A STUDY OF VOLTAGE-CONTROLLED PARALLEL NETWORK CRYSTAL OSCILLATORS

by

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A Thesis Submitted to the Faculty of the

DEPARTMENT OF ELECTRICAL ENGINEERING

In Partial Fulfillment of the Requirements
For the Degree of

MASTER OF SCIENCE

In the Graduate College

THE UNIVERSITY OF ARIZONA

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ABSTRACT

A study was made of the parallel network oscillator with the view of using quartz crystals in the side channels as the frequency determining networks. An oscillator was designed, built, and tested. A mathematical analysis of the inability of this circuit to oscillate properly over the desired tuning range was made and several means of correcting this inability were investigated.

Several suggestions for further research on the topic in order to develop a successful oscillator of this type are made.

ACKNOWLEDGEMENTS

The author wishes to express his appreciation to Dr. J. L. Stewart for his suggestion of the topic and for his advice and guidance which were of great assistance in the preparation of this thesis.

The author also wishes to express his appreciation to his wife, Theresa, whose patience and understanding have made work on this thesis as pleasureable as possible.

Acknowledgement is also given to the University of Arizona, Tucson, Arizona for providing the crystals used in the experimental phase of the thesis work.

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CHAPTER 1

THE PARALLEL NETWORK OSCILLATOR

Many circuit configurations are known to provide oscillation at frequencies below the microwave region. One of the lesser known circuits is the parallel network oscillator. The purpose of this paper is to discuss this type of oscillator and to study its possible use with quartz crystals in the side channels.

1.1 General

The parallel network oscillator is one which has two signal paths; the outputs from these two paths are added and the signal which results from this addition is fed back to the two input circuits. The basic configuration is shown in Fig. 1. Path one consists of a system which is a function of frequency and which also has variable gain. Path two is likewise a function of frequency and it also has variable gain. The outputs from paths one and two are fed to the addition or subtraction block (block three) and the resulting output is returned to the input. Obviously, if the open loop gain of this configuration is equal to or greater than one, and if the open loop phase change from input to output is exactly zero degrees (or some integer multiple of 360°), oscillation will result. As can be derived from Fig. 1, the open loop transfer function between the input of the oscillator and the output of the adder is

$$\frac{E_{out}}{E_{in}} = \left[A_1F_1(p) + A_2F_2(p)\right] F_3(p) \qquad (1\text{-}1)$$

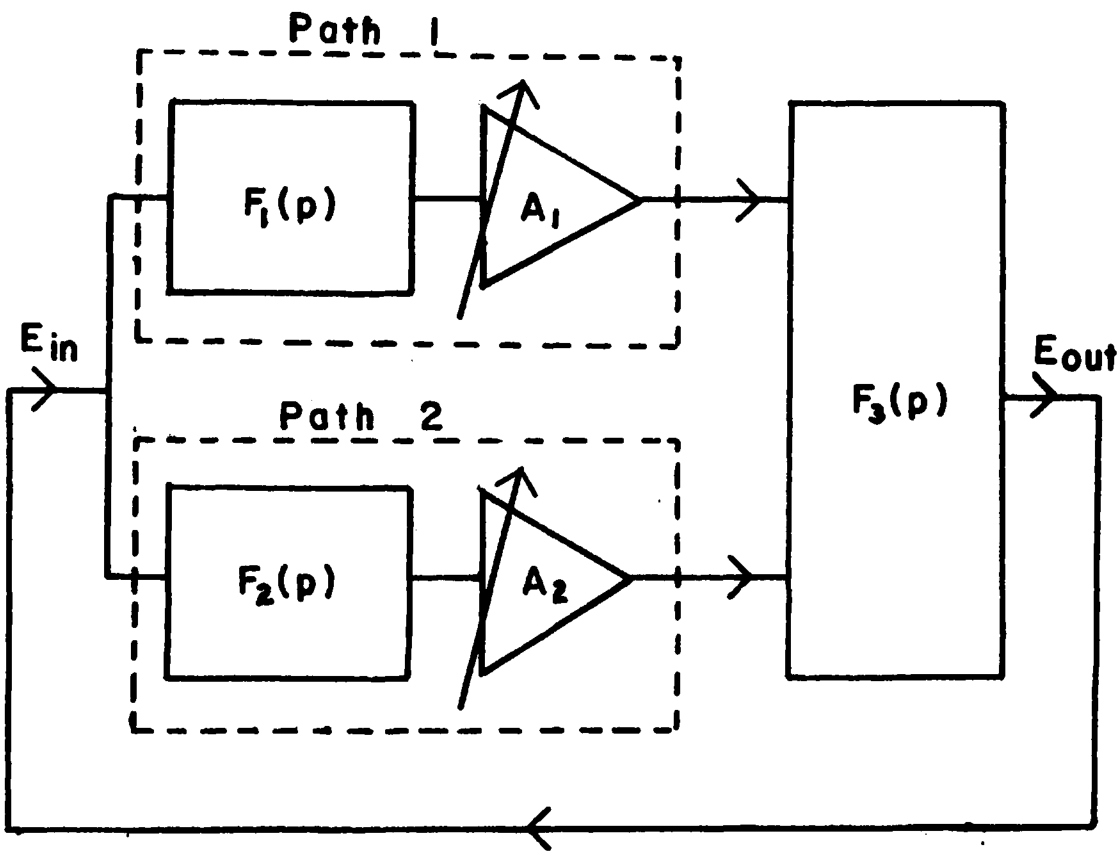

Fig. 1 SCHEMATIC OF PARALLEL NETWORK OSCILLATOR

in which $p = j\omega$ is the steady state variable. If the relative gains of the two channels are varied symmetrically about the mean gain A_o as

$$A_1 = A_o(1+k) \qquad A_2 = A_o(1-k) \qquad (1\text{-}2)$$

where $|k| \leq 1$. By combining Equations 1-1 and 1-2, we obtain

$$\frac{E_{out}}{E_{in}} = A_o\left[(F_1(p) + F_2(p)) + k(F_1(p) - F_2(p))\right] F_3(p) \qquad (1\text{-}3)$$

This equation demonstrates the dependence of the overall loop transfer function on the two functions $F_1(p)$ and $F_2(p)$, as well as on the variation in gain k. If $F_1(p)$ and $F_2(p)$ are appropriate functions, they will, in conjunction with the values of A_o and k, determine the oscillation frequency of the overall system. By careful selection of these networks, the magnitude of the transfer functions can be made to be a fairly linear function of k. $F_1(p)$ and $F_2(p)$ obviously must be quite different functions of frequency if tuning is to be obtained over any wide band of frequencies. The effect of $F_3(p)$, or the adder configuration, may often be less than $F_1(p)$ or $F_2(p)$, although $F_3(p)$ cannot be ignored. Generally, the effect of $F_3(p)$ will be to narrow the overall bandwidth of the oscillator. However, it may be used to some advantage in maintaining the output amplitude of the oscillator fairly constant, as will be discussed later.

1.2 A Qualitative Analysis

The operation of the oscillator may be seen qualitatively in Fig. 2. A voltage E_{in} is applied at the common input to the two circuits. Networks $F_1(p)$ and $F_2(p)$ will produce at their outputs new voltages E_1 and E_2. If E_1 is shifted from the input value E_{in} by some angle Θ_1 and E_2 is shifted by some value Θ_2, obviously some gain function (A_1) times E_1 and some gain function (A_2) times E_2 will

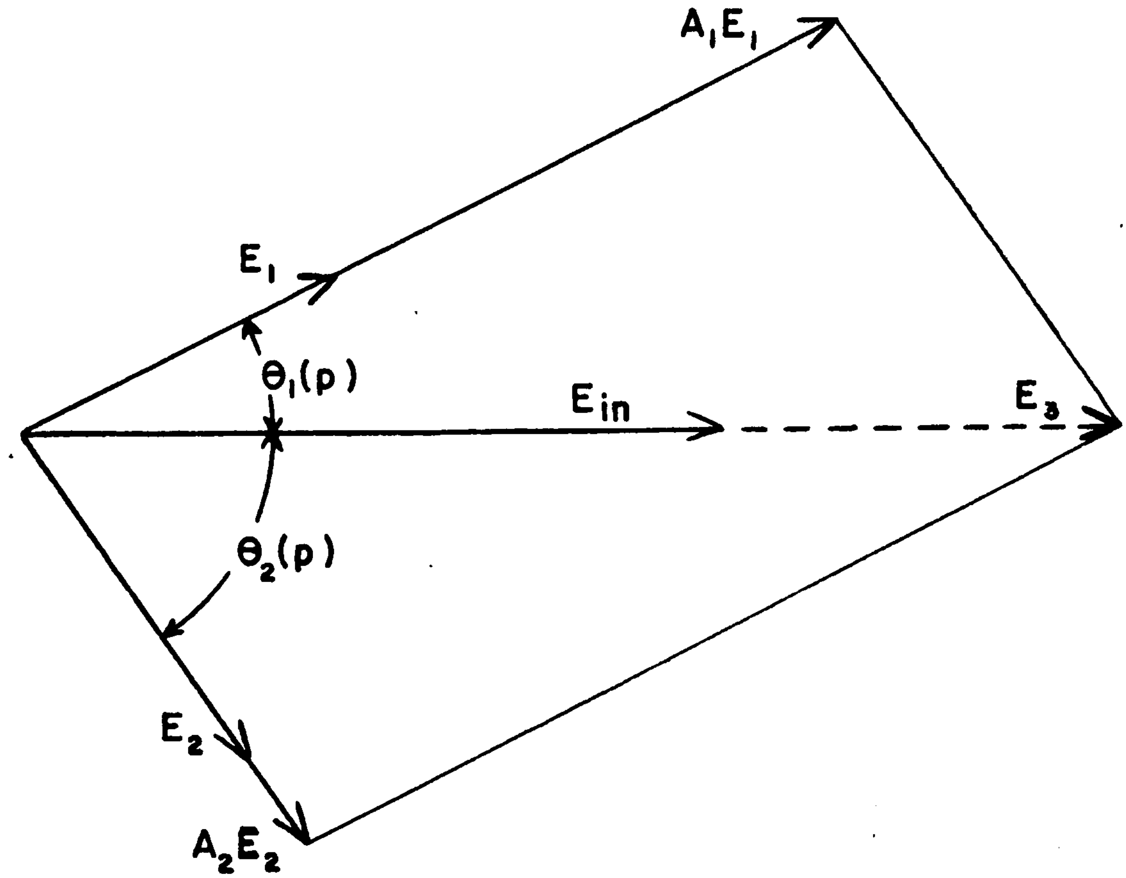

Fig. 2 VOLTAGE RELATIONSHIPS

result in magnitudes A_1E_1 and A_2E_2 which, when added together in the adder circuit, will provide an output voltage with a zero phase angle as compared with E_{in}. Then, if the overall open loop gain of the system is equal to the value one or to a value greater than one, oscillation will take place. It is to be noted here that, if the overall gain of the combined paths is positive, $F_1(p)$ must provide a lagging phase angle and $F_2(p)$ must provide a leading phase angle (or vice versa) in order for oscillation to be obtained. Of course, the gain in each of the individual channels must be adjusted such that the magnitude of the sum has a phase angle with respect to the input of zero, or some multiple of 360°. The weighted outputs, A_1E_1 and A_2E_2, one weighted relative to $(1 - k)$ and the other weighted relative to $(1+k)$ will have a phase angle of zero at some unique frequency. By changing the value of k, the vector addition must have a phase angle of zero at some different frequency, and this frequency will be such as to cause Θ_1 and Θ_2 to be in the correct relationship so that E_3 (the weighted addition of E_1 and E_2) is exactly in phase with the input voltage E_{in}. For proper operation, the values of Θ_1 and Θ_2 should increase in the same directions as k changes as can be seen from Fig. 3. If Θ_1 becomes more negative as Θ_2 becomes more positive, there may be more than one frequency at which the summation of these two vectors has a zero phase angle. However, if Θ_1 becomes more positive as Θ_2 becomes positive, this will occur only for specific ratios of these weighted gains, $A_o(1 - k)$ and $A_o(1+k)$.

