FREQUENCY VARIATIONS OF TRANSISTOR PARAMETERS

by

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ACKNOWLEDGEMENTS

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INTRODUCTION

As this paper is concerned with the transistor as a circuit element, it is apparent that frequency effects are important, and that we must know these effects. In the design of practically any electronic circuit, the frequency response of the system is of vital interest. The usual design requirements specify the desired frequency response of the system and, from these and other requirements, the system can be designed if the parameters of the active elements of the system are known.

Thus, it is necessary that the effects of frequency on the parameters of the transistor must be established before any good design can be developed.

Chapter 1

THE COMMON EMITTER EQUIVALENT CIRCUIT

(1.1) Introduction

The common emitter equivalent circuit was developed to facilitate design of transistor circuits. It is true that the common base hybrid (or "h") parameter equivalent circuit is more widely used, but the author feels that it possesses a disadvantage that necessitates the development of a different form of equivalent circuit. This disadvantage becomes apparent from a consideration of most electronic circuits. Nearly all conventional transistor circuits are designed for the common emitter connection, not for the common base connection from which the h-parameter equivalent circuit was designed.

(1.2) Definition of "A" Parameters [1]

The circuit diagram of the common emitter hybrid parameter equivalent circuit is shown in Figure (1.1). The four hybrid parameters are defined as follows:

A_{11} is the input resistance with the collector shorted to the emitter.

$$A_{11} = \frac{\Delta V_b}{\Delta I_b} = \frac{d V_b}{d I_b}\bigg/_{V_c = 0} \tag{1.1}$$

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1. The original definitions of the hybrid A parameters were made by Dr. Thomas L. Martin, Josef Gartner, and Aladdin Perkins at the University of Arizona in December, 1954.
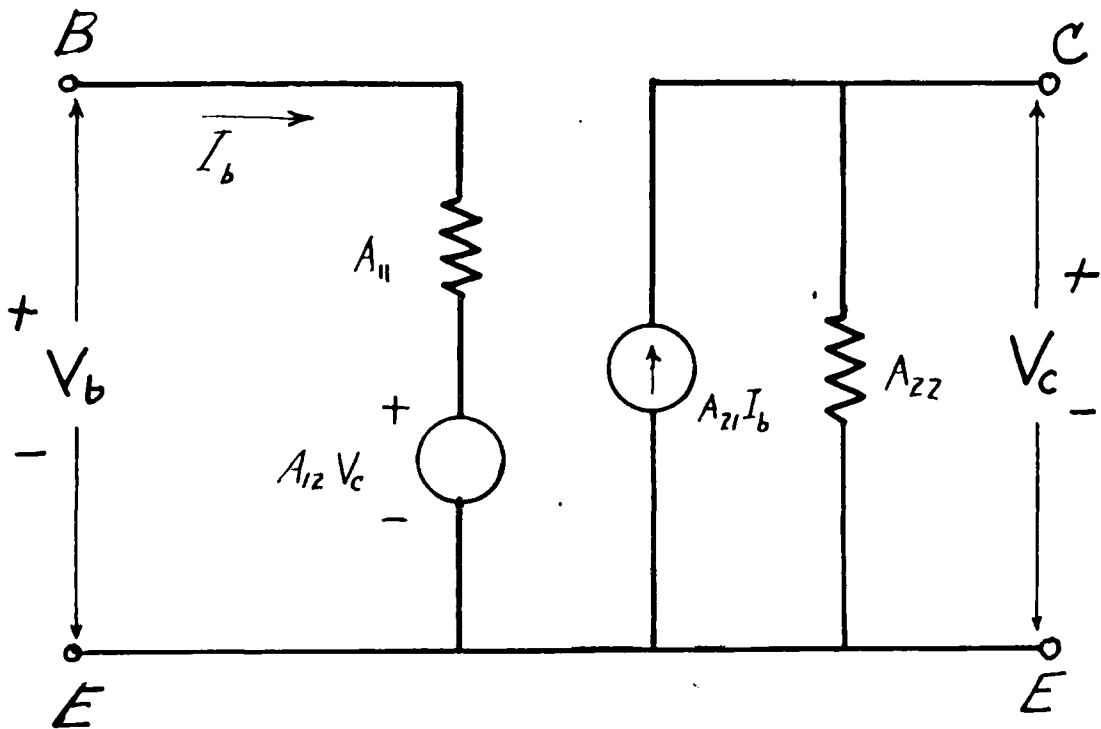
FIG. (1-1)   Common Emitter, Hybrid
Parameter Equivalent Circuit

A_{22} is the output admittance with the base an open circuit.

$$A_{22} = \frac{\Delta I_c}{\Delta V_c} = \frac{d I_c}{d V_c}\Bigg/_{I_b = 0} \qquad (1.2)$$

A_{12} is the voltage feedback factor with the base an open circuit.

$$A_{12} = \frac{\Delta V_b}{\Delta V_c} = \frac{d V_b}{d V_c}\Bigg/_{I_b = 0} \qquad (1.3)$$

A_{21} is the ratio of the collector current to the base current with the collector shorted to the emitter.

$$A_{21} = \frac{\Delta I_c}{\Delta I_b} = \frac{d I_c}{d I_b}\Bigg/_{V_c = 0} \qquad (1.4)$$

The factor A_{21}, which is defined as the current amplification factor, is more commonly identified by the symbol-β. Therefore, the equivalent circuit assumes the form shown in Figure (1.2).

(1.3) Development of The Common Emitter Hybrid Parameter Equivalent Circuit For Circuit Design

The equivalent circuit drawn in Figure (1.2) is not suited to a straightforward circuit design. This seems to be a disadvantage common to all two-generator equivalent circuits. This circuit can be further simplified by evaluating the voltage generator in the input circuit. This is the generator marked $A_{12}V_c$ in Figure (1.2).

Assume that the transistor has some arbitrary three-terminal load connected as shown in Figure (1.3). Three new terms are defined as follows:
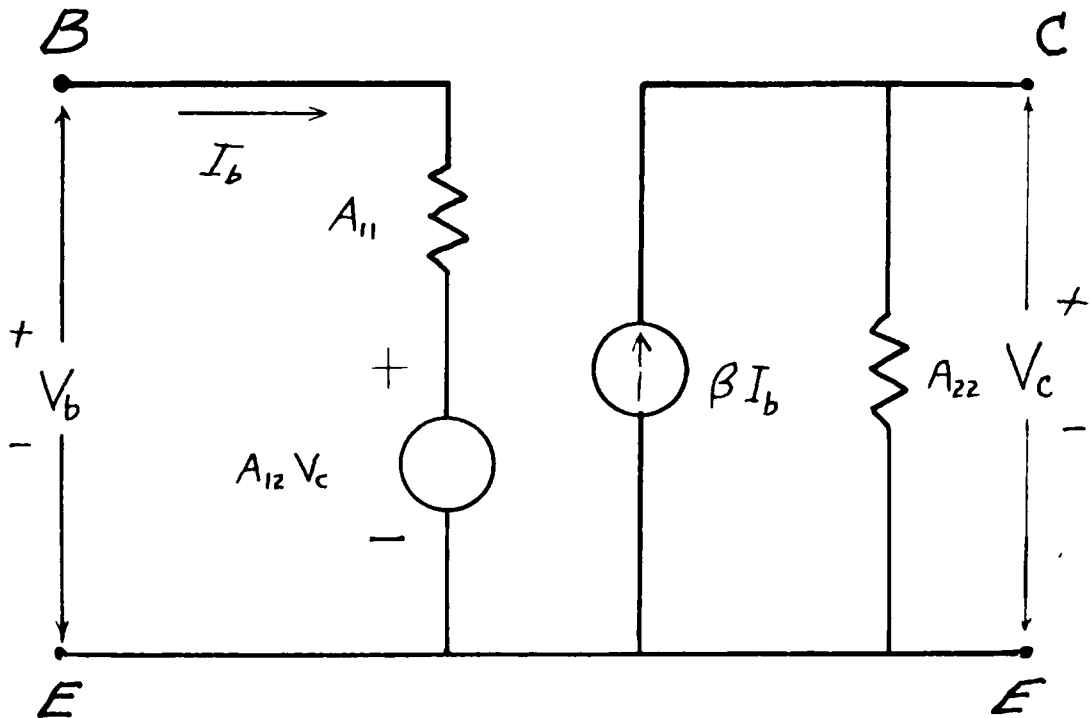
F/G. (1-2)   CHANGE in Nomenclature;
Common Emitter, Hybrid
Parameter Equivalent Circuit
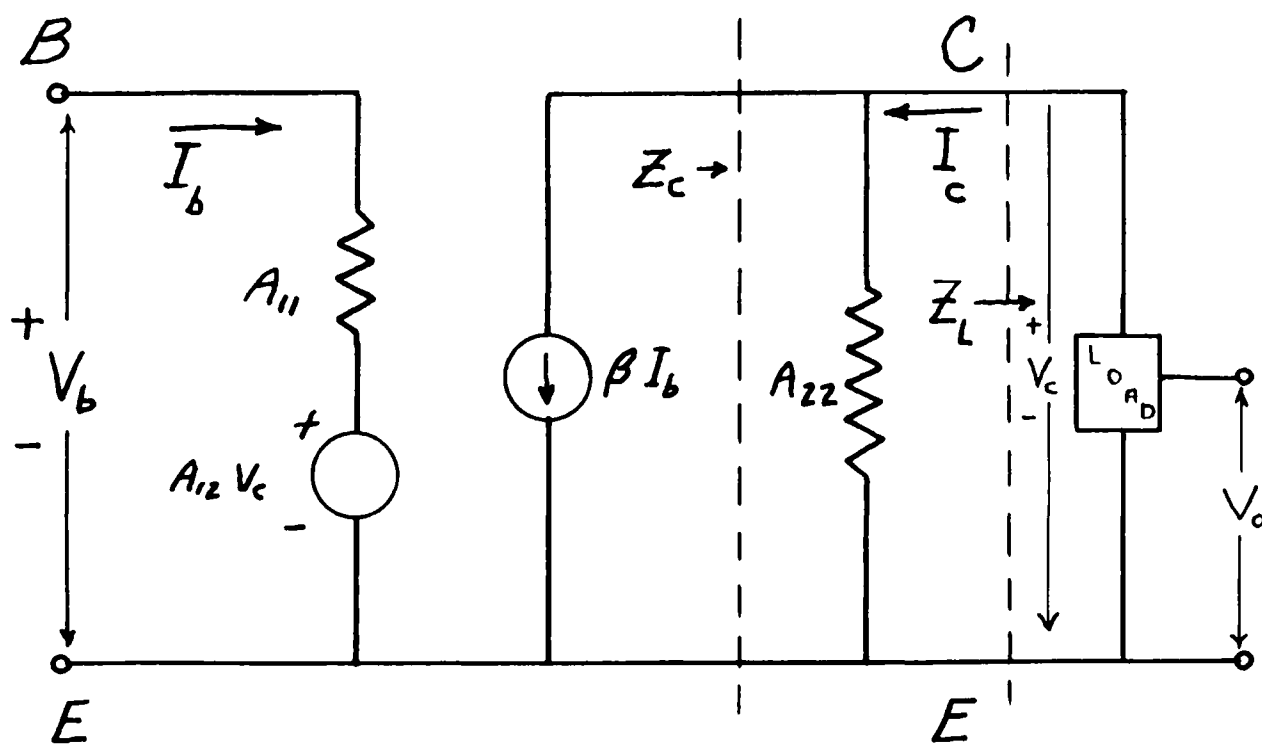
$$Z_m = \frac{V_o}{\beta I_b}$$

FIG. (1-3)   Equivalent Circuit With
Connected Load

Z_L = Input impedance of the connected load circuit.

Z_C = Input impedance of the entire passive collector circuit.

Z_M = Mutual impedance of the entire passive collector circuit.

These terms are indicated in Figure (1.3).

From the circuit, it can be seen that the collector voltage V_C is given by:

$$V_c = -\beta I_b Z_c \tag{1.5}$$

Therefore,

$$A_{12} V_c = I_b \left(-A_{12} \beta Z_c \right) \tag{1.6}$$

Now, define a new parameter so that the circuit can be further simplified into a more useful form. This parameter, A_{bc}, is the voltage amplification, measured from the base to the collector, and is given by:

$$A_{bc} = V_c / V_b \tag{1.7}$$

This ratio is usually a negative, complex number for the common emitter circuit.

From the equivalent circuit of Figure (1.3), and from the equations previously developed, the voltage gain parameter is:

$$A_{bc} = - \frac{\beta Z_c}{A_{11} - A_{12} \beta Z_c} \tag{1.8}$$

Solve for βZ_C.

$$\beta Z_c = -A_{11} \frac{A_{bc}}{1 - A_{12} A_{bc}} \tag{1.9}$$

Thus, the negative component of the input impedance becomes:

$$-A_{12}\,\beta Z_c = A_{11}\,\frac{A_{12}\,A_{bc}}{1-A_{12}\,A_{bc}}$$

(1.10)

Because the base to collector gain, A_{bc}, is normally negative, this is a negative impedance. Therefore, the total input impedance of the circuit is:

$$Z_{IN} = A_{11} - A_{12}\,\beta Z_c$$

(1.11)

or

$$Z_{IN} = \frac{A_{11}}{1-A_{12}\,A_{bc}}$$

(1.12)

And the equivalent circuit assumes the form shown in Figure (1.4).

