PROMPT NEUTRON LIFETIME
FROM
TIME DEPENDENT THERMAL DIFFUSION EQUATION

by
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ABSTRACT

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Useful formulas for calculating the prompt neutron lifetime for reflected reactors are derived from equations that are applicable to the reactor systems where the diffusion coefficients in the core and the reflector are approximately equal. In addition an equation is developed that applies to a reactor system with equal diffusion coefficients and equal thermal neutron diffusion lengths. Finally, an analysis is given that states the relation between the albedo of the reflector and the contribution of the reflector to the neutron lifetime.

The effect of the neutron reflector on the reactor at steady state is associated with the effective neutron lifetime. The discussions are given from the point of view of steady state reactor kinetics and the augmented neutron lifetime.

Calculated values and the experimental measurements of the prompt neutron lifetime in TRIGA, TREAT, and ZEEP reactors are given for comparison.

An experiment was performed using a foil activation technique that gives the contribution of the reflected neutrons to the total prompt neutron lifetime. The experimentally determined contribution to the prompt neutron lifetime was 49.7 micro-seconds that is in good agreement with the calculated results.
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CHAPTER I
INTRODUCTION

The major purpose of this paper is the study of the effect of the neutron reflector on the prompt neutron lifetime in a reactor. This is essentially the kinetics effect of the reflected neutrons under steady state conditions.

The effect of a reflector on the geometric buckling of a reactor, and its effect on increasing the average power output of a reactor is directly associated with the neutron savings. In the effect of a reflector on reactor kinetics, we are concerned not only with the neutron savings, but also with the time the neutrons spend in the reflector before returning to the core.

In a thermal reactor, a reflector is generally chosen to help moderate the neutrons. Consequently, the neutrons that leak out of the core as thermal, epithermal, and fast neutrons are reflected back into the core as thermalized neutrons.

In view of the time delay in the process of diffusing in and out of the reflector, the kinetics effect of a neutron reflector can be thought of as an effective increase in the neutron lifetime in the core under steady state conditions, or as an additional delayed neutron precursor under transient operations.
The scope of this investigation will be limited to the studies concerning the effective increase in lifetime of the neutrons. In order to formulate the kinetics effect of a neutron reflector in an analytical form, it is essential to assume a simplified model of the neutron behavior in the system. As a result of the approximations the answers are also approximate. More precise results can be obtained only from the solution of the transport equation or from multi-group equations.

The life cycle of the neutrons and the effect of the neutron reflector is qualitatively discussed in Fig.(1). The neutrons diffusing through the reflector and returning to the core are designated by \( R \) in the diagram.
Figure (1)
Neutron Cycle in Critical Thermal Reactor
(With the simplified reflector effects)
CHAPTER II
FORMULATION OF THE EFFECT OF THE REFLECTOR ON LIFETIME OF NEUTRONS

A. The Rumsey Method [1]

The two energy group approximation to describe the behavior of neutrons is developed, assuming that the neutrons are either fast or thermal. With,

\[ p(\vec{r}, t) \, d\vec{r} = \text{the probability that a fast neutron which is born at } \vec{r} \text{ will die at } \vec{r} \text{ in } d\vec{r}. \]

\[ t(\vec{r}, t) = \text{the average time between birth at } \vec{r} \text{ and death at } \vec{r} \text{ for fast neutrons.} \]

\[ p(\vec{r}, t) \, d\vec{r} = \text{the probability that a thermal neutron which is born at } \vec{r} \text{ will die at } \vec{r} \text{ in } d\vec{r}. \]

\[ t(\vec{r}, t) = \text{the average time between birth at } \vec{r} \text{ and death at } \vec{r} \text{ for thermal neutrons.} \]

**Note:** The fast neutrons die by being thermalized or by being captured (resonance capture). Thermal neutrons die by being captured.

\[ F(\vec{r}, t) \, d\vec{r} dt = \text{the total number of fast neutrons produced in } d\vec{r} \text{ at } \vec{r}, \text{ during the time } dt \text{ at } t. \]

\[ T(\vec{r}, t) \, d\vec{r} dt = \text{the total number of thermal neutrons produced in } d\vec{r} \text{ at } \vec{r}, \text{ during the time } dt \text{ at } t. \]

\[ p(\vec{r}) = \text{the resonance escape probability at } \vec{r}. \]

**Note:** \( p(\vec{r}) = p \) for points \( \vec{r} \) in the core; \( p(\vec{r}) = 1 \) for points \( \vec{r} \) in the reflector.

\[ k = \text{multiplication factor for fission reaction; } k = 0 \text{ in the reflector.} \]
\[ pF(r, t) = k \text{ times the rate of dying of thermal neutrons at } r \text{ in the core.} \]

\[ pF(r, t) = k \int_{\Omega_r} d\bar{r} T \left[ \int_{\Omega_r} d\bar{r} t - t_0 \right] p(\bar{r}, r) \]
\[ V_\Omega = \text{core plus reflector volume.} \]  

\[ T(\bar{r}, t) = p \int_{\Omega_r} d\bar{r} F \left[ \int_{\Omega_r} d\bar{r} t - t_0 \right] p(\bar{r}, r) \]
\[ V_\Omega = \text{core volume.} \]

\[ pF(\bar{r}, t) = k \int_{\Omega_r} d\bar{r} p \int_{\Omega_r} d\bar{r} F \left[ \int_{\Omega_r} d\bar{r} t - t_0 \right] - t_0 \right] p(\bar{r}, r) \]
\[ p(\bar{r}, r) \cdot p(\bar{r}, r) \]  

From the nature of the neutron fission process, we seek a solution of the form:

\[ F(\bar{r}, t) = \sum_{i} F(\bar{r}) \exp(\omega_t) \text{ for all positive } i. \]  

Except during transients, the most positive \( \omega \), predominates; denoting this value of \( \omega \), by \( \omega \), we have

\[ F(\bar{r}, t) = F(\bar{r}) \exp(\omega t). \]  

Substituting this in the integral equation (3) gives

\[ pF(\bar{r}) = k \int_{\Omega_r} d\bar{r} p \int_{\Omega_r} d\bar{r} F(\bar{r}) \exp \left[ -\omega t_0(\bar{r}, r) - \omega t(\bar{r}, r) \right] \cdot \]
\[ \cdot \int_{\Omega_r} d\bar{r} p(\bar{r}, r) \cdot p(\bar{r}, r) \]  

Suppose that we put \( F(\bar{r}) \) fast neutrons into an otherwise empty system. Then we can develop the concept of effective multiplication factor, \( k_e \), as follows:

\[ pk_e = k \text{ times the fraction of the total number of fast neutrons put into the system which are captured as thermal neutrons in the core.} \]
If \( T(r_f) \) is the distribution of thermal neutrons arising from the fast neutron source distribution \( F(r) \), then,

\[
T(r_f) = \int_{V_c} d\vec{r}_2 F(\vec{r}_2) p(\vec{r}_2, \vec{r}_f) .
\]

