A TRANSISTOR INTEGRATING AMPLIFIER

by

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Approved: ____________________________
Director of Thesis

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SIGNED:  

Harry Ramish
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INTRODUCTION

Modern science, engineering, and technology must solve complex mathematical problems and perform tremendous amounts of computation if their progress is to continue at the present rapid rate. This situation has led, almost inevitably, to the design and development of the modern high speed electronic computer. Research and development in high speed aviation, guided missiles, atomic energy, and many other fields would be slowed down considerably without the use of these electronic devices.

In view of the ever-growing importance and application of the electronic computer, it is proposed to design, build, and test an integrating circuit using a relatively new electronic device, the transistor. The transistor, as compared to the vacuum tube, uses less power, is more rugged, and is much smaller. Therefore, an integrating circuit using the transistor would be an improvement over the present vacuum tube integrating circuit.


Chapter 1.

THE PROBLEM AND ITS LIMITATIONS

(1.1) INTRODUCTION

There are two main classes of computer, digital and analogue. Digital computers consist essentially of counters capable of adding and registering in discrete steps, making use of the so-called binary system. Thus a desk calculator may be considered a digital computer.

By contrast, an analogue computer uses physical quantities such as shaft rotations, displacements, and voltages to simulate mathematical equations. While the analogue computer does not have the accuracy of a digital computer, it is much less expensive and is easier to operate and maintain.

---


(1.2) THE PROBLEM AND ITS LIMITATIONS

In the analogue computer, integration of a variable with respect to time is one of the basic operations. Many circuits have been devised to perform this operation, but all make use of vacuum tube amplifiers. An integrating circuit using the transistor would have certain advantages. As is well known, the transistor is much smaller than a vacuum tube, uses much less power, is extremely rugged both mechanically and electrically, and has a much longer operating life. The latest evidence indicates that the life expectancy of the transistor may well be one million hours. For these reasons, it appears that an integrating amplifier using the transistor would possess certain definite advantages. It was the purpose of the present study to determine the feasibility of such an amplifier.

As the primary object of this investigation was to ascertain the feasibility of a transistor integrating amplifier, no attempt was made to secure the highest possible gain, to secure 100% accuracy of integration, or to make the amplifier response constant to d-c. In general, it can be said that the design requirements were set up mainly to determine the possibility of a transistor integrating amplifier.

(1.3) THE INTEGRATION PROCESS

The process of electrical integration is based on the fundamental property of the capacitor. Figure (1.1) shows a simple RC circuit. Using the Laplace transform, the transfer function is

\[ \frac{E_A(s)}{E_x(s)} = \frac{1}{RC} \times \frac{1}{s + 1/RC} \]  

(1.1)

The desired transfer function for an integrator is

\[ \frac{E_A(s)}{E_x(s)} = \frac{K}{s} \]  

(1.2)

Equations (1.1) and (1.2) will be equal if 1/RC is made small, compared to s, for all frequencies of interest. For RC to be large, requires that R or C, or both, be large.

If R is made large, it is not difficult to show that the input signal will be highly attenuated. If C is made large, not only is the signal attenuated, but the high capacitor leakage resistance effectively decreases the value of the time constant. In addition the output circuit is easily loaded, impairing the validity of the transfer function, and consequently, of the integration.

These disadvantages may be largely overcome through the use of a high gain amplifier with negative feedback. With this system it is possible to perform accurate integration with little attenuation and with reasonable values of resistance and capacitance.

An integrator of this type is shown in block diagram in Figure (1.2). The triangle is the usual symbol for an amplifier with a gain
of A. Assuming negligible current into the amplifier, the nodal equation becomes

$$RCS (1 - 1/A) E_0(s) = C(s)/A - E_i(s)$$  \hspace{1cm} (1.3)

Solving for $E_0(s)$

$$E_0(s) = \frac{A E_i(s)}{RCS (1 - A) + 1}$$  \hspace{1cm} (1.4)

To avoid ambiguity, $A$ represents the absolute value of gain. Where phase inversion occurs, the gain will be represented by $-A$. If $A$ is much larger than unity for all frequencies of interest, equation (1.3) becomes

$$E_o = \frac{1}{RC} \int E_i \, dt$$  \hspace{1cm} (1.5)

It is clear that the output is the integral of the input, modified by the scale factor $1/RC$.

