FERRIMAGNETIC RESONANCE IN DOUBLY TRIANGULAR SYSTEMS

by

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PREFACE

In recent years, the need for magnetic materials other than ferromagnetic has stimulated research and theoretical progress in the field of ferrimagnetism. With the arrival of computers, electronic brains, and other recording devices upon the modern scene, a search was made for magnetic materials capable of operating efficiently in the very high frequency ranges. The answer to this challenge was found in non-metallic crystals such as the ferrites. Since they are non-metallic, the ferrites are poor conductors, and can thus eliminate much of the power losses due to eddy currents. In addition, many have low hysteresis loss. Such advantages as these over a metallic ferromagnet are obvious.

In order to explain the fundamental properties of a ferrimagnetic crystal, L. Néel was led to postulate the existence of separate interpenetrating magnetic sublattices within a crystal, which interact in such a way that a net spontaneous magnetization would result. Néel's original two sublattice model was extended recently to more complicated three sublattice systems. One such system, the triangular, was discussed by R. K. Wangsness, and suggests new avenues of research in this field.

This paper considers an extension of the triangular branches, interacting in such a way as to produce a net magnetization. It is not yet known whether or not such systems actually exist in nature, but in anticipation of new discoveries in the field, we look into the problem of the magnetic characteristics of a doubly triangular ferrimagnetic system.

I wish to express my deepest thanks and appreciation to Professor Roald K. Wangsness of The University of Arizona for his very valuable suggestions and assistance while I was preparing this manuscript.

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F.A.B.

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CHAPTER I

INTRODUCTION

Before the advent of the modern theories of magnetism, there were only a few substances known which possessed a spontaneous magnetization; the most common examples are the magnetic group of iron, colbalt and nickel. A material is said to be spontaneously magnetized when small regions or "domains" of the specimen are magnetized to saturation, but the net magnetization of the whole specimen may be zero if all of the domains are oriented at random so that their magnetizations cancel each other when averaged over the specimen. In 1907, P. Weiss postulated that spontaneous magnetization was a molecular phenomenon in which atoms or ions having a permanent magnetic moment engaged in a collective interaction which resulted in their moments becoming aligned. In this way, different regions of a specimen exhibited a net magnetic moment, or magnetization, even in the absence of an external field. Weiss assumed that the actual field acting on the magnetic atoms in a ferromagnetic substance consisted of an internal "molecular" field proportional to the magnetization in addition to any external field. The internal field arose from the collective interaction of all the atoms in the material. Although Weiss' theory was not completely successful quantitatively, it established an excellent approach to magnetic problems for later workers in the field.

More recently, L. Néel proposed that another type of magnetic interaction in certain crystals could lead to a spontaneous magnetization in a specimen. He assumed the existence of unique sublattices in a crystal, each sublattice possessing a net magnetization in a preferred direction. These sublattices might interact with each other, resulting in a net magnetization for the sample. The total field acting on any magnetic atom consisted of an external field and a Weiss-type molecular field due to neighboring atoms on other sublattices. Néel first conceived of a system having two identical sublattices, oriented antiparallel to each other, so that the magnetizations of each practically cancelled the other. Néel called this type of magnetization "antiferromagnetism".

THE PROBLEM

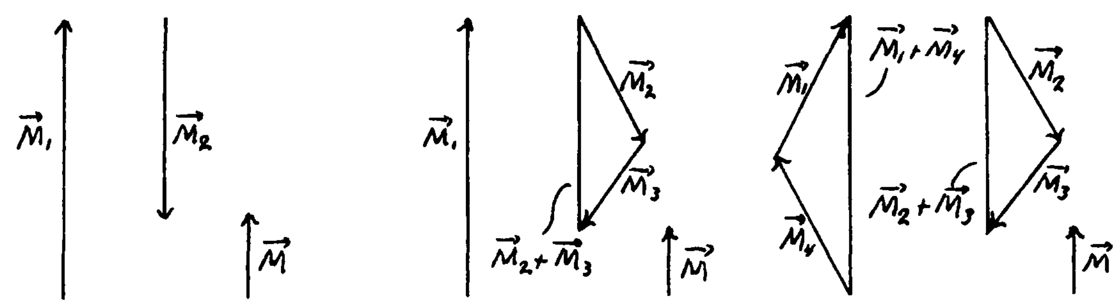
In 1948, Néel suggested still another type of situation within a crystal which could yield a net spontaneous magnetization for the sample. He called this new effect "ferrimagnetism". Ferrimagnetism is attributed to the existence of non-identical interacting magnetic sublattices within a specimen, generally arranged in an antiparallel fashion to yield an observable net magnetization. One possible arrangement of sublattices is the three sublattice triangular system in which one of the antiparallel branches is obtained by the vector addition of two sublattices making arbitrary angles with the third, such that the resultant of this branch is antiparallel to the third sublattice. The triangular ferrimagnetic problem has been investigated very recently,[1] and several unexpected results, which appear completely unique to the triangular system, have been described. By adapting methods which have been successful in previous treatments of simpler arrangements, one might be able to determine the properties of a doubly triangular ferrimagnetic system. It is the purpose of this study to investigate the resonance properties of such a system, and to compare the results with the properties of simpler systems which have been completely solved.

IMPORTANCE OF THE PROBLEM

The doubly triangular ferrimagnetic system consists of four interacting sublattices, arranged in a somewhat symmetrical pattern with respect to some preferred direction in space. In some respects, it is basically similar to both an antiparallel two sublattice system, and to a triangular

[1] R. K. Wangsness, Phys. Rev. 119, 1496 (1960).

ferrimagnetic system, as shown in Fig. 1. This similarity might lead one to suspect that the doubly triangular system should reduce to one of the other two under certain simplifying restrictions. For example, if the vector sums $\vec{M}_1 + \vec{M}_4$ and $\vec{M}_2 + \vec{M}_3$ are considered as the only interacting sublattices in the doubly triangular system, it is apparent that this reduces precisely to the two antiparallel sublattice system.



(a) Two sublattice system (b) Triangular system (c) Doubly triangular system

Fig. 1. Three types of ferrimagnetic systems.

If the combined branch $\vec{M}_1 + \vec{M}_4$ were to interact with both $\vec{M}_2$ and $\vec{M}_3$ separately, this system should be identical with the triangular ferrimagnetic system.

Since systems (a) and (b) have both been completely solved, it should be possible to suspect some possible interesting results for the doubly triangular system by employing such analogies between the various cases. In addition, by treating the doubly triangular system alone, we may be able to determine some of the magnetic properties unique to this system.

METHOD OF TREATMENT

One technique that is extensively used to investigate the magnetic properties of a specimen is known as the method of magnetic resonance, in which the specimen is subjected to various external magnetic fields, and the effects on the specimen are observed. Following the usual theoretical techniques for magnetic resonance problems, we adopt the assumptions of the Weiss molecular field approximation for a magnetic system and set up the semiclassical equations of motion for a system of four coupled magnetic sublattices arranged in a doubly triangular ferrimagnetic pattern. If certain symmetries of the molecular field coefficients are assumed, several conditions on the static magnetizations are obtained. These, combined with other useful relationships among the molecular field coefficients, make it possible to simplify the equations of motion of the system when an external field consisting of a large static component and a small arbitrarily oriented oscillating component is introduced. For certain ratios, these equations reduce exactly to those for a two sublattice or a three sublattice system.

ORGANIZATION

In order to facilitate comprehension of the details of the doubly triangular problem, a brief discussion of magnetic resonance and associated terms is given in Chapter II. This is followed by a presentation in Chapter III of the essential concepts of Néel's theory of ferrimagnetism and Weiss' molecular field approximation. These concepts are illustrated qualitatively by applying them to the simple two antiparallel sublattice system.

