GUST ALLEVIATION IN AIRCRAFT USING FORWARD MOUNTED CONTROL SURFACES

by

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LIST OF SYMBOLS

c_c - mean aerodynamic chord, control surface

c_w - mean aerodynamic chord, wing

$C_{L\alpha}$ - airplane lift-curve slope $\partial C_L/\partial\alpha$

$C_{L\alpha_c}$ - control surface - lift curve slope $\partial C_{L_c}/\partial\alpha$

$C_{L\alpha_w}$ - wing lift curve slope $\partial C_{L_W}/\partial\alpha$

$C_{m\alpha}$ - $\partial C_m/\partial\alpha$

C_{mq} - damping in pitch

c_w - mean aerodynamic chord, wing

D - Drag

$\bar{\epsilon}_n$ - unit normal vector

$\bar{\epsilon}_t$ - unit tangential vector

$\bar{F}$ - external forces

g - gravity constant

H_c - moment of momentum about center

I - moment of inertia

k - radius of gyration

L - lift

L_c - lift, control surface

L_W - lift, wing

ℓ - $\ell_c + \ell_W$

LIST OF SYMBOLS
(Cont'd)

ℓ_c - distance from center of gravity to control surface aerodynamic center

ℓ_s ℓ_c - distance from sensor to control surface aerodynamic center

ℓ_w - distance from center of gravity to wing aerodynamic center

m - mass

M_c - moment of external forces about center

S_c - control surface area

S_w - wing area

t - time

T - thrust

$u(\ )$ - step function

V_c - velocity of center

x_c - distance from hinge point of control surface to its leading edge

α - angle of attack

α_g - v_g/v_0

α_{g_c} - $u(\tau)$

α_{g_s} - $u(\tau-\tau_s)$

α_{g_w} - $u(\tau-1)$

LIST OF SYMBOLS
(Cont'd)

δ - delta function

δ_c - angle made by control surface

θ - pitch angle

γ - flight path angle

ρ - density of air

v_g - gust velocity

V_0 - airplane velocity

τ - non-dimensionalized time

τ_s - l_s/l

ϵ - additional angle of attack induced by the motion of the surface

AR - aspect ratio

T_α - dimensionless thrust parameter, Tl/mV_0^2

L_α - dimensionless lift parameter, $\rho l S_W C_{L\alpha}/2m$

M_q - dimensionless moment parameter, $\rho l S_W C_{mq}\left(\frac{l}{k}\right)^2/2m$

M_α - dimensionless moment parameter, $\rho l S_W C_{m\alpha}\left(\frac{l}{k}\right)^2/2m$

L_{α_W} - dimensionless lift parameter, $\rho l S_W C_{L\alpha_W}/2m$

L_{α_c} - dimensionless lift parameter, $\rho l S_c C_{L\alpha_c}/2m$

M_{α_c} - dimensionless moment parameter, $\rho S_c l_c C_{L\alpha_c}\left(\frac{l}{k}\right)^2/2m$

M_{α_W} - dimensionless moment parameter, $\rho S_W l_W C_{L\alpha_W}\left(\frac{l}{k}\right)^2/2m$

CHAPTER I

INTRODUCTION

An important consideration in the design of an aircraft is the response of the aircraft to gusts or turbulence in the atmosphere. The response of the aircraft will affect many points of view - maintenance of a pre-determined flight path, comfort of passengers and crew, stability as gun and bombing platform, fatigue life of frame and equipment.

Many proposals have been made for alleviating the effects of gusts. Among them are the use of spoiler-deflector controls and sensing devices used to detect gusts and to operate gust-alleviation controls. Systems such as these are discussed by Phillips (10), Tobak (12) and Adams and Mathews (1).

In this study, the system under consideration is an aircraft with a forward mounted or canard control surface. The scheme of alleviation involves the operation of the forward control in response to the output of an angle of attack sensor mounted well forward on the aircraft. The forward control responding to the gust disturbance then reacts so as to ease the impact of the disturbance on the wing.

Since the aircraft is a highly complicated nonlinear dynamic system with at least six degrees of freedom, it is convenient to make a considerable number of assumptions to linearize the motion of the aircraft and to restrict it to only two degrees of freedom ((McClean (8), and Etkin (6)). The restriction limits the analysis to response in the plane of symmetry of the aircraft due to the vertical components of gust velocity. This paper will then assume that the aircraft encounters an idealized sharp-edged gust.

The equations of motion will be developed according to the Newtonian laws of motion. The aerodynamic forces resulting from the disturbance will be evaluated using quasi-steady aerodynamics and considering circulatory forces.

The solution of the equations of motion will be undertaken in two phases. Initially, the forward control will be assumed fixed and a solution will be obtained for the normal acceleration which is directly proportional to the rate of change of the flight path angle. Secondly, the normal acceleration will be determined using feedback control to the forward control surface, and a comparison will be made between the two.

A numerical example, based on a proposed Douglas transport, will then be employed:

1. To determine the effects of the gain of the feedback control system on the normal acceleration.

2. To determine the effects of the location of the sensing device on the normal acceleration.

The rate of change of the flight path angle will be developed in terms of the gain, G, and the location of the sensing device, ℓ_s, from the aerodynamic center of the control surface. The sensing device will be fixed on the control surface, on the nose of the aircraft, and forward of the nose of the aircraft. The gain will be varied. A graph will then be obtained relating the gain, the location of the sensor, and the ratio of the rate of change of the flight path angle with feedback control to that of the control surface fixed.

CHAPTER II

EQUATIONS OF MOTION

2.1 Co-ordinate System

The co-ordinate system used in this analysis is shown in Figure 2.1.