A more detailed analysis of this integrating amplifier is presented in Chapter 4. However, the above general discussion indicates the basic requirements of a good integrating amplifier. The amplifier should have a high gain. The input impedance should be high, indicating a cathode follower input.

In this investigation the following minimum criteria were set up:

1. Gain of the amplifier—100.
2. Input impedance—100,000 ohms.
3. Frequency range—20 to 1,000 cps.
Fig. 1.1: The RC circuit.

Fig. 1.2: Block diagram of integrating amplifier.
(1.4) OVERVIEW OF THE THESIS

The following is an outline of the organization of this study. Chapter 1 has introduced the problem and discussed its importance and its limitations.

Chapter 2 presents the considerations governing the preliminary design of the integrating amplifier.

Chapter 3 consists of the actual amplifier design, including the derivation of the actual design equations.

Chapter 4 contains the theoretical analysis of the feedback amplifier.

Chapter 5 gives the experimental results, including the necessary graphs and oscillograms.

Chapter 6 interprets the results and contains certain recommendations for further investigation.
Chapter 2.

THE PRELIMINARY DESIGN

(2.1) INTRODUCTION

The only transistor available for this investigation was the Motorola XN-2. This is an experimental model and is not in production. The XN-2 is a germanium p-n-p fused junction type transistor, which was designed for use at audio or intermediate frequencies. As there were no manufacturer's curves or data available, it became necessary to determine the characteristics of the transistor experimentally. The input and output characteristics were plotted and from these curves the small signal parameters were obtained.

The small signal parameters are defined as follows: 6

\( a_{11} \) is the input resistance with the collector shorted to ground.

\[
a_{11} = \left. \frac{V_{be}}{i_b} \right|_{V_{ce} = 0}
\]  \hspace{1cm} (2.1)

\( a_{12} \) is the voltage feedback ratio with base an open circuit.

\[
a_{12} = \left. \frac{V_{be}}{V_{ce}} \right|_{i_b = 0}
\]  \hspace{1cm} (2.2)

\( a_{21} \) is the ratio of the base current to the collector current with the collector shorted to ground.

\[
a_{21} = \frac{i_b}{i_c} \bigg| V_{ce} = 0
\]

(2.3)

\( a_{22} \) is the output admittance with the base open-circuited.

\[
a_{22} = \frac{i_c}{V_{ce}} \bigg| i_b = 0
\]

(2.4)

It must be emphasized that these parameters are defined in terms of the grounded emitter as being the reference case.
(2.2) THE CHARACTERISTIC CURVES AND OPERATING POINT

The two defining curves of a transistor are the output and the input curves. The output curve is a plot of output current vs. output voltage—i.e. $I_C$ vs. $V_{ce}$. The input characteristic is a plot of input current vs. input voltage—i.e. $I_B$ vs. $V_{be}$. The circuit used to determine these characteristic curves is shown in Figure (2.1).

Because these characteristics vary from transistor to transistor, ten transistors were tested. Three were finally selected as being fairly similar in operating values. The curves for these three transistors are shown in Figures (2.2) through (2.7).

The Q-point was chosen so that the transistor would operate, at least for small signals, over a fairly linear portion of the characteristic curves. The point chosen was

\[ I_B = -5 \times 10^{-6} \text{ amperes} \]
\[ V_{ce} = -4 \text{ volts} \]

The same point was chosen for all three transistors.
Fig 2.1: Circuit used in making d-c measurements.
Fig. 2.2: Collector current versus collector voltage for Motorola XN-2 transistor. #1.
Fig. 2.3: Base current versus base voltage for Motorola XN-2 transistor M1.
Fig. 2.4: Collector current versus collector voltage for Motorola 2N3904 transistor 

Fig. 2.5: Base current versus base voltage for Motorola XN-2 transistor #2.
Fig 2.6: Collector current versus collector voltage for Motorola KN-2 transistor #3
Fig. 2.7 - Base current versus base voltage for Motorola XN-2 transistor #3.
(2.3) DETERMINATION OF PARAMETERS

Once the operating point is selected, the following values are fixed: \( I_c, I_b, V_{ce}, \) and \( V_{be} \). Hence it is possible to calculate \( a_{11}, a_{12}, a_{21}, \) and \( a_{22} \) using equations (2.1) through (2.4).

