ABSORPTION INDUCED THERMAL NEUTRON FLUX PERTURBATIONS

by

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ABSTRACT

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The flux distribution around a strong thermal neutron absorber is experimentally investigated. Using the method of superposition, the flux is compared with that predicted by diffusion theory for a strong source. The failure of diffusion theory in describing the flux within a distance of one mean free path from the absorber is pointed out and a Monte Carlo model is developed and used to investigate the flux in this region. The flux profile determined by the Monte Carlo method is seen to compare favorably near the source with the experimentally determined flux distribution.

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CHAPTER I

INTRODUCTION

Thermal neutrons are assumed to obey Fick's law in a weak absorber. Fick's law states that the net current of neutrons is in the direction away from the region of greater neutron density and proportional to the spatial rate of change. This fundamental formula of diffusion theory is written (1),*

$$\bar{J} = -D\nabla\phi \quad ^{**}$$

The notable weakness of diffusion theory lies in the failure to describe flux near discontinuities in the diffusing media, e.g., strong sources and absorbers.

The purpose of this thesis is to investigate by experimental and theoretical means the thermal neutron flux in the vicinity of a localized neutron source and sink. A Monte Carlo computer program has been developed and used to obtain a more exact solution than that given by diffusion theory. The flux around a neutron sink has been determined by experiment, and using the method of superposition, this distribution is compared with that predicted by the Monte Carlo and diffusion theory methods for a thermal neutron source having the same geometric size and shape.

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*Numbers in parentheses refer to references
**See glossary for symbol representation

A neutron sink can be thought of as a negative source, the only difference between the flux distribution around a negative and positive source being in sign and not magnitude. One is the mirror image of the other. Since pure thermal neutron emitters are technically impossible to obtain, it is convenient to experimentally measure the flux around a sink in a diffusing medium and use superposition to infer from the results the distribution from a pure thermal neutron source.

An analytical description for the thermal neutron flux distribution around a source in an infinite diffusing medium may be obtained by solving the second order differential equation expressing the conservation of neutrons. This differential equation is known as the diffusion equation and its development and solution is based upon several simplifying assumptions.

The diffusion equation for thermal neutrons in a non-equilibrium system is

$$D\nabla^2\phi - \Sigma_a\phi + S = \frac{\partial n}{\partial t} .$$

At steady state and for a source-free diffusing medium this becomes

$$D\nabla^2\phi - \Sigma_a\phi = 0 .$$

The first term is the expression for the neutron leakage per unit volume per second and is equal to the divergence of the net current density vector $\bar{J}$.

In the formulation of the net current density, and hence the leakage term, the flux is expressed in a Taylor series expansion using only the first order terms.[2] This approximation is valid provided the change in $\overline{J}$ over a distance of two or three mean free paths is small. Near a strong source or absorber or near a boundary between two media with dissimilar neutron diffusing characteristics, the first order approximation to the flux must be used with caution. The second term represents the number of neutron absorptions per unit volume per second.

An alternate solution for the flux distribution around a strong source or sink can be obtained by application of the Monte Carlo method using a suitable model. The physical model used in this treatment is not based upon the diffusion equation for neutrons and is therefore applicable in regions where the spatial distribution of the flux changes greatly with distance. Hence, the solution should be valid near the boundaries of the media. In essence, the Monte Carlo solution is a numerical "experiment" based upon an understanding of first principles and is subject to statistical variations.

Application of the Monte Carlo method may be outlined briefly in terms of the following steps: (1) a neutron is introduced into the diffusing medium from the source traveling in a random direction; (2) the distance traveled to the point of first collision is selected by a random process;

(3) a decision is then made as to the type of collision, i.e., is the neutron absorbed or scattered; (4) if the event is an absorption, a new neutron is introduced and the process is repeated; (5) if the event is a scattering, note is made of the position coordinates and a new direction and path length is selected; (6) these steps are repeated until an absorption occurs, thus terminating the history. The number of collision points as a function of position from the source describes the flux distribution.

For this investigation, the flux will be examined around an infinitely long cylindrical source in a water diffusing medium.

CHAPTER 2

DISCUSSION OF THEORY

2.1 Monte Carlo Treatment

The problem will be treated using a Monte Carlo program for the IBM 650 Computer. The flow chart is shown in Figure 2.1.

