AN INVESTIGATION OF THE TRACKING CAPABILITY OF A HUMAN PILOT

by

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STATEMENT BY AUTHOR

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June 19, 1964
Date

PREFACE

The primary purpose of investigating the problem presented in this thesis is to provide a method of predicting the tracking capability of a human pilot when presented with the task of controlling an aircraft which is subjected to random disturbances. A secondary purpose is to attempt to arrive at some approximate transfer function which will describe the pilot's performance when placed in the feedback loop of a closed control system.

The equations of motion of an aeroplane will be programmed on an analog computer and the resulting attitude and flight characteristics of the aeroplane will be displayed to a pilot seated in a simulator cockpit. The pilot will attempt to maintain a desired attitude of the aeroplane by providing control inputs to the computer. Gust disturbances will make the problem more difficult for the pilot and will provide a means of measuring his response time and tracking accuracy.

The autopilot was adjusted so that its output closely approximated that of the human pilot. The transfer function of the autopilot was easy to establish since it was comprised of simple electrical elements. It was then assumed that the pilot's transfer function was that of the autopilot.

ACKNOWLEDGMENTS

I gratefully acknowledge the Department of Aerospace and Mechanical Engineering at The University of Arizona for making available the equipment used for the experimental portion of this thesis. The author also wishes to acknowledge Dr. E. K. Parks, thesis advisor, for his expert, professional guidance, and counsel during the experimental tests and drafting of the thesis.

Appreciation is also extended to Charles C. Blackwell, a doctoral candidate in the Department of Aerospace and Mechanical Engineering, who was mainly responsible for the development and design of the delay circuit incorporated within this thesis; Mr. Otis O'Brien and Mr. Gerald Fields for their assistance in constructing the simulator; and to the pilots who made it possible to obtain the data.

For her patience and understanding during my graduate studies, I thank my wife, Norma; and Mrs. Meta M. Anderson for being so kind to type the manuscript.

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LIST OF SYMBOLS

$G(s)$	Transfer function
CL_o	Zero lift coefficient
W	Weight of aircraft, pounds
m	Mass of aircraft, slugs
g	32.2 ft/sec^2
S	Wing area, ft^2
$\bar{C}$	Mean aerodynamic chord, ft
b	Wing span, ft
ρ	Air density, slugs
V	Velocity, ft/sec
u_o	Level flight velocity, ft/sec
I_{xx}, I_{yy}, I_{zz}	Moments of inertia about (x, y, z) axis, slug-ft^2
I_{xz}	Product of inertia, slug-ft^2
δ_a, δ_e	Aileron and elevator angle, radians
p, q	Angular velocity in roll and pitch, radians/sec
$\mathring{p}$, $\mathring{q}$	Angular acceleration in roll and pitch, radians/sec^2

$D = t^* \frac{d}{dt}$ where $t^* = \frac{\ell}{u_o}$ and ℓ $= \bar{C}/2$ longitudinal equations

$= b/2$ lateral equations

$\mu = \frac{m}{\rho S \ell}$

$i_A = \frac{I_{xx}}{\rho S \ell^3}$, $i_B = \frac{I_{yy}}{\rho S \ell^3}$, $i_C = \frac{I_{zz}}{\rho S \ell^3}$

$i_E = \frac{I_{xz}}{\rho S \ell^3}$

$(\mathcal{L})$	Laplace transform of A function
α	Angle of attack, radians
β	Angle of side slip, radians
θ, ϕ, ψ	Pitch, roll, and yaw angles, radians
τ	Time constant

$C_{\ell p}$, C_{xu}, C_{mg}, $C_{x\alpha}$, $C_{z\alpha}$, $C_{m\alpha}$, $C_{m\dot{\alpha}}$, $C_{z\delta_e}$, $C_{m\delta e}$,

$C_{\ell \delta a}$	Stability derivatives

ABSTRACT

The idea of substituting a mathematical model for a human pilot has been considered for some period of time in hopes of obtaining better results of prototype models of aircraft which are being constructed every day. In order to make this idea a useful tool, certain characteristics must be known about the behavior of a human pilot. This thesis investigates the tracking capability of human pilots and also arrives at suitable transfer functions that can be used as mathematical representations of a human pilot that is subjected to small perturbations in the pitching and rolling modes of flight. A simulator was constructed from salvaged aircraft parts and once completed represented a basic instrument flying configuration less the fact that the rudders were not free to move. The equations of motion were programmed on an analog computer and disturbances were fed into the system through the use of a function generator. A delay circuit was incorporated into the autopilot that accounted for the delay that is built into the humanbeing. Through the use of various recording instruments, it was possible to determine the pilot's transport delay and also to establish the elevator

and aileron deflections that a pilot applies to the control surface to return the vehicle to straight and level flight. Recordings were also made of the pitch and roll angles, and angle of attack. All of these parameters were used in establishing the autopilot that was considered as the representative of the human transfer function. Having established individual transfer functions for the rolling and pitching modes, some investigation was accomplished within the combined modes. It was determined that the results obtained for the individual degrees of freedom could not be combined, without change, to perform in a fashion similar to that of a pilot in this more complex mode.

This paper only is the beginning of a vast project which should be continued through the use of more and better instrumentation which would find an answer that would be more precise.

CHAPTER 1

INTRODUCTION

In the design of aircraft, the function that the aircraft must perform usually is well known prior to the beginning of its design. If the subject aircraft is to be a bomber type, one can expect the results to be a heavy, large aircraft capable of extended periods of flight. If the main interest is a fighter type aircraft, results usually obtain a fast, highly maneuverable aircraft with the undesired characteristic of short flights, time wise, without refueling. Finally, the fighter bomber is the compromise between the two previously mentioned types, with all types having the characteristics of high airspeed and altitude capabilities. Regardless of any consideration relative to speed, endurance, maneuverability, or any other characteristic of the manned aircraft, the fact that all of the aircraft must be manned is of great concern during the design testing and operation of these vehicles. The fact that the aircraft are manned is justified by two basic functions that must be performed. The pilot is primarily concerned with the actuation of the control systems which in turn provide the two basic functions of:

1. maintaining a prescribed flight attitude regardless of any external disturbances that may be imposed upon the system, and

2. being able to change the flight conditions to any other set of flight conditions when required or desired.

In the area of disturbances, the pilot senses a deviation, interprets it, and makes the necessary adjustment through appropriate application of forces applied to the control system. There will be a time lag involved between sensing and acting. The lag will be accompanied by some form of correction which, the authors feel, should have characteristics which can be studied and reproduced. It is within this area that this thesis subject is concentrated with the aim of attempting to reproduce this correction through use of an analytical model.

CHAPTER 2

ANALYTICAL CONSIDERATIONS

In analyzing a flight control system, one must be concerned with its overall operation which consists of many sub systems including the pilot. The aerodynamical and structural capabilities of these systems are known or can be determined and altered, if necessary, with the pilot as the only unalterable element. Through the use of an electronic computer such as the analog computer, simulation of the complete system, less the pilot, provides an excellent analytical method for predicting the response characteristics of the entire airframe and sub systems. The subject of representing the pilot with a mathematical model is a fairly new concept with more and more interest and emphasis being placed upon it every day.

In the past, and to a lesser extent even at the present time, the technique of analyzing the stability and control characteristics of a prototype aircraft has been accomplished by using a closed loop, but the pilot or a reasonable facsimile thereof has been omitted. This certainly puts a limitation on the evaluation obtained in the experimental stage. It often appears that some aircraft that have been designed and prototyped have been produced with complete disregard to the fact that they will carry a pilot. This is not to say that the airplane performance should be unduly comprised for the sake of pilot's comfort but, on the other hand, when a degree or two more of elevator or rudder deflection could save the life of a pilot or a whole crew and a load of passengers, it appears justifiable that if the pilot can be inserted into the closed loop system during the design phase of the vehicle, the results could prove more satisfactory and more complete. Some valuable and useful modifications are often made to compensate for possible engineering discrepancies by interviewing test pilots, but these pilots are only human and differ in opinion due to personal likes and dislikes, thereby making a final decision very difficult. Costwise, this method of evaluation and analysis is very high because a slight change at the final stage could result in several additional changes of other components. However, if the pilot could be represented in the infant stages, there exists the possibility that he could fit right into the pattern of design similar to the other sub systems.

The desired situation, of course, is to be able to evaluate the integrated pilot-airframe system. To apply a mathematical model would have certain advantages.

First of all, a pilot need not be present each time a test was conducted if his transfer function were included in the loop by use of a mathematical model. Additionally, a mathematical model would eliminate any individual idiosyncrasies that the test pilots might have. It is not the intention of this study to imply that the mathematical model can perform a task such as the pilot's thinking process, but rather those tasks that he has learned while proceeding through a pilot-training course.

It has been shown experimentally (1)* that no single transfer function can be produced that will completely simulate all the responses of a pilot, but it is possible to do it by parts, that is, particular transfer functions can be used for particular response characteristics.

Some questions that may have to be answered during the investigation within this thesis are:

1. Does the magnitude of the signal received affect the response time?

2. Does a pilot react equally fast to disturbances that vary in direction?

Studies have been conducted in this country (1) as well as in England (2), and many other countries to determine the answers to these various questions.

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* Numbers in parentheses refer to REFERENCES.

If we want to compare a pilot with his mechanical counterpart, he can be considered a simple servomechanism. He receives a visual signal which reflects the deviation of the aircraft from the desired attitude or flight path, interprets the signal and then actuates the controls to return the aircraft to its desired position. Should one want to replace the pilot with a servomechanism, his characteristics must be known. This means that his transfer function must be defined to properly represent him with a "black box."

Referring to Fig. 2.1, one observes that either a human pilot or a mechanical pilot is contained in the closed loop. The "black box" that has been mentioned is the auto pilot. The airplane receives a gust of wind, it is displayed on the oscilloscope and the pilot will then correct for it. By studying the pilot and analyzing his reactions, it is then possible to reproduce the pilot with his mathematical counterpart.

