A NOISE MODULATED RADAR ALTIMETER

by

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ABSTRACT

There are errors and ambiguities which are inherent in conventional radar altimeters. These errors and ambiguities can, for the most part, be attributed to the periodicity of the transmitted signal. With this in mind, the practicality of a system which uses random noise as the modulating signal is explored.

If two samples of a random signal, which differ only by a displacement in time, are correlated, the resulting correlation signal will be a function of the time displacement. In the proposed noise modulated altimeter, a sample of the transmitted signal is averaged with an echo signal which is the transmitted signal delayed by the amount of time required for the transmitted signal to reach the earth and return. If the value of the correlation function of the noise used as the modulating signal has a known correspondence to the time displacement between the two samples, the time displacement can be found. Altitude can then

be found through its relation to the velocity of the transmitted signal and the time displacement.

Certain errors peculiar to the noise modulated system are of particular interest. These are error due to finite time averaging and error due to varying the time displacement during averaging. The first of these decreases as the averaging time is increased while the second increases as the averaging time is increased. Using this information a theoretically optimum system can be found for a given set of specifications.

ACKNOWLEDGMENT

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CHAPTER 1
INTRODUCTION

The purpose of this thesis is to design a radar altitude measuring system which uses random noise as the modulating signal. Before discussing this system, a consideration of conventional systems may be useful in pointing out some of the problems to be solved. In general, all radar ranging systems determine range by measuring the time required for a transmitted signal to reach the target and return. In principle this is a very simple arrangement because we know that the transmitted wave in free space travels at the velocity of light. Thus by multiplying the velocity of light be one half the time required for the transmitted signal to reach the target and return, the range to the target is indicated.

1.1. Conventional Radar Altimeters

Radar ranging systems can be divided into two general classes, those which transmit a pulsed signal and those which transmit a continuous signal[1]. In those systems which transmit a pulsed signal, the usual approach is for the transmitter to trigger the sweep of the indicator oscilloscope every time a pulse is transmitted. When the echo of this pulse is received, a pip or displacement of the sweep is caused to appear on the indicator. The displacement of the pip from the starting point of the sweep is the indication of range. To illustrate how some systems determine and indicate range, consider the oscilloscope presentation represented in Figure 1-1. The transmitted signal appears as a rectangular pulse at the beginning of the sweep. The echo is represented by the triangular pulse. The range markers are smaller pulses which are superimposed on the sweep by an internal timing circuit at regular predetermined intervals. Each interval represents a specified increment of range. Thus the target range can be determined by counting the number of range markers between the beginning of the sweep and the echo pip. Such a system as this can be made as accurate as necessary by adjusting the pulse width and the pulse repetition rate of the transmitted signal. From the representation in Figure 1-1, it is obvious that trouble will be encountered at very short ranges. For any practical system to work, the transmitted pulses must contain enough power to produce an echo. This requires that the pulses have some finite duration. The power being transmitted will in all cases be much greater than the power of the echos. Therefore, echos cannot be received while a pulse is being transmitted. For targets which have an echo time less than the duration of the transmitted pulse, no echo can be indicated. In practical pulsed systems this means that no altitudes of less than several hundred feet can be indicated.

Radar ranging systems which transmit continuously can be characterized by a simple frequency modulated system. If the transmitted signal is modulated by a waveform similar to that of Figure 1-2a[2], then each cycle of the frequency modulated wave will differ from the preceding wave by some small increment of frequency. Figure 1-2a shows a linear modulating signal. This could just as easily be a sine wave if a few simple modifications were made to the following analysis. In one scheme of operation the difference in frequency between the frequency being transmitted at some reference time, t_o, and the time, $t_o + \Delta t$, at which the echo of the reference frequency returns to the receiver is sought. To accomplish this both signals, $f_o + \Delta f$, which is being transmitted at $t_o + \Delta t$, and f_o, which is being received at $t_o + \Delta t$, are impressed simultaneously on a detector. The output of the detector is the beat frequency, Δf, which is represented in Figure 1-2b. Since the magnitude of the beat frequency is determined entirely by the time required for the transmitted signal to reach the earth and return, a frequency meter, calibrated in units of altitude, could be used to indicate altitude.

Certain ambiguities are inherent in the system which is modulated by a periodic function. If the delay due to altitude is greater than a period of the modulating signal then obviously an incorrect altitude is indicated. This usually means that it is necessary to change the modulating frequency and the scale of the indicator when going through large changes in altitude. Another type of error is discussed by Hansen[3].

This is a fixed error which is due to a quantizing effect of the frequency meter which is used as the indicator. The error is inversely proportional to the total frequency swing of the modulating signal. At low altitudes and at normal operating frequencies, the maximum fixed error is about two meters.

Elimination of the errors and limitations referred to above implies first that the system must transmit continuously. This would overcome the difficulties encountered with the pulsed radar system. Secondly, the modulating signal should be non-periodic. This would eliminate the errors encountered in the conventional continuous transmission system.

1.2. A Noise Modulated System

One means of obtaining a non-periodic modulating signal is through the use of random noise. Assume that the transmitted signal of an altimeter is modulated by a random process. If the altitude is not very great, then the modulating noise signal would not have time to make any great change before the reflected signal returned. If these two signals are multiplied and averaged, then some signal value relatively close to the mean square value of the modulating signal could be expected. However, if the altitude were greater, then the modulating signal would have made a greater change and the average or expected value of the two signals would be a smaller percentage of the mean square value of the modulating signal. This averaging or finding the expected value of two signals is known as correlation. In this case both signals are the same except for a delay so that the correlation function obtained is the autocorrelation function. If the two signals were not the same the result would be the crosscorrelation function. For example, if the return signal were averaged with the derivative of the transmitted signal, a crosscorrelation would result. Since the value of these correlation functions varies with the delay of one signal with respect to the other, it seems likely that an altimeter using random noise as the modulating signal with some sort of correlating device in the receiver would be feasible.

A block diagram of the proposed system is shown in Figure 1-3. A noise generator feeds Gaussian noise into a filter which is designed such that its output has a specified output autocorrelation function. Part of the filter output is then used to modulate a carrier signal. The modulated signal is then transmitted toward a target from which it is reflected. The reflected signal then appears at the receiver as a delayed version of the transmitted signal. If $x(t)$ is the transmitted signal and τ is the delay corresponding to twice the distance to the target, then the received signal is $x(t + \tau)$. Another part of the filter output or a part of the modulated signal is operated on by $g(t)$ and $h(t)$ and then fed directly to the correlator. This signal is $y(t)$. At the correlator $x(t + \tau)$ and $y(t)$ are multiplied and averaged. If the autocorrelation function of the modulation signal has been properly chosen and the correct operations performed on y(t), then the output of the correlator will be a monotone function of the delay, τ. In general x(t) could be either amplitude of angle modulated.

Two general modes of operation will be considered. In the first of these correlation is performed upon the modulated signals. In the second, post detection correlation is performed. Within each of these general modes the results at the correlator output can be made to appear in any one of several forms depending on g(t), h(t), and the correlator. In particular, g(t) could be either a short circuit or a differentiator and h(t) could be either a short circuit or a delay, λ, which would be controlled by the correlator output. Finally, the correlator could have an output which would be the correlation function between x(t) and y(t), $\phi_{xy}(\tau)$, or the anticorrelation function between x(t) and y(t), $\phi_{xy}(0) - \phi_{xy}(\tau)$. The results of these variations in system parameters will be considered in detail in Chapter 2.

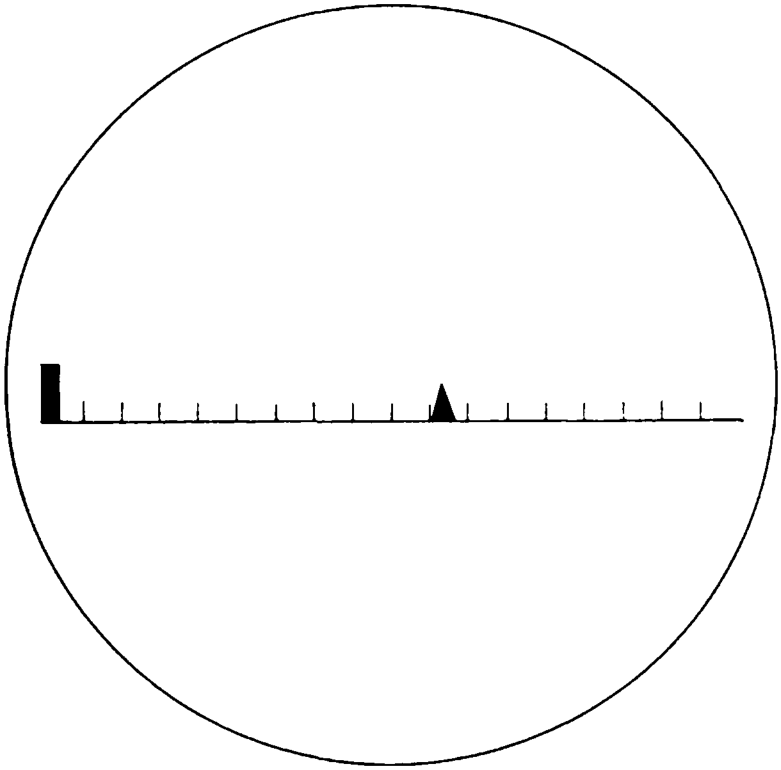
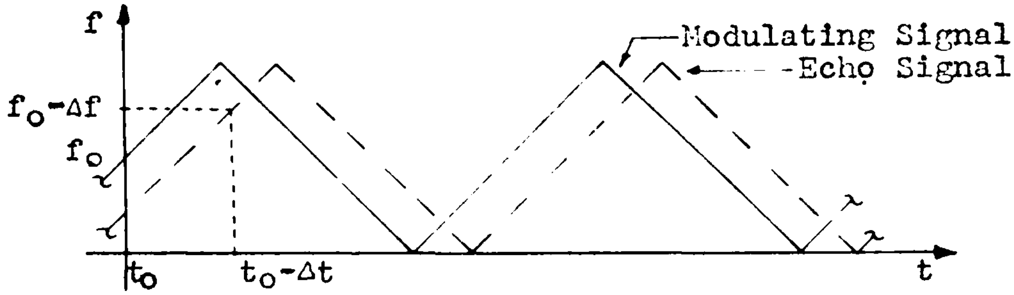
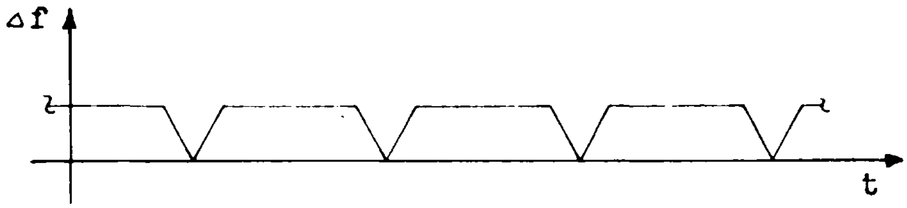


Figure 1-1. Type A Radar Range Indicator



(a) Modulating and Echo Signals



(b) Difference Frequency

Figure 1-2. Range Indicating Signals

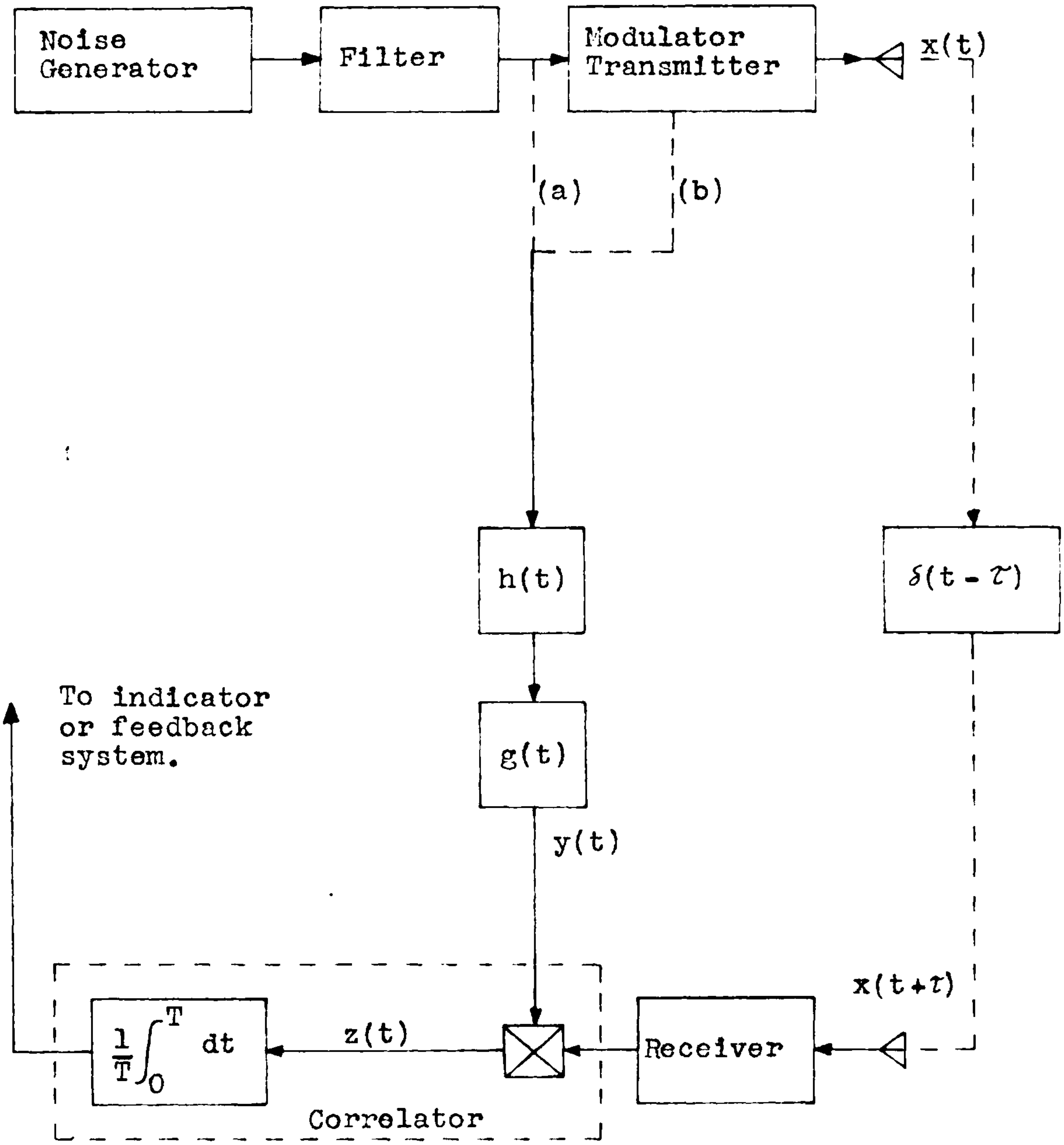


Figure 1-3. Block Diagram, Noise Modulated System

(a) Systems Correlating Unmodulated Signals

(b) Systems Correlating Modulated Signals

CHAPTER 2
SYSTEM CORRELATION FUNCTIONS

In Figure 1-3 it can be seen that the correlator output signal must be determined. This signal could be used in either one of two ways. It can be used to determine altitude directly or it can be used to control a variable delay which continuously trys to match the delay of the return signal. In this chapter the correlator output for several different but related schemes of operation will be derived.

There are three basic schemes of operation; autocorrelation, anticorrelation, and crosscorrelation. The correlator output for the autocorrelation scheme is the expected value of the altitude delayed signal and another version of the same signal which is either undelayed or delayed by some known amount. The anticorrelation scheme has a correlator output which is the same as that of the autocorrelation scheme but subtracts this from the mean square value of the modulating signal. In the crosscorrelation scheme the altitude delayed signal is averaged with the derivative of the transmitted signal.

If the correlation is to be performed on modulated signals, then the different types of modulation must be considered as it is not likely that the correlator output would be the same for different modulation processes. For this reason correlation functions will be derived for amplitude and frequency modulated signals. When the signals are demodulated or detected the results will be the same, at least theoretically, regardless of the modulating process. Of course the noise components may be different and this might result in frequency modulation being more advantageous than amplitude modulation or vice versa. Correlator outputs for the system where detection is used will also be derived.