When a triangular arrangement is introduced into a coupled system, the equations of motion become much more complicated, but the difficulties are compensated by the discovery of completely new and interesting effects. The highlights of this system are introduced and discussed in Chapter IV.

Chapter V is devoted to the detailed description of the doubly triangular ferrimagnetic system, and the reduction of this system to simpler arrangements under the appropriate assumptions. Chapter VI summarizes the study.

CHAPTER II

MAGNETIC RESONANCE

Any system which possesses an intrinsic magnetic moment will be subjected to a torque when placed in a magnetic field. A simple example of such a system is a bar magnet suspended in a homogeneous magnetic field. The field exerts a torque on the magnet, thus tending to align the magnet with the direction of the applied field. The torque is given by the usual expression

$$\text{Torque} = \vec{\mu} \times \vec{H}, \tag{2.1}$$

where $\vec{\mu}$ is the magnetic moment of the magnet, and $\vec{H}$ the applied field. The torque equals the time rate of change of angular momentum, in the usual language of classical mechanics.

In most cases, it is convenient to consider the magnetic effects of a specimen in an external field in terms of its macroscopic porperties, such as the magnetic moment per unit volume (magnetization), instead of in terms of microscopic phenomena like electron spin. If we let $\vec{M}$ denote the magnetization of a sample subjected to a homogeneous field $\vec{H}$, the equation of motion for the sample becomes.

$$\frac{d\vec{J}}{dt} = \vec{M} \times \vec{H}, \tag{2.2}$$

where $\vec{J}$ represents the angular momentum per unit volume. Suppose the sample is oriented in such a way that its magnetization makes an angle θ with a constant field in the z direction, as illustrated in Fig. 2. The angular momentum and magnetic moment are necessarily parallel, their ratio

$$\gamma = M/J \tag{2.3}$$

being known as the gyromagnetic ratio. Substituting this into (2.2) yields

$$\frac{d\vec{M}}{dt} = \gamma \vec{M} \times \vec{H}. \tag{2.4}$$

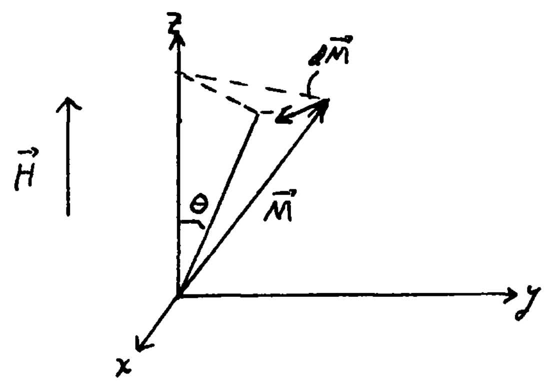


Fig. 2. Precession of a magnetization vector in an external field.

If we assume a time dependent solution such as $M_j \sim e^{i\omega t}$, the three component equations of (2.4) become

$$\begin{aligned} i\omega M_x &= \gamma M_y H \\ i\omega M_y &= -\gamma M_x H \\ \dot{M}_z &= 0 \end{aligned} \tag{2.5}$$

The third equation immediately shows that the z-component of the magnetization is constant in time. Solving the first two for M_x, we get

$$M_x = \gamma^2 H^2 M_x / \omega^2 \quad ,$$

or, cancelling M_x and solving for ω, we find that

$$\omega = \gamma H \quad . \tag{2.6}$$

The solution of (2.4) is a steady precessional motion of $\vec{M}$ about the direction of $\vec{H}$ with an angular velocity $\vec{\omega}$ given by (2.6) and with $\vec{M}$ remaining at a constant angle θ with $\vec{H}$. The precessional frequency ω is called the natural or the resonance frequency of the specimen.

Suppose that, in addition to a constant field in the z direction, a small oscillating field h_x were applied along the x axis. Again assuming a time dependent solution of the form $M_i \sim e^{i\omega t}$, we find that (2.4) yields new component equations as follows:

$$\begin{aligned} i\omega M_x &= \gamma M_y H_z , \\ i\omega M_y &= \gamma (M_z h_x - M_x H_z), \\ i\omega M_z &= -\gamma M_y h_x . \end{aligned} \tag{2.7}$$

If we assume h_x, M_x, and M_y are all small compared with H_z and M_z, then the third equation becomes negligible. As before, we substitute M_y from the second equation into the first and get

$$M_x (1 - \gamma^2 H^2/\omega^2) = -\gamma^2 M_z H_z h_x / \omega^2 .$$

Solving for the x-component of the susceptibility, M_x/h_x we get

$$\chi_x = \frac{M_x}{h_x} = \frac{M_z/H_z}{1-(\omega/\omega_0)^2}, \tag{2.8}$$

where $\omega_0 = \gamma H_z$ denotes the natural or resonance frequency.[2] As the frequency ω of the oscillating field approaches the resonance frequency ω_0, the sample will absorb more and more energy from the field, resulting in its shifting its own orientation in space. It will then have a different component of angular momentum along the z axis, and hence a different energy.

This brief discussion on magnetic resonance and the two examples presented introduce most of the concepts which will be required later on. In the next chapter the pertinent details of ferrimagnetic theory will be discussed.

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[2]Charles Kittel, Introduction to Solid State Physics (John Wiley & Sons, Inc., New York, 1959), 2nd ed., p. 228.

CHAPTER III

THE NATURE OF FERRIMAGNETISM

As introduced in Chapter I, a magnetic material which possesses a net spontaneous magnetization may fall into one of two general classes: ferromagnetic or ferrimagnetic. Ferromagnetism is explained in an elementary way by assuming that the atoms of the material have electrons in various shells, with each electron possessing a magnetic moment due to spin. In some shells, the electron spins are not completely cancelled out, resulting in an uncompensated magnetic moment per atom. The atoms interact so as to favor parallel alignment of their magnetic moments, even in the absence of an external field, yielding a spontaneous magnetization for the material.

THE NÉEL THEORY

L. Néel explained an antiferromagnetic material on a similar basis.[3] He assumed that certain crystals could be divided into two identical interpenetrating sublattices. The magnetic atoms on each sublattice oriented themselves in a parallel arrangement to produce a net spontaneous magneti-

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[3] J. Samuel Smart, Am. J. Phys. 23, 356 (1955).

zation, but the magnetizations of the two sublattices were always antiparallel to each other, so that the net magnetization for the crystal was always zero in the absence of an external field.

The third class of magnetic materials, the ferrimagnets, was also postulated by Néel. A ferrimagnet is essentially the same as an antiferromagnet, except that, in the simplest case, the two interpenetrating sublattices are not identical, due to different kinds of magnetic atoms, or to magnetic atoms occupying different positions in the crystal structures, or both. These differences produce an asymmetry in the sublattices so that the net spontaneous magnetizations are not equal, although they are still antiparallel. Hence, complete cancellation of the sublattice magnetizations does not generally occur, and there will exist a net magnetization for the material, even in the absence of an external field.

The theories of antiferromagnetism and ferrimagnetism are based on the existence of unique sublattices within a crystal. Just as two antiparallel sublattices were assumed in the previous paragraphs, one might postulate the existence of any number of unique, interpenetrating sublattices in a crystal, interacting with each other to produce a net magnetization. Several possible arrangements of sublattices are illustrated schematically in Fig. 3.

The triangular arrangement in Fig. 3(d) was first suggested by Kittel and Yafet in an attempt to explain certain types of magnetic properties which did not appear in Néel's original theory of ferrimagnetism.[4] The details of the triangular ferrimagnetic system will be considered in Chapter IV.