Because the operation of a transistor changes with changes in signal frequency, the equivalent circuit must indicate the cause of the changes. It was assumed that the frequency changes were the result of reactive components actually associated with the transistor. This will be discussed in more detail later: at this point, the final equivalent circuit is assumed to have the form shown in Figure (1.5).
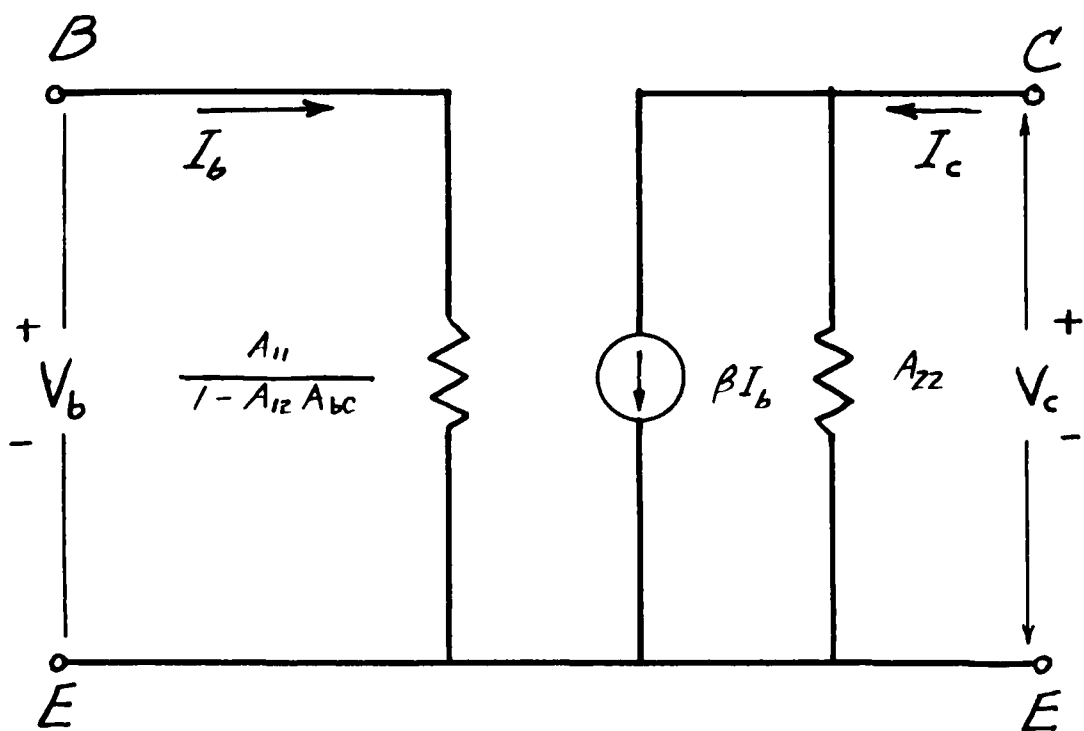
FIG. (1-4) Simplified Equivalent Circuit
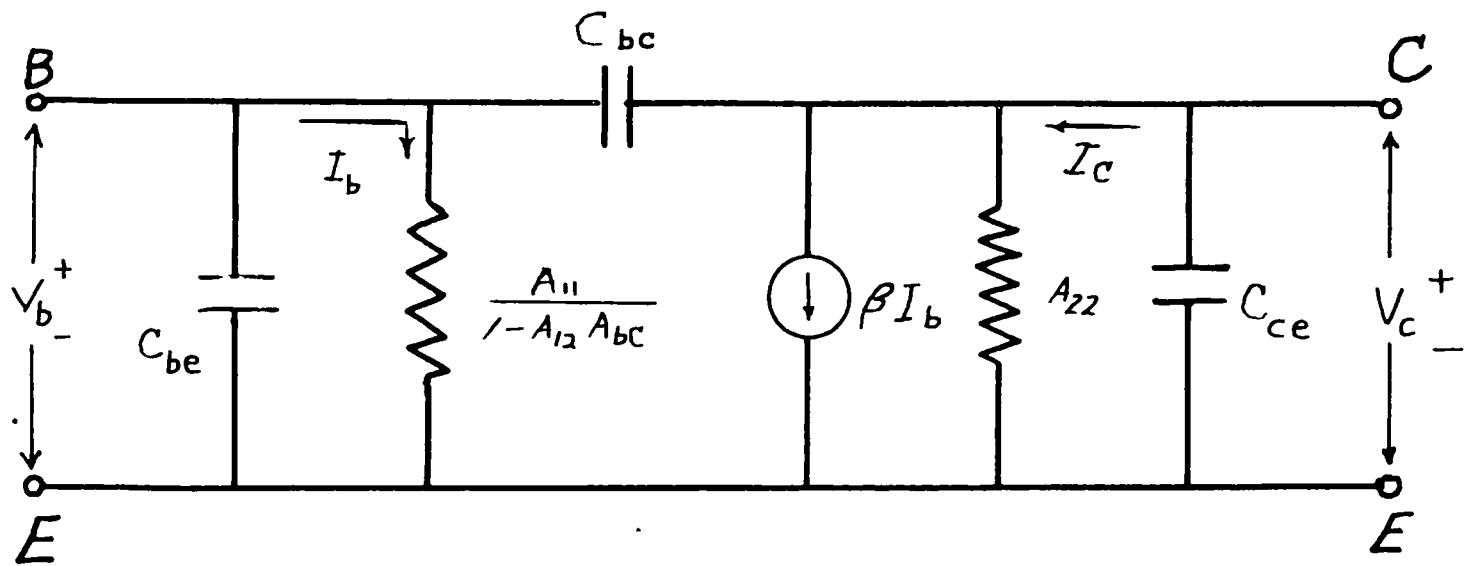
FIG. (1-5)   Final Equivalent Circuit

Chapter 2

STATIC VALUES OF PARAMETERS AND INDEPENDENCE OF FREQUENCY OF A_{11} AND A_{22}

(2.1) Introduction

It is necessary to know the static characteristics of the transistor before the effects of frequency can be determined. The static or direct current values for the previously defined parameters were determined by ordinary methods, described briefly in the paragraphs that follow.

(2.2) Static Measurements of Parameters

The input and output characteristic curves for the transistors were obtained with a Librascope X-Y plotter. For the output characteristics, the transistor was operated at different parametric values of base current, and the collector voltage varied from zero volts to the maximum allowable voltage without exceeding the maximum collector dissapation. The plotter recorded a continuous curve showing collector current as a function of collector voltage for each parametric value of base current.

The input characteristics were obtained in a similar manner, the collector voltage serving as the family parameter. Thus, for each value of collector voltage, the base voltage was varied from zero volts up to the maximum allowable value, and the plotter recorded a continuous
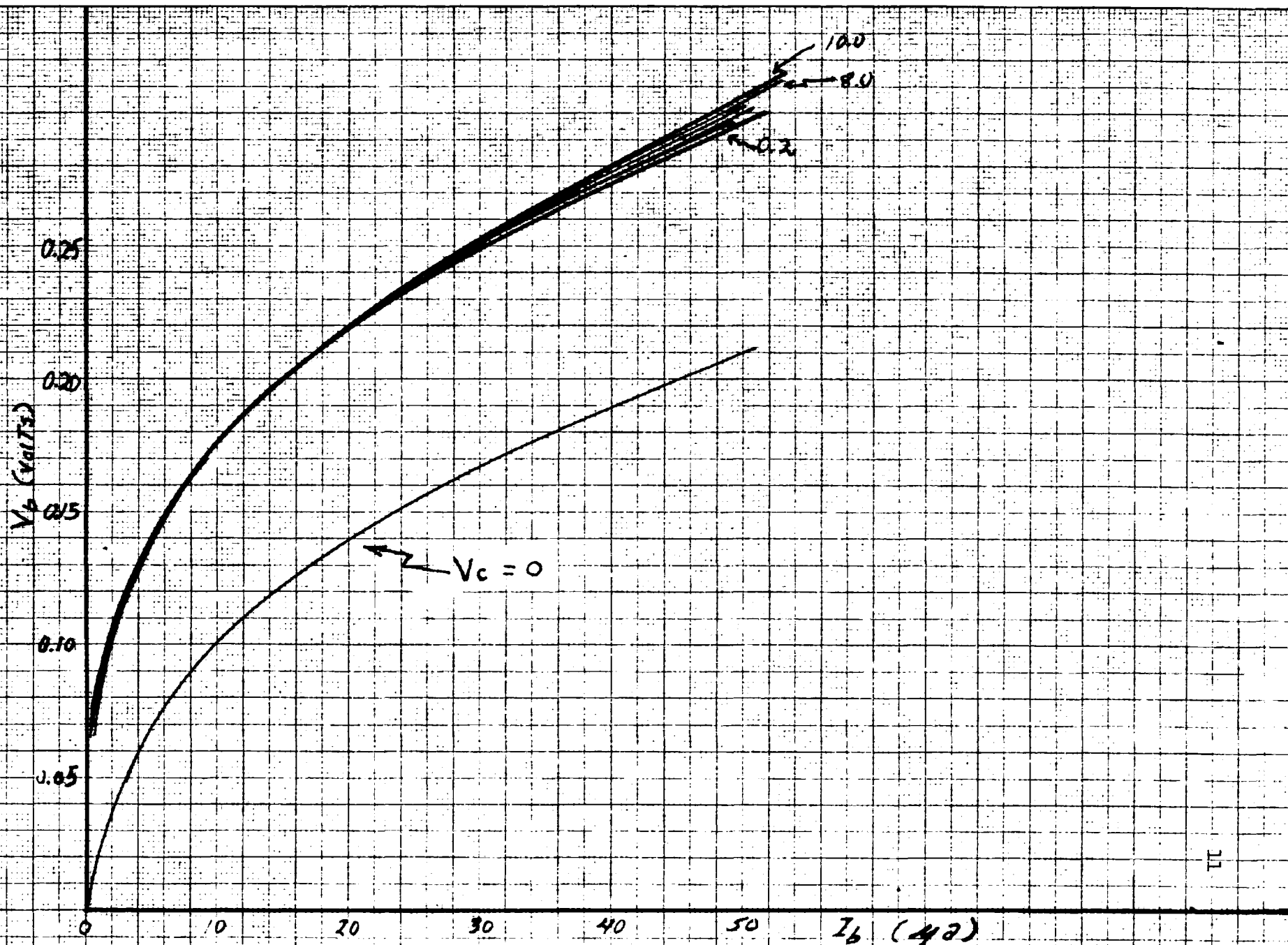
FIG. (2-1) Input Characteristics - Sylvania 2N-35 #54
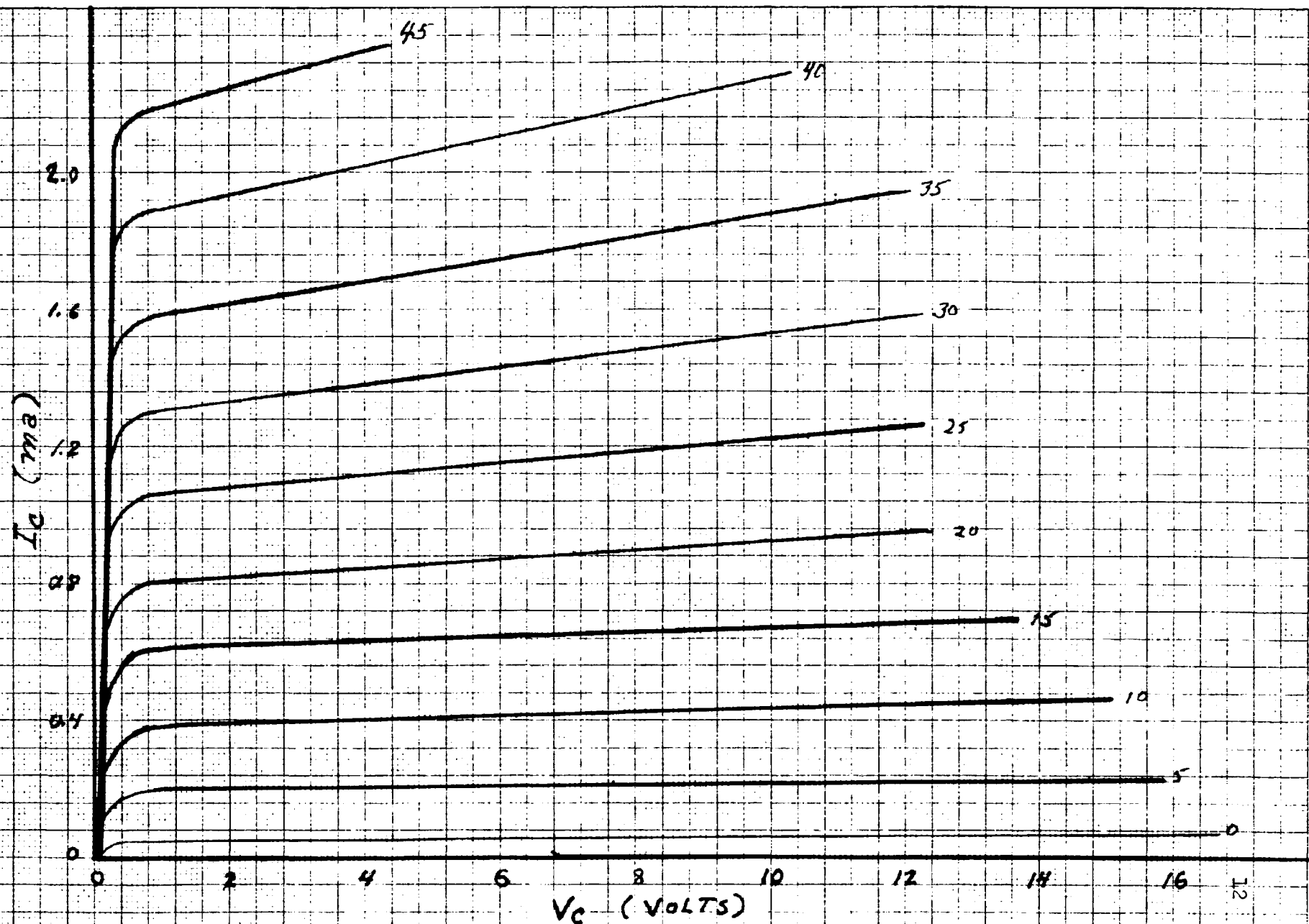
FIG. (2.2)   OUTPUT Characteristics - Sylvania 2N-35 #54

curve of the base current as a function of the base voltage. Figures (2.1) and (2.2) show sample characteristic curves for both the input and output circuits of the transistor. From the previous definitions of the hybrid parameters, it is clear that they can be determined directly from these curves.

(2.3) Effect of Frequency on A_{11} And A_{22}

The dynamic or alternating current values of A_{11} and A_{22} were determined from measurements made by Mr. David J. Sakrison of the Electrical Engineering department at the University of Arizona, and the values obtained are in agreement (within 10 %) with the static values. Therefore, from these results, it can be said that A_{11} and A_{22} are constant with frequency. This is in accordance with the equivalent circuit shown in Figure (1.5). From this you can see that the changes in input and output impedance are accounted for by the capacitances present in these circuits. The voltage feedback factor, A_{12}, also has an effect upon the input impedance. This will be discussed in more detail in a later chapter.

Chapter 3

THE VARIATION OF β WITH FREQUENCY

(3.1) Introduction

In Chapter two, it was noted that the value of the current ampli-
fication factor β, equal to $-A_{21}$, was determined from the static
characteristic curves. The purpose of this chapter is to show, theo-
retically, that β is independent of frequency and, then to verify
this theory by experiment. This assumption that β is not frequency
dependent disagrees with the generally accepted theory which assumes
frequency dependence. However, it is felt that the approach presented
in this chapter merely depends upon a simpler definition of the base
current, I_b. Specifically, in the theory proposed, the base current
is assumed to be only in the impedance designated as $\dfrac{A_{11}}{1-A_{12}A_{bc}}$.
In the more common theory of frequency dependence of β, the base current
is assumed to be all of the current into the base terminals. It should
be clear from the equivalent circuit shown in Figure (1.5) that this
new notion greatly facilitates the design of a multitude of electronic
circuits using transistors.

(3.2) Theoretical Basis For The AC Measurement of β

The circuit used for determining the AC value of β is shown in
Figure (3.1). The operation of the circuit is best understood from
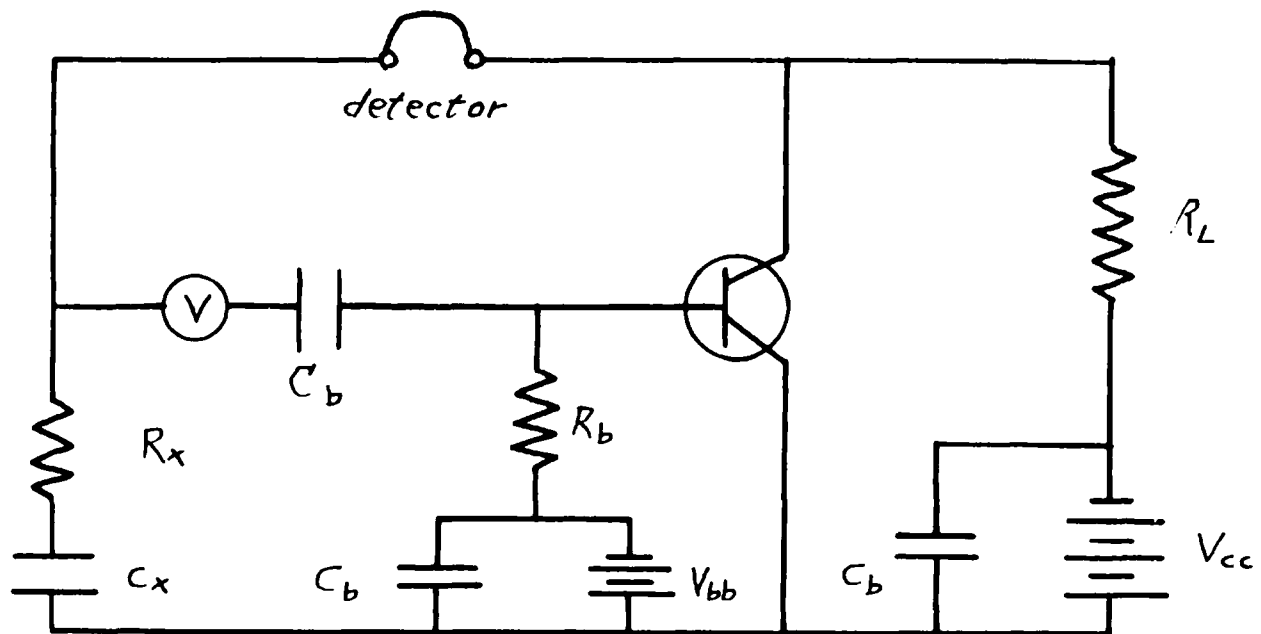
FIG. (3-1)   Circuit Used to Measure
β as a Function of Frequency
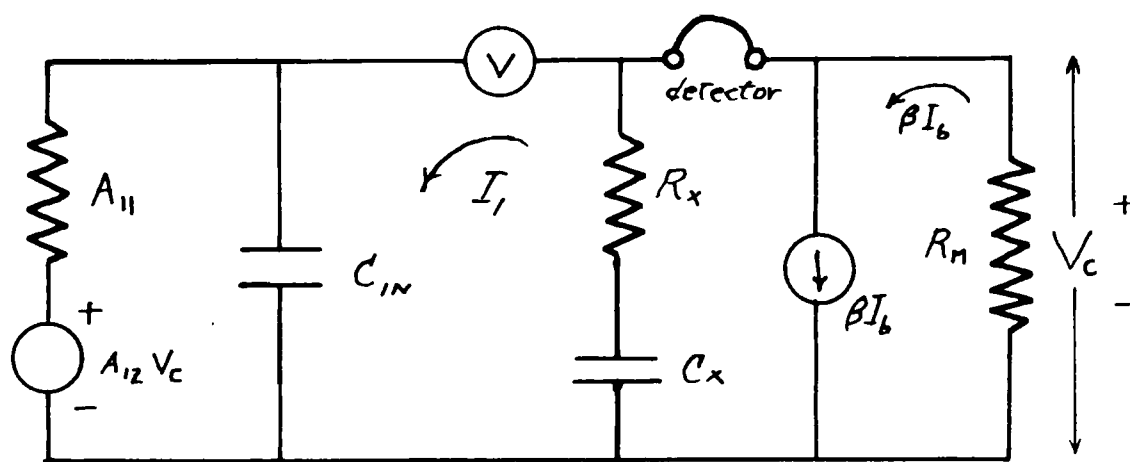
$$R_m = \frac{R_L}{1 + A_{22} R_L}$$

FIG. (3-2) Class A Equivalent Circuit of Figure (3-1)

the Class A equivalent circuit in Figure (3.2). Actually, the circuit is essentially a bridge circuit. R_x and C_x are adjusted to produce a null detector reading at a number of different signal frequencies provided by the signal source V. The collector signal voltage is monitored with a Cathode Ray Oscilloscope at all times to assure that the condition of Class A operation always exists.