The result of these calculations is shown in Table (2.1). The values actually used in designing the amplifier were the average values of the three transistors, rounded off to the nearest convenient value.

<table>
<thead>
<tr>
<th>Transistor #</th>
<th>( a_{11} ) ( \Omega )</th>
<th>( a_{21} ) ( \Omega )</th>
<th>( a_{12} \times 10^{-6} )</th>
<th>( 1/a_{22} ) ( \Omega )</th>
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<tr>
<td>#1</td>
<td>5,500</td>
<td>91</td>
<td>800 x 10^{-6}</td>
<td>18,200</td>
</tr>
<tr>
<td>#2</td>
<td>5,500</td>
<td>100</td>
<td>600 x 10^{-6}</td>
<td>16,670</td>
</tr>
<tr>
<td>#3</td>
<td>5,000</td>
<td>96</td>
<td>600 x 10^{-6}</td>
<td>15,000</td>
</tr>
<tr>
<td>Average</td>
<td>5,300</td>
<td>96</td>
<td>670 x 10^{-6}</td>
<td>16,610</td>
</tr>
<tr>
<td>Value used</td>
<td>5,500</td>
<td>95</td>
<td>600 x 10^{-6}</td>
<td>17,000</td>
</tr>
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</table>
(2.4) THE PRELIMINARY DESIGN

The basic requirements of an integrating amplifier were determined above to be:

(1) High forward gain
(2) High input impedance
(3) Phase inversion for stability

It was obvious from the very beginning that a single stage amplifier could not meet the above requirements. To obtain the necessary high input impedance, the first stage was designed as a grounded collector with an input impedance of 200,000 ohms. The second stage was designed as a grounded emitter stage with a forward gain of 200. This stage has the required phase inversion. The preliminary design is shown in block diagram form in Figure (2.8).
Fig. 2.8: Block diagram of transistor amplifier.

\[ Z_{in} = 2 \times 10^5 \ \Omega \]

- Grounded collector stage
- Grounded emitter stage
(2.5) GENERAL PROCEDURE

It was decided to design the grounded emitter stage first. Then, knowing the gain and the impedances of this stage, it is possible to design the grounded collector stage. The input impedance of the grounded emitter stage is considered part of the output impedance of the grounded collector stage. It is then possible to compute the overall gain of the cascade as the product of the gains of the individual stages.
Chapter 3.

DESIGN OF THE INTEGRATING AMPLIFIER

(3.1) INTRODUCTION

The design requirements of the overall amplifier were set up as

1. Forward gain = 150
2. Input impedance = 150,000 ohms
3. Phase inversion

The gain and the phase inversion were to be achieved by a grounded emitter stage and the high input impedance by a grounded collector stage.

The transistor parameters used were as follows:

\[ a_{11} = 5,500 \text{ ohms} \]
\[ a_{12} = 95 \]
\[ a_{21} = 600 \times 10^{-6} \]
\[ 1/a_{22} = 17,000 \text{ ohms} \]
(3.2) DESIGN OF THE GROUNDED Emitter STAGE

The equivalent circuit of a grounded emitter amplifier is shown in Figure (3.1).

Define $R_m$ as the total equivalent collector circuit resistance:

$$R_m = \frac{R_L / a_{32}}{R_L + 1 / a_{32}}$$  \hspace{1cm} (3.1)

The voltage gain, using standard methods, becomes

$$A = -\frac{a_{31} R_m}{a_{11} - a_{12} a_{31} R_m}$$  \hspace{1cm} (3.2)

where the minus sign indicates phase inversion. Solving equation (3.2) for $R_m$

$$R_m = \frac{a_{11} A}{a_{21} + a_{12} a_{31} A}$$  \hspace{1cm} (3.3)

For the required gain of 200, the value of $R_m$ is 10,340 ohms.