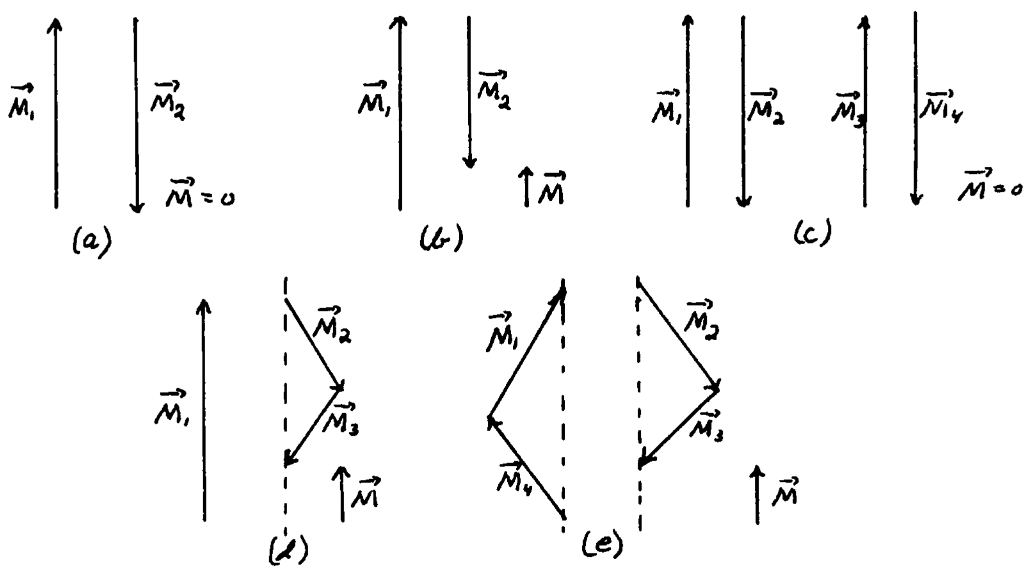

Fig. 3. Schematic diagrams of several possible sublattice arrangements: (a) antiferromagnetic, (b) ferrimagnetic, (c) doubly antiferromagnetic, (d) triangular, and (e) doubly triangular.

THE MOLECULAR FIELD APPROXIMATION

It was pointed out in Chapter I that Weiss, in trying to introduce a mechanism for ferromagnetism, assumed that the actual fields acting on the magnetic atoms of the specimen were composed of two parts, $\vec{H} = \vec{H}_e + \vec{H}_i$, where $\vec{H}_e$ is the ex-

[4] Y. Yafet and C. Kittel, Phys. Rev. 87, 290 (1952).

ternal field, and $\vec{H}_i$ an inner "molecular" field, proportional to the magnetization. That is, $\vec{H}_i = \lambda \vec{M}$, where λ is a coefficient of proportionality called the molecular field coefficient and $\vec{M}$ is the magnetization. More recently, Néel and P. W. Anderson revised the original ideas of Weiss and applied their assumptions to actual magnetic crystals by using Néel's concept of unique interacting sublattices.[5] According to their postulate, the total field acting on a magnetic atom of the j^{th} lattice should be

$$\vec{H}_j = \vec{H} + \sum_k \lambda_{jk} \vec{M}_k \quad , \tag{3.1}$$

where λ_{jk} is the molecular field coefficient for the field of the k^{th} sublattice acting on the atoms of the j^{th} sublattice, $\vec{M}_k$ is the magnetization of the k^{th} sublattice, and $\vec{H}$ is the total external field. It can be shown from the thermodynamical properties of the system, that $\lambda_{ij} = \lambda_{ji}$. The molecular field coefficients are seen to be of two types. The first type λ_{jj} is a measure of the magnitude of the intra-sublattice interaction, while the second kind, λ_{jk} , gives the inter-sublattice effects. Hence, when one sums over all the sublattices as shown in (3.1), the total molecular field will be due to both inter and intra-sublattice fields. In order to agree with experiment, it is found that the molecular fields must be extremely large compared to any ordinary external field.

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[5] P. W. Anderson, Phys. Rev. **79**, 705 (1950).

THE TWO SUBLATTICE MODEL

To illustrate the ideas introduced thus far in this chapter, let us consider a simple two sublattice model, as shown in Fig. 3(b). The total field acting on each sublattice is given by expanding equation (3.1):

$$\begin{aligned} \vec{H}_1 &= \vec{H} + \lambda_{11}\vec{M}_1 + \lambda_{12}\vec{M}_2 , \\ \vec{H}_2 &= \vec{H} + \lambda_{21}\vec{M}_1 + \lambda_{22}\vec{M}_2 . \end{aligned} \tag{3.2}$$

Let us assume $\vec{H}$ to be a constant field in the z direction. It must be mentioned that in such a case as this, we are neglecting all demagnetizing and anisotropy fields. If (3.2) is substituted into the equation of motion

$$d\vec{M}_i/dt = \gamma_i \vec{M}_i \times \vec{H}_i ,$$

one notices that the intra-sublattice fields vanish in the vector product. All that remains are two coupled equations

$$\begin{aligned} \dot{\vec{M}}_1 &= \gamma_1 \left[\vec{M}_1 \times \vec{H} + \lambda_{12}\vec{M}_1 \times \vec{M}_2 \right] , \\ \dot{\vec{M}}_2 &= \gamma_2 \left[\vec{M}_2 \times \vec{H} + \lambda_{21}\vec{M}_2 \times \vec{M}_1 \right] . \end{aligned} \tag{3.3}$$

If each of these equations is divided by its corresponding gyromagnetic ratio and both equations added together, we get

$$\frac{d}{dt}\left[\frac{\vec{M}_1}{\gamma_1} + \frac{\vec{M}_2}{\gamma_2} \right] = (\vec{M}_1 + \vec{M}_2) \times \vec{H}, \tag{3.4}$$

where we have used the relation $\lambda_{12} = \lambda_{21}$. Equation (3.4) is very similar to (2.4) for a single sublattice. If we assume very strong coupling between the two sublattices, we can treat the system in an approximate manner by writing

$$\frac{d\vec{J}'}{dt} = \vec{M}' \times \vec{H} ,$$

where $\vec{J}' = (\vec{M}_1/\gamma_1) + (\vec{M}_2/\gamma_2)$ is the total angular momentum and $\vec{M}' = \vec{M}_1 + \vec{M}_2$ the total magnetization of the system. Referring to (2.6), the natural frequency of the strongly coupled system can be written

$$\omega = \gamma_{eff} H ,$$

where γ_{eff} is an "effective"gyromagnetic ratio for the whole system, given by

$$\gamma_{eff} = \frac{M'}{J'} = \frac{M_1 + M_2}{(M_1/\gamma_1) + (M_2/\gamma_2)} . \qquad (3.5)$$

One might suspect a similar result for more complicated magnetic systems. We shall show that this is the case for a triangular ferrimagnetic system, and from this one might postulate the existence of an effective gyromagnetic ratio for any system, although the form it may take might not be as simple an expression as (3.5). Let us now look at some of the features of the triangular system.

CHAPTER IV

THE THREE SUBLATTICE TRIANGULAR SYSTEM

The general method of approach to magnetic resonance problems should be clear from the material introduced in Chapter II and from the example given toward the end of the last chapter. Two essential concepts which must be used in describing ferrimagnetic systems were seen to be Néel's theory of interpenetrating sublattices and the Weiss molecular field approximation. We are now in a position where we can briefly discuss the important features of the triangular ferrimagnetic system. Since the problem of resonance in triangular systems has already been completely solved,[1] we shall present only the highlights of this system.

