3 d^5 C_p \qquad (2)$$

The propeller efficiency of course, is a measure of the completeness of the conversion of the engine horsepower to thrust horsepower. So:

$$\eta = \frac{TV}{\omega Q} = \frac{1}{2\pi} \cdot \frac{TV}{QN}$$

and according to Equation (1) and Equation (2):

$$\eta = \frac{\rho N^2 d^4 C_t V}{\rho N^3 d^5 C_p} = \frac{C_t}{C_p} J$$

where the dimentionless quantity V/Nd is termed the Advance Ratio. J.

The coefficients of thrust (C_t) and power (C_p) of a given propeller depend on the advance ratio and the Reynolds number, which is of minor importance, and consequently neglected.

The curves then showing the two coefficients (C_t, C_p) as functions of the advance ratio (J) are called propeller characteristics.

If the values of C_t are read from the C_t vs. J curves and are replotted on semi-logarithmic paper it results that, for a given value of C_p, the logarithm of C_t varies linearily with the advance ratio J, as illustrated in Fig. 8. This means then, that:

$$\log C_t = \log A' - BJ \tag{4}$$

or

$$C_t = \epsilon^{(A-BJ)2.3} \tag{5}$$

The above relation holds true for values of J within the range of normal operation of the propeller.[4]

The values of the coefficients A and B can be computed for each value of C_p. If these values of A and B are plotted on Log-Log paper against C_p the result, Fig. 9 , is a straight line. It follows then that the coefficients A and B can be expressed as:

$$\log(-A) = \log \Delta - (\log C_p - \log 0.1)g \tag{6}$$
$$\log(B) = \log \Delta' - (\log C_p - \log 0.1)k \tag{7}$$

where $-A = \log A'$ and Δ is the y-axis intercept.

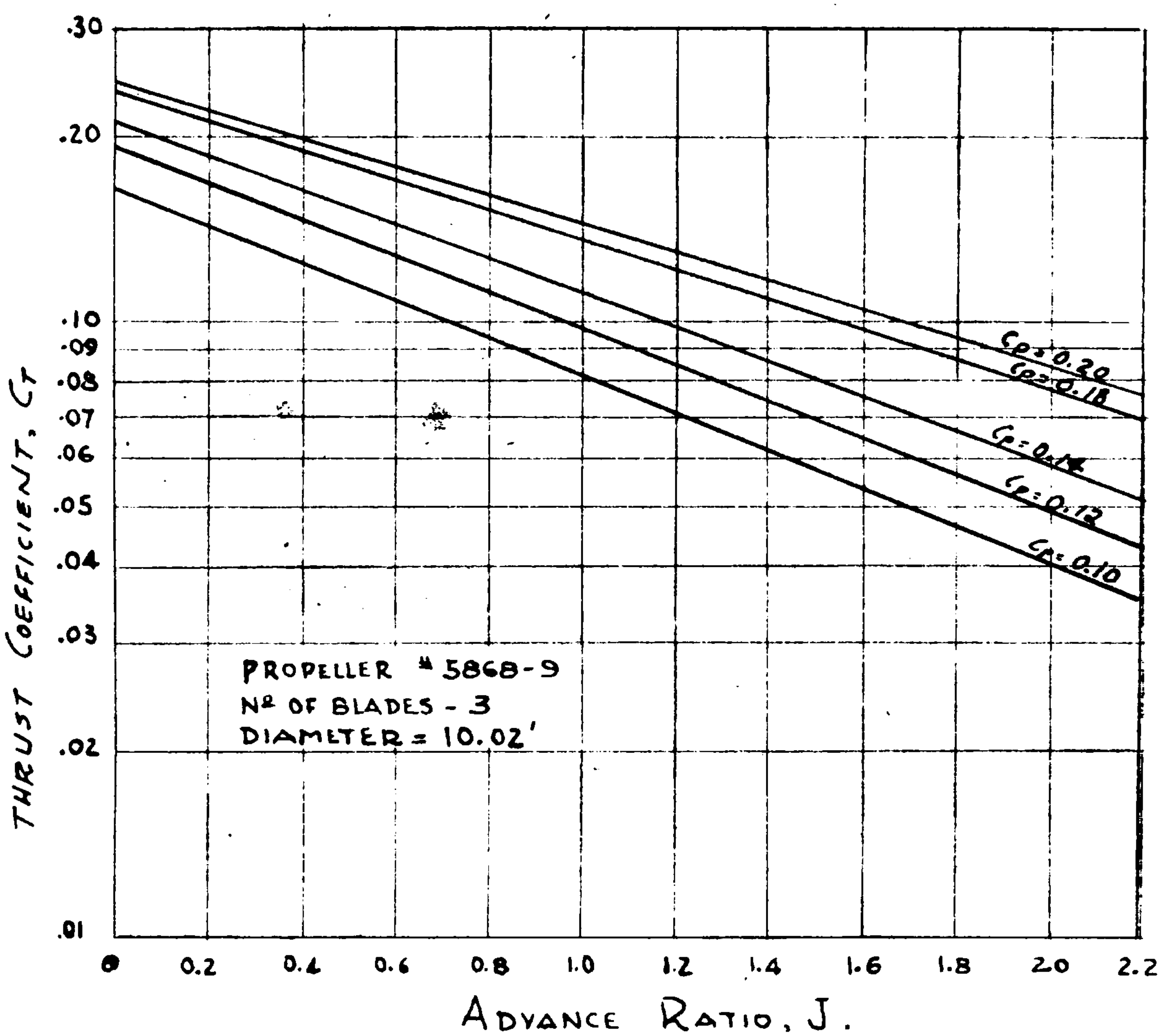
Equations (6) and (7) can be re-written as:

$$A = -fC_p^{-g} \tag{8}$$

and

$$B = hC_p^{-k} \tag{9}$$

[4] E. Kessler, "Propeller Performance Analysis", Aero Digest No. 52, June 1946, pp. 80.



.30
.20
.10
.09
.08
.07
.06
.05
.04
.03
.02
.01
THRUST COEFFICIENT, C_T
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
ADVANCE RATIO, J.
C_P=0.20
C_P=0.18
C_P=0.14
C_P=0.12
C_P=0.10
PROPELLER # 5868-9
Nº OF BLADES - 3
DIAMETER = 10.02'

FIG. 8.

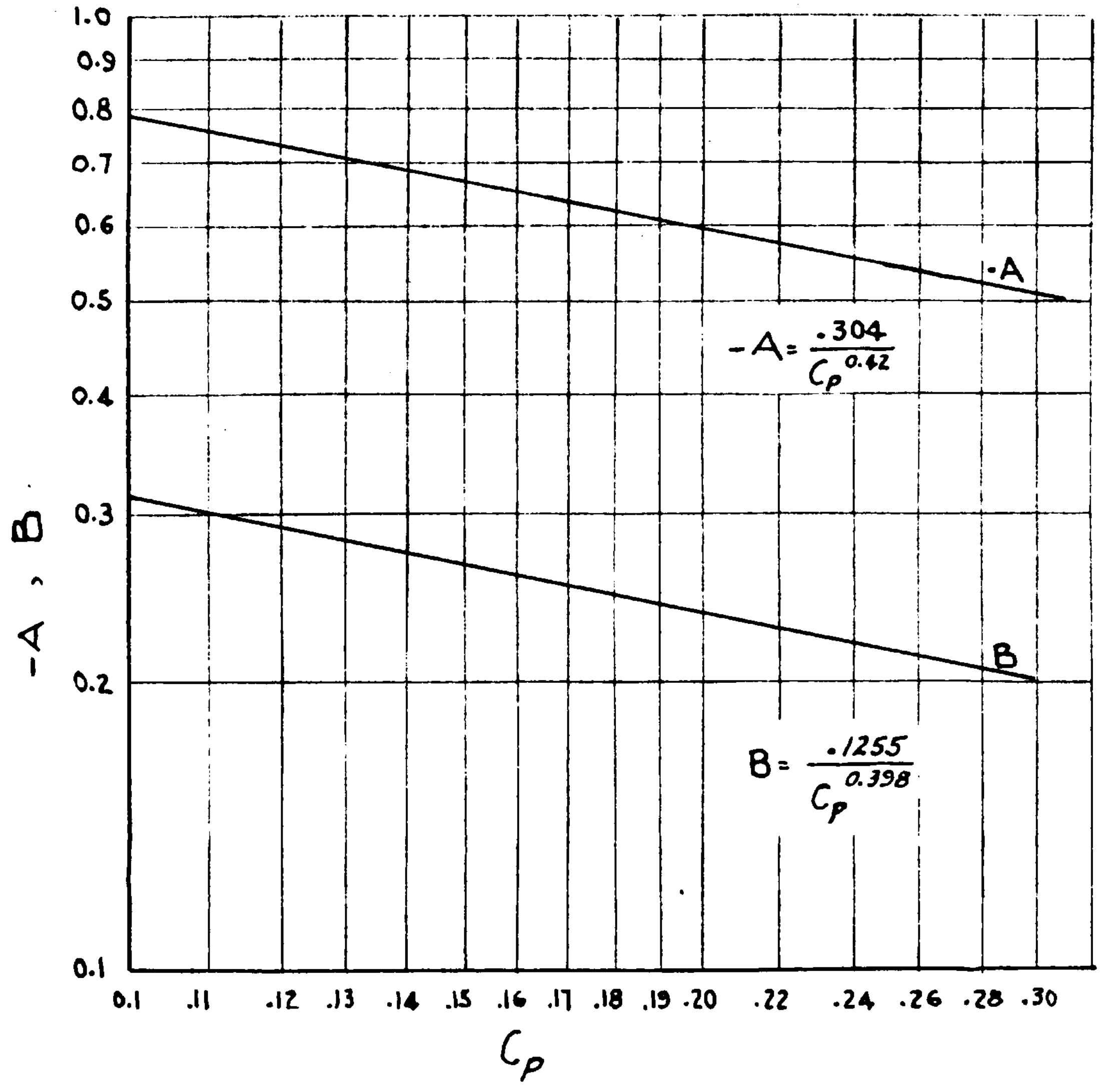

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
-A , B
-A
B
$-A = \frac{.304}{C_p^{0.42}}$
$B = \frac{.1255}{C_p^{0.398}}$
0.1
.11
.12
.13
.14
.15
.16
.17
.18
.19
.20
.22
.24
.26
.28
.30
C_p

FIG. 9

The constants f, g, h, and k, serve not only to identify the propeller, but also to define its performance since:

$$C_T = \epsilon^{-fC_p^{-g} - JhC_p^{-k}}$$

The efficiency now can be expressed as follows:

$$\eta = \frac{C_T}{C_p} J = \frac{J\epsilon^{-fC_p^{-g} - JhC_p^{-k}}}{C_p} \qquad (10)$$

Differentiating Equation (10) with respect to J and setting the derivative equal to zero we find the value of the maximum efficiency for a given C_p value.

Thus:

$$\left.\frac{\partial \eta}{\partial J}\right|_{C_p = \text{Constant}} = \frac{J}{C_p}\left[-hC_p^{-k} \times \epsilon^{-fC_p^{-g} - JhC_p^{-k}}\right] + \left[\frac{\epsilon^{-fC_p^{-g} - JhC_p^{-k}}}{C_p}\right]$$

and for

$$\frac{\partial \eta}{\partial J} = 0 \qquad \left(1 - JhC_p^{-k}\right)\epsilon^{-fC_p^{-g} - JhC_p^{-k}} = 0$$

Therefore for maximum efficiency:

$$J = \frac{C_p^{k}}{h} \qquad (11)$$

and

$$\eta = \frac{J_{MAX}\,\epsilon^{-fC_p^{-g} - 1}}{C_p} \qquad (12)$$