Other methods of operation are described in Figs. 4a and 4b. In Fig. 4a the gains A_1 and A_2 are negative. This is equivalent to A_o being negative with factor $(1 - k)$ for one channel and $(1+k)$ for the other. In this type of operation $F_1(p)$ and $F_2(p)$ have angles Θ_1 and Θ_2

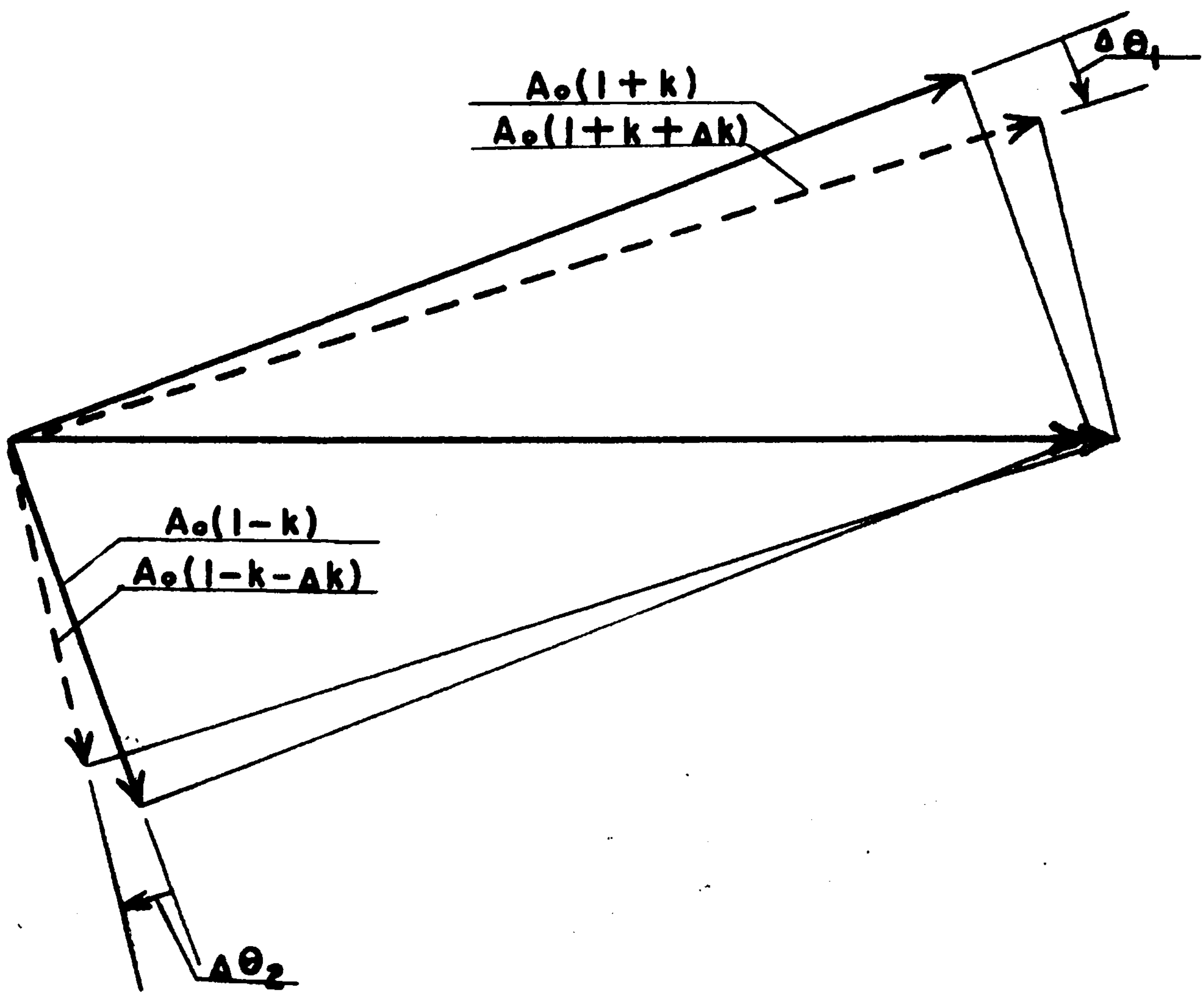

Fig. 3 CHANGES IN Θ_1 and Θ_2 DUE TO A CHANGE IN k

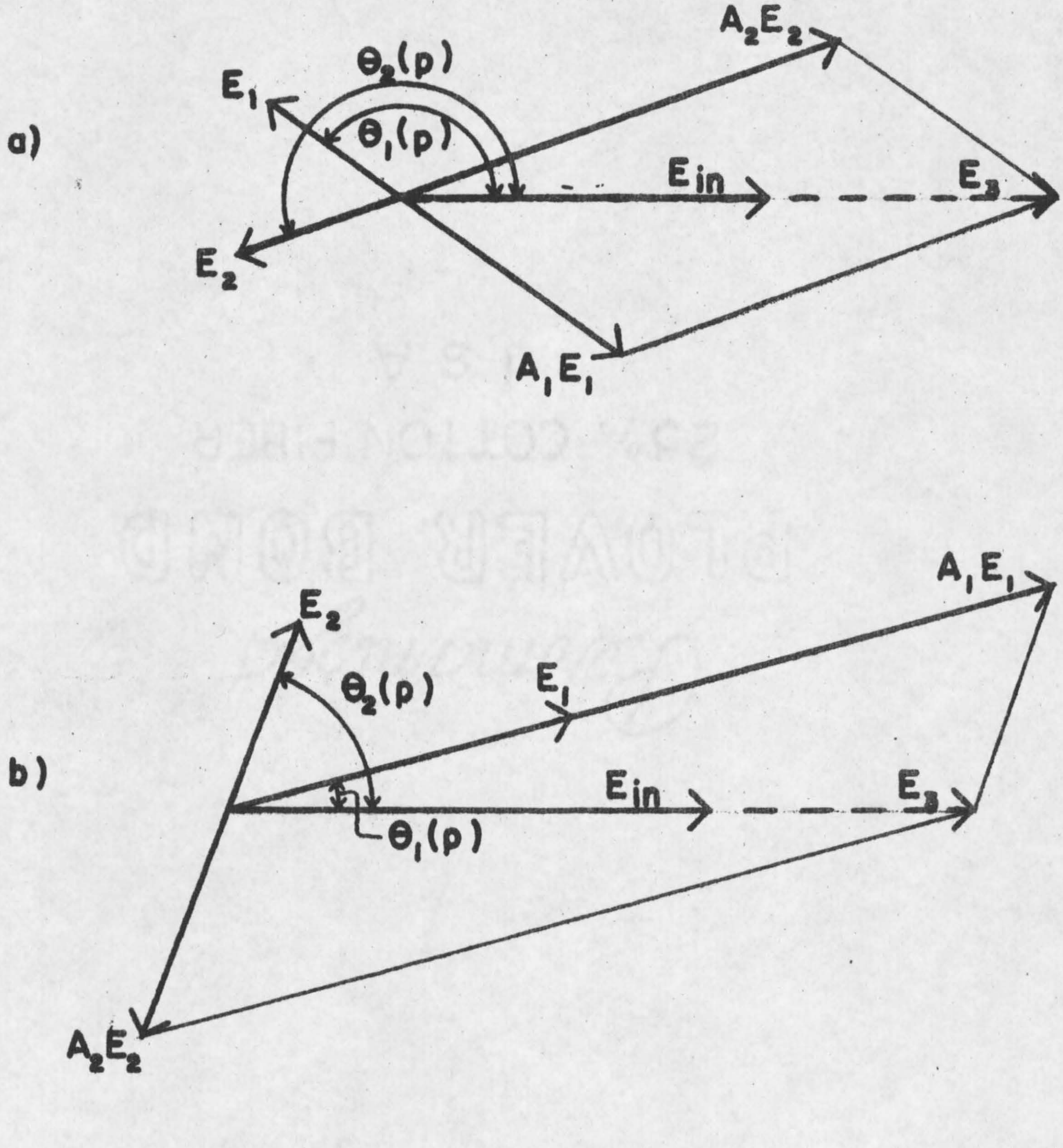

Fig. 4 VOLTAGE RELATIONSHIPS

which vary about 180°. When multiplied by the negative magnitudes A_1 and A_2, this results in the vectors A_1E_1 and A_2E_2, which add as vector E_3, which has a relative phase of zero with the input voltage E_{in}. Another case would be as in Fig. 4b, where A_1 is a positive magnitude but A_2 is a negative magnitude. In this case the vector addition is as shown. In both Figs. 4a and 4b, note that the functions $F_1(p)$ and $F_2(p)$ may provide leading (or lagging) phase angles in both channels.

We generally will be concerned with Fig. 2 with the gains of the two channels being positive and with $F_1(p)$ and $F_2(p)$ varying oppositely in phase. In other words, one will be a lead network and one will be a lag network. These circuits may be provided by the use of RC networks or by parallel resonant circuits operating above or below their resonant frequencies or by series resonant circuits operating above or below their resonant frequencies.

1.3 Type I Circuit [1]

The type of circuit shown in Fig. 4a where A_1 and A_2 are both negative (and thus A_o is negative) can be generalized in terms of utilizing some type of delay line or transmission line, - either distributed types or those built with lumped circuit elements. To consider this case we may think of an idealized transmission line where, theoretically, the output has constant amplitude but has a phase delay proportional to frequency. In this case, the functions are of the form

$$F_1(p) = A_1 \exp^{-pT} \quad F_2(p) = A_2 \exp^{-paT} \quad F_3(p) = 1 \qquad (1\text{-}4)$$

[1] J. L. Stewart, "Parallel-Network Oscillators", Proc, IRE, vol. 43, p. 589; May, 1955.

where $F_3(p)$ has been assumed to constitute a simple addition in order to make calculations easier. Then Equation 1-3 becomes

$$\frac{E_{out}}{E_{in}} = A_o \left[(1+k)\ \exp^{-pT} + (1-k)\ \exp^{-paT} \right] \qquad (1\text{-}5)$$

Now utilizing $p = j\omega$ and combining the real and imaginary parts

$$\frac{E_{out}}{E_{in}} = A_o \left[(1+k)\ \cos\omega T + (1-k)\ \cos a\omega T \right] - j\, A_o \left[(1\ k)\ \sin\omega T + (1-k)\ \sin a\omega T \right] \qquad (1\text{-}6)$$

In order for oscillation to occur, the imaginary part must disappear so that the phase angle will be zero; this is defined by

$$(1+k)\ \sin\omega T + (1-k)\ \sin a\omega T = 0 \qquad (1\text{-}7)$$

The open loop gain (which is the real part of this function) must be equal to or greater than unity.

$$\left| A_o \left[(1+k)\ \cos\omega T + (1-k)\ \cos a\omega T \right] \right| \geqq 1 \qquad (1\text{-}8)$$

The limits of oscillation take place at frequencies ω_1 and ω_2 when $k = \pm 1$ and the center frequency ω_c is given when $k = 0$. Therefore,

$$\sin\omega_2 T = 0 \quad \text{for } k = +1$$
$$\sin a\,\omega_1 T = 0 \quad \text{for } k = -1 \qquad (1\text{-}9)$$
$$\sin\omega_c T + \sin a\,\omega_c T = 0 \quad \text{for } k = 0$$

If we assume that oscillation takes place at the lowest allowable frequency, then

$$\omega_1 = \frac{\pi}{aT}$$
$$\omega_2 = \frac{\pi}{T} \qquad (1\text{-}10)$$
$$\omega_c = \frac{2\,\omega_1\omega_2}{\omega_1 + \omega_2}$$

In this particular case, the magnitude of this function is of importance. At the extreme oscillation frequencies ω_1 and ω_2, the cosine function is unity, and the magnitude of the transfer function is $2A_o$. At the center frequency, it is

$$A_o \left[\cos \frac{2\pi}{a+1} + \cos \frac{2a\pi}{a+1} \right] \qquad (1\text{-}11)$$

which, in general, is less than $2A_o$ except for the trivial case of $a = 1$, in which case the two side channels are identical and the tuning range of the oscillator is zero. For $a = 2$, the gain magnitude at center frequency is $\sqrt{3}A_o$, and the tuning ratio is, of course, 2 to 1. For $a = 3$, the gain magnitude falls to zero at the center frequency and oscillation ceases. Thus, the type of parallel network oscillator that utilizes delay lines is not suited for tuning ratios larger than 3 to 1 and, practically, is only valuable for ratios up to about 2 to 1. It should be noted that the open loop gain varies only by a factor of $\sqrt{3}$ to 2 over the entire tuning range for $a = 2$; for $a \leq 2$, the variation is even less. Thus the amplitude of oscillation is fairly constant with changing frequency for this particular case.

1.4 Type II Circuit[2]

Another type of parallel network oscillator is that shown in Fig. 5. This parallel network oscillator employs parallel resonant circuits for $F_1(p)$, $F_2(p)$ and $F_3(p)$. It is assumed that all the tubes are identical and are ideal current generators. The transfer function at the first tube of channel one is

$$\frac{E_1}{E_{in}} = -\frac{g_{m1}}{C_1} \left[\frac{p}{p^2 + pB_1 + \omega_1^2} \right] \qquad (1\text{-}12)$$

[2] Ibid.