The sublattice arrangement was described earlier, and is shown schematically in Fig. 1(b). In this case, let us assume an applied external field consisting of a large constant component in the z direction, parallel to sublattice 1, plus a small arbitrarily-oriented oscillating component $\vec{h}$. As before, the total fields are given by (3.1), and the same symmetry of the molecular field coefficients mentioned in that paragraph also holds here. In triangular arrangements, we may take advantage of the fact that in the static case, the sublattice magnetizations of the specimen must be parallel to the total fields acting on them. If we do this, when the total external field is zero, we obtain several simplifying conditions which must hold at all times:

$$\lambda_{12} = \lambda_{13} \tag{4.1}$$

$$\lambda_{12}\vec{M}_1^\circ + \lambda_{23}\left(\vec{M}_2^\circ + \vec{M}_3^\circ\right) = 0 \tag{4.2}$$

where $\vec{M}_i^\circ$ denotes the static spontaneous magnetizations. Since (4.2) is a vector equation, and since $\vec{M}_1^\circ$ is in the z direction only, we get the following conditions on the components of the static magnetizations:

$$M_{1z}^\circ + \frac{\lambda_{23}}{\lambda_{12}}\left(M_{2z}^\circ + M_{3z}^\circ\right) = 0, \tag{4.3}$$

$$M_{2x}^\circ + M_{3x}^\circ = M_{2y}^\circ + M_{3y}^\circ = 0; \tag{4.4}$$

One must keep in mind that these conditions hold only for the triangular arrangements.

The equations of motion for the sublattices under the influence of the external fields described above take the usual form

$$d\vec{M}_i/dt = \gamma_i \vec{M}_i \times \vec{H}_i \,. \tag{4.5}$$

The effect of the external field is to induce a small component of magnetization, so that the total sublattice magnetization components can be written

$$\begin{aligned}
M_{1x} &= m_{1x}, & M_{1y} &= m_{1y}, & M_{1z} &= M_{1z}^\circ + m_{1z}^\circ + m_{1z}, \\
M_{2x} &= M_{2x}^\circ + m_{2x}, & M_{2y} &= M_{2y}^\circ + m_{2y}, & M_{2z} &= M_{2z}^\circ + m_{2z}^\circ + m_{2z}, \\
M_{3x} &= M_{3x}^\circ + m_{3x}, & M_{3y} &= M_{3y}^\circ + m_{3y}, & M_{3z} &= M_{3z}^\circ + m_{3z}^\circ + m_{3z}.
\end{aligned} \tag{4.6}$$

where M_{ij}° are the large static components, m_{iz}° are small static components induced by the large constant field $\vec{H}$, and

m_{ij} are small oscillating components. The m_{iz}° must be introduced to satisfy the requirement that $m_{ij} \longrightarrow 0$ as $\vec{h} \longrightarrow 0$. Under this restriction, it is found that the m_{iz}° satisfy the equations

$$\begin{aligned} \lambda_{12} m_{1z}^{\circ} + \lambda_{23}\left(m_{2z}^{\circ} + m_{3z}^{\circ}\right) = 0, \\ \lambda_{13} m_{1z}^{\circ} + \lambda_{23}\left(m_{2z}^{\circ} + m_{3z}^{\circ}\right) = 0. \end{aligned} \tag{4.7}$$

Now when (4.6) is substituted into the equations of motion (4.5), with the conditions (4.1), (4.3), (4.4) and (4.7) kept in mind, and assuming that $m_{ij} \sim e^{i\omega t} \sim h_j$, one gets the complete set of equations for the system. In this process, the equations are linearized by neglecting all terms of second order in m_{ij}.

This set of equations can be solved for several special cases. First, if one assumes that the gyromagnetic ratios for sublattices 2 and 3 are equal, that is, if $\gamma_2 = \gamma_3$, it turns out that the results reduce exactly to those for an antiparallel two sublattice system,[6] described by an effective gyromagnetic ratio equal to (3.5).

When $\gamma_2 \neq \gamma_3$, the general solution of the triangular system yields a more complicated effective gyromagnetic ratio

$$\frac{1}{\gamma_{eff}} = \frac{1}{\beta - 1}\left[\frac{\beta}{\gamma_1^2} + \frac{1}{\gamma_2\gamma_3} - \frac{1}{\gamma_1}\left(\frac{1}{\gamma_2} + \frac{1}{\gamma_3}\right)\right], \tag{4.8}$$

where $\beta = \lambda_{23}/\lambda_{12} = \lambda_{23}/\lambda_{13}$. In addition, it is found that there exist both longitudinal and transverse oscillating

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[6] A. Eskowitz and R. K. Wangsness, Phys. Rev. <u>107</u>, 379 (1957).

magnetization components of the same frequency as the oscillating external field. The longitudinal component is parallel to the z direction, and arises from all three oscillating components of the field h_x, h_y, and h_z. The transverse components lie in the x-y plane, and are also induced by all three components of the oscillating field. These results are indicated conveniently in tensor notation

$$m_i = \chi_{ij} h_j \quad ; \quad i, j = x, y, z. \tag{4.9}$$

Several important special cases may be reviewed. First, consider the case when $h_z = 0$, so that $\vec{h} = h_x\hat{i} + h_y\hat{j}$. Equation (4.9) reveals that there still exists an induced oscillating magnetization component in the z direction, a result that is not intuitively evident at first sight. This new effect was predicted by R. K. Wangsness,[1] and the reader may find a complete discussion of the effect in the liturature.

The second case arises when $h_x = h_y = 0$, so that $\vec{h} = h_z\hat{k}$. Once again, a new effect appears in the production of oscillating magnetization components both parallel and perpendicular to the applied oscillating field.[7]

At this time, these predictions have not yet been experimentally verified. More experimentation is needed.

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[7] R. K. Wangsness, Phys. Rev. (in press).

CHAPTER V

THE DOUBLY TRIANGULAR FERRIMAGNETIC SYSTEM

In the previous chapter, some of the salient features of the triangular ferrimagnetic system have been discussed. Now we would like to extend the ideas of a triangular system to an arrangement consisting of two triangular branches. This arrangement is shown schematically in Fig. 4, where we denote the two triangular branches as A and B. The magneti-

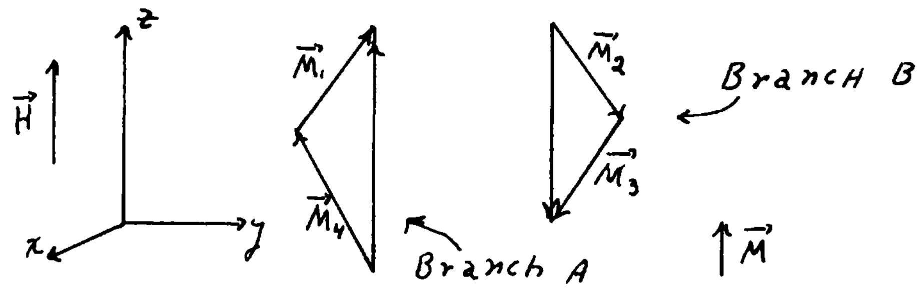

Fig. 4. The doubly triangular arrangement.

zations of the two sublattices of each branch add vectorially to give a resultant magnetization for the branch. The net branch magnetizations are oriented antiparallel to each other. It must be pointed out that the sublattice magnetizations depicted in Fig. 4. are not coplanar in general. However, since $\vec{M}_A$ and $\vec{M}_B$ are antiparallel, they define a plane in space. We choose $\vec{M}_A$ to be in the positive z direction.

Since resonance phenomena pertaining to this doubly triangular system are of primary interest, an external field consisting of a large static component in the z direction and a small arbitrarily-oriented oscillating component is applied over the specimen. If we take the molecular fields into account, the field $\vec{H}_i$ acting on the i^{th} sublattice takes the form

$$\vec{H}_i = \vec{H}_e + \sum_{j=1}^{4} \lambda_{ij} M_j , \qquad (5.1)$$

where $\vec{H}_e = H\hat{k} + \vec{h}$ is the total external, field applied across the specimen. The oscillating field $\vec{h}$ is very small compared to $\vec{H}$, and each component h_x, h_y, h_z is proportional to $e^{i\omega t}$. The second term in (5.1) represents a sum of Weiss-type internal fields arising from sublattice interaction. The λ_{ij} are the molecular field coefficients and $\vec{M}_j$ is the magnetization of the j^{th} sublattice. When (5.1) is written, terms involving both inter and intra-sublattice interactions occur. It can be shown in general that $\lambda_{ij} = \lambda_{ji}$, but that $\lambda_{ii} \neq \lambda_{jj}$ for $i \neq j$ unless the sublattices i and j are identical, which we will assume not to be the case.