Now, at bridge balance:

$$I_1 Z_x = \beta I_b R_m = -V_c \tag{3.1}$$

Hence,

$$I_1 = \frac{\beta R_m}{Z_x} I_b \tag{3.2}$$

The loop equation around the input circuit is:

$$I_b \left(A_{11} + \frac{1}{SC_{IN}} \right) + A_{12} V_c - I_1 \left(\frac{1}{SC_{IN}} \right) = 0 \tag{3.3}$$

Or:

$$I_b \left(A_{11} - A_{12} \beta R_m + \frac{1}{SC_{IN}} \right) - I_1 \left(\frac{1}{SC_{IN}} \right) = 0 \tag{3.4}$$

Substitute equation (3.2).

$$I_b \left(A_{11} - A_{12} \beta R_m + \frac{1}{SC_{IN}} - \frac{\beta R_m}{SC_{IN} Z_x} \right) = 0 \tag{3.5}$$

Cancel I_b, replace Z_x with R_x 1/ SC_x and solve for β. The result is:

$$\beta = \frac{A_{11}}{A_{12} R_m} \frac{(S + \omega_{11})(S + \omega_x)}{S(S + \omega_y)} \tag{3.6}$$

Where:

$$\omega_{11} = \frac{1}{A_{11} C_{IN}} \tag{3.7}$$

$$\omega_x = \frac{1}{R_x C_x} \tag{3.8}$$

$$\omega_y = \omega_x + \frac{1}{A_{12} R_x C_{IN}} \tag{3.9}$$

In the steady state, equation (3.6) reduces to:

$$\beta = \frac{A_{11}}{A_{12} R_M} \frac{(1-j\frac{\omega_{11}}{\omega})(1-j\frac{\omega_x}{\omega})}{(1-j\frac{\omega_y}{\omega})} \tag{3.10}$$

This equation provides the experimental basis for establishing the frequency variation β.

(3.3) Experimental Results— Comparison of AC And DC Values of β

From the circuit presented in the previous section, data were taken for a number of transistors. Plots were then made of β as a function of frequency. These curves are shown in Figure (3.3). From Figure (3.3), it is evident that β is independent of frequency. However, to more firmly establish these results, another method was used, as described below.

If the output circuit of the transistor is broad-banded (R_L made very small) and a constant value of signal base current maintained, it is possible to determine the frequency response of a new parameter, β'. The difference in β and β' lies in the assumption of the path for the base current. In Figure (1.5), the base current is shown only in the impedance $\frac{A_{11}}{1-A_{12}A_{bc}}$, and the parameter β applies. By defining the base current as the current into the input terminals
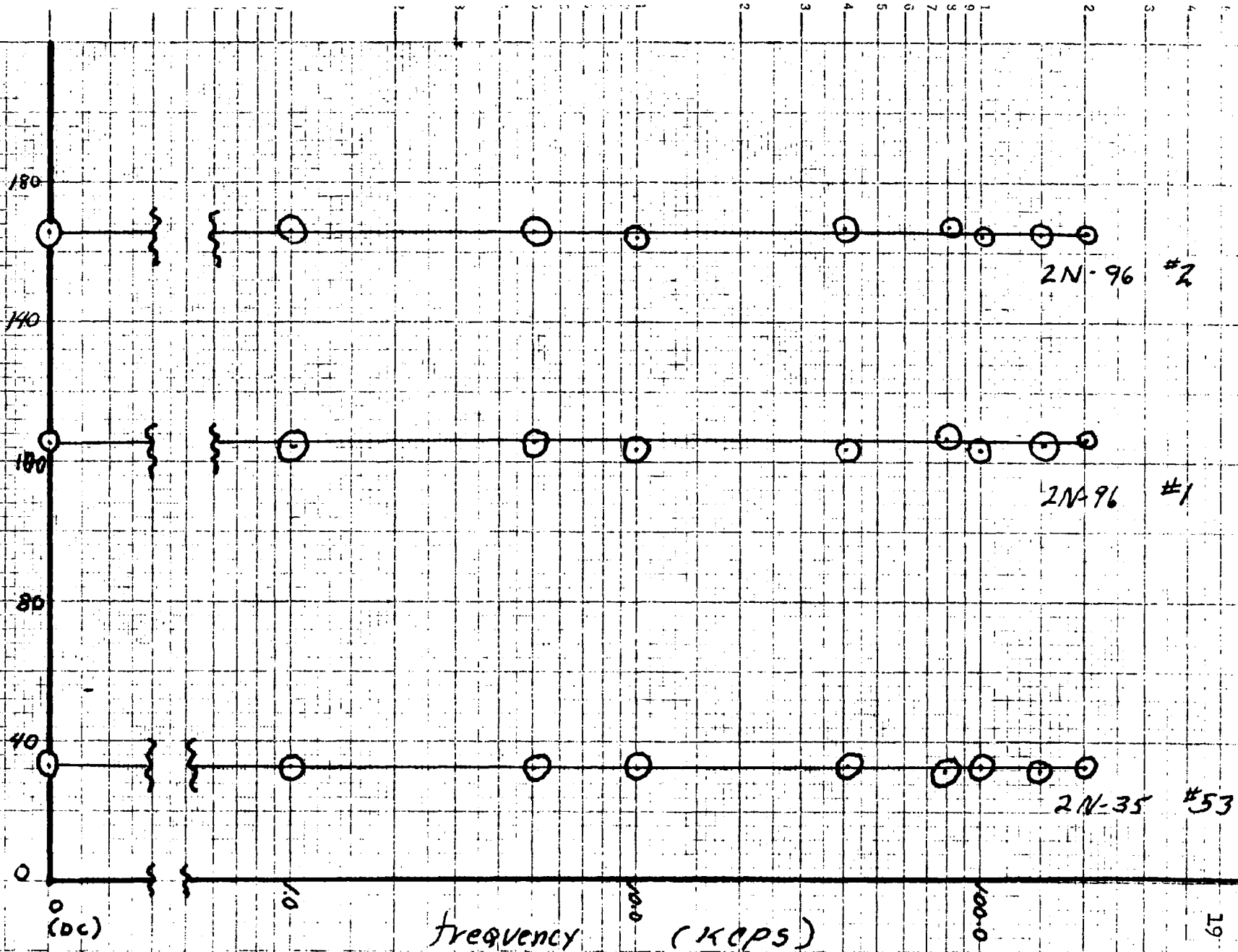
FIG. (3-3) Variation of β With Frequency

of the transistor, we must use another parameter to represent the current amplification factor. This parameter is β'.

The circuit used to determine β' as a function of frequency is given in Figure (3.4). Because R_2 is very small, it is valid to assume that the output circuit is short-circuited. β' was determined for numerous values of signal frequency from the following expression:

$$\beta' = \frac{i_c}{i_b}\Big/ V_c = 0 \tag{3.11}$$

Now, because the output circuit was short-circuited, the roll-off of the frequency response curve is caused solely by the input circuit. The frequency response characteristic of the input circuit was determined theoretically. Figure (3.5) shows the curves for each of the above-mentioned sets of data. These curves are in close agreement and, therefore, it can be said that the apparent change in β' with frequency can be attributed to the input circuit and that β is constant with frequency.
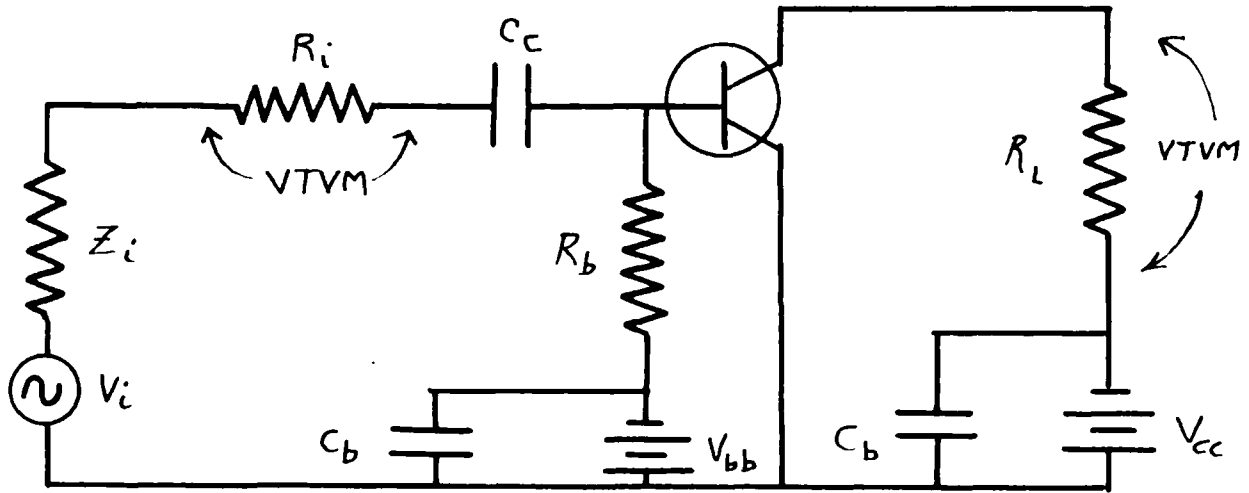
FIG. (3-4)   Circuit Used to Determine
Frequency Response of β'
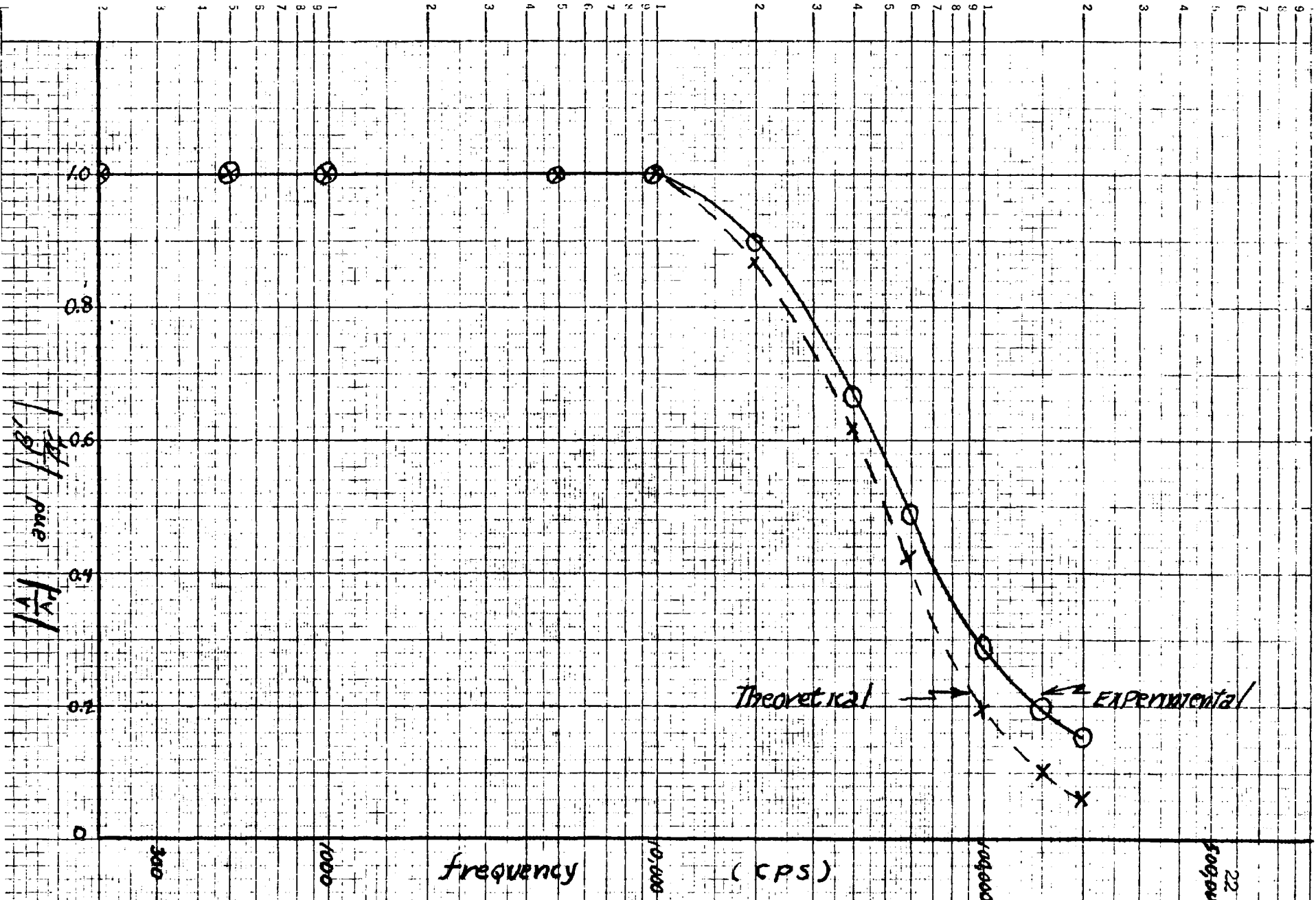
FIG. (3-5) Variation of β' with Frequency

Chapter 4

THE VARIATION OF A_{12} WITH FREQUENCY

(4.1) Introduction

The voltage feedback factor, A_{12}, is the most difficult of the hybrid A parameters to measure. It is possible to determine the static value of A_{12} from the static input characteristic curves, but this is extremely difficult because of the very close spacing of the curves. However, the approximate average value of A_{12} for some 40 different junction transistors is 0.5×10^{-3}. This value is assumed to be accurate within an order of magnitude. This might seem to be a rather poor approximation. However, it should be noted that A_{12} is a very small quantity and, as will be shown in subsequent paragraphs, it can usually be neglected.

The dynamic values of A_{12} were determined at various frequencies. The techniques used in measuring A_{12} and the conclusions drawn from these measurements are discussed in the remainder of this chapter.