It seems appropriate at this time to discuss the meaning of the term "transfer function." Consider a system that is initially quiescent; that is, the input is zero, and the system is at rest in a state of equilibrium. After time zero ($t > o$), an input $x_i(t)$ is applied, and the resulting output $x_o(t)$ is obtained. The transfer function $G(s)$ of this system is defined as the ratio of

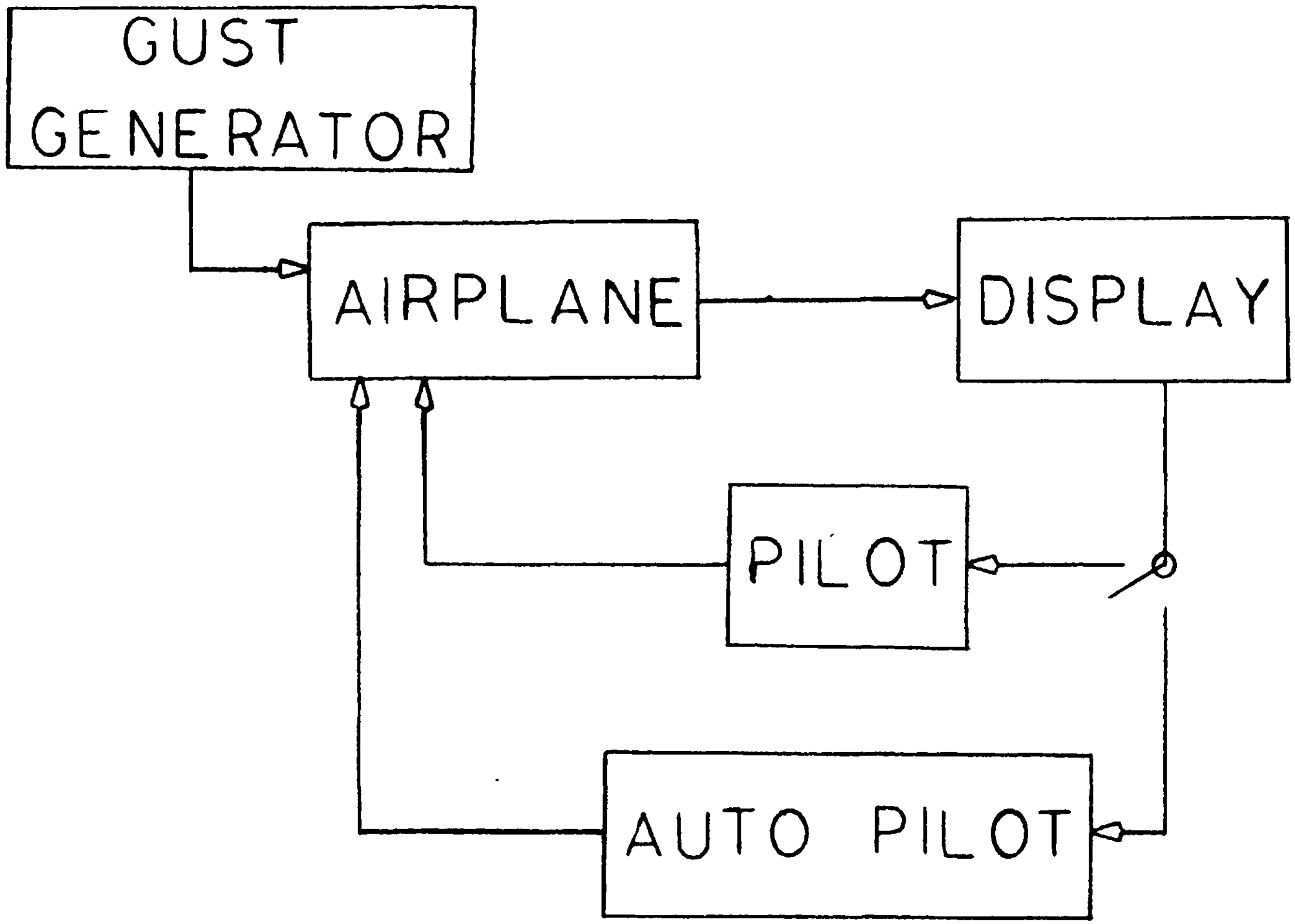
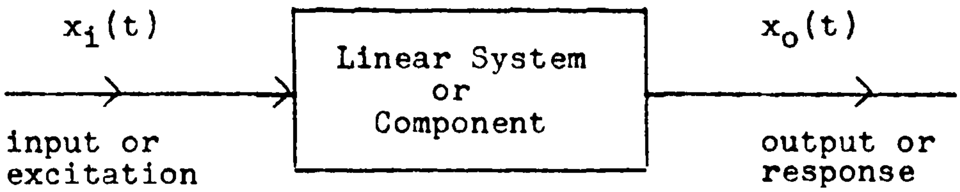


Fig. 2.1. Typical Closed Loop Diagram of an Aircraft System with Applied Disturbance.

the Laplace transforms of the output to the input. Specifying that the system be initially quiescent eliminates any transient that might be associated with non-zero initial conditions (3). As an example of a transfer function, consider the following system:



The transfer function of this system is

$$G(s) = \frac{\mathcal{L}x_o(t)}{\mathcal{L}x_i(t)}$$

Fig. 2.2 Block Diagram

This example is a very fundamental one but it does exemplify the meaning of a transfer function. This method of obtaining a transfer function can be extended to more complicated systems.

CHAPTER 3

ANALYSIS AND SET-UP OF EXPERIMENTAL EQUIPMENT

A realistic aircraft of the transport type was used for this study. The flight characteristics including stability derivatives, etc., were obtained from Ref. 3 and are tabulated in Appendix A. The reference system used in the derivation of the equations of motion were the wind-axis system due to certain advantages that are attached with them. To arrive at the wind axis, one starts with the body axis of the aircraft. The x and y axes of this orthogonal system lie in the plane of symmetry of the aircraft. The x axis is in the fore and aft direction with the positive sense forward, while the sense for the z axis is downward for the airplane in a level flight configuration. By rotating the body axis system about the y axis through an angle equal to the angle of attack α, the resulting new-axis system is the stability axis x_s, y_s, z_s. If the stability axes are then rotated about z_s through β, the angle of sideslip, then the x axis becomes aligned in the direction of the total aircraft velocity V_p, i.e., in the direction of the flight path. The resulting axis x_w, y_w, z_w, is the wind-axis system because the x_w axis is aligned exactly opposite to the relative wind. It is at this point that the advantages become apparent. Terms in the equations of motion that contain V and W are zero since there are no components of velocity in the y_w and z_w directions. Once the axes were established, the equations of motion could be evaluated and programmed into the computer, which represented the aircraft. Two of the six degrees of freedom of the airplane were restrained, the two restraints being on β and γ. The only lateral degree of freedom remaining was in roll or ϕ so no rudder inputs were included. The following stability derivatives are either zero or negligible and were neglected: $Cx\hat{q}$, $Cx\delta e$, $Cx\dot{\delta} e$, $Cz\dot{\delta} e$, $Cy\delta a$, $Cy\dot{\delta} a$.

It should be emphasized that none of these assumptions is basically necessary for the solution of airplane dynamics problems. They are made as a matter of experience and convenience. When it appears necessary to do so, any of the terms dropped can be restored into the equations (3). The remaining equations of motion as listed in Appendix B were represented on the computer and are shown in Figs. 3.1 and 3.2. The δe and δa inputs of Figs. 3.1 and 3.2 are provided by the pilot or by the equivalent autopilot to control the aircraft whenever it is disturbed from the desired flight attitude.

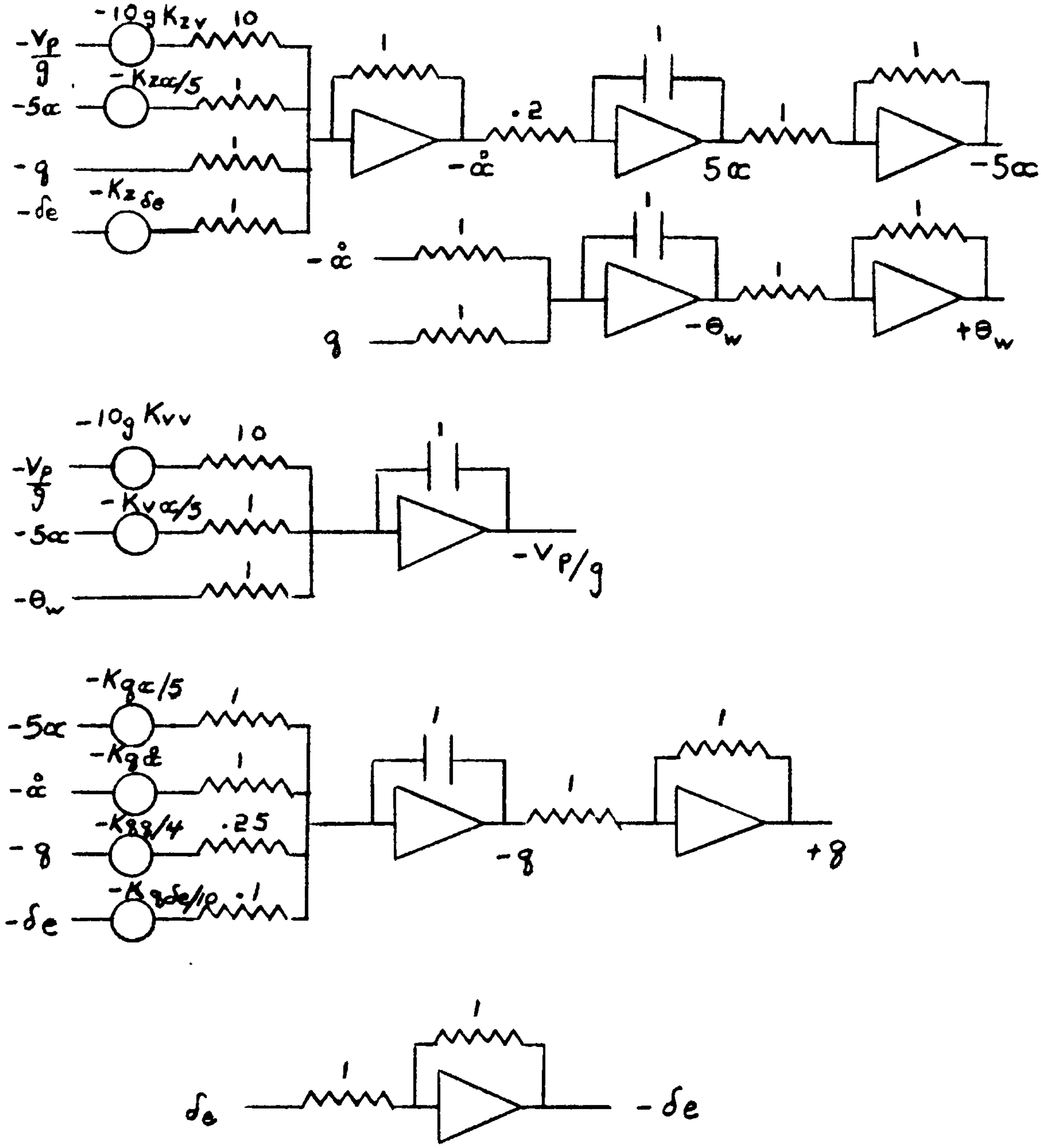


Fig. 3.1. Analog Computer Representation of the Longitudinal Equations of Motion.

