A LINEAR BENDING THEORY OF TAPERED SANDWICH PLATES

by

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ABSTRACT

In this thesis, six partial differential equations are derived which govern the bending of a linearly tapered sandwich plate. The plate is composed of two face layers of constant thickness and a core which is linearly tapered in the direction of one coordinate axis. The face layers are rigidly connected to the core at their natural boundaries.

The core is assumed to be linearly elastic, orthotropic, inextensional in the direction normal to its middle plane, and free from certain stresses acting parallel to its middle plane. The face layers are linearly elastic, plane-isotropic, and obey Kirchhoff's hypothesis.

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LIST OF FIGURES

NOTATION

x, y, z	Rectangular coordinates for the core (Figure 1).
m, y, n	Rectangular coordinates for facing a (Figure 1).
r, y, s	Rectangular coordinates for facing b (Figure 1).
$\sigma_m, \sigma_y, \sigma_n$	Tensile stresses acting on plane elements that are normal to m, y, and n coordinate lines, respectively.
τ_{my}, τ_{mn}	Components of shearing stress in y and n directions, respectively, acting on a plane element that is normal to the m coordinate line. Similarly, τ_{yn} denotes the n component of shearing stress acting on a plane element that is normal to the y coordinate line; $\tau_{my} = \tau_{ym}$, $\tau_{mn} = \tau_{nm}$, $\tau_{yn} = \tau_{ny}$.

M, H	Bending and twisting moments per unit length, respectively, defined by Equations (23) and (24).
N, S	Normal and shearing forces per unit length, respectively, defined by Equations (23) and (24).
Q	Shearing force per unit length defined by Equations (23) and (24).
ϵ_m, ϵ_y, ϵ_n	Unit elongations parallel to the m, y, and n coordinate lines, respectively.
γ_{my}, γ_{yn}, γ_{mn}	Components of shearing strain in the my, yn, and mn planes, respectively.
E	Modulus of elasticity of the face layers.
ν	Poisson's ratio of the face layers.
$G = \frac{E}{2(1+\nu)}$	Modulus of rigidity of the face layers.
G_{xz}, G_{yz}	Moduli of rigidity of the core in the xz and yz planes, respectively.
$D = \frac{E\delta^3}{12(1-\nu^2)}$	Flexural rigidity of the face layers.

$K = \frac{E\delta}{1-\nu^2}$	Extensional rigidity of the face layers.
δ	Thickness of a face layer.
$2t$	Variable thickness of the core.
θ	Angle formed by x and m axes as well as x and r axes.
U^a, V^a, W^a	Displacement components of a general point in facing a parallel to the m, y, and n coordinate lines, respectively.
u^a, v^a, w^a,	Displacement components of a general point on the middle plane of facing a parallel to the m, y, and n coordinate lines, respectively.
τ^a_m, τ^a_y	Distributed tangential loads on the free surface of facing a in the m and y directions, respectively.
q^a	Distributed normal load on the free surface of face a in the n direction.
$2\overset{+}{w} = w^a + w^b$	
$2\overset{-}{w} = w^a - w^b$	

a, b	As superscripts, they refer to facings a and b, respectively.

CHAPTER I

INTRODUCTION

Because of the extensive use of sandwich components in aircraft design, numerous papers have been written on the subject of bending and buckling of sandwich plates and shells [1]*. In many of these papers it is assumed that the flexural rigidities of the face layers are negligible, and that the thicknesses of these face layers are small as compared to the thickness of the core. The above simplifying assumptions are not included in the present work.

Within the author's knowledge, no means of analysis are currently available of a plate composed of two constant-thickness face layers and a linearly tapered core (Figure 1). In this thesis a system of differential equations is derived which describes the small-deflection behavior of the aforementioned tapered sandwich plate.

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*Numbers in brackets designate references at the end of thesis.

Herein, the core is assumed to be linearly elastic, orthotropic, inextensional in the direction normal to its middle plane, and free from x and y components of stress acting on plane elements normal to the x and y coordinate lines (Figure 1). In addition the taper of the core is assumed to be so small that the following approximations are valid:

$$\sin \theta = \theta,$$

$$\cos \theta = 1.$$

These assumptions allow the determination of stresses and displacements in the core to within six unknown functions of x and y.

The two constant-thickness face layers are assumed to be composed of identical, plane-isotropic materials and to obey Kirchhoff's hypothesis [2]. It is also assumed that the face layers are rigidly connected to the core at their natural boundaries (interfaces). This last statement is equivalent to requiring that the displacement components, and certain of the stresses in the face layers and core, be continuous across the interfaces.

Utilization of the foregoing assumptions permits the establishment of twelve independent equations involving twelve unknown functions. These equations are then manipulated to obtain the final system of six differential

equations involving only the six displacement components of the middle planes of the two face layers. The remaining six equations permit a direct determination of the previously mentioned six unknown functions associated with the core.

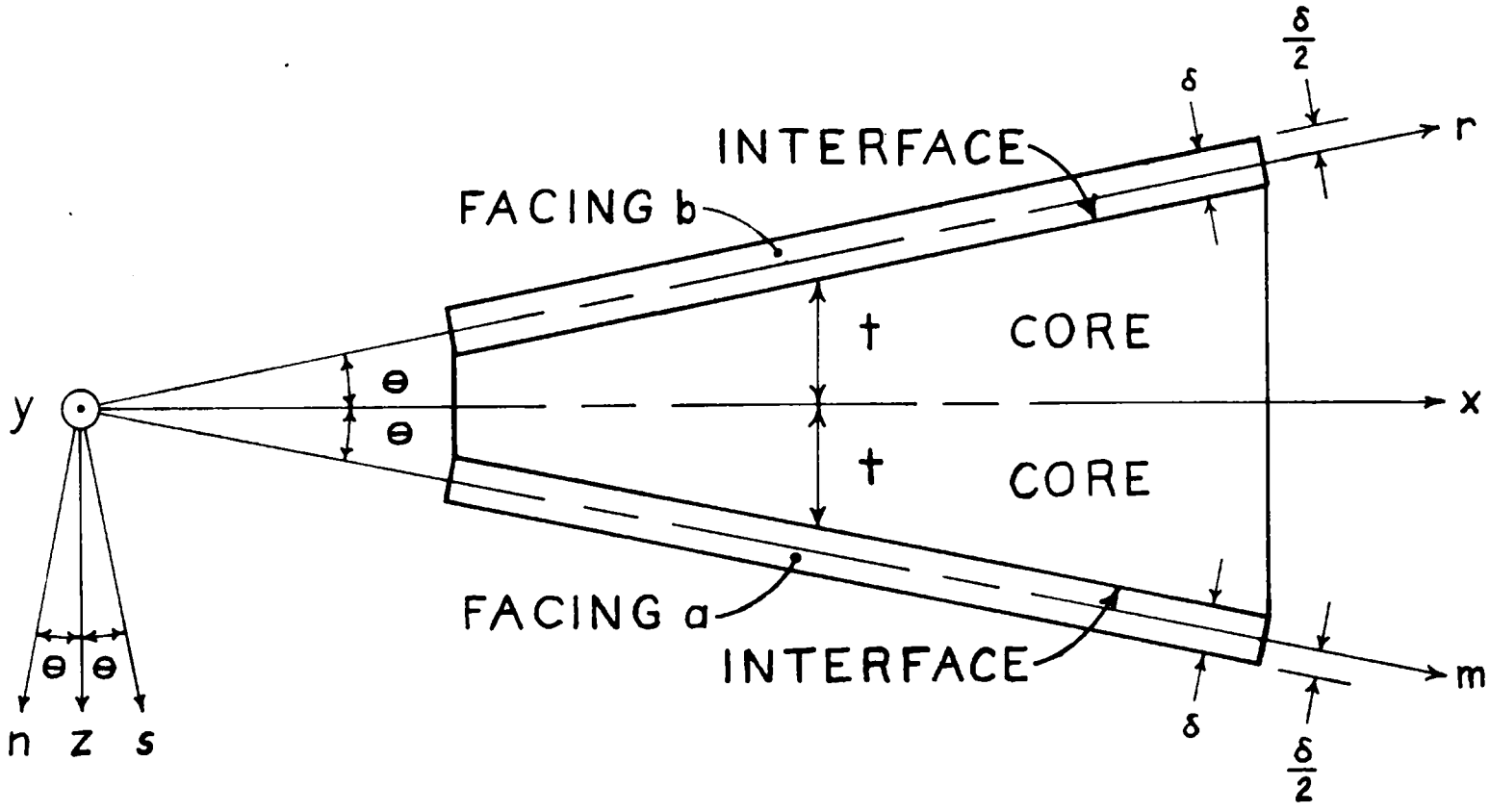
δ/2
δ
INTERFACE
r
FACING b
t
CORE
θ
θ
y
x
t
CORE
FACING a
INTERFACE
m
θ θ
n z s
δ
δ/2