The radius of the infinitely long cylindrical source used is .792 cm. The rectangular coordinate system has its origin at the geometric center of the cylindrical source with the point on the surface from which the neutrons are emitted having the coordinates (0, .792). The geometry and coordinate system used is shown in Figure 2.2.

The initial directional cosines α, β, γ are calculated using the formula

$$\alpha, \beta, \gamma = \frac{(1-2n)}{T} ,$$

where a new random number (n) is chosen for each of the directional cosines.

$$T = \sqrt{(1-2n')^2 + (1-2n'')^2 + (1-2n''')^2} ,$$

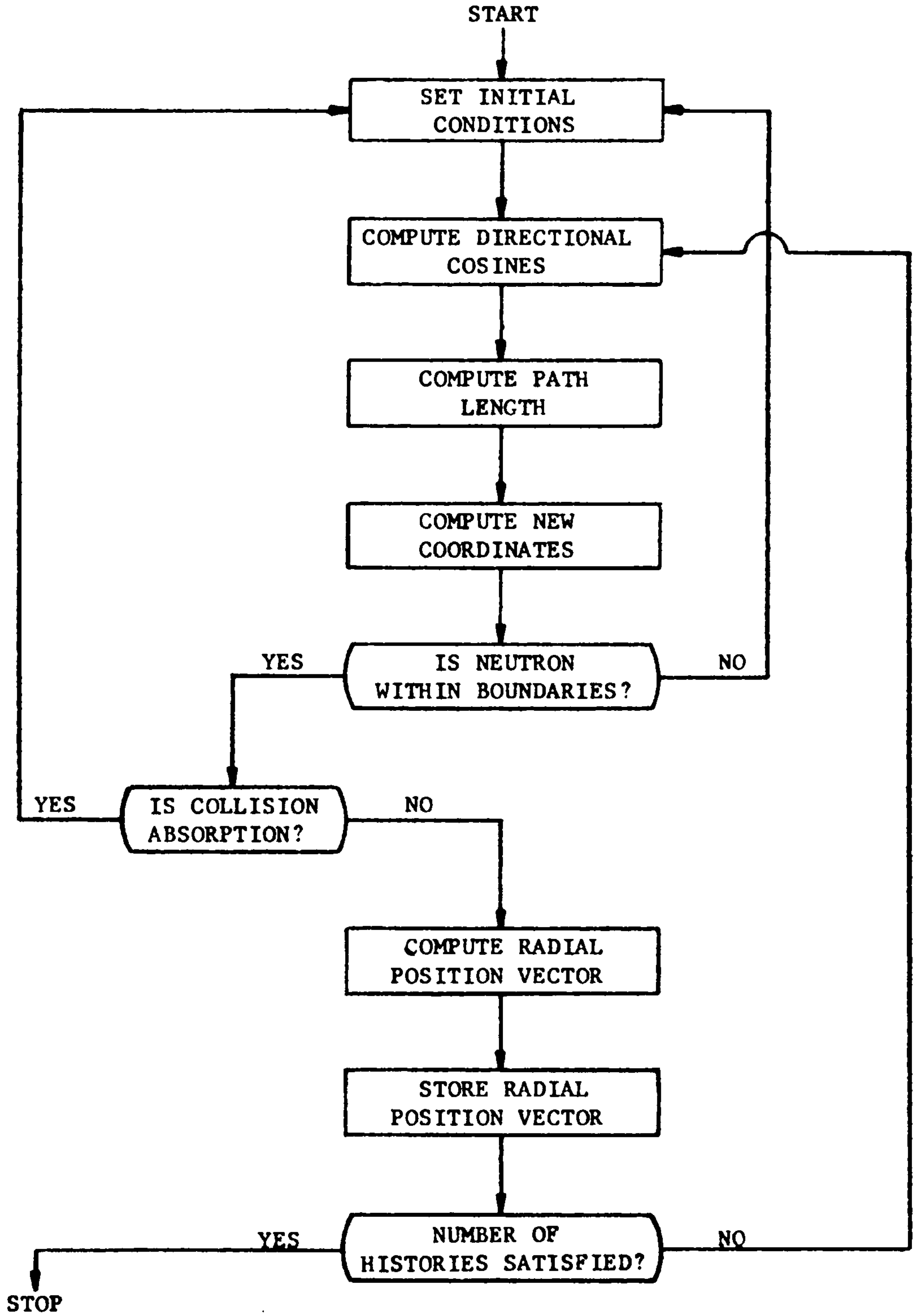
where the primes indicate a random number corresponding to the respective directional cosine.