To find the load impedance, $R_L$, solve equation (3.1) for $R_L$.

$$R_L = \frac{R_m}{1 - a_{32} R_m}$$  \hspace{1cm} (3.4)

Substituting the known values, the calculated size of $R_L$ is 26,100 ohms. The nearest HTMA value is 27,000 ohms.

The next step is to determine the input impedance of this stage. First define a voltage generator in the following manner:

$$V_{ce} = A_{bc} V_{be}$$  \hspace{1cm} (3.5)
Applying the compensation theorem to the output circuit, and solving the input circuit, results in the following expression for the input impedance:

\[ R_{\text{in}} = \frac{a_{11}}{1 + a_{12} A_{bc}} \]  

(3.6)

Since the load is a two-terminal network, \( A_{bc} \) is the same as the gain of the amplifier and is therefore equal to 200. Substituting the proper values in equation (3.6) shows that \( R_{\text{in}} \) has a value of 4,900 ohms. This is the input impedance of the grounded emitter stage with a gain of 200 and a load impedance of 27,000 ohms. This must be considered part of the output impedance of the preceding grounded collector stage.

The output impedance is essentially \( R_m \) or 11,000 ohms.

Figure (3.2) shows the grounded emitter stage in block diagram form.
Fig. 3.1: Equivalent circuit of the grounded emitter amplifier.

Fig. 3.2: Block diagram of grounded emitter stage.
The design equations for this stage are derived in terms of the grounded emitter as the reference case. The complete grounded emitter equivalent circuit is shown in Figure (3.3). For the low frequency and mid frequency cases, \( C_{\text{in}} \) and \( C_{\text{ce}} \) can be neglected. The equation for the voltage gain in terms of \( R_{\text{in}} \) is

\[ A_{r_e} = \frac{a_{21} \, R_m}{R_{\text{in}} + R_L} \]  

(3.7)

where

\[ R_m = \frac{a_{22} \, R_L}{a_{22} + R_L} \]  

(3.8)

\[ R_{\text{in}} = \frac{a_{11}}{1 + a_{12} \, A_{bc}} \]  

(3.9)

\( A_{r_e} \) = voltage gain for the grounded emitter reference case.

Later in this analysis use will be made of the fact that, for a two terminal load, \( A_{r_e} = A_{bc} \).

Grounding the collector terminal and redrawing the equivalent circuit, results in the circuit of Figure (3.4). Define another voltage generator

\[ V_{\text{e c}} = A_{be} \, V_{bc} \]  

(3.10)
Using the compensation theorem, the circuit becomes as shown in Figure (3.5). The loop equation is

$$ e_i (1 - A_{be}) = i_b R_{in c} $$  \hspace{1cm} (3.11)

But

$$ R_{in c} = \frac{e_i}{i_b} = \frac{R_{in c}}{1 - A_{be}} $$  \hspace{1cm} (3.12)

As a result, the equivalent circuit finally simplifies to Figure (3.6). The voltage gain equation can now be derived as follows

$$ i_b = \frac{e_i}{R_{in c}} $$  \hspace{1cm} (3.13)

Therefore

$$ a_{31} i_b = \frac{a_{31} e_i}{R_{in c}} $$  \hspace{1cm} (3.14)

$$ e_o = V_{ec} = \frac{a_{31} e_i}{R_{in c}} \times R_m $$  \hspace{1cm} (3.15)

The voltage gain is

$$ A_c = \frac{e_o}{e_i} = \frac{a_{31} R_m}{R_{in c}} $$  \hspace{1cm} (3.16)

Substituting the value of $R_{in c}$ from equation (3.12),

$$ A_c = \frac{a_{31} R_m}{R_{in c}} (1 - A_{be}) $$  \hspace{1cm} (3.17)
But the first term on the right is equal to $A_r e$ from equation (3.7). Therefore the voltage gain becomes

$$A_c = A_r e (1 - A_{be}) \quad (3.18)$$

For a two terminal load

$$A_{be} = A_c \quad (3.19)$$

Solving equations (3.18) and (3.19) simultaneously, there results finally

$$A_{be} = \frac{A_r e}{1 + A_r e} \quad (3.20)$$

This gives the gain of the grounded collector stage in terms of the grounded emitter stage as reference.
Fig. 3.3: Complete equivalent circuit of the grounded emitter amplifier.