All derivations will be made initially assuming that the signal to be correlated with the altitude delayed signal have been delayed by a controlled amount, $\lambda(t)$. Both the controlled delay and the altitude delay, $\tau(t)$, will be assumed constant. Investigation of the effects of varying these delays will be made in Chapter 5. In addition, all system elements; multiplier, integrator, delay, and differentiator; are assumed ideal. The effects of imperfections and noise will be considered in Chapter 5. All signals are normalized with respect to both phase and amplitude. This can be done without loss of generality. Phase variations of signal inputs to the correlator are absorbed by the delay factors. Certain amplitude variations will be considered where necessary.

2.1. AM Systems

Consider first the system shown in Figure 2-1. If amplitude modulation is assumed then consideration must be given to anything which might affect the amplitude of the signals to be correlated. For this reason, an amplitude factor, k, is impressed on the return signal. This amplitude factor can be expected to have a relatively constant mean value, however it will have a random component due to variations to be discussed later. The modulating signal, n(t), is a random process having an autocorrelation function, $\phi_{nn}(\tau)$. It is assumed ergodic and independent of other signals or noise in the system. The remainder of the system is self-explanatory.

$$f(t) = n(t) \cos w_c t$$

$$x(t) = kn(t + \tau) \cos w_c(t + \tau)$$

$$y(t) = n(y + \lambda) \cos w_c(t + \lambda)$$

$$z(t) = x(t)\, y(t)$$

$$= kn(t + \tau)\, n(t + \lambda) \cos w_c(t + \tau) \cos w_c(t + \lambda)$$

$$\phi_{xy}(\tau - \lambda) = E\left[z(t)\right]$$

$$= k\, E\left[n(t + \tau)\, n(t + \lambda)\right]\ E\left[\cos w_c(t + \tau) \cos w_c(t + \lambda)\right]$$

The expected value of z(t) can be expressed as the product of expected values of different signal components because the noise and the carrier are mutually independent[4].

$$\phi_{xy}(\tau - \lambda) = k\, \phi_{nn}(\tau - \lambda) \lim_{T \to \infty} \frac{1}{T} \int_0^T \cos w_c(t + \tau) \cos w_c(t + \lambda)dt$$

$$= \frac{k\, \phi_{nn}(\tau - \lambda)}{2} \cos w_c(\tau - \lambda) \frac{w_c}{2\pi} \int_0^{2\pi/w_c} \cos w_c (2t + \tau - \lambda)dt$$

The limit operation has been dropped because the average of a periodic function is the same whether the function is averaged over one or over an infinite number of periods.

Thus:

$$\phi_{xy}(\tau - \lambda) = \frac{k\,\phi_{nn}(\tau - \lambda)}{2} \cos w_c(\tau - \lambda)$$

$$= \phi_{ff}(\tau - \lambda) \qquad (2.1)$$

This is the correlator output signal for the AM autocorrelation scheme.

The anticorrelation scheme is shown in Figure 2-2. This is the same as the autocorrelation scheme in all but one respect. The correlator output, $\psi_{xy}(\tau - \lambda)$, is the same as the correlator output of the autocorrelation scheme but subtracted from the zero delay or maximum value of the autocorrelation function[5]. The anticorrelation scheme could be termed the inverse of the autocorrelation scheme since where the autocorrelation system has a maximum output the anticorrelation system will have a minimum. The correlator output for the anticorrelation scheme can be stated by considering the derivation of Equation 2.1, as:

$$\psi_{xy}(\tau - \lambda) = \phi_{xy}(0) - \phi_{xy}(\tau - \lambda)$$

$$= \frac{k}{2}\left[\phi_{nn}(0) - \phi_{nn}(\tau - \lambda)\right] \cos w_c(\tau - \lambda) \qquad (2.2)$$

Figure 2-3 is a representation of the crosscorrelation scheme. Here a differentiation has been introduced into the path of the internally delayed signal. Otherwise signals in this system are the same as in the autocorrelation system.

$$f(t) = n(t) \cos w_c t$$

$$x(t) = kn(t + \tau) \cos w_c(t + \tau)$$

$$= kf(t + \tau)$$

$$y(t) = \frac{d}{dt} f(t + \lambda)$$

$$z(t) = x(t)\, y(t)$$

$$\phi_{xy}(\tau - \lambda) = E\left[z(t)\right]$$

$$= \lim_{T\to\infty} \frac{K}{T} \int_0^T f(t) \frac{d}{dt} f(t + \tau - \lambda)dt$$

Here the differentiation with respect to (t) can be changed to a differentiation with respect to $(\tau - \lambda)$. This allows the differentiation operation to be removed from within the integral.

$$\phi_{xy}(\tau - \lambda) = k \frac{d}{d(\tau - \lambda)} \lim_{T\to\infty} \frac{1}{T} \int_0^T f(t)\, f(t + \tau - \lambda)dt$$

$$= k \frac{d}{d(\tau - \lambda)} \phi_{ff}(\tau - \lambda) \tag{2.3}$$

The above derived correlation functions are the general expressions for the correlator output of noise amplitude modulated systems. More will be said of these functions in the next chapter after specific forms for $\phi_{nn}(\tau - \lambda)$ have been derived.

2.2. FM Systems

The same three schemes of operation will be considered using angle modulation. Specifically, frequency modulation is to be used. The two processes differ only by an integration of the modulating signal. It is assumed that the modulating signal will have a Gaussian amplitude distribution. The integral of a Gaussian signal is itself a Gaussian signal. Thus, for the purpose of this derivation, frequency and phase modulation are equivalent. Since the amplitude of the return signal carries no information, the factor, k, used in considering the amplitude modulated systems will be dropped.

Referring again to Figure 2-1, the correlator output signal for the frequency modulated autocorrelation scheme can be derived.

$$f(t) = \cos\left[w_c t + \theta(t)\right]$$

$$x(t) = \cos\left[w_c(t + \tau) + \theta(t + \tau)\right]$$

$$y(t) = \cos\left[w_c(t + \lambda) + \theta(t + \lambda)\right]$$

$$z(t) = x(t)\, y(t)$$

$$\begin{aligned}\phi_{xy}(\tau - \lambda) &= E\left[x(t)\, y(t)\right] \\ &= E\left\{\cos\left[w_c(t + \tau) + \theta(t + \tau)\right] \cos\left[w_c(t + \lambda) + \theta(t + \lambda)\right]\right\} \\ &= 1/2E\left\{\cos\left[2w_c t + w_c(\tau - \lambda) + \theta(t) + \theta(t + \tau - \lambda)\right]\right\} \\ &\quad + 1/2E\left\{\cos\left[w_c(\tau - \lambda) + \theta(t + \tau - \lambda) - \theta(t)\right]\right\}\end{aligned}$$

The first term on the right side of the above expression is the expected value of a periodic waveform with zero mean and is therefore zero.

Thus:

$$\begin{aligned}\phi_{xy}(\tau - \lambda) &= 1/2E\left\{\cos\left[w_c(\tau - \lambda) + \theta(t + \tau - \lambda) - \theta(t)\right]\right\} \\ &= 1/2E\left\{\mathrm{Re}\exp\left[jw_c(\tau - \lambda) + j\theta(t + \tau - \lambda) - j\theta(t)\right]\right\} \\ &= 1/2\mathrm{Re}\left[\exp jw_c\tau \; E\left\{\exp j\left[\theta(t + \tau - \lambda) - \theta(t)\right]\right\}\right]\end{aligned}$$

let $x = \theta$

$$\phi_{xy}(\tau - \lambda) = 1/2\mathrm{Re}\left[\exp \langle jw_c(\tau - \lambda)\rangle \; E\left\{\exp jx\right\}\right]$$

$$= 1/2\mathrm{Re}\left[\exp \langle jw_c(\tau - \lambda)\rangle \int_{-\infty}^{\infty} \exp jx\rho(x)\, dx\right]$$

$\rho(x)$ is the probability density function associated with $\left[\theta(t+\tau-\lambda) - \theta(t)\right]$. Since this is a function which is the result of linear operations on a function which has a Gaussian probability density function, it must also have a Gaussian probability density function. The probability of x can be stated as[6]:

$$\rho(x) = \frac{1}{\sqrt{2\pi}\,\sigma_x} \exp - \frac{x^2}{2\,\sigma_x^2}$$

The variance of x, σ_x^2, is defined[7] as:

$$\sigma_x^2 = E\left[(x - \bar{x})^2\right], \text{ where } \bar{x} \text{ is the expected value of x.}$$

From which:

$$\sigma_x^2 = E\left\{\theta^2(t) - 2\theta(t)\,\theta(\tau-\lambda) + \theta^2(t+\tau-\lambda)\right\} - E\left\{\theta(t+\tau-\lambda) - \theta(t)\right\}^2$$

$$= \phi_{\theta\theta}(0) - 2\phi_{\theta\theta}(\tau-\lambda) + \phi_{\theta\theta}(0)$$

If $\theta(t) = n(t)$

$$\sigma_x^2 = 2\left[\phi_{nn}(0) - \phi_{nn}(\tau-\lambda)\right]$$

Therefore:

$$\int_{-\infty}^{\infty} \exp(jx)\,\rho(x)dx = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_x} \exp(jx)\exp(-\frac{x^2}{2\,\sigma_x^2})dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left[\frac{-x^2 + 2jx\sigma_x^2 + \sigma_x^4}{2\,\sigma_x^2} - \frac{\sigma_x^4}{2\sigma_x^2}\right]dx$$

$$= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_x}\left[\exp \frac{-(x^2 - j\,\sigma_x^2)^2}{2\,\sigma_x^2}\right]\left[\exp - \frac{\sigma_x^2}{2}\right]dx$$

$$= \exp - \frac{\sigma_x^2}{2}$$

$$= \exp -\left[\phi_{nn}(0) - \phi_{nn}(\tau - \lambda)\right]$$

and:

$$\phi_{xy}(\tau - \lambda) = 1/2 \mathrm{Re}\left[\exp jw_c(\tau - \lambda)\right] \quad \exp -\left[\phi_{nn}(0) - \phi_{nn}(\tau - \lambda)\right]$$
$$= 1/2 \exp -\left[\phi_{nn}(0) - \phi_{nn}(\tau - \lambda)\right]$$
$$\left[\cos w_c(\tau - \lambda)\right] \qquad (2.4)$$

By analogy with the amplitude modulated systems, the correlation functions for the frequency modulated anticorrelation and autocorrelation schemes can be stated.

$$\psi_{xy}(\tau - \lambda) = 1/2 \left[\phi_{ff}(0) - \phi_{ff}(\tau - \lambda)\right] \qquad (2.5)$$

$$\phi_{xy}(\tau - \lambda) = 1/2 \frac{d}{d(\tau - \lambda)} \phi_{ff}(\tau - \lambda) \qquad (2.6)$$

The frequency modulated systems having these correlation functions are represented in Figures 2-2 and 2-3 respectively.

2.3. Systems with Post Detection Correlation

If distortion by the detector is ignored, amplitude and angle modulated systems will have identical correlator inputs and hence identical correlator outputs when detection of the return signal is performed prior to correlation. With post detection correlation, the correlator output signals are quite easily derived. With these systems it is unnecessary to take a sample of the transmitted signal to correlate with the return signal. For this purpose a sample of the noise source output could be used. This would eliminate the need for detecting more than one modulated signal and would greatly decrease extraneous noise and distortion in that channel to the correlator input.

The autocorrelation scheme with post detection correlation is represented in a simplified form by the block diagram of Figure 2-4. Here certain of the signals represented are more simply defined than in the previous derivations. The noise source again has an output, n(t), with autocorrelation function, $\phi_{nn}(\tau)$. The correlator input signals are now defined as:

$$x(t) = n(t + \tau)$$

$$y(t) = n(t + \lambda)$$

$$z(t) = x(t)\, y(t)$$

The correlator output is then:

$$\begin{aligned}\phi_{xy}(\tau - \lambda) &= E\left[z(t)\right] \\ &= E\left[n(t + \tau)\, n(t + \lambda)\right] \\ &= \phi_{nn}(\tau - \lambda) \qquad (2.7)\end{aligned}$$

The anticorrelation scheme is represented in Figure 2-5. The correlator output for this system is:

$$\psi_{xy}(\tau - \lambda) = \phi_{nn}(0) - \phi_{nn}(\tau - \lambda). \qquad (2.8)$$

The crosscorrelation scheme represented in Figure 2-6, has a correlator output that is:

$$\phi_{xy}(\tau - \lambda) = \frac{d}{d(\tau - \lambda)}\, \phi_{nn}(\tau - \lambda) \qquad (2.9)$$

Before any specific comments can be made about the systems for which correlator outputs have been derived, an expression for

$\phi_{nn}(\tau)$ must be obtained. This will be done in the next chapter.