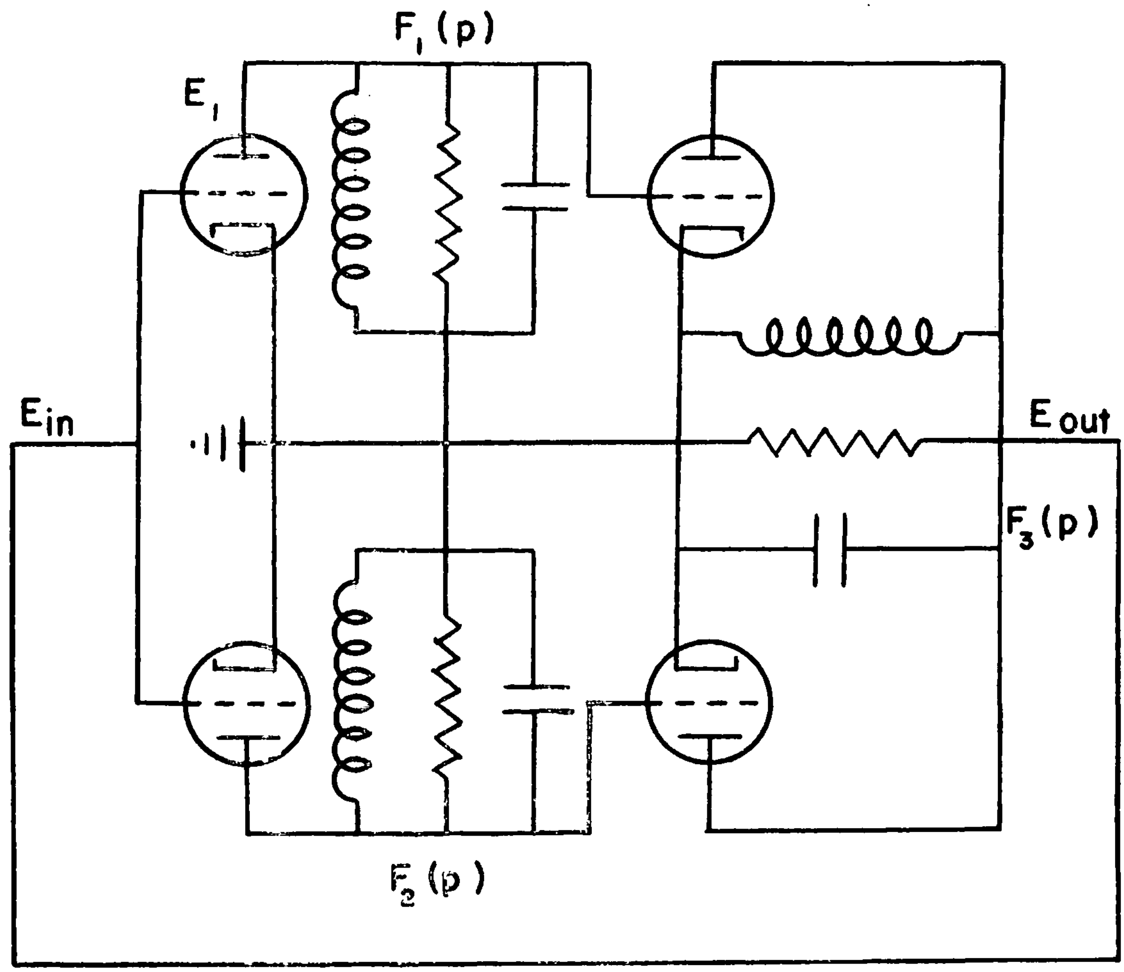

Fig. 5 SCHEMATIC OF TYPE II OSCILLATOR

where g_{m_1} is the transconductance, C_1 is shunt capacity, B_1 is the half-power bandwidth and ω_1 is the resonant frequency. The expressions for $F_2(p)$ and $F_3(p)$ are similar. If it is assumed that

$$C_1 = C_2 = C \qquad \omega_3 = \omega_o$$
$$B_1 = B_2 = B \qquad g_{m_1} = \frac{g_m}{2}(1+k)$$
$$C_3 = C' \qquad\qquad\qquad\qquad (1\text{-}13)$$
$$B_3 = B' \qquad g_{m_2} = \frac{g_m}{2}(1-k)$$

Then the open loop transfer function is

$$\frac{E_{out}}{E_{in}} = \frac{g_m^2}{2CC'}\left[\frac{(1+k)\,p}{p^2 + pB + \omega_1^2} + \frac{(1-k)\,p}{p^2 + pB + \omega_2^2}\right] \cdot \frac{p}{p^2 + pB' + \omega_o^2} \qquad (1\text{-}14)$$

The narrow band approximation may be employed here in order to keep the calculations simple. This has the advantage of simplicity although it may not be sufficiently accurate in every case. Thus, if we assume that $\omega_2 - \omega_1$, B and B' are small compared to ω_o, upon translating the system centered around ω_o to a center frequency of zero (band pass to low pass transformation), Equation 1-14 becomes

$$\frac{E_{out}}{E_{in}} = \frac{g_m^2}{4CC'}\left[\frac{p + B/2 - jk\,\omega_x}{(p + B/2 + j\,\omega_x)\;(p + B/2 - j\,\omega_x)\;(p + B'/2)}\right] \qquad (1\text{-}15)$$

where $\omega_x = \omega_2 - \omega_o = \omega_o - \omega_1$. This function has poles as indicated in Fig. 6 and a zero which moves along a vertical line a distance proportional to k. At the extremes of the tuning range, $k = \pm 1$, in which case the zero cancels a pole and the oscillator is a simple two stage, single channel, resonant device, one of the two parallel channels being cuttoff and the other parallel channel being at full gain. The extreme frequencies are found from the requirement that the two re-

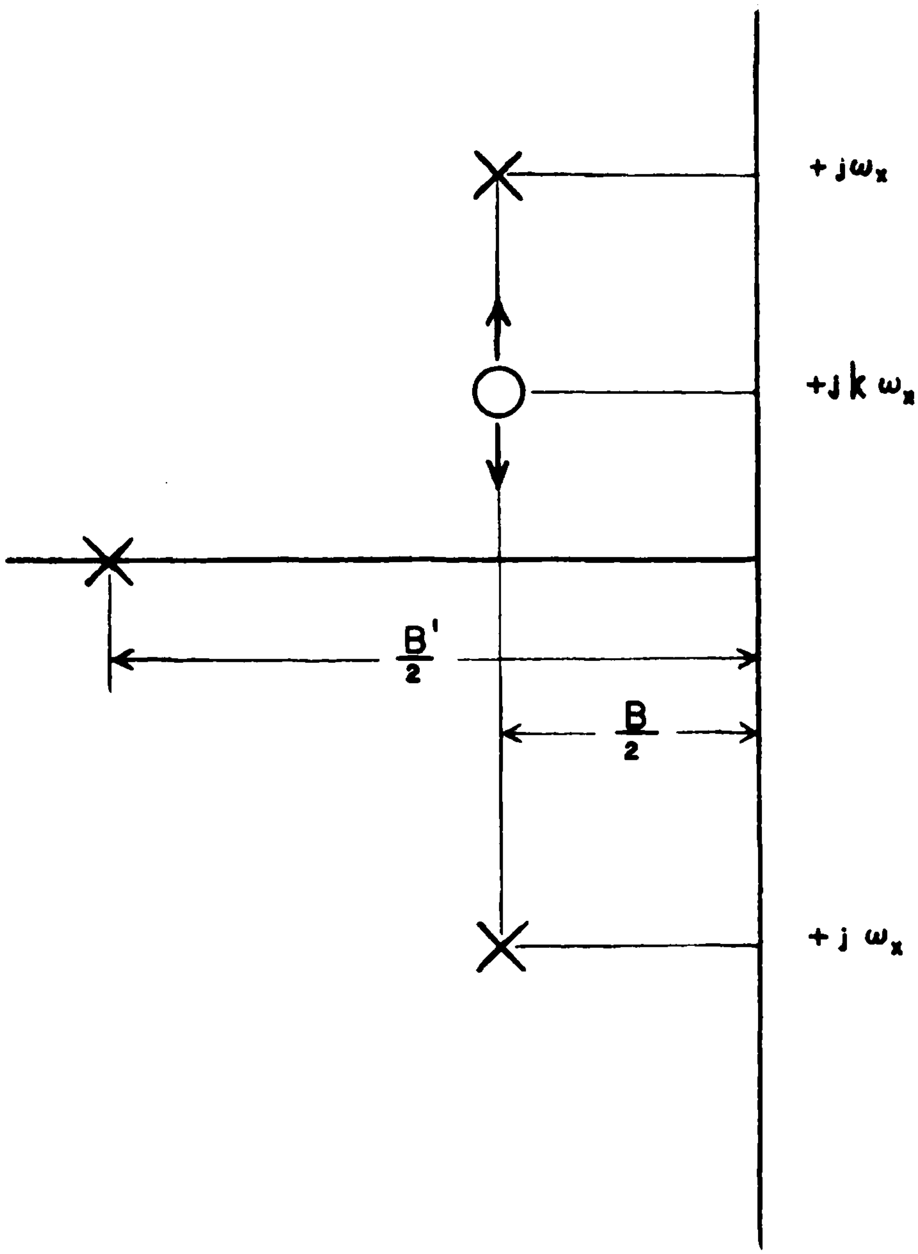

Fig. 6 POLE-ZERO PLOT FOR LOW PASS TRANSFORMATION

maining poles must furnish a phase shift of zero, giving the tuning range

$$\Delta B = \frac{2\,\omega_x}{1+B/B'} = \frac{2\,(\omega_2 - \omega_0)}{1+B/B'} \approx \frac{\omega_2 - \omega_1}{1+B/B'} \qquad (1\text{-}16)$$

which approaches a maximum for B' much greater than B and is one-half of the maximum possible value for B' = B. The open loop gain at the extreme frequencies is

$$\frac{g_m^2}{4CC'}\left[\frac{4\,(B+B')^2}{B\,B'\left[4\omega_x^2 + (B+B')^2\right]}\right] \qquad (1\text{-}17)$$

and the gain at center frequency is

$$\frac{g_m^2}{4CC'}\left[\frac{B/B'}{\omega_x^2 + B^2/4}\right] \qquad (1\text{-}18)$$

The ratio of these two equations is

$$\frac{1+\left(\frac{2\,\omega_x}{B}\right)^2}{1+\left[\frac{2\,\omega_x}{B+B'}\right]^2} \qquad (1\text{-}19)$$

which gives a characteristic dip to the oscillation amplitude as a function of frequency at the center frequency. This variation can be made as small as desired by making the bandwidths B and B' very large compared to ω_x. However, ω_x is primarily determined by the desired tuning range wanted in the oscillator.

1.5 General Pole-Zero Diagram

The pole zero plot of Fig. 6 is typical of parallel network oscillators. The poles are those of both of the two channels plus those common to the two channels. In addition to the fixed poles, zeroes are introduced by the paralleling process and these zeroes move in position as k is varied. It is their motion that causes the changes in the frequency of oscillation since the overall phase contributed by vectors from each of these poles and zeroes must be zero at the resonant frequency of the oscillator.

1.6 Comparison with Other Types of Oscillators

As can be seen from the above discussion, the wide tuning range of the parallel network oscillator is limited in the final analysis by the gain - bandwidth product of the tubes employed. Due to the fact that the gain in both channels is generally voltage controlled, there are no moving parts with their associated problems. Also, the discussion applies to frequencies in any range, and theoretically, oscillators of this type in the microwave frequencies are entirely feasible. Thus, this type of oscillator has certain advantages over each of the following types:

a) Reactance tube - is limited in tuning range.

b) RC - is limited to low frequencies, may use mechanical tuning, generally has varying output amplitude.

c) LC - generally uses mechanical tuning.

d) Ferrite - has hysteresis and temperature problems.

e) Multivibrator - is limited in frequency range, doesn't produce sinusoidal signal (without filtering).

One disadvantage of this type oscillator is that it may require more components than some of the other types. Also the frequency stability is quite dependent on the stability of the screen voltage power supply. Unless crystals can be used, the band edges are not sharply defined; but this is a defect from which several of the other types also suffer.

CHAPTER 2

THE QUARTZ CRYSTAL

After the discovery of the piezoelectric effect in the quartz crystal, it became the general frequency stabilizing device in precision oscillators due to the extremely high Q inherent in this particular device. In recent years the stability of the mechanical mode of vibration has been improved even more and zero temperature coefficient crystals are now available. Many circuits have been developed to utilize the characteristics of the crystal. Some of the more familiar are the Pierce Oscillator, the Miller Oscillator, and types of Transformer Coupled Oscillators, not to mention their use in passive filters.

2.1 Equivalent Circuit

The equivalent circuit for a crystal is shown in Fig. 7a. The series arm branch, (R_1, C_1, L_1), resonates at the series mode frequency, f_r, and has a very low impedance at resonance (essentially R_1). As the driving frequency increases above f_r, the series arm appears inductive and it will resonate with the parallel shunt capacity C_o (from the holder, plating, etc.). This frequency is known as the anti-resonant frequency, f_a. It is always higher in frequency than f_r as will be shown later.[1] By adding capacity in parallel with C_o, the crystal may

[1] W. R. McSpadden, "A Design Procedure for Transistor Crystal Oscillators", thesis for M.S. Degree, University of Arizona, Tucson, Arizona; May, 1959.

