

A SINE-COSINE GENERATOR FOR USE IN
ANALOG COMPUTERS

by

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ABSTRACT

A circuit generating the sine, cosine, and arcsine functions is presented. The accuracy and bandwidth are such that the generator is useful in most slow analog computers. Non-linear modulation is used to obtain the desired functions while maintaining circuit simplicity.

Several components and design techniques utilized in the complete circuit have considerable interest as separate elements. A Schmitt trigger is developed that switches at about zero input with zero hysteresis and a one microsecond rise time. A very simple oscillator with 0.26 per cent distortion and a high degree of amplitude stability is presented. A method of designing low-pass filters for use in conjunction with operational amplifiers is developed. Simple diode function generators are used to increase the accuracy by a factor of two.

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Chapter 1

Introduction

The sine and cosine functions are so often used in the solution of differential equations that special circuits generating these functions are frequently incorporated in analog computers. One of the most important applications of sine-cosine generators is to coordinate conversion.

1.1 Types of Trigonometric Function Generators

Because of the frequency with which these functions are used, many schemes have been devised to generate them. The servo multiplier-resolver is one of the oldest of these. A computer servo-mechanism drives special potentiometers wound so that the output voltage varies proportionally to the sine or cosine of the input voltage. At low frequencies, these devices attain accuracies better than 0.15 per cent.¹ Servo bandwidths are very low, on the order of a few cycles per second. Servo-driven potentiometers also require mechanical adjustment and are subject to wear. They have the added disadvantage of being quite expensive.

These problems have caused circuit designers to look for some way to accomplish the same thing with all electronic circuitry. This has led to the diode function generator, which approximates the desired function with a series of straight-line segments. At the present time there are units available with 0.05 per cent

¹G. A. Korn and T. M. Korn, Electronic Analog Computers, 2nd Ed., McGraw-Hill Book Co., Inc., 1956, p. 333

accuracy and bandwidths of 50 cycles per second.² Devices of this type are very complex, as a large number of segments must be used to obtain a high accuracy.

Sine and cosine functions may also be generated by solving the differential equation,

$$\frac{d^2y}{dx^2} = -a^2y.$$

The equipment required is so complex that this method is not really satisfactory as a special-purpose circuit.³ Because this scheme uses standard analog computer components, a circuit to do this may be set up as part of the problem being solved if no other method is available.

As a fourth method, it is possible to design circuits which modulate a carrier in a non-linear fashion to produce an output proportional to the sine or cosine of the input.^{4,5} It is this last group to which the circuit present here belongs.

1.2 Advantages of the Circuit Discussed in This Paper

The circuit under discussion shares the advantages of others of the modulated-wave type, simplicity of construction, wide bandwidth, and freedom from mechanical wear. In addition,

²"Dual Sinusoid Function Generator Model 1154," Berkley Division, Beckman Instruments, Inc., 1959

³E. G. Gilbert and R. M. Howe, "Trigonometric Resolution in Analog Computers by Means of Multiplier Elements," I.R.E. Transactions on Electronic Computers, Vol. EC-6, June, 1957, pp. 86-92

⁴H. Schmid, "A Transistorized All-electronic Cosine/Sine Function Generator," Western Electronic Show and Convention, 1958

⁵H. Huskey and G. A. Korn, Computer Handbook, McGraw-Hill Book Co., Inc., 1961

it is completely self-contained, requiring no external equipment except power supplies. The use of tubes in this circuit makes it compatible with most large analog computers, which also use tubes. The design utilizes several components which have considerable use as separate elements.

Chapter 2

Principles of Operation

The operation of the circuit described in this paper may be explained with the aid of figure 2.1. A low-distortion oscillator generates a sinusoidal wave of amplitude E, which is added to a voltage X. This sum is applied to a comparator, whose output is V if the sum of the inputs is positive, and -V if it is negative. The average value of the output is $\frac{2V}{\pi} \sin^{-1} \frac{X}{E}$, where the arcsine is expressed in radians. A similar principle has been used by H. Schmid in a transistorized circuit.⁶ The circuitry used here is, however, new. A novel feature is the correction introduced, which permits a substantial increase in accuracy.

2.1 Design Requirements

The block diagram of a circuit enforcing the desired relations is shown in figure 2.2, which also shows the feedback circuit required to obtain the sine function.

The design requirements on each of the components shown in figure 2.2 are rather stringent. The oscillator must have very low distortion, as this will directly effect the accuracy of the complete unit. The sine-wave amplitude must remain constant, since any change will show up as drift at the output. The oscillator, as well as all other components, must be free from hum and noise.

The comparator must switch at zero input voltage. Any drift of the comparator switching point will cause a corresponding

⁶H. Schmid, *ibid* page 2

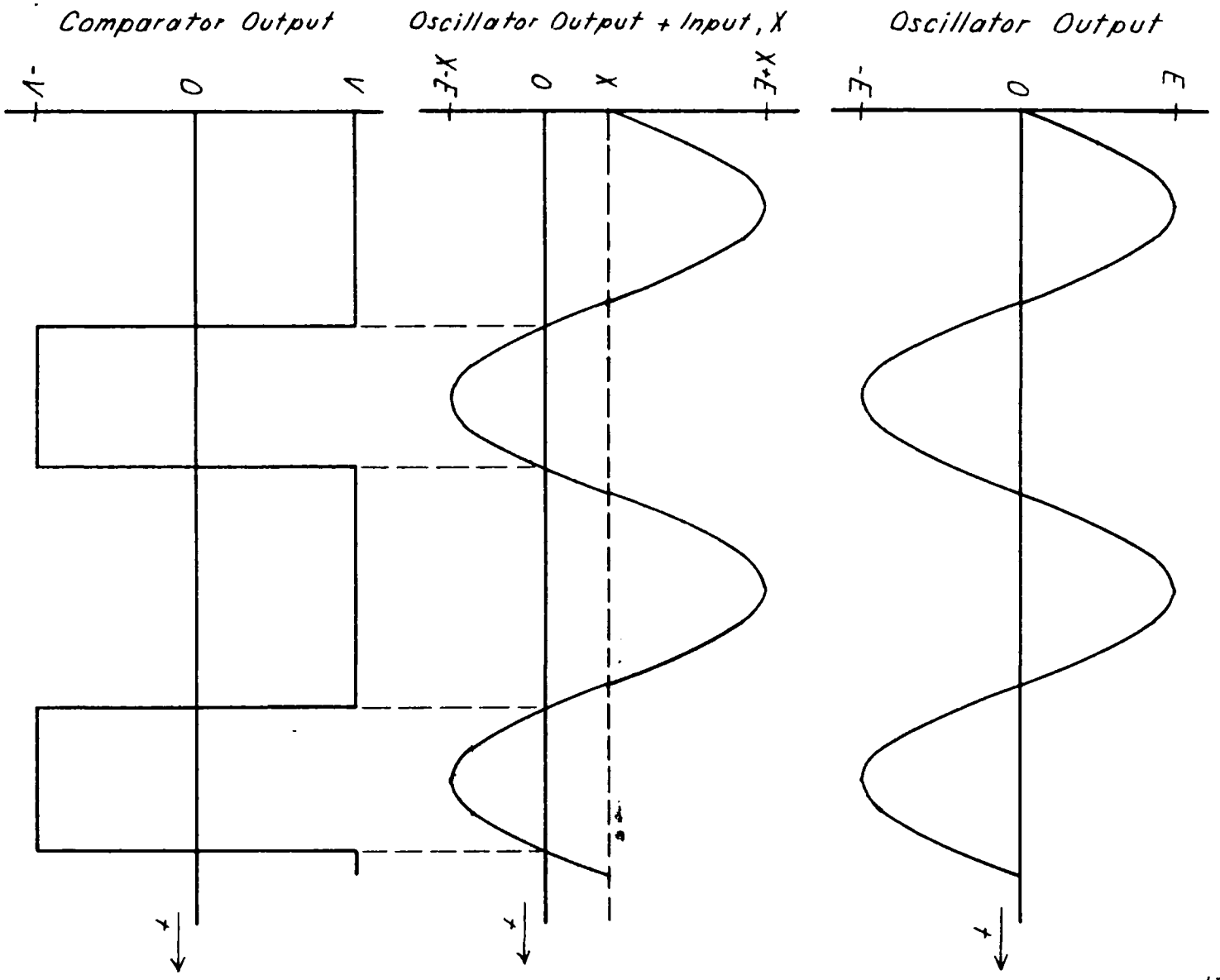
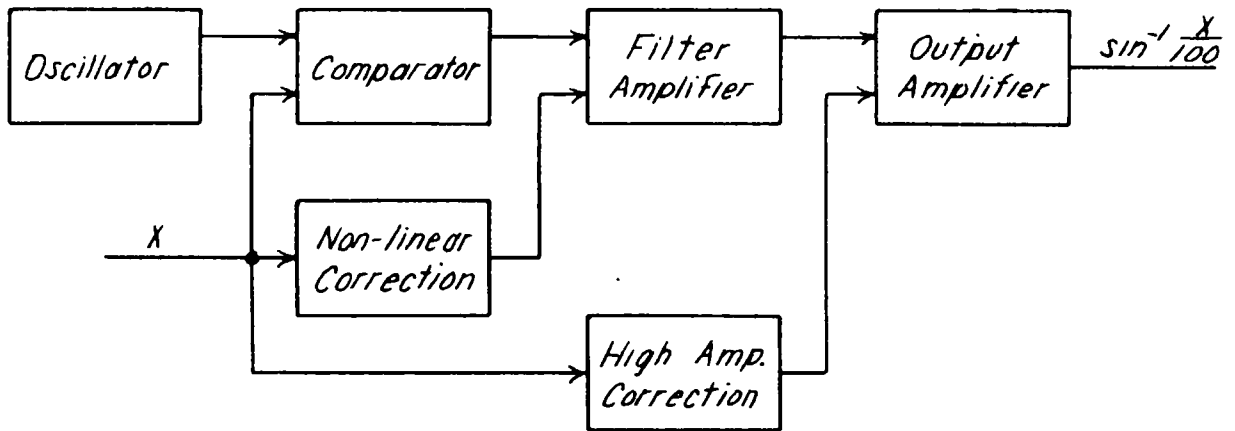
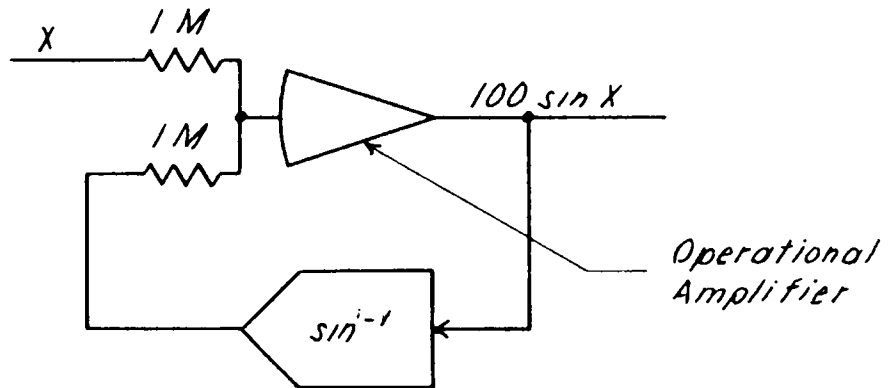


Figure 2.1, Principle of Operation



Arcsine Generator



1 M resistors are wirewound $\pm 1\%$

Sine Generator

Figure 2.2, Block Diagram of Arcsine and Sine Generator

drift at the output. The comparator output must have a short rise time, and the levels between which the output switches must be accurate and free from drift.

The output filter must have a constant input impedance and low phase shift. The attenuation at the carrier frequency and its harmonics must be as high as possible. The filter bandwidth should extend from d-c to as high a frequency as possible with minimum phase shift.