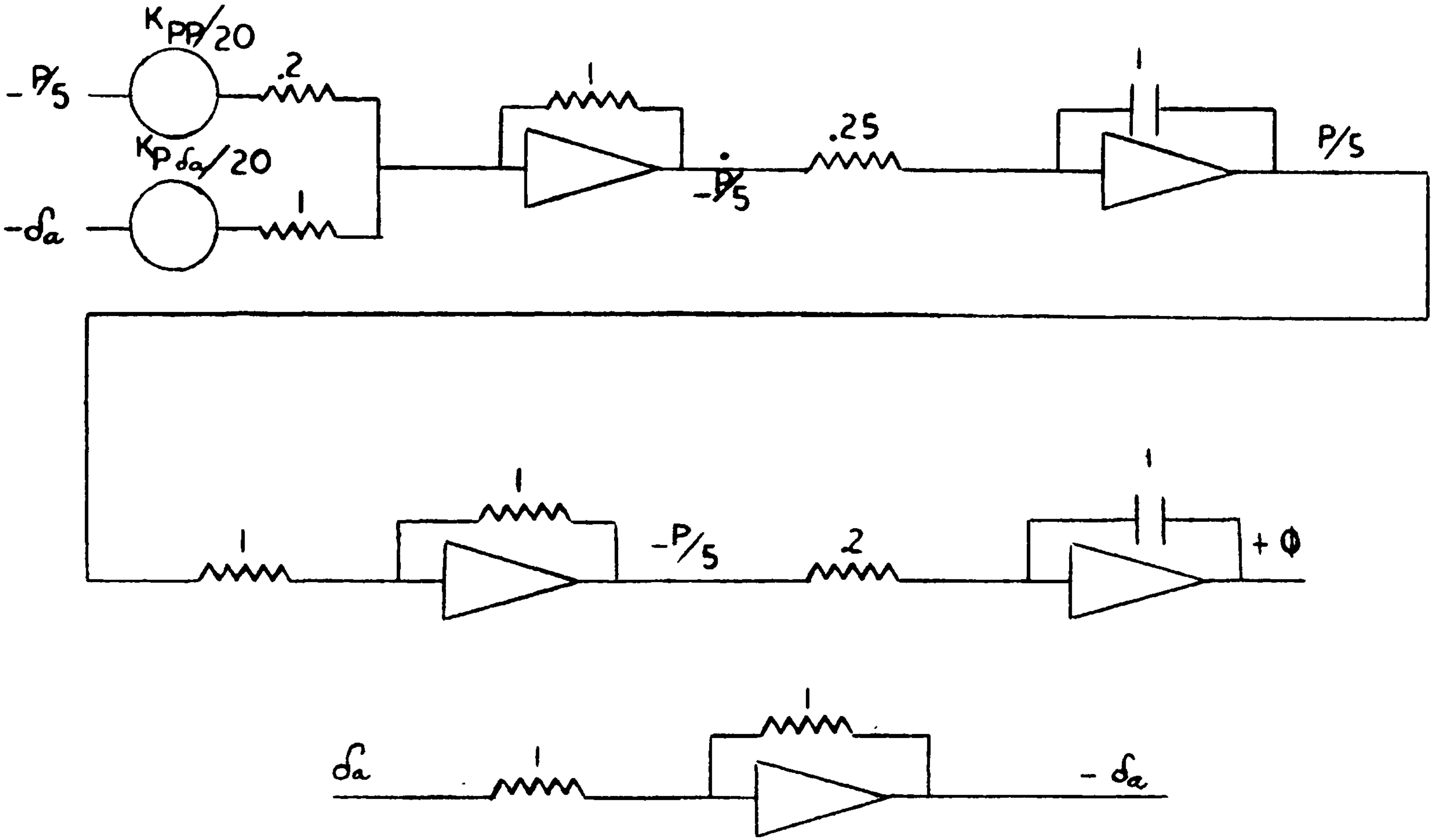


Fig. 3.2. Analog Computer Representation of the Lateral Equation of Motion.

The disturbances were supplied to the circuit in the form of voltages which were available at the computer. The simplest type of gust that can be considered is the sharp gust which is supplied in the form of a step voltage. Although this type of gust is somewhat unrealistic, it is useful in providing an elementary solution from which the response to more complicated types can be obtained from superposition. The gusts were generated by use of the circuits as shown in Fig. 3.3.

Initially, both circuits contained a time constant of 5 seconds but it was discovered that the aircraft would correct itself in the pitching mode without any assistance from the pilot, making it necessary to remove the capacitance from this circuit. By applying gusts to the rolling mode and pitching mode, the effects that the pilot observed on the oscilloscope appeared very similar. If one wanted a different type of gust, say, in the form of a ramp, all that need be done would be to supply a step function to an integrator and the results of a ramp would be obtained.

As the experiment progressed, it was decided to supply the perturbations to the aircraft through the use of a function generator. By so doing, the same pattern could be repeated which helped in analyzing and reducing the obtained data. The function generated could be

Note: Gust perturbation is applied by quickly turning potentiometer 1. and/or 2. to the right or left from the neutral position.

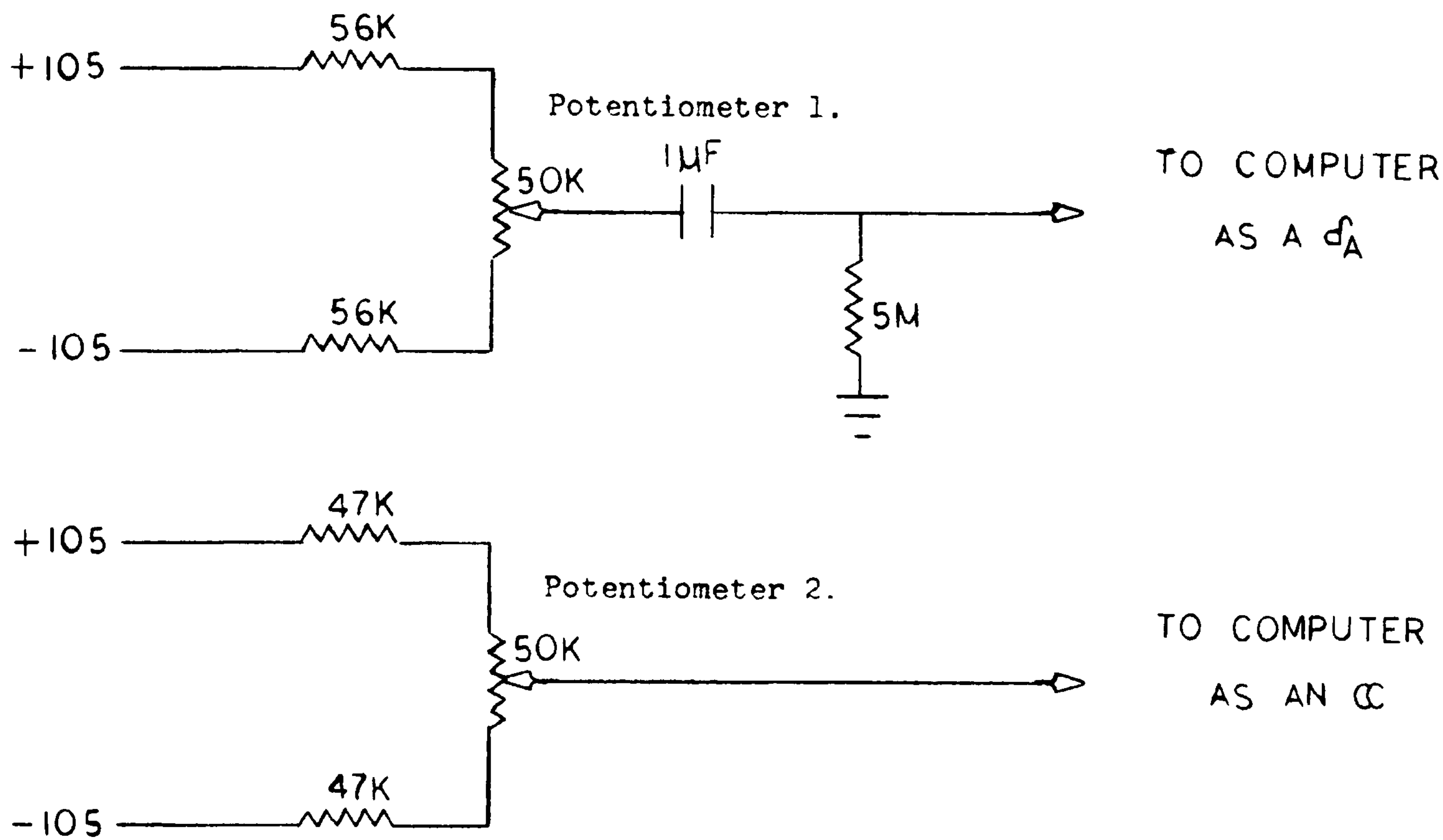


Fig. 3.3. Gust Generation Circuits

supplied to the pilot in a very fast pattern, slow pattern or changed in direction by putting it through a sign changer on the computer, thereby minimizing the possibility that the pilot could memorize the pattern. It was also possible to feed random gusts into the system which further complicated matters for the pilot. All of these methods were used to assure that the obtained data would be valid. The function as generated is shown in Fig. 3.4 along with its first and second derivatives. Initially, an attempt was made to set up a step function on the function generator but this proved futile as the maximum slope of the generator is 2 volts/volt and to set two break points at each slope change was very difficult, resulting in a very poorly shaped function. A ramp function was then selected that varied in slope as well as in amplitude. This permitted the amplitude and duration of the differentiated gusts to vary, resulting in what can be considered realistic flying conditions. Time was generated on the function generator through the use of an integrating amplifier which had -100 volts applied to it as an initial condition. By supplying an additional voltage to the amplifier and integrating, it was possible to vary the function with respect to time. An example is set forth to show the method used:

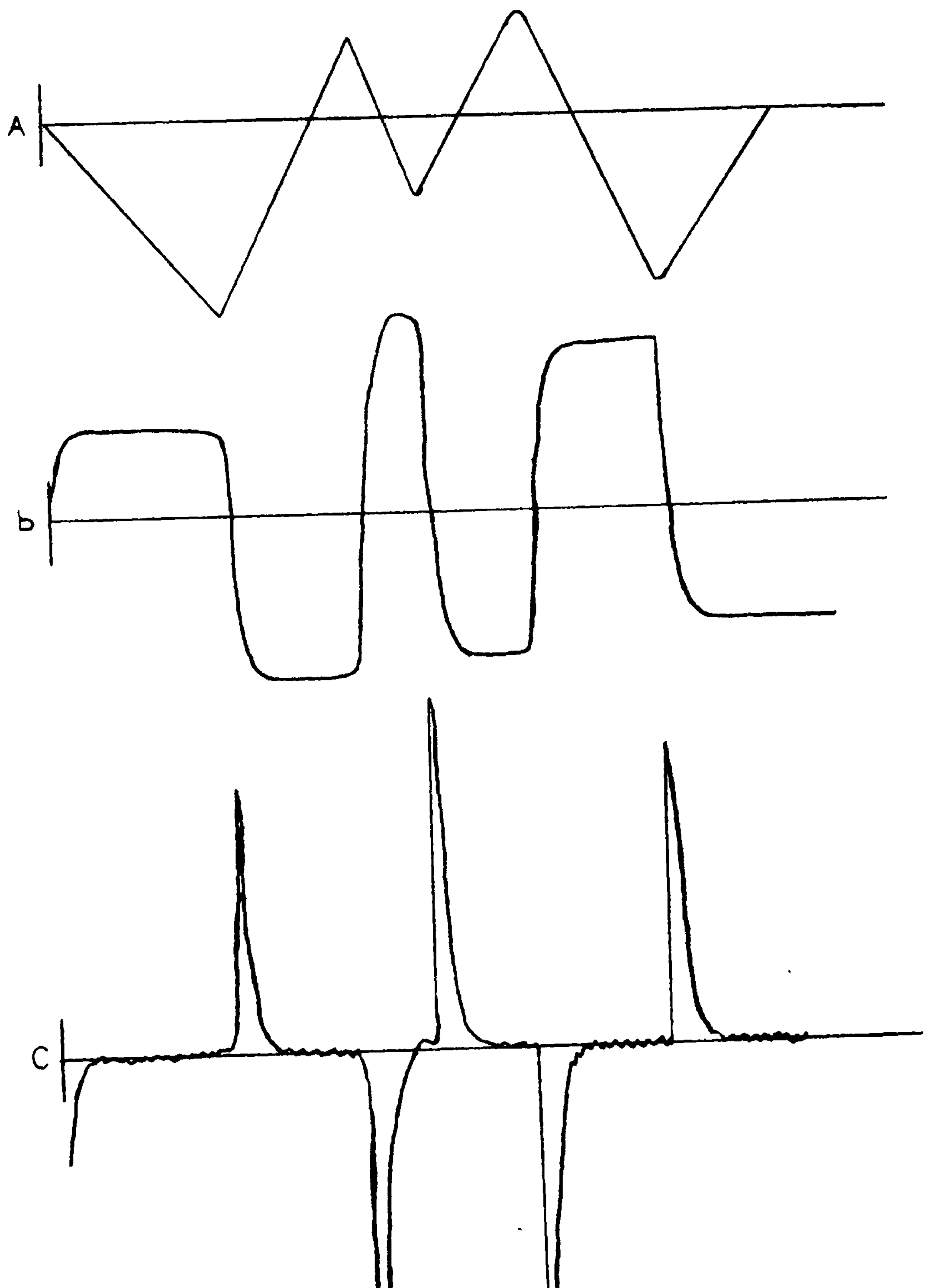


Fig. 3.4. Generated Function (a) with Associated First (b) and Second (c) Derivatives.

$$V = A \int_0^t dt + B$$

$$V = At + B$$

Boundary conditions

1. at $t = 0 \quad V = -100^v$

$\therefore B = -100^v$

2. at $t =$ end of run $V = +100^v$

$100 = At - 100$

$At = 200$

(The values of $\pm 100^v$ for V are equipment limitations).

If the desired duration of the function were to be two minutes, A would be equal to $200/120 = 1.667^v$. The value of A was one of the parameters that was varied while obtaining data in an attempt to prevent the pilot from memorizing the gust pattern.

An observation is brought up at this point which proved most interesting. The author decided to allow the pilots to observe him apply the gusts to the aircraft without any mention made of it. Naturally, the pilot would watch the operator's hands and as soon as the hands moved, a correction would be made. One of three things would occur. First, luck would be with the pilot and a proper correction would be made in a very short time. Secondly, a correction would be made but in the opposite direction. Finally, not being certain as to the direction of the gusts, their eyes would have to shift from the operator's hands to the attitude indicator, interpret the deviation and then make a correction, thereby greatly increasing their reaction times. None of these points was used in the data but it did seem worthy of investigation.

The display on the oscilloscope was obtained as shown in Fig. 3.5. Through the proper selection of the input and feedback resistors, the amount of pitch and roll portrayed on the attitude indicator was made realistic with respect to the displacement of the steering column, or "yoke" and wheel.

Probably the most difficult portion of this thesis was the design of a suitable lag circuit which would account for the pilot's reaction time. The desired lag was one which would be incorporated at the outset of the disturbance, but which was removed throughout the duration of the remaining gust and would be reset at the start of the following disturbance. It has been experimentally determined (1) that a pilot's reaction time is .375 seconds. From the time a pilot detects a change in attitude until the time he is ready to correct for it requires a period of .25 seconds, and his muscular reaction adds

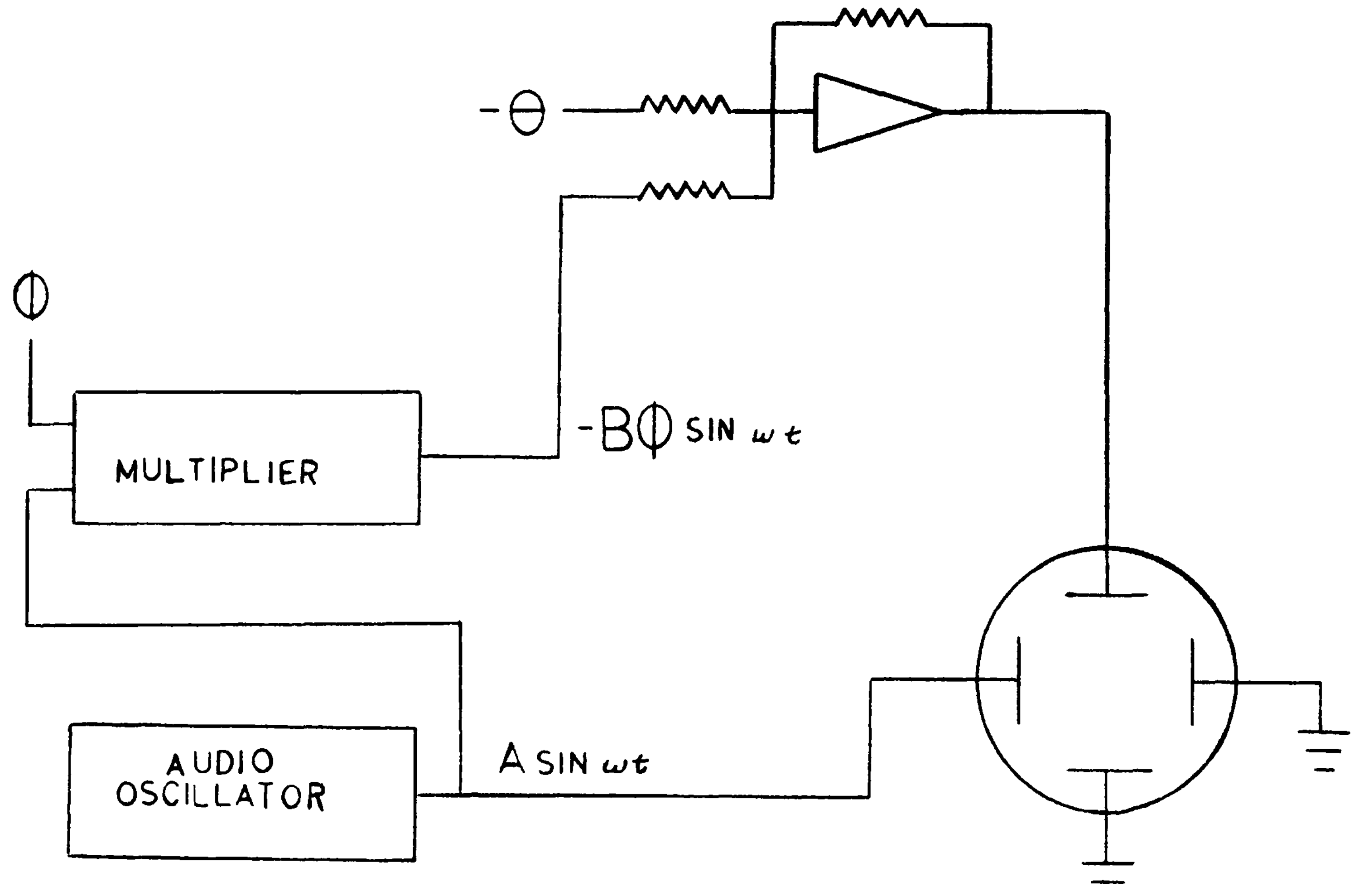


Fig. 3.5. Circuit for Obtaining Display on Oscilloscope.

another .125 seconds to this previous figure. The former is often referred to as delay time and the latter is called the neuromuscular lag combining to make the total reaction time.

Approximately four different circuits were tried of which three will be discussed. In the first circuit, the ramp function of the generator was supplied to an amplifier which had two 56^V zener diodes, back to back, in the feedback circuit. When the voltage of the input signal was anything other than zero, the output of the amplifier would be $\pm 56^V$, depending upon the polarity of the input voltage. This circuit was rejected as unsatisfactory due to the fact that the amplitude of the signals from the amplifier was constant and since this function would eventually be used as a perturbation function, an unrealistic condition existed. The second circuit which was tried used the differentiated function as the triggering signal for the time delay which was used in conjunction with the autopilot. When the autopilot was switched on, the rise time of the signals to trip the relay was approximately .6 seconds which meant that the autopilot was computing for this period of time prior to any delay being incorporated in the circuit. Once again, this situation was considered unrealistic and as a result another circuit was designed which, when used, was found to be very acceptable. This circuit incorporates the features of being able to use a variable amplitude perturbation along with an extremely short rise time for tripping the lag circuit (see Fig. 3.6). Due to the random noise of the input signal, the zener amplifier will conduct at high frequencies, each time the random noise is positive. To obviate this problem a small bias voltage was applied to the input, the value of this voltage being as large as the threshold of the random noise.