Figure 1

CHAPTER II

CORE

1 Stresses

The coordinates for the core are shown in Figure 1. With reference to these coordinates (x, y, z) the usual stress notations σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{xz} will be employed. For example, σ_x denotes the tensile stress acting on a plane element that is normal to the x coordinate line; τ_{xz} denotes the z component of the shearing stress acting on a plane element that is normal to the x coordinate line.

From the classical theory of elasticity [3] the stress equations of equilibrium, in the absence of body forces, have the form:

$$\sigma_{x,x} + \tau_{xy,y} + \tau_{xz,z} = 0 , \tag{1a}$$

$$\tau_{xy,x} + \sigma_{y,y} + \tau_{yz,z} = 0 , \tag{1b}$$

$$\tau_{xz,x} + \tau_{yz,y} + \sigma_{z,z} = 0 \ , \qquad (1c)$$

where subscripts following a comma denote differentiation. That is

$$\sigma_{x,x} \equiv \frac{\partial \sigma_x}{\partial x} \ , \quad \tau_{xy,y} \equiv \frac{\partial \tau_{xy}}{\partial y} \ ,$$

and so on.

The core is assumed to be free from x and y components of stress acting on plane elements normal to the x and y coordinate lines, i.e.,

$$\sigma_x = 0 \ , \quad \sigma_y = 0 \ , \quad \tau_{xy} = 0 \ . \qquad (2)$$

With Equations (2) the foregoing equilibrium equations reduce to

$$\tau_{xz,z} = 0 \ , \qquad (3a)$$

$$\tau_{yz,z} = 0 \ , \qquad (3b)$$

$$\tau_{xz,x} + \tau_{yz,y} + \sigma_{z,z} = 0 \ . \qquad (3c)$$

Equations (3) can be integrated with respect to z. Performing this integration, the following expressions for stresses in the core are obtained:

$$\tau_{xz} = f_1(x,y) \ , \tag{4a}$$

$$\tau_{yz} = f_2(x,y) \ , \tag{4b}$$

$$\sigma_z = -z(f_{1,x} + f_{2,y}) + f_3(x,y) \ , \tag{4c}$$

where f_1, f_2, and f_3 are arbitrary functions of integration; they are independent of the thickness coordinate of the core.

2 Displacements

The strain-stress and strain-displacement relations are required for the determination of displacement components of the core. These relations are taken from the linear theory of elasticity. The strain-stress relations are [4]

$$G_{xz}\gamma_{xz} = \tau_{xz} \ , \tag{5a}$$

$$G_{yz}\gamma_{yz} = \tau_{yz} \ , \tag{5b}$$

where G_{xz} and G_{yz} are the shear moduli of the core in the xz and yz planes, respectively. The remaining strain-stress relations are disregarded. The components of strain are [3]

$$\epsilon_x = U,_x \ , \tag{6a}$$

$$\epsilon_y = V,_y \ , \tag{6b}$$

$$\epsilon_z = W,_z \ , \tag{6c}$$

$$\gamma_{xy} = U,_y + V,_x \ , \tag{6d}$$

$$\gamma_{yz} = V,_z + W,_y \ , \tag{6e}$$

$$\gamma_{xz} = U,_z + W,_x \ , \tag{6f}$$

where U, V, and W are the displacement components of a general point in the core in the directions of the x, y, and z axes, respectively.

Equations (4), (5), (6c, e, and f) are sufficient for the determination of displacements to within five arbitrary functions of x and y. These five functions and the function f_3 [see Equation (4c)] can be determined by using the conditions of continuity of displacements at the interfaces and the equilibrium conditions for the face layers.

The assumption that the core is inextensional in the direction normal to its middle plane is equivalent to the assumption that ϵ_z is zero, or that the modulus of elasticity of the core in the normal direction is infinite. With this assumption, Equation (6c) can be integrated with respect to z to obtain

$$W = F_1(x,y) \ . \qquad (7)$$

In order to obtain expressions for the displacement components V and U, Equations (5) are rewritten with the aid of Equations (4). Hence,

$$G_{xz}\gamma_{xz} = f_1 \ , \quad G_{yz}\gamma_{yz} = f_2 \ . \qquad (8a,b)$$

Equations (6e) and (8b) can be solved for $V_{,z}$, and then integrated with respect to z. Thus,

$$V = \frac{z}{G_{yz}} f_2 - zF_{1,y} + F_2(x,y) \quad . \tag{9}$$

In a similar manner, U is found from Equations (6f) and (8a),

$$U = \frac{z}{G_{xz}} f_1 - zF_{1,x} + F_3(x,y) \quad . \tag{10}$$

The quantities F_1, F_2, and F_3 in Equations (7), (9), and (10) are functions of integration. It is easily seen, by setting $z = 0$ in these equations, that they are the displacement components of the middle plane of the core.

CHAPTER III

FACE LAYERS (FACINGS)

3 Displacements

Three coordinate systems, each of which belongs to one layer of the tapered sandwich plate, are used to describe the behavior of this plate. The notation for quantities associated with the core was explained in Chapter II; the notation for quantities associated with the facings is explained in the following.

All of the components of stress, strain, and displacements of each facing are taken in the directions of the coordinate lines for that facing. The superscripts attached to each component indicate the facing to which this component belongs. The subscripts attached to the stress and strain components have their usual meaning. For example, σ_m^a denotes the tensile stress in facing a acting on a plane element normal to the m coordinate line; σ_r^b denotes the tensile stress in facing b acting on a plane element normal to the r coordinate line.

The components of strain for facing a can now be written using Equations (6) and the notation explained above. They are

$$\epsilon_m^a = U_{,m}^a \ , \tag{11a}$$

$$\epsilon_y^a = V_{,y}^a \ , \tag{11b}$$

$$\epsilon_n^a = W_{,n}^a \ , \tag{11c}$$

$$\gamma_{my}^a = U_{,y}^a + V_{,m}^a \ , \tag{11d}$$

$$\gamma_{yn}^a = V_{,n}^a + W_{,y}^a \ , \tag{11e}$$

$$\gamma_{mn}^a = U_{,n}^a + W_{,m}^a \ , \tag{11f}$$

where U^a, V^a, and W^a denote the displacement components of a general point in facing a in the directions of the m, y, and n axes, respectively. (The notation u^a, v^a, and w^a will be reserved for displacement components of a general point on the middle plane of facing a.)