The number of thermal neutrons captured in the core is then,

\[
\int_{V_c} d\vec{r}_2 \int_{V_T} d\vec{r}_1 T(\vec{r}_f) p(\vec{r}_1, \vec{r}_f) = \int_{V_c} d\vec{r}_2 \int_{V_T} d\vec{r}_1 p(\vec{r}_1) \int_{V_T} d\vec{r}_2 F(\vec{r}_2) p(\vec{r}_2, \vec{r}_f) p(\vec{r}_f, \vec{r}_1) .
\]

Therefore,

\[
k_e = \frac{k f}{\int_{V_c} d\vec{r}_2 F(\vec{r}_2) p(\vec{r}_2)} .
\]

Integrating (6) over the core volume gives:

\[
l = \int_{V_c} d\vec{r}_2 \int_{V_T} d\vec{r}_1 p(\vec{r}_2) \exp \left[ -t(r_f, \vec{r}_f) - t(r_f, \vec{r}_1) \right] p(\vec{r}_2, \vec{r}_1) \frac{p(\vec{r}_1, \vec{r}_f)}{\int_{V_c} d\vec{r}_2 F(\vec{r}_2)} .
\]

Comparing (7) and (8) gives:

\[
\frac{1}{k_e} = \exp \left[ -t(r_f, \vec{r}_f) - t(r_f, \vec{r}_1) \right] .
\]

Since \( k_e \) is of the order of unity, for our case of interest, we can approximate

\[
\exp \left[ -t(r_f, \vec{r}_f) - t(r_f, \vec{r}_1) \right] \approx 1 - w (t + t_f).
\]
Substituting (9) in (8) gives:

\[ l = \frac{k^2}{c} - Tw \]  
where,

\[ T = \left\{ \frac{\int d^2\vec{r} \int d^2\vec{r}' p^2 F(\vec{r}) p(\vec{r},\vec{r}') p(\vec{r}',\vec{r}) t(\vec{r},\vec{r}') t(\vec{r}',\vec{r})}{\frac{P}{k} \int d^2\vec{r} F(\vec{r})} \right\}. \]

Equation (10) is similar to the relation obtained for \( w \) by one-group theory, where \( T \) is the neutron lifetime. From (11), we can calculate the lifetime of neutrons.
Evaluation of prompt neutron lifetime from equation (11).

Let,

\[ p(\vec{r}, \vec{r}') = \text{the rate of dying of neutrons at } \vec{r} \text{ due to a unit source at } \vec{r}' \]

\[ y(\vec{r}, \vec{r}) = p(\vec{r}, \vec{r}) \ell_o(\vec{r}) \]

= the density at \( \vec{r} \) due to a unit source at \( \vec{r}' \).

\[ \ell_o(\vec{r}) = \text{mean lifetime in the medium at } \vec{r}. \]

\[ t(\vec{r}, \vec{r}) = \text{the average time between birth at } \vec{r} \text{ and death at } \vec{r}. \]

\[ p(\vec{r}, \vec{r}) \ t(\vec{r}, \vec{r}) = N \]

= the total number of neutrons present in the system at any instant, which are destined to die at \( \vec{r} \).

\[ N = \int_{\vec{r}} y(\vec{r}, \vec{r}) \ p(\vec{r}, \vec{r}) \ d\vec{r} \]

From the above relations we have,

\[ \frac{y(\vec{r}, \vec{r})}{\ell_o(\vec{r})} \ t(\vec{r}, \vec{r}) = p(\vec{r}, \vec{r}) \ t(\vec{r}, \vec{r}) = \int_{\vec{r}} d\vec{r} \ y(\vec{r}, \vec{r}) \ p(\vec{r}, \vec{r}) = \int_{\vec{r}} \int_{\vec{r}} d\vec{r} \ y(\vec{r}, \vec{r}) / \ell_o(\vec{r}). \]

Also from the reciprocity theorem:

\[ \ell_o(\vec{r}) \ p(\vec{r}, \vec{r}) = y(\vec{r}, \vec{r}) = y(\vec{r}, \vec{r}) = \ell_o(\vec{r}) \ p(\vec{r}, \vec{r}) \]

\[ t(\vec{r}, \vec{r}) = t(\vec{r}, \vec{r}) \].
For a source distribution $S(r)$, we have

$$
\int d^3 r \left( S(\vec{r}) p(\vec{r}, \vec{r}) t(\vec{r}, \vec{r}) \right) = \int d^3 r S(\vec{r}) y(\vec{r}, \vec{r}) / \rho(\vec{r}) \left[ \rho(\vec{r}) y(\vec{r}, \vec{r}) / \rho(\vec{r}) \right] = ^{\text{def}} \int d^3 r \rho(\vec{r}) y(\vec{r}, \vec{r}) / \rho(\vec{r}) ,
$$

where,

$$\rho(\vec{r}) = \int d^3 r S(\vec{r}) y(\vec{r}, \vec{r})$$

and

$$\rho = \rho - \rho S .$$

According to equation (11),

$$T = \frac{\int d^3 r \left( S(\vec{r}) p(\vec{r}, \vec{r}) t(\vec{r}, \vec{r}) \right)}{\int d^3 r S(\vec{r}) p(\vec{r}, \vec{r})} = \frac{\int d^3 r \rho(\vec{r}) y(\vec{r}, \vec{r}) / \rho(\vec{r})}{\int d^3 r S(\vec{r}) y(\vec{r}, \vec{r}) / \rho(\vec{r})} .$$

From which,

$$T = \frac{\int d^3 r q_t}{\int d^3 r \rho_t} ,
$$

where,

$$q_t = \rho_{t} T_{t} \nabla^2 q_t + \rho_{t} \left( \rho_{t} + p q_t / \rho_{t} \right)$$

$p$ = resonance escape probability

$q_t$ = steady state thermal neutron density

$q_f$ = steady state fast neutron density.
We can also express the neutron lifetime in terms of the one-group neutron density as follows.

We shall assume that the fast neutrons slow down to thermal, with an average slowing down time of $\ell$, giving rise to a thermal neutron source distribution $T(\vec{r})$.