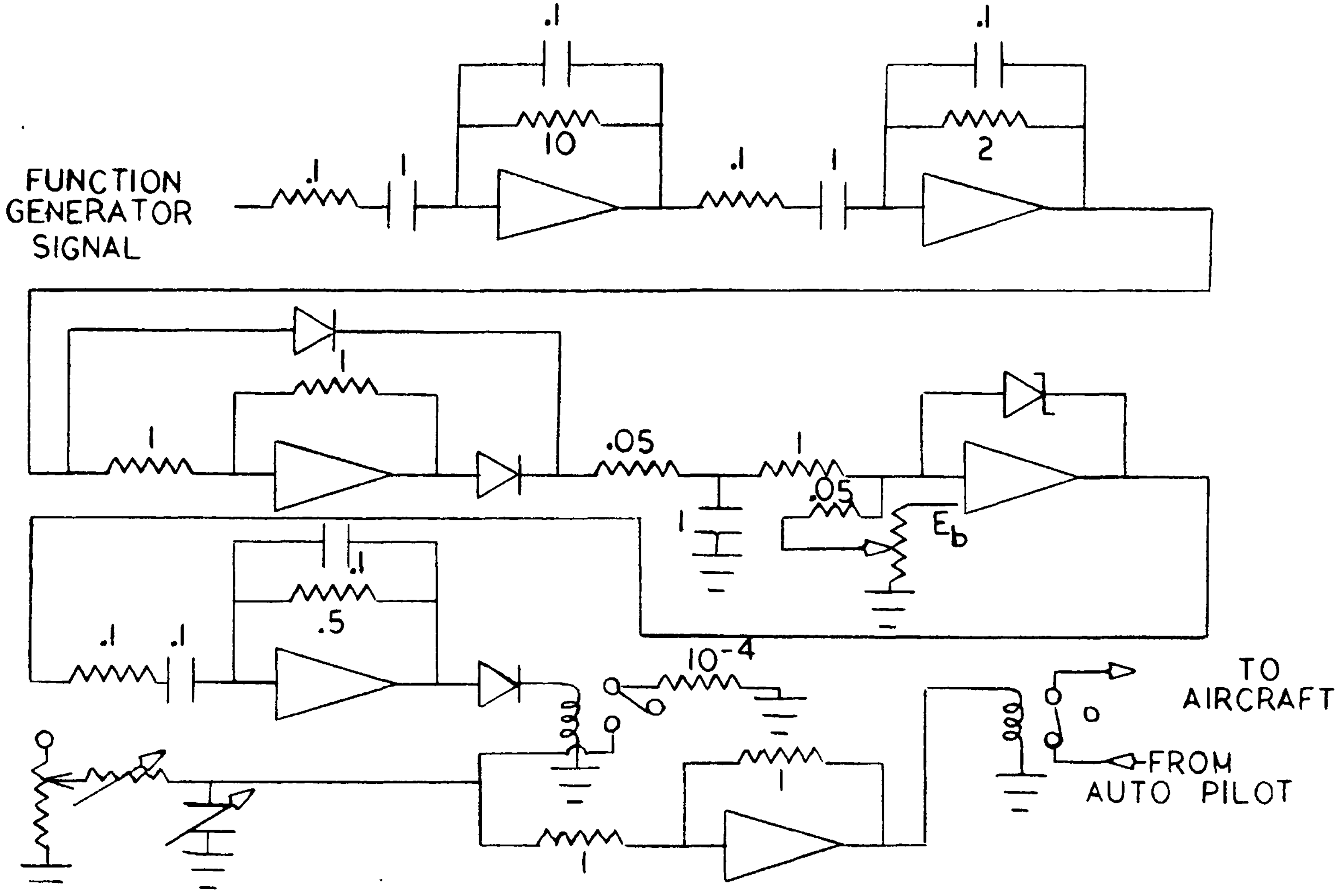


Fig. 3.6. Delay Circuit for Obtaining the Appropriate Time Lag.

CHAPTER 4

SIMULATOR CONSTRUCTION

The simulator (Fig. 4.1) was constructed, based on the principle of instrument flying, i.e., an artificial horizon was the basic instrument used to duplicate an aeroplane flying at altitude. The pilot received video signals of the airplane's attitude (Fig. 4.2) from this instrument, which for this experiment was a D-C oscilloscope. The oscilloscope was placed on a table approximately three feet from the pilot with the horizontal center line of the scope at eye level. The seat and other aircraft parts were obtained from various aircraft located at the storage area on Davis-Monthan Air Force Base, Tucson, Arizona. The seat was not free to move up or down, but could move fore and aft. Therefore, the height of the seat was fixed in relation to the height of the oscilloscope and the centerline of the oscilloscope was at eye level for the average of the personnel used as pilots. Rudder pedals were installed but were not used to obtain data since this degree of freedom was restrained. This fact was withheld from the pilots so as to prevent them from concentrating solely on

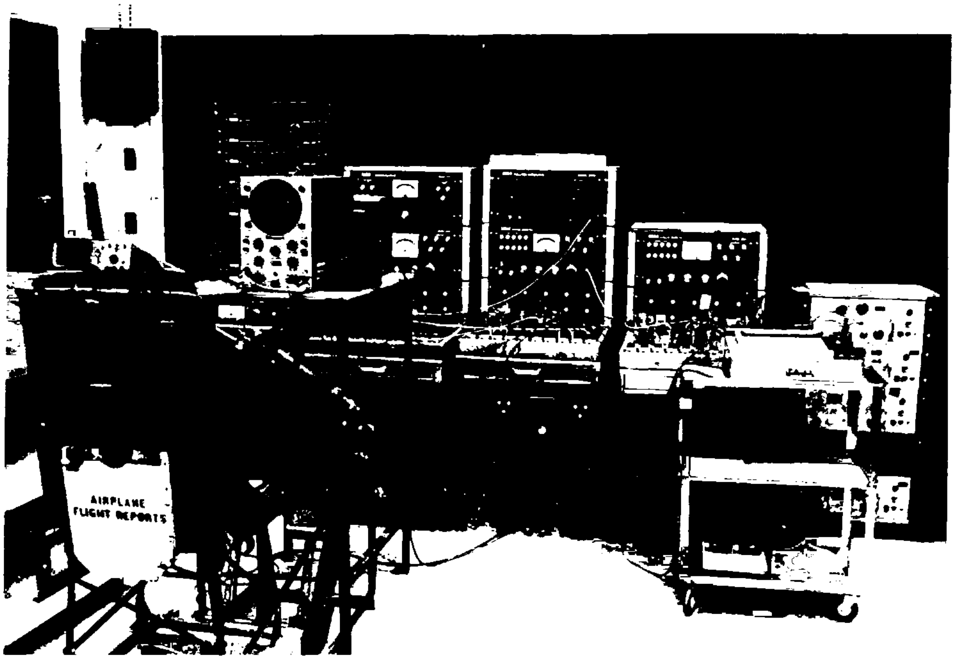


Fig. 4.1. Overall Simulator Set-Up

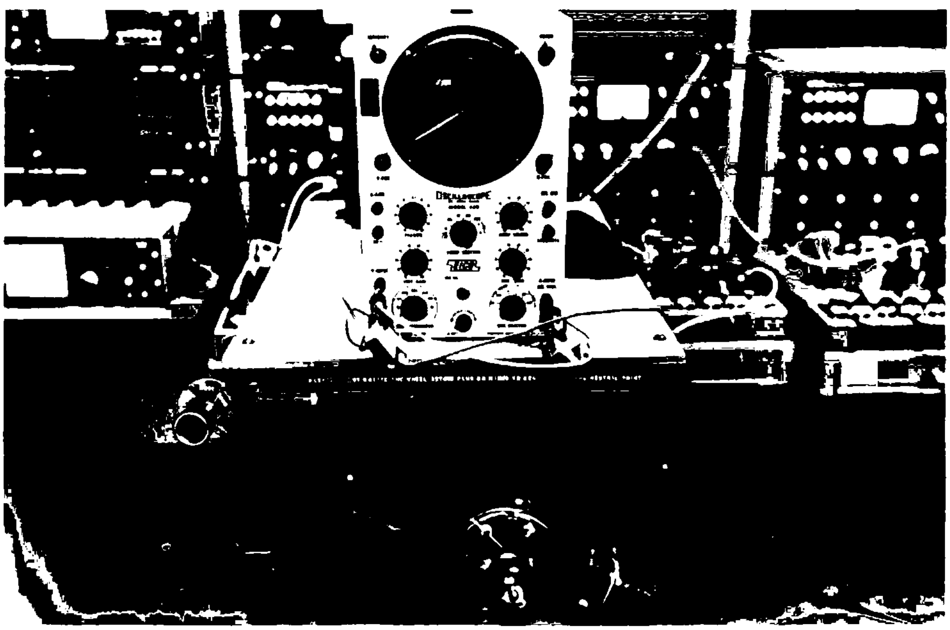


Fig. 4.2. Close-Up View of the Display of the Attitude Indicator

elevator or aileron corrections to maintain straight and level flight. The rudders were designed to return to the neutral position through a spring arrangement.

The ailerons and elevator movements were controlled with a "yoke" obtained from a C-47. The yoke was also centered at the neutral position with the aid of springs. The forward and aft neutral position could be adjusted to the pilot's desires with the aid of turnbuckles which were attached to the frame of the simulator. The springs used were all linear and there was no damping present which meant that if the yoke were given a considerable displacement from the neutral position and then released, it would oscillate fore and aft until the effects of friction brought it to rest. This condition was not considered too serious as this thesis investigates small disturbances which means that the corrections should also be small and of a short duration. Potentiometers were mounted on the axis of rotation of the wheel and the yoke to generate voltages to represent the δ_e and δ_a computer inputs. These voltages were fed through multipliers on the analog computer to scale them and hence make them compatible with the equations of motion (one radian being the equivalent to 100 volts).

Disturbances could be applied to the simulator by introducing external voltages in the form of impulses, ramp or step function, to the desired amplifier on the analog computer.

Various data that was desired could be selected from the appropriate amplifiers and fed to a four channel Sanborn recorder or a Moseley X-Y recorder (Fig. 4.3). As an example, one could plot the elevator deflection (δ_e) and the pitch angle θ versus time simultaneously on the multi-channel recorder to evaluate the reaction times for various individuals and thereby determine a good overall reaction time for a group of pilots.

A Moseley X-Y recorder was also used for recording data. The advantage of using this recorder was that stylus returned to the same zero position after each run, thereby making it very convenient to compare a graph of the autopilot with one obtained from a human pilot.