In order to obtain expressions for the displacement components, the Kirchhoff hypothesis is assumed to be valid. According to this hypothesis

$$\epsilon_n = \gamma_{nm} = \gamma_{yn} = 0$$

in the case of facing a. Introducing these assumptions into Equations (11c, e, and f) one obtains the following expressions which can be integrated with respect to n:

$$W^a_{,n} = 0 \ , \tag{12a}$$

$$V^a_{,n} + W^a_{,y} = 0 \ , \tag{12b}$$

$$U^a_{,n} + W^a_{,m} = 0 \ . \tag{12c}$$

After integration, the displacement components are found to within three arbitrary functions of integration. By setting n = 0 in these resulting expressions, the arbitrary functions are easily interpreted to be the displacement components w^a, v^a, and u^a of the middle plane of facing a. Performing the above manipulations, the following results are obtained:

$$W^a = w^a(m,y) \ , \tag{13a}$$

$$V^a = -nw^a_{,y} + v^a(m,y) \ , \tag{13b}$$

$$U^a = -nw^a_{,m} + u^a(m,y) \ . \tag{13c}$$

The foregoing process, when applied to facing b, results in the following expressions for displacement components:

$$W^b = w^b(r,y) \ , \tag{14a}$$

$$V^b = -sw^b_{,y} + v^b(r,y) \ , \tag{14b}$$

$$U^b = -sw^b_{,r} + u^b(r,y) \ . \tag{14c}$$

4 Stresses

Since the face layers are assumed to be composed of identical, plane-isotropic materials, the stress-strain relations of the classical theory of elasticity are valid. Using the Kirchhoff hypothesis, and employing the notation

for facing a, these relations have the form [3]:

$$(1 - \nu^2)\sigma_m^a = E(\epsilon_m^a + \nu\epsilon_y^a) \ , \tag{15a}$$

$$(1 - \nu^2)\sigma_y^a = E(\epsilon_y^a + \nu\epsilon_m^a) \ , \tag{15b}$$

$$\tau_{my}^a = G\gamma_{my}^a \ , \tag{15c}$$

where E is Young's modulus, ν is Poisson's ratio, and G is the shear modulus.

The stresses σ_m^a, σ_y^a, and τ_{my}^a can be expressed in terms of displacement components of the middle plane of facing a. This is accomplished by substituting Equations (13) in Equations (11a, b, and d) and then substituting the resulting equations in Equations (15). The expressions for stresses obtained in this manner are

$$(1 - \nu^2)\sigma_m^a = E[u_{,m}^a + \nu v_{,y}^a - n(w_{,mm}^a + \nu w_{,yy}^a)] \ , \tag{16a}$$

$$(1 - \nu^2)\sigma_y^a = E[v_{,y}^a + \nu u_{,m}^a - n(w_{,yy}^a + \nu w_{,mm}^a)] \ , \tag{16b}$$

$$\tau_{my}^a = G(u_{,y}^a + v_{,m}^a - 2nw_{,my}^a) \ . \tag{16c}$$

The stress-displacement relations for facing b are obtained by the method outlined above for facing a. They are

$$(1 - \nu^2)\sigma_r^b = E[u^b_{,r} + \nu v^b_{,y} - s(w^b_{,rr} + \nu w^b_{,yy})] \ , \quad (17a)$$

$$(1 - \nu^2)\sigma_y^b = E[v^b_{,y} + \nu u^b_{,r} - s(w^b_{,yy} + \nu w^b_{,rr})] \ , \quad (17b)$$

$$\tau^b_{ry} = G(u^b_{,y} + v^b_{,r} - 2sw^b_{,ry}) \ . \quad (17c)$$

Since it is assumed that the face layers are rigidly connected to the core at their natural boundaries, the following continuity conditions for stresses exist at the interface of facing a and the core:

$$\tau^a_{nm}|_n = -\frac{\delta}{2} = p_{nm} \ ,$$

$$\tau^a_{ny}|_n = -\frac{\delta}{2} = p_{ny} \ ,$$

$$\sigma^a_n|_n = -\frac{\delta}{2} = p_n \ . \quad (18)$$

Here, the stresses denoted by p are the stresses shown in Figure 2a.

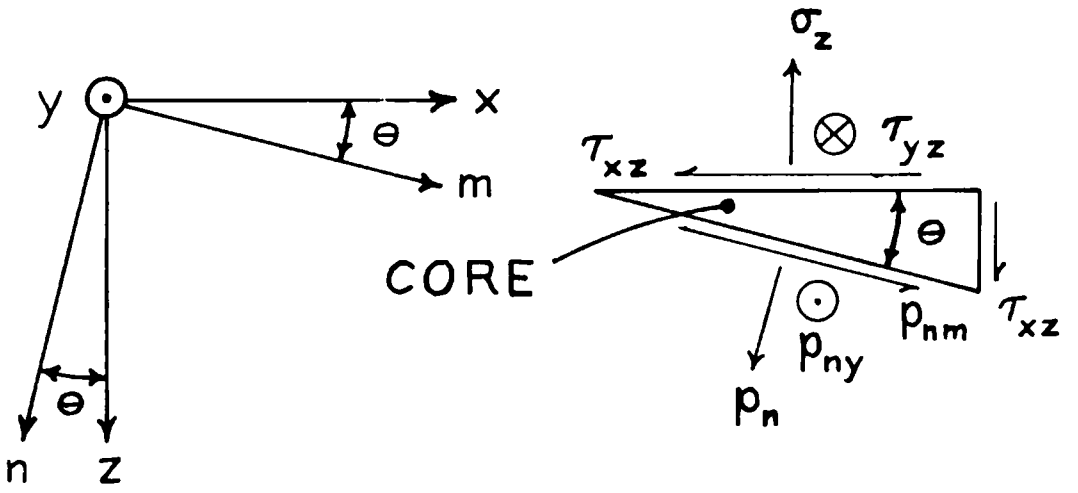

Figure 2a

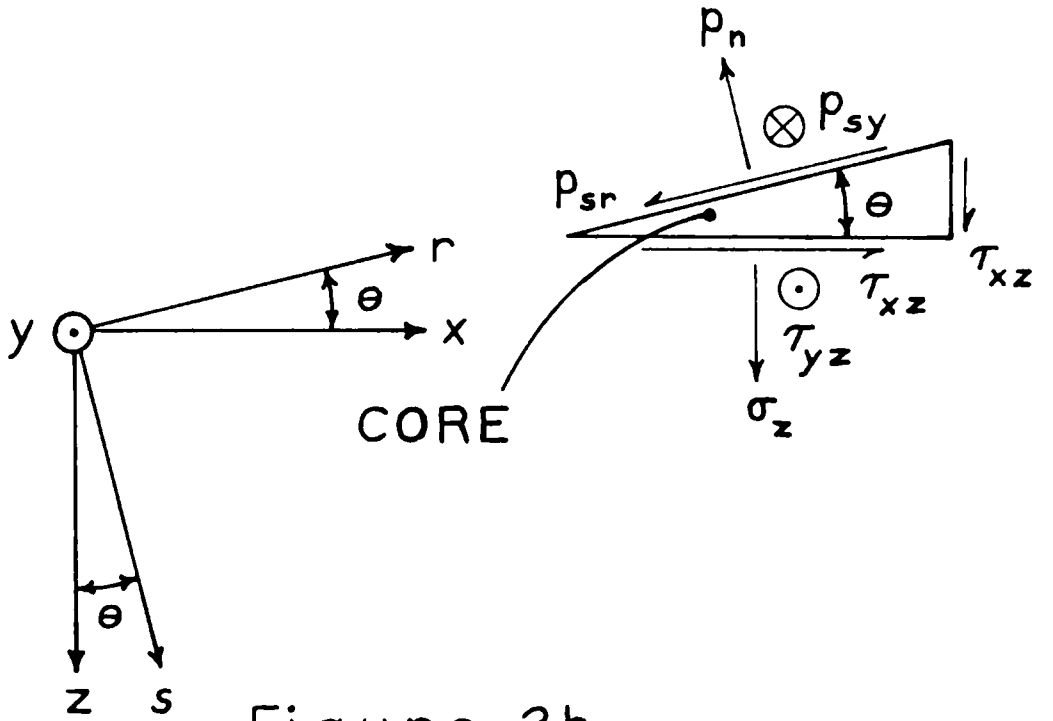

Figure 2b

An element of the core located at the interface of facing a and the core is shown in Figure 2a. Equilibrium considerations for this element yield the following relations:

$$p_{nm} = \theta\sigma_z + \tau_{xz} ,$$

$$p_{ny} = \tau_{yz} ,$$

$$p_n = \sigma_z - 2\theta\tau_{xz} . \qquad (19)$$

The small angle approximations for θ are used in these equations.