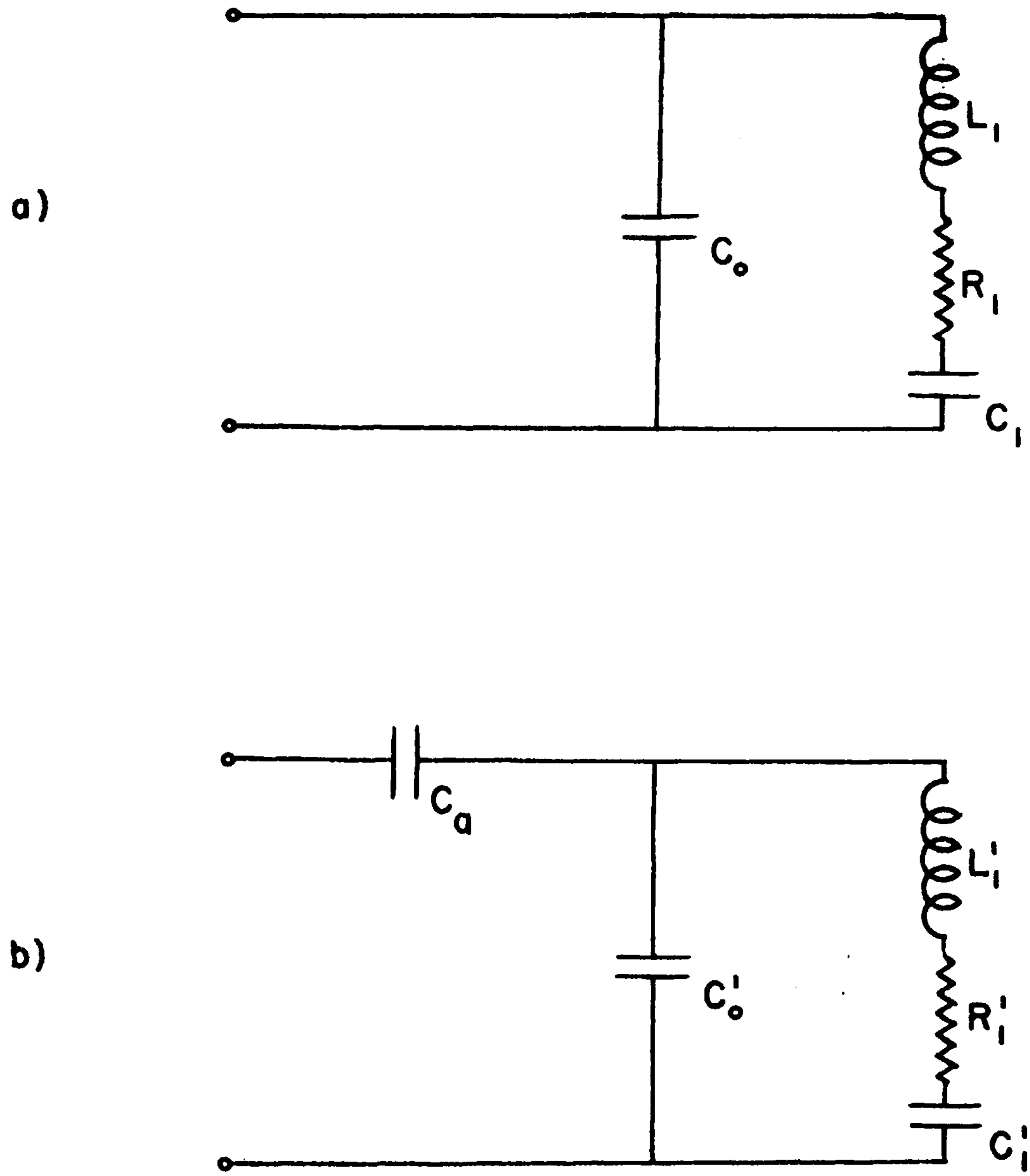

Fig. 7 EQUIVALENT CIRCUITS FOR CRYSTALS

be anti-resonant anywhere between f_r and f_a. Fig. 7b shows the equivalent circuit of a crystal with an air gap type of mounting. This circuit can be reduced to that of Fig. 7a by the use of the following expressions:[2]

$$\frac{C_o}{C_o'} = \frac{C_a}{C_a + C_o'}$$

$$\frac{C_1}{C_1'} = \frac{C_a^2}{(C_a + C_o')\ (C_a + C_o' + C_1')}$$

$$\frac{L_1}{L_1'} = \frac{R_1}{R_1'} = \frac{(C_o' + C_a)^2}{C_a^2}$$

where C_o' is the shunt capacity of the crystal and C_a is the capacity due to the separation of the quartz from the electrodes.

2,2 Typical Parameter Values

Typical values of the parameters for a one megacycle crystal are

$R_1 = 150\ \Omega$	$C_1 = 0.015$ pf.	$f_r = 1{,}000{,}000$ cps.
$L_1 = 2$ h.	$C_o = 5$ pf.	$f_a = 1{,}000{,}500$ cps.
		$Q_c = 50 \times 10^3$ (2-2)

2.3 Placement in Circuits

If a crystal is intended to operate at f_r, it is called a series mode crystal and its series resonant frequency is generally stamped on the case of the crystal. If the crystal is intended to appear as an inductance and resonate with external capacitance added to it in the circuit, then it is called a parallel mode crystal and operates at its anti-resonant frequency. In order to standardize the industry, it

[2] W. G. Cady, "Piezoelectricity", McGraw Hill Book Co., Inc., New York, 1946.

has been agreed in specifying the parallel mode resonant frequency that the external capacity, including wire and stray capacity, shall be 32 pf (20 pf for some low frequency crystals).[3]

Any crystal may operate in either its parallel or its series mode. In any given circuit, its operating mode is determined by the manner by which the crystal is used in the circuit. If it is intended to appear as a minimum impedance (maximum admittance), then it will operate in its series mode; if it is to appear as a maximum impedance (minimum admittance), it will be utilized in its parallel mode. One example of the use of a crystal in its antiresonant mode is in the Pierce Oscillator.

2.4 Series Resonance[4]

The highest possible Q that a crystal can have is the Q of its series arm defined here as Q_c. This is often specified by the manufacturer on the crystal and is always much higher than that of simple L-C networks. The circuit Q (the Q of the circuit including the crystal) is always less than the crystal Q, Q_c, and will be denoted here by a Q with no subscript. For the best frequency stability, the circuit Q must be as high as possible, realizing, of course, that it may never have a Q higher than Q_c. A simple RLC circuit approximates a series mode crystal in the frequency range near the resonant frequency of the crystal. The response of a series RLC circuit, shown in Fig. 8,

[3]MIL - C - 3098 B, "Crystal Units, Quartz", December 1955

[4]McSpadden, op. cit.

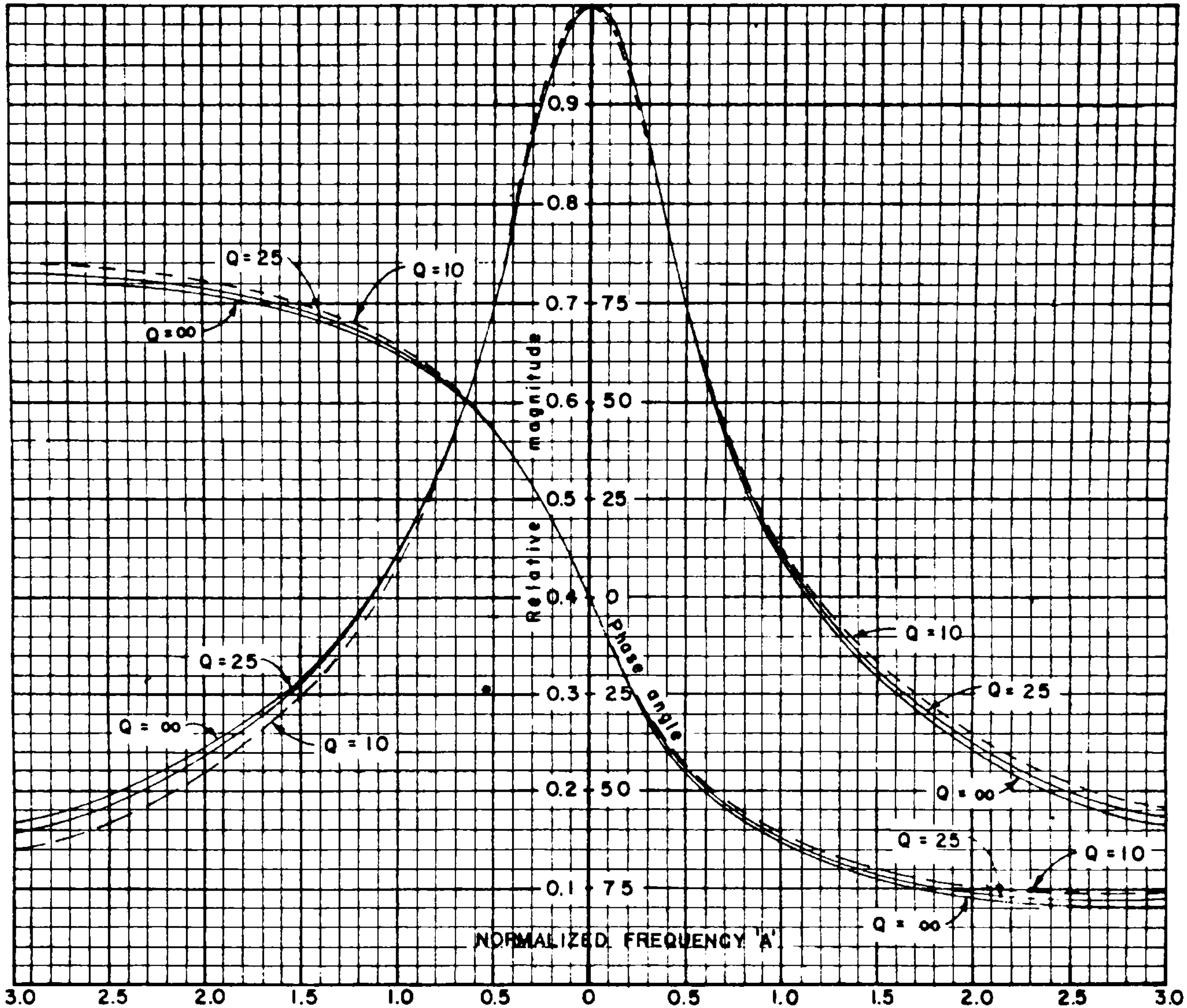

FIGURE 8 THE UNIVERSAL RESONANCE CURVE

is called the normalized universal resonance curve.[5] This curve is quite useful for determining the operating frequency of an oscillator. From Fig. 8, define the following:

Δf = the change in operating frequency from f_r

a = the normalized $\Delta f = Q\,\Delta f/f_r$

Q = circuit Q (not the unloaded crystal Q_c) (2-3)

$$\Delta f = \frac{a f_r}{Q}$$

As an example of finding the operating frequency assume that f_r is one megacycle and Q is 35,000. When the phase shift through the circuit is equal to +20 degrees, a = +0.02 from Fig. 8. Upon substituting these values into Equation 2-3

$$\Delta f = \frac{a f_r}{Q} = \frac{0.02 \cdot 10^6}{35 \cdot 10^3} = 5.7 \text{ c.p.s.} \qquad (2\text{-}4)$$

Thus, the operating frequency will be 5.7 cycles above the resonant frequency of the crystal.

As seen from Fig. 8, if frequency stability is defined as the rate of change of frequency with respect to a given variable (supply voltage, load, temperature, time, phase angle, etc.), and if the rate of change of frequency with respect to phase angle is to be a minimum, the rate of change of phase angle with respect to frequency must be a maximum. This occurs where the slope of the curve of phase angle versus frequency is the greatest which is at exactly its resonance frequency. In fact, a relative measure of frequency stability can be obtained from the ratio of Q_c to Q. This factor is called

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[5]W. R. Le Page and S. Seely, "General Network Analysis," McGraw Hill Book Company, Inc., New York, 1942.

the degradation factor F_d since it shows how much the circuit has degraded the natural crystal stability.

In order to have F_d as near unity as possible, Q should be as large as possible. With the series mode crystal, this means keeping the resistance in series with the crystal as small as possible. For example, if a series mode crystal is to be utilized in conjunction with a transformer, it is obviously better to put the crystal in series with the low impedance winding rather than in series with high impedance winding.