If we expand (5.1) and rearrange the terms, using the fact that $\vec{M}_1 + \vec{M}_4 = \vec{M}_A$ and $\vec{M}_2 + \vec{M}_3 = \vec{M}_B$, we get

$$\vec{H}_1 = \vec{H}_e + (\lambda_{11} - \lambda_{14})\vec{M}_1 + \lambda_{14}\vec{M}_A + \lambda_{13}\vec{M}_B + (\lambda_{12} - \lambda_{13})\vec{M}_2 , \qquad (5.2.a)$$

$$\vec{H}_2 = \vec{H}_e + (\lambda_{22} - \lambda_{23})\vec{M}_2 + \lambda_{24}\vec{M}_A + \lambda_{23}\vec{M}_B + (\lambda_{12} - \lambda_{24})\vec{M}_1 , \qquad (5.2.b)$$

$$\vec{H}_3 = \vec{H}_e + (\lambda_{33} - \lambda_{23})\vec{M}_3 + \lambda_{34}\vec{M}_A + \lambda_{23}\vec{M}_B + (\lambda_{13} - \lambda_{34})\vec{M}_1 , \qquad (5.2.c)$$

$$\vec{H}_4 = \vec{H}_e + (\lambda_{44} - \lambda_{14})\vec{M}_4 + \lambda_{14}\vec{M}_A + \lambda_{34}\vec{M}_B + (\lambda_{24} - \lambda_{34})\vec{M}_2 . \qquad (5.2.d)$$

In the static case, when the external field is zero, the sublattice magnetizations must be parallel to the total fields acting on them. Applying this condition to equations (5.2), and denoting the static magnetization components by $\vec{M}_i^\circ$, we get four conditions on the static components:

$$\lambda_{14}\vec{M}_A^\circ + \lambda_{13}\vec{M}_B^\circ + (\lambda_{12}-\lambda_{13})\vec{M}_2^\circ = 0, \tag{5.3.a}$$

$$\lambda_{24}\vec{M}_A^\circ + \lambda_{23}\vec{M}_B^\circ + (\lambda_{12}-\lambda_{24})\vec{M}_1^\circ = 0, \tag{5.3.b}$$

$$\lambda_{34}\vec{M}_A^\circ + \lambda_{23}\vec{M}_B^\circ + (\lambda_{13}-\lambda_{34})\vec{M}_1^\circ = 0, \tag{5.3.c}$$

$$\lambda_{14}\vec{M}_A^\circ + \lambda_{34}\vec{M}_B^\circ + (\lambda_{24}-\lambda_{34})\vec{M}_3^\circ = 0. \tag{5.3.d}$$

These are vector equations. If the x, y, and z components of (5.3.a) are separated, we obtain these three conditions:

$$(\lambda_{12}-\lambda_{13})M_{2x}^\circ = 0, \tag{5.4.a}$$

$$(\lambda_{12}-\lambda_{13})M_{2y}^\circ = 0, \tag{5.4.b}$$

$$\lambda_{14}M_A^\circ + \lambda_{13}M_B^\circ = 0. \tag{5.4.c}$$

Equations (5.4.a) and (5.4.b) immediately indicate that $\lambda_{12} = \lambda_{13}$. If a similar process is caried out for (5.3.b) through (5.3.d), it is found that $\lambda_{12} = \lambda_{24}$, $\lambda_{13} = \lambda_{34}$, and $\lambda_{24} = \lambda_{34}$, and that when all are combined we get

$$\lambda_{12} = \lambda_{13} = \lambda_{24} = \lambda_{34} = \lambda. \tag{5.5}$$

From (5.4.c) and a similar result obtained from the z component of (5.3.b), two simultaneous equations relating the three remaining independent molecular field coefficients are obtained:

$$\lambda_{14}\vec{M}_A + \lambda\vec{M}_B = 0, \tag{5.6.a}$$

$$\lambda\vec{M}_A + \lambda_{23}\vec{M}_B = 0. \tag{5.6.b}$$

Since any set of n simultaneous linear homogeneous equations in n unknowns has nontrivial solutions only if the determinant of the coefficients vanishes, a final relationship if found:

$$\lambda^2 = \lambda_{14} \lambda_{23} . \tag{5.7}$$

It will be convenient to define

$$\beta = \lambda/\lambda_{14} = \lambda_{23}/\lambda , \tag{5.8.a}$$

so that

$$\vec{M}_A + \beta \vec{M}_B = 0 . \tag{5.8.b}$$

We have previously chosen $\vec{M}_A$ and $\vec{M}_B$ to be parallel to the z axis. This being the case, we know immediately that

$$M_{1x}^{\circ} + M_{4x}^{\circ} = 0 , \quad M_{1y}^{\circ} + M_{4y}^{\circ} = 0, \tag{5.9.a}$$

$$M_{2x}^{\circ} + M_{3x}^{\circ} = 0 , \quad M_{2y}^{\circ} + M_{3y}^{\circ} = 0. \tag{5.9.b}$$

Equations (5.5), (5.8), and (5.9) will be of great importance in the remainder of this paper. These relationships were all extracted from the static case, when the external field over the specimen was zero.

Now let the external field $\vec{H}_e = H\hat{k} + \vec{h}$ be applied. The sublattice equations of motion for the system are of the usual form

$$d\vec{M}_i/dt = \gamma_i \vec{M}_i \times \vec{H}_i , \tag{5.10}$$

where γ_i is the gyromagnetic ratio of the i^{th} sublattice, $\vec{M}_i$ is the magnetization of the i^{th} sublattice, and $\vec{H}_i$ is the total field acting on the i^{th} sublattice, and is given explicitly by (5.1). This field causes the sublattice magnetizations to be displaced slightly from their static equilibrium positions, and induces a small component of magnetization in each sublattice. The sublattice magnetizations can then be written as follows:

$$M_{ij} = M^{\circ}_{ij} + m'_{ij}; \quad i = 1,2,3,4; \quad j = x,y,z. \tag{5.11}$$

Here we have denoted M°_{ij} as the large static components, and $m'_{ij} = m^{\circ}_{ij} + m_{ij}$, where m°_{ij} are very small static components induced when $\vec{H} \neq 0$, and m_{ij} are the induced oscillating components. We have to introduce the small induced static components m°_{ij} so that the induced oscillating components will go to zero as $\vec{h} \longrightarrow 0$. This is a requirement unique to triangular arrangements, and is necessary to preserve the consistency of equations (5.10) when the small oscillating field is turned off. We assume that all the small oscillating components m_{ij} are proportional to $e^{i\omega t}$; that is, they vary as the field that induces them.