By substituting for the stresses σ_z, τ_{xz}, and τ_{yz} in Equations (19) their expressions from Equations (4), and evaluating the resulting equations at $z = t$ (the interface), one can write the stress continuity conditions, Equations (18), in the form:

$$\tau^a_{nm}\Big|_{n=-\frac{\delta}{2}} = f_1 + \theta f_3 - \theta t(f_{1,x} + f_{2,y}) , \qquad (20a)$$

$$\tau^a_{ny}\Big|_{n=-\frac{\delta}{2}} = f_2 , \qquad (20b)$$

$$\sigma_n^a\Big|_{n=-\frac{\delta}{2}} = f_3 - 2\theta f_1 - t(f_{1,x} + f_{2,y}) \ . \quad (20c)$$

Proceeding along the lines of the process just outlined, one obtains for the stress continuity conditions at the other interface

$$\tau_{sr}^b\Big|^{s=\frac{\delta}{2}} = f_1 - \theta f_3 - \theta t(f_{1,x} + f_{2,y}) \ , \quad (21a)$$

$$\tau_{sy}\Big|^{s=\frac{\delta}{2}} = f_2 \ , \quad (21b)$$

$$\sigma_s\Big|^{s=\frac{\delta}{2}} = f_3 + 2\theta f_1 + t(f_{1,x} + f_{2,y}) \ . \quad (21c)$$

Equations (20) and (21) will be used in the following article.

5 Equilibrium Equations

The equilibrium of the face layers is considered in this article. With the notation for facing a, the stress equations of equilibrium [Equations (1)] are

$$\sigma^a_{m,m} + \tau^a_{my,y} + \tau^a_{mn,n} = 0 \ , \qquad (22a)$$

$$\tau^a_{my,m} + \sigma^a_{y,y} + \tau^a_{yn,n} = 0 \ , \qquad (22b)$$

$$\tau^a_{mn,m} + \tau^a_{yn,y} + \sigma^a_{n,n} = 0 \ . \qquad (22c)$$

For the purpose of eliminating the variable n from these equations, the stress resultants for facing a are defined as follows:

$$N^a_m = \int \sigma^a_m dn \ , \qquad (23a)$$

$$N^a_y = \int \sigma^a_y dn \ , \qquad (23b)$$

$$S^a = \int \tau^a_{my} dn \ , \qquad (23c)$$

$$Q^a_m = \int \tau^a_{mn} dn \ , \qquad (23d)$$

$$Q^a_y = \int \tau^a_{yn} dn \ , \qquad (23e)$$

$$M^a_m = \int \sigma^a_m n dn \ , \qquad (23f)$$

$$M^a_y = \int \sigma^a_y n dn \ , \qquad (23g)$$

$$H^a = \int \tau^a_{my} n dn \ , \qquad (23h)$$

where all the integrals are definite and are evaluated between the lower and upper limits of $n = -\frac{\delta}{2}$ and $n = \frac{\delta}{2}$, respectively.

Similarly, the stress resultants for facing b are defined as follows:

$$N^b_r = \int \sigma^b_r ds \ , \qquad (24a)$$

$$N^b_y = \int \sigma^b_y ds \ , \qquad (24b)$$

$$S^b = \int \tau^b_{ry} ds \ , \qquad (24c)$$

$$Q^b_r = \int \tau^b_{rs} ds \ , \qquad (24d)$$

$$Q^b_y = \int \tau^b_{ys} ds \ , \qquad (24e)$$

$$M^b_r = \int \sigma^b_r s ds \ , \qquad (24f)$$

$$M^b_y = \int \sigma^b_y s ds \ , \qquad (24g)$$

$$H^b = \int \tau^b_{ry} s ds, \qquad (24h)$$

where all the integrals are definite and are evaluated between the lower and upper limits of $s = -\frac{\delta}{2}$ and $s = \frac{\delta}{2}$ respectively.

In the future it will be convenient to use the following notations:

$$\tau^a_{nm}\Big|^{n = \frac{\delta}{2}} = \tau^a_m \,, \tag{25a}$$

$$\tau^a_{ny}\Big|^{n = \frac{\delta}{2}} = \tau^a_y \,, \tag{25b}$$

$$\sigma^a_n\Big|^{n = \frac{\delta}{2}} = q^a \,. \tag{25c}$$

The terms τ^a_m, τ^a_y, and q^a represent the components of the distributed load acting on the free surface of face a in the positive directions of the m, y, and n axes, respectively.

Three force equilibrium equations for facing a are obtained by integrating Equations (22) through the thickness of that facing and using Equations (20), (23), and (25). For example, from Equation (22a)

$$\int_{-\delta/2}^{\delta/2} \frac{\partial}{\partial m}\, \sigma^a_m \, dn + \int_{-\delta/2}^{\delta/2} \frac{\partial}{\partial y}\, \tau^a_{my} \, dn$$

$$+ \int_{-\delta/2}^{\delta/2} \frac{\partial}{\partial n}\, \tau^a_{mn} \, dn = 0 \,. \tag{26a}$$

This equation can be rewritten

$$\frac{\partial}{\partial m}\int_{-\delta/2}^{\delta/2}\sigma_m^a\,dn + \frac{\partial}{\partial y}\int_{-\delta/2}^{\delta/2}\tau_{my}^a\,dn + \tau_{nm}^a\Big|^{n=\frac{\delta}{2}}$$

$$-\tau_{nm}^a\Big|_{n=-\frac{\delta}{2}} = 0\,, \qquad (26b)$$

where the last two terms in Equation (26b) are obtained by integration of the last term in Equation (26a). With the aid of Equations (20a), (23a,c), and (25a) the foregoing equation can be expressed as follows:

$$N_{m,m}^a + S_{,y}^a - f_1 - \theta f_3 + \theta t(f_{1,x} + f_{2,y})$$

$$= -\tau_m^a\,. \qquad (27a)$$

In a similar manner one obtains from Equations (22b) and (22c) the remaining force equilibrium equations for facing a. They are

$$S_{,m}^a + N_{y,y}^a - f_2 = -\tau_y^a\,, \qquad (27b)$$

$$Q^a_{m,m} + Q^a_{y,y} - f_3 + 2\theta f_1 + t(f_{1,x} + f_{2,y}) = - q^a \quad . \quad (27c)$$

The previous method outlined for facing a is also valid for facing b. Thus, one obtains the following force equilibrium equations for facing b:

$$N^b_{r,r} + S^b_{,y} + f_1 - \theta f_3 - \theta t(f_{1,x} + f_{2,y}) = \tau^b_r \quad , \quad (28a)$$

$$S^b_{,r} + N^b_{y,y} + f_2 = \tau^b_y \quad , \quad (28b)$$

$$Q^b_{r,r} + Q^b_{y,y} + f_3 + 2\theta f_1 + t(f_{1,x} + f_{2,y}) = q^b \quad . \quad (28c)$$

Here, τ^b_r, τ^b_y, and q^b represent the components of the distributed load acting on the free surface of face b in the negative directions of the r, y, and s axes, respectively.