Equation (11) can be written in terms of the thermal neutron source term, as follows.

\[
T = \frac{\int d\vec{r} \int d\vec{r} \, T(\vec{r}) \, p(\vec{r}, \vec{r}) \left[ t(\vec{r}, \vec{r}) + \ell \right]}{\int d\vec{r} \int d\vec{r} \, T(\vec{r}) \, p(\vec{r}, \vec{r})}
\]

since $\ell \ll t(\vec{r}, \vec{r})$; using (12) and assuming $\ell_0$ is constant,

\[
T = \frac{\int d\vec{r} \int d\vec{r} \, \rho(\vec{r}) \, y(\vec{r}, \vec{r}) / \ell_0}{\int d\vec{r} \int d\vec{r} \, T(\vec{r}) \, y(\vec{r}, \vec{r}) / \ell_0}
\]

\[
T = \frac{\int d\vec{r} \, q(\vec{r}) / \ell_0}{\int d\vec{r} \, \rho(\vec{r}) / \ell_0} = \frac{\int d\vec{r} \, q(\vec{r})}{\int d\vec{r} \, \rho(\vec{r})}
\]

(14)

where,

\[
\ell_0 \frac{\partial}{\partial t} \rho = \rho - \ell_0 T(\vec{r})
\]

\[
\ell_0 \frac{\partial}{\partial t} q = q - \ell_0 \rho
\]

$T = T_0 + \ell_0$, including slowing down time.

A mono–energetic equation is adopted with a minor correction for the reflector flux. The one–group model underestimates the reflector flux, but can be corrected by multiplying the reflector flux by a factor $Q$. If we assume the equal diffusion coefficients for the core and the reflector [2],

$$Q = \frac{k_\infty - 1}{B \cdot L_c}$$

where,

$k_\infty$ = infinite multiplication factor $\geq 1.2$

$B$ = buckling

$L_c$ = thermal diffusion length in the core.

The steady state neutron densities for the core and the reflector for this case are given by:

$$\rho = A \cos m z J_0 (\lambda_c r)$$

for the core

$$\rho^c = C \cos m z K_0 (\lambda_R r)$$

for the reflector

$$m = \frac{n}{H}$$

$H$ = the height of the cylinder

$$\lambda_c^2 = B^2 - m^2$$

$$\lambda_R^2 = \frac{L_c^2}{L_R^2} - m^2$$

Then,

$$\ell_c \cdot D_c \nabla^2 q = q - \ell_c \rho_c$$

for the core

$$\ell_R \cdot D_R \nabla^2 q = q - \ell_R \rho_R$$

for the reflector.
Using the variation of parameters method, and assuming the same z-dependence of the neutron densities, we have [for derivations see Appendix A]

\[ T_i = \left[ \frac{1}{1 + \frac{Z_e}{L_e} B^2} + \frac{\lambda_c R_o Q}{2 \lambda_R \beta} \frac{1 - (K_o/K_i)^2}{\lambda_R I_o I_i + \beta \frac{K_o}{K_i}} \right] \tag{15} \]

where,

- \( T_i = T_1 + l_x \)
- \( l_c^o = \) mean neutron lifetime in the core
- \( L_e = \) thermal diffusion length in the core
- \( B = \) buckling
- \( \lambda^2_e = B^2 - m^2 \)
- \( m = \frac{\pi}{H} \)
- \( R_o = \) radius of the core
- \( \lambda_R^2 = \frac{1}{L_R^2} + m^2 \)
- \( L_R = \) thermal diffusion length in the reflector
- \( \beta = \frac{1}{L_c^2} + m^2 \)
- \( K_o = K_o(\lambda R_o) \) the Bessel functions
- \( K_i = K_i(\lambda R_o) \)
- \( I_o = I_o(\beta R_o) \); \( I_i = I_i(\beta R_o) \).

The first term in (15) represents the neutron lifetime for bare finite reactor; the second term represents the contribution from the reflector. Rumsey[2] found, in analysis of ZEEP experiments, that this contribution amounts to an increase in the prompt neutron lifetime of approximately 20%.
A.2. Spherical reactor with finite reflector.

As in the previous case, we shall assume the one-group theory with a corrected neutron density in the reflector. The diffusion coefficients in the core and the reflector are assumed to be the same.

The neutron densities for this case are given by,

$$\rho_c = A \frac{\sin Br}{r} \quad \text{for the core}$$

$$\rho_R = C \frac{\sinh \kappa (\tilde{R}_i - r)}{r} \quad \text{for the reflector}$$

$$\mathcal{B} = \text{buckling}$$

$$\kappa^2 = \frac{1}{L_n^2}$$

$$\tilde{R}_i = \text{extrapolated outside boundary.}$$

Then,

$$\ell_c^c D_c^c \nabla q_c^c = q_c^c - \ell_c^c \rho_c$$ \quad \text{for the core}

$$\ell_R^k D_R^k \nabla q_R^k = q_R^k - \ell_R^k \rho_R$$ \quad \text{for the reflector.}

Using the variation of parameters method, we have the following formula for the lifetime $T$:

$$T = T_i + \ell_s$$

$$T_i = \ell_c^c \left[ \frac{1}{1 + L_c^2 B^2} \frac{\phi B^2}{2 \pi \alpha^2 L_R^2} \frac{H^I}{G^I} \frac{K'G - KG}{HG' - H'G} \right]$$ (16)
where,

- \( \tau_c \) = mean neutron lifetime in the core
- \( L \) = thermal diffusion length in the core
- \( B \) = buckling
- \( Q = \frac{k_e - 1}{B L_c} \)
- \( \lambda = \frac{1}{L_c} \)
- \( \delta = \frac{1}{L_c} \)
- \( L_r \) = thermal diffusion length in the reflector

The first term in (16) represents the finite bare core neutron lifetime; the second term represents the contribution of the reflector on the neutron lifetime in the core. [For derivation see Appendix B.]
B. **Reflector Effects on Kinetics with One-group Delayed Neutrons: The Galanin Method.** [3]

B.1. **Kinetics with one-group delayed neutrons.**

Let the number of delayed neutrons with period equal to $\frac{1}{\lambda}$ be such that the fraction of them in the multiplication factor is $\nu$, that is,

$$k = k_0 (1 + \nu)$$

$k_0$ = multiplication factor without delayed neutrons.

The steady state one-group equation can be written as,

$$\frac{\partial^2 \rho_c(\vec{r}, \rho)}{\partial z^2} + \frac{k_{\text{cm}} - 1}{M^2} \rho_c(\vec{r}) = 0$$

$M^2 = L^2 + \tau^2$ migration area

$\tau$ = Fermi age

$L_\tau$ = thermal diffusion length in the core.