2.5 Parallel Resonance

When the crystal is used in its anti-resonant or parallel mode, the shunt resistance should be as large as possible in order to hold circuit Q high and increase or maintain frequency stability. When the parallel mode crystal resonates with a 32 picofarad capacitance (added shunt capacitance C_T), the combination appears as a parallel resonant circuit. The effective resistance of this circuit at resonance is known as the Performance Index (PI) of the crystal. The performance index may be calculated from the equation for a parallel resonant circuit.[6]

$$PI = Z = R_1 Q^2 \left[1 - \frac{1}{jQ}\right] \approx Q^2 R_1 \qquad (2\text{-}5)$$

Crystals with a high performance index are desirable since they indicate a high Q and hence provide better frequency stability. Note that the same factors that produce a good series mode crystal also produce a good parallel mode crystal. In both cases, small values of R_1, C_1, and C_o, and large values of L_1 are desirable. Note that PI is not the resistance of the crystal at parallel resonance, but rather is the

[6] Le Page and Seely, op. cit.

resistance of the resonant circuit, including the external capacitance C_T which has been added across C_o. This resistance is very high, generally in the order of megohms.

2.6 Choice of crystals

A quartz crystal may be operated at its fundamental frequency up to about 30 megacycles. Because the crystal slab becomes extremely thin and fragile, use of fundamental mode crystals above this frequency is difficult. Quartz crystals, however, may operate on mechanical overtones of the fundamental, and these crystals may operate up to frequencies in the vicinity of 200 Mcs. Overtone crystals are generally not operated beyond the 7th to 9th overtones because of mechanical problems in the crystal plates themselves. Also, the crystal Q drops off as the number of the overtone increases.

The crystals chosen for this investigation of the use of quartz crystals in a parallel network oscillator were selected in the vicinity of one megacycle and four crystals were obtained as follows:

$f_1 =$ 1000 Kc. $\qquad f_3 =$ 1010 Kc.

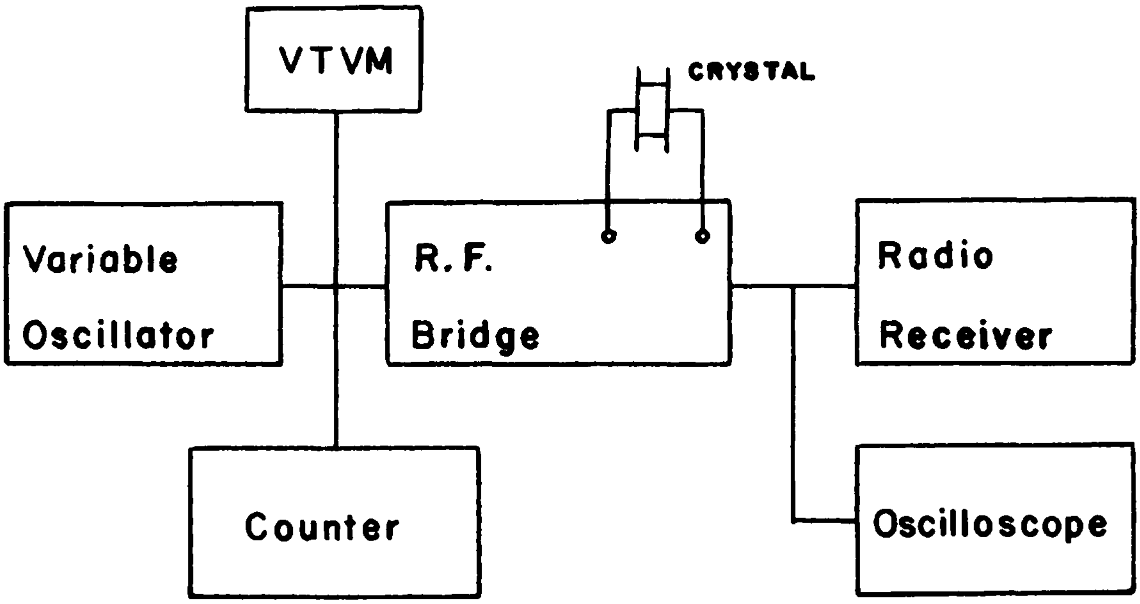
$f_2 =$ 1005 Kc. $\qquad f_4 =$ 1020 Kc.

These crystals were chosen because, with a few multiplier stages, a frequency variation or shift of 200 kilocycles (which provides good signal to noise ratio in typical FM systems) could be easily obtained. Also one megacycle is a reasonable frequency at which to operate since problems due to exceptionally high frequency are not encountered and crystals may be ordered in the fundamental series mode without worrying about operating on overtones.

2.7 Measurement of Parameters

The test arrangement shown in Fig. 9 was used in an attempt to measure the parameters of the crystals. The RF bridge utilized a series substitution method of measurement so that inductance or capacitance of the leads to the crystal played little or no part in the measurements. The radio receiver with AVC turned off and the oscilloscope were utilized as null detecting instruments. Unfortunately, this method did not operate well since the oscillator was unstable and could not be held on the resonant frequency of the crystal for any length of time. In addition, after difficulty was encountered in maintaining balance with the RF bridge, it was disassembled and a discovery was made that a precision resistor in one of the bridge arms had been replaced by a 10% carbon composition resistor. No other RF bridge or oscillator in this frequency range was available.

Therefore, since exact measurement of L_1, R_1, C_1, and C_0 was impossible, an attempt was made with the test arrangement of Fig. 10 to at least measure R_1 accurately and to determine the resonant frequency of the crystals accurately. At resonance the crystal impedance becomes a minimum and thus, the voltage across resistance R is a maximum. By varying the oscillator through resonance and finding the frequency at which maximum voltage is developed across R and by comparing the magnitude of this voltage with the voltage across the crystal and R in series, it was possible to measure R_1 and also the resonant frequency fairly accurately. Also, an estimate of the bandwidth of the crystal could be obtained, and this, in conjunction with the resonant frequency, provided the Q of the crystal. Then, knowing



OSCILLATOR	-	HEWLETT PACKARD 650A
COUNTER	-	HEWLETT PACKARD 524D
R.F. BRIDGE	-	GENERAL RADIO 916A
OSCILLOSCOPE	-	TEKTRONIX 512 (or 514)
RADIO RECEIVER	-	NATIONAL NC - 127

Fig. 9 TEST ARRANGEMENT FOR MEASURING CRYSTAL PARAMETERS

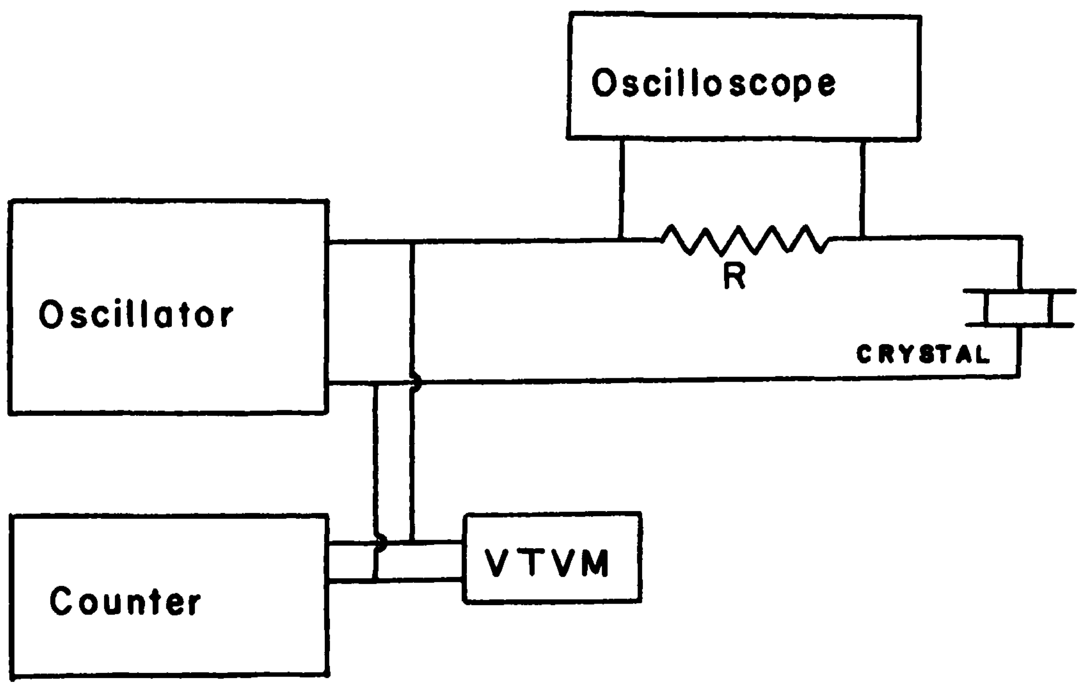


Fig. 10 SECONDARY TEST ARRANGEMENT

R_1, estimates could be made of the magnitude of damping resistors required for desired bandwidths when utilizing these crystals in the parallel network oscillator. Unfortunately, due to the instability of the only available oscillator, these measurements were only partially successful. However, in conjunction with another method whereby the voltage across R was used to drive the horizontal sweep of the oscilloscope and the voltage across the crystal was utilized to drive the vertical sweep of the oscilloscope, Lissajou figures were obtained for various values of R which also tended to determine more accurately the resonant frequency of the crystal and the value of R_1. By the same general method shown in Fig. 10, the antiresonant frequency of the crystal was obtained by finding the frequency at which the minimum voltage across R occurred.

In operational notation, the impedance of the equivalent circuit shown in Fig. 7a is

$$Z = \frac{1}{pC_0 + \dfrac{1}{R + pL_1 + \dfrac{1}{pC_1}}} = \qquad (2\text{-}6)$$

$$\frac{p^2 + p\dfrac{R_1}{L_1} + \dfrac{1}{L_1C_1}}{C_2 \; p \; \left(p^2 + p\dfrac{R_1}{L_1} + \dfrac{1}{L_1\left[\dfrac{C_1C_2}{C_1 \; C_2}\right]}\right)}$$

These polynominals are of the form $(p+\alpha)^2 + \beta^2$, which in the narrow band (high Q) approximation is $p^2 + pB + \omega^2$. If we employ values determined (approximately) by use of the circuits in Figs. 9 and 10 as

$$R_1 = 200 \text{ ohms}$$
$$B = 100 \text{ cps.} \qquad (2\text{-}7)$$
$$f_r = 999{,}793 \text{ cps.}$$
$$f_a = 1{,}000{,}667 \text{ cps.}$$

the following may be calculated from Equation 2-6 for the parameters of the one megacycle crystal:

$$R_1 = 200 \text{ ohms}$$
$$L_1 = 2\text{h.} \qquad (2\text{-}8)$$
$$C_1 = 0.0127 \text{ pf.}$$
$$C_2 = 6.35 \text{ pf.}$$

The values of bandwidth and Q for the other three crystals were generally of the same magnitudes.

CHAPTER 3

A POSSIBLE CONFIGURATION AND PROBLEMS ENCOUNTERED

3.1 Development of the Circuit and Mathematical Analysis

It was decided to utilize the crystals in their series resonant mode of operation, damping them with appropriate series resistance so that the 3 db points would be at approximately the midpoint of the desired tuning range. The extreme frequencies of this tuning range are, obviously, the resonant frequencies of the crystals in each of the two side channels. It was also decided to utilize two stages of amplification in order that the gain would be positive. The second stage of the amplification would be utilized as an adder circuit (by using a common load for both paths). The gain in the two paths can be controlled by varying about some average voltage E_o the screen voltages applied to the two pentodes used in each path. The circuit in Fig. 11 was developed appropriately.

Resistor R_3 at the input of each path is the damping resistor for the two crystals and is also utilized as the resistor across which voltage is developed to apply to the grids of the first tubes in each path. In the following derivation of the open loop transfer function, it is assumed that

a) the pentodes are ideal current sources,

b) R_G and the crystals in parallel do not affect the loading of the pentodes appreciably, and

c) R_1 of both crystals is negligible as compared with damping resistors R_3.

All Resistors in Ohms

All Capacitors in μf.