The path length (ℓ) to a collision center is given by

$$\ell = \lambda_t \ \ln \frac{1}{n} .$$

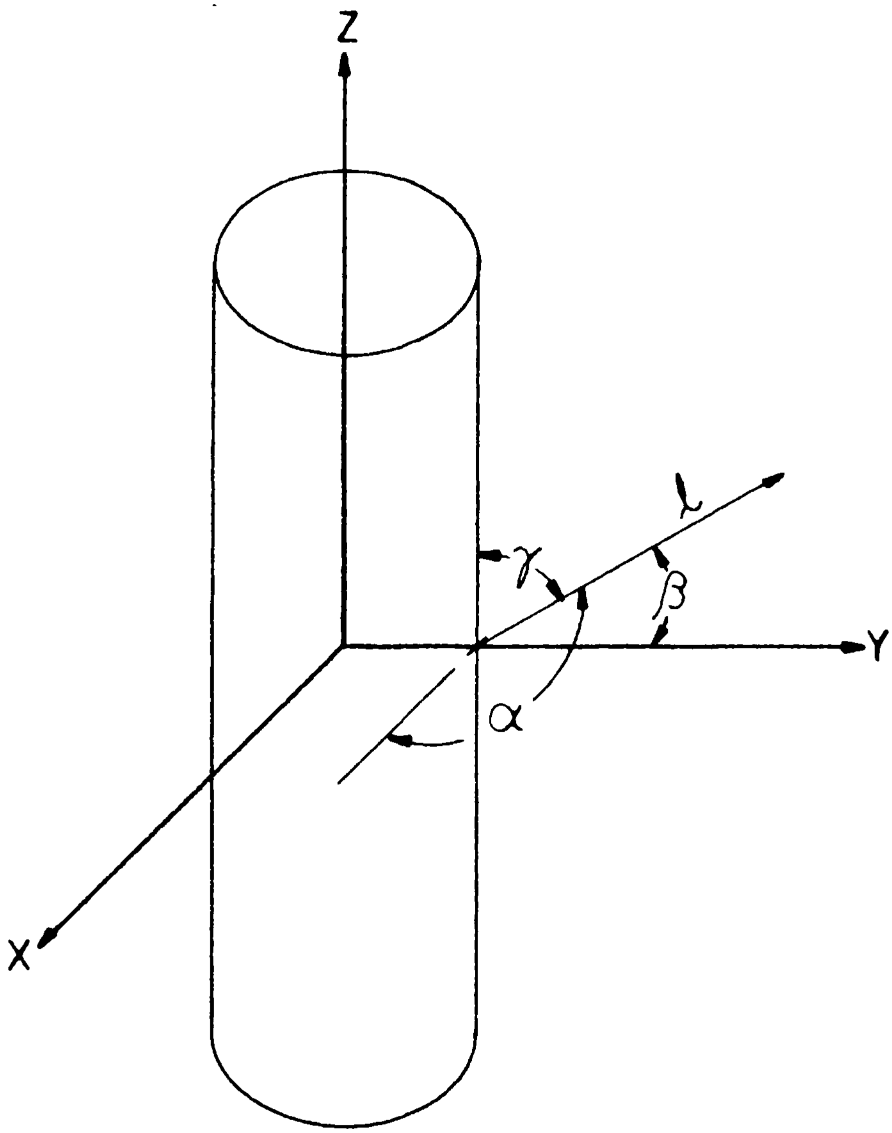
GENERAL FLOW DIAGRAM

FIGURE 2.1



GEOMETRY AND COORDINATE SYSTEM

FIGURE 2.2



For a water diffusing medium λ_t = .48 cm.[3]

The new coordinates are:

$$X_n = X_{n-1} + \alpha \ell$$

$$Y_n = Y_{n-1} + \beta \ell$$

The radial position vector (R) is then found to be

$$R_n = \sqrt{X_n^2 + Y_n^2}.$$

If R is less than .792 cm., i.e., the neutron is within the source, or is greater than 8.1 cm. which corresponds to the outer limit to which the neutrons are followed, then the history is terminated and a new neutron history is started.