We now apply a perturbation to the reactor, such that the multiplication factor changes by $\delta k$ (either positive, or negative). The new multiplication factor is then,

$$k = k_0 + \delta k = k_0 (1 + \delta k).$$

Now,

$$D_{\rho_c} \frac{\partial^2 \rho_c(\vec{r}, t)}{\partial z^2} + \frac{k_{\text{cm}} - 1}{T_0} \rho_c(\vec{r}, t) + \frac{k_{\text{cm}} \lambda}{T_0} \int \rho_c(\vec{r}, t-u) e^{-\lambda u} du$$

$$= \frac{\partial \rho_c(\vec{r}, t)}{\partial t}$$
where the integral term represents the total number of neutrons produced per unit time at time \( t \) as a result of neutron emitting nuclei before time \( t \), with,

\[
\rho_c(\vec{r}, t) = \rho_c(\vec{r}) \exp( tw)
\]

\[
\nabla^2 \rho_c(\vec{r}) + \left[ \frac{k_o - 1}{M^2} + \frac{k_o \mu}{M^2} \cdot \frac{\lambda/\omega}{1 + \lambda/\omega} \right] \rho_c(\vec{r}) = 0
\]

\[
\frac{k_o (1 + \mu) - \delta k - 1}{M^2} = \left[ \frac{k_o - 1}{M^2} + \frac{k_o \mu}{M^2} \right] \frac{\lambda/\omega}{1 + \lambda/\omega} - \frac{\omega}{D_c}
\]

From which, we have

\[
\omega = \frac{\delta k}{T_o} - \frac{k_o \mu}{T_o} \frac{1}{1 + \lambda/\omega}.
\]  

(17)

For \( \delta k > 0 \)  
\[ w = \omega_p > 0 \]  inverse runaway time, or inverse period \( T \).

\[ w = w_t < 0 \]  transient effect.

For \( \delta k < 0 \)  
\[ w = w_t < 0 \]  transient effect.
B.2. **The effect of the reflector.**

The neutron density in the reflector can be given by,

\[
\vec{\nabla}^2 \rho_R - \frac{l}{L_R^2} \rho_R = \frac{l}{D_0^R} \frac{\partial \rho_R}{\partial t}.
\]

Let

\[
\rho_R(\vec{r}, t) = \rho_R(\vec{r}) \exp(\omega t).
\]

Then,

\[
\frac{l}{L_R^2} \left( \frac{1}{L_R^2} + \frac{\omega_p}{D_0^R} \right) \rho_R = 0. \tag{18}
\]

This is the same as the steady state distribution,

\[
\vec{\nabla}^2 \rho_{R, 0} - \frac{l}{L_{R, eff}^2} \rho_{R, 0} = 0,
\]

with,

\[
L = L + \delta L_R \tag{19}
\]

\[
\delta L_R = \text{small perturbation in thermal diffusion length}.
\]

From (18) and (19) we have,

\[
\frac{\delta L_R}{L_R} = -\frac{l^2}{2L_K^2 D_0^R}
\]

Here we assumed that \[ \left| \frac{\delta L_R}{L_R} \right| = \frac{T_p}{2T_p^2} \ll 1. \]
From the steady state one-group diffusion equation

\[ \nabla^2 \rho(\vec{r}) + \frac{k_{\text{eff}} - 1}{M^2} \rho(\vec{r}) = 0 \quad ; \quad M^2 = \frac{L^2}{\lambda} + \frac{\omega^2}{\lambda} \]

and with perturbed

\[ k = k_c + \delta k - \delta k' = k_0 (1 + \mu) \]

where, \( \delta k' \propto \delta L \), and \( \mu \) is the delayed neutron contribution; we have as in the bare case

\[ \frac{k(1 + \mu) - \delta k + \delta k' - 1}{M^2} = \frac{k_0 - 1}{M^2} + \frac{k_0 \mu}{1 + \lambda / \omega} - \frac{\omega}{D_0} \]  \hspace{1cm} (20)

The value of \( \delta k' \) can be obtained from the one-group equation giving the critical size of a reactor with a reflector. For the case of a spherical reactor, assuming the same diffusion coefficients in the core and the reflector we find that [See Appendix C for derivation]

\[ \delta k' = \frac{\lambda^2 L^2}{R/L} \left( 1 + \alpha^2 L^2 \right)^{-1} T_0^{\text{c}} \omega \]  \hspace{1cm} (21)

where,

- \( R_0 \) = core radius
- \( \alpha \) = buckling
- \( T_0^{\text{c}} \) = core mean neutron lifetime
- \( D_0^{\text{c}} \) = effective diffusion coefficient.
From (20) and (21) we can arrive at the inverse runaway time, \( w \), formula exactly the same as in (17), but with an effective thermal neutron lifetime, viz.

\[
T_e = T^0 \left[ 1 + \frac{\alpha^2 L^2}{R/L \left( 1 + \alpha^2 L^2 \right) + 1} \right].
\]  

(22)

Thus the effect of the reflected neutrons on the kinetics of the reactor amounts to an effective increase in the lifetime of the thermal neutrons [3].
CHAPTER III
FOIL ACTIVATION ANALYSIS OF NEUTRON CURRENT
AT THE CORE AND REFLECTOR BOUNDARY

A. Experimental Method

A.1. Calibration

\textsuperscript{79}Au\textsuperscript{197} foils were placed around the edge of a plastic cup, then activated at the top of the TRIGA reactor core. By rotating the plastic cup the foils were then exposed to the same integrated neutron flux. The activities of the foils were then monitored using a scintillation counter. This gives the foil activities normalized to a constant flux.

A.2. Activation

After several days decay time the same foils were then activated at the core-reflector boundary. To account for the respective fraction of the thermal and epithermal neutrons standard cadmium filter techniques were used. The set-up of the \textsuperscript{79}Au\textsuperscript{197} foils and Cd. is given in Fig. (2).

The activities were then counted in a scintillation counter. With the data from the calibration run, the normalized activities were calculated and expressed in terms of a standard foil having the highest activity.

Assuming that the neutrons enter the reflector at epithermal energies and that the fraction which return to the core are at thermal energy, the fractional neutron current in and out of the reflector were then calculated.
The albedo, $\bar{\psi}$, was then calculated from these fractions. The albedo obtained from the data is 0.820. The data given in Table I. are believed to be reliable, since large Cd. plates and Cd. rings were used to protect the gold foils from any neutron streaming or moderation in the plastic.

The albedo was also calculated from the one-group equations for the TRIGA reactor, and a value of 0.76 was obtained.
Figure (2)

Foil Activation Experiment Set-up
Table I

**Calibration of Au foils: Data**

<table>
<thead>
<tr>
<th>Foil#</th>
<th>$\bar{x}$ c/m</th>
<th>$\bar{x}/\bar{x}_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>667</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>614</td>
<td>0.922</td>
</tr>
<tr>
<td>3</td>
<td>613</td>
<td>0.920</td>
</tr>
<tr>
<td>4</td>
<td>623</td>
<td>0.935</td>
</tr>
<tr>
<td>5</td>
<td>600</td>
<td>0.900</td>
</tr>
</tbody>
</table>

**Activation of Au foils: Data**

<table>
<thead>
<tr>
<th>Foil#</th>
<th>$\bar{y}$ c/m</th>
<th>$\bar{y}/\bar{y}_1$</th>
<th>$(\bar{y}/\bar{y}_1)/(\bar{x}/\bar{x}_1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2864</td>
<td>1.000</td>
<td>1.000</td>
</tr>
<tr>
<td>2</td>
<td>1474</td>
<td>0.516</td>
<td>0.588</td>
</tr>
<tr>
<td>3</td>
<td>2665</td>
<td>0.933</td>
<td>1.010</td>
</tr>
<tr>
<td>4</td>
<td>2312</td>
<td>0.808</td>
<td>0.864</td>
</tr>
<tr>
<td>5</td>
<td>1270</td>
<td>0.444</td>
<td>0.493</td>
</tr>
</tbody>
</table>