Fig. 3.4: Equivalent circuit of the grounded collector amplifier.
Fig. 3.5: Simplified equivalent circuit.

Fig. 3.6: Final form of the equivalent circuit.
The input impedance of the grounded collector stage is:

\[
R_{in c} = \frac{R_{in e}}{1 - A_{be}} \quad (3.21)
\]

Solving for \( A_{be} \):

\[
A_{be} = \frac{R_{in c} - R_{in e}}{R_{in c}} \quad (3.22)
\]

Equating equations (3.20) and (3.22):

\[
\frac{A_{be}}{1 + A_{be}} = \frac{R_{in c} - R_{in e}}{R_{in c}} \quad (3.23)
\]

Use equation (3.20) and solve for \( A_{bc} \):

\[
A_{bc} = \frac{R_{in c} - a_{11}}{a_{21} - a_{12} R_{in c}} \quad (3.24)
\]

Solve equations (3.23) and (3.24) simultaneously for \( R_m \), yielding

\[
R_m = \frac{R_{in c} - a_{11}}{a_{21} (1 - a_{12})} \quad (3.25)
\]

where

\[
R_m = \frac{\kappa_{21} R_e}{\kappa_{22} + R_e} \quad (3.26)
\]

\( R_e \) is the parallel combination of the load impedance and the input impedance of the following grounded emitter stage.
Thus, knowing the parameters of the transistor and the desired input resistance, it is possible to calculate the required load impedance.
(3.5) CALCULATED VALUES OF THE GROUNDED COLLECTOR STAGE

The required input impedance for the cascaded amplifier was 200,000 ohms. Using this value, $R_m$ has a value of 2,100 ohms. Therefore $R_e$ becomes equal to 3,400 ohms and $R_L$, the load impedance must be 11,100 ohms.

The calculated gain is .96.
BIAS CIRCUIT DESIGN EQUATIONS

The d-c connections for a transistor amplifier is shown in Figure (3.7). The loop equation around the base-emitter circuit is

\[ I_a = \frac{V_{be} + I_e R_e}{R_a} \]  

(3.27)

The node equation at the base node is

\[ I_1 = I_b + I_a = I_b + \frac{V_{be} + I_e R_e}{R_a} \]  

(3.28)

Finally

\[ V_{cc} = I_1 R_1 + I_a R_2 \]  

(3.29)

Solving for \( R_1 \)

\[ R_1 = \frac{V_{cc} - I_a R_2}{I_1} \]  

(3.30)

Substituting the values for \( I_1 \) and \( I_2 \) in equation (3.30)

\[ R_1 = \frac{V_{cc} - V_{be} - I_e R_e}{V_{be} + I_e R_e + I_b R_2} \times R_2 \]

From the determination of the Q-point, \( I_b, I_c, V_{ce}, \) and \( V_{be} \) are known. From other considerations \( I_e, V_{cc}, \) and \( R_e \) are fixed. Thus, only \( R_1 \) and \( R_2 \) are unknown. The maximum value of \( R_1 \) is

\[ R_1 (\text{max}) = \frac{V_{cc}}{I_e} \]

Now \( R_1 \) and \( R_2 \) are both in parallel with the input impedance of the transistor, and therefore they should both be chosen much larger than...
the input impedance of the transistor. However, in the final analysis, the choice of $R_1$ and $R_2$ is a cut and try procedure, subject to the limitations outlined above.

The value of $V_{cc}$ was set at 30 volts. For the grounded collector stage $R_1$ becomes $1.8 \times 10^6$ ohms and $R_2 = 56 \times 10^6$ ohms. For the grounded emitter stage, $R_1$ is $0.82 \times 10^6$ ohms and $R_2$ is $1 \times 10^6$ ohms.

As a result, the input impedance of the cascade becomes 136,000 ohms, which is greater than the minimum design criterion of 100,000 ohms.
Fig. 3.7: Bias circuit of the grounded emitter amplifier.