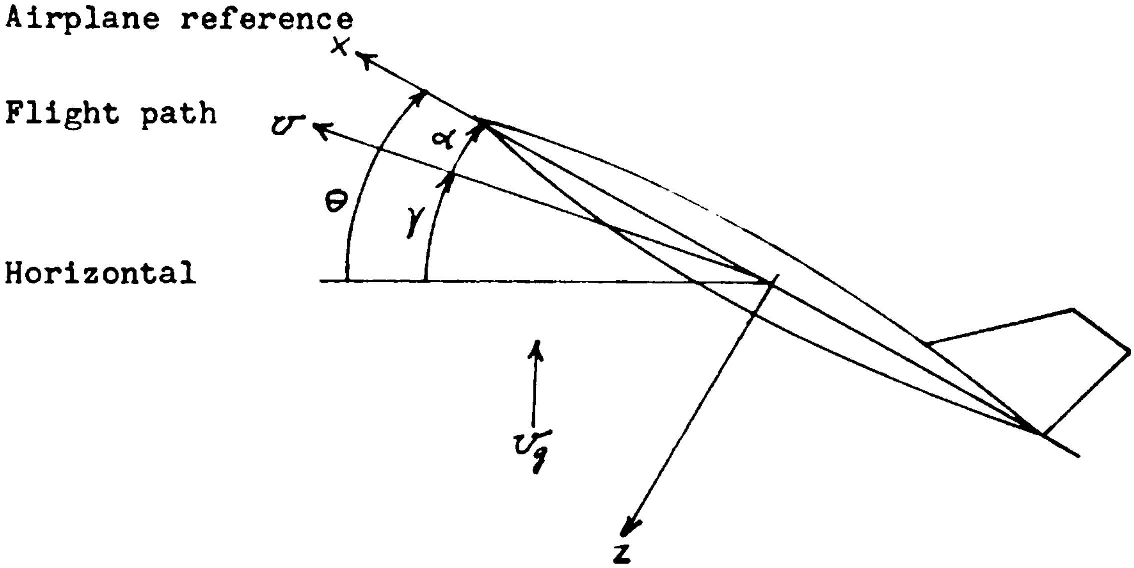

Figure 2.1 Body Axes

An x, y, z co-ordinate system is placed in the airplane; the origin fixed at the center of gravity. The instantaneous direction of the path of the center of gravity relative to still air is indicated by the direction of the flight velocity vector U. The positive branch of the x-axis is forward.

The y-axis coincides with the lateral axis running through the center of gravity. The z-axis lies in the vertical plane of symmetry, perpendicular to the xy plane. We define the angle of attack α as the angle between the xy plane and the plane of the velocity vector; the angle of pitch θ as the angle between the xy plane and the horizontal plane; the flight path angle γ as the angle between the horizontal plane and the plane of the velocity vector. γ is equal to $\theta - \alpha$. The gust velocity is indicated by U_g. All quantities in Figure 2.1 are indicated in their positive directions.

Forces and moments are as indicated in Figure 2.2.

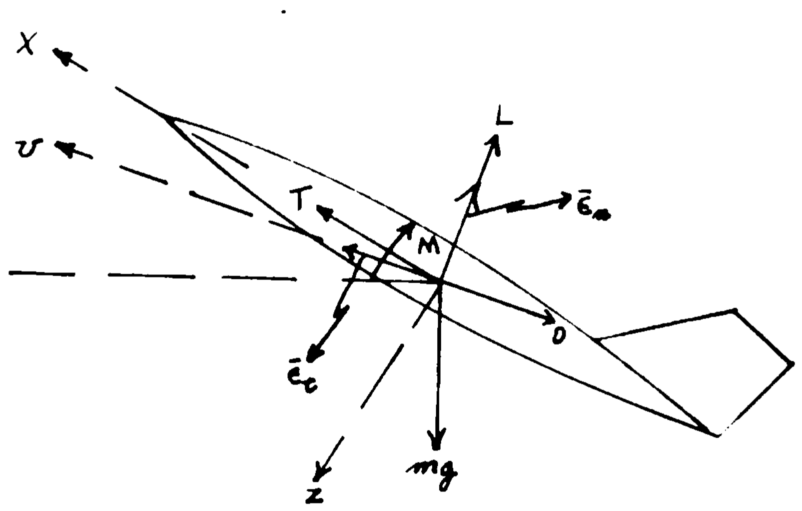

Figure 2.2 Applied Forces and Moments

Forces are positive when measured upward; pitching moments, positive when tending to increase the angle of pitch. We define $\bar{e}_t$ as a unit vector in the plane of the velocity vector and coinciding with the line of action of the velocity vector; $\bar{e}_n$, as a unit vector perpendicular to the plane of the velocity vector and coinciding with the line of action of the lift forces. The positive directions of these unit vectors are as shown in Figure 2.2.

2.2 Development of the Equations of Motion

The equations of motion can be written according to the Newtonian laws of motion which state that the summation of external forces must equal the time rate of change of momentum, and that the summation of the moments of the external forces must equal the time rate of change of the moment of momentum.

By these laws, we have the following equations of motion:

$$\bar{F} = \frac{d}{dt}(m\bar{V}_c)$$

$$M_c = \frac{d}{dt} H_c \qquad (2.1)$$

where $\bar{F}$ is equal to the summation of the external forces; $\bar{M}_c$, the summation of the moments of the external forces about the center; $\bar{H}_c$, the moment of momentum about the center.

The mass m is assumed to be constant and $\bar{V}_c$ is equal to $U\bar{e}_t$. The external forces may be given as:

$$\bar{F} = m\dot{U}\bar{e}_t + mU\dot{\bar{e}}_t$$

where $\bar{e}_t$ changes with a change in direction of the velocity U (Figure 2.3)

Figure 2.3 Unit Tangential and Normal Vectors

For a small change in the flight path angle,

$$d\bar{e}_t = d\gamma\bar{e}_n$$

$$\frac{d}{dt}\bar{e}_t = \frac{d}{dt}\gamma\,\bar{e}_n$$

$$\dot{\bar{e}}_t = \dot{\gamma}\,\bar{e}_n$$

and

$$\bar{F} = m\dot{U}\bar{e}_t + mU\dot{\gamma}\,\bar{e}_n \tag{2.2}$$

Summing the external forces,

$$\bar{F} = L\bar{e}_n - D\bar{e}_t - mg\sin\gamma\,\bar{e}_t - mg\cos\gamma\,\bar{e}_n + T\sin\alpha\,\bar{e}_n + T\cos\alpha\,\bar{e}_t \tag{2.3}$$

Substituting the value obtained from equation (2.3) into equation (2.2), and equating coefficients, we then have the equations for translational motion.

$$
\begin{aligned}
m\dot{U} &= T\cos\alpha - D - mg\sin\gamma \\
mU\dot{\gamma} &= L + T\sin\alpha - mg\cos\gamma
\end{aligned}
\tag{2.4}
$$

The moment of momentum $\overline{H}_c$ about the center is equal to the sum of its components about the OX, OY, OZ axes. These may be expressed as

$$
\begin{Bmatrix} H_x \\ H_y \\ H \end{Bmatrix} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix} \begin{Bmatrix} 0 \\ \dot{\theta} \\ 0 \end{Bmatrix} \tag{2.5}
$$

where the quantities I are the moments and products of inertia and where $\dot{\theta}$ is equal to the angular velocity about the y axis.