We are now ready to substitute (5.1) and (5.11) into the equations of motion (5.10), and obtain four equations, one for each sublattice. The first equation, for sublattice 1 becomes

$$d\vec{M}_1/dt = \gamma_1 \vec{M}_1 \times \vec{H}_1 = \gamma_1\left[\vec{M}_1 \times (H\hat{k} + \vec{h} + \lambda_{11}\vec{M}_1 + \lambda\vec{M}_2 + \lambda\vec{M}_3 + \lambda_{14}\vec{M}_4)\right]$$

$$= \gamma_1\left[\vec{M}_1 \times H\hat{k} + \vec{M}_1 \times \vec{h} + \lambda\vec{M}_1 \times \vec{M}_2 + \lambda\vec{M}_1 \times \vec{M}_3 + \lambda_{14}\vec{M}_1 \times \vec{M}_4\right], \tag{5.12}$$

where we have used condition (5.5). This is a vector equation, and contains equations for the three components M_{1x}, M_{1y}, and M_{1z}. If we write these out, taking into account (5.8), (5.9) and (5.11), plus the fact that $dM_{ij}/dt = i\omega m_{ij}$, and keeping only first order terms in small quantities, we find that they take the form:

$$i\omega m_{1x}/\gamma_1 = M^\circ_{1y}\left[H + h_z + \lambda(m'_{2z}+m'_{3z}) + \lambda_{14}(m'_{1z}+m'_{4z})\right]$$
$$- M^\circ_{1z}\left[h_y + \lambda(m'_{2y}+m'_{3y}) + \lambda_{14}(m'_{1y}+m'_{4y})\right] + m'_{1y}H, \qquad (5.13.a)$$

$$i\omega m_{1y}/\gamma_1 = -M^\circ_{1x}\left[H + h_z + \lambda(m'_{2z}+m'_{3z}) + \lambda_{14}(m'_{1z}+m'_{4z})\right]$$
$$+ M^\circ_{1z}\left[h_x + \lambda(m'_{2z}+m'_{3z}) + \lambda_{14}(m'_{1x}+m'_{4z})\right] - m'_{1x}H, \qquad (5.13.b)$$

$$i\omega m_{1z}/\gamma_1 = M^\circ_{1x}\left[h_y + \lambda(m'_{2y}+m'_{3y}) + \lambda_{14}(m'_{1y}+m'_{4y})\right]$$
$$- M^\circ_{1y}\left[h_x + \lambda(m'_{2x}+m'_{3x}) + \lambda_{14}(m'_{1x}+m'_{4x})\right]. \qquad (5.13.c)$$

There are nine more equations similar to (5.13), corresponding to the three components of $\vec{M}_4$, $\vec{M}_2$, and $\vec{M}_3$.

At this point, it is wise to check our system of equations to see if they comprise a consistent set, so that a solution would be possible. It is convenient to do this for a special case by letting $\vec{h} \longrightarrow 0$, so that all the induced oscillating magnetization components $m_{ij} \longrightarrow 0$. When this is done, all twelve equations become:

$$m_{1x}:\quad 0 = M^\circ_{1y}\left[H + \lambda(m^\circ_{2z}+m^\circ_{3z}) + \lambda_{14}(m^\circ_{1z}+m^\circ_{4z})\right]$$
$$- M^\circ_{1z}\left[\lambda(m^\circ_{2y}+m^\circ_{3y}) + \lambda_{14}(m^\circ_{1y}+m^\circ_{4y})\right] + m^\circ_{1y}H, \qquad (5.14.a)$$

$$m_{1y}:\quad 0 = -M^\circ_{1x}\left[H + \lambda(m^\circ_{2z}+m^\circ_{3z}) + \lambda_{14}(m^\circ_{1z}+m^\circ_{4z})\right]$$
$$+ M^\circ_{1z}\left[\lambda(m^\circ_{2x}+m^\circ_{3x}) + \lambda_{14}(m^\circ_{1x}+m^\circ_{4x})\right] - m^\circ_{1x}H, \qquad (b)$$

$$m_{1z}:\quad 0 = M^\circ_{1x}\left[\lambda(m^\circ_{2y}+m^\circ_{3y}) + \lambda_{14}(m^\circ_{1y}+m^\circ_{4y})\right]$$
$$- M^\circ_{1y}\left[\lambda(m^\circ_{2x}+m^\circ_{3x}) + \lambda_{14}(m^\circ_{1x}+m^\circ_{4x})\right], \qquad (c)$$

$$m_{2x}:\quad 0 = M^\circ_{2y}\left[H + \lambda(m^\circ_{1z} + m^\circ_{4z}) + \lambda_{23}(m^\circ_{2z} + m^\circ_{3z})\right]$$
$$- M^\circ_{2z}\left[\lambda(m^\circ_{1y} + m^\circ_{4y}) + \lambda_{23}(m^\circ_{2y} + m^\circ_{3y})\right] + m^\circ_{2y} H, \qquad \text{(d)}$$

$$m_{2y}:\quad 0 = -M^\circ_{2x}\left[H + \lambda(m^\circ_{1z} + m^\circ_{4z}) + \lambda_{23}(m^\circ_{2z} + m^\circ_{3z})\right]$$
$$+ M^\circ_{2z}\left[\lambda(m^\circ_{1x} + m^\circ_{4x}) + \lambda_{23}(m^\circ_{2x} + m^\circ_{3x})\right] - m^\circ_{2x} H, \qquad \text{(e)}$$

$$m_{2z}:\quad 0 = M^\circ_{2x}\left[\lambda(m^\circ_{1y} + m^\circ_{4y}) + \lambda_{23}(m^\circ_{2y} + m^\circ_{3y})\right]$$
$$- M^\circ_{2y}\left[\lambda(m^\circ_{1x} + m^\circ_{4x}) + \lambda_{23}(m^\circ_{2x} + m^\circ_{3x})\right], \qquad \text{(f)}$$

$$m_{3x}:\quad 0 = M^\circ_{2y}\left[H + \lambda(m^\circ_{1z} + m^\circ_{4z}) + \lambda_{23}(m^\circ_{2z} + m^\circ_{3z})\right]$$
$$- M^\circ_{3z}\left[\lambda(m^\circ_{1y} + m^\circ_{4y}) + \lambda_{23}(m^\circ_{2y} + m^\circ_{3y})\right] + m^\circ_{3y} H, \qquad \text{(g)}$$

$$m_{3y}:\quad 0 = -M^\circ_{3x}\left[H + \lambda(m^\circ_{1z} + m^\circ_{4z}) + \lambda_{23}(m^\circ_{2z} + m^\circ_{3z})\right]$$
$$+ M^\circ_{3z}\left[\lambda(m^\circ_{1x} + m^\circ_{4x}) + \lambda_{23}(m^\circ_{2x} + m^\circ_{3x})\right] - m^\circ_{3x} H, \qquad \text{(h)}$$

$$m_{3z}:\quad 0 = M^\circ_{3x}\left[\lambda(m^\circ_{1y} + m^\circ_{4y}) + \lambda_{23}(m^\circ_{2y} + m^\circ_{3y})\right]$$
$$- M^\circ_{3y}\left[\lambda(m^\circ_{1x} + m^\circ_{4x}) + \lambda_{23}(m^\circ_{2x} + m^\circ_{3x})\right], \qquad \text{(i)}$$

$$m_{4x}:\quad 0 = M^\circ_{4y}\left[H + \lambda(m^\circ_{2z} + m^\circ_{3z}) + \lambda_{14}(m^\circ_{1z} + m^\circ_{4z})\right]$$
$$- M^\circ_{4z}\left[\lambda(m^\circ_{2y} + m^\circ_{3y}) + \lambda_{14}(m^\circ_{1y} + m^\circ_{4y})\right] + m^\circ_{4y} H, \qquad \text{(j)}$$

$$m_{4y}:\quad 0 = -M^\circ_{4x}\left[H + \lambda(m^\circ_{2z} + m^\circ_{3z}) + \lambda_{14}(m^\circ_{1z} + m^\circ_{4z})\right]$$
$$+ M^\circ_{4z}\left[\lambda(m^\circ_{2x} + m^\circ_{3x}) + \lambda_{14}(m^\circ_{1x} + m^\circ_{4x})\right] - m^\circ_{4x} H, \qquad \text{(k)}$$

$$m_{4z}:\quad 0 = M^\circ_{4x}\left[\lambda(m^\circ_{2y} + m^\circ_{3y}) + \lambda_{14}(m^\circ_{1y} + m^\circ_{4y})\right]$$
$$- M^\circ_{4y}\left[\lambda(m^\circ_{2x} + m^\circ_{3x}) + \lambda_{14}(m^\circ_{1x} + m^\circ_{4x})\right]. \qquad \text{(l)}$$