APPENDIX IV - Definition of Symbols

A, B, Δ, h, f, g, K_t, and K .... Constants

C_d .... Drag coefficient.

C_L .... Lift coefficient.

C_p $\left(C_p = \dfrac{P}{\rho N^3 d^5}\right)$ ...... Propeller power coefficient.

C_t $\left(C_t = \dfrac{T}{\rho N^2 d^4}\right)$ .... Propeller thrust coefficient.

D $D = \frac{1}{2}\rho C_d V^2 S$ .... Airfoil drag.

d -(feet) ...... Propeller diameter.

e_1, e_2, e_{01}, e_{02} .... Electrical signals (volts)

F (Lbs/hr) .... Fuel flow rate.

J $\left(J = \dfrac{V}{Nd}\right)$ .... Propeller advance ratio.

BHP .... Brake Horsepower

L $L = \frac{1}{2}\rho C_L V^2 S$ ...... Airfoil lift.

M Servo motor.

M_{br} (lbs-ft) ..... Engine brake moment.

N (rpm) .... Propeller rotational speed.

N_c (rpm) .... Engine rotational speed.

P_{req} (ft-lbs/hr) .. Power required.

P_{av} (ft-lbs/hr) .. Power available.

Q (lbs-ft) ..... Torque moment.

R (Mi/lbs) ..... Range parameter.

r, r_1 (ohms) ..... Resistance.

S (ft^2) ..... Servo amplifier of airfoil area.

s LaPlace transform variable.

SFC $\frac{lbs}{hr\text{-}HP}$ ...... Specific fuel consumption

T (lbs) .... Propeller thrust

T_p Torquemeter oil pressure - $\frac{lbs}{in^2}$

W (lbs) ...... Airplane gross weight.

V $\frac{mi}{hr}$ ...... Airplane velocity (airspeed)

<u>Greek alphabet symbols:</u>

α Propeller angle of attack (degrees)

β Propeller blade angle (degrees)

η Propeller efficiency

θ Angular displacement in radians

ρ Air density $\frac{slugs}{ft^3}$

ω Angular velocity

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