Multiplying the right side of the above equation,

$$
\begin{Bmatrix} H_x \\ H_y \\ H_z \end{Bmatrix} = \begin{Bmatrix} -I_{xy}\,\dot{\theta} \\ I_{yy}\,\dot{\theta} \\ -I_{zy}\,\dot{\theta} \end{Bmatrix}
$$

For an airplane, the plane of motion is a plane of symmetry, and the products of inertia, $I_{xy} = I_{zy} = 0$.

Letting $I_{yy} = I$, then

$$
H_c = H_y = I\,\dot{\theta}
$$

and

$$\bar{H}_c = I\dot{\theta}\bar{j}.$$

Therefore,

$$\dot{\bar{H}}_c = \frac{d}{dt}(I\dot{\theta}\bar{j}) + \bar{\omega} \times \bar{H}_c$$

where: $\bar{\omega} = \dot{\theta}\,\bar{j}$

$\bar{\omega} \times \bar{H}_c$ is an additional term due to the fact that we have a moving set of reference axes

and

$$\dot{\bar{H}}_c = I\ddot{\theta}\bar{j} = \bar{M}_c$$

The equation for angular motion becomes

$$M_c = I\ddot{\theta} \tag{2.6}$$

Considering steady state, straight and level flight where

m, I, T, g are constant

$U = U_o$

$\alpha = \theta = \alpha_o = \theta_o \ll 1$

$\gamma = \gamma_o = 0$

$L = L_o$

$D = D_o$

$M_c = 0$

the equations of motion, equations (2.4) and (2.6), become

$$0 = T - D_o$$

$$0 = L_o + T\alpha_o - mg \tag{2.7}$$

$$0 = 0$$

Considering the disturbed motion resulting from the gust and assuming that the changes in induced drag and in velocity are negligible

$$U = U_o$$
$$\alpha = \alpha_o + \bar{\alpha}$$
$$\gamma = \bar{\gamma}$$
$$\theta = \theta_o + \bar{\theta}$$
$$L = L_o + \bar{L}$$
$$M_c = \bar{M}_c$$
$$D = D_o$$
$$\bar{\alpha}, \bar{\gamma}, \bar{\theta} \ll 1$$

and where changes are denoted by the barred quantities, the equations of motion become

$$0 = T - D_o$$
$$mU_o \dot{\bar{\gamma}} = L_o + \bar{L} + T\,(\alpha_o + \bar{\alpha}) - mg \tag{2.8}$$
$$I\,\ddot{\bar{\theta}} = \bar{M}_c$$

Subtracting equations (2.7) from (2.8) we then have as equations of motion,

$$mU_o \dot{\bar{\gamma}} = \bar{L} + T\bar{\alpha}$$
$$I\,\ddot{\bar{\theta}} = \bar{M}_c \tag{2.9}$$

2.3 Aerodynamic Forces

In order to solve the simultaneous equations of motion, it is necessary to evaluate the aerodynamic forces resulting from the disturbance. This will be accomplished using quasi-steady aerodynamics and considering circulatory forces.

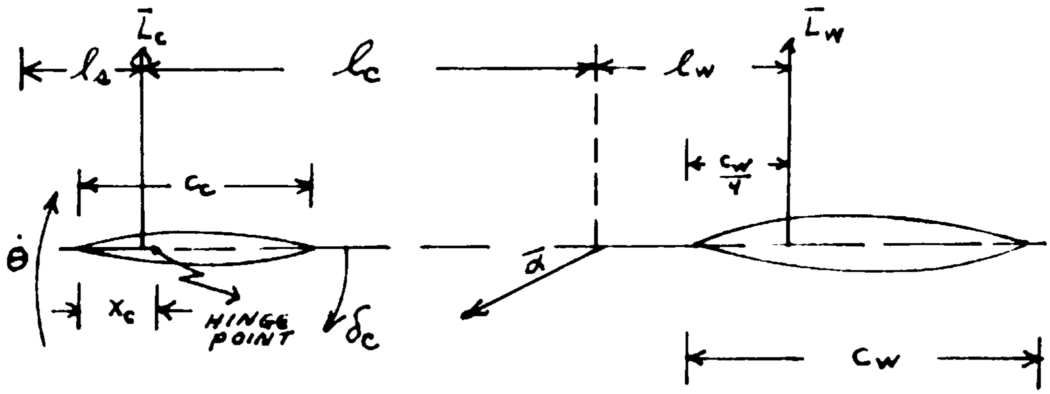

Figure 2.4 Surfaces

The lift on a surface has the form

$$L = \tfrac{1}{2} \rho V^2 S C_L$$

where:

$$C_L = C_{L\alpha} (\bar{\alpha} + \alpha_g + \epsilon)$$

α_g = ratio of the gust velocity to the airplane velocity

ϵ = additional angle of attack at the 3/4 chord induced by the motion of the surface.

With reference to Figure 2.4, the lift of the control surface is

$$\bar{L}_c = \frac{1}{2} \rho V_0^2 S_c C_{L\alpha_c} \left[\bar{\alpha} + \alpha_g + \delta_c - \frac{\dot{\theta}}{V_0}\left(l_c - \frac{c_c}{2}\right) + \frac{\dot{\delta}_c}{V_0}\left(\tfrac{3}{4} c_c - X_c\right) \right] \tag{2.10}$$

and the lift of the wing is

$$\bar{L}_W = \tfrac{1}{2}\rho V_0^2 S_W C_{L\alpha_W}\left[\bar{\alpha} + \alpha_g + \frac{\dot{\bar{\theta}}}{V_0}\left(\ell_W + \frac{c_W}{2}\right)\right] \qquad (2.11)$$

The total lift,

$$\bar{L} = \bar{L}_c + \bar{L}_W \qquad (2.12)$$

Carrying out the summation, we have

$$\begin{aligned}\bar{L} = \tfrac{1}{2}\rho V_0^2 \Big\{ & S_c C_{L\alpha_c}\left[\delta_c + \frac{\dot{\delta}_c}{V_0}\left(\tfrac{3}{4}c_c - x_c\right)\right] \\ & + \left(S_c C_{L\alpha_c} + S_W C_{L\alpha_W}\right)\bar{\alpha} \\ & + \left(S_c C_{L\alpha_c} + S_W C_{L\alpha_W}\right)\alpha_g \\ & + \left[-S_c C_{L\alpha_c}\left(\ell_c - \frac{c_c}{2}\right) + S_W C_{L\alpha_W}\left(\ell_W + \frac{c_W}{2}\right)\right]\frac{\dot{\bar{\theta}}}{V_0}\Big\}\end{aligned}$$

At this point, the time lapse between the gust input to the control surface and that to the wing should be recognized (Figure 2.5).