The high-amplitude correction network insures that the circuit is stable regardless of the input voltage. The non-linear correction network generates a function complementary to the error and thus increases the accuracy. Both of these circuits must be as simple as possible.

2.2 Scaling

The unit is scaled so that one volt corresponds to one degree, and so that 100 volts corresponds to one unit. For example, the sine of 30 degrees is 0.5. The sine generator would represent this as 50 volts being the sine of 30 volts.

Chapter 3

A Low-distortion Oscillator

A Wien-bridge type of oscillator is used because it combines the required low distortion with circuit simplicity. Such an oscillator has the added advantage of using only resistors and capacitors as frequency-determining components. These are cheap, light, and available in a great variety of values.

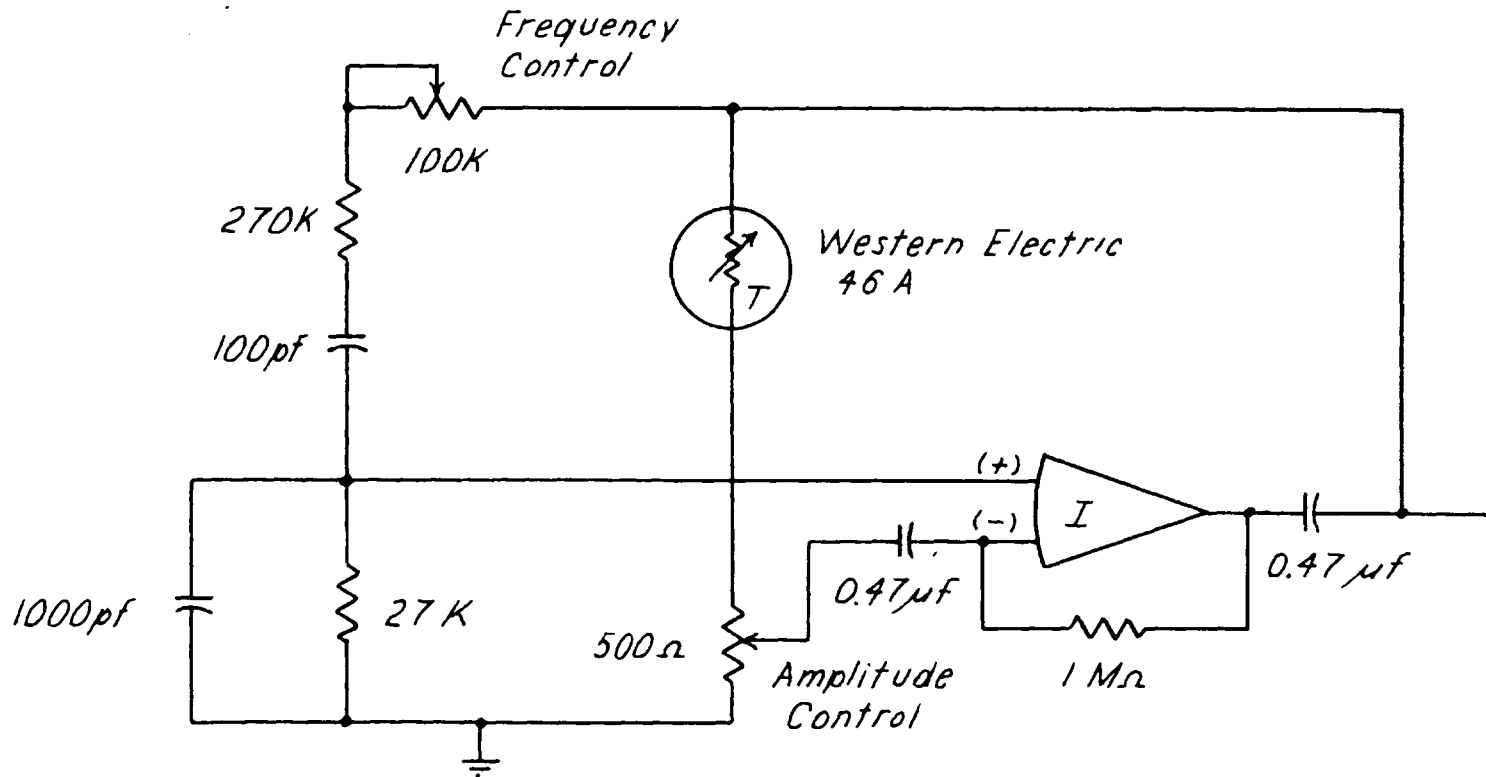
3.1 Operation of the Oscillator Circuit

The circuit used, shown in figure 3.1, requires only a single plug-in operational amplifier with differential inputs, so as to permit both regenerative and degenerative feedback. The gain of the regenerative feedback circuit is a function of frequency, and at the desired frequency of operation a maximum of about one-tenth of the output voltage is fed back. The degeneration increases with increasing output amplitude; again, about one-tenth of the output is fed back at the desired output amplitude.

3.2 Oscillator Circuit Design

The gain of ten is not arbitrary. The amplifier used is linear only as long as the input remains within ± 2 volts of zero.⁷ The maximum output voltage is ten volts. The maximum input voltage is then one volt. This is well within the linear range of the amplifier.

⁷H. Koerner, "Plug In Operational Amplifiers with Extended High Frequency Response," ACL Memo 13, Electrical Engineering Department, The University of Arizona, 1960



All resistors are $\frac{1}{2}$ watt, $\pm 10\%$.
 Operational amplifier is a U.A. Model I compensated with 10 pf
 and boosted with a 47 K, 2 watt resistor (see footnote 7).

Figure 3.1, 5 Kc Wien-bridge Oscillator

A thermistor is used as the amplitude-sensitive element in the degenerative loop. In circuits of this type, it is customary to use a lamp bulb for this element. It was found that with the lamp there is an intolerable amplitude jitter. Again, most lamps have an impedance level far too low to permit them to be heated to a temperature that provides sufficient regulation for use in this application. The particular thermistor used has a resistance of five kilohms at the operating point of the oscillator, which may be compared to the 500 to 2000 ohm resistance of most lamps.

The use of the thermistor raises some problems not encountered with lamps. These result from the thermistor's extreme sensitivity to temperature. The desired regulating action depends on the current through the thermistor causing a change in temperature which results in a change in the resistance of the element. Proper operation also depends on a change in the ambient temperature not causing a significant change in the resistance. These conditions are obtained in two ways. A large current is run through the thermistor to cause maximum heating due to the current, and the ambient temperature is lowered to the point where it can no longer effect the thermistor. In the experimental model, this is done by cooling the thermistor by evaporation. It is possible to hold the temperature constant by placing the thermistor in a crystal oven. However, this raises the temperature so high that the effect of the current on the resistance is negligible.

The ten volt amplitude used results from the current limitations of the amplifier.⁸ The thermistor requires a considerable current to keep it's resistance stable. If the output is raised much above ten volts, then the amplifier can no longer supply this required current and severe clipping results.

⁸Ibid, H. Koerner, p. 8

Larger amplitudes may possibly be obtained with different thermistors; under no conditions, however, can the amplifier sustain oscillations above 80 volts without severe clipping.

As shown in figure 3.1, the oscillator is a-c coupled. The use of the differential inputs prevents the amplifier from being chopper stabilized, hence the d-c level of its output is subject to drift. This drift is blocked by the capacitor. Because the feedback networks are a-c coupled, the output level is not influenced by a d-c current flowing through the thermistor. The one megohm resistor provides the d-c feedback necessary to keep the amplifier stable and the output close to zero. The impedance of this resistor is so large compared to that of the degenerative feedback loop that its effect on a-c voltages is negligible.

Note that the output blocking capacitor is inside the feedback loop. It may be shown that any non-linearities are divided by the ratio of the degenerative gain to regenerative gain at the frequency of the harmonics that the distortion generates. This is a desirable and indeed a necessary property in a low-distortion circuit. The input blocking capacitor is not in this fortunate position, but because the voltage swing at this point is low, there should be no noticeable non-linearity from this source.

3.3 Oscillator Performance

The distortion, noise, and hum present are equal to 0.26 per cent of the total output amplitude. The amplitude is adjustable from 8.5 to 12 volts. The frequency is adjustable from 4.2 to 5 kilocycles. In this configuration the amplitude and frequency controls do not appreciably interact. The offset at the output of the amplifier is 1 to 2 volts.

Several sine generators may be supplied by the same oscillator, resulting in an appreciable saving in cost. The low-distortion oscillator circuit is of considerable interest in itself, and may well find use in other applications.

Chapter 4

A Precision Fast Comparator

It is not possible to use the standard operational-amplifier comparator shown in figure 4.1, because it's response time is too slow to make it practical.⁹ With regeneration added to this device, it is possible to obtain a rise time on the order of three microseconds. This regeneration causes ringing to the extent that the switching point is not stable. For this reason, the sine generator uses a Schmitt trigger preceded by a linear preamplifier to reduce the effects of hysteresis and drift in the switching point.

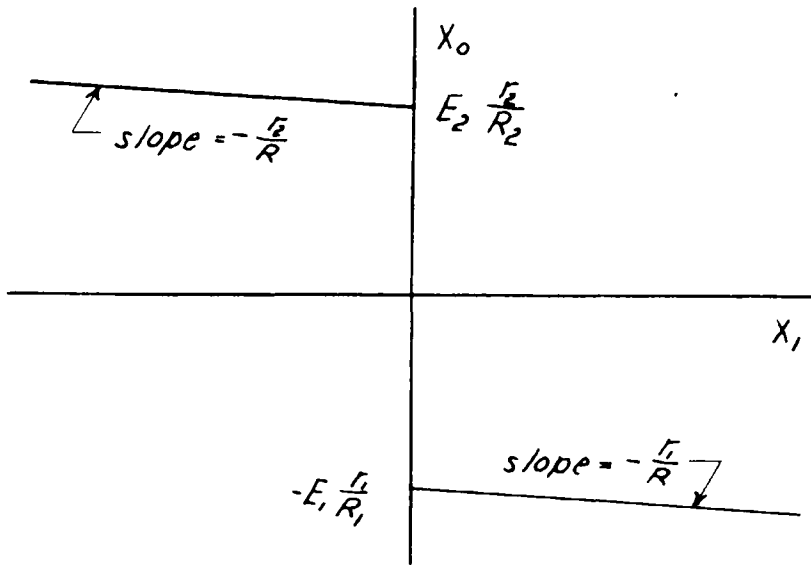
4.1 Design of the Preamplifier

The preamplifier must have very low phase shift and as high a gain as possible. The configuration shown in figure 4.2 represents the maximum gain obtainable without making the compensating capacitor shown excessively large.

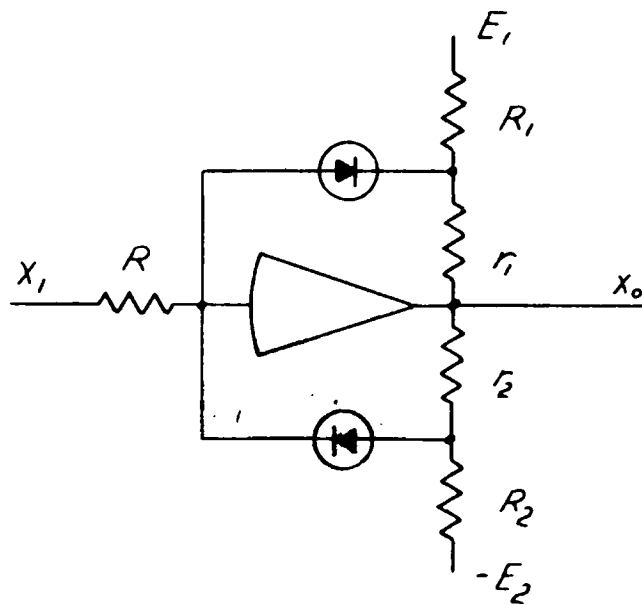
The high gain is necessary only in the vicinity of zero. To keep the amplifier from overloading, it is necessary to limit the output. Near their break-points, the diodes introduce phase shift and storage effects, making it desirable to keep the break-points as far from zero as possible. With the circuit shown, the amplifier is linear throughout the range over which the Schmitt trigger can drift.