The two moment equilibrium equations for facing a are obtained by multiplying Equations (22a) and (22b) by n, integrating them through the thickness of that facing, and then utilizing Equations (21), (23), and (25). For example, from Equation (22a)

$$\int_{-\delta/2}^{\delta/2} \frac{\partial}{\partial m}(\sigma_m^a) n dn + \int_{-\delta/2}^{\delta/2} \frac{\partial}{\partial y}(\tau_{my}^a) n dn$$

$$+ \int_{-\delta/2}^{\delta/2} \frac{\partial}{\partial n}(\tau_{nm}^a)\, n dn = 0 \,. \quad (29a)$$

This equation can be rewritten

$$\frac{\partial}{\partial m} \int_{-\delta/2}^{\delta/2} \sigma_m^a \, n dn + \frac{\partial}{\partial y} \int_{-\delta/2}^{\delta/2} \tau_{my}^a \, n dn + \frac{\delta}{2} \tau_{nm}^a \Big|_{n=\frac{\delta}{2}}$$

$$+ \frac{\delta}{2} \tau_{nm}^a \Big|_{n=-\frac{\delta}{2}} - Q_m^a = 0 \,, \quad (29b)$$

where the last three terms are the result of integration by parts of the last term in Equation (29a). With the aid of Equations (20a), (23f,h), and (25a) the foregoing equation can be expressed as follows:

$$Q_m^a = M_{m,m}^a + H_{,y}^a + \frac{\delta}{2} f_1 + \frac{\theta\delta}{2} f_3$$

$$- \frac{\theta t \delta}{2} (f_{1,x} + f_{2,y}) + \frac{\delta}{2} \tau_m^a \,. \quad (29c)$$

Similarly, from Equation (22b),

$$Q_y^a = H_{,m}^a + M_{y,y}^a + \frac{\delta}{2} f_2 + \frac{\delta}{2} \tau_y^a \ . \qquad (29d)$$

The moment equilibrium equations for facing b are obtained by the method just outlined for facing a. They are

$$Q_r^b = M_{r,r}^b + H_{,y}^b + \frac{\delta}{2} f_1 - \frac{\theta\delta}{2} f_3$$

$$- \frac{\theta t\delta}{2} (f_{1,x} + f_{2,y}) + \frac{\delta}{2} \tau_r^b \ , \qquad (30a)$$

$$Q_y^b = H_{,r}^b + M_{y,y}^b + \frac{\delta}{2} f_2 + \frac{\delta}{2} \tau_y^b \ . \qquad (30b)$$

The system of five equilibrium equations for facing a, Equations (27), (29c), and (29d), can be reduced to a system of three equations. This is accomplished by substituting for Q_m^a and Q_y^a in Equation (27c), their expressions from Equations (29c) and (29d). Performing this substitution one obtains

$$M_{m,mm}^a + 2H_{,my}^a + M_{y,yy}^a - f_3 + 2\theta f_1 + \frac{\theta\delta}{2} f_{3,x}$$

$$+ t(f_{1,x} + f_{2,y}) + \frac{\delta}{2}(f_{1,m} + f_{2,y})$$

$$- \frac{\theta\delta}{2} \frac{\partial}{\partial m}[t(f_{1,x} + f_{2,y})] = -q^a$$

$$- \frac{\delta}{2}(\tau_{m,m}^a + \tau_{y,y}^a). \qquad (31)$$

Similarly, by substituting for Q_r^b and Q_y^b in Equation (28c) their expressions from Equations (30) one obtains

$$M^b_{r,rr} + 2H^b_{,ry} + M^b_{y,yy} + f_3 + 2\theta f_1 - \frac{\theta\delta}{2} f_{3,r}$$

$$+ t(f_{1,x} + f_{2,y}) + \frac{\delta}{2}(f_{1,r} + f_{2,y})$$

$$- \frac{\theta\delta}{2} \frac{\partial}{\partial r}[t(f_{1,x} + f_{2,y})] = q^b$$

$$- \frac{\delta}{2}(\tau^b_{r,r} + \tau^b_{y,y}) \ . \qquad (32)$$

It is convenient to replace all derivatives with respect to m and r by derivatives with respect to x. Using the small angle approximations for θ it is seen that the following transformations are valid:

$$\frac{\partial}{\partial m} = \frac{\partial}{\partial x} \ , \qquad \frac{\partial^2}{\partial m^2} = \frac{\partial^2}{\partial x^2} \ ,$$

$$\frac{\partial}{\partial r} = \frac{\partial}{\partial x} \ , \qquad \frac{\partial^2}{\partial r^2} = \frac{\partial^2}{\partial x^2} \ . \qquad (33)$$

With Equations (33), the equations of equilibrium for the facings can be rewritten as follows:

$$N^a_{m,x} + S^a_{,y} - f_1 - \theta f_3 + \theta t(f_{1,x} + f_{2,y}) = - \tau^a_m \ , \quad (34a)$$

$$S^a_{,x} + N^a_{y,y} - f_2 = - \tau^a_y \ , \quad (34b)$$

$$M^a_{m,xx} + 2H^a_{,xy} + M^a_{y,yy} - f_3 + 2\theta f_1 + \frac{\theta\delta}{2} f_{3,x}$$

$$+ (t + \frac{\delta}{2})(f_{1,x} + f_{2,y}) - \frac{\theta\delta}{2} \frac{\partial}{\partial x}[t(f_{1,x} + f_{2,y})]$$

$$= - q^a - \frac{\delta}{2}(\tau^a_{m,x} + \tau^a_{y,y}) \ , \quad (34c)$$

$$N^b_{r,x} + S^b_{,y} + f_1 - \theta f_3 - \theta t(f_{1,x} + f_{2,y}) = \tau^b_r \ , \quad (35a)$$

$$S^b_{,x} + N^b_{y,y} + f_2 = \tau^b_y \ , \quad (35b)$$

$$M^b_{r,xx} + 2H^b_{,xy} + M^b_{y,yy} + f_3 + 2\theta f_1 - \frac{\theta\delta}{2} f_{3,x}$$

$$+ (t + \frac{\delta}{2})(f_{1,x} + f_{2,y}) - \frac{\theta\delta}{2} \frac{\partial}{\partial x}[t(f_{1,x} + f_{2,y})]$$

$$= q^b - \frac{\delta}{2} (\tau^b_{r,x} + \tau^b_{y,y}) \ . \quad (35c)$$

The stress resultants for facing a that appear in Equations (34) can be expressed in terms of the displacement components u^a, v^a, and w^a of the middle plane of the facing by substituting Equations (16) in Equations (23) and performing the indicated integrations. The relations thus obtained are

$$N_m^a = K(u^a_{,x} + \nu v^a_{,y}) \ , \tag{36a}$$

$$N_y^a = K(v^a_{,y} + \nu u^a_{,x}) \ , \tag{36b}$$

$$S^a = \frac{1-\nu}{2} K(u^a_{,y} + v^a_{,x}) \ , \tag{36c}$$

$$M_m^a = -D(w^a_{,xx} + \nu w^a_{,yy}) \ , \tag{36d}$$

$$M_y^a = -D(w^a_{,yy} + \nu w^a_{,xx}) \ , \tag{36e}$$

$$H^a = -(1-\nu) D w^a_{,xy} \ , \tag{36f}$$

where $D = \frac{E\delta^3}{12(1-\nu^2)}$ and $K = \frac{E\delta}{1-\nu^2}$. Here, the transformations of variables, Equations (33), have been used.