**Normalized ratio of Au foil activities**

<table>
<thead>
<tr>
<th>Foil#</th>
<th>Normalized ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.000 ± 0.009</td>
</tr>
<tr>
<td>2</td>
<td>0.558 ± 0.006</td>
</tr>
<tr>
<td>3</td>
<td>1.010 ± 0.007</td>
</tr>
<tr>
<td>4</td>
<td>0.864 ± 0.002</td>
</tr>
<tr>
<td>5</td>
<td>0.493 ± 0.005</td>
</tr>
</tbody>
</table>
A.3. Calculation of albedo

Assume that neutrons leave the reactor core at epithermal energies, and that the fraction returning to the core from the reflector are at thermal energy. This assumption has been based upon the preliminary calculations.

Foil locations

<table>
<thead>
<tr>
<th>Reflector</th>
<th>Core</th>
</tr>
</thead>
<tbody>
<tr>
<td>#5</td>
<td></td>
</tr>
<tr>
<td>#1</td>
<td></td>
</tr>
<tr>
<td>#2</td>
<td></td>
</tr>
<tr>
<td>#3</td>
<td></td>
</tr>
<tr>
<td>#4</td>
<td></td>
</tr>
</tbody>
</table>

Foil#   Activity Contributions
-------   -------   ---------
  1       1.000  0.009  \( A_e^\circ + A_t^i \)
  2       0.558  0.006  \( A_e^\circ + A_t^i \)
  3       1.010  0.007  \( A_e^\circ + A_t^i \)
  4       0.864  0.002  \( A_e^\circ + A_t^i \)
  5       0.493  0.005  \( A_e^\circ + A_t^i \)

\( A_e^\circ \) Epicadmium activity from core
\( A_t^i \) Thermal activity from reflector

\[
\begin{align*}
\hat{A}_e^\circ + A_t^i &= #1 = 1.000 \\
\hat{A}_e^\circ + A_t^i &= #3 = 1.010 \\
\hat{A}_e^\circ + A_t^i &= #4 = 0.864 \\
\hat{A}_e^\circ &= #5 = 0.493 \\
\hat{A}_e^\circ &= #2 = 0.558 \\
\end{align*}
\]

Average \( \hat{A}_e^\circ + A_t^i = 0.955 \)
Average \( \hat{A}_e^\circ = 0.525 \)

\[
\beta = \frac{A_t^i}{A_e^\circ} = \frac{0.430}{0.525} = 0.820
\]
B. Formulation of the Effect of the Reflector on Neutron Lifetime from Albedo \[4\]

B.1. Analysis

Frankel and Nelson \([5]\) give the following analysis in their "Albedo of an Isotropic Surface Source" concept:

Assuming that the scattering medium produces only isotropic and elastic scattering, the fraction of the incident neutrons returning across the boundary after any specified number of collisions can be calculated.

Using the probability of finite sets and mathematical induction, the fraction of neutrons returning after \(n\) collisions is given by:

\[
F_n = \frac{1}{4} \cdots \frac{3}{6} \frac{5}{8} \frac{7}{10} \cdots \frac{2n-1}{2(n+1)}
\]

or,

\[
F_n = \frac{\binom{2n}{n}}{2^{2n}} \frac{(2n)!}{(n+1) (n!)^2}
\]

We shall use this formula to formulate the reflector effect on neutron lifetime as follows:

Since the effect of the reflector is due to the time delay in the diffusion process in and out of the reflector, the time spent by the neutrons in the reflector until return to the core multiplied by a weighting factor is essentially the contribution of the reflector to the total prompt neutron lifetime.
Table II

Values of $F_n$

<table>
<thead>
<tr>
<th>Collisions $n$</th>
<th>$F_n$</th>
<th>$\sum_{n=1}^{N} F_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.2500</td>
<td>0.2500</td>
</tr>
<tr>
<td>2</td>
<td>0.1250</td>
<td>0.3750</td>
</tr>
<tr>
<td>3</td>
<td>0.0780</td>
<td>0.4530</td>
</tr>
<tr>
<td>4</td>
<td>0.0546</td>
<td>0.5076</td>
</tr>
<tr>
<td>5</td>
<td>0.0397</td>
<td>0.5473</td>
</tr>
<tr>
<td>6</td>
<td>0.0343</td>
<td>0.5816</td>
</tr>
<tr>
<td>7</td>
<td>0.0280</td>
<td>0.6090</td>
</tr>
<tr>
<td>8</td>
<td>0.0232</td>
<td>0.6322</td>
</tr>
<tr>
<td>9</td>
<td>0.0198</td>
<td>0.6520</td>
</tr>
<tr>
<td>10</td>
<td>0.0171</td>
<td>0.6691</td>
</tr>
<tr>
<td>11</td>
<td>0.0150</td>
<td>0.6840</td>
</tr>
<tr>
<td>12</td>
<td>0.0132</td>
<td>0.6972</td>
</tr>
<tr>
<td>13</td>
<td>0.0118</td>
<td>0.7090</td>
</tr>
<tr>
<td>14</td>
<td>0.0106</td>
<td>0.7196</td>
</tr>
<tr>
<td>15</td>
<td>0.0097</td>
<td>0.7293</td>
</tr>
<tr>
<td>16</td>
<td>0.0088</td>
<td>0.7381</td>
</tr>
<tr>
<td>17</td>
<td>0.0078</td>
<td>0.7459</td>
</tr>
<tr>
<td>18</td>
<td>0.0071</td>
<td>0.7530</td>
</tr>
<tr>
<td>19</td>
<td>0.0066</td>
<td>0.7596</td>
</tr>
<tr>
<td>20</td>
<td>0.0061</td>
<td>0.7657</td>
</tr>
<tr>
<td>21</td>
<td>0.0057</td>
<td>0.7714</td>
</tr>
<tr>
<td>22</td>
<td>0.0053</td>
<td>0.7767</td>
</tr>
<tr>
<td>23</td>
<td>0.0050</td>
<td>0.7810</td>
</tr>
<tr>
<td>24</td>
<td>0.0047</td>
<td>0.7857</td>
</tr>
<tr>
<td>25</td>
<td>0.0044</td>
<td>0.7901</td>
</tr>
<tr>
<td>26</td>
<td>0.0041</td>
<td>0.7942</td>
</tr>
<tr>
<td>27</td>
<td>0.0039</td>
<td>0.7981</td>
</tr>
<tr>
<td>28</td>
<td>0.0037</td>
<td>0.8018</td>
</tr>
<tr>
<td>29</td>
<td>0.0035</td>
<td>0.8053</td>
</tr>
<tr>
<td>30</td>
<td>0.0033</td>
<td>0.8086</td>
</tr>
<tr>
<td>31</td>
<td>0.0031</td>
<td>0.8117</td>
</tr>
<tr>
<td>32</td>
<td>0.0030</td>
<td>0.8147</td>
</tr>
<tr>
<td>33</td>
<td>0.0029</td>
<td>0.8176</td>
</tr>
<tr>
<td>34</td>
<td>0.0028</td>
<td>0.8204</td>
</tr>
</tbody>
</table>
From the foil activation analysis, at the reflector and core boundary, the albedo is calculated. This can be used to calculate the number of collisions in the reflector until return to the core, from equation (23). That is,

$$\sum_{n=1}^{N} F_n = \beta$$

(24)

$N$ = the number of collisions.