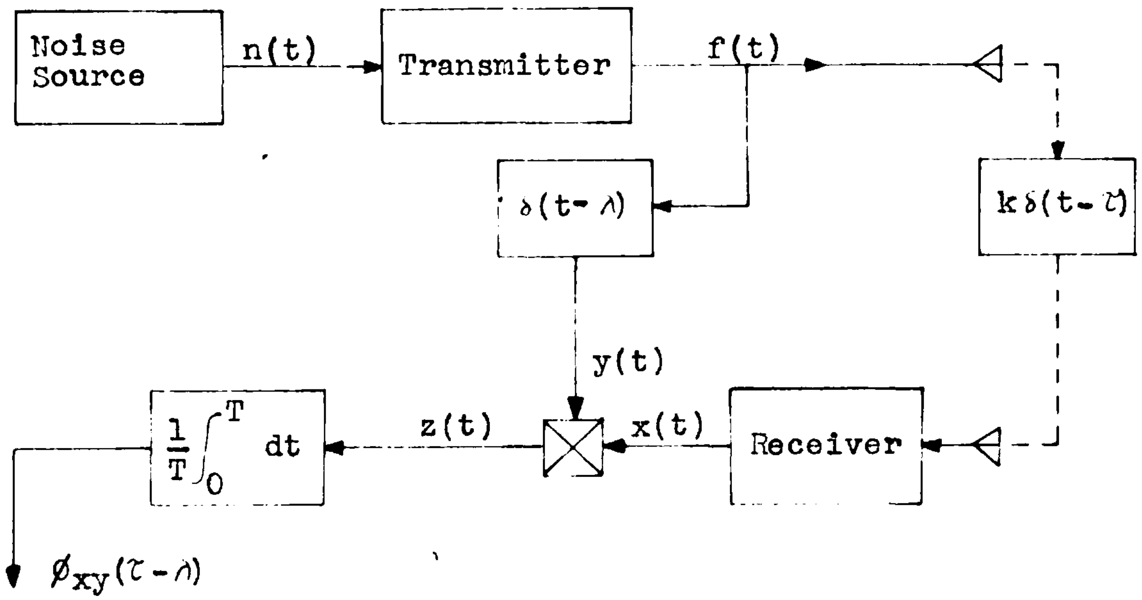


Figure 2-1. Autocorrelation Scheme

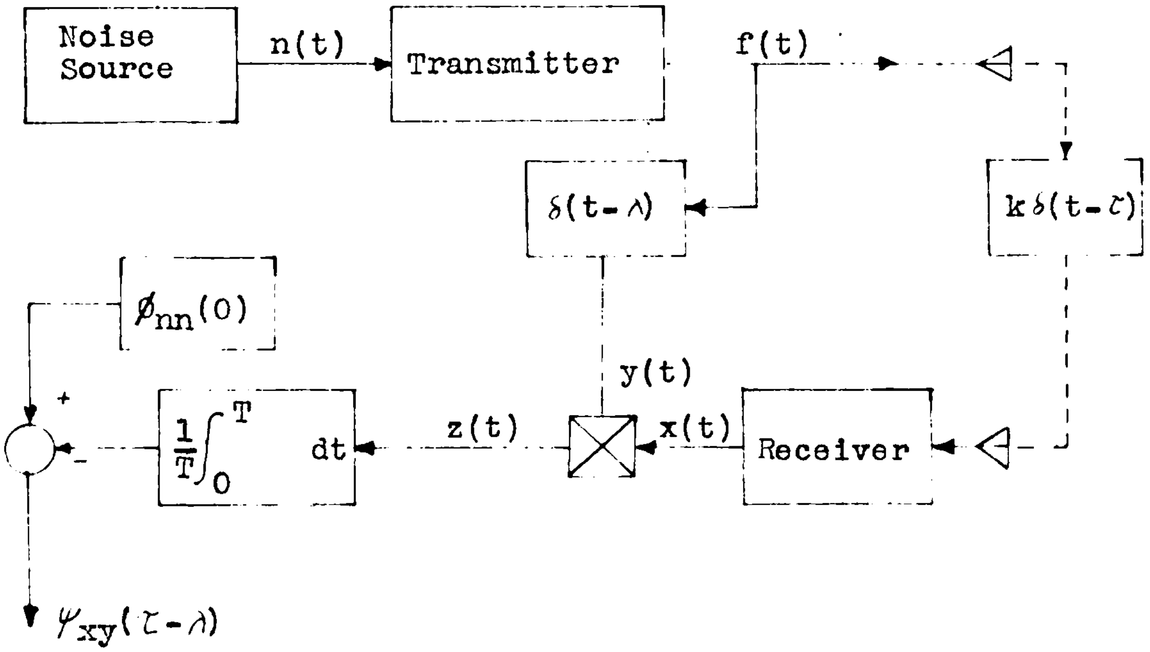


Figure 2-2. Anticorrelation Scheme

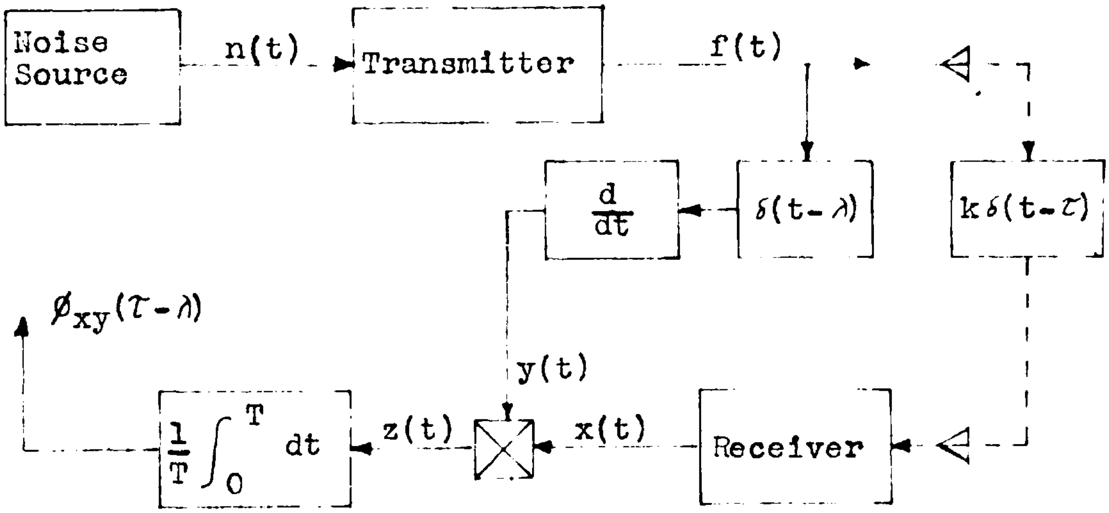


Figure 2-3. Crosscorrelation Scheme

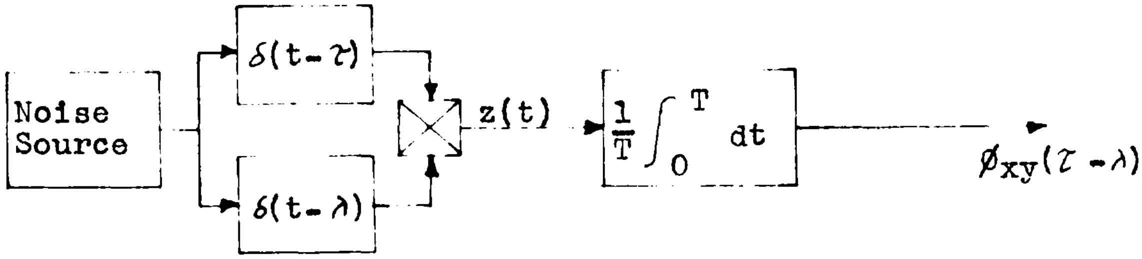


Figure 2-4. Autocorrelation Scheme With Post Detection Correlation

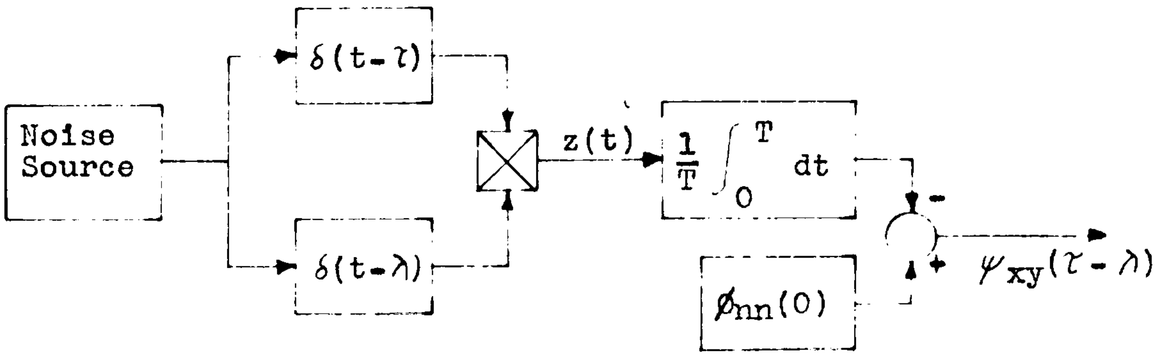


Figure 2-5. Anticorrelation Scheme With Post Detection Correlation

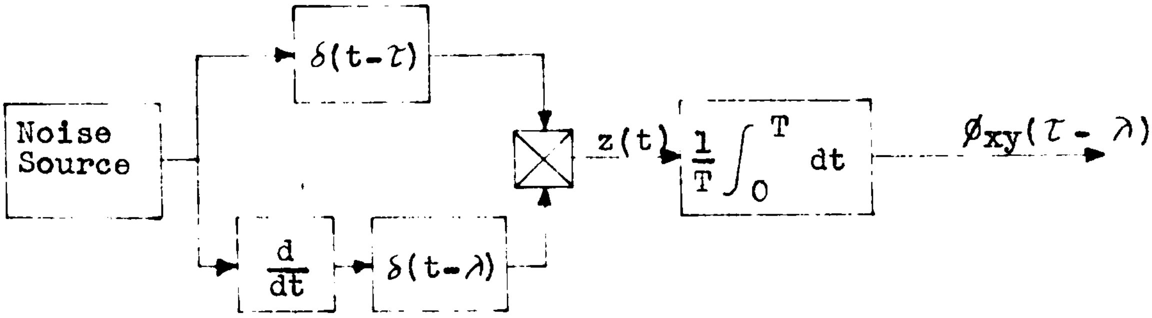


Figure 2-6. Crosscorrelation Scheme With Post Detection Correlation

CHAPTER 3
NOISE AUTOCORRELATION FUNCTIONS

In this chapter a means of obtaining noise with a specified autocorrelation function will be sought. Since Gaussian, or white, noise has an autocorrelation function which is just $N^2 \delta(\tau)$, it would seem that a reasonable way of arriving at a suitable noise autocorrelation function would be to filter white noise. The approach used here will be to start with an autocorrelation function having the desired characteristics and, assuming a white noise filter input, derive the filter which causes the output to have the specified autocorrelation function.

3.1. Properties of Autocorrelation Functions

For the purpose of approaching the problem intelligently and eliminating unnecessary calculations with functions which are not valid autocorrelation functions it is instructive to consider certain properties of autocorrelation functions[8]. Autocorrelation functions are even functions of delay. This means that it makes no difference whether a signal is averaged with a delayed version of itself or an advanced version of itself, the result will be the same if the magnitude of the advance is equal to the magnitude of the delay. As previously stated, the value of the autocorrelation function with no delay is greater than or equal to the value of the autocorrelation function for any other value of delay.

If one considers the type of system to be designed it is easily seen that certain other specific constraints must be placed on the noise autocorrelation function. The noise autocorrelation function must be a monotone decreasing function of delay. Without this condition there could be little advantage gained over the conventional radar altimeter where it is possible for a single output value to correspond to two or more different altitudes.

In the correlator output signals for the crosscorrelation systems of Chapter 2, a derivative of the noise autocorrelation function appeared in each case. Since the noise autocorrelation function is an even function of delay, its derivative must be an odd function. For this reason it would be desirable to have a noise filter output with an autocorrelation function which has a continuous derivative at zero delay. This condition is of course unnecessary for the noise autocorrelation function to be used with the autocorrelation and anticorrelation schemes.

3.2. Obtaining a Specified Autocorrelation Function

White noise is defined as noise which has a flat, or constant, spectral density for all frequencies[9]. Since the autocorrelation function is the inverse Fourier transform of the spectral density, white noise has an autocorrelation function which is $N^2 \delta(\tau)$. Thus the autocorrelation function of white noise has value only when there is no delay and is zero for all other values of delay. Such an autocorrelation function would be unsuitable for the proposed altimeter. However, if white noise is filtered then the autocorrelation function of the filter output can be made to have any correspondence with delay subject only to filter realizability. Such a scheme is shown in Figure 3-1. Here r(t) is the white noise input to the filter. The filter transfer function is h(t) and the output is n(t). Using the convolution integral[10] the filter output is seen to be:

$$n(t) = \int_0^{\infty} h(t_1)\, r(t - t_1)\, dt_1 .$$

The autocorrelation function of the filter output is defined as the expected value of n(t) n(t + τ) where τ is delay. Thus:

$$\begin{aligned} \phi_{nn}(\tau) &= E\left[n(t)\, n(t+\tau)\right] \\ &= E\left[\int_0^{\infty} h(t_1)\, r(t - t_1)dt_1 \int_0^{\infty} h(t_2)\, r(t + \tau - t_2)dt_2\right] \end{aligned}$$

Since E is an operator that is independent of t_1 and t_2, it can be moved inside the integral.

$$\begin{aligned} \phi_{nn}(\tau) &= \int_0^{\infty}\int_0^{\infty} h(t_1)\, h(t_2)\, E\left[r(t - t_1)\, r(t + \tau - t_2)\right] dt_1 dt_2 \\ &= \int_0^{\infty}\int_0^{\infty} h(t_1)\, h(t_2)\, \phi_{rr}(\tau + t_1 - t_2) dt_1 dt_2 \end{aligned} \qquad (3.1)$$

If (3.1) is transformed the spectral density, $\Phi_{nn}(s)$, of the filter output is obtained.

$$\begin{aligned} \Phi_{nn}(s) &= \int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} h(t_1)\, h(t_2)\, \phi_{rr}(\tau + t_1 - t_2)e^{-s\tau} dt_1 dt_2 d\tau \\ &= H(-s)\, H(s)\, \Phi_{rr}(s) \end{aligned} \qquad (3.2)$$

Since $\phi_{rr}(\tau) = \delta(\tau)$

And $\Phi_{rr}(s) = 1$

Then $\Phi_{nn}(s) = H(s)\ H(-s)$

Now consider a function, x(t), such that:

$$\phi_{xx}(\tau) = g(|\tau|)$$

Then

$$\Phi_{xx}(s) = \int_{-\infty}^{\infty} g(|\tau|)\ \exp(-s\,\tau)d$$
$$= \int_{0}^{\infty} g(\tau)\ \exp(s\,\tau)d\tau + \int_{0}^{\infty} g(\tau)\ \exp(-s\,\tau)d\tau$$

$$= G(-s) + G(s) \qquad (3.3)$$

Note here that H(-s) and G(-s) represent Fourier transforms of convergent functions of $(-\tau)$ rather than divergent functions of $(+\tau)$. Comparing (3.3) with (3.2), it can be seen that if $\phi_{nn}(\tau)$ is equal to $\phi_{xx}(\tau)$, then H(s) H(-s) can be expanded by partial fractions so that:

$$H(s)\ H(-s) = G(s) + G(-s).$$

Using the above equations then a filter transfer function can be found which has a specified output autocorrelation function when the input is white noise.

3.3. Examples

Considering the specifications previously discussed for autocorrelation functions, it would appear that the function shown in Figure 3-2 would be quite practical for either an autocorrelation or an anticorrelation system. The equation for this autocorrelation function is:

$$\phi_{nn}(\tau) = \frac{1}{T}(T - |\tau|) \qquad \tau < T$$

$$= 0 \qquad \text{elsewhere}$$

The spectral density is:

$$\Phi_{nn}(s) = \frac{\exp(sT) - \exp(-sT) - 2}{Ts^2}$$

$$= \frac{1 - \exp(sT)}{\sqrt{T}(-s)} \cdot \frac{1 - \exp(-sT)}{\sqrt{T}\, s}$$

and

$$H(s) = \frac{1 - \exp(-sT)}{\sqrt{T}\, s} \tag{3.5}$$

This filter is realizable, however, delay lines may be necessary to realize it physically. If linearity of the autocorrelation function with respect to τ is not too important, it might be possible to obtain an autocorrelation function which requires only a simple RC filter. One such autocorrelation function is shown in Figure 3-3. This autocorrelation function is represented by the function;

$$\phi_{nn}(\tau) = \exp(-w_o |\tau|) \tag{3.6}$$

The spectral density is:

$$\Phi_{nn}(s) = \frac{1}{w_o + s} + \frac{1}{w_o - s}$$

$$= \frac{2w_o}{(w_o + s)(w_o - s)}$$

Thus:

$$H(s)\,H(-s) = \frac{\sqrt{2w_o}}{w_o + s} \cdot \frac{\sqrt{2w_o}}{w_o - s}$$

$$H(s) = \frac{\sqrt{2w_o}}{w_o + s} \tag{3.7}$$

Equation 3.7 is a transfer function which corresponds to a simple RC filter with half power bandwidth w_o. The autocorrelation function from which this filter was derived meets all of the specifications previously set down except that its first derivative is not continuous at τ equal to zero. Therefore it would not be suitable for use with the crosscorrelation systems.

Figure 3-4 shows an autocorrelation function which is mathematically suitable for use with the crosscorrelation systems. Note that this is suitable for use with the other schemes of operation, but it is more complex and not so easy to manipulate. The autocorrelation function represented in Figure 3-4 has the equation:

$$\phi_{nn}(\tau) = \left[1 + w_o |\tau|\right] \exp(-w_o |\tau|) \tag{3.8}$$

The spectral density is:

$$\Phi_{nn}(s) = \frac{4w_o^{\,3}}{(w_o + s)^2\,(w_o - s)^2}$$

and:

$$H(s) = \frac{2w_o^{\,3/2}}{(w_o + s)^2} \tag{3.9}$$

The transfer function can be realized by a pair of RC filters in cascade.

The bandwidth of this filter, B_t, can be found from the "bandwidth-narrowing equation"[11] for cascaded Butterworth functions as:

$$B_t = w_o(2^{1/2} - 1)^{1/2}$$

$$\doteq 0.642\, w_o \tag{3.10}$$

The derivative of the autocorrelation function of equation 3.7, is shown in Figure 3-5, and can be written as:

$$\frac{d\, \phi_{nn}(\tau)}{d\, \tau} = -w_o^{\,2}\, \tau\, \exp(-w_o|\tau|) \tag{3.11}$$

In summary it appears that it will be most convenient to use two noise autocorrelation functions in further investigations. One, which is given in equation 3.6, will be used when considering either the autocorrelation or anticorrelation schemes. The other is given in equation 3.8, and will be used when considering the crosscorrelation scheme.