The choice of the preamplifier impedance level is a compromise; the feedback circuit impedance must be low enough to swamp out the stray capacitance of the circuit, but high enough to avoid overloading the amplifier.

⁹Korn and Korn, op. cit., p. 298

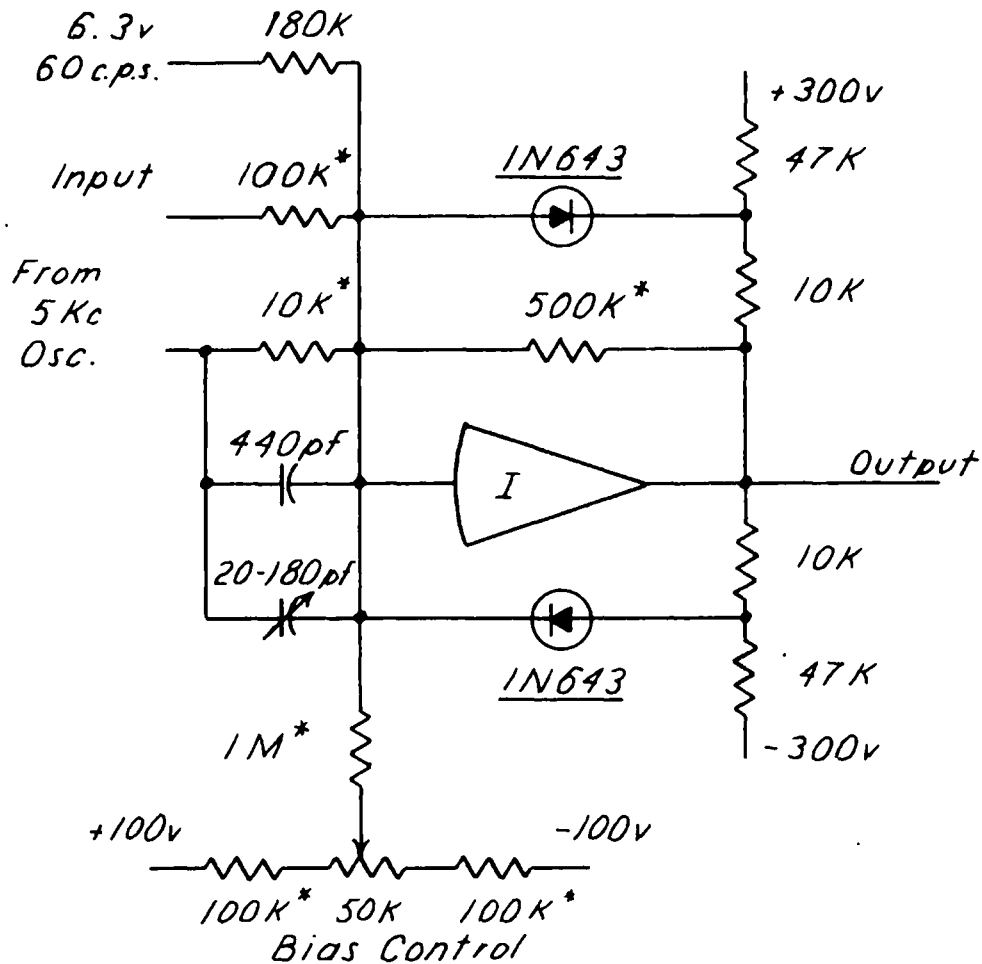


Transfer Characteristic



Circuit

Figure 4.1 , Operational-amplifier Comparator



Operational amplifier is U.A. Model I compensated with 1.8 pf and boosted with a 47K, 2 watt resistor (see Footnote 7).

All resistors are $\frac{1}{2}$ watt, $\pm 10\%$ unless otherwise noted.

* Donotes $\pm 1\%$ wirewound resistor.

Figure 4.2 , Preamplifier

The bias control is necessary, not for the amplifier, which is chopper stabilized, but to zero the small offset of the Schmitt trigger.

4.2 Hum Problems

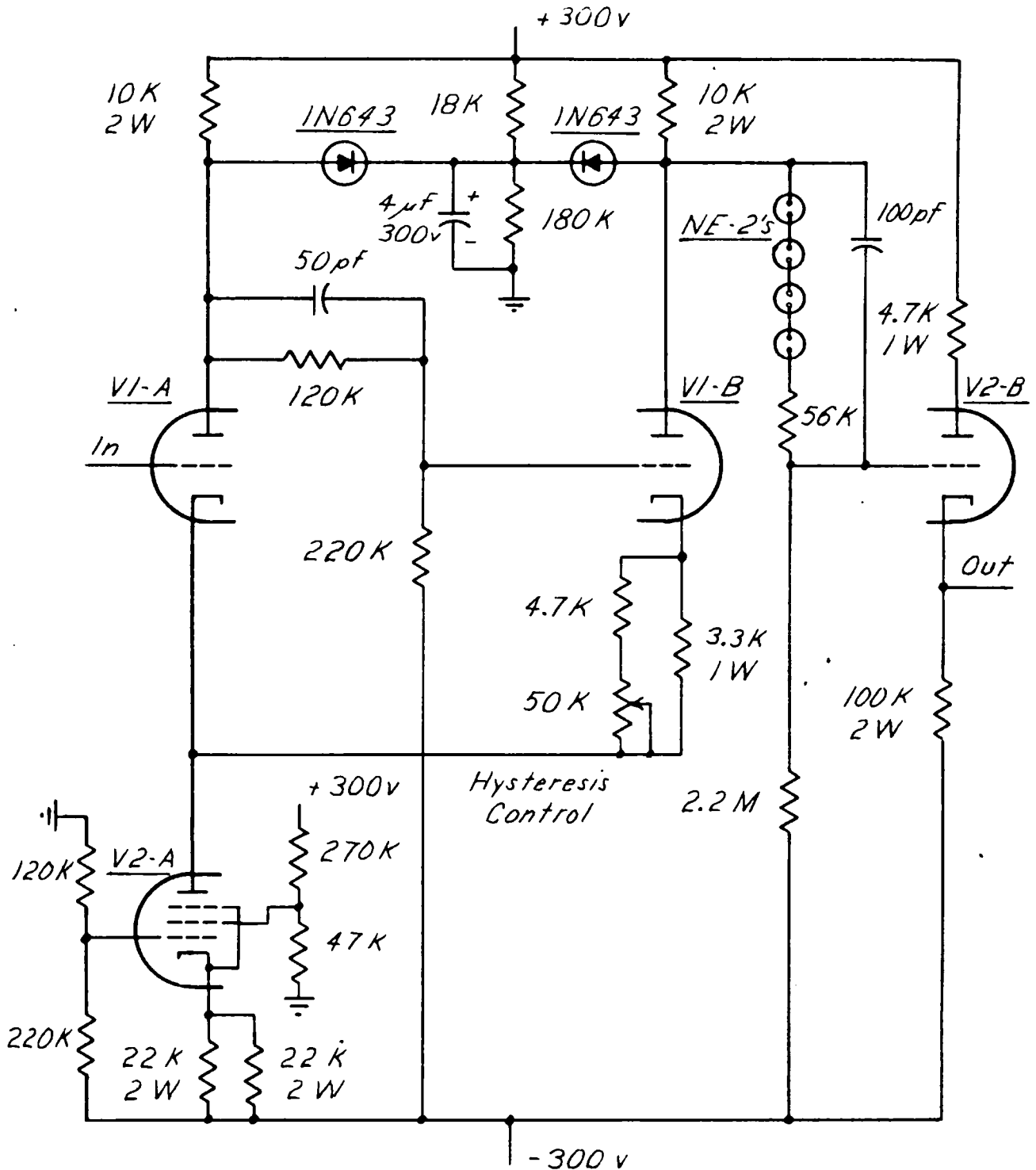
The hum-bucking input is the solution to one of the most serious problems in this design. The size of the hum-bucking input resistor, or indeed whether it is a resistor at all, depends on the power supplies used. In addition to this, all filament leads are run through coaxial conductors. The hum is critically dependent on the placement of circuit elements and lengths of certain critical leads. The present solution is not completely satisfactory, and further efforts should be made to reduce hum in other ways.

4.3 Design of Schmitt Trigger

The Schmitt trigger shown in figure 4.3 is a conventional design.¹⁰ The output cathode follower reduces the output voltage to the desired level and lowers the output impedance.

Because the device is to switch at zero, with the ± 300 volt power supplies available it is necessary that the entire 600 volts be impressed across the circuit. To avoid large resistors and problems with stray capacitance, a dynamic cathode resistance, consisting of a pentode, is used. This provides a current of about 17 milliamperes and the required voltage for the unit to switch at about zero. The dynamic cathode resistance also improves the rise time and stabilizes the switching point. It is possible to adjust the switching point to zero with a suitable potentiometer in parallel with the cathode resistor of the pentode, but because of the drastic effect on the current and hence on the ON voltage of the tubes, it is not practical. This control does provide a method of adjusting the lower of the two output levels, if this is desirable.

¹⁰J. Millman and H. Taub, Pulse and Digital Circuits, McGraw-Hill Book Co., Inc., 1956, pp. 164-172



All resistors $\pm 10\%$, $\frac{1}{2}$ watt unless otherwise noted
 VI-12AU7, V2-6AU8

Figure 4.3, Schmitt Trigger

The trigger portion of the circuit uses an adjustable cathode resistor to reduce the loop gain to unity and thus the hysteresis to zero. The plate-catching diodes are used because of the gradual cutoff characteristics of the tubes. With the nominal loop gain equal to unity, the gain in the region near cutoff is less than unity. The circuit here acts as an amplifier. The plate catching diodes prevent this. Silicon diodes are used because they can stand the high back voltage.

Neon bulbs d-c couple the Schmitt trigger to the cathode follower to prevent the output d-c level from changing as the duty cycle of the trigger changes. The neon bulbs drop a constant voltage without attenuating the signal. The output current capabilities of the comparator may be increased to about eight milliamperes by placing a 47 kilohm, two watt, booster resistor from the output to -300 volts. Without this booster the unit will supply about three milliamperes. It should also be noted that the cathode follower isolates the trigger from any capacitance in the driven circuit.

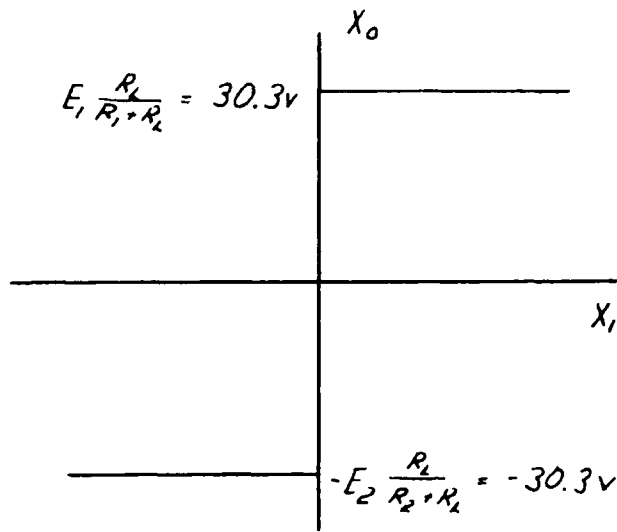
4.4 Performance of the Schmitt Trigger

The rise time of the unit depends somewhat on the speed of the triggering voltage, but in no case is it less than one microsecond. The output switches between +40 and -60 volts. As noted previously, the lower of these voltage levels may be adjusted with a potentiometer from the cathode of the pentode to either the plus or minus supply as necessary. The input switches within three volts of zero. The drift of the switching point is about 0.5 volt. The entire circuit may be mounted in a plug-in can. This has considerable utility in analog-computer work as a separate circuit.