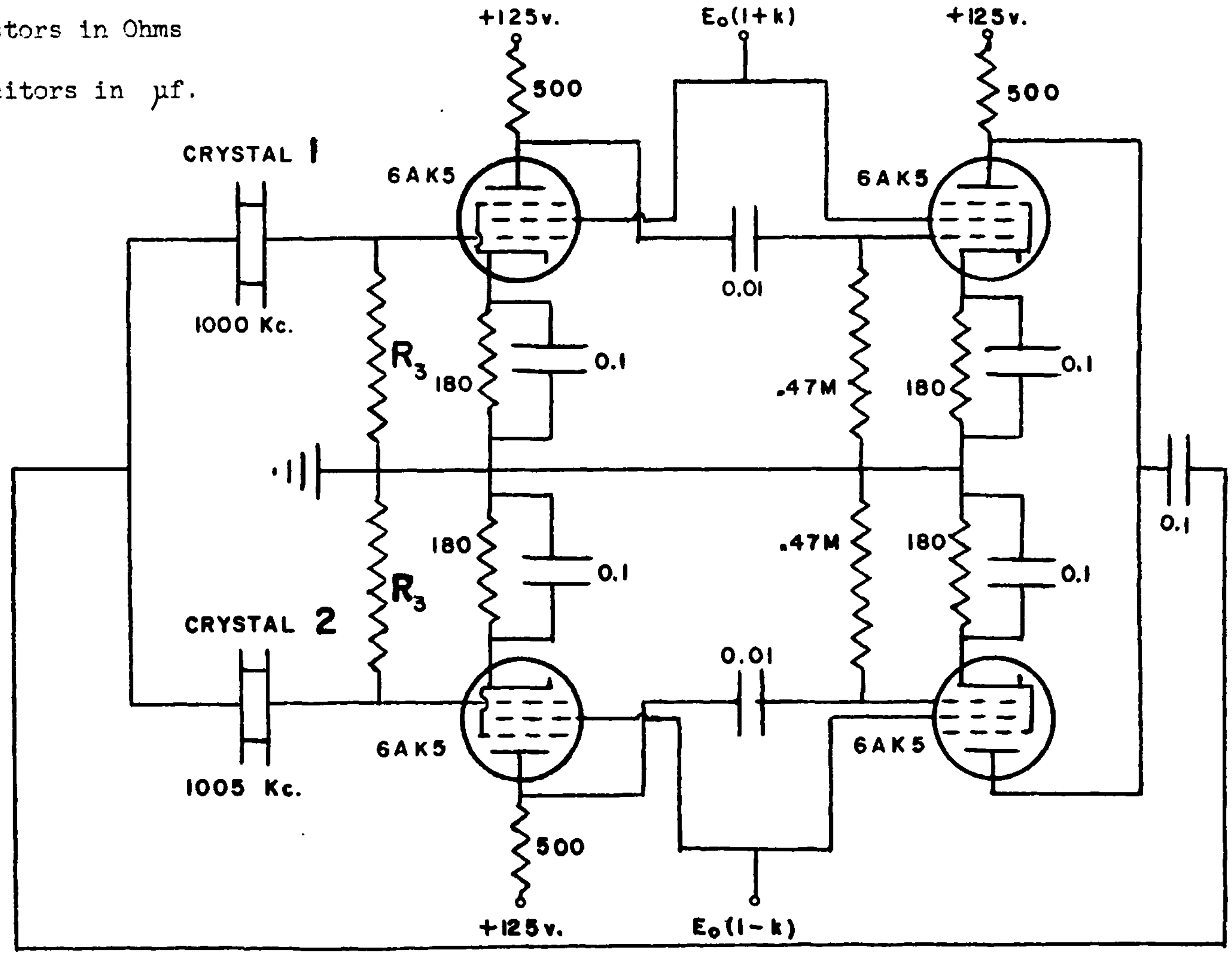


Fig. 11 INITIAL CIRCUIT

With these assumptions, the open loop transfer function is of the form

$$\frac{E_{out}}{E_{in}} = g_{m_1}{}^2 R_L{}^2 \frac{R_3}{Z_1(p)} + g_{m_2}{}^2 R_L{}^2 \frac{R_3}{Z_2(p)} \qquad (3\text{-}1)$$

where $p = j\omega$ and Z = impedance of the damped crystal. Transconducatance g_m is not exactly a linear function of the applied screen voltage. If we assume

$$g_{m_1}{}^2 = g_{m_o}{}^2 \ (1+k) \qquad g_{m_2}{}^2 = g_{m_o}{}^2 \ (1-k)$$
$$L_1 = L_2 = L \qquad (3\text{-}2)$$

then the equation for the loop gain becomes

$$\frac{E_{out}}{E_{in}} \approx \frac{g_{m_o}{}^2 \ R_L{}^2 \ R_3}{L} \cdot \left[\frac{(1+k)\,p}{p^2 + \frac{R_3}{L_1}p + \frac{1}{L_1C_1}} + \frac{(1-k)\,p}{p^2 + \frac{R_3}{L_2}\cdot p + \frac{1}{L_2C_2}} \right] \qquad (3\text{-}3)$$

This approximation provides a fair idea of the expected response. This equation is similar to the form of Equation 1-14 for the type II oscillator. Its pole-zero diagram for positive frequency is shown in Fig. 12. As may be seen, the zero is at the same distance from the $j\omega$ axis as the two poles and moves a distance proportional to the value k. Specific values are

$$k = +1 \text{ gives } \omega_x = \omega_1$$
$$k = -1 \text{ gives } \omega_x = \omega_2 \qquad (3\text{-}4)$$
$$k = 0 \text{ gives } \omega_x = \sqrt{\frac{\omega_1{}^2 + \omega_2{}^2}{2}}$$

This circuit was observed to oscillate very well when the gain of one channel was turned up with the other channel gain turned completely

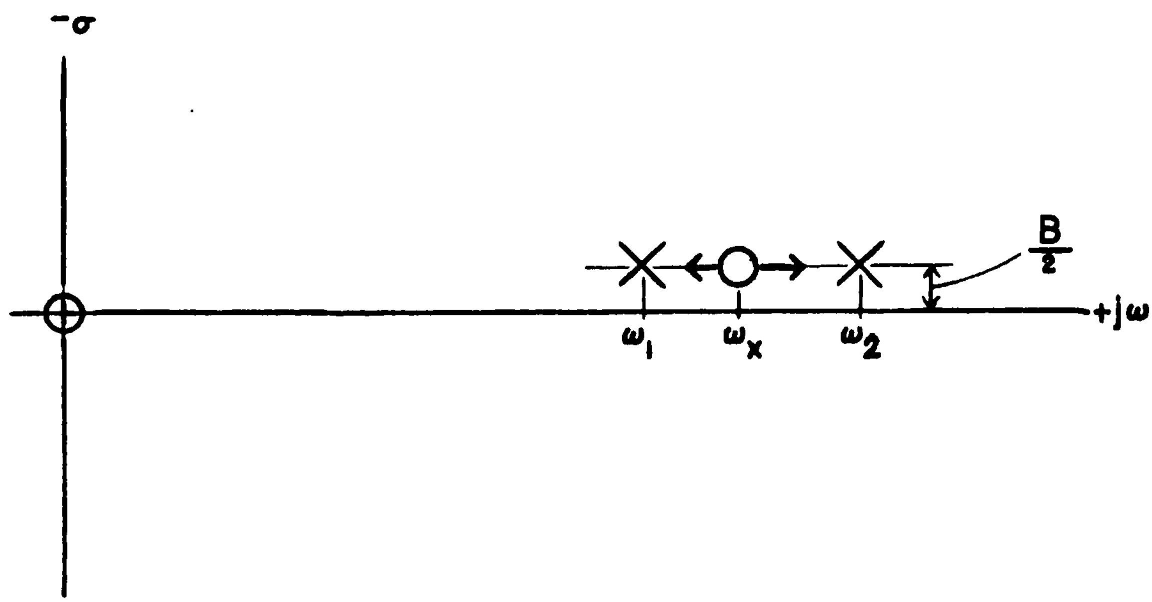


Fig. 12 POLE-ZERO PLOT (FOR POSITIVE FREQUENCY)

down (and vice versa), but it refused to oscillate properly at frequencies between these two extremes.

3.2 Analysis of Difficulty

The reason for this difficulty may be seen in Fig. 13. In Fig. 13a, the reactive component of the impedance of a series RLC resonant circuit is shown as a function of frequency (with the poles and zeroes of the function also plotted on the same scale). Note that one circuit provides inductive or lagging phase and the other circuit provides capacitive or leading phase in the region of interest between the two resonant frequencies. This is one of the requirements for oscillation as shown in Fig. 2a. In Fig. 13b the reactive component of the impedance of the crystal is shown as a function of frequency. Note that the anti-resonant frequency of the crystal introduces a pole very near the resonant frequency of the crystal.

This pole causes the phase angle of the crystal in the first channel to go from a leading phase angle below the resonant frequency to a lagging angle above the resonant frequency; but immediately thereafter (above the frequency of anti-resonance), the phase angle becomes leading again. Successful operation of the oscillator requires that this pole at the anti-resonant frequency of the first crystal either be eliminated or moved up the $j\omega$ axis well beyond the resonant frequency of the second crystal.

Since this anti-resonant pole is caused by the shunt capacitance C_o of the crystal, it was first attempted to eliminate this pole by placing inductance across the crystal. This inductance was chosen so as to anti-resonate with C_o at the series resonant frequency of the

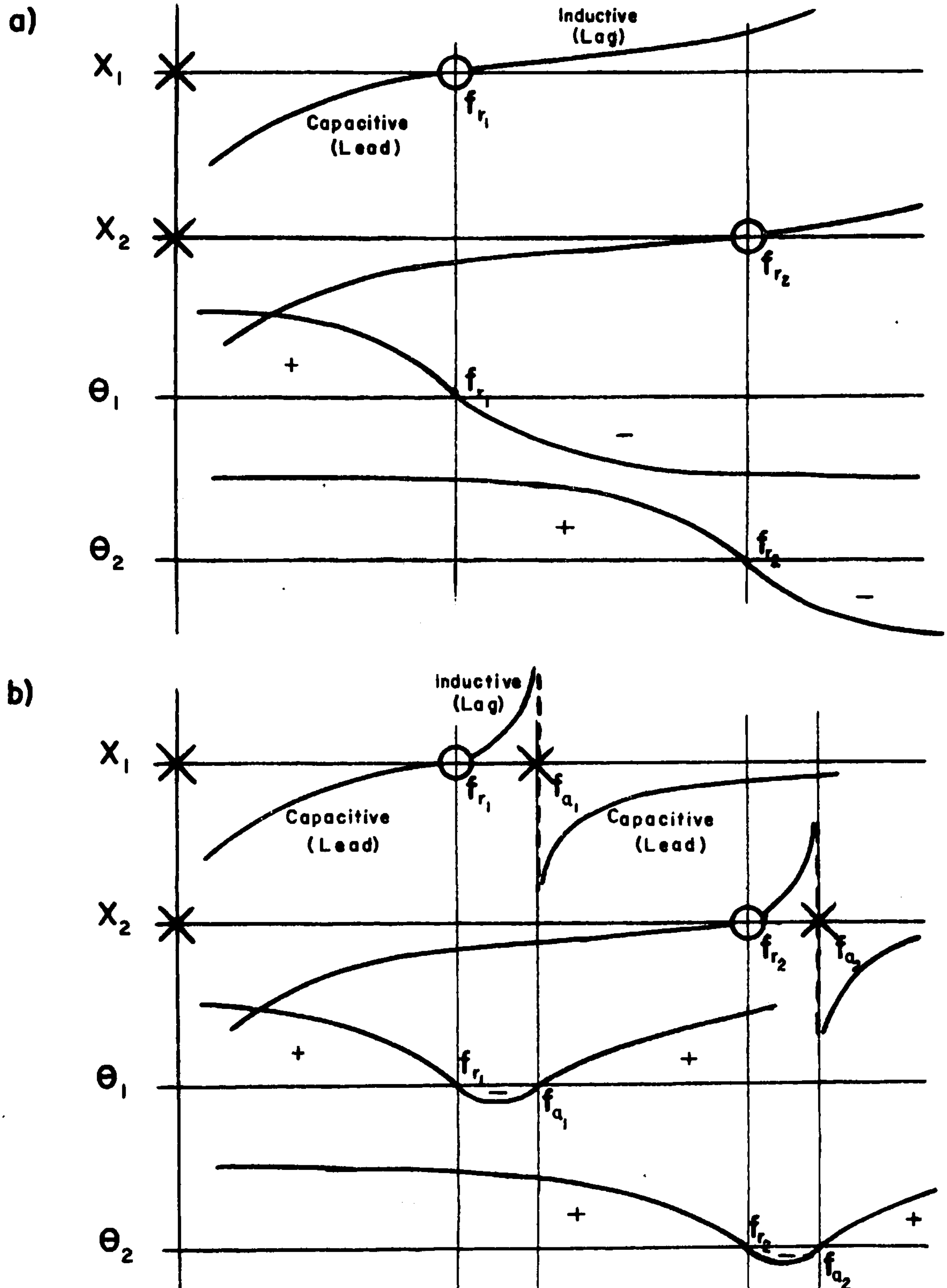


Fig. 13 REACTANCE AND PHASE PLOTS

crystal. This maneuver did not succeed. Various values of inductance and damping resistance were attempted, none of which succeeded in causing the oscillator to operate properly. A mathematical treatment is provided next which explains this failure.