(4.2) Derivation of A_{12} From The Input Circuit

A_{12} was previously defined as the ratio of the base voltage to the collector voltage with the base an open circuit (equation 1.3). From this definition, it seems logical to apply a constant signal voltage. to the collector and measure the resulting open-circuit

base voltage. The circuit used is shown in Figure (4.1). The Class A equivalent circuit of the input circuit of the transistor is shown in Figure (4.2). From Figure (4.2), it is easily seen that:

$$V_b = A_{12} V_c \left(\frac{1/SC_{IN}}{A_{11} + 1/SC_{IN}} \right)$$

(4.1)

Re-arranging terms:

$$V_b \left[A_{11} + 1/SC_{IN} \right] = A_{12} V_c \left[1/SC_{IN} \right]$$

(4.2)

And:

$$A_{12} = \frac{V_b}{V_c} \left[\frac{A_{11} + 1/SC_{IN}}{1/SC_{IN}} \right]$$

(4.3)

Hence:

$$A_{12} = \left(1 + A_{11} SC_{IN} \right) \frac{V_b}{V_c}$$

(4.4)

In a more convenient form:

$$A_{12} = \left(S + \frac{1}{A_{11} C_{IN}} \right) A_{11} C_{IN} \frac{V_b}{V_c}$$

(4.5)

In the steady-state, Equation (4.5) becomes:

$$A_{12} = \left(j\omega + \frac{1}{A_{11} C_{IN}} \right) A_{11} C_{IN} \frac{V_b}{V_c}$$

(4.6)
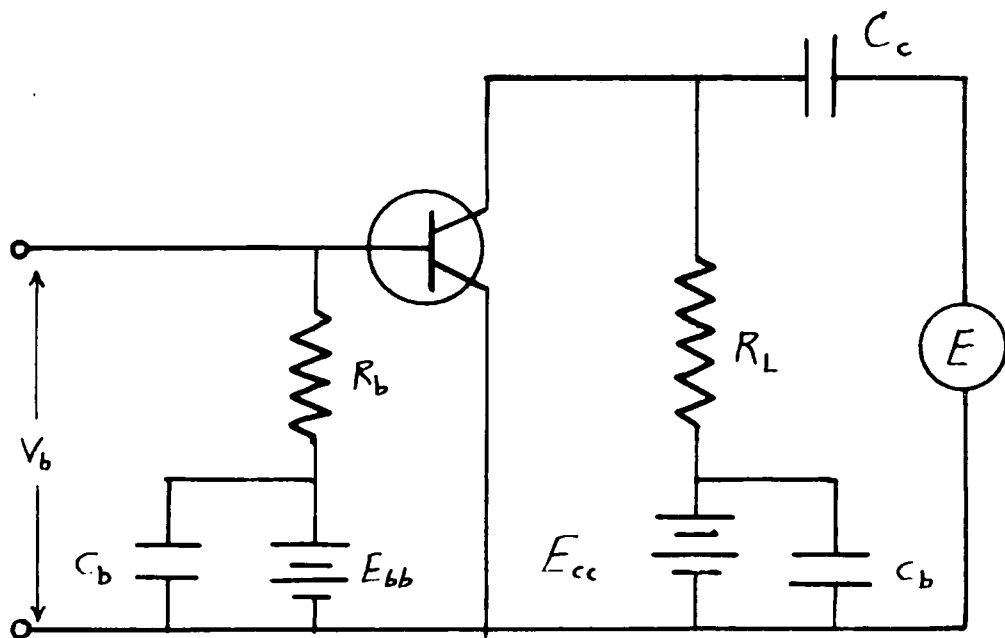
FIG. (4-1) Circuit Used to Determine The Effect of Frequency on A_{12}
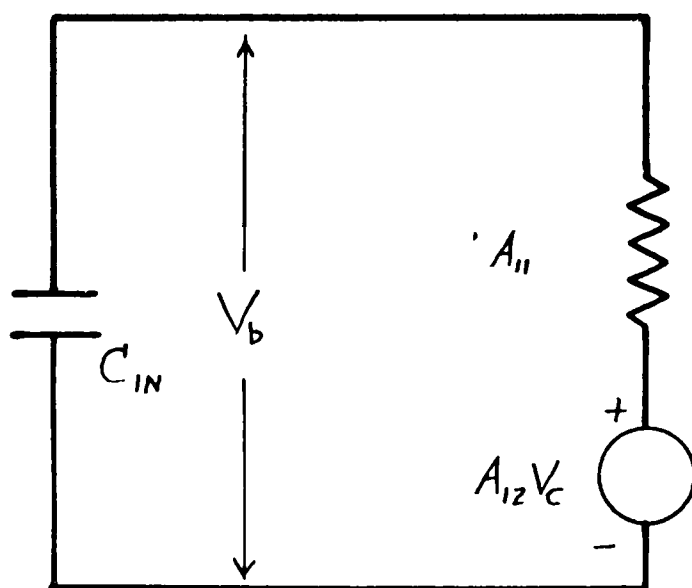
FIG. (4-2)   Class A Equivalent –
            Input Circuit

From Equation (4.6), it is apparent that A_{12} is frequency dependent. It is interesting to note that the curve of A_{12} as a function of frequency would follow the reciprocal of the curve of the base to collector voltage gain as a function of frequency for the lower values of frequency. This can be seen more clearly from Equation (4.4).

As the frequency of operation is further increased, the factor

$$j \omega A_{11} C_{IN} \frac{V_b}{V_c} \tag{4.7}$$

which appears in Equation (4.6), becomes more important. Figure (4.3) is a plot of Equation (4.6) and indicates the variation of A_{12} with frequency.

(4.3) The Effect of A_{12} on The Upper Cutoff Frequency of The Transistor

The effect of the voltage feedback factor, A_{12}, on the upper cutoff frequency of the transistor is illustrated by the curve of Figure (4.4). The base to collector voltage gain was computed theoretically by assuming the cutoff frequency was controlled entirely by the input circuit parameters and A_{12} was neglected. These assumptions are valid for rather low-gain circuits.

The curve of the ratio of the collector voltage to the base voltage as a function of frequency is also shown in Figure (4.4). The cutoff frequency of the actual curve is seen to be lower than that of the theoretical curve,,and it can be assumed that A_{12} does lower the upper cutoff frequency. Figure (4.4) is a representative curve for one particular type of transistor, the Bell 2N-27. However, similar curves
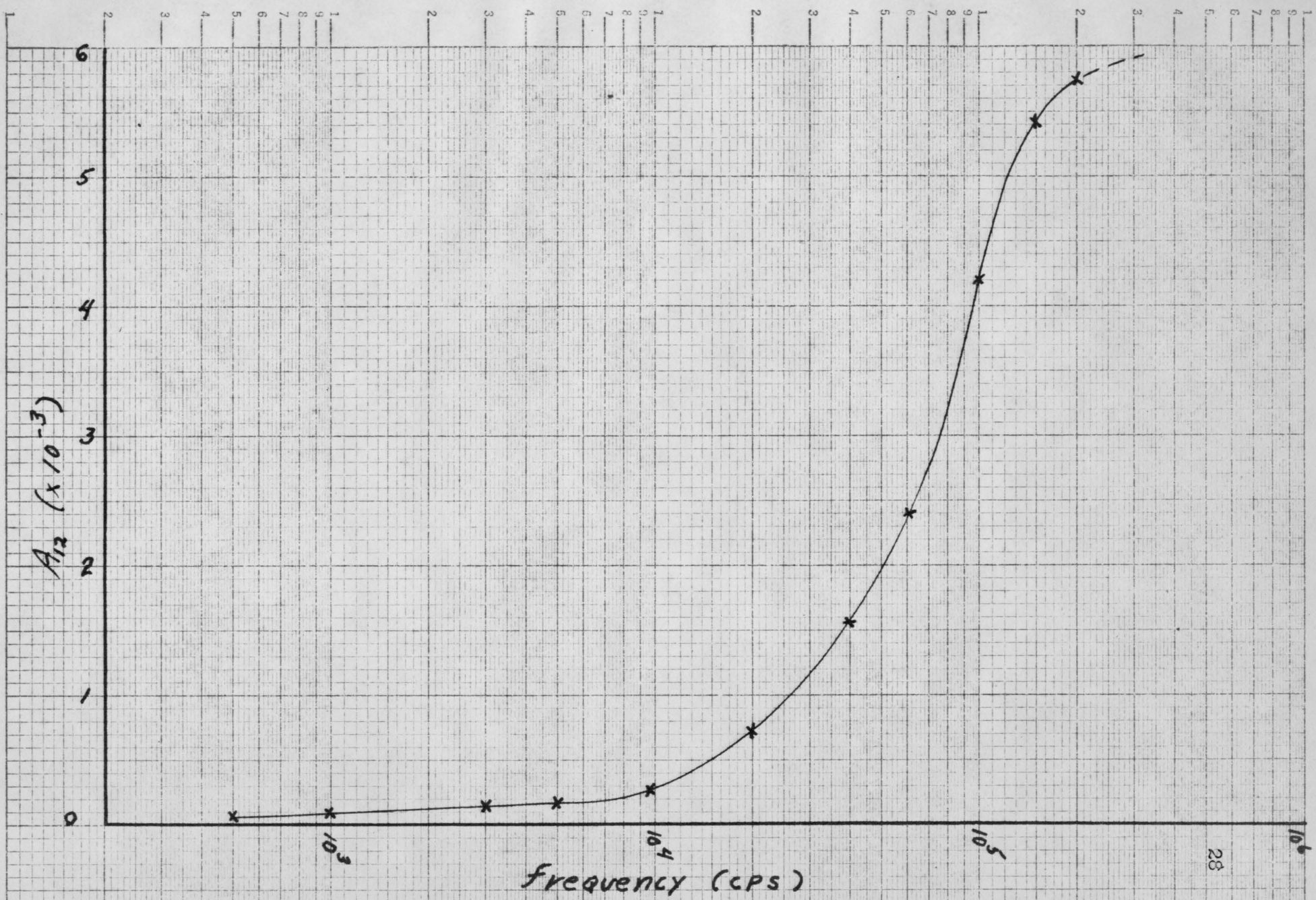
FIG. (4.3) The Variation of A_{12} With Frequency (Texas Inst. 903 #9)

were obtained for the Sylvania 2N-35 and the Texas Instrument Type

903, and the results were in accordance with results presented here.

A_{12} is seen to reduce the upper cutoff frequency of the transistor but, as can be seen from Figure (4.4), it is a very small reduction. Therefore, it is generally safe to neglect this effect in design work.

FIG. (4.4) The Effect of A_{12} on the Upper Cut off Frequency

Axis labels: $\left|\frac{A}{A_o}\right|$ (vertical), Frequency (cps) (horizontal)

Theoretical

Experimental
(Sylvania 2N-35 #60)
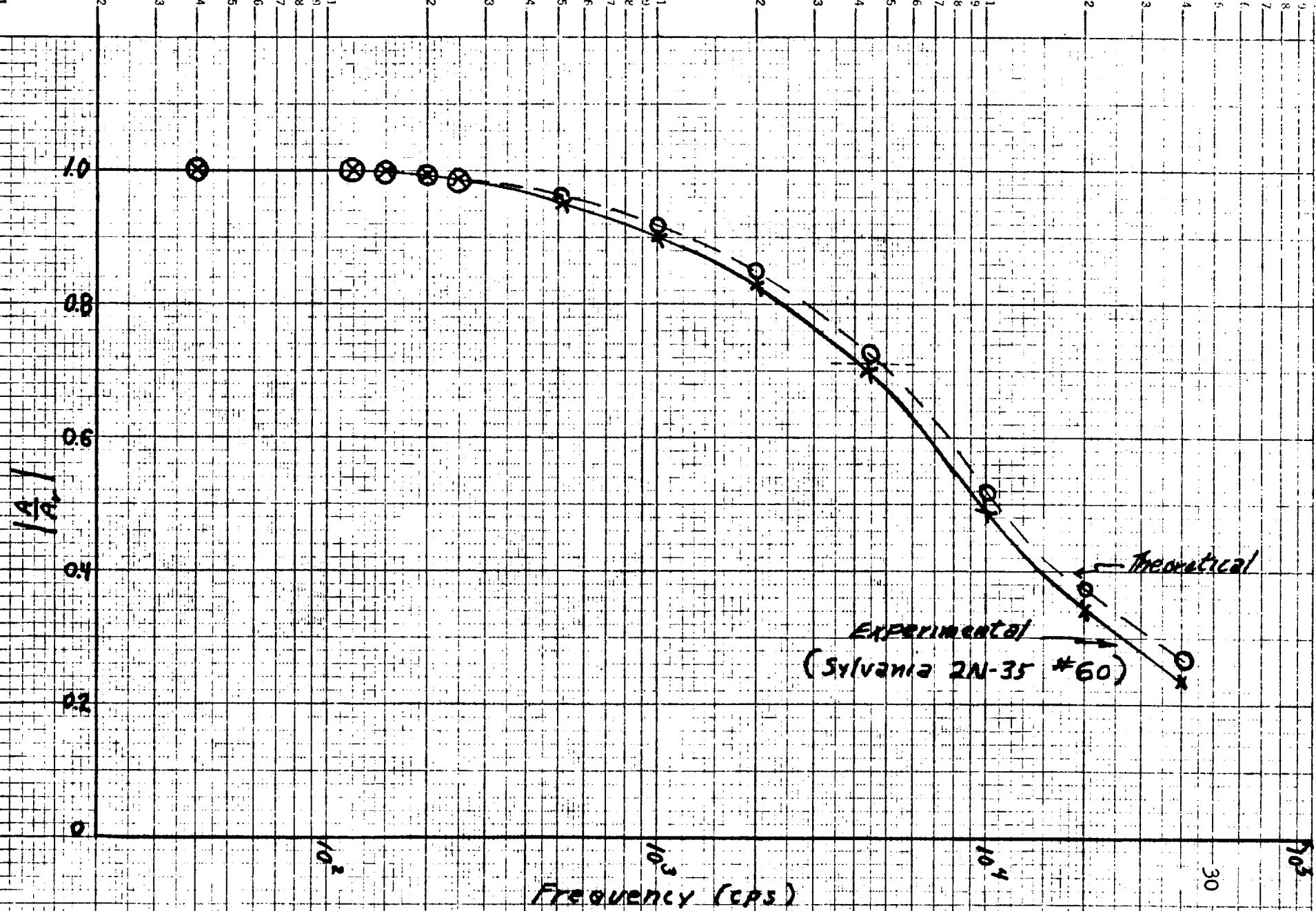
Chapter 5

MODIFICATION OF THE COMMON EMITTER HYBRID A PARAMETER EQUIVALENT CIRCUIT

(5.1) Introduction

The form of the common emitter hybrid A parameter equivalent cir-
cuit that has been used in the previous chapters is not entirely correct.
It can be seen from Figure (1.5) that the input circuit was assumed
to consist of a resistor, A_{11}, in parallel with a capacitor, C_{in}.
The actual circuit consists of these two elements in parallel plus an
additional resistance, r_{11}, in series with the base of the transistor.
This new resistance is termed the "base spreading resistance," and
its usual value is about one-fourth the value of A_{11}. The circuit will
simply be presented in this chapter, and no detailed analysis will be
performed. For a further study of this form of the equivalent circuit,
refer to "Development of a Grounded Emitter Equivalent Circuit for The
Junction Transistor," by Mr. David J. Sakrison, Department of Elec-
trical Engineering, University of Arizona. The correct form of the
common emitter hybrid A parameter equivalent circuit is shown in
Figure (5.1).