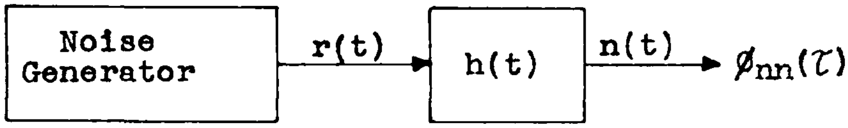


Figure 3-1. Noise Source

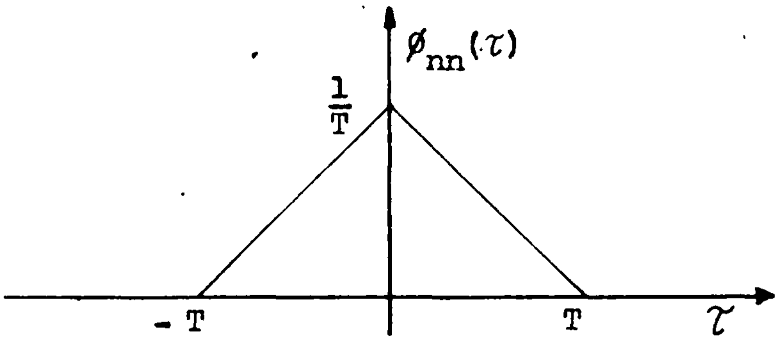


Figure 3-2. Noise Autocorrelation Function

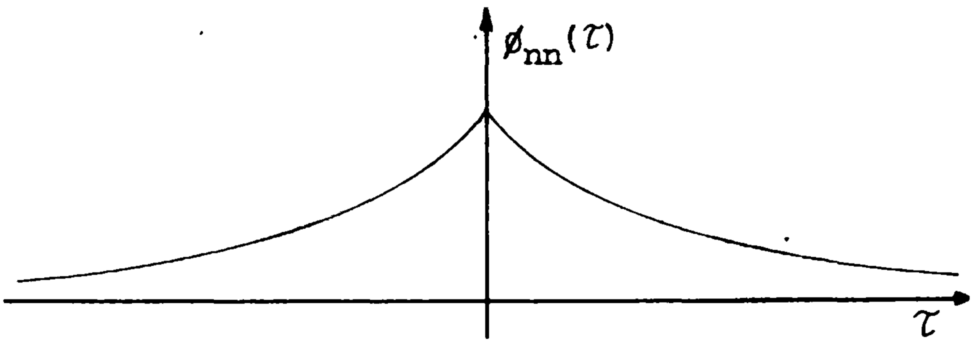


Figure 3-3. Noise Autocorrelation Function
(Equation 3.6)

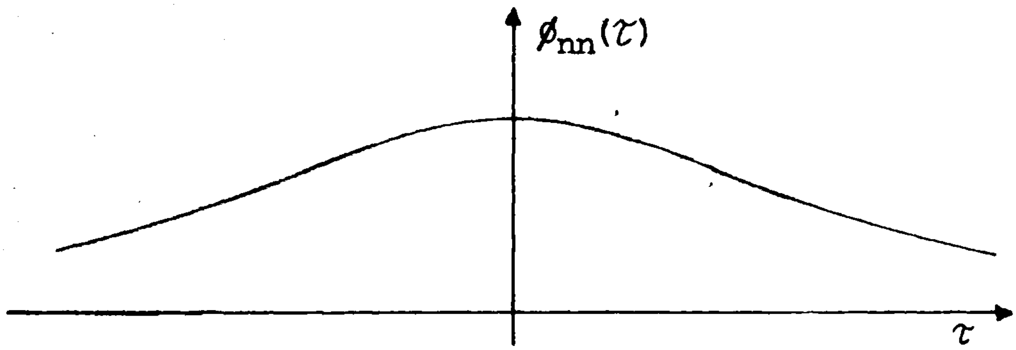


Figure 3-4. Noise Autocorrelation Function (Equation 3.8)

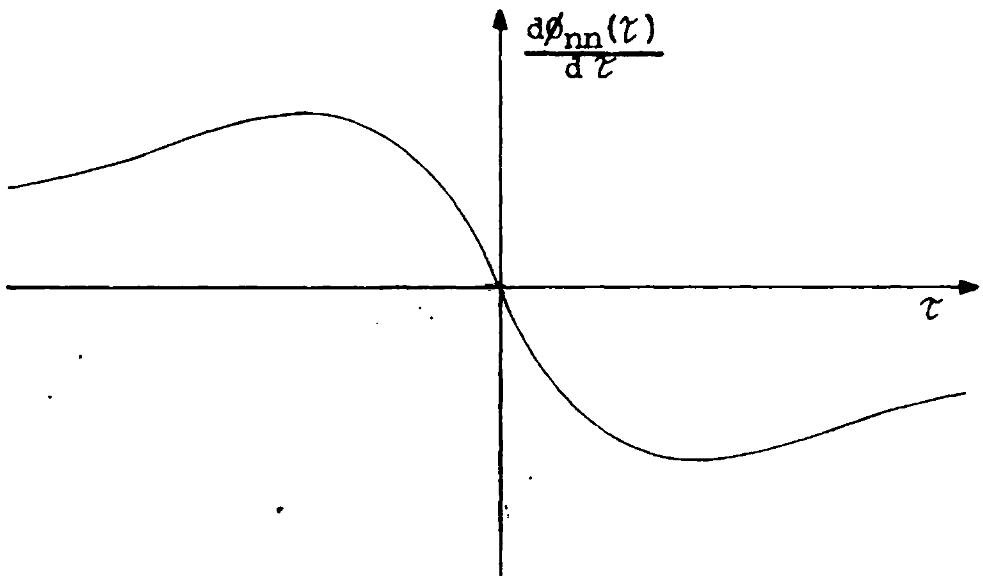


Figure 3-5. Derivative Of Noise Autocorrelation Function (Equation 3.11)

CHAPTER 4
COMPARISON OF IDEAL SYSTEMS

In this chapter the correlation functions derived in Chapter 2 will be investigated and compared. Using the noise autocorrelation functions found in Chapter 3 an attempt will be made to determine which of the schemes are the most practical. Any which are determined to be unsuitable will be discarded so that it will be necessary to consider practical problems only with those schemes which could ultimately provide a workable system. To give some basis for comparison it is assumed that the system must be capable of measuring altitudes from zero to 45,000 meters. Since the altitude delay, $\tau(t)$, represents the time required for the radar wave to traverse the distance to the earth twice, the maximum delay can be written as:

$$\begin{aligned}\tau_{max} &= 2(d)/c \\ &\doteq 2(45,000)/3 \times 10^8 \\ &\doteq 300 \text{ microseconds}\end{aligned}$$

The minimum delay is of course zero. As is normal with radar systems, the carrier frequency will be assumed to be in the megacycle per second range.

4.1. Systems Without Feedback

Consider first the case where λ is zero. Since τ varies directly with altitude and the correlator output is a function of τ, a measuring device could be attached directly to the correlator to indicate altitude. Although those schemes which correlate modulated signals do not have correlator outputs which are monotone decreasing functions of τ, it is possible that they may have this property over a sufficiently wide range to permit their use. The correlator output must be monotone decreasing, or increasing, over some specified range of τ so that a particular correlator output value will not correspond to more than one altitude.

The amplitude modulated systems where λ is always zero and which do not demodulate the return signal prior to correlation can be eliminated without the necessity of plotting the correlator output. Each of these has a correlator output which depends upon an attenuation factor, k, as well as delay and carrier frequency. If k were predictable then it might be possible to determine altitude merely by measuring the return signal strength. However, k depends not only on distance but also on the qualities of the reflecting surface and atmospheric conditions. Thus k is a factor which has two parts, one which is deterministic and another which is, for the most part, random. If an altimeter using the return signals available to these schemes of operation were to have a reasonable accuracy then perfect gain control of the return signal would be required. This would in turn introduce other noise problems. Since certain of the other schemes can provide accurate altitude determinations without any unusual amount of gain control, the AM schemes without feedback will be ignored in further considerations.

As was stated earlier, amplitude variations carry no information in the frequency modulation schemes. It may be possible then to obtain altitude measurements from one of the frequency modulation schemes where λ is zero at all times. The expression of the correlator output for the autocorrelation scheme is:

$$\phi_{xy}(\tau) = \exp\left[-\phi_{nn}(0) - \phi_{nn}(\tau)\right] \cos w_c \tau$$

From Chapter 3:

$$\phi_{nn}(\tau) = \exp w_o \tau$$

The correlator output is then:

$$\phi_{xy}(\tau) = \exp\left[\exp(-w_o \tau) - 1\right] \cos w_c \tau \qquad (4.1)$$

Similarly, the correlator output for the anticorrelation scheme is:

$$\psi_{xy}(\tau) = 1 - \exp\left[\exp(-w_o \tau) - 1\right] \cos w_c \tau \qquad (4.2)$$

The noise autocorrelation function to be used with the crosscorrelation scheme is:

$$\phi_{nn}(\tau) = \left[1 - w_o \tau\right] \exp(-w_o \tau)$$

The correlator output is:

$$\phi_{xy}(\tau) = \frac{d}{d\tau} \left[\exp\left[\phi_{nn}(\tau) - \phi_{nn}(0)\right] \cos w_c \tau \right]$$

$$= -w_o \exp(-w_o \tau) \exp\left[\exp(-w_o \tau) - 1\right] \cos w_c \tau$$
$$- w_c \exp\left[\exp(-w_o \tau) - 1\right] \sin w_c \tau \qquad (4.3)$$

Since $w_c >> w_o$, (4.3) can be approximated by its second term:

$$\phi_{xy}(\tau) \cong -w_c \exp\left[\exp(-w_o \tau) - 1\right] \sin w_c \tau \qquad (4.4)$$

Notice in the above expressions that the absolute value signs about τ have been omitted. This is possible because τ can never be negative since it is impossible to have negative altitude. The correlation functions of (4.1), (4.2), and (4.4) are plotted versus $w_o \tau$ in Figures 4-1, 4-2, and 4-3, respectively. In these plots it is assumed that $w_c = 10w_o$.

A study of Figures 4-1, 4-2, and 4-3, reveals that the functions represented are not suitable as output functions for an altimeter. In the plots the ratio of w_c to w_o was ten. In a practical system this ratio is more likely to be on the order of 1000 or more. Otherwise the bandwidth of the transmitted signal would be too great. If the ratio of w_c to w_o were 1000, the period of the oscillations shown in the graphs would be decreased by 100. This would obviously lead to impractical correlator output functions.

4.2. Systems With Feedback

Consider now the tracking systems where λ (t) attempts to match continuously τ(t) and there is no detection. For the amplitude modulation schemes the attenuation factor, k, is still present, however its importance is somewhat diminished. The tracking systems try to achieve the condition, $(\tau - \lambda) = 0$, which means that they try to obtain a maximum, a minimum, or zero for the autocorrelation, the anticorrelation, and the crosscorrelation schemes respectively. Although the value of the maximum or the minimum may change the tracking system will still seek the maximum or the minimum. The expressions for the correlator outputs for the three amplitude modulation schemes and the three frequency modulation schemes are given below. The variable parameter is now $(\tau - \lambda)$ which is represented by (δ). Since (δ) may take on both positive and negative values, the magnitude signs must be retained in these expressions. The noise autocorrelation function to be used with the autocorrelation and anticorrelation schemes is:

$$\phi_{nn}(\delta) = \exp(-w_o|\delta|),$$

and for the crosscorrelation schemes:

$$\phi_{nn}(\delta) = \left[1 - w_o|\delta|\right] \exp(-w_o|\delta|).$$

The correlator outputs are:

AM autocorrelation;

$$\phi_{xy}(\delta) = \frac{k}{2}\phi_{nn}(\delta) \cos w_c(\delta)$$

$$= \frac{k}{2}\left[\exp(-w_o|\delta|)\right] \cos w_c \delta \qquad (4.5)$$

AM anticorrelation;

$$\psi_{xy}(\delta) = \frac{k}{2}\left[1 - \left[\exp(-w_o|\delta|)\right] \cos w_c \delta\right] \qquad (4.6)$$

AM crosscorrelation;

$$\phi_{xy}(\delta) \cong \frac{k\,w_c}{2}\left[(1 + w_o|\delta|)\exp(-w_o|\delta|)\right]\sin w_c\,\delta \tag{4.7}$$

FM autocorrelation;

$$\phi_{xy}(\delta) = \exp\left[\exp(-w_o|\delta|) - 1\right]\cos w_c\,\delta \tag{4.8}$$

FM anticorrelation;

$$\psi_{xy}(\delta) = 1 - \exp\left[\exp(-w_o|\delta|) - 1\right]\cos w_c\,\delta \tag{4.9}$$

FM crosscorrelation;

$$\phi_{xy}(\delta) \cong -w_c\exp\left[\exp(-w_o|\delta|) - 1\right]\sin w_c\,\delta \tag{4.10}$$

The above functions are plotted in Figures 4-4 through 4-9. Again it was assumed that the ratio of w_o to w_c was ten. It is obvious from a study of these graphs that the same comments made of the functions plotted in Figures 4-1, 4-2, and 4-3, are applicable here. Near $\delta = 0$, the correlation functions could be represented by their sinusoidal components only without any great error. For this reason the noise modulation gives no great advantage over systems modulated by a periodic function. It appears then that it will be necessary to go to a system in which post detection correlation is used.

4.3. Post Detection Correlation Systems

So far as the ideal correlator output signal is concerned it makes no difference whether the system is amplitude or frequency modulated. Since there are no gain factors to consider here it is not necessary to consider the tracking and nontracking systems separately.

For those systems where $\lambda(t)$ is zero for all time it is necessary only to let $\lambda(t) = 0$, in the expressions below. The correlator output signals for those systems which use post detection correlation are:

autocorrelation;

$$\phi_{xy}(\delta) = \phi_{nn}(\delta)$$

$$= \exp(-w_o|\delta|) \qquad (4.11)$$

anticorrelation;

$$\psi_{xy}(\delta) = \phi_{nn}(0) - \phi_{nn}(\delta)$$

$$= 1 - \exp(-w_o|\delta|) \qquad (4.12)$$

crosscorrelation;

$$\phi_{xy}(\delta) = \frac{d}{d\delta}\,\phi_{nn}(\delta)$$

$$= -w_o^2\delta\,\exp(-w_o|\delta|) \qquad (4.13)$$

Here again $\delta = (\tau - \lambda)$, and the noise autocorrelation functions used for a particular system are the same as before. The above correlation functions are plotted versus $w_o\delta$ in Figures 4-10, 4-11, and 4-12. For the case where $\lambda(t) = 0$, let $\delta = \tau$, and consider only the portions of the graphs representing positive $w_o\delta$.

Consider first the case where $\lambda(t)$ is zero for all time. All three of the functions plotted above could be used to indicate altitude. As an example, if τ_{max} is 300 microseconds, then the expressions representing the correlator outputs can be used to obtain a suitable value of w_o. It is obviously necessary that there be a significant difference in values of the correlation function for small changes in $w_o \tau$. One way of assuring that this condition is met is to use only those parts of the graphs for which $w_o \tau$ is less than three in the case of the autocorrelation and the anticorrelation schemes, and less than one in the crosscorrelation scheme. Thus a suitable w_o for the autocorrelation and anticorrelation schemes is:

$$\begin{aligned} w_o \tau &= 3 \\ w_o &= 3/300 \times 10^{-6} \\ &= 10 \times 10^{3} \end{aligned} \qquad (4.14)$$

For the crosscorrelation scheme a suitable w_o is:

$$\begin{aligned} w_o \tau &= 1 \\ w_o &= 1/300 \times 10^{-6} \\ &= 3.33 \times 10^{3} \end{aligned} \qquad (4.15)$$

These values will be used in later examples. It is unnecessary to calculate w_o on the basis used above for the tracking systems since they always operate on that part of the curves in the neighborhood of $w_o \tau = 0$. Another criterion for determining w_o for the tracking systems might be determined by noise and bandwidth.

The stated purpose of this chapter was to decide which schemes of operation might ultimately lead to a practical altimeter. As a result of observations made earlier in this chapter further investigations will be made only for those schemes which yield the correlator outputs of equations (4.11), (4.12), and (4.13).