The stress resultants for facing b can be obtained by the method just outlined. They are

$$N_r^b = K(u^b_{,x} + \nu v^b_{,y}) \ , \tag{37a}$$

$$N_y^b = K(v^b_{,y} + \nu u^b_{,x}) \ , \tag{37b}$$

$$S^b = \frac{1-\nu}{2} K(u^b_{,y} + v^b_{,x}) \ , \tag{37c}$$

$$M_r^b = -D(w^b_{,xx} + \nu w^b_{,yy}) \ , \tag{37d}$$

$$M_y^b = -D(w^b_{,yy} + \nu w^b_{,xx}) \ , \tag{37e}$$

$$H^b = -(1-\nu)Dw^b_{,xy} \ . \tag{37f}$$

Equations (36) and (37) are used in Chapter IV.

CHAPTER IV

CONTINUITY CONDITIONS FOR DISPLACEMENTS AT THE INTERFACE

It is required that the displacement components of the face layers and core be continuous across the interfaces. This follows from the assumption that the face layers are rigidly connected to the core at their natural boundaries. Thus, at the interface between facing a and the core the following continuity conditions exist:

$$U = U_x^a \ ,$$

$$V = V_y^a \ ,$$

$$W = W_z^a \ . \tag{38}$$

Here, U, V, and W are the displacement components of a general point in the core in the x, y, and z directions,

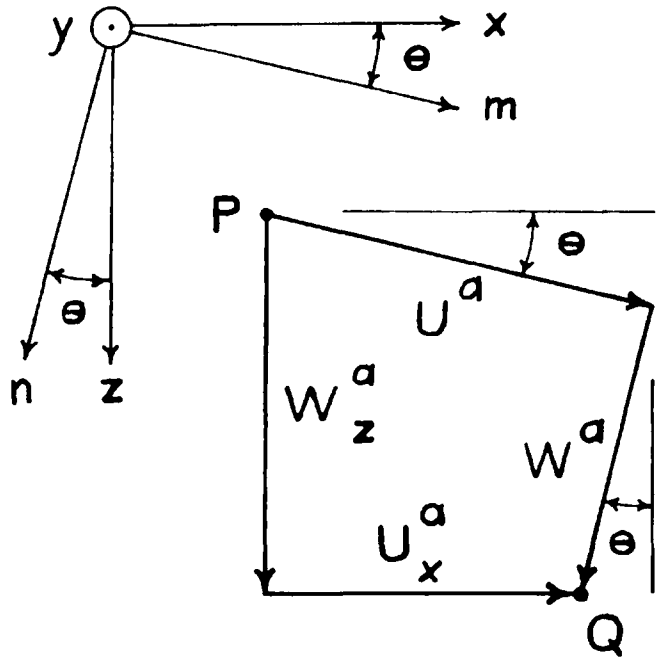

Figure 3a

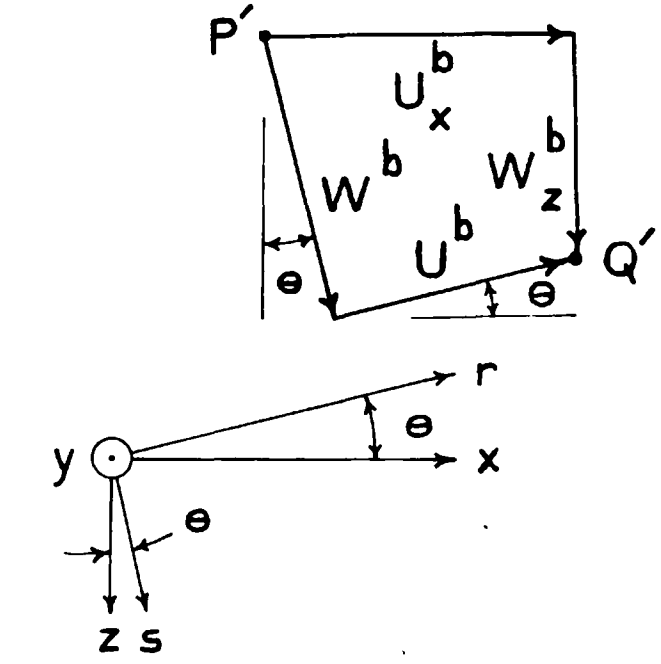

Figure 3b

respectively; the quantities U_x^a, V_y^a, and W_z^a are the displacement components of a general point in facing a in the x, y, and z directions, respectively. The quantities U_x^a and W_z^a are illustrated in Figure 3a.

A general point P in facing a that has displaced to point Q is shown in Figure 3a. From the figure the following relations can be obtained:

$$U_x^a = U^a - \theta W^a ,$$

$$V_y^a = V^a ,$$

$$W_z^a = \theta U^a + W^a , \qquad (39)$$

Here, the small angle approximations for θ are used.

By substituting for U^a, V^a, and W^a in Equations (39) their expressions from Equations (13) and evaluating the resulting expressions at $n = -\frac{\delta}{2}$ (the interface), the right hand sides of Equations (38) are determined in terms of u^a, v^a, and w^a. The left hand sides of these equations are obtained by evaluating Equations (7), (9), and (10) at $z = t$ (the interface). The foregoing method yields the following expressions for the continuity conditions at the interface between facing a and the core:

$$u^a + \frac{\delta}{2} w^a_{,x} - \theta w^a = F_3 - tF_{1,x} + \frac{t}{G_{xz}} f_1 , \qquad (40a)$$

$$v^a + \frac{\delta}{2} w^a_{,y} = F_2 - tF_{1,y} + \frac{t}{G_{yz}} f_2 , \qquad (40b)$$

$$w^a + \frac{\theta\delta}{2} w^a_{,x} + \theta u^a = F_1 . \qquad (40c)$$

Here, the transformations of variables, Equations (33), have been used.

Proceeding along the lines of the method outlined above, the following displacement-continuity conditions are obtained for the other interface:

$$u^b - \frac{\delta}{2} w^b_{,x} + \theta w^b = F_3 + tF_{1,x} - \frac{t}{G_{xz}} f_1 , \qquad (41a)$$

$$v^b - \frac{\delta}{2} w^b_{,y} = F_2 + tF_{1,y} - \frac{t}{G_{yz}} f_2 , \qquad (41b)$$

$$w^b + \frac{\theta\delta}{2} w^b_{,x} - \theta u^b = F_1 . \qquad (41c)$$

CHAPTER V

GOVERNING EQUATIONS

Before deriving the final system of governing equations it is convenient to introduce the following notations:

$$2\overset{+}{w} = w^a + w^b \,,$$

$$2\bar{w} = w^a - w^b \,,$$

$$2\overset{+}{M}_x = M_m^a + M_r^b \,,$$

$$2\bar{M}_x = M_m^a - M_r^b \,, \tag{42}$$

$$2\overset{+}{q} = q^a + q^b \,,$$

$$2\bar{q} = q^a - q^b \,,$$

and so on.

With the aid of Equations (42) a system of twelve independent equations involving twelve unknown functions can be obtained from the continuity conditions for displacements at the interfaces and the equilibrium equations for the face layers.