(5.2) The Effect of The Base Spreading Resistance on Previous Results

Chapters 3 and 4 dealt with the frequency variation of the current
amplification factor, β, and of the voltage feedback factor, A_{12}.
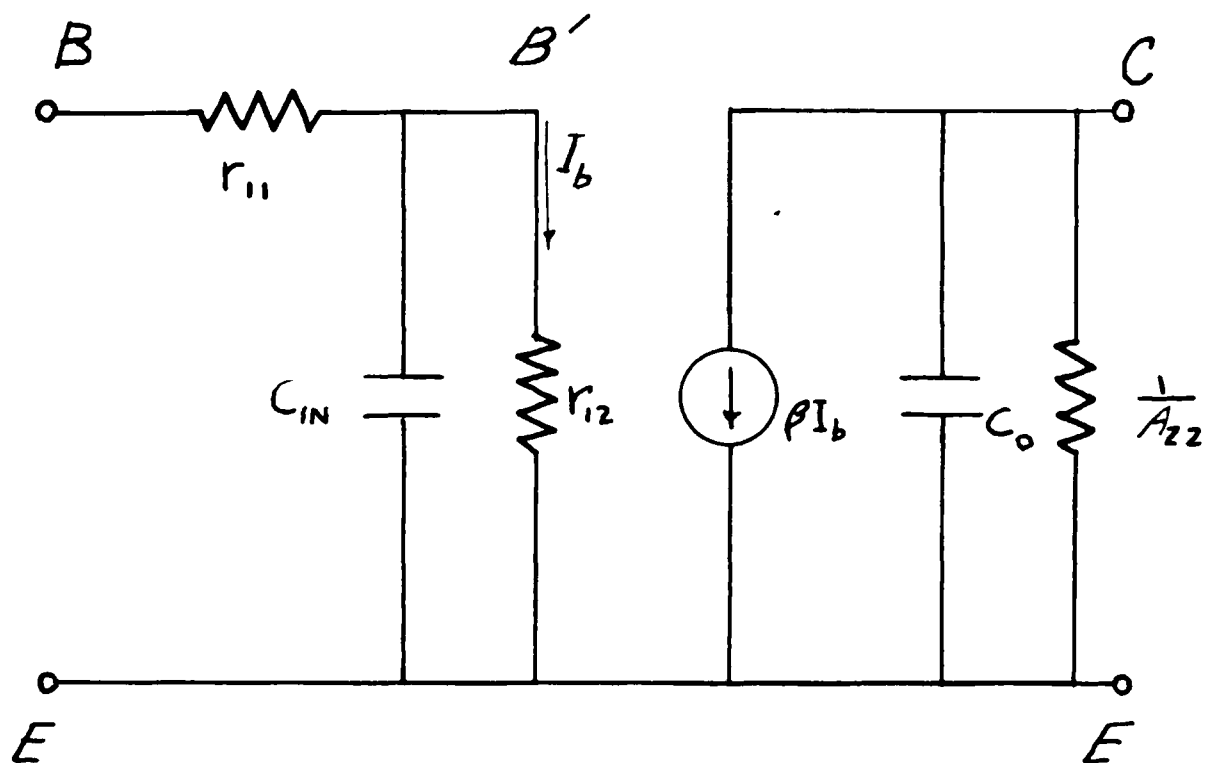
FIG. (5-1)   Correct Form of The Common
Emitter Hybrid A Parameter
Equivalent Circuit

The results presented in these chapters were obtained through the use of the common emitter hybrid A parameter equivalent circuit with the base-spreading resistance neglected. The effect of this omission on the results of chapters 3 and 4 will be discussed in this section from a qualitative standpoint.

In chapter 3, the current amplification factor β was shown to be constant with frequency. The basis for this conclusion was shown to lie in the definition of the current I_b as being only in the impedance designated as $\dfrac{A_{11}}{1 - A_{12}\,A_{bc}}$. Therefore, even with the addition of the base-spreading resistance, it can be seen that, since I_b is not the current into the base terminals, β is independent of frequency.

Chapter 4 was concerned with the effects of frequency on the voltage feedback factor, A_{12}. In this case, the base spreading resistance would simply introduce a constant term in the expression for A_{12} (Equation 4.6), and the frequency effects would not be altered. Therefore, the conclusion that A_{12} is frequency dependent is valid for the complete common emitter hybrid A parameter equivalent circuit.

Chapter 6

BANDPASS TRANSISTOR AMPLIFIERS

(6.1) Introduction

In the previous chapters, the common emitter, hybrid A parameter equivalent circuit was presented, and the frequency variations of its parameters were investigated. The remainder of this paper will deal with the design and operation of some bandpass transistor amplifiers in the megacycle range. The equivalent circuit discussed in Chapter 5 will be investigated in so far as its usefulness in the design of tuned amplifiers is concerned.

The majority of the previous results were obtained with junction transistors, although some surface barriers were used. However, in the higher range of frequencies, the surface barrier transistors were used almost exclusively.

The width of the base region and the interelectrode capacitances of the surface barrier transistors are very much smaller than those of the junction transistors. For these two reasons, which impose the upper limit on the frequency of operation of transistors[2], the surface barrier transistors were used.

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2. W. E. Bradley, and others, "The Surface Barrier Transistor," Proc., IRE., Vol. 41. pp. 1702 – 1720, December, 1953.

(6.2) Derivation of The Expression For The Gain of Tuned Transistor Amplifiers

The expression for the gain of one single-tuned stage will be developed first. Then, this derivation will be extended to include the input coupling circuit, and then, for "n" identical stages in cascade. The equivalent circuit presented in Chapter 5 shall be used for these derivations.

The equivalent circuit for one tuned state assumes the form shown in Figure (6.1). The gain of one stage shall be considered as being from B_1' to B_2'. This definition of stage gain provides a basis for the development of design equations for any number of cascaded stages, and is rather similar to the procedure used with vacuum tubes.

Consider the equivalent circuit shown in Figure (6.1). The nodal equations for the output circuit are:

$$ -\beta I_b = e_3 \left[SC_o + \frac{1}{R_3} + \frac{1}{SL} \right] + \frac{e_3 - e_4}{r_{11}} \tag{6.1}$$

$$ \frac{e_3 - e_4}{r_{11}} = e_4 \left[SC_{IN} + \frac{1}{r_{12}} \right] \tag{6.2}$$

Or:

$$ \frac{e_3}{r_{11}} = e_4 \left[SC_{IN} + \frac{1}{r_{11}} + \frac{1}{r_{12}} \right] \tag{6.3}$$
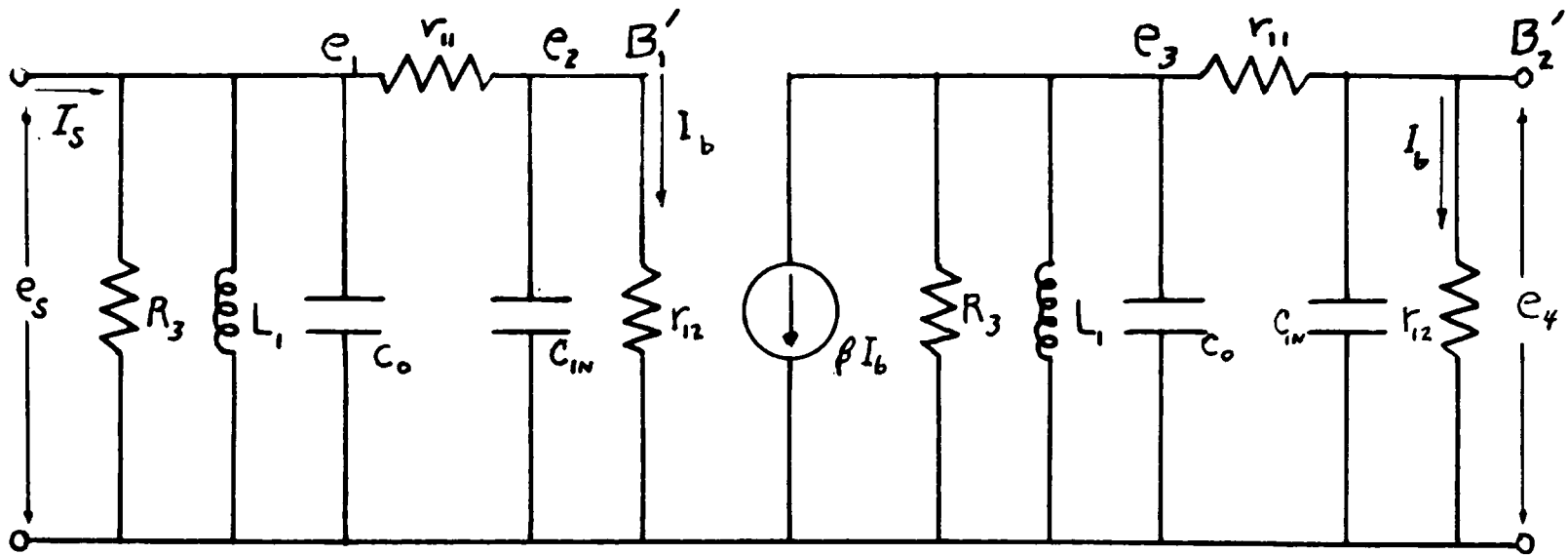
FIG. (6.1)   Equivalent Circuit for a Tuned Transistor Amplifier (One Stage).

So that:

$$e_3 = r_{11} e_4 \left(sC_{IN} + \frac{1}{r_{11}} + \frac{1}{r_{12}} \right)$$

(6.4)

Substitute equation (6.4) into equation (6.1).

$$-\beta I_b = r_{11} e_4 \left[sC_{IN} + \frac{1}{r_{11}} + \frac{1}{r_{12}} \right] \left[sC_0 + \frac{1}{r_{11}} + \frac{1}{R_3} + \frac{1}{sL} \right] - \frac{e_4}{r_{11}}$$

(6.5)

Or:

$$-\beta I_b = e_4 \left[r_{11} \left(sC_{IN} + \frac{1}{r_{11}} + \frac{1}{r_{12}} \right) \left(sC_0 + \frac{1}{r_{11}} + \frac{1}{R_3} + \frac{1}{sL} \right) - \frac{1}{r_{11}} \right]$$

(6.6)

Now, from the input circuit:

$$\frac{e_2}{r_{12}} = I_b$$

(6.7)

Substitute equation (6.7) into equation (6.6).

$$-\beta \frac{e_2}{r_{12}} = e_4 \left[r_{11} \left(sC_{IN} + \frac{1}{r_{11}} + \frac{1}{r_{12}} \right) \left(sC_0 + \frac{1}{r_{11}} + \frac{1}{R_3} + \frac{1}{sL} \right) - \frac{1}{r_{11}} \right]$$

(6.8)

The gain for one stage was defined as:

$$A(s) = \frac{e_4(s)}{e_2(s)}$$

(6.9)

Therefore:

$$A(s) = \frac{-\beta}{\left[r_{11} r_{12} \left(sC_{IN} + \frac{1}{r_{11}} + \frac{1}{r_{12}} \right) \left(sC_0 + \frac{1}{r_{11}} + \frac{1}{R_3} + \frac{1}{sL} \right) - \frac{r_{12}}{r_{11}} \right]}$$

(6.10)

Let

$$\frac{1}{r_{11}} + \frac{1}{r_{12}} = \frac{1}{R_1} \tag{6.11}$$

And

$$\frac{1}{r_{11}} + \frac{1}{R_3} = \frac{1}{R_2} \tag{6.12}$$

Therefore, equation (6.10) becomes

$$A(s) = \frac{-\beta}{\left[r_{11} r_{12} \left(sC_{IN} + \frac{1}{R_1} \right)\left(sC_0 + \frac{1}{R_2} + \frac{1}{sL} \right) - \frac{r_{12}}{r_{11}} \right]} \tag{6.13}$$

Rearranging terms, equation (6.13) can be expressed as

$$A(s) = \frac{-\beta}{\left[r_{11} r_{12} C_0 C_{IN} \left(s + \frac{1}{R_1 C_{IN}} \right)\left(\frac{s^2 + s/R_2 C_0 + 1/L C_0}{s} \right) - \frac{r_{12}}{r_{11}} \right]} \tag{6.14}$$

The term r_{12}/r_{11} was calculated and found to be around 3. This term, upon expansion of the denominator, will be part of the coefficient of the "s" term in the cubic equation. The other components of this coefficient were determined to be about 115. Therefore, the r_{12}/r_{11} term may be neglected.

The expression for the gain becomes

$$A(s) = \frac{-\beta s}{r_{11} r_{12} C_0 C_{IN} \left(s + \frac{1}{R_1 C_{IN}} \right)\left(s^2 + \frac{s}{R_2 C_0} + \frac{1}{L C_0} \right)} \tag{6.15}$$

Consider the coupling circuit in the input of the amplifier. The nodal equations are

$$I(s) = e_1 \left(sC_0 + \frac{1}{R_3} + \frac{1}{sL} \right) + \frac{e_1 - e_2}{r_{11}} \tag{6.16}$$

$$\frac{e_1 - e_2}{r_{11}} = e_2 \left(sC_{IN} + \frac{1}{r_{12}} \right) \qquad (6.17)$$

Hence

$$\frac{e_1}{r_{11}} = e_2 \left(sC_{IN} + \frac{1}{r_{12}} + \frac{1}{r_{11}} \right) \qquad (6.18)$$

And

$$\frac{e_2}{e_1} = \frac{1}{r_{11} \left(sC_{IN} + \frac{1}{r_{11}} + \frac{1}{r_{12}} \right)} \qquad (6.19)$$

Also

$$I_s(s) = e_2 \left[r_{11} \left(sC_{IN} + \frac{1}{r_{11}} + \frac{1}{r_{12}} \right)\left(sC_0 + \frac{1}{R_3} + \frac{1}{sL} \right) - \frac{1}{r_{11}} \right] \qquad (6.20)$$

For the input signal

$$I_s = \frac{e_s}{R_s} \qquad (6.21)$$

Therefore

$$\frac{e_2}{e_s} = \frac{1}{R_s r_{11} \left(sC_{IN} + \frac{1}{r_{12}} + \frac{1}{r_{11}} \right)\left(sC_0 + \frac{1}{R_3} + \frac{1}{sL} \right) - \frac{R_s}{r_{11}}} \qquad (6.22)$$

The R_s/r_{11} term in this expression can be neglected as the ratio
of this term to the other terms in the coefficient of "s" is about
3 to 115. Therefore, the expression for the gain becomes

$$A(s) = \frac{s}{r_{11} R_s C_0 C_{IN} \left(s + \frac{1}{R_1 C_{IN}} \right)\left(s^2 + \frac{s}{R_2 C_0} + \frac{1}{L C_0} \right)} \qquad (6.23)$$

To determine the overall voltage gain of a cascade of "n" identical tuned stages, it is necessary to consider only the output or collector circuit of the last stage. The output voltage will appear at the collector terminals of the last stage, and the ratio of this voltage to the input signal voltage will be the overall voltage gain of the cascade.

From the initial definition it is clear that the voltage gain of the last stage will be

$$A(s) = \frac{-\beta s}{r_{12} C_0 \left(s^2 + \frac{s}{R_3 C_0} + \frac{1}{L C_0}\right)} \tag{6.24}$$

Therefore, the overall voltage gain of "n" identical tuned stages in cascade is

$$A(s) = \frac{(-\beta)^n s^{n+1}}{R_S C_0 \left(s^2 + \frac{s}{R_3 C_0} + \frac{1}{L C_0}\right)\left(r_{11} r_{12} C_0 C_{IN} \left\{s^2 + \frac{s}{R_2 C_0} + \frac{1}{L C_0}\right\}\right)^n} \tag{6.25}$$

The significance of this result shall be discussed later in this chapter.

(6.3) Center Frequency of Tuned Transistor Amplifiers

One of the most important design considerations for bandpass amplifiers is the center or resonant frequency of the amplifier. The expression for the center frequency of one stage of a tuned transistor amplifier shall be derived.