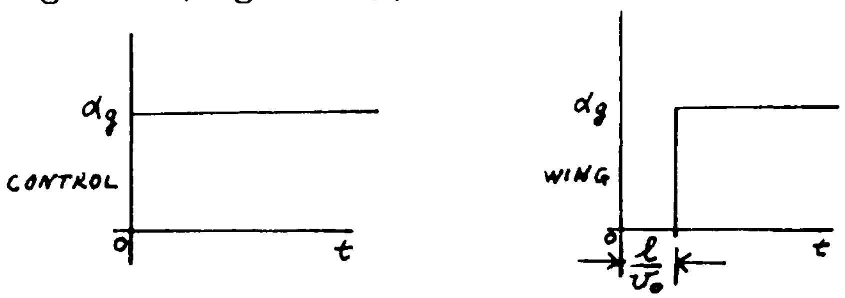

Figure 2.5 Steps Due to Disturbance

To permit a later solution of the equations of motion, we define the gust input to the control surface,

$$\alpha_{g_c} = \alpha_g\, u(t)$$

and the gust input to the wing,

$$\alpha_{g_W} = \alpha_g\, u\left(t - \frac{\ell}{v_0}\right)$$

where:

$$u(t) = 0 \qquad t < 0$$
$$\quad = 1 \qquad t > 0$$
$$u\left(t - \frac{\ell}{v_0}\right) = 0 \qquad t < \frac{\ell}{v_0}$$
$$\quad = 1 \qquad t > \frac{\ell}{v_0}$$

Then,

$$\bar{L} = \tfrac{1}{2}\rho v_0^2 \left\{ S_c C_{L\alpha_c}\left[\delta_c + \frac{\dot{\delta}_c}{v_0}\left(\tfrac{3}{4}c_c - x_c\right) + \alpha_{g_c}\right] + S_W C_{L\alpha}(\bar{\alpha}) + S_W C_{L\alpha_W}\alpha_{g_W} + S_W \ell C_{L_q}\frac{\dot{\bar{\theta}}}{v_0}\right\}$$

where:

$$S_W C_{L\alpha} = S_c C_{L\alpha_c} + S_W C_{L\alpha_W}$$

$$S_W \ell C_{L_q} = -S_c C_{L\alpha_c}\left(\ell_c - \frac{c_c}{2}\right) + S_W C_{L\alpha_W}\left(\ell_W + \frac{c_W}{2}\right)$$

Since the angle of the control surface, its rate of change and the rate of change of the pitch angle have negligible effect on the lift,

$$\bar{L} = \tfrac{1}{2}\rho v_0^2\left[S_W C_{L\alpha}(\bar{\alpha}) + S_c C_{L\alpha_c}\alpha_{g_c} + S_W C_{L\alpha_W}\alpha_{g_W}\right] \tag{2.13}$$

The total moment is equal to the lift forces multiplied by their respective moment arms.

$$\bar{M}_c = \bar{L}_c l_c - \bar{L}_w l_w \tag{2.14}$$

Substituting into equation (2.14) the values obtained from equations (2.10) and (2.11),

$$\begin{aligned}\bar{M}_c = \tfrac{1}{2}\rho v_0^2 \Big\{ & S_c C_{L\alpha_c} l_c \left[\delta_c + \frac{\dot{\delta}}{v_0}\left(\tfrac{3}{4}c_c - x_c\right) + \alpha_{g_c}\right] \\ & + \left(S_c C_{L\alpha_c} l_c - S_w C_{L\alpha_w} l_w\right)\bar{\alpha} \\ & + \left(-S_w C_{L\alpha_w} l_w\right)\alpha_{g_w} \\ & + \left[-S_c C_{L\alpha_c} l_c\left(l_c - \frac{c_c}{2}\right) - S_w C_{L\alpha_w} l_w\left(l_w + \frac{c_w}{2}\right)\right]\frac{\dot{\bar{\theta}}}{v_0}\Big\}\end{aligned}$$

and

$$\begin{aligned}\bar{M}_c = \tfrac{1}{2}\rho v_0^2 \Big\{ & S_c l_c C_{L\alpha_c}\left[\delta_c + \frac{\dot{\delta}_c}{v_0}\left(\tfrac{3}{4}c_c - x_c\right) + \alpha_{g_c}\right] \\ & + S_w l C_{m\alpha}(\bar{\alpha}) - S_w C_{L\alpha_w} l_w \alpha_{g_w} \\ & + S_w l^2 C_{mq}\frac{\dot{\bar{\theta}}}{v_0}\Big\}\end{aligned} \tag{2.15}$$

where:

$$S_w l C_{m\alpha} = S_c C_{L\alpha_c} l_c - S_w C_{L\alpha_w} l_w$$

$$S_w l^2 C_{mq} = -S_c C_{L\alpha_c} l_c\left(l_c - \frac{c_c}{2}\right) - S_w C_{L\alpha_w} l_w\left(l_w + \frac{c_w}{2}\right)$$

Since the rate of change of the angle of the control surface will have negligible effect on the total moment,

$$\bar{M}_c = \tfrac{1}{2}\rho V_0^2 \Big[S_c l_c C_{L\alpha_c}(\delta_c + \alpha_{g_c}) + S_w l C_{m\alpha}(\bar{\alpha}) - S_w C_{L\alpha_w} l_w \alpha_{g_w} + S_w l^2 C_{mq} \frac{\dot{\theta}}{V_0} \Big] \tag{2.16}$$

Substituting the values obtained for $\bar{L}$ and $\bar{M}_c$ into equations (2.9),

$$m V_0 \dot{\gamma} = T\bar{\alpha} + \tfrac{1}{2}\rho V_0^2 \Big[S_w C_{L\alpha}\bar{\alpha} + S_c C_{L\alpha_c}\alpha_{g_c} + S_w C_{L\alpha_w}\alpha_{g_w} \Big]$$

$$I\ddot{\theta} = \tfrac{1}{2}\rho V_0^2 \Big[S_c l_c C_{L\alpha_c}(\delta_c + \alpha_{g_c}) + S_w l C_{m\alpha}\bar{\alpha} - S_w C_{L\alpha_w} l_w \alpha_{g_w} + S_w l^2 C_{mq} \frac{\dot{\theta}}{V_0} \Big] \tag{2.17}$$