Thus, the continuity conditions, Equations (40) and (41), can be rewritten in the following form:

$$\overset{+}{u} + \frac{\delta}{2}\,\bar{w}_{,x} - \theta\bar{w} = F_3 \,, \tag{43a}$$

$$\bar{u} + \frac{\delta}{2}\,\overset{+}{w}_{,x} - \theta\overset{+}{w} = -\,tF_{1,x} + \frac{t}{G_{xz}}\,f_1 \,, \tag{43b}$$

$$\overset{+}{v} + \frac{\delta}{2}\,\bar{w}_{,y} = F_2 \,, \tag{43c}$$

$$\bar{v} + \frac{\delta}{2}\,\overset{+}{w}_{,y} = -\,tF_{1,y} + \frac{t}{G_{yz}}\,f_2 \,, \tag{43d}$$

$$\overset{+}{w} + \frac{\theta\delta}{2}\,\overset{+}{w}_{,x} + \theta\bar{u} = F_1 \,, \tag{43e}$$

$$\bar{w} + \frac{\theta\delta}{2}\,\bar{w}_{,x} + \theta\overset{+}{u} = 0 \,, \tag{43f}$$

where, for example, Equation (43a) is obtained by adding Equations (40a) and (41a) and dividing this sum by two.

In a similar manner, the equilibrium equations, Equations (34) and (35), can be rewritten as follows:

$$\overset{+}{N}_{x,x} + \overset{+}{S}_{,y} - \theta f_3 = - \bar{\tau}_x \ , \tag{44a}$$

$$\bar{N}_{x,x} + \bar{S}_{,y} - f_1 + \theta t(f_{1,x} + f_{2,y}) = - \overset{+}{\tau}_x \ , \tag{44b}$$

$$\overset{+}{S}_{,x} + \overset{+}{N}_{y,y} = - \bar{\tau}_y \ , \tag{44c}$$

$$\bar{S}_{,x} + \bar{N}_{y,y} - f_2 = - \overset{+}{\tau}_y \ , \tag{44d}$$

$$\overset{+}{M}_{x,xx} + 2\overset{+}{H}_{,xy} + \overset{+}{M}_{y,yy} + 2\theta f_1 + (t + \tfrac{\delta}{2})(f_{1,x} + f_{2,y}) - \tfrac{\theta\delta}{2} \tfrac{\partial}{\partial x}[t(f_{1,x} + f_{2,y})] = - \bar{q} - \tfrac{\delta}{2}(\overset{+}{\tau}_{x,x} + \overset{+}{\tau}_{y,y}) \ , \tag{44e}$$

$$\bar{M}_{x,xx} + 2\bar{H}_{,xy} + \bar{M}_{y,yy} - f_3 + \tfrac{\theta\delta}{2} f_{3,x} = - \overset{+}{q} - \tfrac{\delta}{2}(\bar{\tau}_{x,x} + \bar{\tau}_{y,y}) \ , \tag{44f}$$

where, for example, Equation (44a) is obtained by adding Equations (34a) and (35a) and dividing this sum by two.

The expressions for the stress resultants in terms of displacements, Equations (36) and (37), can also be re-written with the aid of Equations (42). Thus,

$$
\begin{aligned}
\overset{+}{N}_x &= K(\overset{+}{u}_{,x} + \nu\overset{+}{v}_{,y}) \ , \\
\overset{-}{N}_x &= K(\overset{-}{u}_{,x} + \nu\overset{-}{v}_{,y}) \ , \\
\overset{+}{N}_y &= K(\overset{+}{v}_{,y} + \nu\overset{+}{u}_{,x}) \ , \\
\overset{-}{N}_y &= K(\overset{-}{v}_{,y} + \nu\overset{-}{u}_{,x}) \ , \\
\overset{+}{S} &= \frac{1-\nu}{2} K(\overset{+}{u}_{,y} + \overset{+}{v}_{,x}) \ , \\
\overset{-}{S} &= \frac{1-\nu}{2} K(\overset{-}{u}_{,y} + \overset{-}{v}_{,x}) \ , \\
\overset{+}{M}_x &= - D(\overset{+}{w}_{,xx} + \nu\overset{+}{w}_{,yy}) \ , \\
\overset{-}{M}_x &= - D(\overset{-}{w}_{,xx} + \nu\overset{-}{w}_{,yy}) \ , \\
\overset{+}{M}_y &= - D(\overset{+}{w}_{,yy} + \nu\overset{+}{w}_{,xx}) \ , \\
\overset{-}{M}_y &= - D(\overset{-}{w}_{,yy} + \nu\overset{-}{w}_{,xx}) \ , \\
\overset{+}{H} &= -(1 - \nu)D\overset{+}{w}_{,xy} \ , \\
\overset{-}{H} &= -(1 - \nu)D\overset{-}{w}_{,xy} \ .
\end{aligned}
\tag{45}
$$

The functions F_3, F_2, and F_1 are determined directly in terms of displacement components from Equations (43a, c, and e), respectively; the functions f_3 and f_2 are obtained in terms of displacement components from Equations (44a) and (44d), respectively, with the aid of Equations (45).

The function f_1 is determined by eliminating F_1 between Equations (43b) and (43e). Thus,

$$f_1 = \frac{G_{xz}}{t}(\bar{u} + \theta t\bar{u}_{,x} - \theta\overset{+}{w} + \theta x\overset{+}{w}_{,x} + \frac{\theta t\delta}{2}\overset{+}{w}_{,xx}) \quad . \qquad (46)$$

Here, the approximation

$$t + \frac{\delta}{2} = \theta x \ ,$$

has been used. For emphasis,

$$t = t(x) \ .$$

Equations (47) are the final system of six simultaneous differential equations; they are obtained by eliminating f_1, f_2, f_3, F_1, F_2, and F_3 from Equations (43f), (44b, c, d, e, and f). Two of these equations, Equations (43f) and (44c), involve only the unknown displacement components, and therefore require no manipulations; they are the first and third governing equations, respectively.

The second governing equation is obtained by eliminating f_1 and f_2 from Equation (44b) by means of Equations

(46) and (44d). The final form of this equation is obtained with the aid of the fourth governing equation.

The fourth governing equation is obtained by eliminating F_1 between Equations (43d) and (43e), and eliminating f_2 from the resulting expression by means of Equation (44d). The fifth governing equation is obtained by eliminating the term

$$\frac{\partial}{\partial x}[t(f_{1,x} + f_{2,y})]$$

in Equation (44e) by means of Equation (44b), substituting for $f_{1,x}$ in the resulting expression its expression obtained from Equation (44b), and then eliminating f_1 and f_2 from the expression obtained by the foregoing manipulation by means of Equations (46) and (44d).

The last governing equation is obtained by eliminating f_3 from Equation (44f) by means of Equation (44a).

$$\bar{w} + \frac{\theta\delta}{2}\bar{w}_{,x} + \theta\overset{+}{u} = 0 , \tag{47a}$$

$$\bar{N}_{x,x} + \bar{S}_{,y} - \frac{G_{xz}}{t}(\bar{u} - \theta^2 t^2 \bar{u}_{,xx} - \theta\overset{+}{w} + \theta x\overset{+}{w}_{,x}$$

$$- \theta t^2 \overset{+}{w}_{,xx} - \frac{\theta^2 t^2 \delta}{2}\overset{+}{w}_{,xxx}) + \theta\, G_{yz}(\bar{v}_{,y} + \theta t\bar{u}_{,yy}$$

$$+ \theta x\overset{+}{w}_{,yy} + \frac{\theta t\delta}{2}\overset{+}{w}_{,xyy}) = -\overset{+}{\tau}_x , \tag{47b}$$

$$\overset{+}{S}_{,x} + \overset{+}{N}_{y,y} = - \bar{\tau}_y \ , \tag{47c}$$

$$\bar{v} + \theta t \bar{u}_{,y} + \theta x \overset{+}{w}_{,y} + \frac{\theta t \delta}{2} \overset{+}{w}_{,xy} - \frac{t}{G_{yz}} (\bar{S}_{,x} + \bar{N}_{y,y})$$