The circuit diagram is shown in Figure (6.2) The circuit can

also be expressed in the form of Figure (6.3). From Figures (6.2) and (6.3), it is immediately obvious that

$$Y_1 = G_1 + j\omega C_1 + \frac{1}{j\omega L} \tag{6.26}$$

$$Y_2 = G_2 + j\omega C_2 \tag{6.27}$$

$$Y_3 = G_3 \tag{6.28}$$

The total admittance is given by

$$Y_T = Y_1 + \frac{Y_2 Y_3}{Y_2 + Y_3} \tag{6.29}$$
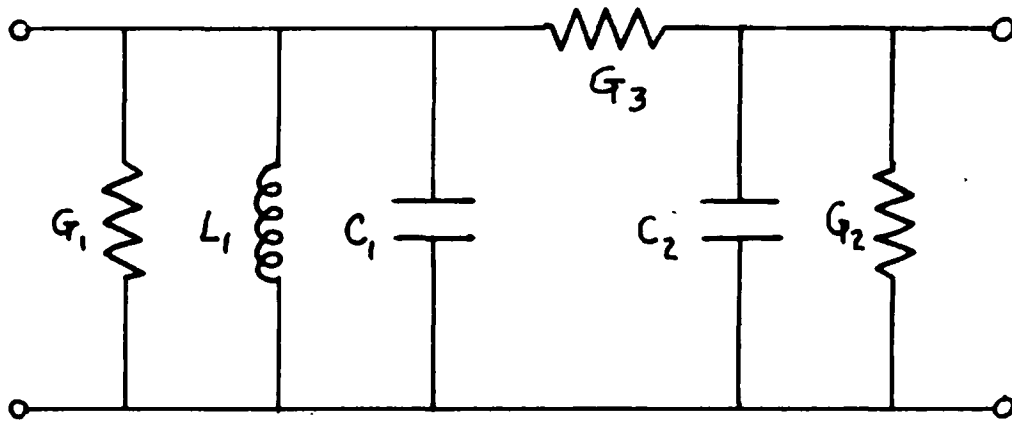
Substitute for Y_1, Y_2, and Y_3

$$Y_T = \left(G_1 + j\omega C_1 + \frac{1}{j\omega L}\right) + \frac{G_3 (G_2 + j\omega C_2)}{G_3 + G_2 + j\omega C_2} \tag{6.30}$$

Multiply through by the denominator

$$Y_T = \frac{\left(G_1 + j\omega C_1 + \frac{1}{j\omega L}\right)(j\omega C_2 + G_2 + G_3) + G_2 G_3 + j\omega C_2 G_3}{j\omega C_2 + G_2 + G_3} \tag{6.31}$$

Let $G_2 + G_3 = G_4$. Equation (6.31) becomes

$$Y_T = \frac{\left(G_1 + j\omega C_1 + \frac{1}{j\omega L}\right)(j\omega C_2 + G_4) + G_2 G_3 + j\omega C_2 G_3}{j\omega C_2 + G_4} \tag{6.32}$$

FIG. (6.2.) Circuit Used to Determine
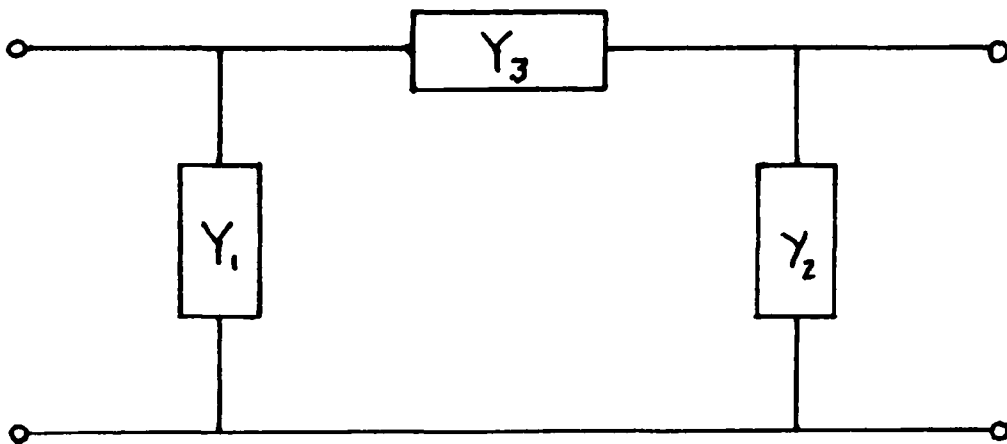The Center Frequency of
Tuned Transistor Amplifiers.

FIG.(6.3.) Alternate Form of Figure (6.2).

Rationalize equation (6.32)

$$Y_T = \frac{\left(G_1 + j\omega C_1 + \frac{1}{j\omega L}\right)\left(G_y^2 + \omega^2 C_z^2\right) + \left(G_2 G_3 + j\omega C_2 G_3\right)\left(G_y - j\omega C_z\right)}{G_y^2 + \omega^2 C_z^2} \quad (6.33)$$

Now, at resonance, the imaginary part of the previous expression is equal to zero. Therefore, the expression is

$$\left(j\omega C_1 + \frac{1}{j\omega L}\right)\left(G_y^2 + \omega^2 C_z^2\right) + j\omega C_2 G_3 G_y - j\omega C_2 G_2 G_3 = 0 \quad (6.34)$$

Or, in more convenient form

$$\omega^2 L C_1 G_y^2 + \omega^4 L C_1 C_z^2 - G_y^2 - \omega^2 C_z^2 + \omega^2 L C_2 G_3 G_y - \omega^2 L C_2 G_2 G_3 = 0 \quad (6.35)$$

Factor equation (6.35)

$$\omega^4 \left(L C_1 C_z^2\right) + \omega^2 \left[L C_1 G_y^2 - C_z^2 + L C_2 G_3 G_y - L C_2 G_2 G_3\right] - G_y^2 = 0 \quad (6.36)$$

Divide by $L C_1 C_z^2$

$$\omega^4 + \omega^2 \left[\frac{G_y^2}{C_z^2} - \frac{1}{L C_1} + \frac{G_3 G_y}{C_1 C_2} - \frac{G_2 G_3}{C_1 C_2}\right] - \frac{G_y^2}{L C_1 C_z^2} = 0 \quad (6.37)$$

Therefore:

$$\omega^2 = \frac{-b \pm \sqrt{b^2 - 4C}}{2} \quad (6.38)$$

Where:

$$b = \frac{G_4^2}{C_2^2} - \frac{1}{LC_1} + \frac{G_3 G_4}{C_1 C_2} - \frac{G_2 G_3}{C_1 C_2} \tag{6.39}$$

$$C = \frac{G_4^2}{L C_1 C_2^2} \tag{6.40}$$

Hence, the resonant frequency of the circuit is

$$\omega_r = \sqrt{\frac{(-b) + \sqrt{b^2 - 4C}}{2}} \tag{6.41}$$

The significance of this expression shall also be discussed in the next paragraph.

(6.4) Proposed Method of Design For Tuned Transistor Amplifiers

The expressions developed for the gain and center frequency of bandpass transistor amplifiers are very cumbersome, and their complexity presents many problems to the circuit designer. Approximations were attempted, but they were found to be invalid. For example, the design procedure could be greatly simplified if the real pole in the complex s plane could be neglected (the pole-zero diagram for equation 6.15 is shown in Figure 6.4). Unfortunately, the location of the real pole, which is determined by the physical structure of the transistor, is such that it can not be neglected.

At the present time, the most practical solution of this problem seems to be the use of an effective equivalent circuit. By using this method, the pole-zero diagram assumes the form shown in Figure (6.5).

The real pole is eliminated by assuming all the impedances of the equivalent circuit are in parallel. It should be emphasized that the base-spreading resistance, r_{11}, is not being neglected. This will become more clear from the method used to determine the effective parameters, which will be described below.

The pole-zero diagram of the effective equivalent circuit is practically of the same form as that obtained in vacuum tube circuits. The center frequency and the bandwidth are immediately obvious, and the design is, therefore, greatly simplified. The effective equivalent circuit is assumed to be of the form shown in Figure (6.6). The values of the effective parameters were determined in the following manner.

The circuit diagram of the circuit used to determine the effective values of the parameters in the input circuit is shown in Figure (6.7). The load resistance in the collector circuit is set equal to zero, and the output circuit is therefore broadbanded - the base to collector voltage gain being equal to zero. A coil, whose inductance and resistance is accurately known, is placed in shunt with the input circuit of the transistor. By changing the signal frequency, a maximum value of e_2 is found. This maximum occurs at the resonant frequency of the input circuit of the transistor. The bandwidth of the input circuit is then found by varying the frequency on either side of the resonant frequency.

The input capacitance, C_{in}, is then given by

$$C_{IN} = \frac{1}{\omega_o^2 L}$$
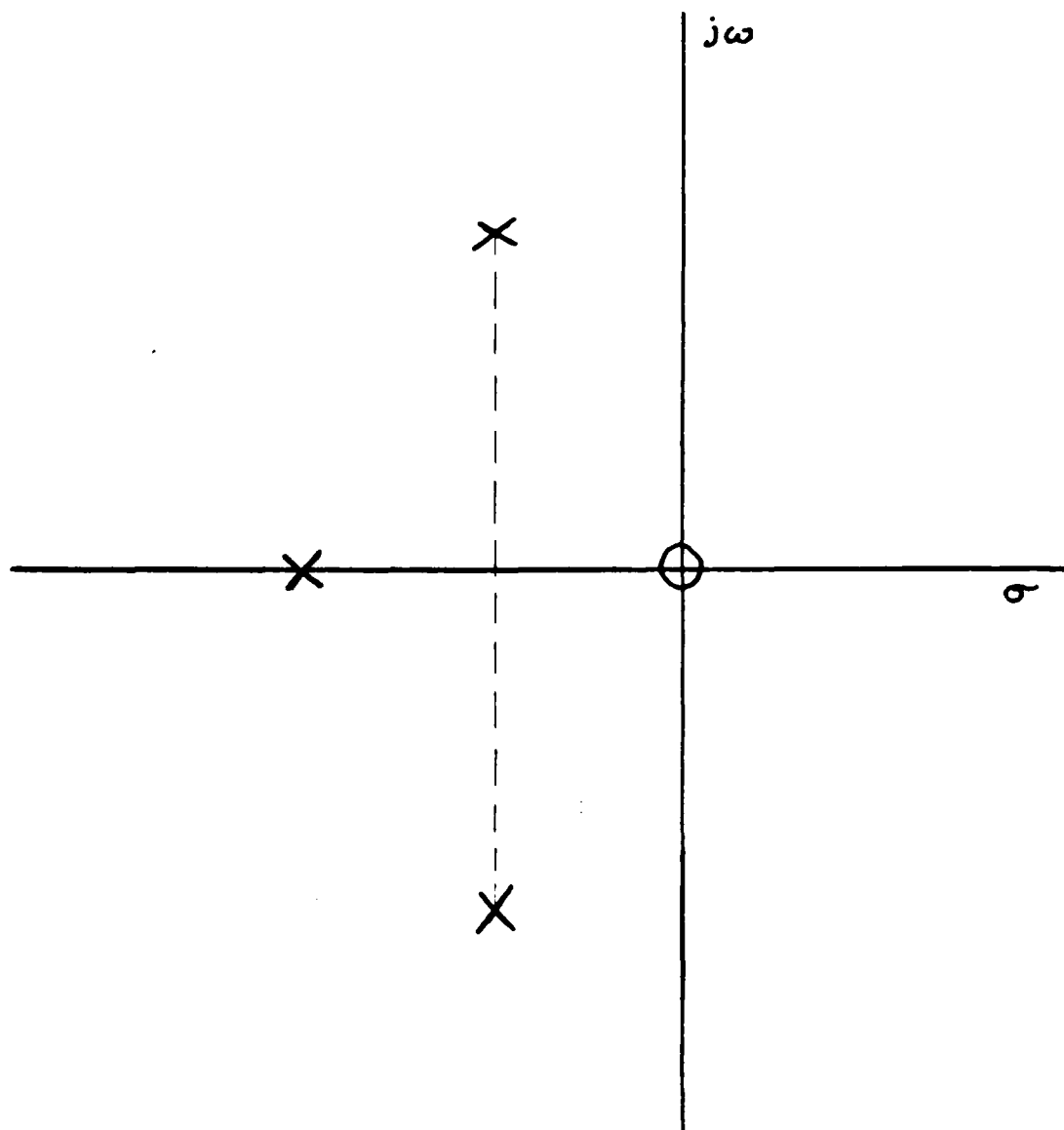
(6.42)

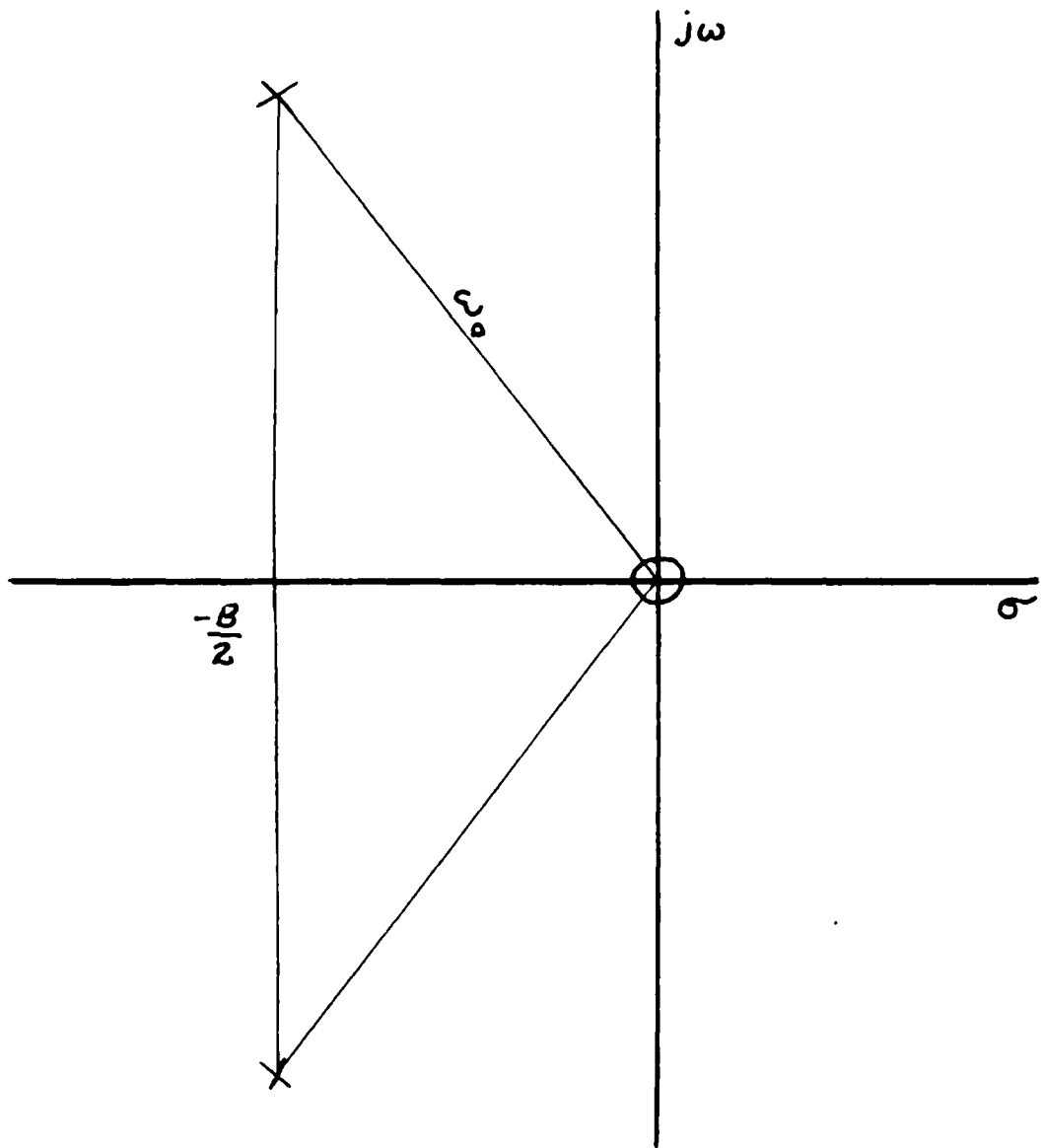FIG. (6.4)   Location of Poles and Zeroes —
Actual Circuit