Introducing

$$\tau = \frac{V_0 t}{l}$$

$$\frac{d(\ )}{d\tau} = (\ )'$$

$$I = mk^2$$

where $l = l_c + l_w$ and k = radius of gyration,

$$\bar{\gamma}' - \left(\frac{T\ell}{m v_0^2} + \ell\rho \frac{S_W C_{L\alpha}}{2m} \right) \bar{\alpha} = \left(\rho\ell \frac{S_c C_{L\alpha_c}}{2m} \right) \alpha_{g_c} + \left(\rho\ell \frac{S_W C_{L\alpha_W}}{2m} \right) \alpha_{g_W}$$

$$\bar{\theta}'' - \rho\ell \frac{S_W}{2m} C_{mq} \left(\frac{\ell}{k} \right)^2 \bar{\theta}' - \rho \frac{S_W \ell C_{m\alpha}}{2m} \left(\frac{\ell}{k} \right)^2 \bar{\alpha} =$$

$$\rho \frac{S_c \ell_c C_{L\alpha_c}}{2m} \left(\frac{\ell}{k} \right)^2 (\delta_c + \alpha_{g_c}) - \rho \frac{S_W \ell_W C_{L\alpha_W}}{2m} \left(\frac{\ell}{k} \right)^2 \alpha_{g_W} \tag{2.18}$$

and,

$$\bar{\gamma}' - (T_\alpha + L_\alpha) \bar{\alpha} = L_{\alpha_c} \alpha_{g_c} + L_{\alpha_W} \alpha_{g_W} \tag{2.19a}$$

$$\bar{\theta}'' - M_q \bar{\theta}' - M_\alpha \bar{\alpha} = M_{\alpha_c} \delta_c + M_{\alpha_c} \alpha_{g_c} - M_{\alpha_W} \alpha_{g_W} \tag{2.19b}$$

where:

$$T_\alpha = \frac{T\ell}{m v_0^2}$$

$$L_\alpha = \rho \frac{\ell S_W C_{L\alpha}}{2m}$$

$$M_q = \rho \frac{\ell S_W C_{mq}}{2m} \left(\frac{\ell}{k} \right)^2$$

$$M_\alpha = \rho \frac{\ell S_W C_{m\alpha}}{2m} \left(\frac{\ell}{k} \right)^2$$

$$L_{\alpha_c} = \rho \frac{\ell S_c}{2m} C_{L\alpha_c}$$

$$L_{\alpha_w} = \rho \frac{\ell S_w}{2m} C_{L\alpha_w}$$

$$M_{\alpha_c} = \rho \frac{S_c \ell_c}{2m} C_{L\alpha_c} \left(\frac{\ell}{k}\right)^2$$

$$M_{\alpha_w} = \rho \frac{S_w \ell_w}{2m} C_{L\alpha_w} \left(\frac{\ell}{k}\right)^2$$

Recalling that $\gamma = \theta - \alpha$,

$$\bar{\theta}' - \bar{\alpha}' - (T_\alpha + L_\alpha)\bar{\alpha} = L_{\alpha_c} \alpha_{g_c} + L_{\alpha_w} \alpha_{g_w} \tag{2.20a}$$

$$\bar{\theta}'' - M_q \bar{\theta}' - M_\alpha \bar{\alpha} = M_{\delta_c} \delta_c + M_{\alpha_c} \alpha_{g_c} - M_{\alpha_w} \alpha_{g_w} \tag{2.20b}$$

CHAPTER III

SOLUTION OF THE EQUATIONS OF MOTION

3.1 Front Control Surface Fixed

The control surface-fixed condition will be studied first. The nature of the airplane's motion after a gust encounter will be investigated by solution of the following reduced equations of motion:

$$\bar{\theta}' - \bar{\alpha}' - (T_\alpha + L_\alpha)\bar{\alpha} = L_{\alpha_c}\alpha_{g_c} + L_{\alpha_w}\alpha_{g_w} \quad (3.1a)$$

$$\bar{\theta}'' - M_q\bar{\theta}' - M_\alpha\bar{\alpha} = M_{\alpha_c}\alpha_{g_c} - M_{\alpha_w}\alpha_{g_w} \quad (3.1b)$$

These equations follow from equations (2.20) since $\delta_c = 0$. From equation (3.1a)

$$\bar{\theta}'' = \bar{\alpha}'' + (T_\alpha + L_\alpha)\bar{\alpha}' + (L_{\alpha_c}\alpha'_{g_c} + L_{\alpha_w}\alpha'_{g_w})$$

By substitution into equation (3.1b)

$$\bar{\alpha}'' + (T_\alpha + L_\alpha - M_q)\bar{\alpha}' - [M_q(T_\alpha + L_\alpha) + M_\alpha]\bar{\alpha} =$$
$$(M_{\alpha_c} + M_q L_{\alpha_c})\alpha_{g_c} + (M_q L_{\alpha_w} - M_{\alpha_w})\alpha_{g_w}$$
$$+ (L_{\alpha_c}\alpha'_{g_c} + L_{\alpha_w}\alpha'_{g_w})$$

Since we will be interested in the peak response which may be expected to occur early, we will ignore the effect of damping. Thus, the term involving $\bar{\alpha}'$ will be ignored.

$$\bar{\alpha}'' - [M_q(T_\alpha + L_\alpha) + M_\alpha]\bar{\alpha} = (M_{\alpha_c} + M_q L_{\alpha_c})\alpha_{g_c} + (M_q L_{\alpha_w} - M_{\alpha_w})\alpha_{g_w} + (L_{\alpha_c}\alpha'_{g_c} + L_{\alpha_w}\alpha'_{g_w}) \tag{3.2}$$

Equation (3.2) has a complementary solution,

$$\bar{\alpha}(\tau)_c = A \sin \omega_o \tau + B \cos \omega_o \tau$$

where:

$$\omega_o = [-M_q(T_\alpha + L_\alpha) - M_\alpha]^{1/2}$$

Applying the initial conditions

$$\bar{\alpha}(\tau) = 0 \qquad \tau = 0$$
$$\bar{\alpha}'(\tau) = 1 \qquad \tau = 0$$

corresponding to a unit impulse, the complementary solution becomes

$$\bar{\alpha}(\tau)_c = \frac{1}{\omega_o} \sin \omega_o \tau$$

Applying Duhamel's integral, the particular solution of equation (3.2) is

$$\bar{\alpha}(\tau)_p = \int_0^\tau F(\tau_1)\, h(\tau - \tau_1)\, d\tau_1$$

where:

$$f(\tau_1) = (M\alpha_c + M_q L\alpha_c)\alpha_{g_c} + (M_q L\alpha_w - M\alpha_w)\alpha_{g_w} + (L\alpha_c \alpha'_{g_c} + L\alpha_w \alpha'_{g_w})$$

$$k(\tau-\tau_1) = \frac{1}{\omega_0} \sin \omega_0 (\tau-\tau_1)$$

Prior to the disturbance, the airplane was in straight and level flight. Thus, the complete solution of equation (3.2) is