The time the neutrons spent in the reflector until return to the core is given by:

$$\frac{T^r}{\tau} = \frac{N \lambda_s(\bar{v})}{\bar{v}}$$

(25)

$\lambda_s(\bar{v})$ = scattering mean free path at an average velocity $\bar{v}$.

In order to determine $\bar{v}$ and $\lambda_s(\bar{v})$, we shall assume that the neutron flux leaving the core is inversely proportional to energy. That is,

$$\varphi = \frac{A}{E}$$

$$d\varphi = -\frac{A}{E^2} dE$$

$$\bar{E} = \frac{\int_{E}^{E^*} E d\varphi}{\int_{E}^{E^*} d\varphi} = \frac{\int_{E}^{E^*} \frac{dE}{E}}{\int_{E}^{E^*} \frac{dE}{E^2}}$$
Since the reflector acts as a moderator, the neutron spectrum returning to the core will be at thermal energy i.e., Maxwellian about 0.025 electron volts.

Thus, it can be concluded that the average neutron energy in the reflector can be given by:

\[
\overline{E} = \frac{0.025 + \overline{E}'}{2}
\]  
(27)

From (24), (25), and (27) \( T' \) can be determined.

The effective contribution of the reflected neutrons to the total prompt neutron lifetime, \( T \), can be given by:

\[
T = T' \left(1 - e^{-\frac{\beta}{\overline{E}'}}\right)
\]

where,

\( (1 - e^{-\frac{\beta}{\overline{E}'}}) \beta \) = fraction of neutrons leaving the core at epithermal energies, and return to the core at thermal energy.

\[
T = T' \left(1 - e^{-\frac{\beta}{\overline{E}'}}\right) + \frac{\ell^c}{1 + L^2 B^2} + T_R
\]

B.2. Calculation of \( T \)

With, \( \beta = 0.820 \) \( N = 34 \)

\[
E' = \left[\frac{1}{E}\right]_{0.025}^{2 \times 10^6} 0.453 \text{ ev.}; \quad \overline{E} = \frac{0.453 + 0.025}{2} = 0.239 \text{ ev.}
\]
\[ \bar{v} = 2.2 \times 10^5 \times \sqrt{\frac{0.239}{0.025}} = 6.82 \times 10^5 \text{ cm/sec.} \]

Assuming that the reflector has 80% graphite, 10% water, and 10% aluminum,

\[
\sum_i (c) = 0.385 \times 0.8 = 0.307
\]
\[
\sum_i (H_2O) = 1.4 \times 0.1 = 0.140
\]
\[
\sum_i (Al) = 0.084 \times 0.1 = \frac{0.0084}{\sum_s 0.4554}
\]

\[ \lambda = 2.2 \text{ cm.} \]

\[ T' = \frac{2.2 \times 34}{6.82 \times 10^5} = 109 \text{ micro seconds.} \]

\[ B^2 = \left[ \frac{\bar{u}}{H} \right]^2 + \left[ \frac{2.405}{R} \right]^2 = \left[ \frac{\bar{u}}{33.6} \right]^2 + \left[ \frac{2.405}{17.8} \right]^2 = 0.02696 \]

\[ \tau = 30 \text{ cm}^2 ; \quad (1 - e^{-\frac{B^2 \tau}{\bar{v}^2}}) = 0.555 \]

\[ T' = (109) \times (0.555) \times (0.82) = 49.7 \text{ micro sec.} \]

\[ T = 10 + 91.5 + 49.7 = 151.2 \text{ micro sec.} \]
Calculated values of neutron lifetime for TRIGA, TREAT, and ZEEP reactors are given in Table III. The corresponding reflector contributions and the experimental values are also given.

The agreement between the calculated values and experimental values are good. Nevertheless, the calculations have been based upon the assumed values of various reactor parameters. Better comparison will be obtained when more accurate reactor parameters are available.

Both spherical approximation and cylindrical forms are used for TRIGA giving the same results. The reflector contributes a lifetime of $39.5 \mu$ sec. The result from the foil activation experiment is $49.7 \mu$ sec.

The calculated result of $141 \mu$ sec. for the prompt neutron lifetime for TRIGA is in good agreement with the foil activation experiment's result of $151.2 \mu$ sec.
### Table III

<table>
<thead>
<tr>
<th>Reactor:</th>
<th>TRIGA (graphite reflector)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental T</td>
<td>$150 \mu \text{sec.}^*$</td>
</tr>
<tr>
<td>Calculated T</td>
<td>$141 \mu \text{sec.}$</td>
</tr>
<tr>
<td>Reflector $T_R$</td>
<td>$28%$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reactor:</th>
<th>ZEEP (heavy water reflector)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental T</td>
<td>$86 \mu \text{sec.}^{**}$</td>
</tr>
<tr>
<td>Calculated T</td>
<td>$84 \mu \text{sec.}$</td>
</tr>
<tr>
<td>Reflector $T_R$</td>
<td>$20%$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reactor:</th>
<th>TREAT (graphite reflector)$^{**}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental T</td>
<td>$860 \mu \text{sec.} (\text{ii}) 880 \mu \text{sec.} (\text{iii}) 900 \mu \text{sec.}$</td>
</tr>
<tr>
<td>Calculated T</td>
<td>$870 \mu \text{sec.}$</td>
</tr>
<tr>
<td>Reflector $T_R$</td>
<td>$17%$</td>
</tr>
</tbody>
</table>

---


** V.H. Rumsey: Canadian Journal of Physics. 32, 443 (1954)

*** (i) PICPUAE 10, 471 (1958); ANL- 6174
 (ii) ANL- 6173, p40.
 (iii) ANL- 6173, p41.
The agreement between the theoretical results and the experimental values of neutron lifetime and the contributions of the reflector has been shown to be good.

In view of the fact that diversity of results exists even in the experimental determinations, the formulas given are recommended for useable approximate values, provided that reliable estimates or experimental values of the reactor parameters are used.