3.3 Mathematical Analysis

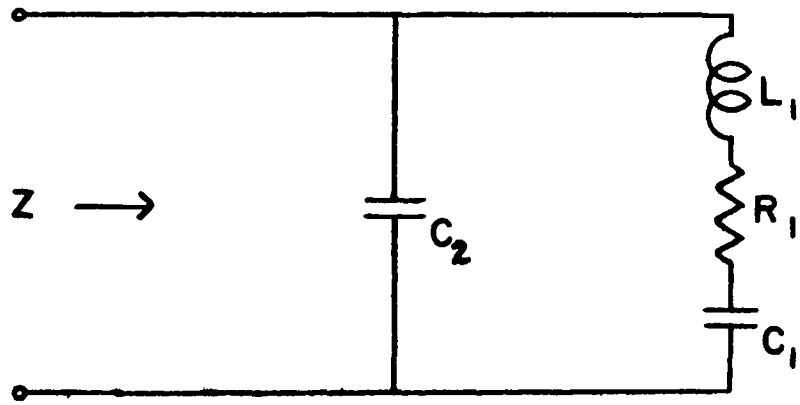
Consider the equivalent circuit for the crystal in Fig. 14a. The input impedance of this circuit is

$$Z = \frac{1}{\dfrac{1}{pL_1 + R_1 + \dfrac{1}{pC_1}} + pC_2} = \frac{p^2 + \dfrac{R_1}{L_1}p + \dfrac{1}{L_1C_1}}{C_2 \cdot p \left[p^2 + \dfrac{R_1}{L_1}p + \dfrac{1}{L_1\left(\dfrac{C_1C_2}{C_1 + C_2}\right)} \right]} \qquad (3\text{-}5)$$

The pole-zero plot of this impedance is shown in Fig. 14b. The zero and the pole along the $j\omega$ axis are the same distance from the $j\omega$ axis; but the zero is at the frequency where L_1 is series resonant with C_1 and the pole is at the frequency where the same inductor L_1 anti-resonates with the series combination of C_2 and C_1. If additional capacitance is added across this circuit, C_2 is effectively increased and the pole moves closer to the zero. This, obviously, does not provide a solution.

An heuristic solution is to damp out the parallel capacitance with an inductor as shown in Fig. 15a. However, while this does result in providing exactly a pure resistance R_1, at the resonant frequency

a)



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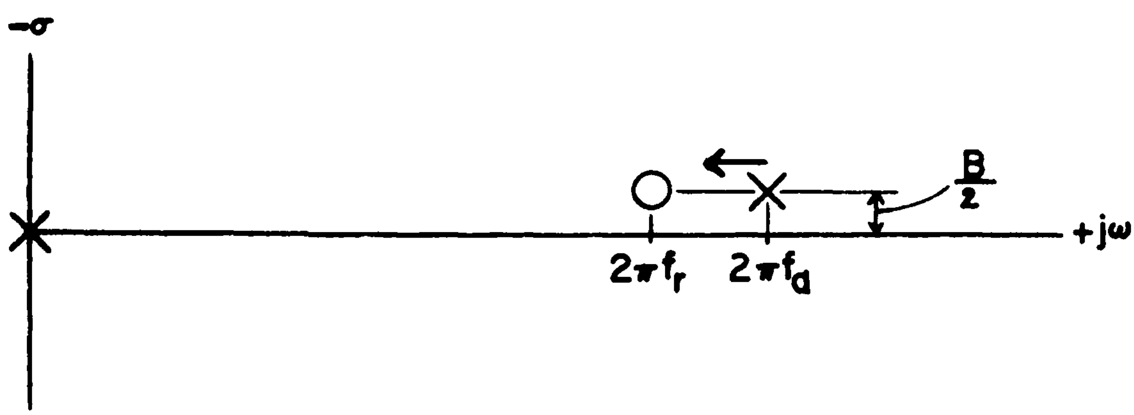
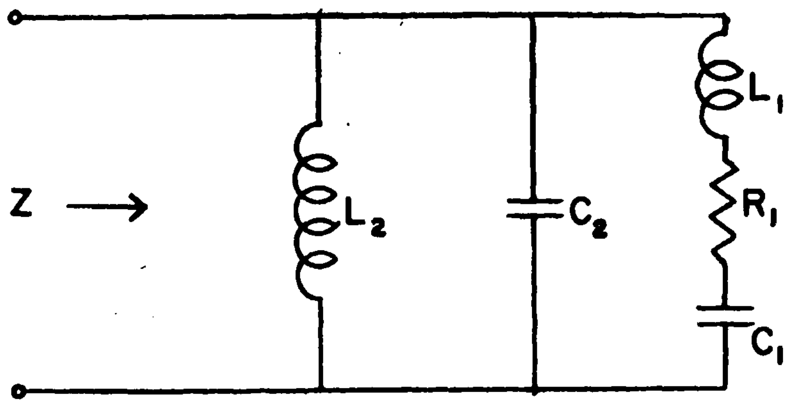


Fig. 14 EQUIVALENT CIRCUIT AND POLE-ZERO PLOT FOR CRYSTAL

a)



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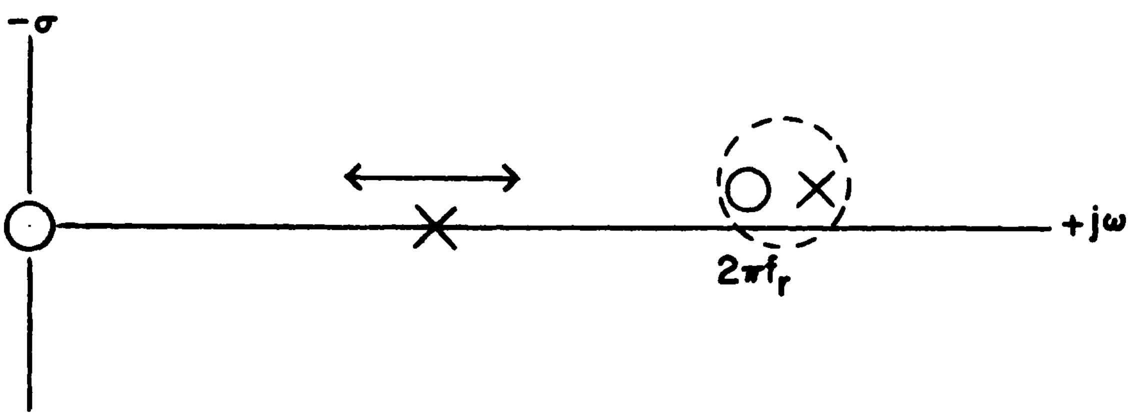

Fig. 15 EQUIVALENT CIRCUIT AND POLE-ZERO PLOT FOR CRYSTAL WITH SHUNT INDUCTANCE

(if L_2 is chosen properly), it does not effect the phase angle of the impedance in the desired manner, as will be shown. The input impedance of this circuit is

$$Z = \frac{1}{pC_2 + \frac{1}{pL_2} + \frac{1}{R_1 + pL_1 + \frac{1}{pC_1}}} = \frac{\frac{1}{C_2}\, p \left[p^2 + \frac{R_1}{L_1}\, p + \frac{1}{L_1\, C_1}\right]}{\left[p^4 + \frac{R_1}{L_1}\, p^3 + \left(\frac{L_2C_2 + L_1C_1 + L_2C_1}{L_1L_2C_1C_2}\right) p^2 + \frac{R_1}{L_1L_2C_2}\, p + \frac{1}{L_1L_2C_1C_2}\right]} \tag{3-6}$$

Now, for the crystals under consideration L_1 is fairly large, C_1 is relatively small (especially as compared with C_2 which is approximately 500 or more times greater) and R_1 has a nominal value of about 200 ohms. Then the term $(L_2C_1p^2)$ in the denominator polynomial may be neglected, since it is extremely small as compared with the other two terms in the expression. L_2 is generally smaller than L_1, and C_2 is much greater than C_1. Due to this approximation, the polynomial may be factored as

$$Z \approx \frac{1/C_2 \cdot p \cancel{\left[p^2 + \frac{R_1}{L_1}\, p + \frac{1}{L_1C_1}\right]}}{\left[p^2 + \frac{1}{L_2C_2}\right] \cancel{\left[p^2 + \frac{R_1}{L_1}\, p + \frac{1}{L_1C_1}\right]}} \tag{3-7}$$

and hence, a pole and a zero are so close together that they effectively cancel one another. All that is left is a zero at the origin and a pole on the $j\omega$ axis, which travels along the axis as L_2 is varied. This is shown in the pole - zero diagram of Fig. 15b, where the pole and zero which are represented by the cancelled polynomials are shown in the circle.

Adding shunt conductance does not help as may be seen from Fig. 16a. The admittance of this circuit, which neglects the effect of R_1 (the idealized case), is

$$Y = G_2 + pC_2 + \frac{1}{pL_2} + \frac{1}{pL_1 + \frac{1}{p\,C_1}} =$$

$$\frac{C_2\left[p^4 + \frac{G_2}{C_2}\,p^3 + \left(\frac{L_2C_2 + L_1C_1 + L_2C_1}{L_1L_2C_1C_2}\right)p^2 + \frac{G_2}{L_1C_1C_2}\,p + \frac{1}{L_1L_2C_1C_2}\right]}{p\left(p^2 + \frac{1}{L_1C_1}\right)} \tag{3-8}$$

Again neglecting the term L_2C_1, which is extremely small when compared with the other coefficients of p^2, the normalized admittance is approximately

$$Y \approx \frac{K\,(p^2 + pB + \omega^2)\,\cancel{(p^2+1)}}{p\cancel{(p^2+1)}} \tag{3-9}$$

which has the inverse

$$Z \approx \frac{1}{K} \cdot \frac{p}{p^2 + pB + \omega^2}$$

The pole-zero plot for this impedance is shown in Fig. 16b. As compared with Fig. 15b, the only difference to be noted is that the pole has shifted off the $j\omega$ axis. Mathematical analyses pertaining to addition of series capacitance, series resistance, or series inductance have similar results; the phase angle of the impedance is not affected in the desired manner (that is, to eliminate the pole at the anti-resonant frequency or to shift it well away from the zero). The next seemingly logical step is to eliminate the shunt capacitance which is causing this trouble by adding negative capacitance (-C) in parallel.

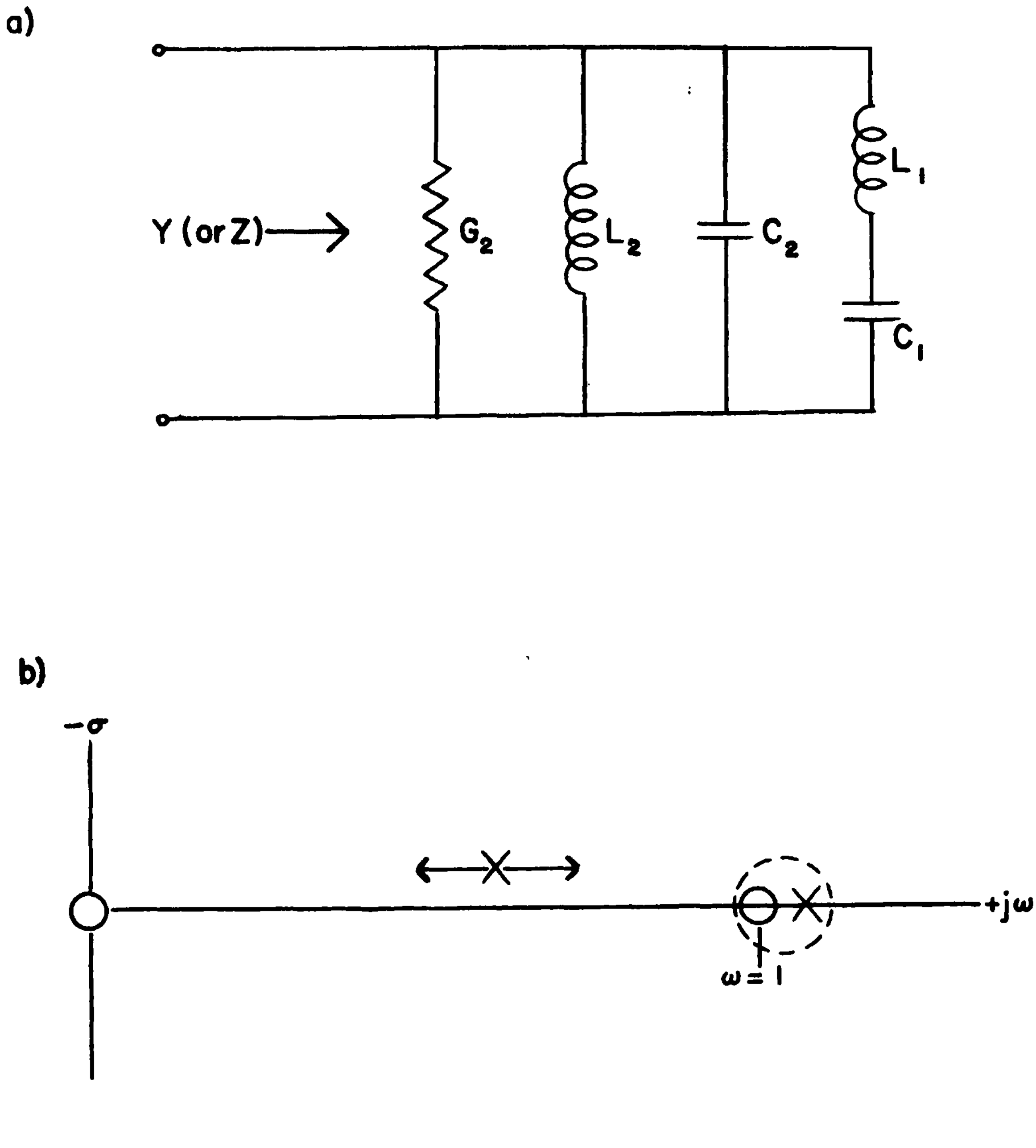

Fig. 16 EQUIVALENT CIRCUIT AND NORMALIZED POLE-ZERO PLOT FOR CRYSTAL WITH SHUNT INDUCTANCE AND SHUNT CONDUCTANCE

CHAPTER 4

CANCELLATION BY NEGATIVE CAPACITANCE AND PROBLEMS

4.1 A Method for Obtaining Negative Capacitance

A method for obtaining negative capacitance is through use of "Miller effect".[1,2] Qualitatively, Miller effect can be understood easily from Fig. 17a, where A_1 is gain. If a voltage E is applied to this system, C_1 will charge to E; but C_2 will have a voltage impressed across it of $E - A_1E = E\,(1-A_1)$. Thus, the effective charging at the input is as if due to a capacitor of value $C_2(1-A_1)$ in parallel with C_1. Overall input capacitance is then

$$C_{total} = C_1 + C_2\,(1-A_1) \qquad (4\text{-}1)$$

If a grounded cathode amplifier stage is used, A_1 is negative and C_{total} is therefore positive. A positive gain can be obtained from a cathode follower but then $A_1 < 1$ and C_{total} is still positive.