$$\begin{aligned}\bar{\alpha}(\tau) &= \frac{1}{\omega_0}\int_0^\tau f(\tau_1) \sin \omega_0(\tau-\tau_1)\, d\tau_1 \\ &= \frac{1}{\omega_0}\Big\{\int_0^\tau \left[(M\alpha_c + M_q L\alpha_c)\alpha_g + L\alpha_c \alpha_g \delta(\tau)\right] \sin \omega_0(\tau-\tau_1)\, d\tau_1 \\ &\quad + \int_1^\tau \left[(M_q L\alpha_w - M\alpha_w)\alpha_g + L\alpha_w \alpha_g \delta(\tau-1)\right] \sin \omega_0(\tau-\tau_1)\, d\tau_1 \Big\}\end{aligned} \qquad (3.3)$$

where:

$$\alpha'_{g_c} = \delta(\tau)\alpha_g$$

$$\alpha'_{g_w} = \delta(\tau-1)\alpha_g$$

At time, $\tau < 1$,

$$\bar{\alpha}(\tau) = \frac{\alpha_g}{\omega_0^2}(M\alpha_c + M_q L\alpha_c)\cos \omega_0\tau + \frac{\alpha_g}{\omega_0} L\alpha_c \sin \omega_0 \tau - \frac{\alpha_g}{\omega_0^2}(M\alpha_c + M_q L\alpha_c) \qquad (3.4)$$

and, at time, $\tau \geqslant 1$,

$$\bar{\alpha}(\tau) = \frac{\alpha_g}{\omega_o^2}\left[M_{\alpha_c} + M_q L_{\alpha_c} - M_{\alpha_W}\cos\omega_o + M_q L_{\alpha_W}\cos\omega_o - \omega_o L_{\alpha_W}\sin\omega_o\right]\cos\omega_o\tau$$
$$+ \frac{\alpha_g}{\omega_o^2}\left[\omega_o L_{\alpha_c} - M_{\alpha_W}\sin\omega_o + M_q L_{\alpha_W}\sin\omega_o + \omega_o L_{\alpha_W}\cos\omega_o\right]\sin\omega_o\tau$$
$$+ \frac{\alpha_g}{\omega_o^2}\left(-M_{\alpha_c} - M_q L_{\alpha_c} + M_{\alpha_W} - M_q L_{\alpha_W}\right) \tag{3.5}$$

Then the solutions for $\bar{\theta}$ and $\bar{\gamma}$ may be obtained, making use of equations (2.19) and (2.20). Since we are interested in the normal acceleration, we can determine $\bar{\gamma}'$ directly from equation (2.19a).

3.2 Feedback Control of Control Surface

From equation (2.20a),

$$\bar{\theta}'' = \bar{\alpha}'' + (T_\alpha + L_\alpha)\bar{\alpha}' + \left[L_{\alpha_c}\alpha'_{gc} + L_{\alpha_W}\alpha'_{gW}\right]$$

the control deflections are given by

$$\delta_c = G(\bar{\alpha} + \alpha_{gs})$$

where:

G = gain of the system

$\alpha_{gs} = \alpha_g u(\tau - \tau_s)$ at $\tau_s = -\frac{\ell_s}{\ell}$

By substitution into equation (2.20b),

$$\bar{\alpha}'' + (T_\alpha + L_\alpha - M_q)\bar{\alpha}' - [M_q(T_\alpha + L_\alpha) + M_\alpha + M_{\alpha_c} G]\bar{\alpha} =$$
$$L_{\alpha_c} \alpha'_{g_c} + L_{\alpha_W} \alpha'_{g_W} + (M_q L_{\alpha_c} + M_{\alpha_c})\alpha_{g_c}$$
$$+ (M_q L_{\alpha_W} - M_{\alpha_W})\alpha_{g_W} + M_{\alpha_c} G \alpha_{g_s}$$

Since we will be interested in the peak response which may be expected to occur early, we will ignore the effect of damping, thus, the term involving $\bar{\alpha}'$ will be ignored.

$$\bar{\alpha}'' - [M_q(T_\alpha + L_\alpha) + M_\alpha + M_{\alpha_c} G]\bar{\alpha} = L_{\alpha_c}\alpha'_{g_c} + L_{\alpha_W}\alpha'_{g_W}$$
$$+ (M_q L_{\alpha_c} + M_{\alpha_c})\alpha_{g_c}$$
$$+ (M_q L_{\alpha_W} - M_{\alpha_W})\alpha_{g_W}$$
$$+ M_{\alpha_c} G \alpha_{g_s} \tag{3.6}$$

Equation (3.6) has a complementary solution

$$\bar{\alpha}(\tau)_c = A \sin \omega_0 \tau + B \cos \omega_0 \tau$$

where:

$$\omega_0 = [-M_q (T_\alpha + L_\alpha) - M_\alpha - M_{\alpha_c} G]^{1/2}$$

Applying the initial conditions

$$\bar{\alpha}(\tau) = 0 \qquad \tau = 0$$

$$\bar{\alpha}(\tau) = 1 \qquad \tau = 0$$

corresponding to a unit impulse, the complementary solution becomes

$$\bar{\alpha}(\tau)_c = \frac{1}{\omega_0} \sin \omega_0 \tau$$

Applying Duhamel's integral, the particular solution of equation (3.6) is

$$\bar{\alpha}(\tau)_p = \int_{\tau_s}^{\tau} f(\tau_1)\, h(\tau-\tau_1)\, d\tau_1$$

where:

$$\begin{aligned} f(\tau_1) = {} & L_{\alpha_c} \alpha'_{g_c} + L_{\alpha_w} \alpha'_{g_w} \\ & + (M_{\dot{\theta}} L_{\alpha_c} + M_{\alpha_c}) \alpha_{g_c} \\ & + (M_{\dot{\theta}} L_{\alpha_w} - M_{\alpha_w}) \alpha_{g_w} \\ & + M_{\alpha_c} G \alpha_{g_s} \end{aligned}$$

$$h(\tau-\tau_1) = \frac{1}{\omega_0} \sin \omega_0 (\tau-\tau_1)$$

Prior to the disturbance, the airplane was in straight and level flight. Thus, the complete solution of equation (3.6)