If we add (5.14.a) to (5.14.j) and (5.14.d) to (g), using once again conditions (5.9), we will get two equations:

$$-(M_{1z}^{\circ}+M_{4z}^{\circ})\left[\lambda(m_{2y}^{\circ}+m_{3y}^{\circ})+\lambda_{14}(m_{1y}^{\circ}+m_{4y}^{\circ})\right]+(m_{1y}^{\circ}+m_{4y}^{\circ})H=0, \quad (5.15.a)$$

$$-(M_{2z}^{\circ}+M_{3z}^{\circ})\left[\lambda(m_{1y}^{\circ}+m_{4y}^{\circ})+\lambda_{23}(m_{2y}^{\circ}+m_{3y}^{\circ})\right]+(m_{2y}^{\circ}+m_{3y}^{\circ})H=0. \quad (5.15.b)$$

If we add these and rearrange terms, this becomes

$$-(m_{1y}^{\circ}+m_{4y}^{\circ})\left[\lambda_{14}(M_{1z}^{\circ}+M_{4z}^{\circ})+\lambda(M_{2z}^{\circ}+M_{3z}^{\circ})\right]$$
$$-(m_{2y}^{\circ}+m_{3y}^{\circ})\left[\lambda(M_{1z}^{\circ}+M_{4z}^{\circ})+\lambda_{23}(M_{2z}^{\circ}+M_{3z}^{\circ})\right]+(m_{1y}^{\circ}+m_{2y}^{\circ}+m_{3y}^{\circ}+m_{4y}^{\circ})H=0.$$

The first two terms are identically zero because of (5.6). We are left with the relation

$$(m_{1y}^{\circ}+m_{4y}^{\circ})=-(m_{2y}^{\circ}+m_{3y}^{\circ}).$$

If this is substituted back into (5.15) and the terms rearranged, we get the important result that

$$(m_{1y}^{\circ}+m_{4y}^{\circ})=0=-(m_{2y}^{\circ}+m_{3y}^{\circ}). \quad (5.16)$$

If a similar process is applied to equations (5.14.b), (k), (e), and (h), we also find that

$$(m_{1x}^{\circ}+m_{4x}^{\circ})=0=-(m_{2x}^{\circ}+m_{3x}^{\circ}). \quad (5.17)$$

These two results, when substituted into equation (5.14) enable us to considerably simplify the twelve equations of motion. It is immediately apparent that each of the z component equations are consistent. By now subtracting (5.14.j) from (a), (g) from (d), (k) from (b), and (h) from (e), we find that the small static magnetization components must satisfy the following conditions:

$$H(1+m_{1x,y}^{\circ}/M_{1x,y}^{\circ})+\lambda(m_{2z}^{\circ}+m_{3z}^{\circ})+\lambda_{14}(m_{1z}^{\circ}+m_{4z}^{\circ})=0, \quad (5.18.a)$$

$$H(1+m_{2x,y}^{\circ}/M_{2x,y}^{\circ})+\lambda(m_{1z}^{\circ}+m_{4z}^{\circ})+\lambda_{23}(m_{2z}^{\circ}+m_{3z}^{\circ})=0. \quad (5.18.b)$$

These four conditions must be satisfied in order that a general solution to the coupled system exists.

In order to focus our attention on the equations of motion, let us put them in a compact form, using (5.18) and the following notation:

$$A_{1,2,3,4} = i\omega\, \eta_{1,2,3,4} \tag{5.19.a}$$

$$B_i = \lambda M^{\circ}_{iz}; \qquad i = 1,2,3,4 \tag{b}$$

$$C_i = \lambda M^{\circ}_{ix}; \qquad i = 1,2 \tag{c}$$

$$D_i = \lambda M^{\circ}_{iy}; \qquad i = 1,2 \tag{d}$$

$$E_i = \lambda_{14} M^{\circ}_{iz}; \qquad i = 1,4 \tag{e}$$

$$F = \lambda_{14} M^{\circ}_{1x}; \qquad G = \lambda_{14} M^{\circ}_{1y} \tag{f}$$

$$H_i = \lambda_{23} M^{\circ}_{iz}; \qquad i = 2,3 \tag{g}$$

$$I = \lambda_{23} M^{\circ}_{2x}; \qquad J = \lambda_{23} M^{\circ}_{2y}. \tag{h}$$

The equations of motion are a set of twelve simultaneous nonhomogeneous equations in twelve unknowns, the m_{ij}'s. In order to save space, the complete set is written below in matrix notation, with the coefficient matrix given in terms of (5.19):

$$\mathbf{AX} = \mathbf{Y}, \tag{5.20}$$

$$A = \begin{bmatrix}
A_1 & E_1-H & -G & 0 & B_1 & -D_1 & 0 & B_1 & -D_1 & 0 & E_1 & -G \\
-E_1+H & A_1 & F & -B_1 & 0 & C_1 & -B_1 & 0 & C_1 & -E_1 & 0 & F \\
G & -F & A_1 & D_1 & -C_1 & 0 & D_1 & -C_1 & 0 & G & -F & 0 \\
0 & B_2 & -D_2 & A_2 & H_2-H & -J_1 & 0 & H_2 & -J_1 & 0 & B_2 & -D_2 \\
-B_2 & 0 & C_2 & -H_2+H & A_2 & I_1 & -H_2 & 0 & I_1 & -B_2 & 0 & C_2 \\
D_2 & -C_2 & 0 & J_1 & -I_1 & A_2 & J_1 & -I_1 & 0 & D_2 & -C_2 & 0 \\
0 & B_3 & D_2 & 0 & H_3 & J_1 & A_3 & H_3-H & J_1 & 0 & B_3 & D_2 \\
-B_3 & 0 & C_2 & H_3 & 0 & -I_1 & -H_3+H & A_3 & -I_1 & -B_3 & 0 & -C_2 \\
-D_2 & C_2 & 0 & -J_1 & I_1 & 0 & -J_1 & I_1 & A_3 & -D_2 & C_2 & 0 \\
0 & E_4 & G & 0 & B_4 & D_1 & 0 & B_4 & D_1 & A_4 & E_4-H & G \\
-E_4 & 0 & F & -B_4 & 0 & -C_1 & -B_4 & 0 & -C_1 & -E_4+H & A_4 & -F \\
-G & F & 0 & -D_1 & C_1 & 0 & -D_1 & C_1 & 0 & -G & F & A_4
\end{bmatrix},$$

$$X = \begin{bmatrix} m_{1x} \\ m_{1y} \\ m_{1z} \\ m_{2x} \\ m_{2y} \\ m_{2z} \\ m_{3x} \\ m_{3y} \\ m_{3z} \\ m_{4x} \\ m_{4y} \\ m_{4z} \end{bmatrix} \qquad \text{and} \qquad Y = \begin{bmatrix} M_{1y}^{\circ} h_z - M_{1z}^{\circ} h_y \\ -M_{1x}^{\circ} h_z + M_{1z}^{\circ} h_x \\ M_{1x}^{\circ} h_y - M_{1y}^{\circ} h_x \\ M_{2y}^{\circ} h_z - M_{2z}^{\circ} h_y \\ -M_{2x}^{\circ} h_z + M_{2z}^{\circ} h_x \\ M_{2x}^{\circ} h_y - M_{2y}^{\circ} h_x \\ M_{3y}^{\circ} h_z - M_{3z}^{\circ} h_y \\ -M_{3x}^{\circ} h_z + M_{3z}^{\circ} h_x \\ M_{3x}^{\circ} h_y - M_{3y}^{\circ} h_x \\ M_{4y}^{\circ} h_z - M_{4z}^{\circ} h_y \\ -M_{4x}^{\circ} h_z - M_{4z}^{\circ} h_x \\ M_{4x}^{\circ} h_y - M_{4y}^{\circ} h_x \end{bmatrix}.$$

If the sublattice gyromagnetic ratios are all different, the solution of (5.20) entails the complete solution of a twelve by twelve system, which is completely beyond the scope of this paper. This is as far as one can go with the general problem of the doubly triangular system without explicitly solving (5.20). However, we can ask whether or not our equations for this general system will reduce to those of simpler systems under certain assumptions.