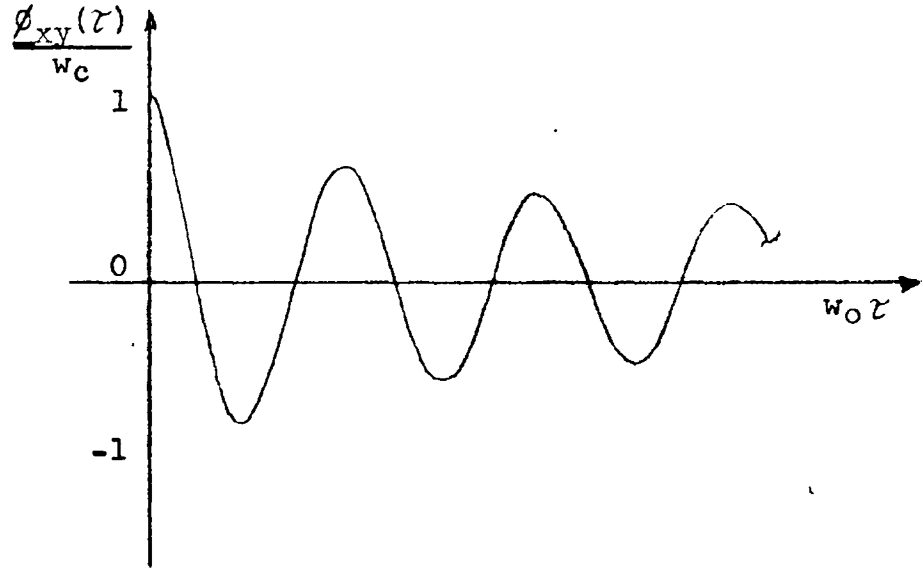


Figure 4-1. Correlator Output, FM Autocorrelation Scheme

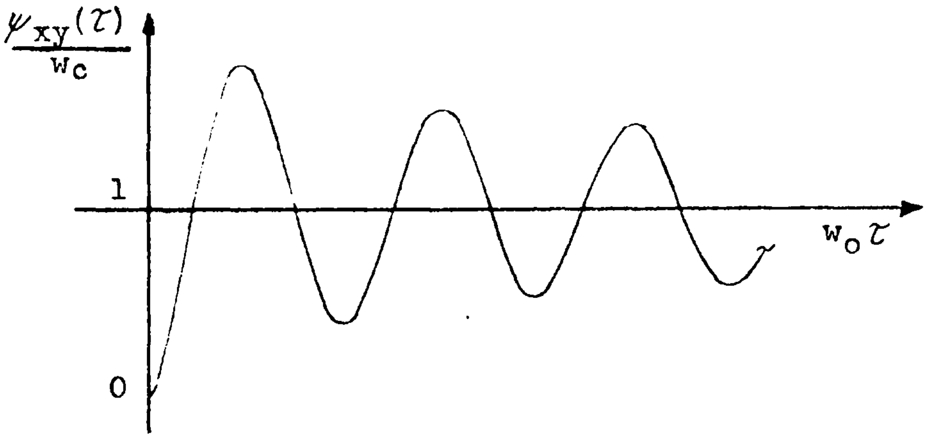


Figure 4-2. Correlator Output, FM Anticorrelation Scheme

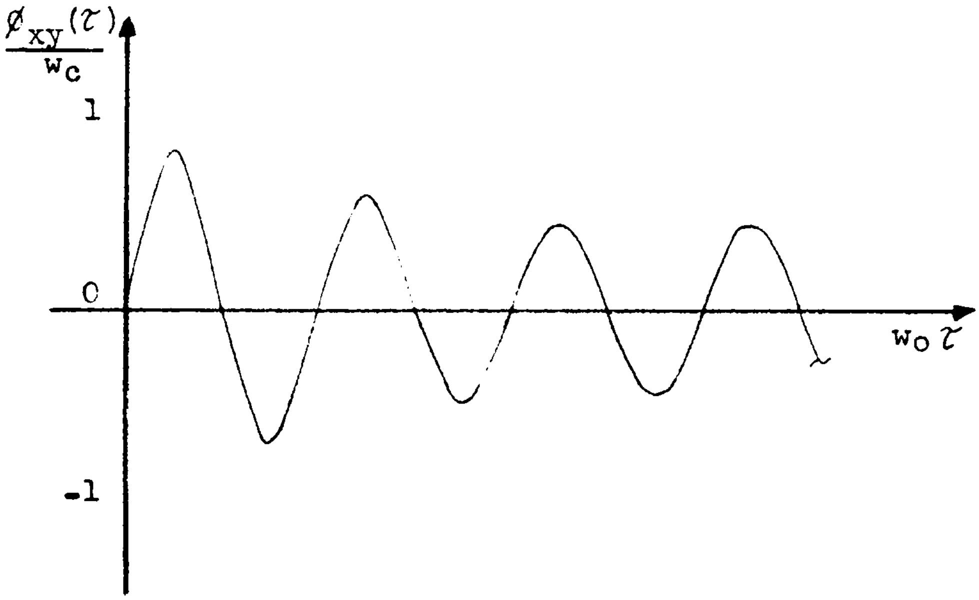


Figure 4-3. Correlator Output, FM Crosscorrelation Scheme

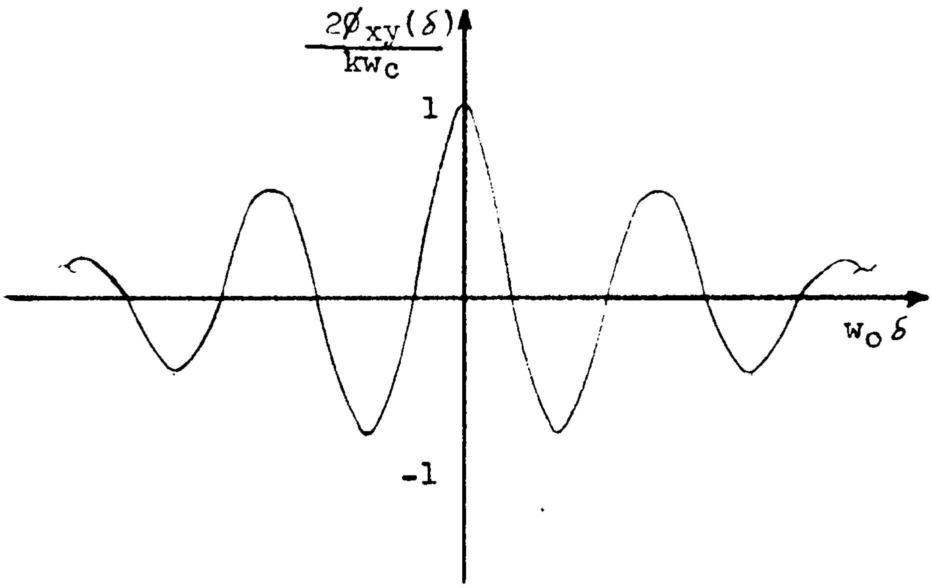


Figure 4-4. Correlator Output, AM Autocorrelation Scheme With Internal Delay

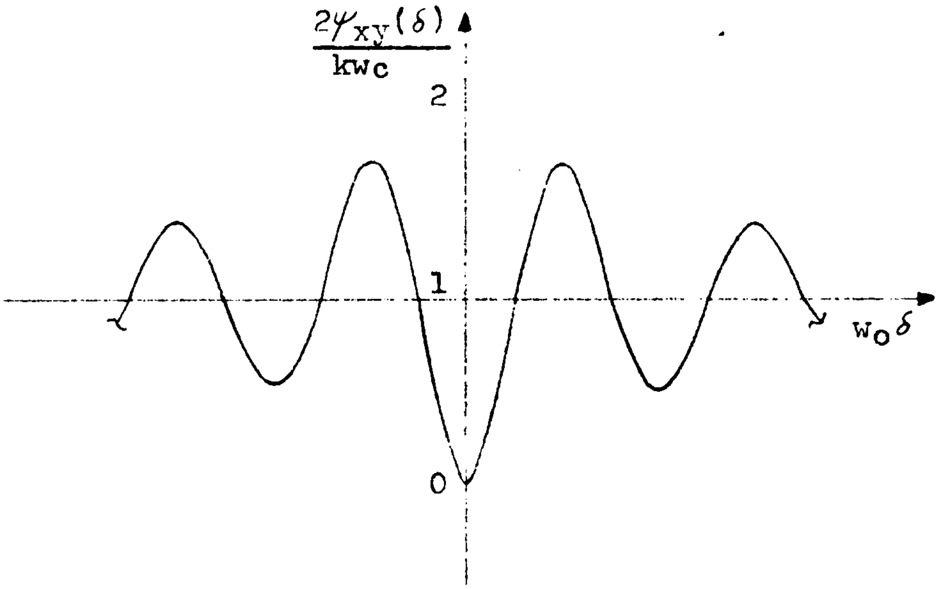


Figure 4-5. Correlator Output, AM Anticorrelation Scheme With Internal Delay

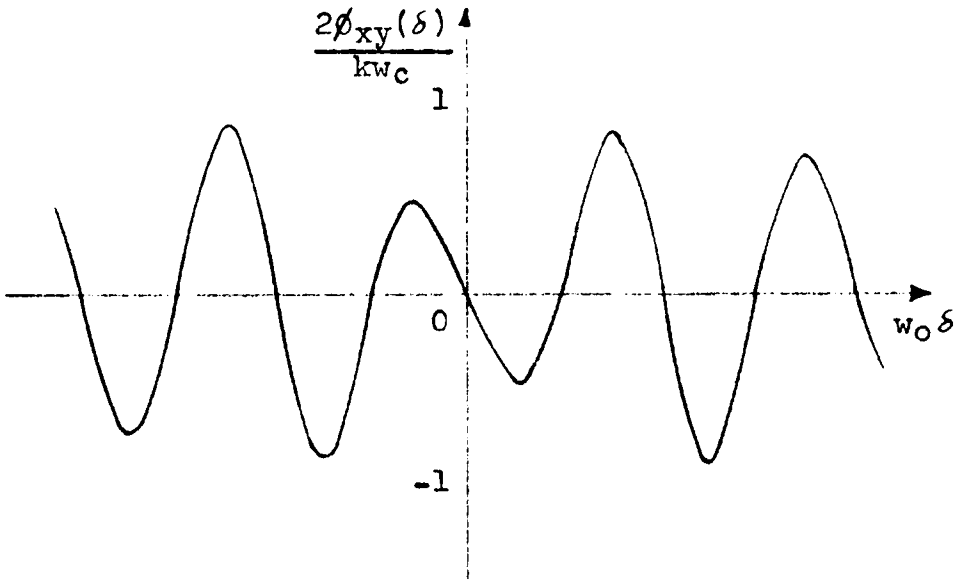


Figure 4-6. Correlator Output, AM Crosscorrelation Scheme With Internal Delay

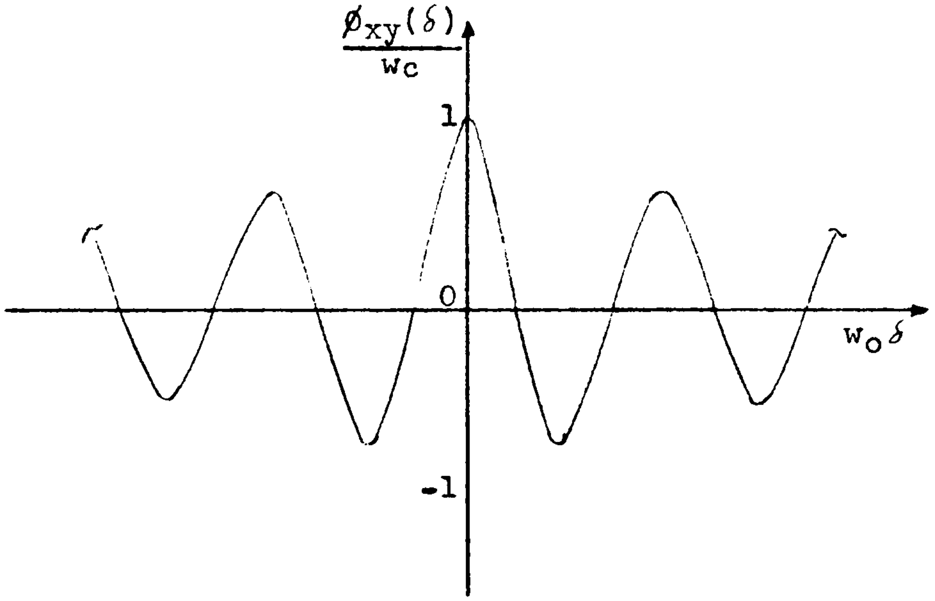


Figure 4-7. Correlator Output, FM Autocorrelation Scheme With Internal Delay

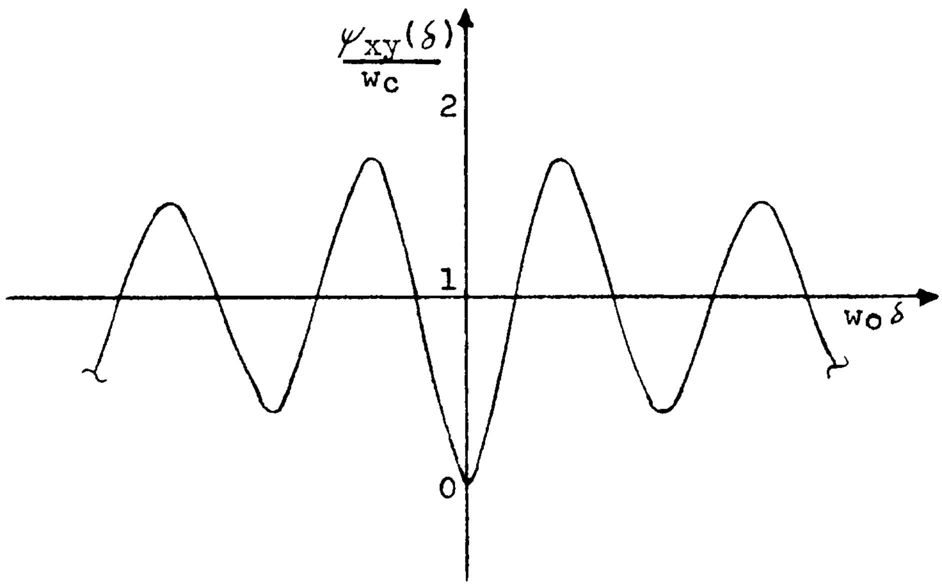


Figure 4-8. Correlator Output, FM Anticorrelation Scheme With Internal Delay

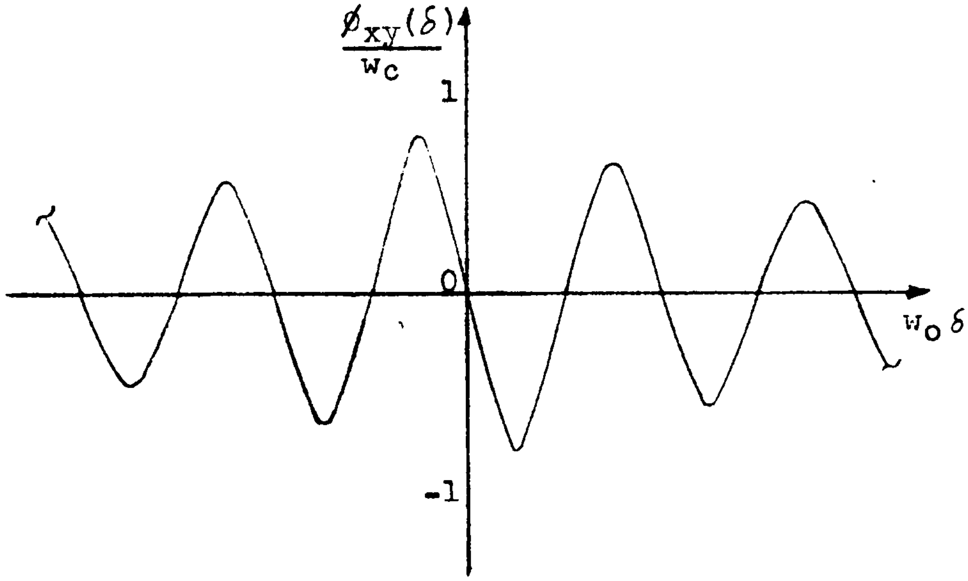


Figure 4-9. Correlator Output, FM Crosscorrelation Scheme With Internal Delay

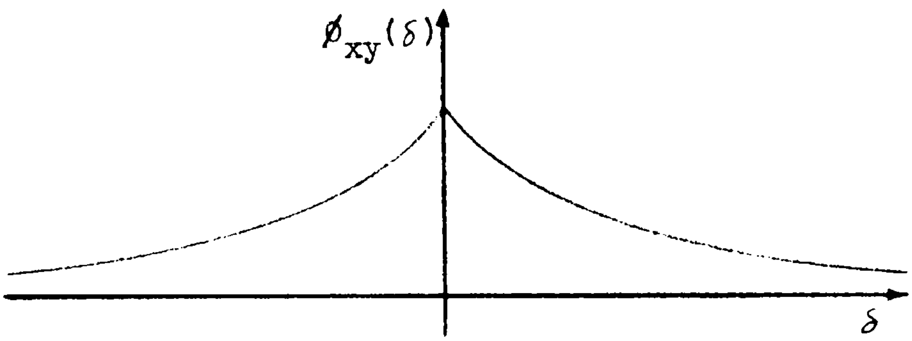


Figure 4-10. Correlator Output, Autocorrelation Scheme With Post Detection Correlation