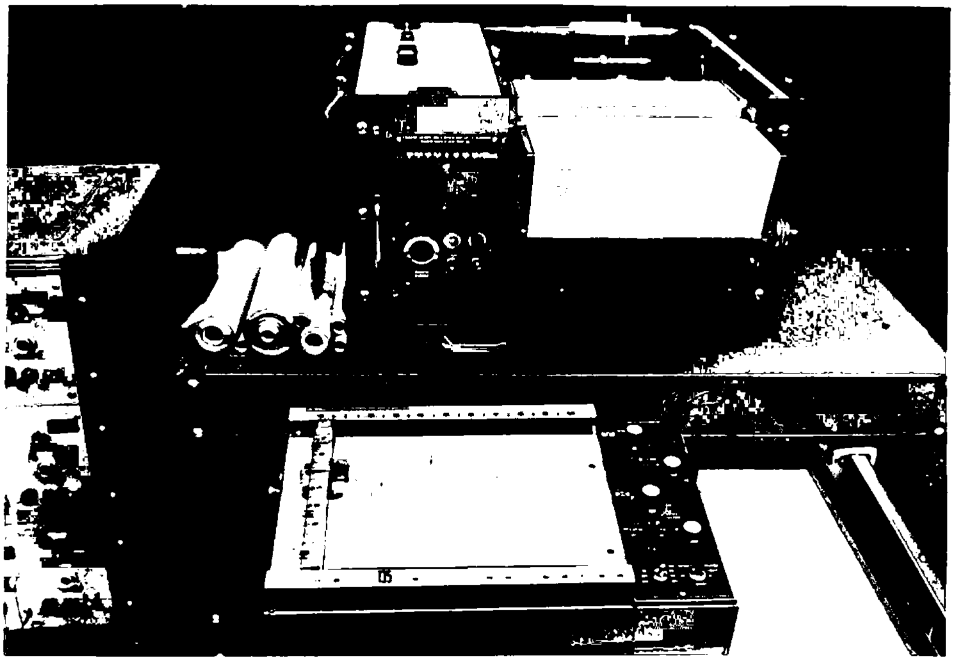


Fig. 4.3. Equipment Used in Recording Experimental Test Data

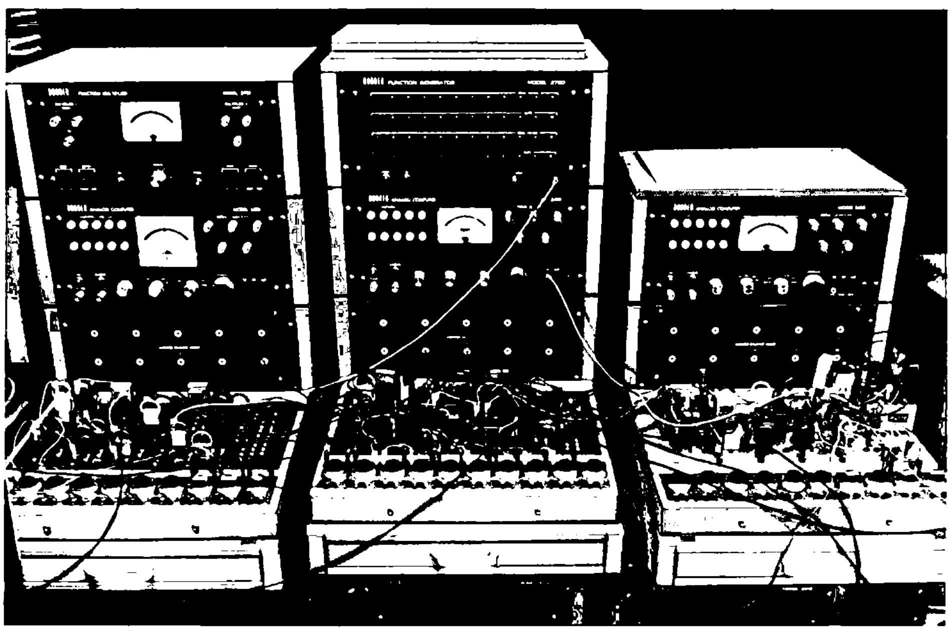


Fig. 4.4. Equations of Motions as Represented on the Analog Computers

CHAPTER 5

DISCUSSION OF RESULTS

Once the experimental set up was completed and all the equipment was working properly, a check was made to establish the average reaction time of the pilots who took part in the experimental work. Numerous points were plotted in the rolling mode, pitching mode, and combined rolling and pitching mode. The delay that was determined was .365 seconds in duration as compared with .375 seconds, as mentioned earlier. The difference may be attributed to the fact that the pilots were experienced and well trained in instrument flying as they were pilots from the United States Air Force. Another factor that could account for the difference may be in the method of obtaining the time delay. The method used was to allow the pilot to fly the aircraft and then superimpose the curve produced by use of the autopilot. The deviation of 10 milliseconds could very well be hidden in the width of the curve traced by the stylus. The value of .365 seconds was therefore considered acceptable and was used throughout the remainder of the experiment.

Data were then obtained by plotting pilot outputs such as elevator deflection, pitch angle, roll angle, and angle of attack. Approximately 100 data curves were obtained for each of the above outputs and the typical pilot response curves are shown in Fig. 5.1. Once these curves were obtained, it was a matter of trial and error to obtain the proper potentiometer settings for the autopilot. Some arbitration was done in arriving at the final values for the pitching mode as no compatible combination of potentiometer settings could be obtained which simultaneously satisfied the shapes of the curves for elevator deflection, pitch angle and angle of attack. That is to say that if the potentiometer settings were adjusted to satisfy elevator deflection, the amplitude of the pitch angle and angle of attack curves would be too great for curves that were similar in shape to those of the human pilot. The converse was also true in establishing autopilot curves that matched those of the human pilot for angle of attack and pitch angle. The final decision was a matter of arbitration with more emphasis being placed on the pitch angle and angle of attack as these parameters depict the motion of the complete aircraft, whereas, the elevator deflection portrays the motion of one of the smaller, but yet important, sub systems. One might say that this is a matter of the

Fig. 5.1. Pilot Response Curves for Small Disturbances

end justifying the means rather than the means justifying the end. To help substantiate this conclusion, some curves of velocity and time rate change of angle of attack that were recorded indicated that this decision seemed to be justified. However, sufficient data of these parameters are not available, thereby making it impossible to draw any concrete conclusions from such, but the trend that they depicted was very satisfying. The final results arrived at are those shown in Fig. 5.2.

One of the questions that was to be answered was whether the magnitude of the perturbations had any effect on the reaction time. Much data were available to investigate this area and after a close analysis of these data, it was determined that once the magnitude of the disturbance obtained a value above the minimum value for detection, no correlation existed. The pilot would react to a small disturbance just as quickly as to a large disturbance. Another question to be answered was whether or not the direction of the disturbance had any effect upon the length of the delay time. Again, once the magnitude of the perturbation reached a value above the pilot's minimum threshold level, the delay was independent of direction.

It is also interesting to compare the curves of pilots that have had different flying backgrounds. One

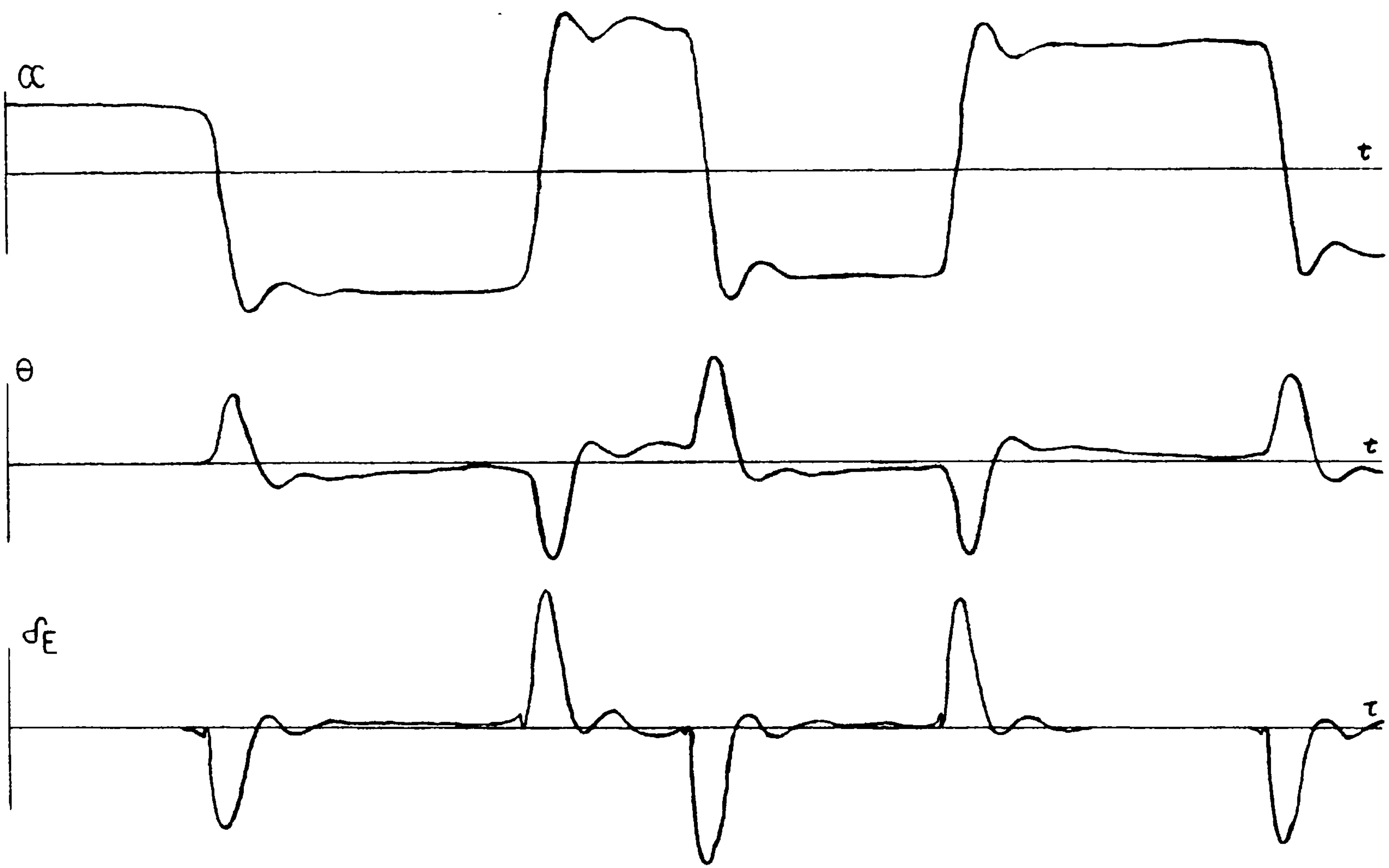


Fig. 5.2. Auto Pilot Response Curves for Identical Small Disturbances

person had never had any flying experience, one pilot had been a single-engine jet-aircraft pilot, another had obtained most of his flying experience in reciprocating, multi-engine aircraft, and the last one to be mentioned had time in both single engine and multi-engine aircraft. If one did not know the personal backgrounds of these individuals, it would be impossible to establish which was which by the curves, due to the striking similarity of all of the plotted data. Personal conversation with the pilots indicated that all of them would oftentimes not correct for a small disturbance at higher altitudes, but would correct for any disturbance during the landing approach as any deviation from the desired flying attitude could prove disastrous when in close proximity to the ground.