Fig. 3.8: Normalized exponential curve for $(1-e^{-\frac{t}{T}})$. 
(3.7) DESIGN OF THE COUPLING CIRCUIT

To couple the two stages together, a simple RC network was designed. Here R is the input impedance of the grounded emitter stage, which was calculated to be 4,900 ohms. The transfer function is

\[
\frac{E_o(s)}{E_i(s)} = \frac{R C S}{R C S + 1} \tag{3.31}
\]

For ideal coupling, it is desirable that \(e_0/e_1 = 1\). This requires that RCS be much greater than one for all frequencies of interest. If RCS = 20, the transfer function will differ from unity by less than 5%.

With the lower frequency limit set at 20 cps, the necessary value of C is \(32 \times 10^{-6}\) farads. A capacitor of \(100 \times 10^{-6}\) farads was used.

A similar analysis shows that the coupling capacitor for the first stage should be \(4 \times 10^{-6}\) farads.

This completed the design of the amplifier proper. The next step was to design the feedback loop.
(3.8) DESIGN OF THE FEEDBACK LOOP

There are several possible approaches to this problem. The one chosen here is based on the similarity which exists between a time integral and a very long time constant exponential. From this point of view, integration operations may be considered as infinite time constant exponential operations.

The transfer function of an RC network is given by

\[ \frac{e_o(s)}{e_i(s)} = \frac{1}{RC} \frac{1}{s + 1/RC} \]  \hspace{1cm} (1.1)

where \( e_o(s) \) is the transform voltage across the capacitor. Using operational methods, the response in the time domain to a unit step of voltage is easily shown to be

\[ e_o = 1 - e^{-\frac{t}{RC}} \]  \hspace{1cm} (3.32)

A plot of this function is shown in Figure (3.8).

The time interval for an exponential function to decrease to 37% of its initial value, or to increase to 63% of its final value, is known as the time constant of the function. From an examination of equation (3.32) it is apparent that the time constant of an RC network is simply the product RC.

The slope of the output voltage is obtained by the process of differentiating equation (3.32). The result is

\[ \frac{de_o}{dt} = \frac{1}{RC} e^{-\frac{t}{RC}} \]  \hspace{1cm} (3.33)
If the output is to be the true integral of the step input, it must be a straight line. This requires that the slope given in equation (3.33) be a constant. For this to occur, the time constant must be large in comparison with the time of the frequencies of interest.

Let it be required that the integrated function be within 5% of the true integral. The slope at \( t = 0 \) is \( 1/RC \). For the output to be within 5% of the true integral requires that

\[
(1 - \frac{x}{RC}) = 0.95
\]  

(3.34)

Solving this equation results in the requirement that

\[
RC = 20 \times \frac{T}{x}
\]  

(3.35)

Choosing \( C = 1 \times 10^{-6} \) farads as a reasonable value, the calculated size of \( R \) at the lowest frequency of interest is \( 5 \times 10^6 \) ohms. Substituting these values in equation (3.32) shows that there will be a signal attenuation of 95%. This points up one of the primary drawbacks of the simple RC circuit as an integrator. The situation is remedied by the use of the feedback amplifier.

Figure (1.2) shows the integrating amplifier. Making use of the Miller effect, the amplifier can be redrawn as in Figure (3.9). Here the feedback capacitor is multiplied by the gain of the amplifier. Making use of equations (3.32) to (3.35), with \( C \) replaced by \( (A+1)C \), the value of \( R \) is calculated to be 26,000 ohms. The nearest RTMA value is 27,000 ohms, which was used.

The output voltage can now be represented by

\[
E_0 = A \left( 1 - \frac{x}{RC} \right) \left( 1 - \frac{x}{(1+1)RC} \right)
\]  

(3.36)
The amplifier forward gain, $A$, not only increases the time constant, but also amplifies the capacitor output voltage. In this way, the excessive attenuation of the simple RC circuit is considerably reduced. This factor is discussed and verified experimentally in Section (5.3).
Fig. 3.9: Equivalent circuit of Miller integrator.
(3.9) THE COMPLETE INTEGRATING AMPLIFIER

The complete integrating amplifier is shown in Figure (3.10). Transistor number three was used in the grounded collector stage, and transistor number two in the grounded emitter stage. All the experimental work was done using this configuration.
Fig. 3.10 Complete transistor integrating amplifier.
Chapter 4.