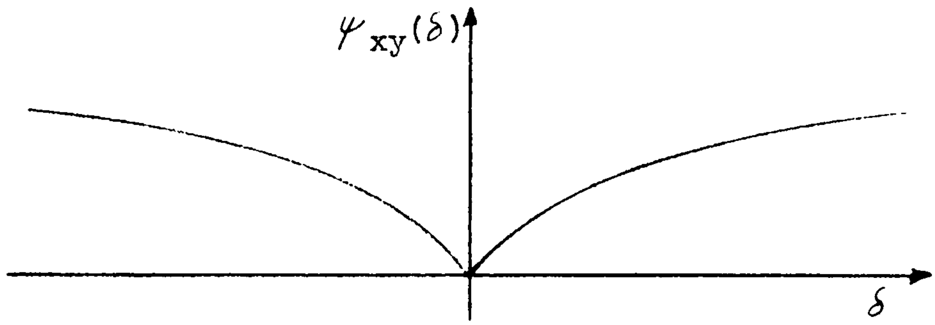


Figure 4-11. Correlator Output, Anticorrelation Scheme With Post Detection Correlation

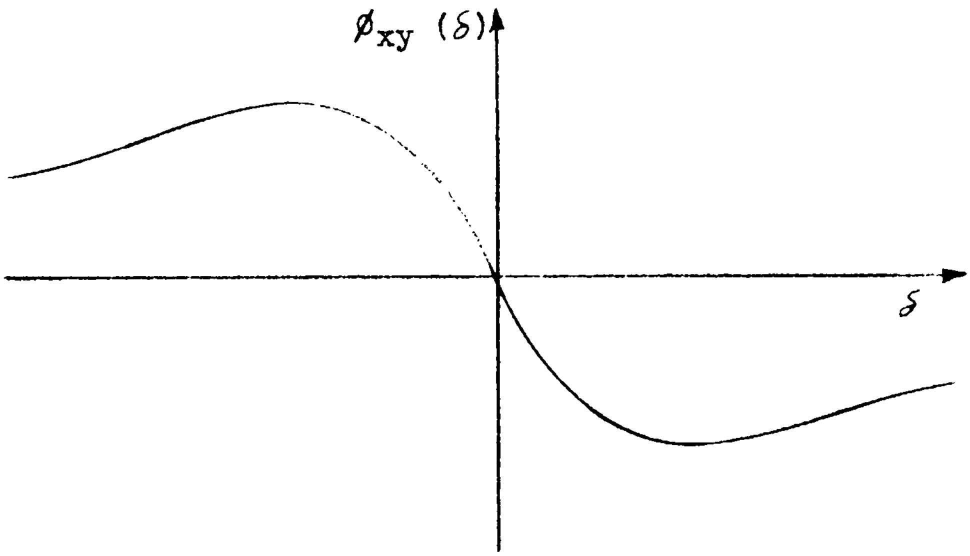


Figure 4-12. Correlator Output, Crosscorrelation Scheme With Post Detection Correlation

CHAPTER 5
CORRELATOR OUTPUT ERRORS

Six schemes of operation have been found which could be used to measure altitude if ideal circuits could be used and if there were no extraneous noise present. However, such is not the case in the practical sense. It is not possible to determine exactly all errors or disturbances which would be present in a physical system, but we can investigate certain disturbing influences from a statistical point of view. The first of these is the effect of finite time integration in the correlator. Consideration will next be given to the effects of extraneous noise, and finally to the effect of varying the correlation point during measurement. At the end of the chapter is a brief discussion of power requirements of an operating system.

5.1. Signal Error Due to Finite Integration Time

Figure 5-1 is a representation of the system to be investigated. As previously assumed $x(t + \tau)$ and $y(t)$ are ergodic signals with Gaussian statistics which are independent of any extraneous noise which may latter be added. Of interest here are any errors which may be caused by the finite time integrator. Recall that the ideal signal output of the integrator is the crosscorrelation function of $x(t + \tau)$ and $y(t)$.

It is easily shown that the ideal output is actually the average output of the integrator.

$$E\left[c(t)\right] = E\left[\frac{1}{T}\int_0^T x(t+\tau)\,y(t)dt\right]$$

$$= \frac{1}{T}\int_0^T \phi_{yx}(\tau)dt$$

$$= \phi_{yx}(\tau)$$

Thus if a large number of samples of c(t) were available or if there were an infinite time for integration, one could expect the correlator output to be ideal. The fact that a large number of ensembles of the correlator output must be averaged to arrive at the ideal output leads one to suspect that there is some fluctuation of the actual output about the ideal output. That this is true can be proven by finding the mean square error, or mean square variation, of the output signal about the ideal value.

Perhaps the most direct way of finding the mean square error of the correlator output will be to find the output autocorrelation function. If the autocorrelation function of the output is known then the mean square value of the output can be found by allowing the delay variable of the autocorrelation function to go to zero. The mean square error would then be this value less the signal squared. Consider first the system shown in Figure 5-2. The output of this system is easily found using the convolution integral.

$$c(t) = \int_0^{\infty} h(\alpha)\, r(t - \alpha)\, d\alpha$$

Next find the autocorrelation function, $\phi_{cc}(v)$.

$$\phi_{cc}(v) = E\left[c(t)\, c(t + v)\right]$$

$$= E\left[\int_0^{\infty}\int_0^{\infty} h(\alpha_1)\, h(\alpha_2)\, r(t - \alpha_1)\, r(t + v - \alpha_2) d\alpha_1 d\alpha_2\right]$$

$$= \int_0^{\infty}\int_0^{\infty} h(\alpha_1)\, h(\alpha_2)\, \phi_{rr}(v + \alpha_1 - \alpha_2) d\alpha_1 d\alpha_2 \qquad (5.1)$$

This last expression shows that the autocorrelation function of the output of the system in Figure 5-1 can be easily found once the autocorrelation function of the integrator input is known.

The autocorrelation function of the integrator input can be found by finding the expected value of $z(t)\, z(t + v)$.

$$\phi_{zz}(v) = E\left[z(t)\, z(t + v)\right]$$

$$= E\left[x(t + \tau)\, x(t + \tau + v)\, y(t)\, y(t + v)\right]$$

Laning and Battin[12] have shown that where x(t) and y(t) have Gaussian statistics and zero means that $\phi_{xx}(v)$ can be stated as:

$$\phi_{zz}(\tau, v) = \phi_{xy}{}^2(\tau) + \phi_{xx}(v)\, \phi_{yy}(v) + \phi_{xy}(v - \tau)\, \phi_{yx}(v + \tau) \qquad (5.2)$$

The first term of (5.2) is obviously the signal squared and the two remaining terms are error. The mean square output error is easily found by letting v go to zero and substituting into equation (5.1), remembering that the transfer function in this case is a finite integration.

$$\sigma_{es}^{2}(\tau, T) = \phi_{cc}(\tau, T, v)\Big|_{v=0}$$

$$= \frac{1}{T^2}\int_0^T\int_0^T \Big[\phi_{xx}(\alpha_1 - \alpha_2)\,\phi_{yy}(\alpha_1 - \alpha_2) +$$

$$\phi_{xy}(\alpha_1 - \alpha_2 - \tau)\,\phi_{yx}(\alpha_1 - \alpha_2 + \tau)\Big]d\alpha_1 d\alpha_2 \qquad (5.3)$$

Making the substitution $(\alpha_1 - \alpha_2) = v$ and using calculus techniques to change the order of integration it can be shown that this expression can be somewhat simplified.

$$\sigma_{es}^{2}(\tau, T) = \frac{2}{T^2}\int_0^T (T - v)\Big[\phi_{xx}(v)\,\phi_{yy}(v) + \phi_{xy}(v - \tau)\,\phi_{yx}(v + \tau)\Big]\,dv$$

This is the same result obtained by Bendat[13] using a different approach. The subscript, es, is used to denote that this is, so far, purely a signal error.

5.2. Extraneous Noise Error

If there is noise in either or both multiplier input channels then there is almost sure to be some contribution to the total mean square error at the output. In recalling the sources of the multiplier input signals, it seems reasonable to expect that the return signal will have a much greater noise content than will the other signal which remained within the system. For this reason only the case where noise is added to the channel with the echo signal will be considered. Assume that the noise added to the x channel is white with autocorrelation function, $\phi_{nene}(\tau)$, and that it is independent of either $x(t + \tau)$ or $y(t)$. Considering equation (5.3) it is obvious that the only place that extraneous noise in the x channel can make a contribution is in the $\phi_{xx}(v)$ term. The mean square error due to extraneous noise can then be quite easily stated.

$$\sigma_{ene}^{2}(\tau, T) = \frac{1}{T^2} \int_0^T \int_0^T \phi_{nene}(\alpha_1 - \alpha_2)\, \phi_{yy}(\alpha_1 - \alpha_2) d\alpha_1 d\alpha_2 \qquad (5.4)$$

The total mean square error from both signal and extraneous noise is then:

$$\sigma_{esne}^{2}(\tau, T) = \sigma_{es}^{2} + \sigma_{ene}^{2}$$

$$= \frac{1}{T^2} \int_0^T \int_0^T \Big[\phi_{xx}(\alpha_1 - \alpha_2)\, \phi_{yy}(\alpha_1 - \alpha_2) +$$

$$\phi_{nene}(\alpha_1(\alpha_1 - \alpha_2)\, \phi_{yy}(\alpha_1 - \alpha_2) +$$

$$\phi_{xy}(\alpha_1 - \alpha_2 - \tau)\, \phi_{yx}(\alpha_1 - \alpha_2 + \tau)\Big]\, d\alpha_1 d\alpha_2 \qquad (5.5)$$

5.3. Error Due to Moving Correlation Point

There is one other source of error which warrants consideration. This error results from moving the correlation point while the integration is taking place. All derivations made so far have assumed that the relative delay between the two signals being correlated was constant. That this is not so in a practical case is obvious. Even if the altimeter were being carried by an aircraft in level flight, there would be small variations in the delay of the return signal due to contour changes in the terrain over which the aircraft was flying. It is to be expected that the most significant variations in delay can be attributed to changes in position. This amounts to a velocity change in delay. If $\tau(t)$ is the relative delay between the inputs to the correlator then it is reasonable to represent $\tau(t)$ as $\tau_o + \beta t$. Higher orders of t are possible but it is fairly certain that $\tau(t)$ could not vary at the higher rate for a prolonged period of time. It is also reasonable to assume that β is very small. This is fortunate because it allows $x\left[t + \tau(t)\right]$ to be represented by the first two terms of its Taylor series.

$$x\left[t + \tau(t)\right] \cong x(t + \tau_o) + \beta t \frac{d}{d\tau(t)} x\left[t + \tau(t)\right] \Big|_{\tau(t) = \tau_o}$$

Returning to equation (5. 5) it can be seen that changing $\tau(t)$ can contribute to the mean square error only in the third term of the integrand. All other terms are independent of $\tau(t)$. With $x\left[t + \tau(t)\right]$ represented as above, new expressions must be found for ϕ_{xy} and ϕ_{yx}.

$$\phi_{xy}\left[\tau(t)\right] = \phi_{xy}\left[\tau(t), v\right] \Big|_{v = 0}$$

$$= E\left[y(t + v)\, x(t + \tau_o) + \beta t\, y(t + v)\, x'(t + \tau_o)\right] \Big|_{v = 0}$$

$$= \left[\phi_{xy}(v - \tau_o) + \frac{\beta T}{2} \phi_{xy}'(v - \tau_o)\right] \Big|_{v = 0}$$

$$\phi_{yx}\left[\tau(t)\right] = \phi_{yx}\left[\tau(t), v\right] \Big|_{v = 0}$$

$$= \left[\phi_{xy}(\tau_o - v) + \frac{\beta T}{2} \phi_{xy}'(\tau_o - v) \right] \Big|_{v=0}$$

The additional error due to varying τ (t) can now be written.

$$\nabla_{ecp}^2 = \frac{1}{T^2} \int_0^T \int_0^T \Big[\phi_{xy}[\alpha_1 - \alpha_2 - \tau(t)] \, \phi_{yx}[\alpha_1 - \alpha_2 + \tau(t)] -$$

$$\phi_{xy}(\alpha_1 - \alpha_2 - \tau_o) \, \phi_{yx}(\alpha_1 - \alpha_2 + \tau_o) \Big] d\alpha_1 d\alpha_2$$

$$= \frac{1}{T^2} \int_0^T \int_0^T \Big[\frac{\beta T}{2} \phi_{xy}(\alpha_1 - \alpha_2 - \tau_o) \, \phi_{yx}'(\alpha_1 - \alpha_2 + \tau_o)$$

$$+ \frac{\beta T}{2} \phi_{xy}'(\alpha_1 - \alpha_2 - \tau_o) \, \phi_{yx}(\alpha_1 - \alpha_2 + \tau_o)$$

$$+ \frac{\beta^2 T^2}{4} \phi_{xy}'(\alpha_1 - \alpha_2 - \tau_o) \, \phi_{yx}'(\alpha_1 - \alpha_2 + \tau_o) \Big] d\alpha_1 d\alpha_2 \quad (5.6)$$

In the above equations, the prime notation indicates differentiation with respect to τ_o.

Perhaps the best way to show the usefulness of the information obtained so far would be to work an example problem. Assume that an altimeter using either the autocorrelation of the anticorrelation scheme is mounted in an aircraft which is, at the time of measurement, at an altitude of 9,000 meters. The aircraft is also assumed to be changing altitude at a rate of 150 meters per second. These figures will be used shortly to determine τ_o and β .

5.4. White Noise Representation of System Errors

To simplify calculations white noise representations of the errors will be obtained. This method will not yield exact results, but if the representation chosen gives a slightly greater error than would an exact solution, then one could expect the resulting system to perform better than specified. No performance specifications will be made initially. Instead, an expression relating signal amplitude, noise amplitude, T, and allowable error will be obtained.