$$= + \frac{t}{G_{yz}} \overset{+}{\tau}_y \ , \tag{47d}$$

$$\overset{+}{M}_{x,xx} + 2\overset{+}{H}_{,xy} + \overset{+}{M}_{y,yy} + \frac{\delta}{2} (\bar{N}_{x,xx} + 2\bar{S}_{,xy} + \bar{N}_{y,yy})$$

$$- \frac{1}{\theta} (\bar{N}_{x,x} + \bar{S}_{,y}) + \frac{G_{xz}}{\theta t} (\bar{u} + \theta t \bar{u}_{,x} - \theta \overset{+}{w} + \theta x \overset{+}{w}_{,x}$$

$$+ \frac{\theta t \delta}{2} \overset{+}{w}_{,xx}) = - \bar{q} + \frac{1}{\theta} \overset{+}{\tau}_x - \delta (\overset{+}{\tau}_{x,x} + \overset{+}{\tau}_{y,y}) \ , \tag{47e}$$

$$\bar{M}_{x,xx} + 2\bar{H}_{,xy} + \bar{M}_{y,yy} + \frac{\delta}{2} (\overset{+}{N}_{x,xx} + \overset{+}{S}_{,xy})$$

$$- \frac{1}{\theta} (\overset{+}{N}_{x,x} + \overset{+}{S}_{,y}) = - \overset{+}{q} + \frac{1}{\theta} \bar{\tau}_x$$

$$- \delta (\bar{\tau}_{x,x} + \tfrac{1}{2} \bar{\tau}_{y,y}) \ . \tag{47f}$$

The approximation

$$1 \pm \theta^2 = 1 \ ,$$

has been used in Equations (47).

By utilizing Equations (45) the foregoing system can be expressed entirely in terms of the six displacement components of the middle planes of the facings. Thus,

$$\bar{w} + \frac{\theta\delta}{2}\bar{w}_{,x} + \theta\overset{+}{u} = 0 \ , \tag{48a}$$

$$\bar{u}_{,xx} + \frac{1-\nu}{2}\bar{u}_{,yy} + \frac{1+\nu}{2}\bar{v}_{,xy} - \frac{G_{xz}}{Kt}\left(\bar{u} - \theta^2t^2\bar{u}_{,xx} - \theta\overset{+}{w} + \theta x\overset{+}{w}_{,x} - \theta t^2\overset{+}{w}_{,xx} - \frac{\theta^2t^2\delta}{2}\overset{+}{w}_{,xxx}\right) + \frac{\theta G_{yz}}{K}\left(\bar{v}_{,y} + \theta t\bar{u}_{,yy} + \theta x\overset{+}{w}_{,yy} + \frac{\theta t\delta}{2}\overset{+}{w}_{,xyy}\right) = -\frac{1}{K}\overset{+}{\tau}_x \ , \tag{48b}$$

$$\overset{+}{v}_{,yy} + \frac{1-\nu}{2}\overset{+}{v}_{,xx} + \frac{1+\nu}{2}\overset{+}{u}_{,xy} = -\frac{1}{K}\bar{\tau}_y \ , \tag{48c}$$

$$\bar{v}_{,yy} + \frac{1-\nu}{2}\bar{v}_{,xx} + \frac{1+\nu}{2}\bar{u}_{,xy} - \frac{G_{yz}}{Kt}\left(\bar{v} + \theta t\bar{u}_{,y} + \theta x\overset{+}{w}_{,y} + \frac{\theta t\delta}{2}\overset{+}{w}_{,xy}\right) = -\frac{1}{K}\overset{+}{\tau}_y \ , \tag{48d}$$

$$\nabla^4 \overset{+}{w} - \frac{6}{\delta}\left(\nabla^2 \bar{u}_{,x} + \nabla^2 \bar{v}_{,y}\right) + \frac{12}{\delta^2\theta}\left(\bar{u}_{,xx} + \frac{1-\nu}{2}\,\bar{u}_{,yy}\right.$$

$$\left. + \frac{1+\nu}{2}\,\bar{v}_{,xy}\right) - \frac{G_{xz}}{Dt\theta}\left(\bar{u} + \theta t\bar{u}_{,x} - \theta \overset{+}{w} + \theta x \overset{+}{w}_{,x}\right.$$

$$\left. + \frac{\theta t\delta}{2}\,\overset{+}{w}_{,xx}\right) = \frac{1}{D}\,\bar{q} - \frac{1}{D\theta}\,\overset{+}{\tau}_x + \frac{\delta}{D}\left(\overset{+}{\tau}_{x,x} + \overset{+}{\tau}_{y,y}\right) , \quad (48e)$$

$$\nabla^4 \overset{-}{w} - \frac{6}{\delta}\,\frac{\partial}{\partial x}\left(\overset{+}{u}_{,xx} + \frac{1-\nu}{2}\,\overset{+}{u}_{,yy} + \frac{1+\nu}{2}\,\overset{+}{v}_{,xy}\right)$$

$$+ \frac{12}{\theta\delta^2}\left(\overset{+}{u}_{,xx} + \frac{1-\nu}{2}\,\overset{+}{u}_{,yy} + \frac{1+\nu}{2}\,\overset{+}{v}_{,xy}\right) = \frac{1}{D}\,\overset{+}{q}$$

$$- \frac{1}{D\theta}\,\bar{\tau}_x + \frac{\delta}{D}\left(\bar{\tau}_{x,x} + \tfrac{1}{2}\bar{\tau}_{y,y}\right) . \quad (48f)$$

Here, ∇^2 is the plane-harmonic operator,

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} ,$$

and

$$\nabla^4 = \frac{\partial^4}{\partial x^4} + 2\,\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4} .$$

Equation (49) below is an alternate, more symmetric form for Equation (48f) and may be used instead of the latter equation; it is obtained from Equation (48f) with the aid of Equation (48c),

$$\nabla^4 \bar{w} - \frac{6}{\delta}\left(\nabla^2 \overset{+}{u}_{,x} + \nabla^2 \overset{+}{v}_{,y}\right) + \frac{12}{\theta\delta^2}\left(\overset{+}{u}_{,xx} + \frac{1-\nu}{2}\overset{+}{u}_{,yy}\right.$$

$$\left. + \frac{1+\nu}{2}\overset{+}{v}_{,xy}\right) = \frac{1}{D}\overset{+}{q} - \frac{1}{D\theta}\bar{\tau}_x + \frac{\delta}{D}\left(\bar{\tau}_{x,x} + \bar{\tau}_{y,y}\right) . \quad (49)$$

CHAPTER VI

DISCUSSION

The system of governing equations, Equations (48), is certainly not a simple system of equations. However, for many sandwich plates it may be possible to neglect certain terms as small compared to other terms. One criterion, other than

$$1 \gg \theta^2 ,$$

that can be used to simplify the foregoing equations by neglecting small terms, rests on the possibility that

$$1 \gg \frac{G_{xz}}{E} , \text{ and } 1 \gg \frac{G_{yz}}{E} ,$$

where G_{xz} and G_{yz} are the shear moduli of the core, and E is Young's modulus for the facings. In certain sandwich plates with honeycomb cores [5] the foregoing inequalities are valid.

The boundary value problem to be solved is completely formulated if the governing equations are supplemented by appropriate boundary conditions.

By solving the given boundary value problem the displacement components $\overset{+}{u}$, $\bar{u}$, $\overset{+}{v}$, $\bar{v}$, $\overset{+}{w}$, and $\bar{w}$ are determined. Once these components are known all other quantities of interest can be calculated by means of expressions presented in the text.

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