First, let us assume that the gyromagnetic ratios for both sublattices of the A branch in Fig. 4. are equal, that is, $\gamma_1 = \gamma_4$. Then, one can add corresponding component equations (5.13) for these two sublattices. If we define $m_{1j} + m_{4j} = m_{Aj}$, $j = x,y,z$, and add the x component equations together, we will get

$$i\omega m_{Ax}/\gamma_1 = m_{Ay}\left[H + \lambda\left(M_{2z}^{\circ} + M_{3z}^{\circ}\right)\right] - \lambda M_{Az}^{\circ}\left(m_{2y} + m_{3y}\right) - M_{Az}^{\circ} h_y. \quad (5.21)$$

Similarly, combining the y and z component equations yields

$$i\omega m_{Ay}/\gamma_1 = -m_{Ax}\left[H + \lambda\left(M_{2z}^{\circ} + M_{3z}^{\circ}\right)\right] + \lambda M_{Az}^{\circ}\left(m_{2x} + m_{3x}\right) + M_{Az}^{\circ} h_x, \quad (5.22)$$

$$m_{Az} = 0. \quad (5.23)$$

These three equations and six others for sublattices 2 and 3 reduce exactly to those previously found for the three sublattice triangular ferrimagnetic system.[1] Hence, in the special case when $\gamma_1 = \gamma_4$, the system acts just like a single triangular system, with an effective gyromagnetic ratio given by (4.8).

Next consider the special case when $\gamma_1 = \gamma_4 = \gamma_A$ and $\gamma_2 = \gamma_3 = \gamma_B$. Then, by letting $\vec{m}_1 + \vec{m}_4 = \vec{m}_A$ and $\vec{m}_2 + \vec{m}_3 = \vec{m}_B$. one can appropriately combine the corresponding component equations (5.13) such that the equations of motion reduce identically to those for a system of two antiparallel sublattices. This system would behave exactly like a two sublattice system having an effective gyromagnetic ratio given by (3.5).[6]

We can therefore conclude that the resonance behavior of the doubly triangular system is the same as that of simpler systems under appropriate conditions, in spite of the fact that its basic structure is more complex. However, nothing more specific can be determined unless the complete coupled set of equations (5.20) are solved.

CHAPTER VI

DISCUSSION AND SUMMARY

Although the complete solution of the equations of motion for the doubly triangular ferrimagnetic system was not found, several interesting properties of the system were uncovered in the process of our investigation. First, by considering the static case, when the total external field over the specimen was zero, we found that several of the molecular field coefficients had to be equal in order to satisfy the condition that the sublattice magnetizations be parallel to the local fields acting on them. This yielded the result that $\lambda_{12} = \lambda_{13} = \lambda_{24} = \lambda_{34} = \lambda$. Since $\lambda_{ij} = \lambda_{ji}$, this tells us that the internal field acting on either sublattice of one triangular branch as a result of interaction with the two sublattices of the other branch will be equal. Moreover, this field enters into the problem as though it were an "effective" field produced by a single sublattice whose magnetization is the sum of the two branch sublattices. It is also found that the intra-sublattice fields have no effect whatever on the resonance properties of the system, since they vanish in the vector product in the equations of motion (5.10).

Another interesting result from the static equilibrium case is the manner in which the three different molecular field coefficients are related. From equation (5.7) we have $\lambda^2 = \lambda_{14}\lambda_{23}$. This is a result that is unique to a doubly triangular system. It also reduces the number of independent molecular field coefficients for the system to two, an effect which would not be suspected at first glance.

The equilibrium state also reveals that the sum of the x components and the sum of the y components of the static magnetizations of each branch are zero. This result should have been qualitatively apparent from the manner in which the doubly triangular system was set up, for we required that the net branch magnetizations be antiparallel to each other.

When an external field consisting of a large constant component in the z direction and a small arbitrarily-oriented oscillating component of circular frequency ω was applied across the system, two effects occurred. First, the static sublattice magnetizations were displaced slightly from their natural positions. Second, the small oscillating field induced small oscillating magnetization components that varied directly as the field. Hence, each sublattice magnetization component became a sum of three terms: $M_{ij} \doteq M^{\circ}_{ij} + m^{*}_{ij} + m_{ij}$. A similar situation was found to exist in the single triangular system.[1]

The complete set of equations of motion were then written out in detail. Using the fact that all the small induced oscillating components of magnetization had to vanish as $\vec{h} \to 0$, we obtained four conditions (5.18) which simplified the general equations of motion. Thus, the set of twelve linear nonhomogeneous equations of motion in twelve unknowns (the oscillating components of magnetization) were reduced to their final form. The complete solution of the system requires the solution of these twelve simultaneous equations, a problem much beyond the scope of this paper.

It might be remarked that it would be possible to reduce the set of twelve equations to a set of six equations in six unknowns by eliminating, say, all the m_{iy} from the x and z component equations, and by combining the z component equations to get

$$\frac{m_{1z}}{\gamma_1} = -\frac{m_{4z}}{\gamma_4} \; ; \quad \frac{m_{2z}}{\gamma_2} = -\frac{m_{3z}}{\gamma_3} .$$

However, the set is still so unwieldly that it cannot be solved very easily. Therefore, a complete solution to the doubly triangular system was not attempted. Rather, it was shown that our equations can be reduced exactly to those for a three sublattice triangular system and a two antiparallel sublattice system under the appropriate conditions.

We could not calculate the resonance frequency, the components of the susceptibility tensor, or an effective gyromagnetic ratio without explicitly solving the equations of motion. However, we can discuss some interesting phenomena on a qualitative basis. Fig. 5. shows schematically what happens to the system in the presence of the small oscillating field. The individual sublattice magnetizations are set into forced precession about their static equilibrium positions. Each sublattice will possess an oscillating com-

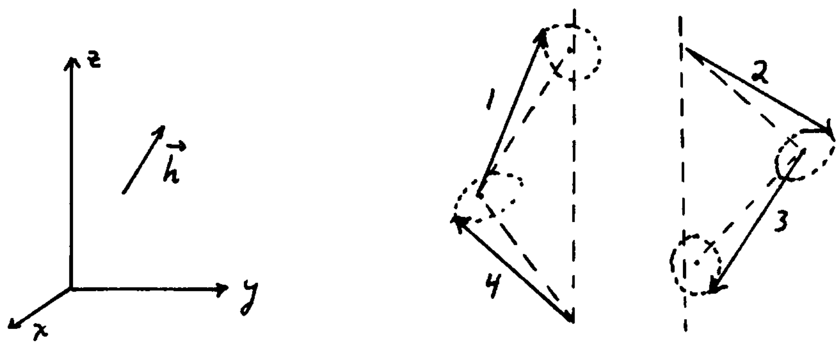

Fig. 5. Origin of oscillating magnetization components.

ponent in the z direction plus a component in the x-y plane. This should occur regardless of the orientation of $\vec{h}$, even in the special cases when $\vec{h}$ lies along the z axis or in the transverse x-y plane. Under these circumstances, one might expect all the components of the susceptibility tensor to be different from zero, although the exact values cannot be known without the complete solution of the equations of motion.

Finally, it might be mentioned that there is as yet no conclusive experimental evidence for the existence of doubly triangular ferrimagnetic crystals. However, with the accelerated investigations going on in this field at present, it may be desirable at some future time to carry through the complete solution of this problem, in order to obtain results of value in the study of ferrimagnetism.

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