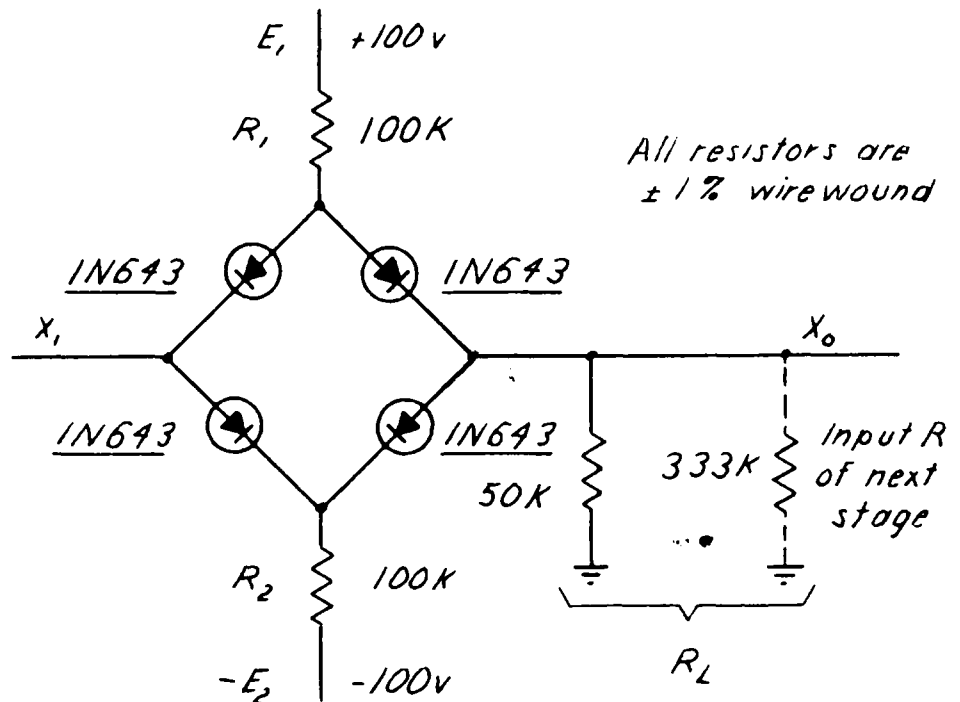
4.5 Design of a Diode Bridge Limiter

The comparator output has some overshoot, and the voltages between which it switches are not accurate. The comparator is therefore followed by the diode bridge limiter shown in 4.4.¹¹ To allow fast rise times, the impedance level should be as low as possible. Precision noninductive wire-wound resistors provide for accurate limiting levels. The voltages must be very well regulated and preferably should be obtained directly from the reference supplies of the computer in which the device will be used. The particular circuit shown here limits at 30.3 volts. This is low enough to insure that the trigger can decisively switch the bridge.

¹¹Korn and Korn, op. cit., p. 294



Transfer Characteristic



Circuit

Figure 4.4, Diode Bridge Limiter

Chapter 5

Output Amplifier and Filter Design

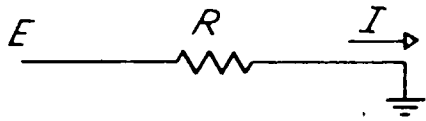
The output element of the complete sine-cosine generator must be an amplifier capable of supplying the required gain and low output impedance. The output low-pass filter is placed in the feedback loop of this amplifier; this greatly simplifies the filter design. The very low output impedance and the virtual ground at the input of the operational amplifier result in effective isolation of parallel sections. The following procedure may be used whenever a low-pass filter of similar design is required.

5.1 Design of the Filter

The four basic sections and their transfer admittances are shown in figure 5.1. The R-section should be chosen first. The resistor, in general, should be as large as possible, 1 megohm being the most common value used. Along with the input resistor, this determines the low frequency gain.

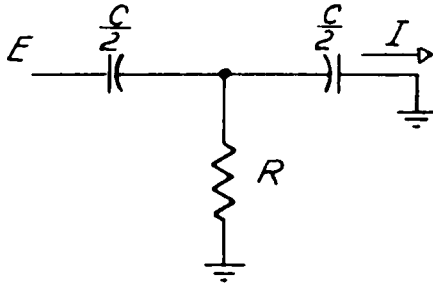
Next the R-C T-section is designed. This attenuates all harmonics higher than second. R and C are chosen to provide the necessary attenuation at the third harmonic of the oscillator frequency. For example, if the admittance of the R-section is 10^{-6} mhos, and the desired attenuation at the third harmonic is 10^{-4} , the admittance of the R-C T-section at this frequency should be 10^{-2} mhos.

Finally, the two L-C T-sections are designed. These are resonant at the fundamental and second harmonic of the oscillator frequency. The attenuation of these sections is limited only by the Q of the capacitors and inductors used. The admittance at all frequencies below resonance should be lower than that of the R-C T-section. This results in minimum phase shift and simplifies



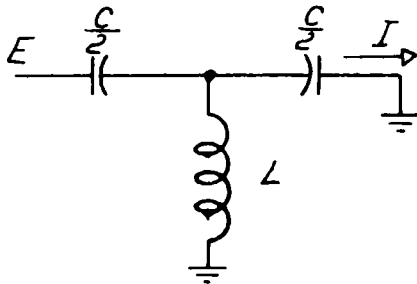
$$Y = \frac{I}{E} = \frac{1}{R}$$

R-section



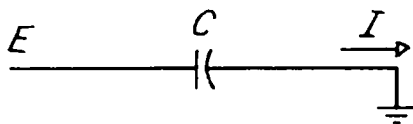
$$Y = \frac{I}{E} = \frac{Cs^2}{4(s + \frac{1}{RC})}$$

R-C T-section



$$Y = \frac{I}{E} = \frac{Cs^3}{4(s^2 + \frac{1}{LC})}$$

L-C T-section



$$Y = \frac{I}{E} = Cs$$

C-section

Figure 5.1, Filter Sections

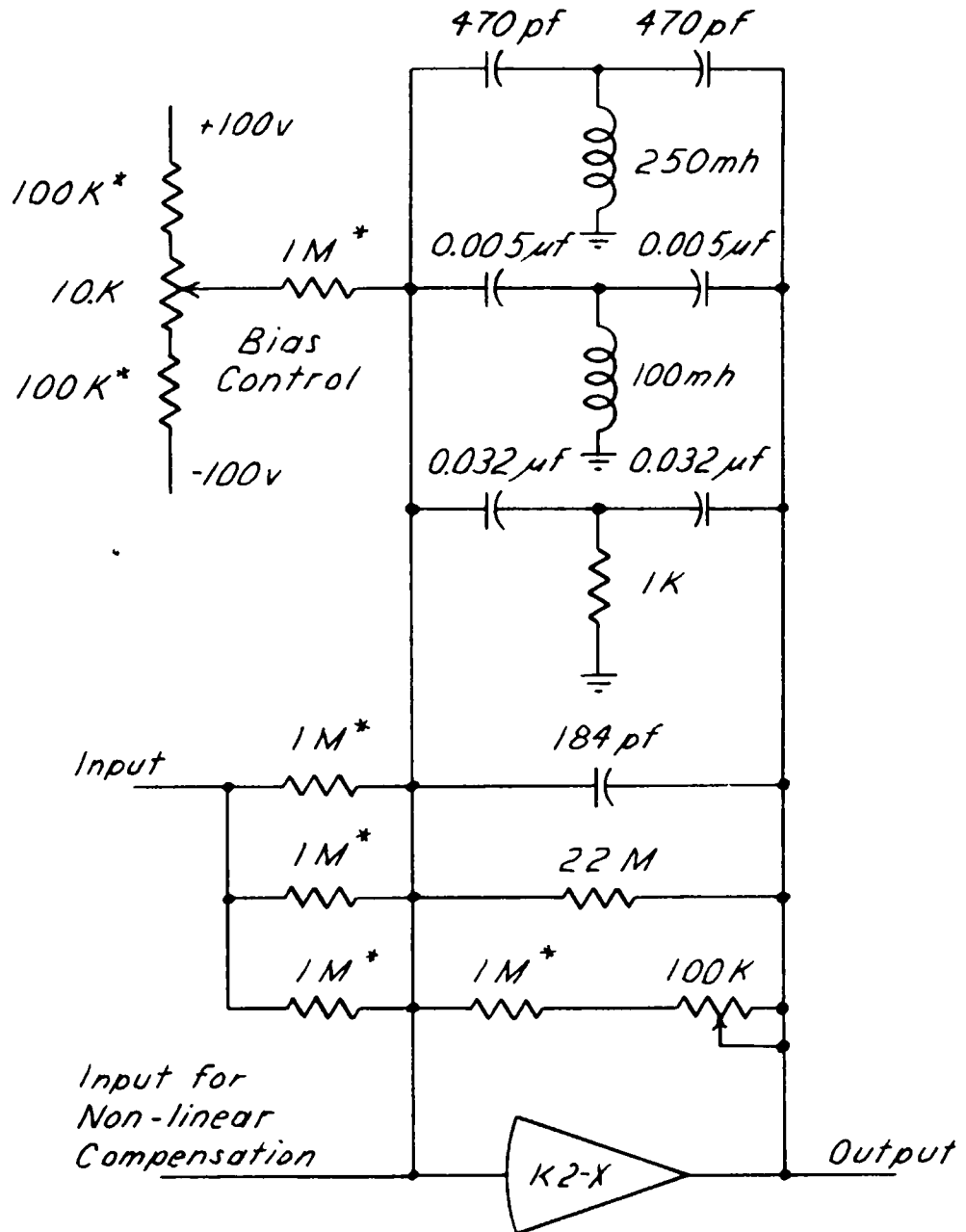
compensation. If the admittance is too low however, the attenuation will not be sufficient at the resonant frequencies. Two L-C T-sections were used here to simplify explanation, but any number may be used.

5.2 Filter Compensation

The filter-amplifier combination described so far will be unstable. The network required for compensation may be computed analytically; this is, however, a very cumbersome procedure. It has been found that if the amplifier gain is less than about 50 at low frequencies, the required compensation is a simple capacitor. A trial-and-error compensation of the complete filter with a capacitor decade is feasible. This method is indeed useful as the required compensation will vary as the complete circuit varies. For example in this design, it was found that 184 pf were sufficient to compensate the unit in the arcsine mode. In the sine mode, 300 pf of additional capacitance were necessary. The complete filter is shown in figure 5.2.

5.3 Filter Characteristics

The complete amplifier has several inputs. The bias control is necessary to remove any offset of the diode bridge. An input for non-linear correction is also available. The performance of this circuit is shown in figure 5.3.



Operational amplifier is a Philbrick K2-X boosted with a 100K, 2 watt resistor and stabilized with a K2-P.

* Denotes $\pm 1\%$ wirewound resistor.