If R falls between these inner and outer limits a random number is chosen such that if $n \geq \frac{\Sigma_s}{\Sigma_t}$ an absorption has occurred and a new neutron is emitted. In the case where $n \leq \frac{\Sigma_s}{\Sigma_t}$ a scattering collision has occurred and the position vector is stored in the memory of the computer. New directional cosines and path length are then selected at random and the neutron is followed onward from this point until the history is terminated either by the physical boundary limitations or an absorption event.

The calculation ended with 23,000 scattering events and the position vectors were grouped in numbers according to their value in a one dimensional array beginning at .792 cm. and increasing by .1 cm. from .8 cm. to 8.1 cm.

Since $N = V \Sigma_s \phi$, the number of scattering events is proportional to the thermal flux. V is the volume of each cylindrical shell in which the N scatterings take place. The shells were then grouped in .4 cm. radial increments to provide better statistics in plotting the flux distribution.

2.2 Diffusion Theory Treatment

The diffusion kernel in an infinite medium for a cylindrical shell source geometry is given by [4]

$$G = \frac{K_0(kr) I_0(kr')}{2\pi D} \quad ,$$

where r' is the radius of the cylindrical shell. For a fixed value of r', $I_0(kr')$ is a constant and the flux is given by

$$\phi = cK_0(kr).$$

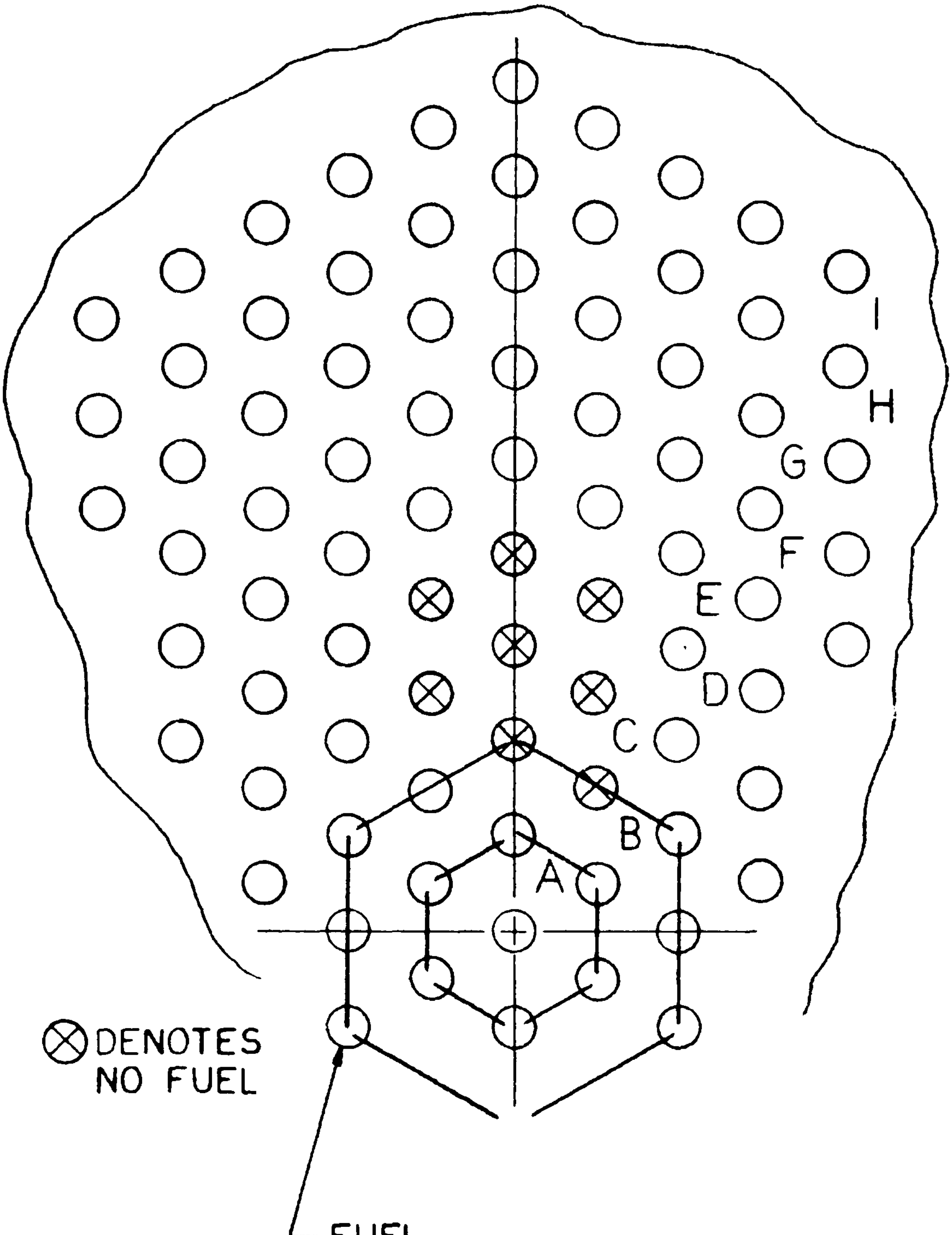
CHAPTER III

EXPERIMENTAL DETERMINATION OF FLUX

A water void was arranged in the University of Arizona light water moderated subcritical assembly so that the flux was nearly constant across the void region[5]. This void was arranged as shown in Figure 3.1. The flux was then perturbed by introducing a cylindrical absorber