THEORETICAL ANALYSIS OF THE INTEGRATING AMPLIFIER

(4.1) INTRODUCTION

Having designed the amplifier, it became important to subject it to a theoretical analysis to determine its response as certain parameters were varied. In this chapter, the effect of certain variables, such as input impedance and gain, on the operation of the integrating amplifier, is analyzed. In Chapter 5, the experimental results are presented and are compared with the theoretical predictions.
(4.2) EFFECT OF AMPLIFIER INPUT IMPEDANCE

The integrating amplifier, together with the amplifier input impedance represented by $R_g$, can be shown as in Figure (4.1). Writing the nodal equation at the input node

$$\frac{e_i - e_0/A + (e_0 - e_0/\beta)SC}{R_i} = \frac{e_0}{A R_g}$$  \hspace{1cm} (4.1)

Solving for $e_0$

$$e_0 = \frac{A e_i}{(1-A)RC\tau + 1 + R_i/R_g} \hspace{1cm} (4.2)$$

The desired transfer function is given by

$$\frac{e_o}{e_i} = \frac{K}{S}$$  \hspace{1cm} (1.2)

For good integration, the ratio $R_i/R_g$ should be small. In other words, the input impedance should be as large as possible, as it has the effect of shunting the input resistor, $R_i$. 
Fig. 4.1: Integrating amplifier with input impedance.
The amplifier forward gain—the gain of the amplifier without the feedback loop—affects the operation of the circuit in two important ways. In the first place, it increases the time constant of the circuit, as shown in Figure (3.11). This, in turn, greatly increases the quality of integration, as demonstrated in Section (3.8).

In the second place, the forward gain affects the overall amplifier gain. Define a term as follows:

\[
\text{Percentage Error} = \frac{E_0 (A_{finite}) - E_0 (A_{infinite})}{E_0 (A_{finite})} \times 100\% \tag{4.3}
\]

In the case of the integrating amplifier

\[
E_0 (A_{finite}) = \frac{A e^s}{(1-A)RCs + 1} \tag{4.4}
\]

\[
E_0 (A_{infinite}) = \frac{E_0 s}{RCs} \tag{4.5}
\]

Substituting these two values in equation (4.3) results in

\[
P.E. = \frac{100}{A} \% \tag{4.6}
\]

For a gain of 200 used in this study, the percentage error is less than 1%. By adding two more grounded emitter stages, a gain of 5,000 could easily be achieved. In such a case the percentage error would be less than .02%. This is probably sufficient for most engineering work.
EXTENDING THE USEFUL APPLICATION OF THE AMPLIFIER

If generalized impedances are used in the feedback loop, the transfer function, assuming high gain, becomes

\[ C_o = -\frac{Z_f}{Z_j} e^s \]  \hspace{1cm} (4.7)\]

By the appropriate choice of \( Z_f \) and \( Z_j \), a great variety of transfer functions can be obtained. As a simple example, let \( Z_f \) be a resistor and \( Z_j \) be a capacitor. Then equation (4.7) becomes

\[ C_o = R C e^s e_j = R C \frac{dC_j}{ds} \]  \hspace{1cm} (4.8)\]

The output is proportional to the derivative of the input, and the amplifier operates as a differentiator. Experimental verification of this fact is presented in Chapter 5.

In addition, the feedback amplifier can be used for addition and multiplication, and their inverses. By the proper interconnection of whole series of these amplifiers many complex problems involving algebraic and differential equations can quickly be solved. The results are displayed on a cathode ray oscilloscope.


EXPERIMENTAL RESULTS

(5.1) INTRODUCTION

To check the validity of the theoretical analysis presented in Chapter 3 and Chapter 4, the amplifier was built exactly as designed, using standard RTMA values with 5% tolerances. The amplifier was then subjected to several experimental tests. The gain of the amplifier, its response to a step input, and the accuracy of its integration were measured. The results are presented in the form of tables, graphs, and pictures of cathode ray oscillograms.
(5.2) AMPLIFIER FREQUENCY RESPONSE