FIG. (6.5)   Location of Poles and Zeroes —
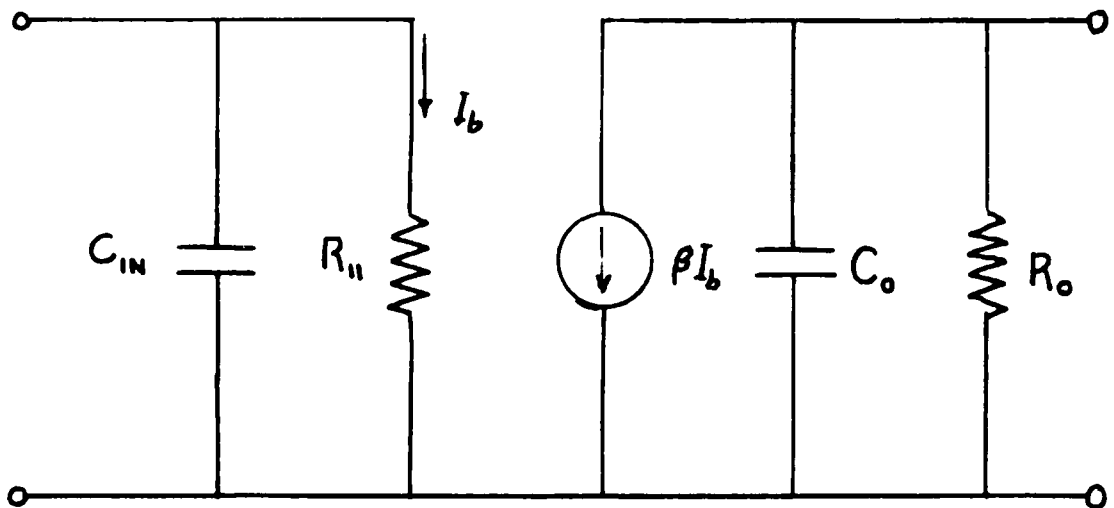   Effective Equivalent Circuit

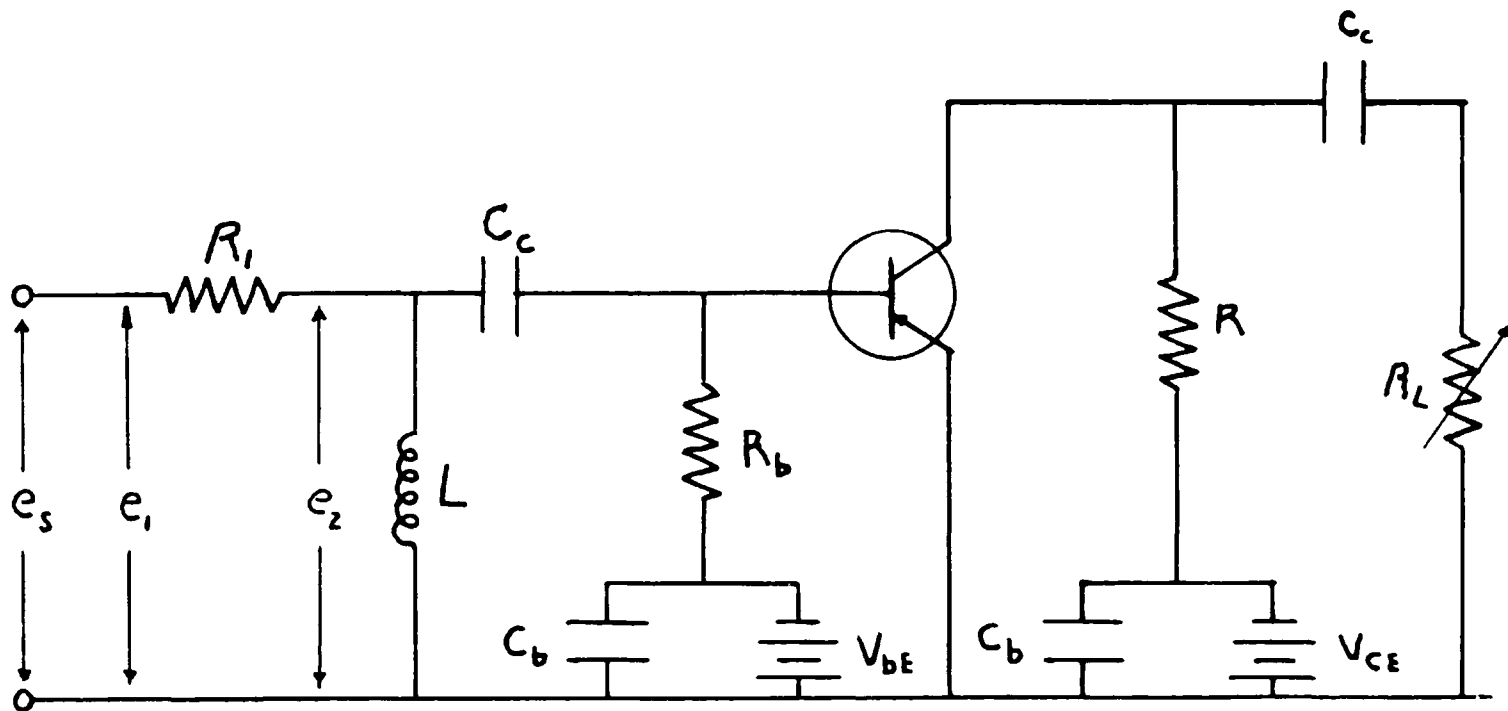FIG. (6.6)   Effective Equivalent Circuit
Grounded Emitter Amplifier.

FIG. (6.7)   Circuit Used to Determine Effective
Values of Parameters.

The input resistance, R_{11}, is easily calculated from:

$$R = \frac{1}{2 \pi B C_T} \qquad (6.43)$$

Where

$$C_T = C_{IN} \qquad (6.44)$$

$$R = \frac{R_{11} R_{ar}}{R_{11} + R_{ar}} \qquad (6.45)$$

R_{ar} being the parallel resistance
of the coil at this frequency.

A curve of the input circuit response is shown in Figure (6.8). The value of the input capacitance is a function of the base to collector gain. This is analagous to the Miller effect observed in vacuum tubes, and the equations are of the same form[3]. The curve of the input capactiance as a function of voltage gain is shown in Figure (6.9).

All the information required to design a tuned transistor amplifier can be determined in the preceeding manner. This method was used and an amplifier constructed in the laboratory. The characteristics of this amplifier will be discussed in the next section.

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3. A. N. Perkins, "A Common Emitter Equivalent Circuit For Transistor Design," (Thesis), University of Arizona, 1955.
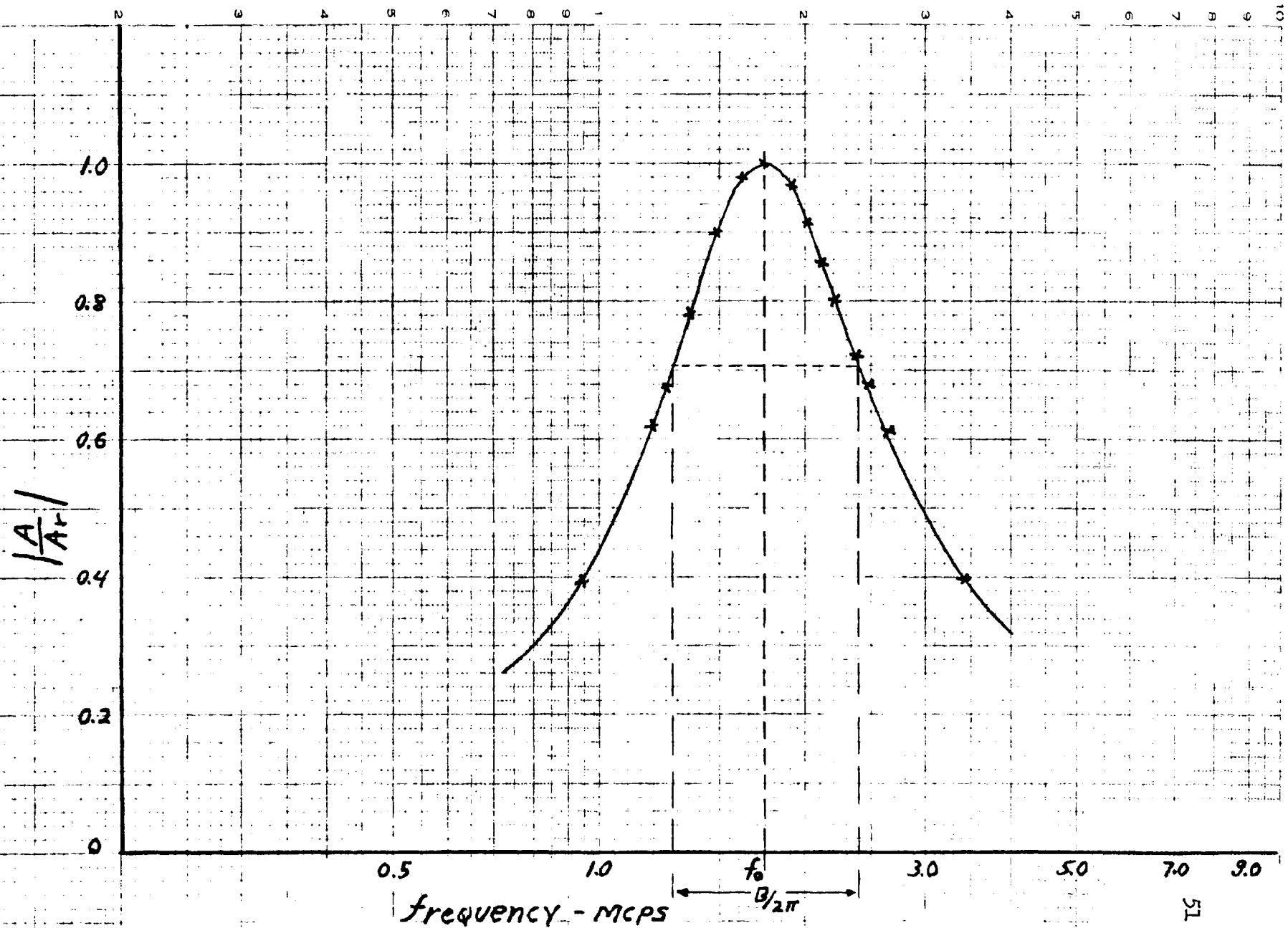
FIG (6·8)   Frequency Response of The Input Circuit for
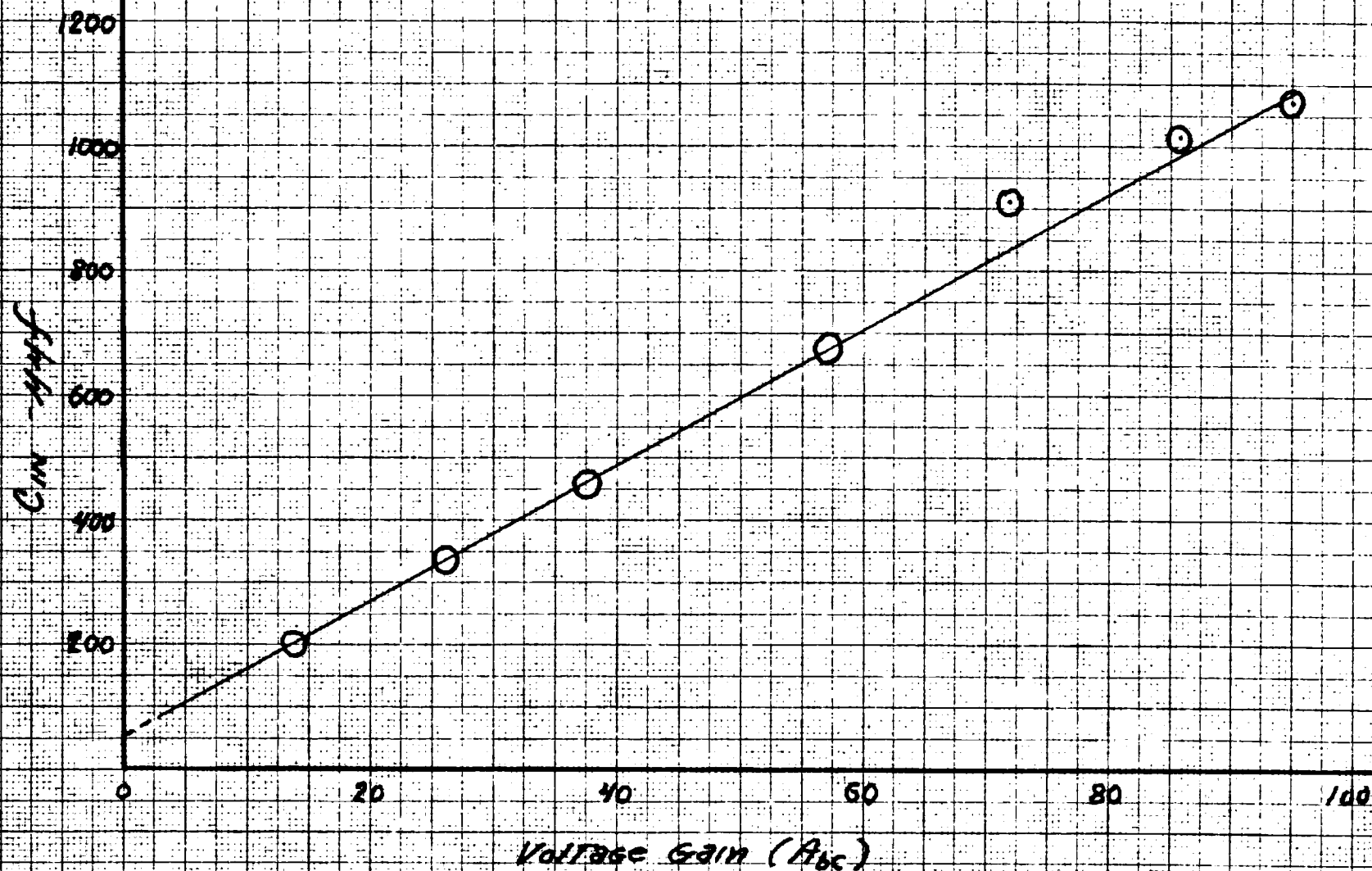   Philco Surface Barrier 2N-128 #5

51

FIG. (6.9) Input Capacitance vs. Voltage Gain for Philco Surface Barrier 2N128 # 4

(6.5) Neutralization of Tuned Transistor Amplifiers

The problem of neutralization arises when using a one stage amplifier with both input and output tuned or when cascading tuned amplifiers. This can be seen from the circuit diagram shown in Figure (6.10).

The base to collector feedback capacitor, C_{bc}, provides positive feedback when the input conductance is negative (the input conductance becomes negative when the impedance in the collector circuit is inductive.) If the positive feedback is of sufficiently large amplitude, the amplifier will oscillate.

The method of neutralizing this type of tuned transistor amplifier is analogous to the Hazeltine system used in grounded cathode vacuum tube amplifiers. The circuit diagram of a typical neutralized grounded emitter amplifier is shown in Figure (6.11). The equivalent circuit of this amplifier is shown in Figure (6.12).