The Rumsey formulation is quite powerful and can be extended to multi-group approach. The problem, however, is to solve the resulting differential equations.

The solutions given in this paper make use of the Variation of Parameters method, a useful tool for non-homogeneous differential equations with variable coefficients.

The standard foil activation technique has been shown to be very useful.

It is recommended that the foil activation be extended to reduce the experimental errors and to examine any systematic errors in the experimental technique.
APPENDICES
APPENDIX A

Analytical Formulation of Prompt Neutron Lifetime for Cylindrical reactor with Infinite Side Reflector [6]

Steady state neutron densities for the core and the reflector for this case are given by:

\[ \rho_c = A \cos m z \, J_0(\lambda_c r) \]
\[ \rho_R = C \cos m z \, K_0(\lambda_R r) \]

\[ m = \frac{\pi}{H} \]
\[ \lambda_c^2 = B^2 - m^2 \]
\[ \lambda_R^2 = \frac{1}{L_R^2} - m^2 \]

Also,

\[ l_c^c D_c^c \nabla^2 q_c = q_c - l_c^c \rho_c \]
\[ l_R^c D_R^c \nabla^2 q_R = q_R - l_R^c \rho_R \]
\[ l_c^c D_c^c = l_R^c D_R^c = \nabla^2 \cdot \]

Thus,

\[ l_c^c \nabla^2 q_c = q_c - l_c^c \rho_c \]
\[ l_R^c \nabla^2 q_R = q_R - l_R^c \rho_R \]
From (2) and (1),
\[ \nabla^2 \phi - \frac{1}{\ell_c^2} \phi = - \ell_c^\ell \frac{A}{\ell_c^2} \cos m \varphi J(\lambda r) \]
\[ = A \cos m \varphi J(\lambda r). \]

Let \( \phi = X(r) Z(\varphi) \) \quad \& \quad \lambda^2 = \frac{1}{\ell_c^2}.

Then (3) becomes,
\[ \frac{1}{X} \left[ \frac{d^2X}{dr^2} + \frac{1}{r} \frac{dX}{dr} \right] + \frac{1}{Z} \frac{d^2Z}{d\varphi^2} = \lambda^2 \left( \frac{A \cos m \varphi J(\lambda r)}{X(r) Z(\varphi)} \right). \]

The homogeneous equation is then,
\[ \frac{1}{X} \left[ \frac{d^2X}{dr^2} + \frac{1}{r} \frac{dX}{dr} \right] + \frac{1}{Z} \frac{d^2Z}{d\varphi^2} = \lambda^2 = 0. \]

Let
\[ \frac{1}{X} \left[ \frac{d^2X}{dr^2} + \frac{1}{r} \frac{dX}{dr} \right] = \beta^2 \]
and
\[ \frac{1}{Z} \frac{d^2Z}{d\varphi^2} = - m^2 \]
assuming the same \( z \)-dependence.

The solutions of (6) are then,
\[ X_1 = I_0(\beta r), \quad X_2 = K_0(\beta r), \quad Z = \cos m \varphi \]
\[ \beta^2 = \lambda^2 + m^2 = \frac{1}{\ell_c^2} + m^2. \]
From (7), (6), and (4) the nonhomogeneous equation in $X$ is

$$\frac{1}{X} \left[ \frac{d^2X}{dY^2} + \frac{1}{Y} \frac{dX}{dY} \right] - \beta^2 = \frac{A_2 J_0 (\lambda r)}{X}$$  \hspace{1cm} (8)

or,

$$\frac{d^2X}{dY^2} + \frac{1}{Y} \frac{dX}{dY} - \beta^2X = A_2 J_0 (\lambda r).$$

We shall make use of the variation of parameters method to solve for the general solution of (8).

The variation of parameters method states that:

if $X$ and $X'$ are the solutions of the homogeneous equation

$$L \left[ X(r) \right] = f(r)$$

then, the general solution of the differential equation is given by:

$$X = X_1 \left[ D_1 - \int_a^y \frac{f(\xi)X_2(\xi)}{W(\xi)} d\xi \right] + X_2 \left[ D_2 + \int_b^y \frac{f(\xi)X_2(\xi)}{W(\xi)} d\xi \right]$$ \hspace{1cm} (9)

where $a, b, D_1, D_2$ are arbitrary constants; and

$$W(r) \equiv X_1 \frac{dX_2}{dY} - X_2 \frac{dX_1}{dY}.$$

with,

$$X_1 = I_0 (\beta r), \quad X_2 = K_0 (\beta r), \quad f(r) = \frac{A_2 J_0 (\lambda r)}{X}$$

$$W(r) = \beta I_0 (\beta r) K_0' (\beta r) - \beta K_0 (\beta r) I_0' (\beta r)$$

$$= -\beta [I_0 K_1 + K_0 I_1'] = -\frac{\beta}{\beta r} = -\frac{1}{Y}$$
\[ X = I_o(\beta r) \left[ D_1 + \int_a^r A_z J_0(\gamma \xi) K_0(\beta \xi) \xi d\xi \right] + K_0(\beta r) \left[ D_2 - \int_b^r A_z J_0(\gamma \xi) I_0(\beta \xi) \xi d\xi \right] \]

\[ \int_a^r J_o(\gamma \xi) K_0(\beta \xi) \xi d\xi = \frac{r}{\beta^2 + \lambda^2} \left[ \frac{1}{\beta J_o(\gamma r) K_0(\beta r)} \right] + \frac{1}{\beta^2 + \lambda^2} \]

\[ \int_b^r J_o(\gamma \xi) I_0(\beta \xi) \xi d\xi = \frac{r}{\beta^2 + \lambda^2} \left[ \frac{1}{\beta J_o(\gamma r) I_0(\beta r)} + \frac{1}{\beta J_o(\gamma r) J_0(\beta r)} \right]. \]

Since \( X \) is finite at the origin, the coefficients of the terms containing \( K_0 \) should be zero; then we have,

\[ X = A_1 I_0(\beta r) - \frac{A_2}{\beta^2 + \lambda^2} J_0(\gamma r) \]

The same procedure can be applied to find \( q \) from (2), remembering that \( q \) is zero at infinity; that is, \( I_0 \) terms should not enter.
with, \( q = Y(r) \frac{\partial Z}{\partial r} \), \( Z(r) = \cos m z \)

\[ Y(r) = A_{\kappa} K_{\nu}(\kappa r) + A_{\xi} r K_{\nu}(\xi r) \]

\[ \kappa^2 = \frac{1}{\kappa_r^2} + m^2 = \lambda^2 \]

Using the boundary conditions of \( \rho_c \) and \( \rho_r \) - at the core and reflector boundary - and extending these conditions to \( q_c \) and \( q_r \): viz.

\[ X(R) = \frac{l_c}{l_r} Y(R) \]

\[ X'(R) = \frac{l'_c}{l'_r} Y'(R). \]

Note that \( l_c \) and \( l_r \) are included, since \( q_c/l_c \) is the density of neutrons that are destined to die in the core, and \( q_r/l_r \) is for that of reflector.