Addition of a second stage of gain as shown in Fig. 17b will permit negative capacitance to be obtained. Now,

$$C_{total} = C_1 + C_2\,(1-A_1) + C_3\,(1-A_1A_2) \qquad (4\text{-}2)$$

and if A_1 and A_2 are both negative (grounded cathode amplifier stages), the third term can be made negative. By appropriate choice of magnitudes for A_1, A_2, and C_3, C_{total} can be made negative.

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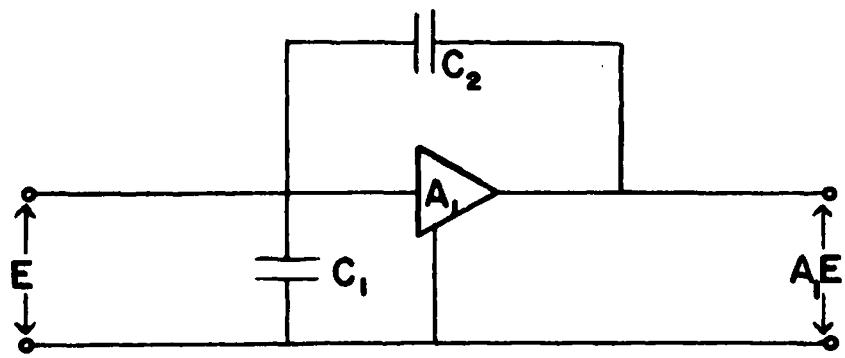
[1] H. J. Zimmerman and S. J. Mason, "Electronic Circuit Theory", John Wiley & Sons, Inc., New York, 1959.

[2] J. L. Stewart, "Circuit Theory and Design", John Wiley & Sons, Inc., New York, 1956.

a)



b)

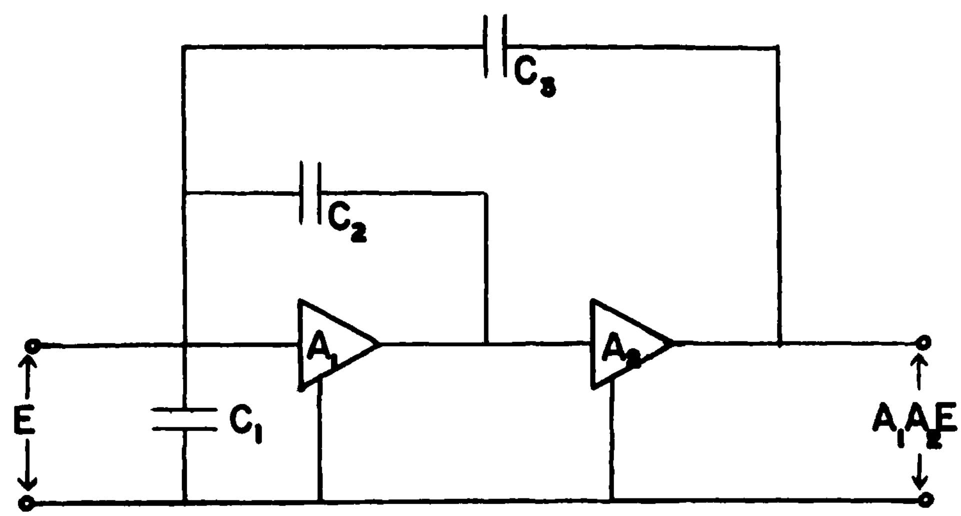


Fig. 17 MILLER EFFECT SCHEMATICS

4.2 A Circuit

Fig. 17b can be converted to an actual circuit by letting $C_1 = C_{gk} + C_{wiring}$, $C_2 = C_{gp}$, and C_3 be some physical capacitor. The circuit in Fig. 18 was developed for this purpose. Several points are worthy of note. A_1 should be kept small in order to minimize the positive term $C_2(1-A_1)$ but should not be too small since A_1A_2 must be greater than 1 for overall negative capacitance. Pentodes were used to further minimize this term ($C_2 = C_{gp}$). A_2 should be variable since C_1 and A_1 may vary. A_2 can be adjusted then to allow for these variations in order to exactly cancel the shunt capacitance of the crystal. The time constant of the interstage coupling network should be long so as to introduce little phase shift; but the time constant of the C_3 charging circuit should be short compared with $1/f$ so that C_3 can charge and discharge during a cycle of applied signal. This criterion is difficult to meet. For a crystal shunt C of 10 pf, $C_{wiring} = 4$ pf, $|A_1| = 1$, $|A_2| = 2$:-

$$C_{total} = C_{wiring} + C_{gk} + C_{gp}(1 + |A_1|) + C_3(1 - |A_1||A_2|)$$

$$-10 = 4 + 4 + 0.02(2) + C_3(-1)$$

$$C_3 \approx 18\text{pf}$$

A capacitor of 25pf was used with A_2 reduced from its nominal value of 2. This reduction of A_2 reduces the load resistor of the second tube which is also one of the resistors in the C_3 charging circuit. The other impedance in the RC circuit is the input R, which is shunted by the crystal. The input R must be much greater than R_1 in order to prevent swamping out the effect of the crystal by shunt cur- through R. At crystal resonance, R is shunted by R_1 so the time constant is short; but off resonance, the crystal impedance increases and hence

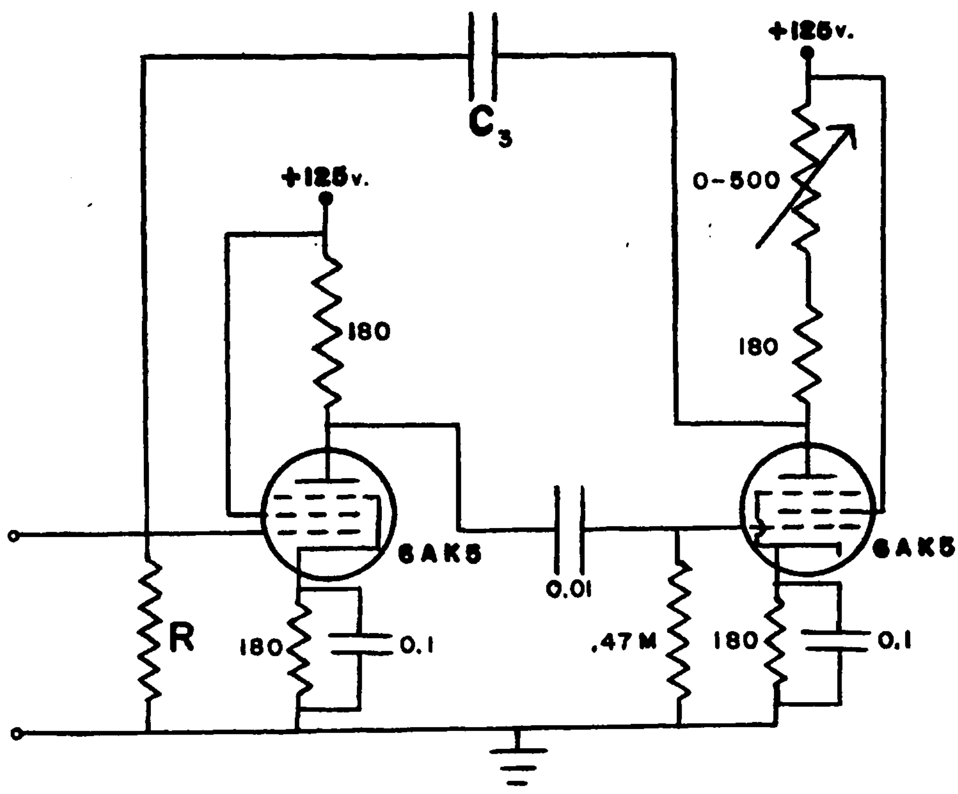


Fig. 18 NEGATIVE CAPACITANCE CIRCUIT

the time constant increases. For the optimium case, the time constant is approximately

$$(R_1 + R_{load_2})\ C_3 \approx (200 + 400)\ (25 \times 10^{-12}) = 0.015\ \mu sec \qquad (4\text{-}4)$$

which is only a factor of 67 different from $T = 1/f = 1\ \mu sec$ at 1 Mc. If a lower value of C_3 is used, A_2 must be increased in order for C_{total} to be negative. This increase in A_2 requires a larger load resistor which increases the time constant whereas we took C_3 smaller in order to shorten the time constant.

It should be noted that the criterion for negative input C is also the same as for positive feedback if the time constants of both "interstages" are long compared to $1/T$; this circuit, in fact, oscillated occasionally at approximately 10 Mc (apparently when the crystal was off resonance or not vibrating at all).

Another problem involved here is that this circuit must be placed across the crystal and therefore the circuit ground must "float" with reference to the chassis ground of the parallel network oscillator. This requires separate power supplies, filament transformers and so forth.

This system has not provided the solution desired to date; further development efforts are needed.

CHAPTER 5

RECOMMENDATIONS FOR FURTHER STUDY AND EXPERIMENTATION

Several recommendations for further study and experimentation can be made.

Continuation of research to develop negative capacitance should succeed after problems of isolating the chassis and positive feedback are solved. This will probably not be an economically satisfactory solution, however, since a two tube circuit has to be added to the basic oscillator. It is doubtful that the two tubes in path one (or path two) could be used for this purpose as well as for amplification. In fact, only one tube in each path is really essential for positive gain since another 180° phase shift can be provided by a transformer.

Further research on the structure and operation of crystals and methods of their use may provide a solution. Damping a crystal by running it under gaseous pressure or in a liquid should increase R_1 since the resistance in the equivalent series arm is partially due to the crystal slab striking air molecules inside the crystal holder. An increase in R_1 should increase bandwidth and may (in conjunction with unknown changes in L_1 and C_1) provide the desired shifting apart of the pole and zero. A new type of crystal, in which C_2 or L_1 is smaller (and thus, C_1 larger for the same frequency) would not permit the term $L_2C_1p^2$ to be neglected in Equation 3-6. C_2 would be more difficult to reduce than L_1 since it is due to the holder, plating, leads, etc. and has obvious limitations on its reduction.

The oscillator circuit described in this paper was originally based on the criterion in Fig. 2a; but, since both channels provide leading phase shift as shown in Fig. 13b over most of the range of frequencies under consideration, perhaps use of Fig. 4b for the design criterion would be more appropriate. This would require, in general, quite different magnitudes of gain in each channel and also negative gain in one of the channels.

Another possibility is to provide a constant 90° phase shift network in order to shift the leading phase of one side channel into the fourth quadrant ($-90^\circ < \theta < 0^\circ$) and thus make it effectively lagging.

Attempting an entirely new configuration in which one crystal is utilized in its anti-resonant mode and the other channel has its crystal utilized in a series mode appears also to be a likely field of endeavor. One could operate above the anti-resonant frequency of channel 1 (a pole) and below the resonant frequency of channel 2 (a zero) and in this way use the pole, which causes so much trouble in the configuration chosen by the author.

Last of all, of course, would be methods of transistorising and miniaturizing a successful circuit. The transistor is a current device and the pentode is often represented as a current source, so there are strong similarities. A wide range, voltage-tunable, crystal controlled, transistorized oscillator surely has a place in today's world of electronic devices.

Although this particular parallel network oscillator with crystals in the side channels did not actually operate as desired, it is felt that the reasons for this are now known and that an operating circuit can be designed and built after further research.

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