$$\begin{aligned} \bar{\alpha}(\tau) = \frac{1}{\omega_0} \Big\{ & \int_{\tau_s}^{\tau} M_{\alpha_c} G \alpha_g \sin \omega_0 (\tau-\tau_1)\, d\tau_1 \\ & + \int_0^{\tau} \left[(M_{\dot{\theta}} L_{\alpha_c} + M_{\alpha_c}) \alpha_g + L_{\alpha_c} \alpha_g\, \delta(\tau) \right] \\ & \quad \sin \omega_0 (\tau-\tau_1)\, d\tau_1 \\ & + \int_1^{\tau} \left[(M_{\dot{\theta}} L_{\alpha_w} - M_{\alpha_w}) \alpha_g + L_{\alpha_w} \alpha_g\, \delta(\tau-1) \right] \\ & \quad \sin \omega_0 (\tau-\tau_1)\, d\tau_1 \end{aligned} \tag{3.7}$$

where:

$$\alpha'_{g_c} = \delta(\tau)\,\alpha_g$$

$$\alpha'_{g_w} = \delta(\tau-1)\,\alpha_g$$

At time, $\tau < 1$,

$$\bar{\alpha}(\tau) = \frac{\alpha_g}{\omega_0^2}\left[M_{\alpha_c} + M_q L_{\alpha_c} - M_{\alpha_c} G \cos\omega_0\tau_s\right]\cos\omega_0\tau$$

$$+ \frac{\alpha_g}{\omega_0^2}\left[\omega_0 L_{\alpha_c} + M_{\alpha_c} G \sin\omega_0\tau_s\right]\sin\omega_0\tau$$

$$- \frac{\alpha_g}{\omega_0^2}\left(M_{\alpha_c} + M_q L_{\alpha_c} - M_{\alpha_c} G\right)$$

At time, $\tau \geqslant 1$,

$$\bar{\alpha}(\tau) = \frac{\alpha_g}{\omega_0^2}\Big[M_{\alpha_c} + M_q L_{\alpha_c} - M_{\alpha_w}\cos\omega_0 + M_q L_{\alpha_w}\cos\omega_0$$

$$- \omega_0 L_{\alpha_w}\sin\omega_0 - M_{\alpha_c} G\cos\omega_0\tau_s\Big]\cos\omega_0\tau$$

$$+ \frac{\alpha_g}{\omega_0^2}\Big[\omega_0 L_{\alpha_c} - M_{\alpha_w}\sin\omega_0 + M_q L_{\alpha_w}\sin\omega_0$$

$$+ \omega_0 L_{\alpha_w}\cos\omega_0 + M_{\alpha_c} G\sin\omega_0\tau_s\Big]\sin\omega_0\tau$$

$$+ \frac{\alpha_g}{\omega_0^2}\left[-M_{\alpha_c} - M_q L_{\alpha_c} + M_{\alpha_w} - M_q L_{\alpha_w} + M_{\alpha_c} G\right]$$

Then the solutions for $\bar{\theta}$ and $\bar{\gamma}$ may be obtained, making use of equations (2.19) and (2.20). Since we are interested in the normal acceleration, we can determine $\bar{\gamma}'$ directly from equation (2.19a)

3.3 Numerical Example

Consider a proposed Douglas Supersonic Transport System (5) (Figure 3.1) Velocity is computed as the proposed constant sub-sonic descent speed. The gust velocity is assumed to be 29.3 feet per second. Calculations are based on measurements from the scaled drawing. The gain, G, and the distance to the sensor from the control surface will be varied.

Gross weight = 393,000 lbs.

U_o = 550 ft/sec

U_g = 29.3 ft/sec

Altitude = 15,000 ft

α_g = .0533

AR = 2.31 - aspect ratio

$C_{L\alpha_W}$ = 2.64 rad. - wing lift - curve slope $\delta C_{LW}/\delta\alpha$

$C_{L\alpha_C}$ = 2.64 rad. - control surface - lift-curve slope $\partial C_{Lc}/\partial\alpha_c$

$C_{L\alpha}$ = 2.91 rad. - airplane - lift-curve slope $\partial C_L/\partial\alpha$

C_{mq} = $-.0475$ rad. - damping in pitch

$C_{m\alpha}$ = $-.338$ rad. - $\partial C_m/\partial\alpha$

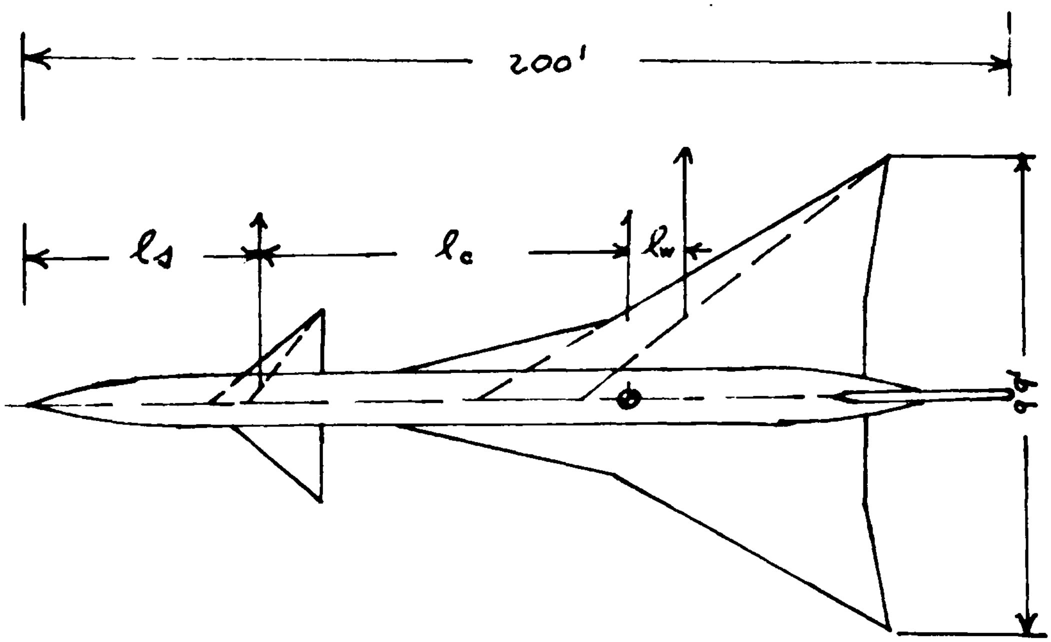

Figure 3.1 Douglas Transport

S_w = 3800 square feet - wing area

S_c = 348 square feet - control surface area

c_w = 48.3 feet - mean aerodynamic chord, wing

c_c = 19.9 feet - mean aerodynamic chord, control surface

ℓ_w = 14.0 feet - distance from center of gravity to wing aero-dynamic center

ℓ_c = 76.75 feet - distance from center of gravity to control surface aero-dynamic center

ℓ = 90.75 feet - $\ell_c + \ell_w$

ℓ_s = 37.5 feet - distance from sensor to control surface aero-dynamic center

k = 38.2 feet - radius of gyration

ρ = .001496 slugs/cubic foot - density of air

Evaluating the parameters:

$T_\alpha \ll 1$ and will have negligible effect on the angle of attack and the normal acceleration.