The circuit will be neutralized when E_{be} is equal to zero, or:

$$E_{bE} = E_{cN} + E_{L3} = E_{c_{bc}} + E_{L2} = 0 \qquad (6.46)$$

Therefore, for proper neutralization

$$E_{cN} = -E_{L2} \qquad (6.47)$$
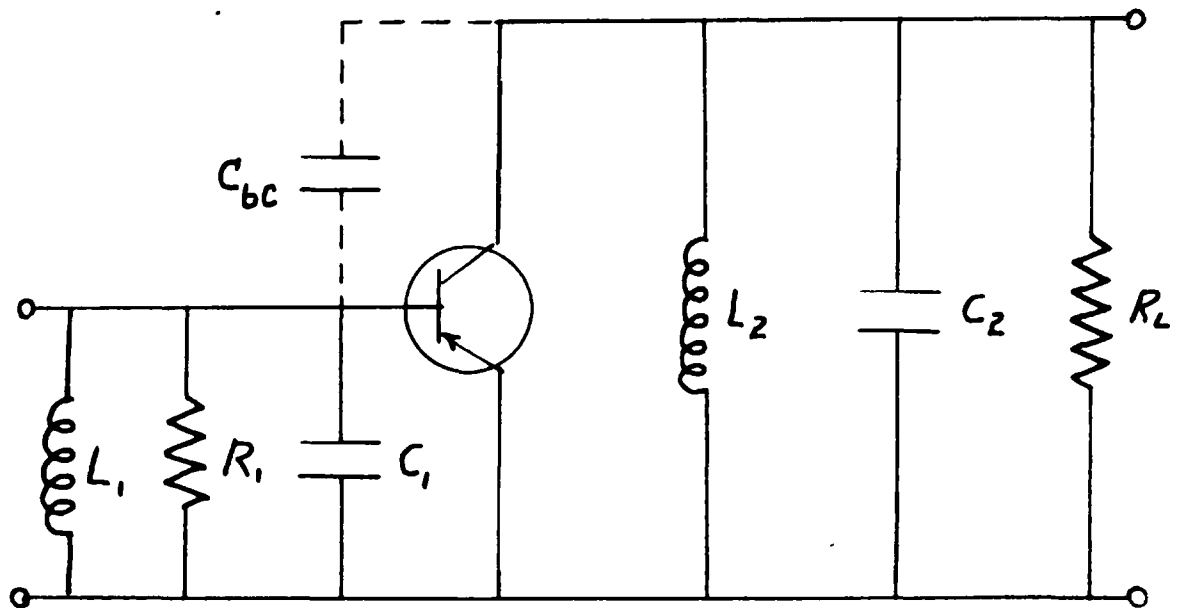
And

$$E_{c_{bc}} = -E_{L1} \qquad (6.48)$$

FIG. (6.10) One Stage Transistor Amplifier
With Tuned Input and Output
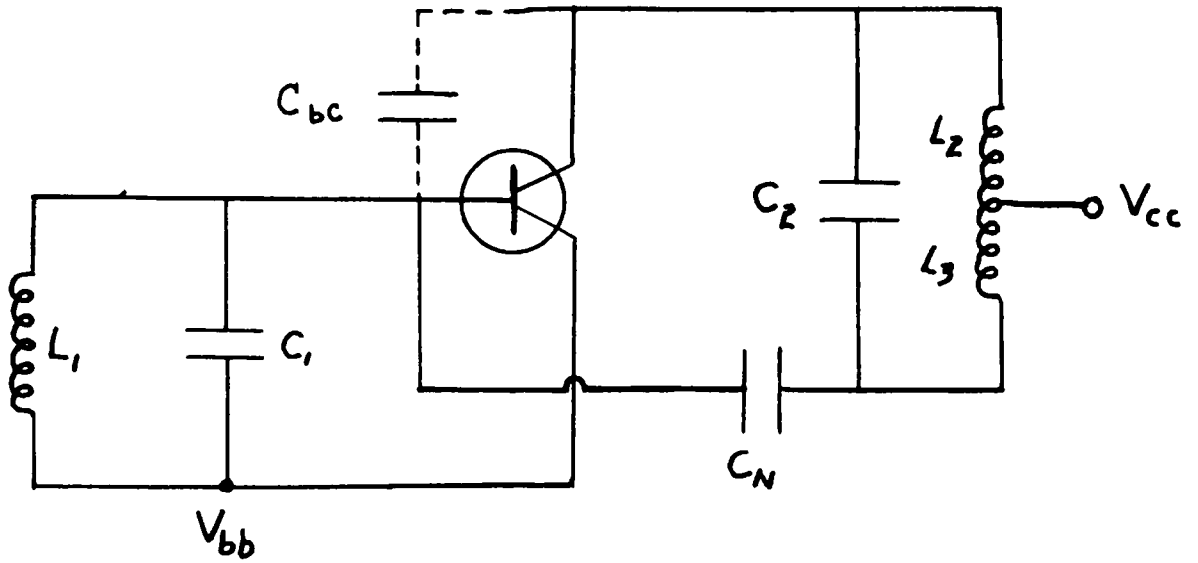Circuits.

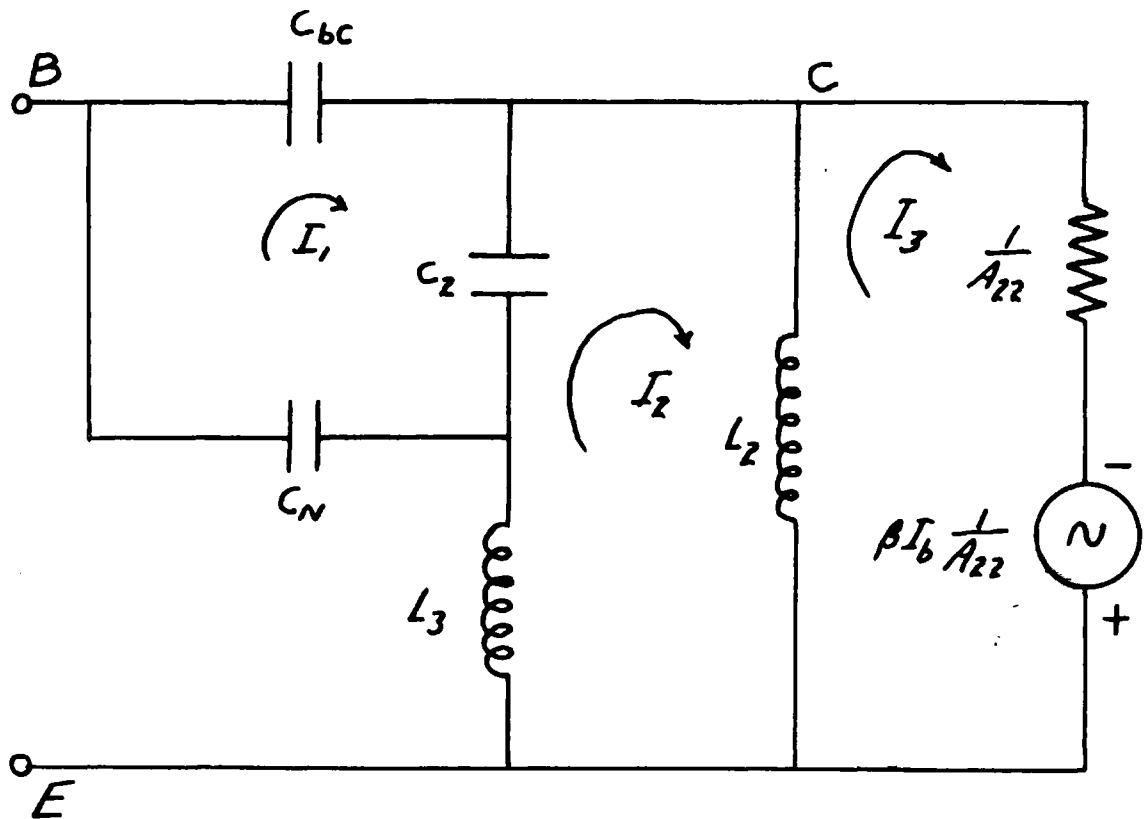FIG. (6.11) Neutralized Grounded Emitter Tuned Amplifier.

FIG. (6-12)   Equivalent Circuit of a
              Neutralized Grounded
              Emitter Amplifier.

If I_2 is much larger than I_3, the currents through L_2 and L_3 will be the same. This is assumed to be true for high Q circuits. Then, for steady-state operation:

$$\frac{-I_1}{j\omega C_N} = I_2 \left(j\omega L_3\right) \tag{6.49}$$

And

$$\frac{I_1}{j\omega C_{bc}} = -I_2 \left(j\omega L_2\right) \tag{6.50}$$

Also, from equation (6.49):

$$-I_1 = I_2 \left(j\omega L_3\right) j\omega C_N \tag{6.51}$$

Substitute equation (6.51) into equation (6.50).

$$\frac{-I_1 \left(j\omega L_3\right) j\omega C_N}{j\omega C_{bc}} = -I_2 \left(j\omega L_2\right) \tag{6.52}$$

Therefore:

$$C_N = C_{bc} \frac{L_2}{L_3} \tag{6.53}$$

Using the above results, a grounded emitter amplifier was constructed with both the input and the output circuits tuned to the same frequency. Without neutralization, the circuit immediately burst into sustained oscillation upon the application of bias potentials. The inductances in both the input and the output circuits were varied, but with no effect.

The circuit was then altered by first center-tapping the inductor

in the collector circuit and connecting the neutralization capacitor, C_n, from one side of this coil to the base of the transistor. This is the circuit that is shown in Figure (6.12). Bias potentials were applied and the circuit no longer oscillated but operated as an amplifier.

(6.6) <u>Characteristics of An Experimental Tuned Transistor Amplifier</u>

The design procedure for tuned transistor amplifiers was outlined in section (6.4). Using this method, a one stage tuned amplifier was constructed and its characteristics studied. The amplifier was designed to have a center frequency of 4.3 megacycles, a bandwidth of 200 kilocycles, and a gain of at least 16. The circuit diagram is shown in Figure (6.13).

The theoretical and experimental response curves are shown in Figure (6.14). The experimental amplifier had a center frequency of 4.1 megacycles, a bandwidth of 240 kilocycles, and a gain of 19.4. Although the results are not exactly equal to the calculated values, they are rather encouraging.

Theoretically, the gain-bandwidth product was 3.2×10^6 cycles per second, while the gam-bandwidth product determined experimentally was 4.66×10^6 cycles per second. As in vacuum tube circuits, the gain-bandwidth product may be thought of as a figure of merit for the circuit in question. Because the figure of merit of the actual circuit was greater than the theoretically calculated value, the design appears to be a rather pessimistic one. Perhaps one of the more

outstanding reasons for this could be the use of average values for the parameters of the effective equivalent circuit.

The error in the center frequency is less than 5 %. This could be caused by the values of the parameters of the effective equivalent circuit, which are not accurate to more than 5 % because of instrumentation. Also, the inductances used were hand-wound on ceramic coil forms and it is possible that the values for the inductance and effective resistance of the coils could be slightly in error.

Although the results seem to validate the proposed method of designing tuned transistor amplifiers, the author feels that the subject bears further investigation.
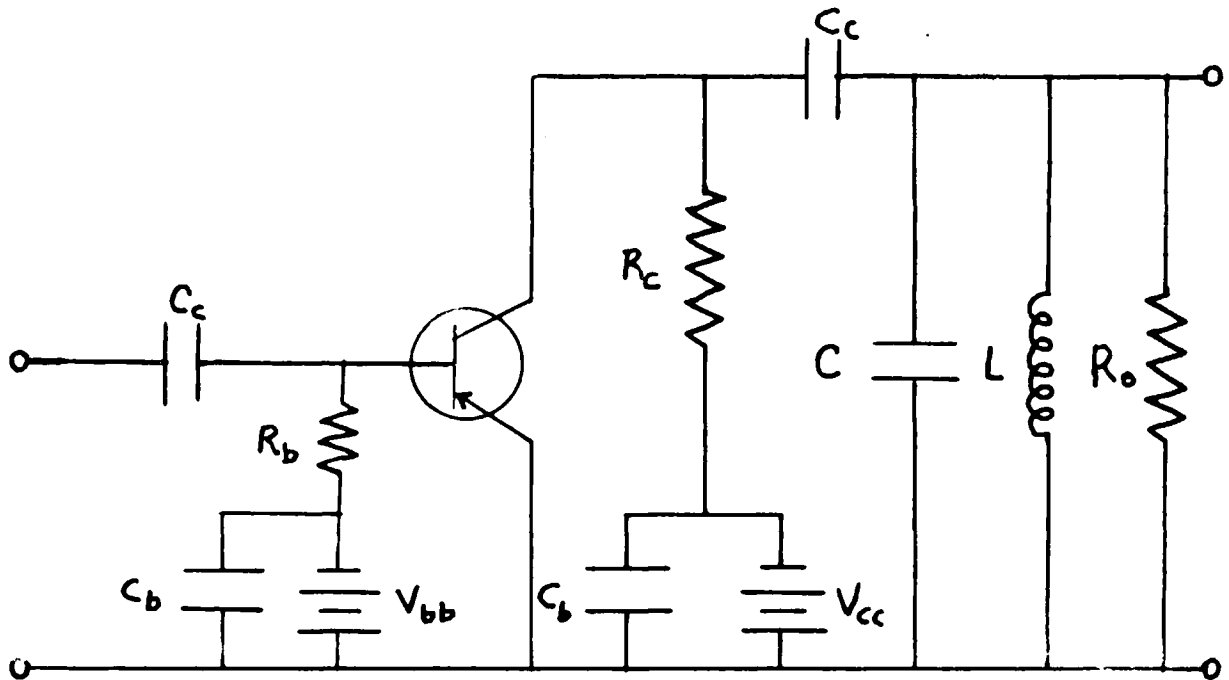
FIG (6-13). Grounded Emitter Tuned
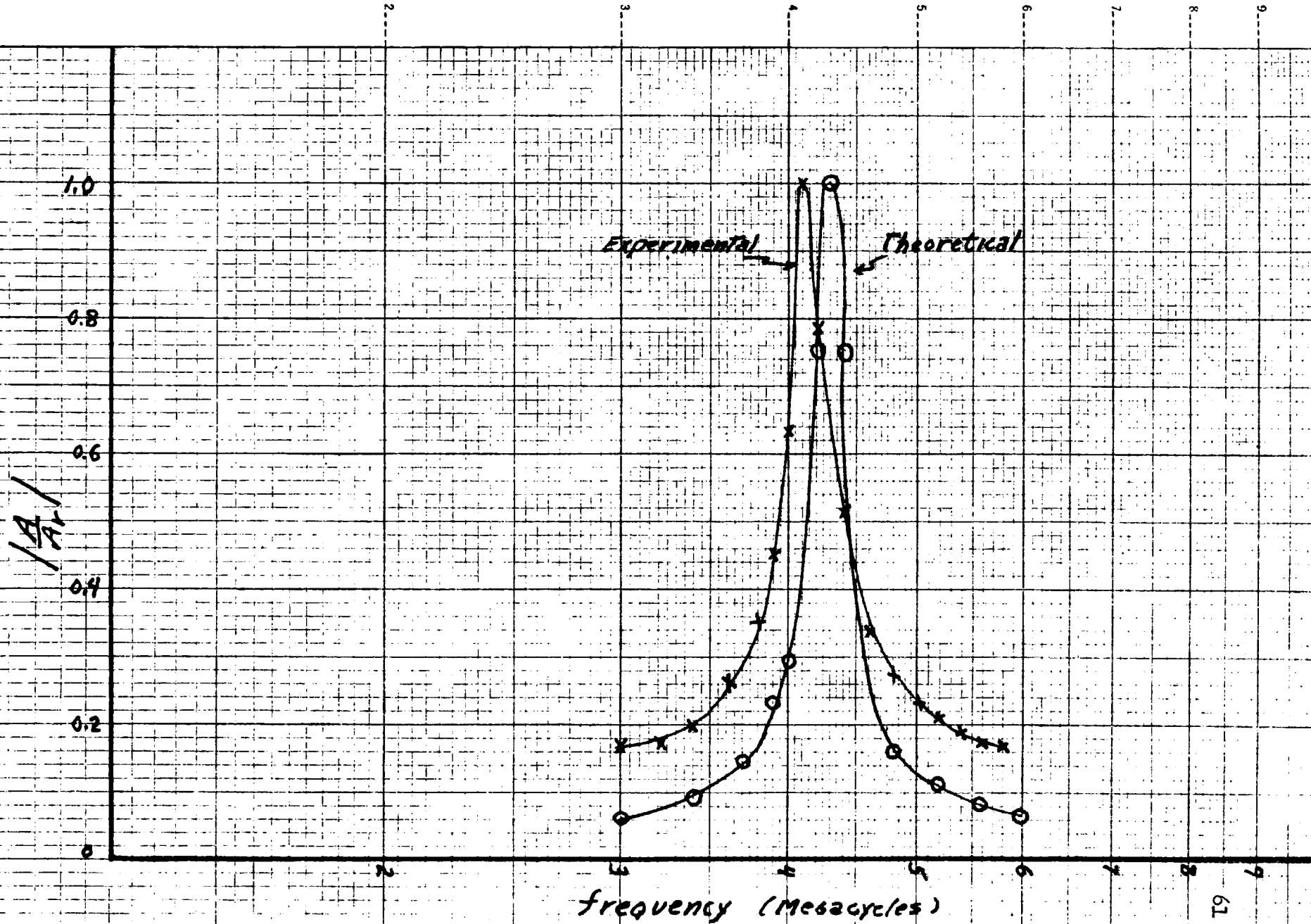Transistor Amplifier.

FIG (6-14) Frequency Response of a Grounded Emitter Tuned Transistor Amplifier.

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