From the above information:

$$\tau_o = \frac{(2)\ (9000)}{3 \times 10^8} = 60 \text{usec}$$

$$\beta = \frac{2\ (150)}{3 \times 10^8} = 10^{-6}$$

From Chapter 4:

$$w_o = 10 \times 10^3$$

$$\phi_{nn}(\tau) = A^2 \exp(-w_o |\tau|)$$

Using (5.5) and the information above the mean square error due to the finite integration and extraneous noise can be found.

$$\phi_{xx}(v) = \phi_{yy}(v) = \phi_{nn}(v)$$

$$\phi_{xy}(v + \tau) = \phi_{nn}(v + \tau)$$

$$\phi_{xy}(v - \tau) = \phi_{nn}(v - \tau)$$

$$\phi_{nene}(v) = N^2\ \delta(v)$$

$$\phi_{ze_1ze_1}(v) = \phi_{xx}(v)\,\phi_{yy}(v) + \phi_{nene}(v)\,\phi_{yy}(v) + \phi_{xy}(v+\tau)\,\phi_{yx}(v-\tau)$$
$$= A^4 \exp\left[-2w_o|v|\right] + A^2N^2 \exp\left[-w_o|v|\right]\,\delta(v) +$$
$$A^4 \exp\left[-w_o|v+\tau|\right]\exp\left[-w_o|v-\tau|\right]$$

Since the last term of the above expression has its greatest value with respect to τ when τ is zero:

$$\phi_{ze_1ze_1}(v) \leqq 2A^4 \exp\left[-2w_o|v|\right] + A^2N^2 \exp\left[-w_o|v|\right]\,\delta(v)$$

The spectral density of this function can be found by taking the Fourier transform.

$$\Phi_{e_1e_1}(s) = \frac{8A^4 w_o}{4w_o^2 - s^2} + A^2N^2$$

Since $\Phi_{e_1e_1}(s)$ has its greatest value at $s = 0$, we can use $\Phi_{e_1e_1}(0)$ as the Fourier transform of the equivalent white noise for the signal and noise inputs to the correlator.

$$\Phi_{e_1e_1}(s) \leqq \frac{2A^4}{w_o} + A^2N^2$$

Thus:

$$\phi_{e_1e_1}(v) = F^{-1}\left[\frac{2A^4}{w_o} + A^2N^2\right]$$
$$= \left[\frac{2A^4}{w_o} + A^2N^2\right]\delta(v)$$

And since $v = \alpha_1 - \alpha_2$:

$$\sigma_{es_1N}^2 \cong \sigma_{e_1}^2$$

$$\sigma_{e_1}^2 = \frac{1}{T^2}\int_0^T\int_0^T\left[\frac{2A^4}{w_o} + A^2N^2\right]\delta(\alpha_1 - \alpha_2)\,d\alpha_1\,d\alpha_2$$

$$= \frac{1}{T}\left[\frac{2 A^4}{w_o} + A^2N^2 \right] \qquad (5.7)$$

Quite obviously this error is zero for very large values of T. Such is not the case for the error caused by moving the correlation point during the integration. Using (5.6) the autocorrelation function of the error due to varying $\tau(t)$ can be written.

$$\begin{aligned} \phi_{ze_2ze_2}(v) = {} & \frac{\beta T}{2} \phi_{xy}(v - \tau_o)\, \phi_{yx}'(v + \tau_o) + \\ & \frac{\beta T}{2} \phi_{yx}(v + \tau_o)\, \phi_{xy}'(v - \tau_o) + \\ & \frac{\beta^2 T^2}{4} \phi_{xy}'(v - \tau_o)\, \phi_{yx}'(v + \tau_o) \end{aligned}$$

Although it is known that the value of the derivatives is actually zero for τ_o equal to zero because the derivative is an odd function, a very small displacement of τ_o will give the required maximum value. The mathematical expressions being used in this example have either a maximum or a minimum value depending upon the direction from which τ_o approaches zero. For the purpose of finding the greatest possible error the positive value of the derivatives at τ_o equal to zero will be used.

$$\phi_{ze_2ze_2}(v) \lesseqgtr \left[\beta T\, w_o A^4 + \frac{\beta^2 T^2 w_o^2 A^4}{4}\right] \exp - 2\, w_o |v|$$

$$\Phi_{e_2e_2}(s) = \left[\beta T\, w_o A^4 + \frac{\beta^2 T^2 w_o^2 A^4}{4}\right] \frac{4\, w_o}{4\, w_o^2 - s^2}$$

$$\leq \left[A^4 \beta T + \frac{A^4 \beta^2 T^2 w_o}{4} \right]$$

Thus the white noise representation for the error due to the moving correlation point can be written as:

$$\phi_{e_2 e_2}(v) \doteq A^4 \beta T \left[1 + \frac{\beta T w_o}{4} \right] \delta(v)$$

$$\sigma_{e_2}^{2} \doteq \frac{1}{T^2} \int_0^T \int_0^T A^4 \beta T \left[1 + \frac{\beta T w_o}{4} \right] \delta(\alpha_1 - \alpha_2) d\alpha_1 d\alpha_2$$

$$\doteq A^4 \beta \left(1 + \frac{\beta w_o T}{4} \right) \tag{5.8}$$

The total mean square error is obtained by combining (5.7) and (5.8)

$$\sigma_{eT}^{2} = \sigma_{e_1}^{2} + \sigma_{e_2}^{2}$$

$$\doteq \frac{1}{T} \left(\frac{2 A^4}{w_o} + A^2 N^2 \right) + A^4 \beta \left(1 + \frac{\beta w_o T}{4} \right) \tag{5.9}$$

Notice that the first part of (5.9) contains a term which is very small as compared to the other term in that part. If the negligible term is dropped, a very simple expression results.

$$\sigma_{eT}^{2} \cong \sigma_{e}^{2} \doteq \frac{A^2 N^2}{T} + \frac{A^4 \beta^2 w_o T}{4} + A^4 \beta$$

When values for β and w_o are substituted in this expression, we have,

$$\sigma_{e}^{2} \doteq \frac{A^2 N^2}{T} + (2.5 \times 10^{-9}) A^4 T + 10^{-6} A^4 \tag{5.10}$$

If we now let $\Delta N \doteq A$, an expression which gives the desired relation between T, signal amplitude, and noise amplitude can be written.

$$\sigma_e^2 = \frac{A^4}{T\Delta^2} + (2.5 \times 10^{-9})\, A^4 T + 10^{-6} A^4 \qquad (5.11)$$

This equation relates specifically to the present example where β is 10^{-6}. A more general expression is given below which is good for any rate of change in $\tau(t)$.

$$\sigma_e^2 = \frac{A^4}{T\Delta^2} + (2.5 \times 10^{3})\, \beta^2 A^4 T + \beta A^4 \qquad (5.12)$$

A graph of (5.11) is shown in Figure 5-3 for $A/N = 1$. Figure 5-4 is the more general graph of (5.12), showing σ_e^2 versus T for several values of Δ and β .

A convenient way of expressing the allowable error for a system is as a percentage of the true altitude. If p represents the percent of the true altitude and τ_o is the delay at that altitude, then an expression for σ_e^2/A^4 can be found which can be used with Figures 5-3 and 5-4 to find the percent error for various values of the system parameters.

$$\epsilon_{max} = \frac{p}{100}\, \phi_{xy}(\tau_o)$$

$$\frac{\sigma_e^2}{A^4} = \left[\frac{1}{A^4} \quad \frac{p}{100} \quad A^4 \exp(-2w_o \tau_o)\right]^2$$

$$= p^2 \times 10^{-4} \exp(-2 \times 10^4 \tau_o) \qquad (5.13)$$

or

$$p = \left[\frac{\sigma_e^2}{A^4} \right]^{1/2} 100 \exp(10^4 \tau_o) \qquad (5.14)$$

Thus for the example above; when $\Delta = .01$, the smallest error occurs when T is approximately 1.8×10^{-6}. This corresponds to an error of about 18% of the true altitude or 1600 meters. This is obviously unacceptable. From Figure 5-4 we see that by increasing the signal to noise ratio, A/N, to 0.1, an error of only 5.75% or 520 meters, at $T = 1.8 \times 10^5$, results. If the signal to noise ratio could be further increased to 1, an error of only 1.82%, or 16.5 meters, with $T = 1.8 \times 10^4$, results.

Several important conclusions can be drawn from Figure 5-4:

(1) For a given signal to noise ratio there is no dependence of mean square error on altitude.

(2) For a given rate of altitude change the mean square error is determined primarily by the noise error for small values of T, and by the moving correlation point error for large values of T.

(3) A decrease in the rate of altitude change will decrease the minimum mean square error while increasing the value T required to produce the minimum only a small amount.

Although no variation of the mean square error with w_o is plotted, (5.9) from which the equation for Figure 5-4 was derived, shows that decreasing w_0 will also decrease the minimum mean square error.

Another interesting consideration is the variation of the squared signal to noise ratio with altitude.

$$\Delta^2 = A^2/N^2$$

$$= \frac{\phi_{xy}^{\,2}(\tau)}{\sigma_e^{\,2}}$$

$$\cong \frac{\exp(-w_0 \tau)}{\frac{\beta^2 w_0 T}{4}}$$

$$\cong \frac{\exp(-w_0 \tau)}{w_0 K} \qquad (5.15)$$

$$\text{where } K = \frac{\beta^2 T}{4}$$

A graph of (5.15) is shown in Figure 5-5. Notice that Δ decreases very rapidly with increasing altitude. There is also a verification of previous results. Recall that decreasing w_0 decreased the minimum mean square error for a given signal to noise ratio. Here, since the w_0 within the exponential term has a greater effect than the w_0 in the denominator, decreasing w_0 causes Δ to increase.

5.5. Power Considerations

We have seen that to achieve an acceptably small error in altitude indication that a signal to noise power ratio of at least one is required. Knowing this it should be relatively easy to estimate the power required at the transmitter for the system to work in an acceptable manner. Figure 5-6 shows the situation represented in the example. If P_T is the signal power transmitted and P_R is the signal power received, then through the use of a rather simplified version of the radar range equation[14] an expression relating P_T and P_R can be found.

$$P_T = \frac{(4\pi)^3 a^4 P_R}{\lambda^2 G_T G_R \sigma} \tag{5.16}$$

where:

a = altitude

= 9000 meters

λ = wavelength

= 0.03 meters ($f = 10 \times 10^3$ mc)

G_T = transmitting antenna gain

= 1

G_R = receiving antenna gain

= 1

σ = radar cross section of the earth

$= \frac{1}{\lambda^2}(4 \times 10^6)$

The quantities assumed above are merely estimates made for the purpose of illustrating the method of determining the transmitted power required. Thus:

$$P_T \cong P_R \left[40 \times 10^{10} \right]$$

The requirement of a signal to noise power ratio of one can now be used to determine P_R. A reasonable estimate of the noise power at the receiving antenna can be stated as[15]:

$P_n = KT \Delta f$

where: K = Boltzman's constant

$= 1.374 \times 10^{-23}$

T = noise temperature (absolute)

$= 300^\circ$ Kelven

Δf = effective receiver bandwidth in cps

$= 4 \pi \times 10^5$

Thus; $P_n = 5 \times 10^{-15}$ watts

Now for a signal to noise power ratio of one set P_R equal to P_n and solve for P_T.

$$P_T \cong P_n \left[40 \times 10^{10} \right]$$

$$\cong 2 \times 10^{-3} \text{ watts}$$

This is a very rough estimate which indicates only that for an actual system the power required will be of a reasonable order of magnitude. This calculation assumes a receiver noise figure of one which is of course unobtainable.

The example just concluded was worked without benefit of improvements which could result from use of the feedback loop for the systems employing an internal delay. Figure 5-7 shows a system model which might be used to obtain an optimum post correlation filter, H(s). A solution to the problem of finding the optimum H(s) has been illustrated by Sage[16] for a very similar situation. The approach used results in a Wiener optimization of H(s) with appropriate constraints on certain system parameters.