having a diameter of 1.58 cm. and a length of 71 cm. A BF_3 counter was used to map the flux in the void region, both in the presence and absence of the absorber. By subtracting the perturbed flux distribution from the undisturbed, the resulting flux profile assumes the shape of the flux around a source of thermal neutrons having the same size and shape as the absorber.

The absorbers used were: (1) a 20 mil sheet of cadmium (Σ_a= 114 cm.$^{-1}$) wrapped around a polyethylene core; (2) a solution of cadmium chloride (Σ_a= 3.8 cm.$^{-1}$) in polyethylene tubing. The flux profiles around both are nearly identical in shape.

The neutron detector used in the experiment was a Nuclear Chicago, boron trifluoride counter. The detector is enclosed in a probe 42 inches long with a 5/16 inch diameter.



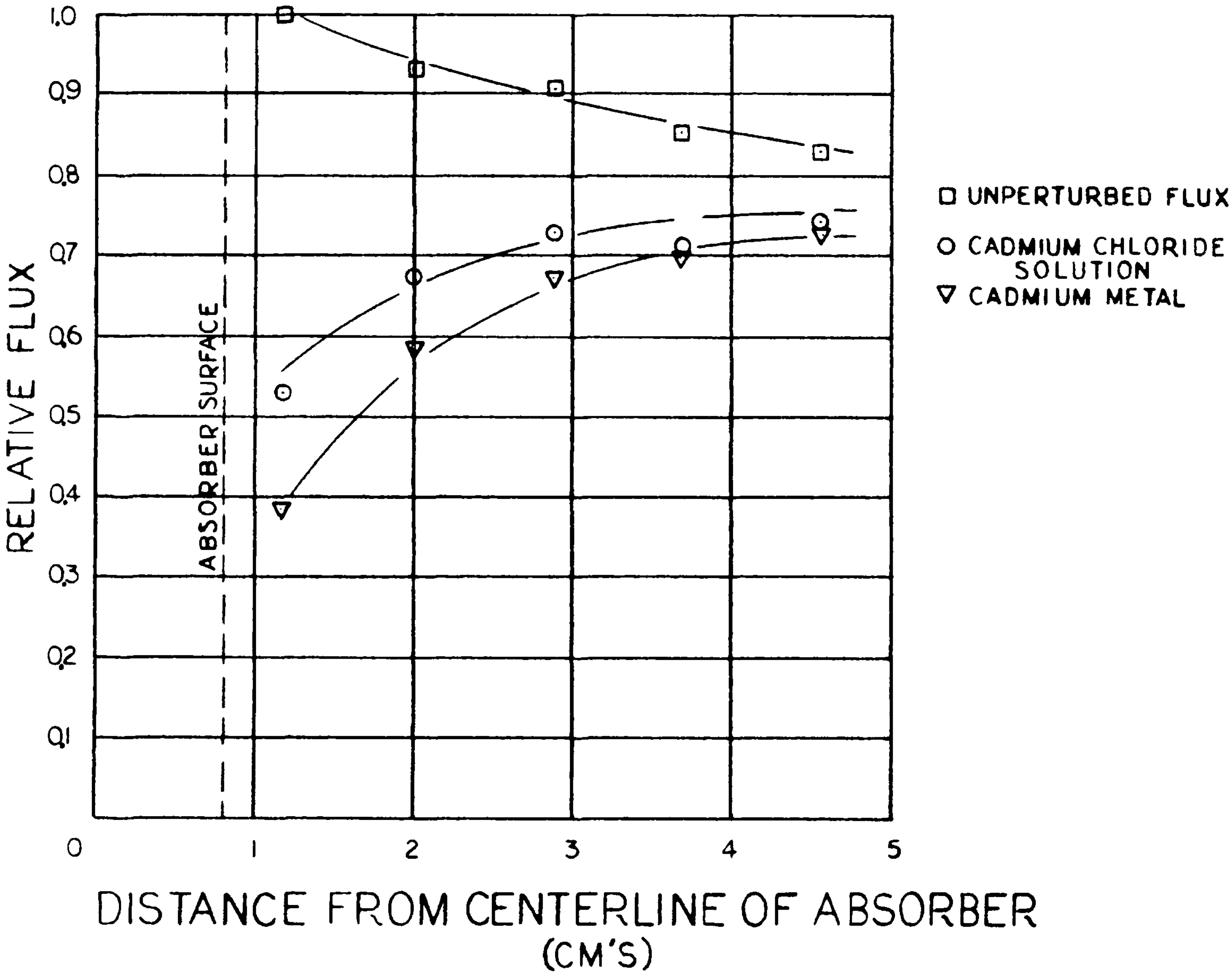
VOID ARRANGEMENT IN SUBCRITICAL ASSEMBLY