During the investigation of the obtained data, another trait was discovered. During the initial portion of the perturbation, the pilot would be out of phase with the input signal due to his own built-in time lag. He would attempt to overcome this situation by trying to obtain a maximum stick deflection at the same time the pitch angle was a maximum. This situation was not always achieved but inspection of the data showed that the time lag, or delay, was greatly reduced.

It is interesting to note the shape of the curve in the area following the initial correction. This is the area in which the pilot provides the appropriate corrections to return the aircraft to the straight and level attitude. As the correction was made, the aircraft would approach straight and level flight and instead of stopping at a zero pitch angle, the aircraft would overshoot. Two small additional corrections were normally associated with this phenomenon at which time the damping had established straight and level flight. Also appearing are some small deviations which follow the initial corrections. It appears that these are small oscillations that the pilot is making, unknowingly, due to the fact that any deviation that they may cause is below the pilot's minimum threshold level. Occasionally, the magnitude of these deviations was of sufficient strength to create a visual stimulus, necessitating a slight correction on behalf of the pilot.

Probably the most interesting and important portion of this experiment was devoted in an effort to determine a suitable transfer function for a human pilot.

Once sufficient data were obtained from the human pilots, the values of the coefficients of the autopilot had to be determined. The configurations of the autopilots are shown in Figs. 5.3 and 5.4. The values of the various coefficients were established as follows:

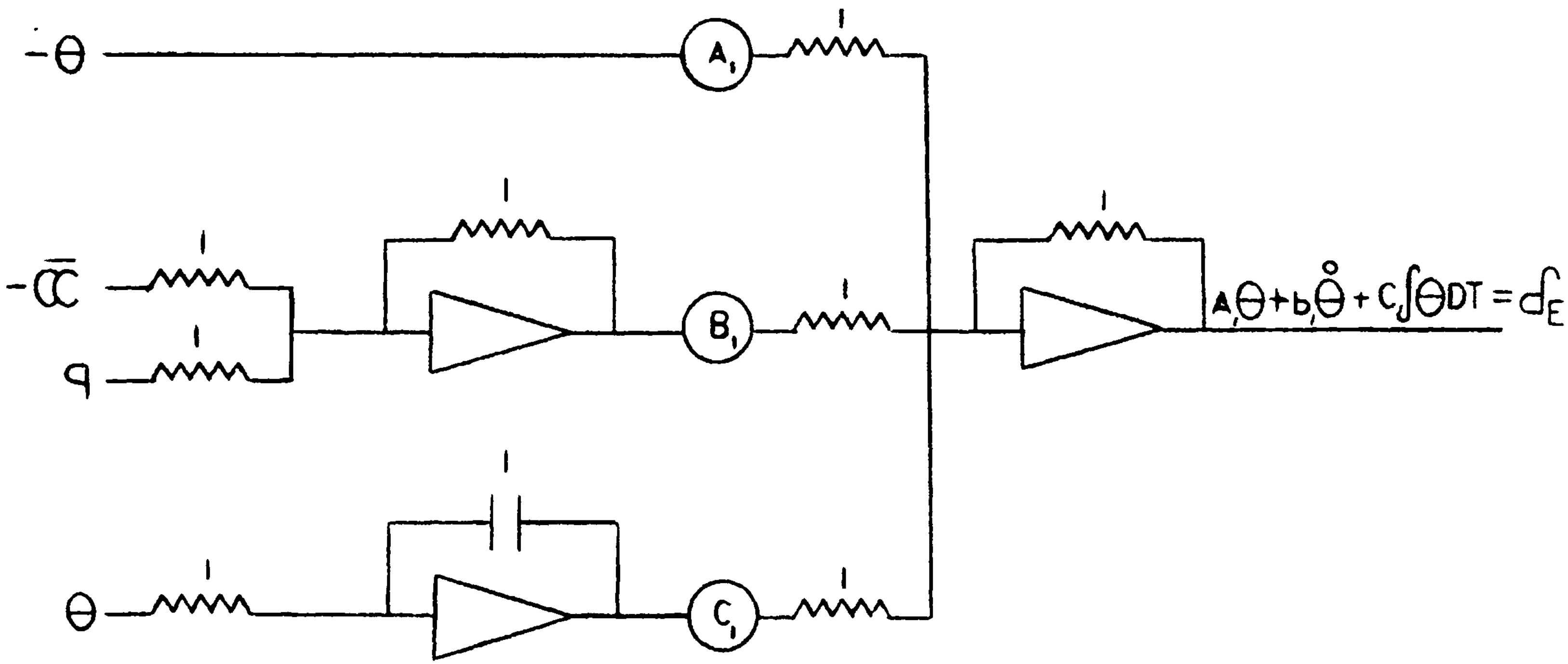


Fig. 5.3. Circuit Diagram for Autopilot Associated with Pitching Mode

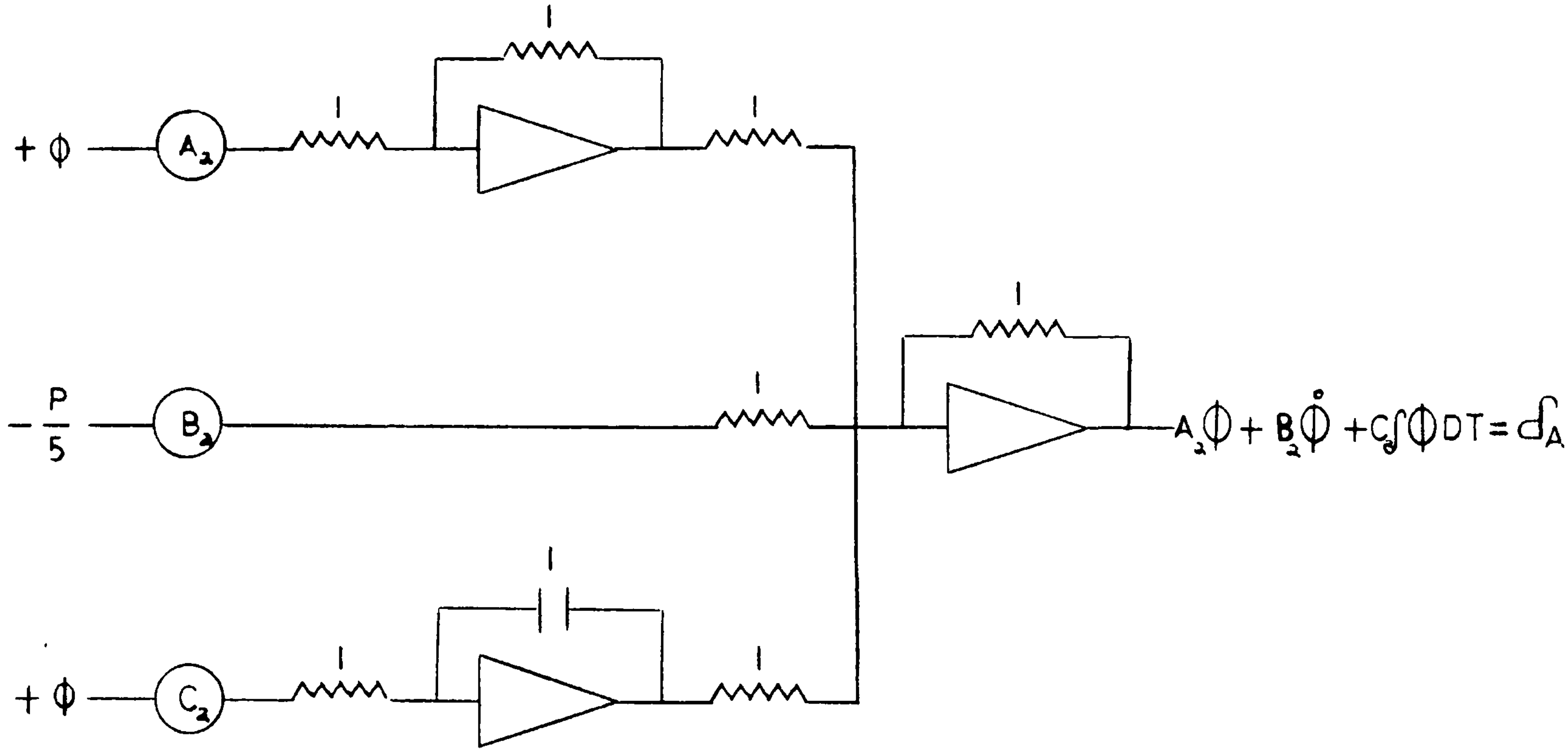


Fig. 5.4. Circuit Diagram for Autopilot Associated with Rolling Mode

$A_1 = .300 \qquad A_2 = .600$

$B_1 = .100 \qquad B_2 = .100$

$C_1 = .015 \qquad C_2 = .060$

The values represent the fraction of the maximum value of the potentiometer setting. The human transfer function was determined as follows:

$$\delta e_{PILOT} = f(t)\, u(t-\tau) = \left[A_1\theta + B_1\dot{\theta} + C_1\int \theta\, dt\right] u(t-\tau)$$

WHERE $\tau = .365$

$$\mathcal{L}\,\delta e_{PILOT} = \mathcal{L}\, f(t)\, u(t-\tau) = \int_{\tau}^{\infty} e^{-st} f(t)\, dt$$

LET $u = t - \tau$ AND $du = dt$

THEN $\mathcal{L}\, f(t)\, u(t-\tau) = \int_{0^+}^{\infty} e^{-s(u+\tau)} f(u+\tau)\, du$

$$\mathcal{L}\, f(t)\, u(t-\tau) = e^{-s\tau}\,\mathcal{L}\, f(u+\tau)$$

SINCE $\tau + u = t$, CLEARLY

$$\mathcal{L}\, f(t)\, u(t-\tau) = e^{-s\tau}\,\mathcal{L}\, f(t)$$

The transfer function, $G_{(s)} = \dfrac{\mathcal{L}\, f(t)\, u(t-\tau)}{\mathcal{L}\, \theta(t)}$

$$G_{(s)} = e^{-\tau s} \frac{\left[A_1\, \theta_{(s)} + B_1\, s\, \theta_{(s)} + \frac{C_1}{s}\, \theta_{(s)}\right]}{\theta_{(s)}} \tag{5.1}$$

Finally

$$G_{(s)\,\theta\,\delta_e} = e^{-.365s} \frac{\left[0.015 + 0.30\,s + 0.10\,s^2\right]}{s} \tag{5.2}$$

Similarly for the rolling mode,

$$\delta_{a_{PILOT}} = f(t)\, u(t-\tau) = \left[A_2 \phi + B_2 \dot{\phi} + C_2 \int \phi\, d\tau\right] u(t-\tau)$$

$$G_{(s)\,\phi\,\delta_a} = \frac{\mathcal{L}\, f(t)\, u(t-\tau)}{\mathcal{L}\, \phi(t)} = e^{-\tau s} \frac{\left[A_2 \phi_{(s)} + B_2\, s\, \phi_{(s)} + \frac{C_2}{s}\, \phi_{(s)}\right]}{\phi_{(s)}} \tag{5.3}$$

Substituting for A_2, B_2, C_2, and τ, yields

$$G_{(s)} = e^{-.365s} \frac{\left[0.060 + 0.60\,s + 0.10\,s^2\right]}{s} \tag{5.4}$$

As a check on these results, the pilot was permitted to fly the aircraft with just the pitching or rolling mode being in effect, and then in some level flight attitude the autopilot would be inserted in the closed loop, replacing the pilot. In the rolling mode, this proved most effective. Once the pilot had established the wings level, there was little tendency for him to make any small corrections about zero which meant that the pilot was behaving very similar to his computer counterpart. The results for the pitching mode were not as gratifying, but, nevertheless, were most useful in this study. The autopilot would make the initial corrections similar to the pilot but then would become very steady near zero. This was not so for the pilot as he would make slight corrections about zero as mentioned earlier. He would not be aware of the fact that he was not controlling the flight of the aircraft until the very last portion of the correction required for the imposed perturbation. Suddenly, the pilot would become aware of the fact that he was sitting in the seat doing nothing, and as a check to be certain that he was still flying the aircraft, he would apply a small stick force that should create a deviation from the straight and level flight path. When this movement of the control was accomplished and no results obtained, the "game" was over and the procedure repeated. It was at this point that the author felt the main objective of this study had been obtained.