Figure 5.2 , Filter Amplifier

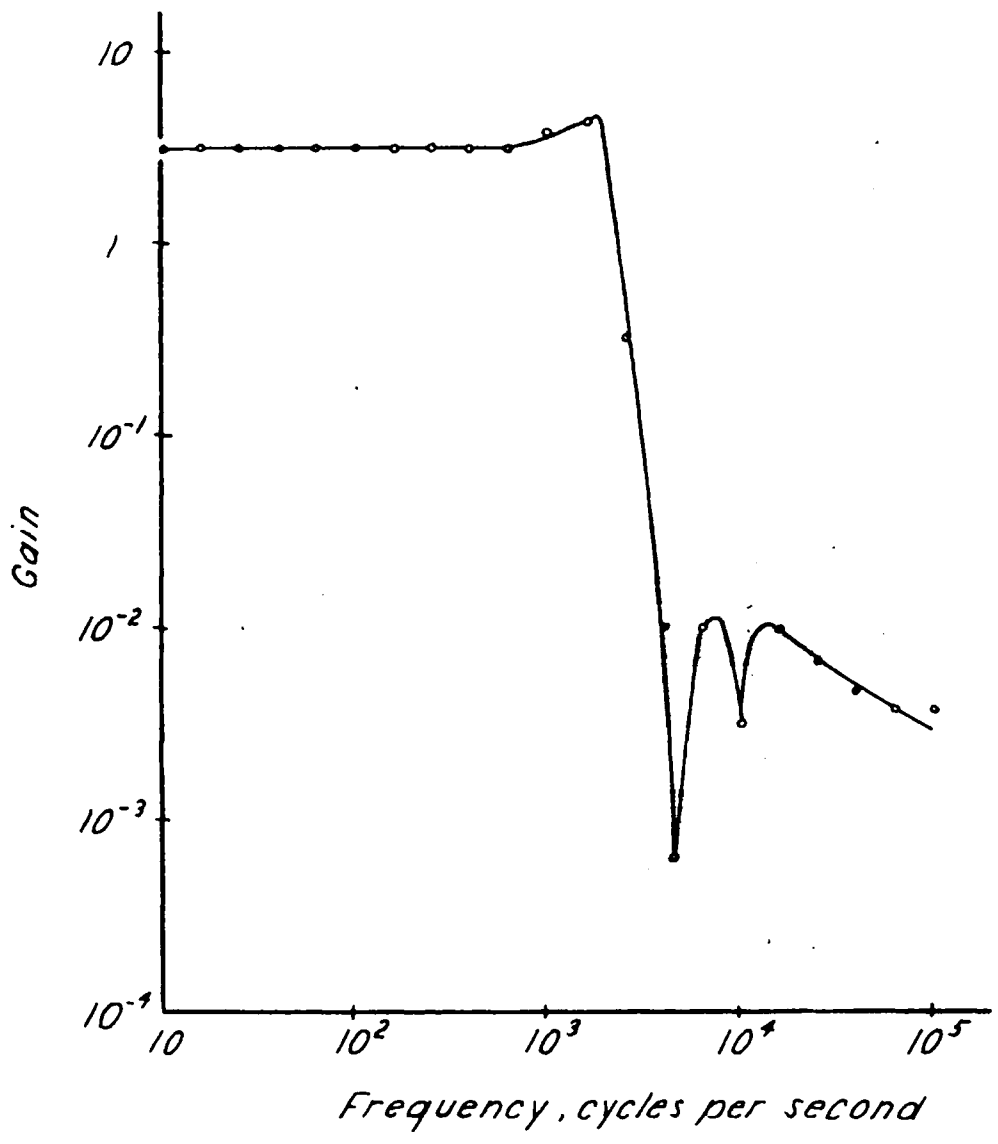


Figure 5.3, Response of Filter Amplifier

Chapter 6

Non-linear Correction Networks

The arcsine function is not suitable for direct use in the feedback circuit of figure 2.2. This can be seen by reference to the arcsine function as generated by this circuit in figure 6.1. If the input exceeds 90 degrees by some small amount, the output of the amplifier will increase beyond 1.00 in an attempt to cause the output of the arcsine generator to increase beyond 90 degrees, and the amplifier overloads.

6.1 Generating the Modified Function

This difficulty may be overcome by using the modified arcsine function shown by the dashed lines in figure 6.1. If the input exceeds 90 degrees, only a very small increase in the output of the amplifier is necessary to cause the required increase in the output of the arcsine generator. The resulting small error is not objectionable.

As can be seen from figure 6.2, a very simple circuit provides the required auxiliary function. This is a standard dead-space circuit, often used in simulation problems. The break points must be placed very accurately, hence R must be small for ease of adjustment. It is necessary to use precision resistors, so that R will still be large enough to compensate for the tolerance of the other resistors.

6.2 Non-linear Correction

Without correction, the errors are distributed about the three straight-line segments shown in figure 6.3. To reduce these errors, the complementary function shown by the dashed lines is generated. The simple circuit providing this function is shown

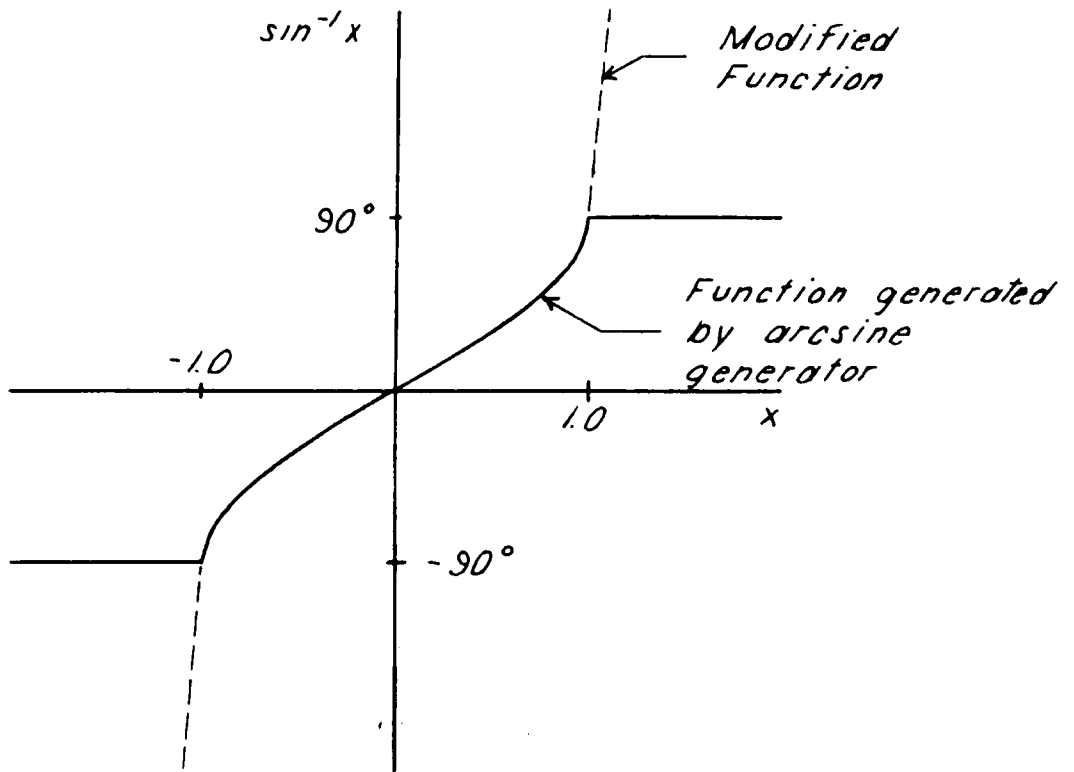


Figure 6.1, Modified Arcsine Function

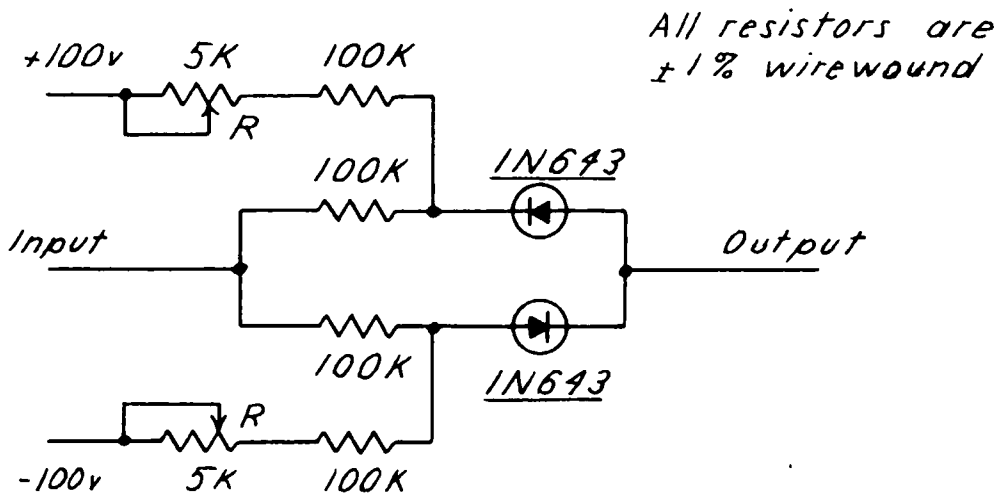


Figure 6.2, Auxiliary Function Generator

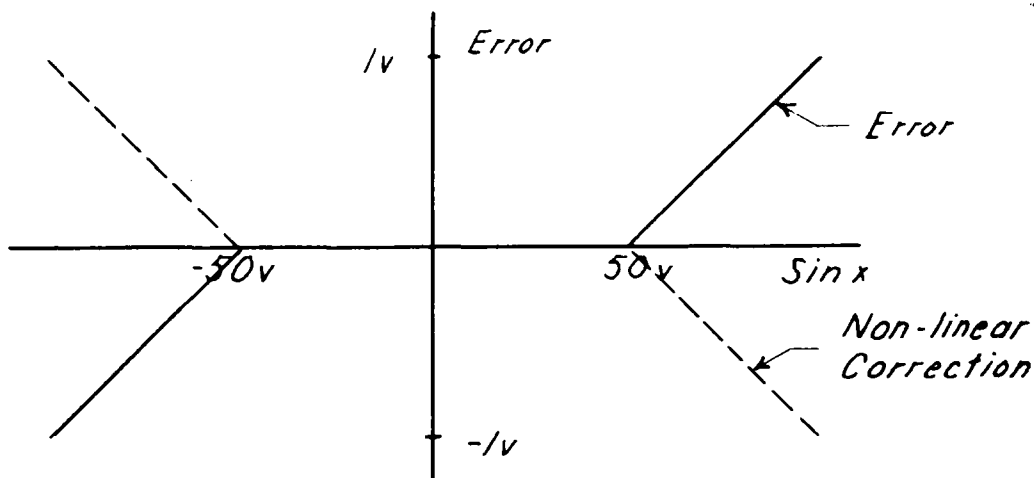


Figure 6.3, Error and Non-linear Correction Functions

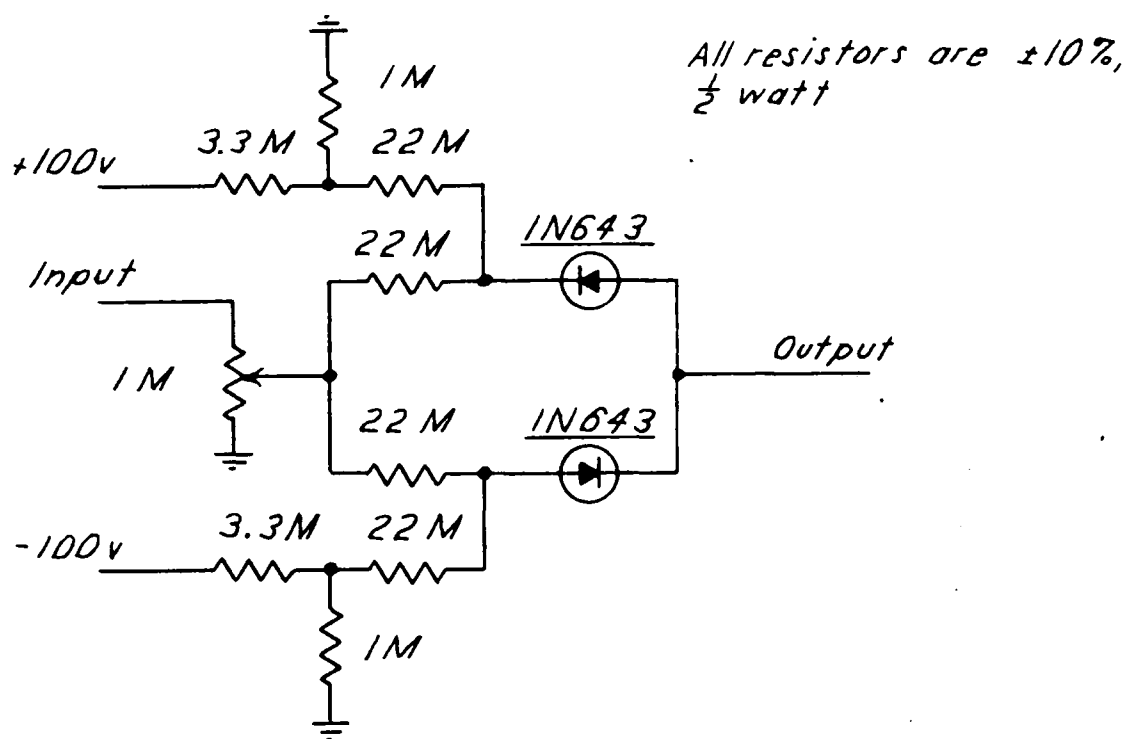


Figure 6.4, Non-linear Correction Generator

in figure 6.4. Because the slope is small, the break points are not critical, and ten per cent resistors are acceptable. The gain control permits adjustment for optimum correction. The high back resistance of silicon diodes is required because of the relatively high impedance level of the circuit. It is interesting to note that the addition of this simple circuit doubles the accuracy of the sine-cosine generator.