Finally, it can be shown that,

\[ T_i = \frac{\int_{V_c} q_c dV}{\int_{V_c} \rho_c dV} \]

\[ = l_c^2 \left[ \frac{1}{1 + l_c^2 B^2} + \frac{\lambda_z^2 R_0 Q}{2 \lambda_r \beta l_r^2} \frac{1 - (K_0/K_1)}{\lambda_r I_r^8 + \beta K_0 K_1}. \right] \]
APPENDIX B

Analytical Formulation of Prompt Neutron Lifetime for Spherical reactor with Finite Reflector

In this case:

\[
\nabla^2 q - \alpha^2 q = - \frac{A \ell_o^c}{L_c^2} \frac{\sin Br}{Y} = A_1 \frac{\sin Br}{Y}
\]

\[
\nabla^2 q - \sigma^2 q = - \frac{C \ell_o^R}{L_R^2} \frac{\sinh \pi (\tilde{R}_i - Y)}{Y}
\]

\[
\alpha^2 = \frac{1}{L_c^2} \quad ; \quad \sigma^2 = \frac{1}{L_R^2}.
\]

Solutions of homogeneous equation are,

\[
q = \frac{e^{\alpha_1 r}}{Y} \quad ; \quad q = \frac{e^{-\alpha_1 r}}{Y}
\]

\[
q = \frac{e^{\alpha_2 r}}{Y} \quad ; \quad q = \frac{e^{-\alpha_2 r}}{Y}
\]

\[
q(q) = \frac{e^{\alpha r}}{Y} \left[ D_1 + \frac{A_i}{2\alpha} \int_0^r \sin B_\ell \ e^{-\alpha_\ell t} \ dt \right]
\]

\[
+ \frac{e^{-\alpha r}}{Y} \left[ D_2 - \frac{A_i}{2\alpha} \int_0^r \sin B_\ell \ e^{\alpha_\ell t} \ dt \right]
\]

\[
= A_2 \frac{\sinh \alpha r}{r} + A_3 \frac{\sin Br}{r}.
\]
\[ \varphi_k(r) = \frac{e^{\pi r}}{r} \left[ F_1 + \frac{e^{-\pi r}}{2\alpha} \int_0^r \sinh \alpha (\tilde{R}_i - \tilde{\xi}) e^{-\frac{\pi \tilde{\xi}}{r}} d\tilde{\xi} \right] + \frac{e^{-\pi r}}{r} \left[ F_2 - \frac{e^{-\pi r}}{2\alpha} \int_0^r \sinh \alpha (\tilde{R}_i - \tilde{\xi}) e^{-\frac{\pi \tilde{\xi}}{r}} d\tilde{\xi} \right] \]

\[ = F_3 \frac{\sinh \alpha (\tilde{R}_i - r)}{r} + F_4 \cosh \alpha (\tilde{R}_i - r). \]

Finally, it can be shown that – as in the previous case:

\[ T = \lambda^c \left[ \frac{1}{1 + L^2 \frac{B^2}{c^2}} + \frac{\Phi B^2}{2 \kappa x^2 L^2} \frac{H'}{G'} \frac{\kappa^'G - \kappa G'}{H G' - H' G} \right] \]

\[ H = \frac{\sinh \alpha r}{r} \bigg|_{r=R_0} \]

\[ G = \frac{\sinh \alpha (\tilde{R}_i - r)}{r} \bigg|_{r=R_0} \]

\[ K = \cosh \alpha (\tilde{R}_i - r) \bigg|_{r=R_0} \]

The derivatives are taken at \( r = R_0 \) (core radius).
Appendix C

Formulation of $\delta k' \sim \delta L_R$ for Galanin's Approach

$$\nabla^2 \rho + \alpha^2 \rho = 0$$

$$\nabla^2 \rho + \frac{k-1}{M^2} \rho = 0$$

$$\alpha^2 = \frac{k-1}{M^2} = \frac{k-1}{L^2 + \varepsilon}$$

$$\delta \alpha^2 = \frac{(L^2 + \varepsilon) \delta k - (k-1) \delta L^2}{(L^2 + \varepsilon)^2}$$

$$= \frac{\delta k}{L^2 + \varepsilon} - \frac{\delta L^2}{L^2} \cdot \frac{L}{(L^2 + \varepsilon)^2} \cdot (k-1).$$

For spherical reactor with infinite reflector with equal diffusion coefficients, we have

$$\tan \alpha R_o = -\alpha L_R$$

$$- \frac{L_R}{L} = \frac{\tan \alpha R_o}{\alpha}$$

$$- \frac{\delta L_R}{\delta \alpha} = \frac{\alpha R_o (1 + \tan^2 \alpha R_o) - \tan \alpha R_o}{\alpha^2 L^2}$$

$$= \frac{\alpha R_o (1 + \frac{\alpha^2 L^2}{L^2}) + \alpha L_R}{\alpha}$$

$$= \frac{R_o (1 + \frac{\alpha^2 L^2}{L^2}) + L_R}{\alpha}.$$
\[
\frac{\delta \alpha}{\alpha} = - \frac{SL_R}{R_0 \left( 1 + \alpha^2 L_R^2 \right) + L_R}
\]
\[
= - \frac{SL_R/L_R}{R_0/L_R \left( 1 + \alpha^2 L_R^2 \right) + 1}
\]
\[
\frac{\delta k'}{M^2} = 2 \alpha \delta \alpha + \frac{2 L_R \delta L_R}{L_R^2} \frac{L_R^2}{M^4} (k-1)
\]
\[
= 2 \alpha \delta \alpha + 2 \left| \frac{\delta L_R}{L_R} \right| \frac{L_R^2}{M^4} (k-1)
\]
\[
k-1 \ll 1 \ ; \quad \left| \frac{\delta L_R}{L_R} \right| \ll 1
\]
\[
\frac{\delta k'}{M^2} = 2 \alpha \delta \alpha = \frac{-2 \alpha^2 \delta L_R/L_R}{R_0/L_R \left( 1 + \alpha^2 L_R^2 \right) + 1}
\]
\[
\frac{\delta L_R}{L_R} = - \frac{L_R^2 \omega_p}{2 D_0^c}
\]
\[
\frac{\delta k'}{M^2} = \frac{\alpha^2 L_R^2 \omega_p / D_0^c}{R_0/L_R \left( 1 + \alpha^2 L_R^2 \right) + 1} \quad ; \quad \frac{M^2}{D_0^c} = T_0^c
\]
\[
\delta k' = \frac{\alpha^2 L_R^2}{R_0/L_R \left( 1 + \alpha^2 L_R^2 \right) + 1} T_0^c \omega_p
\]
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