L_α = .0642

M_q = -.00559

M_α = -.0399

L_{α_c} = .0051

L_{α_w} = .0582

M_{α_c} = .02305

M_{α_w} = .0459

With controls fixed, we have no gain, G. Solving for $\bar{\alpha}_{MAX}$ with ℓ_s = 37.5 ft.

The frequency, ω_o, is equal to .1998.

Solving equation (3.4),

$$\bar{\alpha}_{MAX} = .00009$$

Solving equation (3.5),

$$\bar{\alpha}_{MAX} = .064$$

Solving for the rate of change in flight path angle:

At time, $\tau < 1$.

$$\bar{\gamma}' = L_\alpha \bar{\alpha} + L_{\alpha_c} \alpha_g$$

$$= .000274$$

At time, $\tau \geqslant 1$,

$$\bar{\gamma}' = L_\alpha \bar{\alpha} + L_{\alpha_c} \alpha_g + L_{\alpha_W} \alpha_g$$

$$= .007482$$

We note that the rate of change in flight path angle will be greater at a time $\tau \geqslant 1$.

In order to determine the rate of change in flight path angle by varying G and ℓ_s, we may then set up our equations for ω_o and $\bar{\alpha}$ (τ) as functions of the gain, G, and the distance to the sensor from the aero-dynamic center of the control surface, ℓ_s.

$$\omega_o = \left[3.994\ (10^{-2}) - 2.305\ (10^{-2})\ G \right]^{\frac{1}{2}}$$

$$\bar{\alpha}(\tau) = \frac{5.33(10^{-2})}{\omega_o^2}\Big\{\Big[2.302(10^{-2}) - 4.59(10^{-2})\cos\omega_o - 3.25(10^{-4})\cos\omega_o - \omega_o(5.82)10^{-2}\sin\omega_o - 2.305(10^{-2})\,G\cos\omega_o\tau_s\Big]\cos\omega_o\tau$$
$$+\Big[\omega_o\,5.1(10^{-3}) - 4.59(10^{-2})\sin\omega_o - 3.25(10^{-4})\sin\omega_o + \omega_o 5.82(10^{-2})\cos\omega_o + 2.305(10^{-2})\,G\sin\omega_o\tau_s\Big]\sin\omega_o\tau$$
$$+\Big[2.32(10^{-2}) + 2.305(10^{-2})\,G\Big]\Big\}$$

Fixing ℓ_s at 0, 37.5, and 68 and varying G, Figure 3.2 is obtained. $\bar{\gamma}_o'$ is the rate of change in flight path angle with control surface fixed; $\bar{\gamma}'$, with feedback control to the control surface.

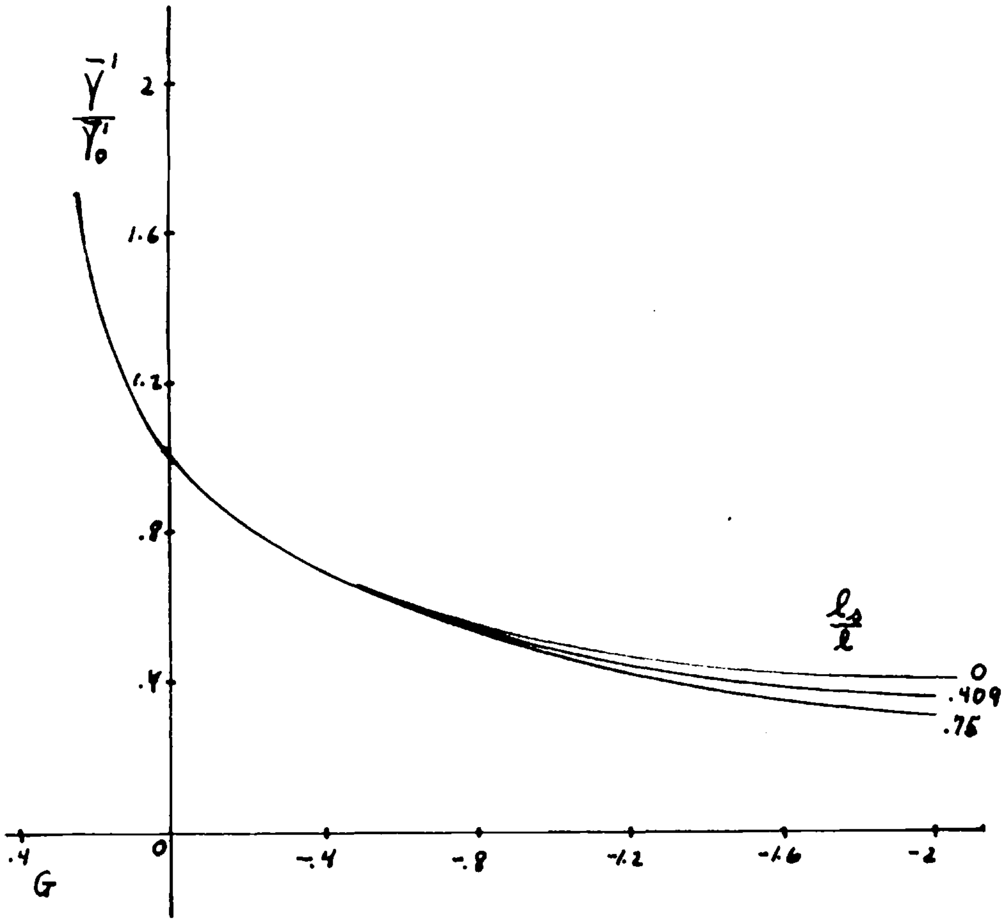

Figure 3.2 Rate of Change in Flight Path Angle, Varying Feedback Control

CHAPTER IV

CONCLUSIONS

The theory has been developed for the behavior of an aircraft with a forward mounted control actuated by an angle of attack sensor. The results of the numerical example permit certain conclusions on the alleviation of the gust-induced normal accelerations. As may be seen in Figure 3.2, a negative gain on the feedback control results in a reduction of the normal acceleration to as little as .4 of the value for no control. It may be reasoned physically that the negative gain causes the aircraft to pitch in the direction resulting in a decrease of angle of attack, and in a reduction of the peak response. It might be expected that a positive gain would have the opposite effect, which is verified by Figure 3.2. The effects of varying the location of the sensing device are small, considering locations forward of the control. As might be expected, there is some advantage in moving the sensor forward from the control surface.

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