FIGURE 3.1

The probe was positioned on 5/16 inch centers from the absorber surface with an aluminum positioning device which rested on the top grid plate. A change in flux of approximately a factor of two occurs in moving 5/16 of an inch from the absorber surface. However, the gradient of the flux is nearly constant in this region and average values will give an accurate representation. The flux profiles measured with and without the absorbers present are shown in Figure 3.2. At large distances, the effect of the localized sink on the neutron flux is vanishingly small and as a consequence the experimental data used to define the image source are based upon the difference in two numbers of nearly equal value. Hence, the scatter in data as the detector is moved away from the image source is not surprising.

RADIAL NEUTRON FLUX

FIGURE 3.2



CHAPTER IV

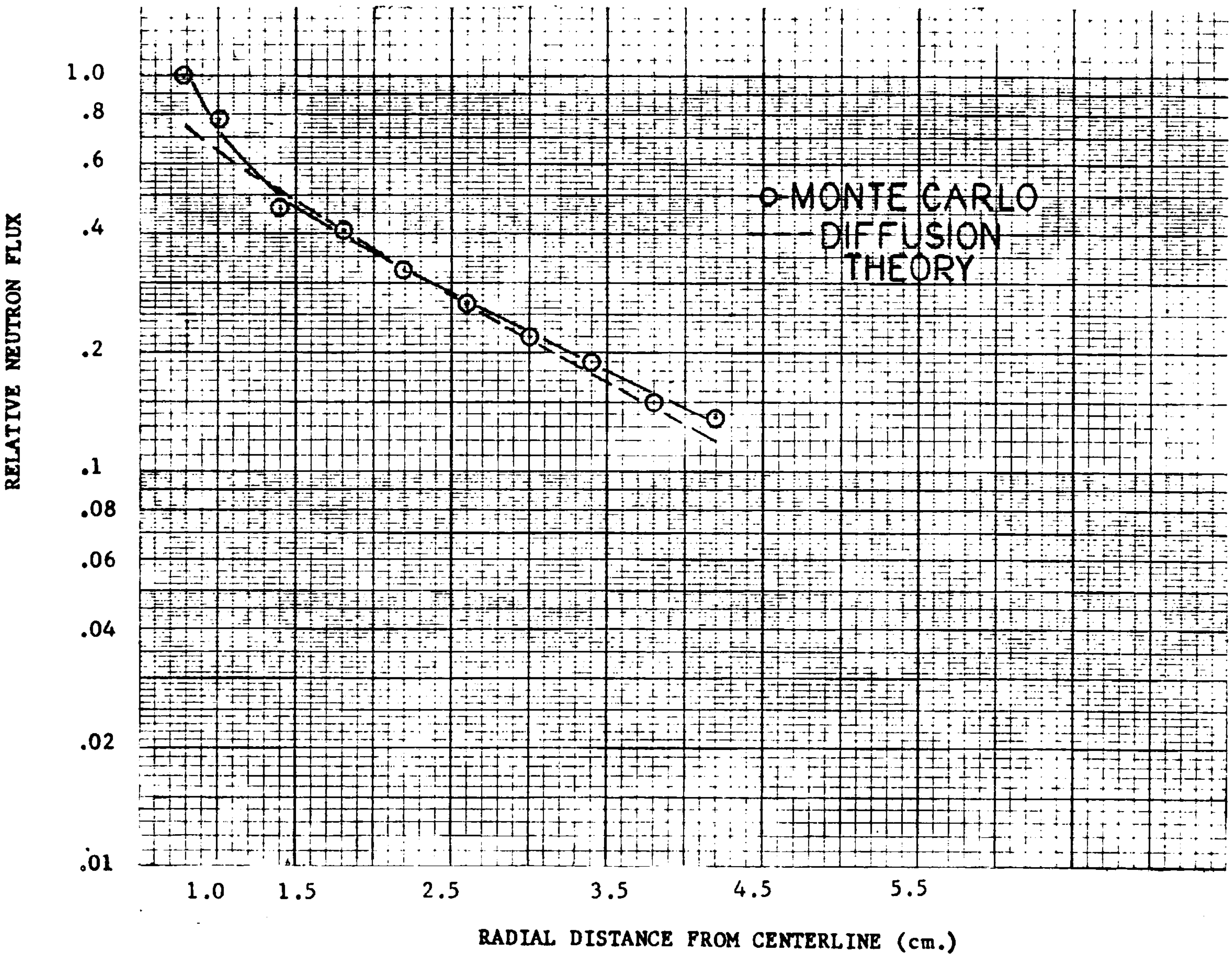
RESULTS AND CONCLUSIONS

A comparison between the experimentally measured flux and that predicted by diffusion theory indicates that diffusion theory tends to underpredict the magnitude of the flux at distances less than 1.5 cm. from the source. It is noteworthy that this is slightly greater than three mean free paths in water. Hence, an analysis using diffusion theory at distances closer than three mean free paths to a strong source or sink does not adequately describe the flux, the reason being that the gradient of the flux is large in this region and the first order terms in the Taylor series expansion do not result in a valid description of the net current density. The deviation is shown in Figure 4.1.

It would be expected that the Monte Carlo determination of the flux would accurately describe the spatial distribution in all regions of the diffusing medium. The physical model used does not suffer from the assumption that the change in net current density be small near the source or any of the boundary conditions used in solving the diffusion equation. To verify this conclusion, the experimentally determined flux profiles based on the difference measurements with and without the two absorbers have been compared with the results of the diffusion theory and Monte Carlo calculations for the image source. The results of these comparisons are shown in Figure 4.2