A short investigation into the combined rolling and pitching modes was conducted to determine if any trends could be detected that were greatly different from the study conducted using the two modes separately. First, it should be noted that the reaction time was not changed when the complexity of the task was made more difficult. This should be obvious as there is a minimum time required to react which was established as .365 seconds, which did not change. The amplitude of the various parameters such as elevator deflection, pitch and roll angles all appeared to be of the same amplitude but the time spent away from zero was of a greater duration. To compensate for this in the design of an auto pilot, it appears that the coefficient for the value of the integral would need to be increased. The amount of oscillation after the initial correction was much greater, particularly in the pitching mode, which means that the damping factor would not be of the same magnitude as for the combined mode. It is emphasized at this point that these results are not intended to be conclusive or inconclusive for that matter, but rather to be considered as a trend which could be used as a starting point for a thesis similar to this one.

CHAPTER 6

CONCLUSIONS

In summary, certain definite conclusions and some strong trends can be abstracted from the work contained herein. Through the use of discretion and appropriate elimination of certain terms in the equations of motion based on experimental evidence, some valid information can be obtained with respect to a human pilot. There exists within the human pilot, regardless of his flying experience, a factor of a time delay that is the same for all. This delay was established as .365 seconds and occurs at the onset of the disturbance. Once the initial correction is made, lag is present which is the time difference between the input and output signal. This lag is not of a constant duration as evidenced by the fact that the pilot applies the stick motion in a manner apparently designed to overcome this situation which is most evident at the onset of the disturbance and tapers off quite readily as time progresses to some finite value. Within the realm of small perturbation theory, it would be very feasible to substitute a mathematical pilot for a human pilot in a closed loop system. To do this, one should use the following transfer functions for the pitching and rolling modes respectively:

$$\text{pitching mode - } G(s) = e^{-\tau s}\frac{(A_1 s + B_1 s^2 + C_1)}{s}$$

$$\text{rolling mode - } G(s) = e^{-\tau s}\frac{(A_2 s + B_2 s^2 + C_2)}{s}$$

where $\tau = .365$ $\quad A_1 = .300 \quad B_1 = .100 \quad C_1 = .015$

$A_2 = .600 \quad B_2 = .100 \quad C_2 = .060$

The quantities A_1, B_1, C_1, A_2, B_2, C_2, are slightly different when the pilot is required to perform a tracking task involving both pitch and roll than for each task involving both pitch and roll than for each task performed individually.

In conclusion, it should be pointed out that the present work opens up a vast area which could and should be extended to contain additional degrees of freedom as well as more elaborate instrumentation with the end objective being that of obtaining a more precise determination of a human transfer function.

APPENDIX A

PARAMETERS OF THE EXPERIMENTAL AIRCRAFT (NOTATION IS THAT OF REFERENCE 3)

Weight - 100,000 lbs.

Wing area - 1,667 sq. ft.

Mean aerodynamic chord - 15.4 ft.

Aspect ratio - 7

$i_A = 3.69$	$i_E = -.39$
$i_B = 1{,}900$	$\mu = 272$ longitudinal mode
$i_C = 9.22$	$\mu = 38.8$ lateral mode

$\tau^* = .0105$ longitudinal mode

$\tau^* = .0737$ lateral mode

Altitude = 30,000 ft.	$\rho = .000889$ slugs/ft^3
$u_o = 500$	mph = 733 FPS
$C_{l_p} = -.43$	$C_{m_\alpha} = -.488$
$C_{L_o} = .25$	$C_{m_{\dot{\alpha}}} = -4.20$
$C_{x_u} = -.0376$	$C_{z_{\delta_e}} = -.24$/radian
$C_{m_q} = -22.9$	$C_{m_{\delta_e}} = -.72$/radian
$C_{x_\alpha} = .14$	$C_{z_{\dot{\delta}_e}} = 0$
$C_{z_\alpha} = -4.90$	$C_{l_{\delta_a}} = -.065$/radian

APPENDIX B

(NOTE - FOR A DETAILED DEVELOPMENT OF THESE EQUATIONS, SEE CHAPTER 4, REFERENCE 3).

$$(2\mu D - \cancel{2C_{L_0} \tan\theta_0}^{\,0} - C_{x_u})\hat{u} - C_{x_\alpha}\bar{\alpha} + C_{L_0}\theta + 0 = 0$$

$$2\mu D\hat{u} = C_{x_u}\hat{u} + C_{x_\alpha}\bar{\alpha} - C_{L_0}\theta$$

$$\mu = m/\rho S\ell \,,\; D = \tau^* \frac{d}{dt} \,,\; \tau^* = \ell/u_0 \,,\; \ell = \bar{c}/2 \,,\; \hat{u} = u/u_0$$

$$\frac{2m}{\rho S\ell}\frac{\ell}{u_0^2}\mathring{u} = C_{x_u}\frac{u}{u_0} + C_{x_\alpha}\bar{\alpha} - C_{L_0}\theta$$

$$C_{L_0} = L/\tfrac{1}{2}\rho S u_0^2$$

$$\frac{C_{L_0}\mathring{u}}{g} = \frac{C_{x_u} u}{u_0} + C_{x_\alpha}\bar{\alpha} - C_{L_0}\theta$$

$$\underline{\mathring{u}/g = K_{vv} u + K_{v\alpha}\bar{\alpha} - \theta} \qquad \text{(A.1)}$$

$$K_{vv} = \frac{C_{x_u}}{C_{L_0} u_0} = -\frac{.0376}{(.25)(733} = -.000205$$

$$\log K_{vv} = -.066$$

$$K_{v\alpha} = \frac{C_{x\alpha}}{C_{L_0}} = -\frac{.14}{.25} = -.56$$

$$K_{v\alpha}/5 = -.112$$

$$-(C_{m\dot{\alpha}} D + C_{m\alpha})\bar{\alpha} + (i_B D^2 - C_{mq} D)\theta - C_{m\delta_e}\delta_e = 0$$

$$i_B D^2 \theta = C_{m\dot{\alpha}} D\bar{\alpha} + C_{m\alpha}\alpha + C_{mq} D\theta + C_{m\delta_e}\delta_e$$

$$\underline{q = K_{q\dot{\alpha}}\overset{\circ}{\bar{\alpha}} + K_{q\alpha}\alpha + K_{qq} q + K_{q\delta_e}\delta_e} \qquad \text{(A.2)}$$

$$K_{q\dot{\alpha}} = \frac{C_{m\dot{\alpha}}}{i_B t^*} = -\frac{4.2}{(1900)(.0105)} = -.21$$

$$K_{q\alpha} = \frac{C_{m\alpha}}{i_B t^{*2}} = -\frac{.488}{(1900)(.0105)^2}$$

$$K_{q\alpha}/5 = -.465$$

$$K_{qq} = \frac{C_{mq}}{i_B t^*} = -\frac{22.9}{(1900)(.0105)}$$

$$K_{qq}/4 = -.287$$

$$K_{q\delta_e} = \frac{C_{m\delta_e}}{i_B t^{*2}} = -\frac{.72}{(1900)(.0105)^2}$$

$$K_{q\delta_e}/10 = -.344$$

$$\underline{\mathring{\theta} = q - \mathring{\bar{\alpha}}} \tag{A.3}$$

$$2C_{L_0}\hat{u} + (2\mu D - C_{z_{\dot{\alpha}}} D - C_{z_\alpha})\bar{\alpha} - 2\mu D\theta - C_{z_{\delta e}}\delta e = 0$$

$$2\mu D\bar{\alpha} = -2C_{L_0}\hat{u} + C_{z_\alpha}\bar{\alpha} + 2\mu D\theta + C_{z_{\delta e}}\delta e$$

$$\underline{\mathring{\bar{\alpha}} = K_{zv}u + K_{z\alpha}\bar{\alpha} + K_{zq}q + q + K_{z\delta e}\delta e} \tag{A.4}$$

$$K_{zv} = -\frac{C_{L_0}\rho S}{m} = -\frac{(.25)(.889)(1.667)}{(10)^5/32.2}$$

$$\log K_{zv} = -.0385$$

$$K_{z\alpha} = \frac{\rho S u_0 C_{z_\alpha}}{2m} = -\frac{(.889)(733)(1.667)(4.90}{2(10)^5/32.2}$$

$$K_{z\alpha}/5 = -.1712$$

$$K_{z\delta e} = \frac{C_{z_{\delta e}}\rho S u_0}{2m} = -\frac{(.24)(.889)(1.667)(733)}{2(10)^5/32.2}$$

$$K_{z\delta e} = -.042$$

$$(i_A D - C_{\ell_p})\hat{p} - C_{\ell_{\delta a}} \delta a = 0$$

$$i_A D \hat{p} = C_{\ell_p} \hat{p} + C_{\ell_{\delta a}} \delta a$$

$$\underline{\mathring{p} = K_{pp}\, p + K_{p\delta a}\, \delta a} \qquad (A.5)$$

$$K_{pp} = \frac{C_{\ell_p}}{i_A \tau^*} = - \frac{.43}{(3.69)(.0737)}$$

$$K_{pp}/20 = -.07915$$

$$K_{p\delta a} = \frac{C_{\ell_{\delta a}}}{i_A \tau^{*2}} = - \frac{.065}{(3.69)(.0737)}$$

$$K_{p\delta a}/20 = -.162$$

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