The response of the amplifier was measured as the frequency was varied from 20 cps to 1,000 cps. At all times, the output was observed on an oscilloscope to make sure the output was undistorted. The results are shown in Figure (5.1). The mid-band gain is 182, which is within 5% of the design gain of 192. The lower cutoff frequency is well below 20 cps. The response, then, can be considered flat over the design frequency range of 20 to 1,000 cps.
Fig. 51. Gain versus frequency response of transistor amplifier.
(5.3) RESPONSE TO A STEP INPUT

To measure the rise time of the amplifier itself, without feedback, a step input was applied. The results are presented in the oscillograms of Figure (5.2). The rise time appears to be small in comparison with the time of the pulse.
To test the accuracy of the amplifier as an integrator, a unit square was applied. Oscillograms of the integration at three different frequencies are shown in Figure (5.3). These appear to be linear to within the 5% design criterion.

In addition, the output was measured and the gain computed at each frequency. The theoretical gain can be calculated using equation (3.36). A comparison of the theoretical and experimental values is shown in Table (5.1). The error is well within 5%.

**TABLE 5.1**

**COMPARISON OF CALCULATED AND EXPERIMENTAL GAIN**

<table>
<thead>
<tr>
<th>Frequency</th>
<th>Cal. Gain</th>
<th>Exper. Gain</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 cps</td>
<td>3.80</td>
<td>3.90</td>
<td>2.6</td>
</tr>
<tr>
<td>500 cps</td>
<td>0.380</td>
<td>0.390</td>
<td>2.6</td>
</tr>
<tr>
<td>1,000 cps</td>
<td>0.190</td>
<td>0.185</td>
<td>2.6</td>
</tr>
</tbody>
</table>
Fig. 5.2: Response of amplifier to step input.

Fig. 5.3: Response of integrating amplifier to step input.
(5.5) DIFFERENTIATION OF A SQUARE WAVE

As discussed in Section (4.4) a high gain amplifier can serve as a differentiator by a proper choice of impedances. The feedback circuit was arranged as shown in Figure (5.4), and a square wave fed into the input. The output is shown in Figure (5.5). This indicates that the square wave has been differentiated.

This would indicate that a high gain transistor amplifier can operate as an operational amplifier as effectively as a vacuum tube amplifier.
Fig. 5.4: Block diagram of amplifier as a differentiator.

Fig 5.5: Response of the differentiating amplifier to step input.
Chapter 6.

SUMMARY

In general, the transistor is looked upon as a current actuated device. However, in this investigation, to make full use of vacuum tube theory and equivalent circuits, the transistor was looked upon as a voltage actuated device. The main purpose of the study was to determine whether an integrating amplifier could be designed and built using this approach.

To meet the design criteria, a two stage amplifier was used, using the grounded emitter circuit as reference. The necessary design equations were derived.

Using the theory of the Miller Effect, a theoretical analysis of the amplifier was carried out, showing the effect of variation in the design parameters such as gain and input impedance.

Oscillograms were taken of the response of the feedback amplifier to a square wave input, both as an integrator and as a differentiator. The results indicate that a transistor amplifier can operate in a computing circuit.

However, before it can be concluded that the transistor can operate as a truly operational amplifier, there are certain factors which must be more fully investigated. The frequency range must be extended. In particular, the amplifier should be d-c coupled. That is, the amplifier should handle d-c signals.
D-c coupling in vacuum-tube amplifiers leads to several problems, of which the most serious is probably that of drift. Any random changes in heater and other voltages, aging of cathodes, and other similar changes, can cause slow drifts which are amplified and appear at the output. These effects may also appear in the transistor. In addition, the transistor is sensitive to temperature change. This whole area requires further study.

The accuracy of the integration was shown to be proportional to the forward gain of the amplifier. In vacuum tubes gains as high as one million are used. Theoretically, this can be achieved with the transistor by cascading a sufficient number of stages. It would be of value to determine experimentally whether any upper limit on the cascaded gain of a transistor amplifier exists.

It has also been shown that the input impedance should be very high. In the transistor, this is made difficult by the inherent low impedances of the device in comparison with the vacuum tube, and by the shunting effect of the biasing circuit. This area deserves further study.

In general, none of the problems mentioned above appear insurmountable. There appears little doubt that the transistor, with its many inherent advantages, will in the near future replace the vacuum tube in operational amplifiers.
BIBLIOGRAPHY

Books