Chapter 7

Performance of the Sine-cosine Generator

The complete block diagram of the sine-cosine generator is shown in figure 7.1. The 220 pf capacitor across the feedback amplifier is necessary for stability. The 220 pf compensating capacitor shunting the input resistor prevents the stabilizing capacitor from causing excessive phase shift. This extra capacitor degrades performance in the arcsine mode because it is in parallel with the stabilizing capacitor. It will also be recalled that an extra 300 pf were required to stabilize the filter amplifier in the sine mode. These remain in the circuit in the arcsine mode, which is only rarely used as such.

7.1 Performance of the Arcsine Generator

As an arcsine generator, the unit has a static accuracy of 0.3 volts.¹² The hum and ripple at zero volts are 0.2 volts and 40 millivolts peak-to-peak respectively. At 50 degrees, these same quantities are 0.5 volts and 60 millivolts peak-to-peak. The hum continues to rise so that the circuit is not really practical above 80 degrees. In addition to hum, drift is a problem in this circuit. At zero volts, the short time drift is less than 0.1 volt after the circuit has warmed up. At 50 degrees, the drift is quite serious, as figure 7.2 shows. This drift appears to be the result of a storage effect in the Schmitt trigger. The change in duty cycle causes a temporary change in the switching point. When the input is returned to zero, the error decays in the same manner in which it builds up. The frequency response of this circuit is shown in figure 7.3