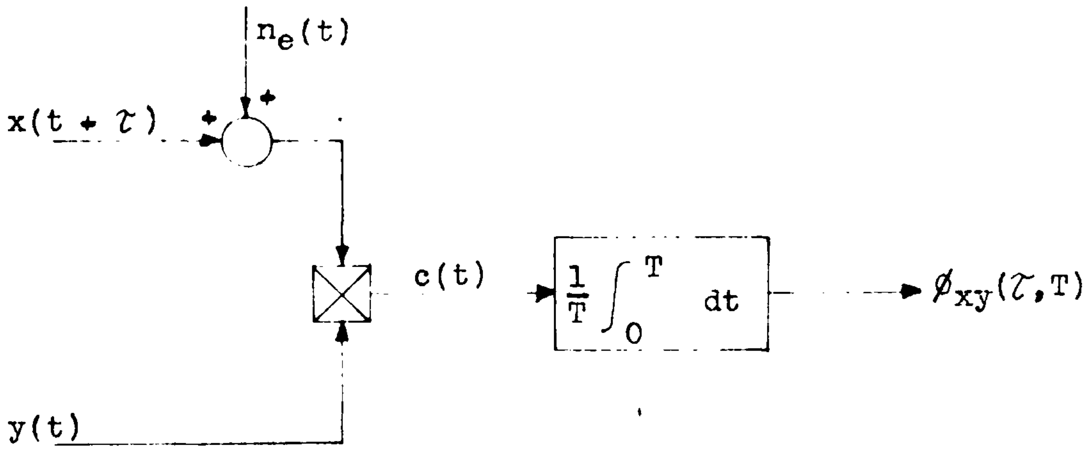


Figure 5-1. Output Circuit

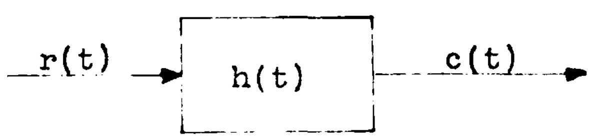


Figure 5-2. Transfer Function

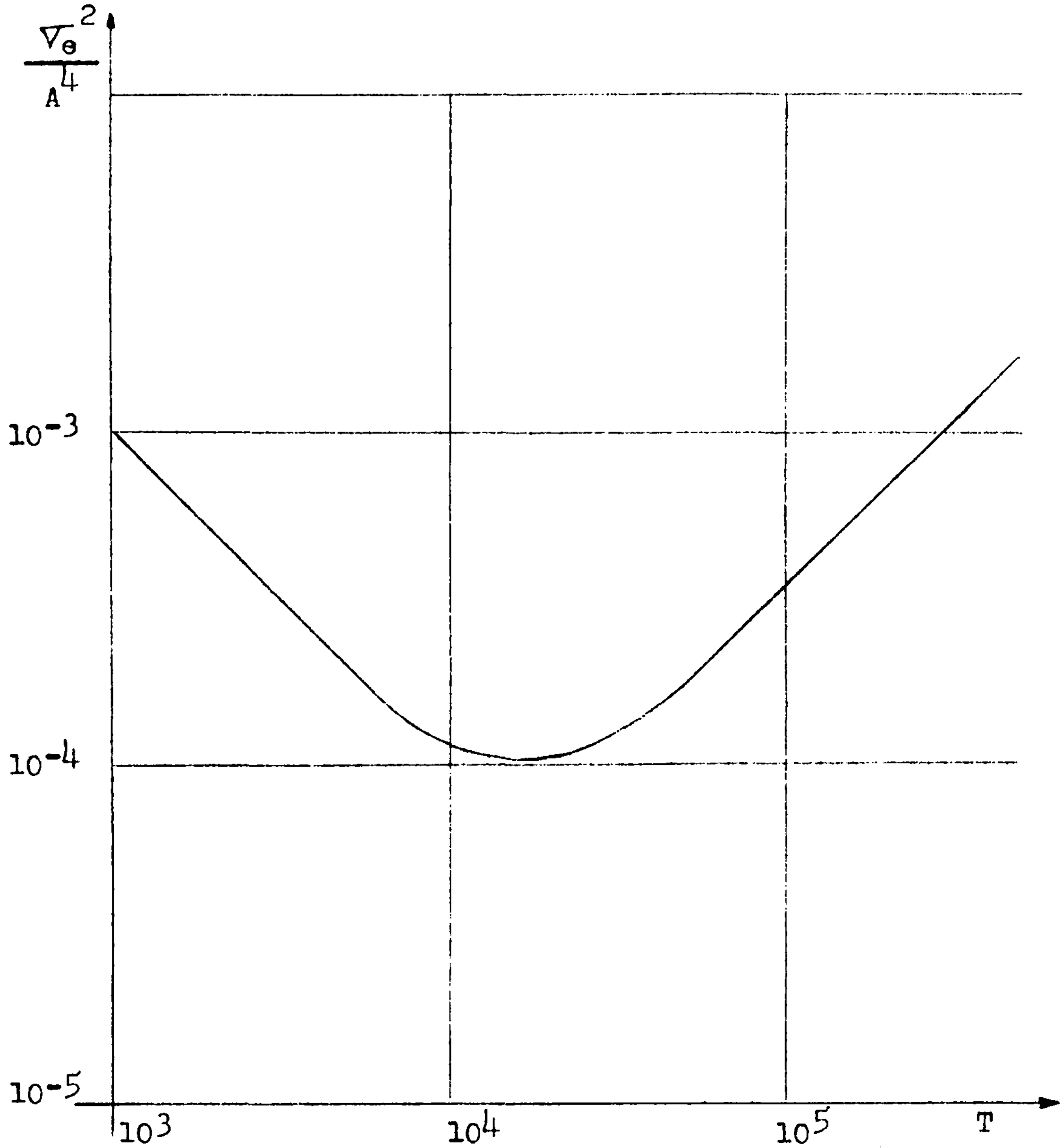


Figure 5-3. Mean Square Error vs Integration Time, Signal To Noise Ratio = 1

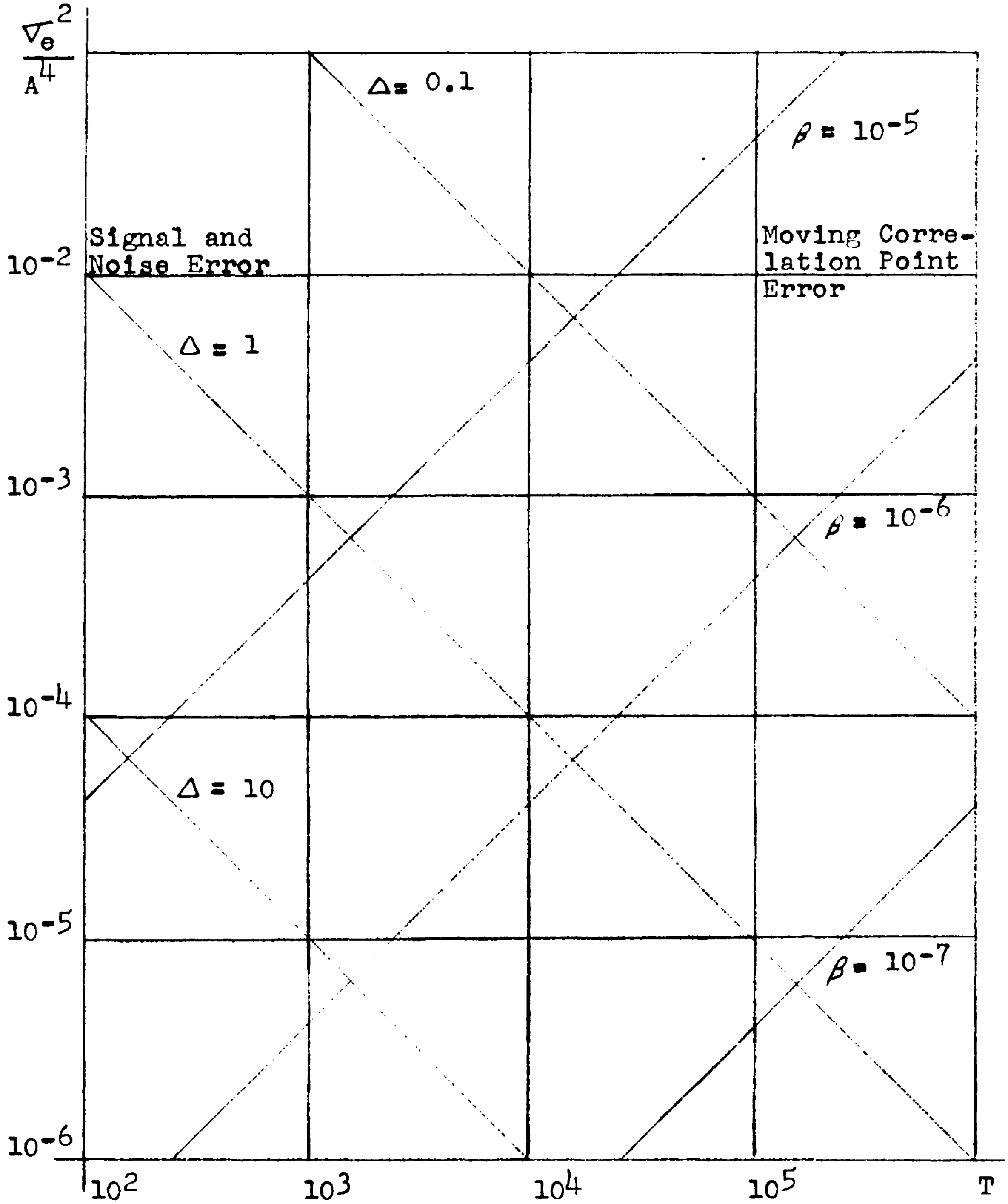


Figure 5-4. Mean Square Error versus Integration Time

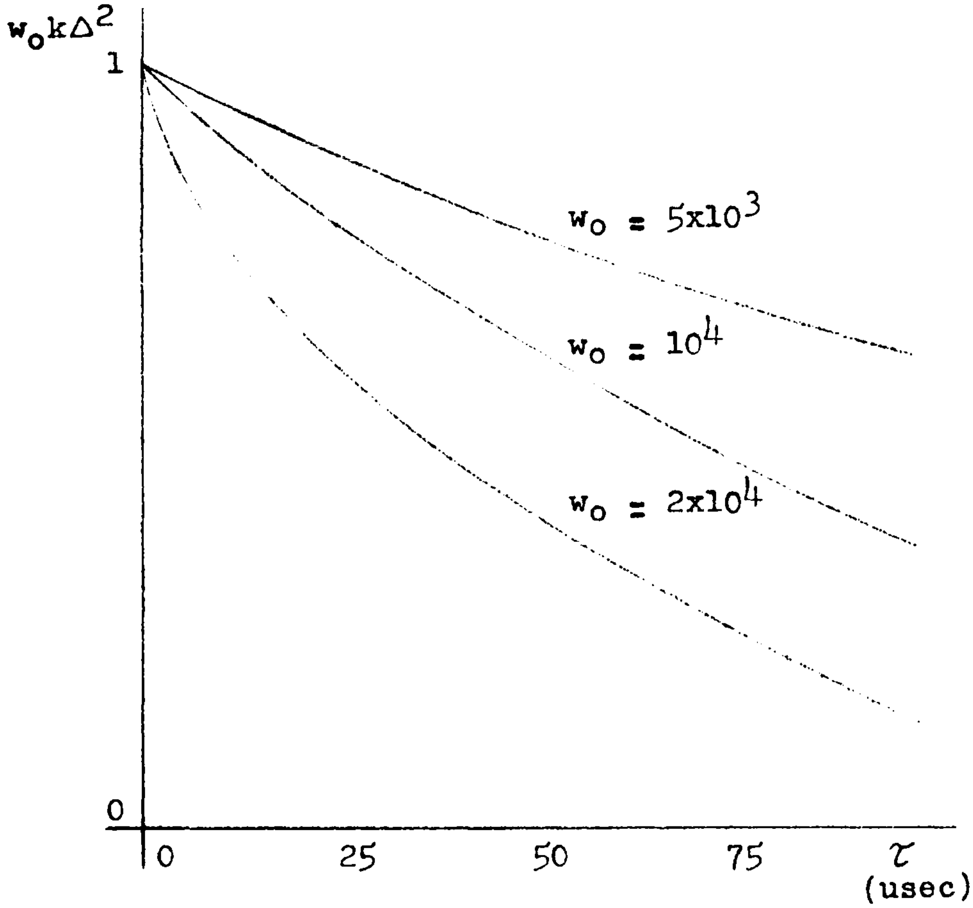


Figure 5-5. Squared Signal to Noise Ratio versus Altitude

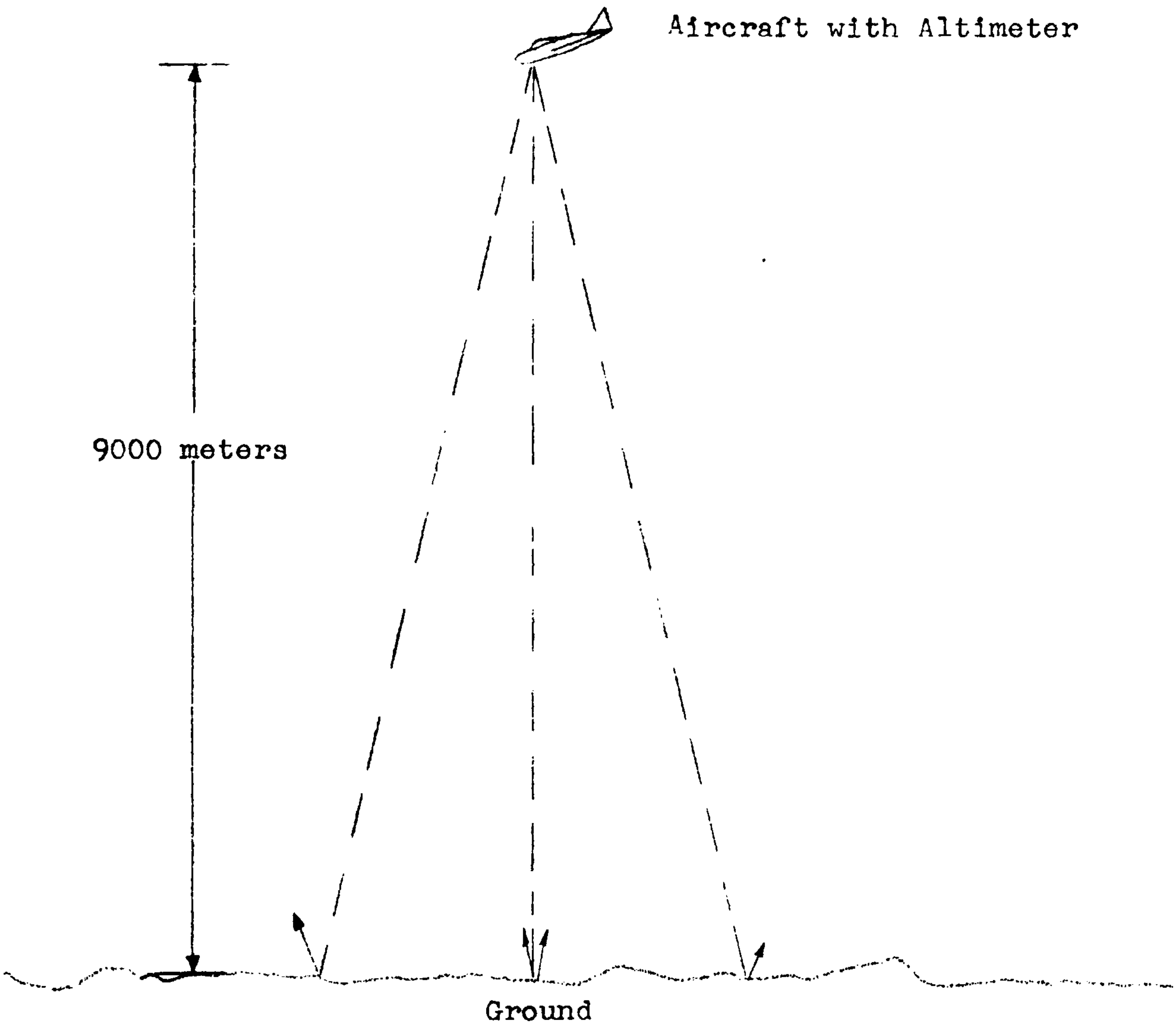


Figure 5-6. Example Situation, Determination of System Power Requirements

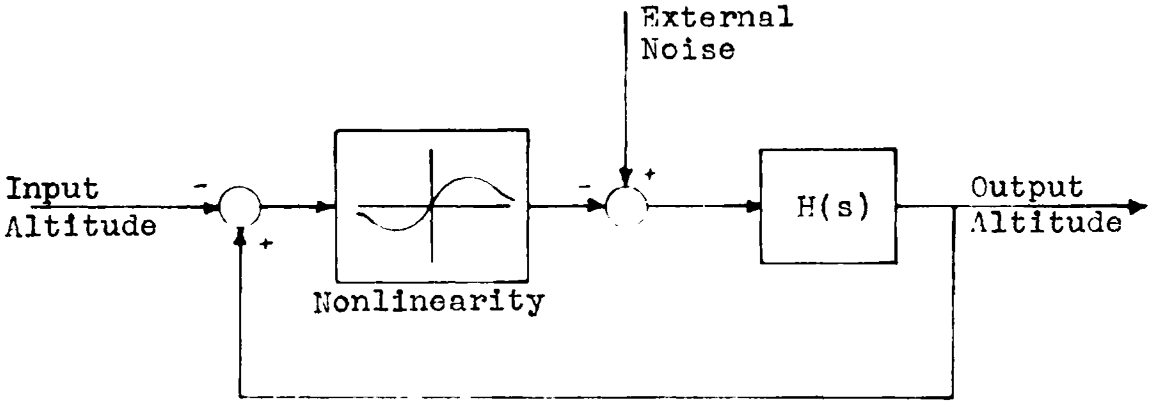


Figure 5-7. Quasi-linear Model For System With Feedback

CHAPTER 6
CONCLUSIONS

Considerations in the design of a noise modulated radar altimeter have been discussed. Initially eighteen different modes of operation were considered. The system output functions for these modes were derived in Chapter 2. Here it was found that those schemes which correlated two versions of the modulated signal all contained a component which varied as the cosine of the carrier frequency times the relative delay between the two signals. Since the carrier frequency was expected to be in the megacycle per second range it became apparent that very little was to be gained by using noise modulation. A system which used unmodulated signals would have worked almost as well. For this reason it was decided to continue the design only with those systems which employed post detection correlation.

Since the correlator outputs of the post detection schemes were actually the modulating noise autocorrelation functions and their derivatives it was necessary to find a means of obtaining suitable autocorrelation functions. In Chapter 3, it was found that this could be done by passing white noise through a suitable filter. It was found that filters could be constructed from simple R - C networks in conjunction with suitable linear amplifiers and isolation circuits.

In Chapter 4, expressions for errors due to finite correlation time, extraneous noise, and moving correlation point were derived. These three sources were picked for investigation because they can be limited to some degree by correct choice of system parameters. It was found that if a long enough correlation time were allowed that the errors due to finite integration and due to extraneous noise could be practically eliminated. On the other hand the error caused by variations of altitude during correlation caused the total error to increase in proportion to the correlation time. Physically, the moving correlation point error is caused by two things. First, there is a component of height change due to variations in terrain over which the altimeter is passing. There is nothing that can be done to eliminate this error. However altitude and velocity can be expected to have considerable effect on this particular source of error. Since the return signal is reflected by a much larger ground area at higher altitudes, and the return signal is a measure of average height, it is expected that terrain variations will have their greatest effect at relatively low altitudes. Certainly the terrain will appear to vary at a greater rate when the altimeter is being carried at higher velocities. Thus the combination of low altitude and high velocity may combine to produce large moving correlation point errors. Second there may be deliberate changes in altitude as during landings or takeoffs. However since the rate of ascent or descent can be controlled this source of error is not considered to be too important.

The factors discussed in the above paragraph are all taken into account in the curves plotted in Figure 5-4. If specifications are given which include allowable error, system bandwidth, vehicle velocity, and range of operation design of an adequate altimeter should be quite simple.

In the design of an actual system there are several factors which require considerable more thought and study than they have been given in this thesis. These include selection of a good carrier frequency based on atmospheric and terrain attenuation effects, accurate determination of system power requirements, system bandwidth, and the selection of system components. Of particular interest here is the mechanization of the variable internal delay. In addition the selection of the best type of modulation to be used is of great importance.

Another area that warrants further study is that of the systems using automatic feedback control of the internal delay. It appears that through the use of such a system the system bandwidth could be considerably decreased because the bandwidth of the noise filter in the transmitter could be decreased. In connection with this, equation (5.15) could probably be used to determine the lower bound on w_0.

CHAPTER 7
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