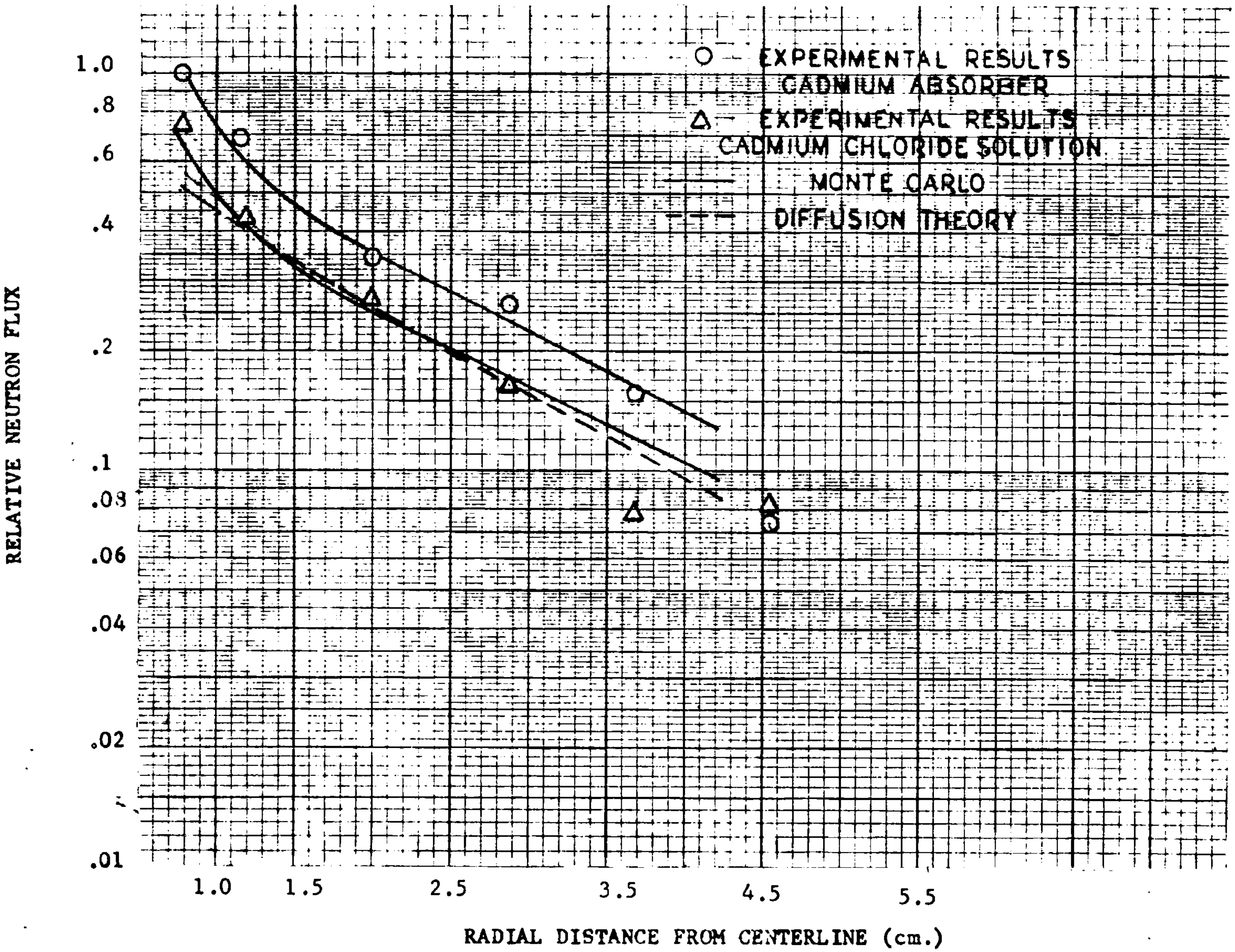
RADIAL NEUTRON FLUX

FIGURE 4.1



RADIAL NEUTRON FLUX

FIGURE 4.2



The Monte Carlo calculation predicts the variation of the flux near both absorbers since it closely matches that determined experimentally. The flux curves are normalized on the basis of the number of absorptions occurring in each of the absorbers.

This simple Monte Carlo analysis has demonstrated advantages over diffusion theory technique for analysing the influence of absorbers in a reactor. Further application of this approach to evaluating the influence of control rods on the flux distribution in reactors appears to be in order. Also, the use of a Monte Carlo calculation of the flux distribution in heterogeneous reactor cells should be investigated.

GLOSSARY

D = diffusion coefficient, cm.

ϕ = thermal neutron flux, neutrons/cm^2/sec.

J = net current density, neutrons/cm^2/sec.

ℓ = neutron path length, cm.

λ_t = transport mean free path, cm.

$\frac{\partial n}{\partial t}$ = time rate of change of neutron density, neutrons/cm^3/sec.

S = source strength, neutrons/cm^3/sec.

Σ_a = macroscopic absorption cross section, cm^{-1}.

Σ_s = macroscopic scattering cross section, cm^{-1}.

Σ_t = total macroscopic cross section, cm^{-1}

c = denotes a constant

$k = \sqrt{\frac{\Sigma_a}{D}}$

$K_o(kr)$ = modified Bessel function of the second kind, of zero order.

$I_o(kr')$ = modified Bessel function of the first kind, of zero order.

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