¹²Appendix, p. 41

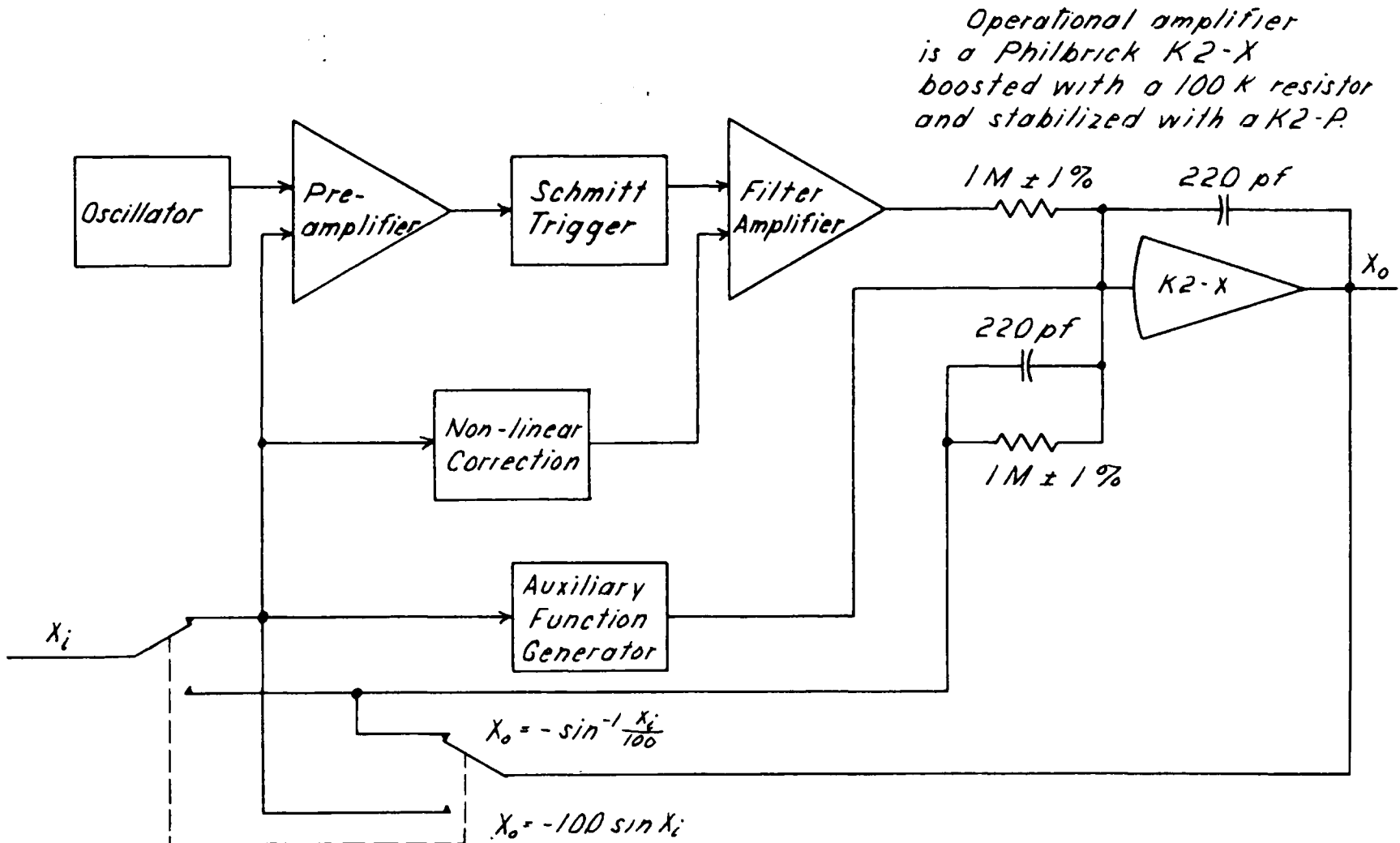


Figure 7.1, Complete Block Diagram of Sine-arc sine Generator

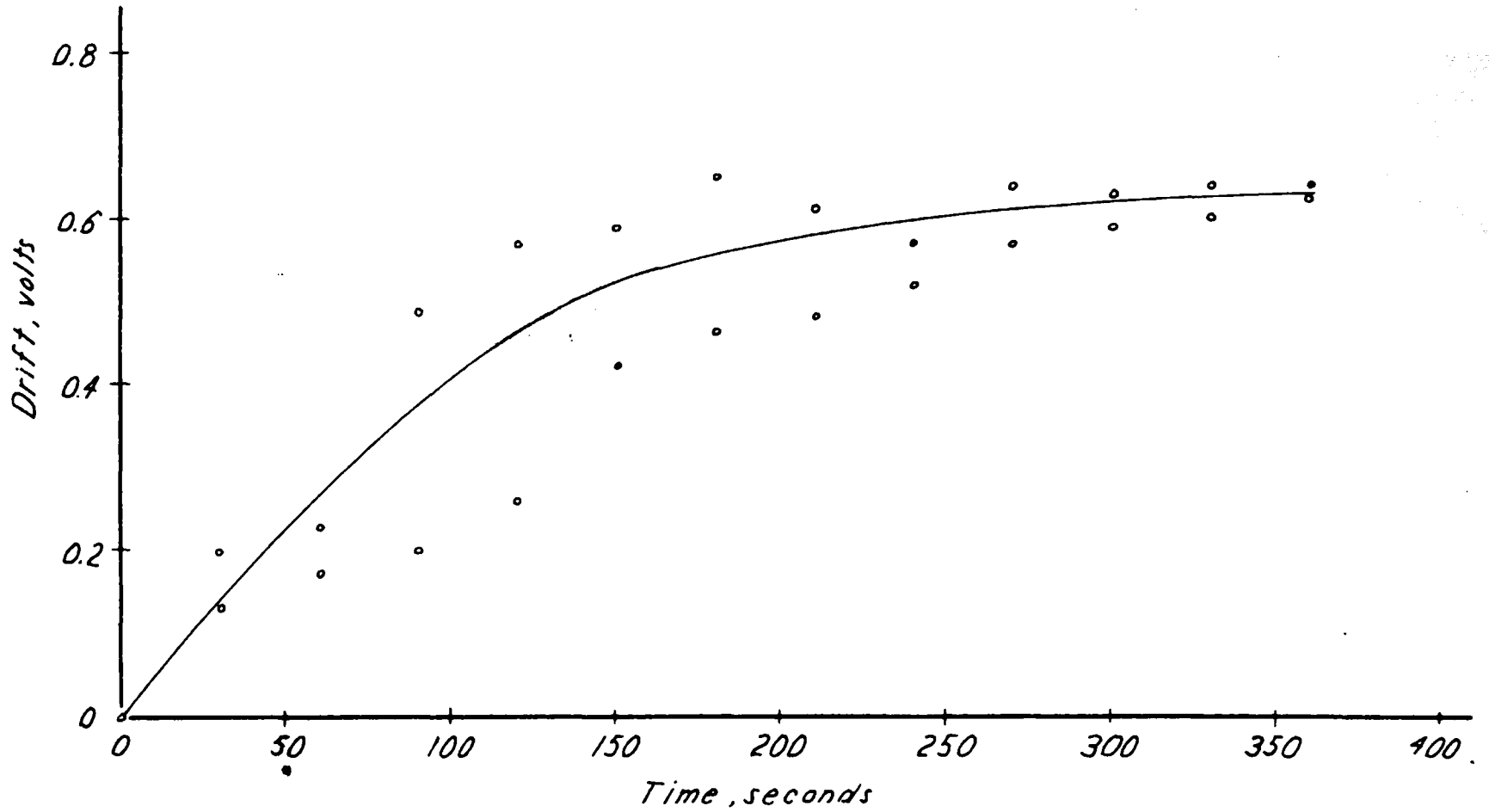


Figure 7.2, Drift of Arcsine Generator at 50 Degrees

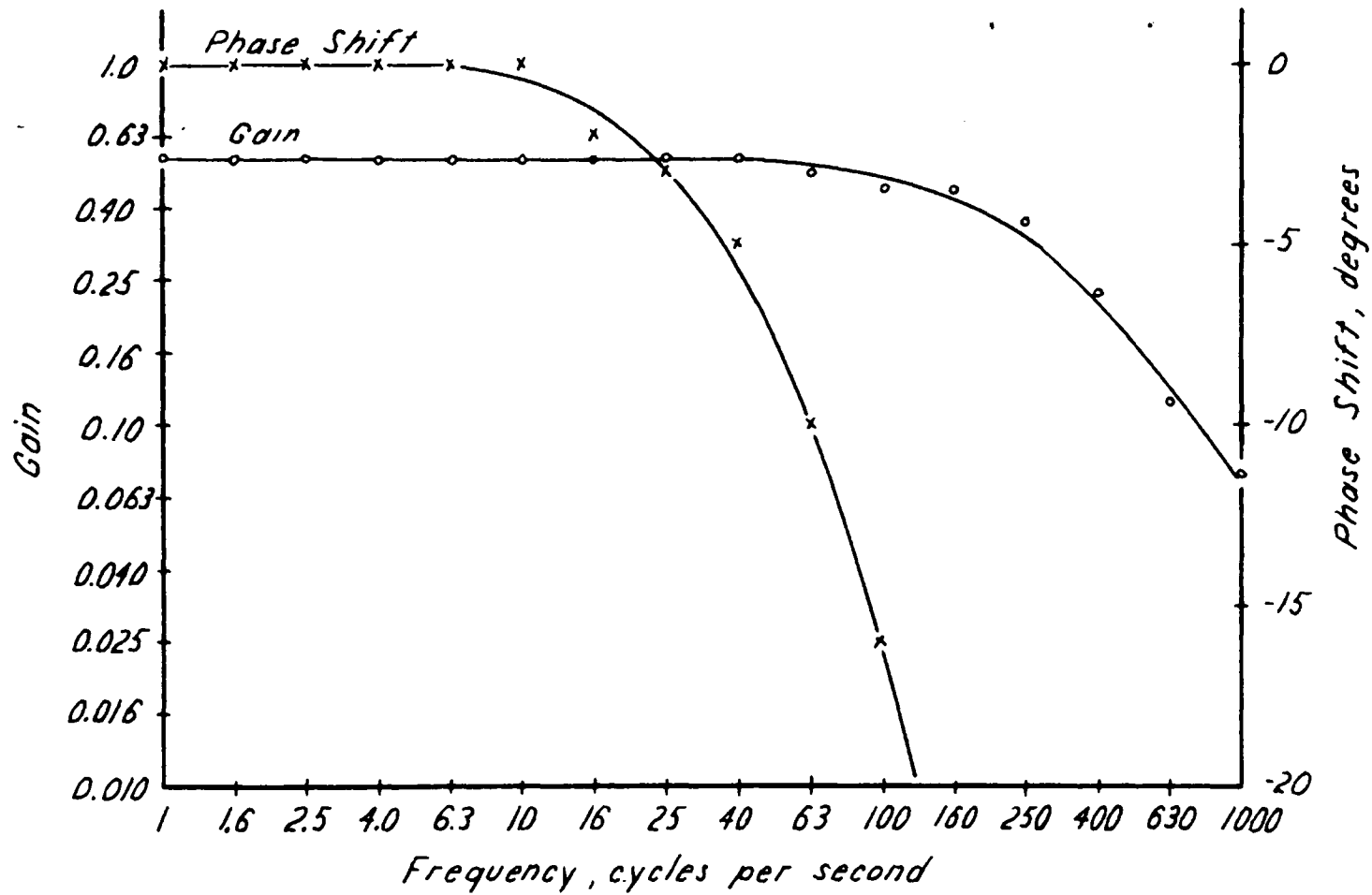


Figure 7.3, Frequency Response of Arcsine Generator

7.2 Performance of the Sine Generator

As a sine generator the static accuracy is also 0.3 volts.¹³ The hum and ripple are 0.4 volts and 90 millivolts peak-to-peak respectively at zero. At 50 degrees, these same quantities are 0.2 volts and 0.15 volts peak-to-peak. Hum is not the serious problem here that it is in the arcsine mode. Drift, however, is still a problem. It behaves in the same way as in the arcsine mode and is shown in figure 7.4. The frequency response is shown in figure 7.5.

¹³Appendix, p. 42

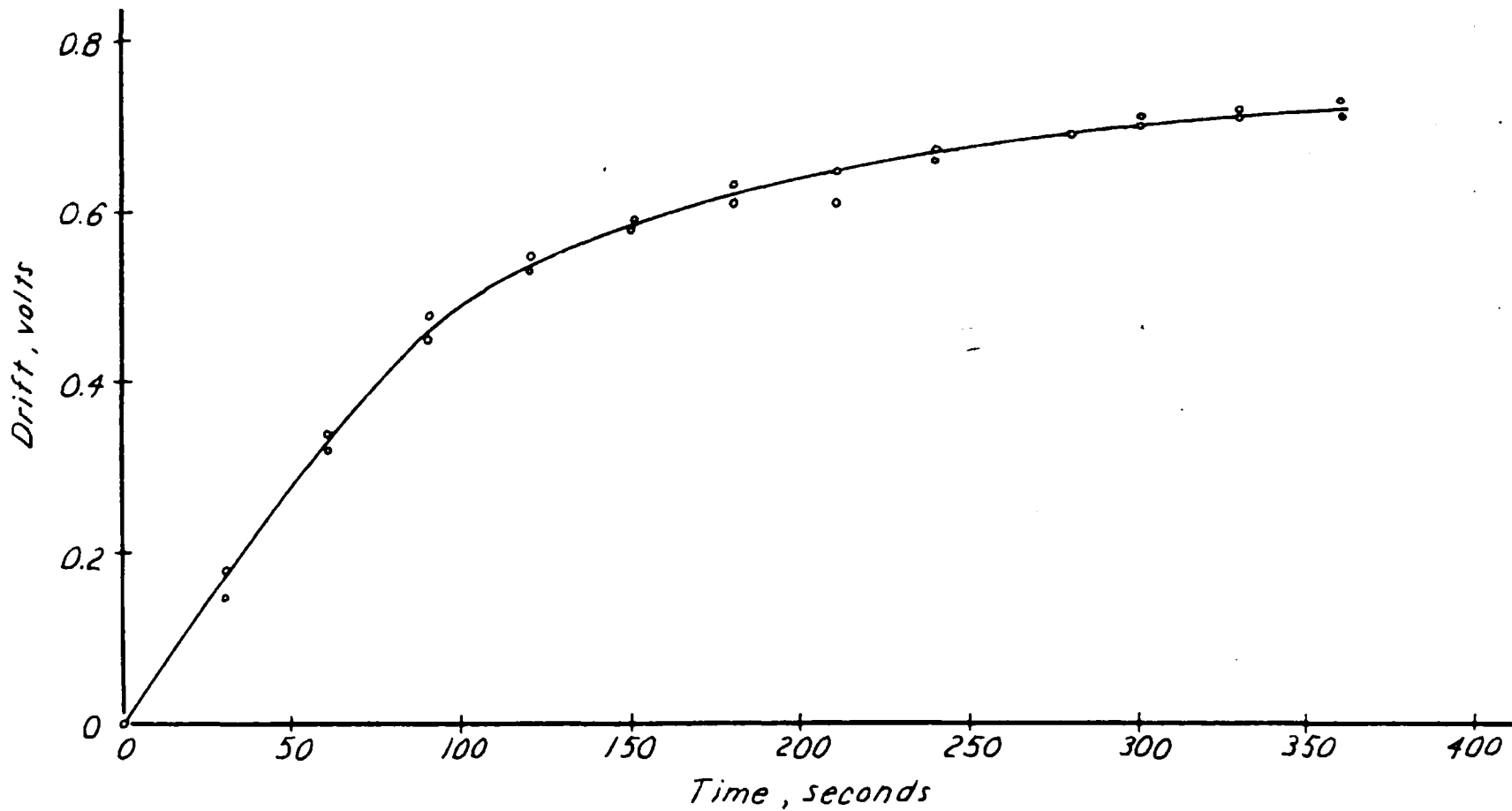


Figure 7.4, Drift of Sine Generator at 50 Degrees

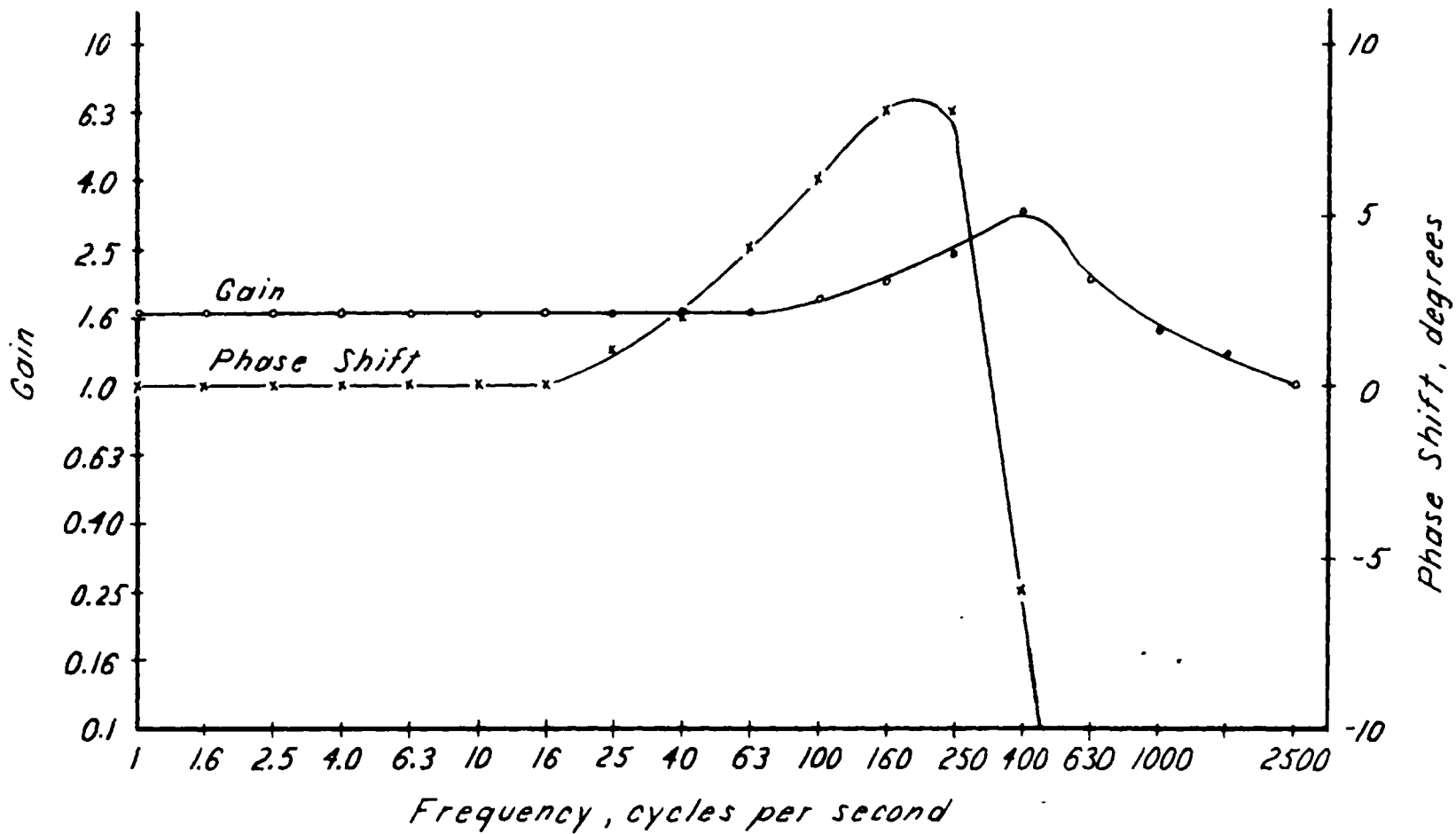


Figure 7.5, Frequency Response of Sine Generator

Chapter 8 Applications

There are several methods by which the usefulness of the sine-cosine generator may be increased.

8.1 Generating Related Functions

The inverse of any function may be generated on an analog computer as shown in figure 8.1. If a circuit is available to generate either sine or cosine the other function is easily obtained by adding the proper constant (phase angle) to the input. Through the use of trigonometric identities, other circular functions may also be generated. These properties greatly increase the utility of the sine-cosine generator.

8.2 Extending the Range of the Sine-cosine Generator

The range of the sine generator circuit is easily extended to 540 degrees. The circuit doing this and its transfer characteristic are shown in figure 8.2. The second amplifier shown here is actually the input amplifier of the sine generator, so that only one extra amplifier is required. This circuit is made possible by the development of the precision dead-space circuit, which has a static accuracy of 0.05 volt and excellent frequency response.¹⁴

8.3 Examples of the Use of the Sine-cosine Generator

The computer set ups for several applications of this circuit are shown in figure 8.3. There are, of course, many more, but these illustrate the utility of the device.

¹⁴G. A. Korn, "A Precision Dead-space Limiter for Triangle-integration Multipliers," ACL Memo 14, Electrical Engineering Department, The University of Arizona, 1960

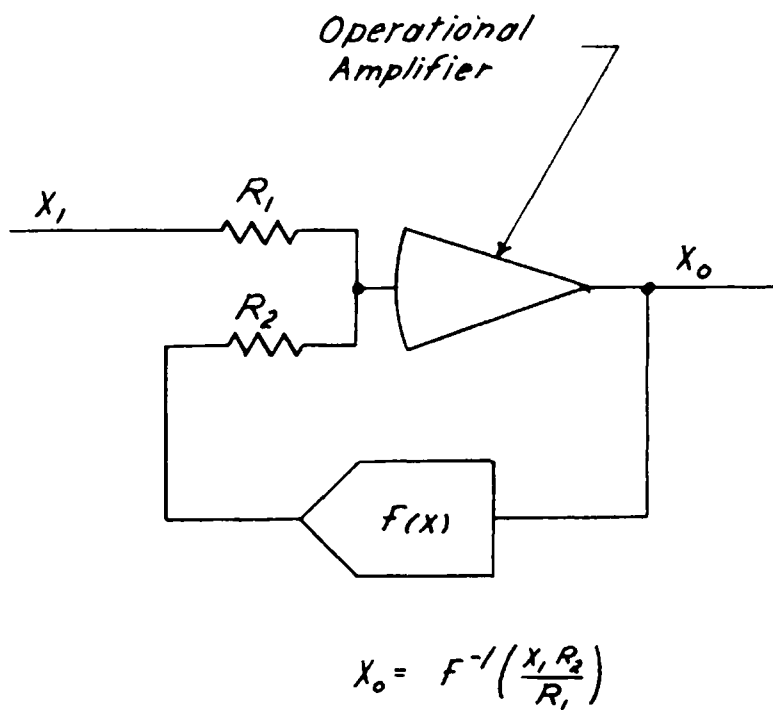
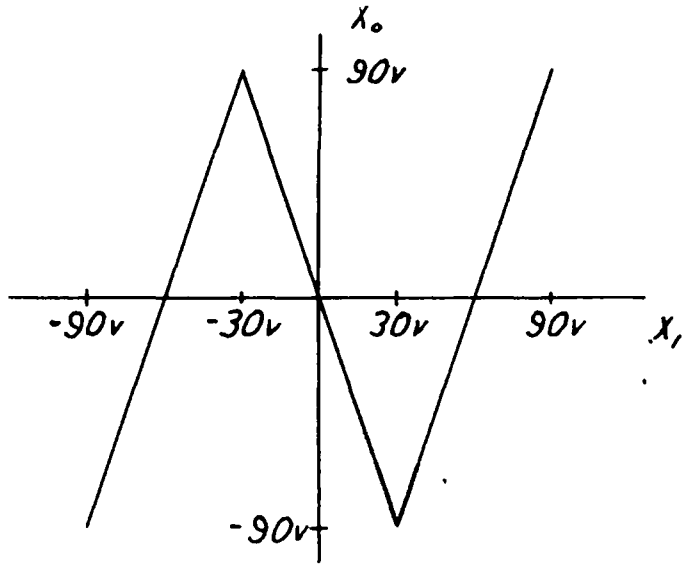
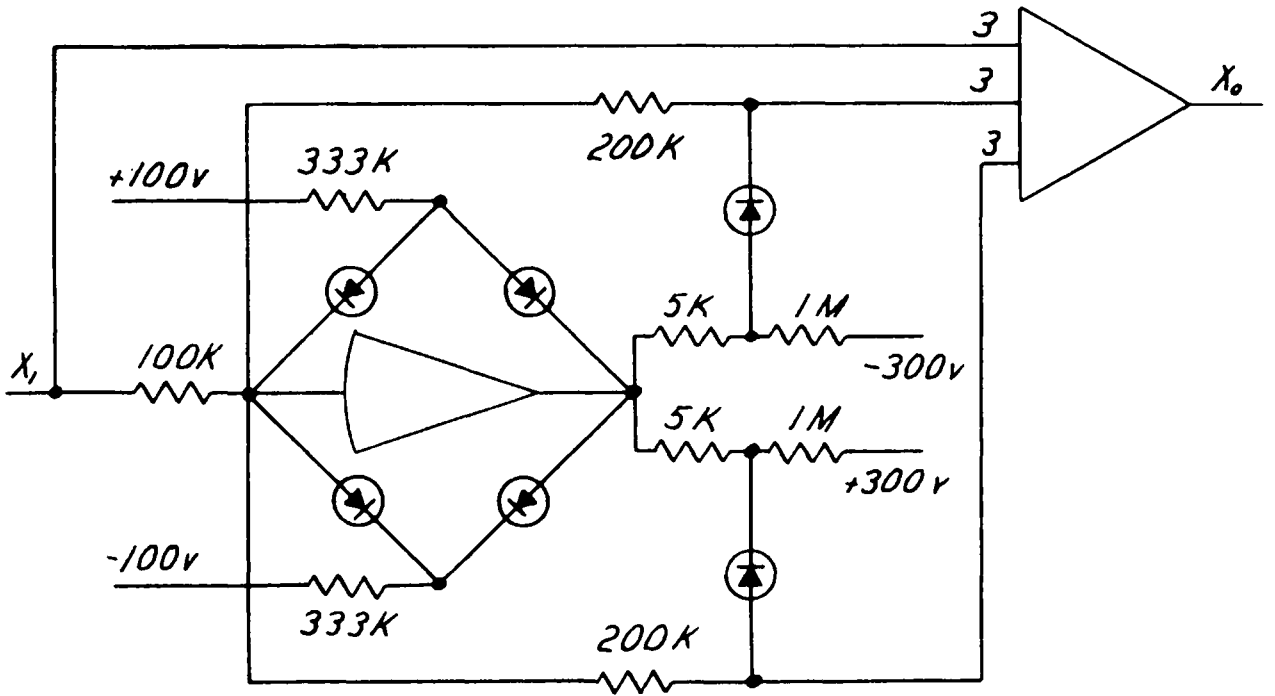


Figure 8.1, Generation of Inverse Functions

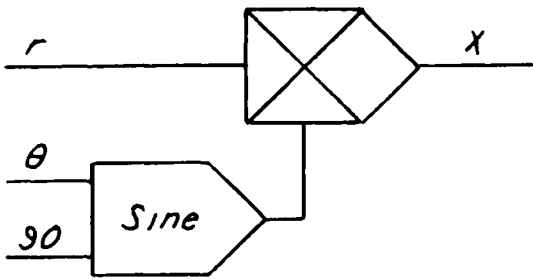


Transfer Characteristic



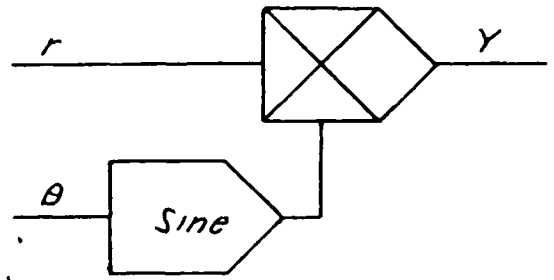
Circuit

Figure 8.2, Range Extension Circuit

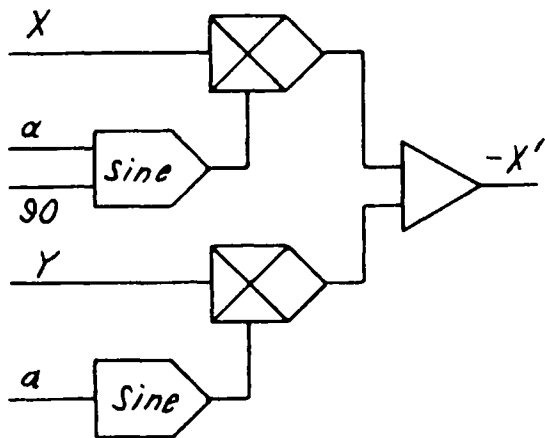


$$X = r \cos \theta$$

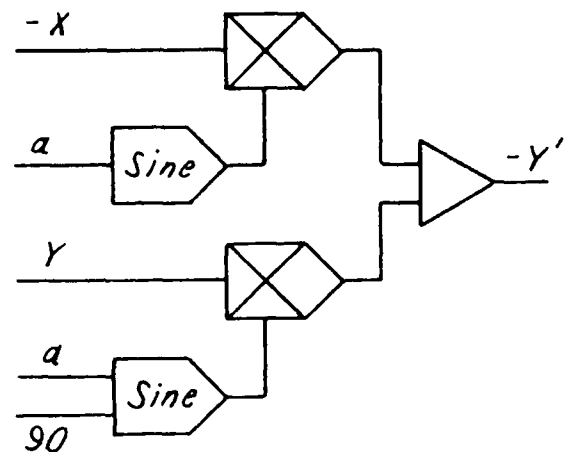
Convert From Polar to Rectangular Coordinates



$$Y = r \sin \theta$$



$$X' = X \cos a + Y \sin a$$



$$Y' = -X \sin a + Y \cos a$$

Convert from Rectangular to Rectangular Coordinates

Figure 8.3, Applications of Sine Generator

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Table I Static Test of Arcsine Generator

X_1 , volts	X_0 , volts	$-\sin^{-1} \frac{X_1}{100}$	Error, volts
-100.0	89.7	90.0	0.3
-90.0	64.3	64.3	0.0
-80.0	53.3	53.1	0.2
-70.0	44.6	44.4	0.2
-60.0	37.0	36.9	0.1
-50.0	30.1	30.0	0.1
-40.0	23.7	23.6	0.1
-30.0	17.5	17.5	0.0
-20.0	11.6	11.5	0.1
-10.0	5.8	5.7	0.1
0.0	0.0	0.0	0.0
10.0	-5.8	-5.7	0.1
20.0	-11.6	-11.5	0.1
30.0	-17.6	-17.5	0.1
40.0	-23.7	-23.6	0.1
50.0	-30.2	-30.0	0.2
60.0	-37.0	-36.9	0.1
70.0	-44.7	-44.4	0.3
80.0	-53.2	-53.1	0.1
90.0	-64.2	-64.3	0.1
100.0	-90.1	-90.0	0.1

Table II Static Test of Sine Generator

X_1 , volts	X_0 , volts	$-100\sin X_1$	Error, volts
-90.0	100.0	100.0	0.0
-80.0	98.6	98.4	0.2
-70.0	93.8	93.6	0.2
-60.0	86.6	86.6	0.0
-50.0	76.6	76.6	0.0
-40.0	64.2	64.3	0.1
-30.0	50.0	50.0	0.0
-20.0	34.2	34.2	0.0
-10.0	17.4	17.4	0.0
0.0	0.0	0.0	0.0
10.0	-17.4	-17.4	0.0
20.0	-34.2	-34.2	0.0
30.0	-50.0	-50.0	0.0
40.0	-64.3	-64.3	0.0
50.0	-76.6	-76.6	0.0
60.0	-86.6	-86.6	0.0
70.0	-93.9	-93.6	0.3
80.0	-98.6	-98.4	0.2
90.0	-100.0	-100.0	0.0